

# Graph-based Machine Learning for Wireless Communications

Santiago Segarra<sup>\*</sup>, Ananthram Swami<sup>†</sup>, Zhongyuan Zhao<sup>\*</sup>

<sup>\*</sup>Rice University, <sup>†</sup>US Army's DEVCOM Army Research Laboratory

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Slides PDF



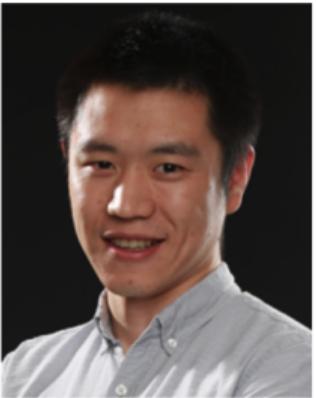
# The team



Santiago Segarra  
ECE, Rice



Ananthram Swami  
DEVCOM, ARL



Zhongyuan Zhao  
ECE, Rice



- ▶ Published 30+ papers in the area in the last 5 years

# Acknowledgements

- ▶ Rice University
  - ⇒ Arindam Chowdhury
  - ⇒ Boning Li
  - ⇒ Ashu Sabharwal
- ▶ Army Research Lab
  - ⇒ Gunjan Verma
  - ⇒ Jake Perazzzone
  - ⇒ Kevin Chan
  - ⇒ Chirag Rao

# Key Takeaways

By 5:30 pm today, you will be able to answer the following questions:

- ▶ What are graph neural networks (GNNs)?
- ▶ Why are GNNs well suited to tackle problems in wireless communications?
- ▶ How have GNNs been applied to specific problems?
- ▶ What are open problems/challenges to which you can contribute?

# Outline

## Part I: Introduction to Graph Neural Networks

- a) Graph-based ML and GNNs
- b) Graphs, GNNs, and Wireless Networks

## Part II: GNNs at the Physical Layer

- a) Introduction to issues at the physical layer
- b) Optimal Power Allocation & Beamforming: SISO and MIMO cases
- c) Optimal Power Allocation: Federated Learning

## Part III: Graph-based ML for Wireless Networking

- a) Introduction to networking tasks
- b) Link scheduling
- c) Graph-based actor-critic reinforcement learning framework
- d) GNNs for Backpressure Routing
- e) Digital twin of wireless networks

## Conclusions and Future Directions

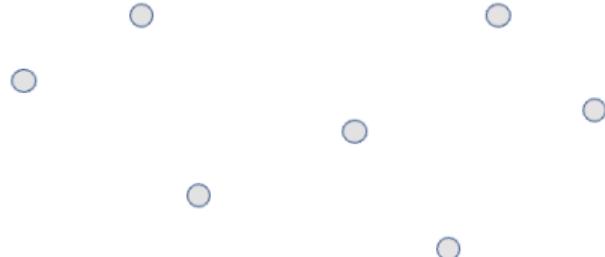
# Part I: Introduction to Graph Neural Networks

# Graph-based Machine Learning

Graph-based ML leverages the network structure of the data to improve learning and processing of these data

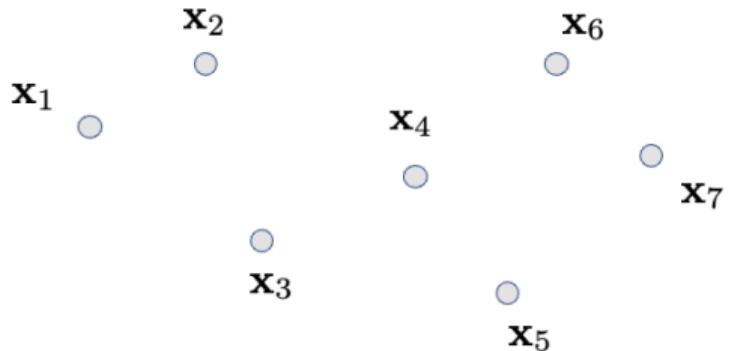
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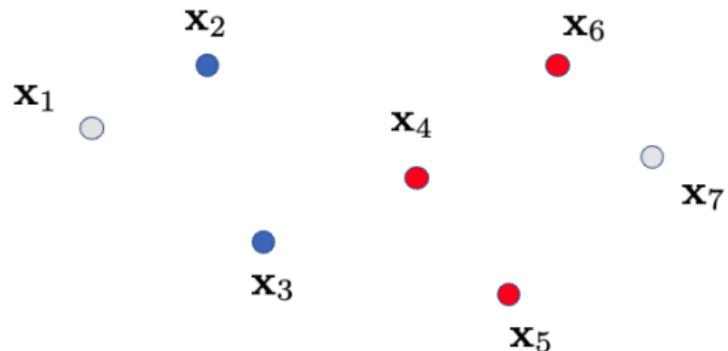
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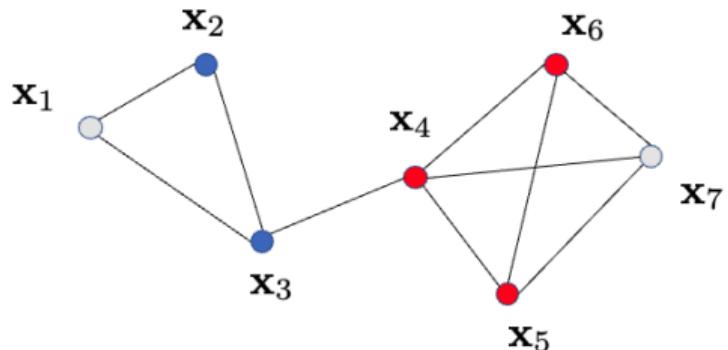
Graph-based ML leverages the network structure of the data to improve learning and processing of these data



- ▶ Classical **supervised learning** setting  
⇒ Learn a **parametric** function that estimates the labels ⇒  $\hat{y}_i = f_{\theta}(\mathbf{x}_i)$

# Graph-based Machine Learning

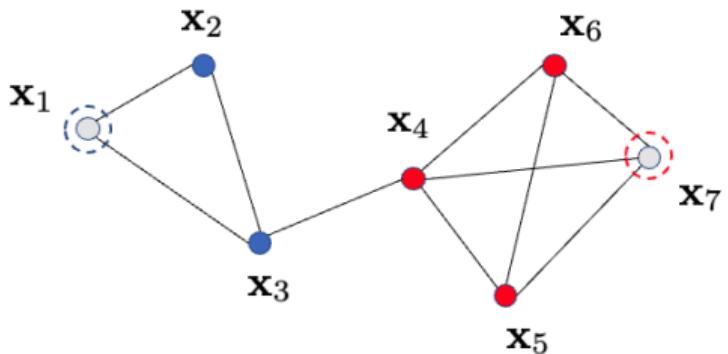
Graph-based ML leverages the network structure of the data to improve learning and processing of these data



- ▶ In some settings, relational structures between nodes are available
  - ⇒ Friendship in social networks or inhibition in protein networks
  - ⇒ Interference in comms networks

# Graph-based Machine Learning

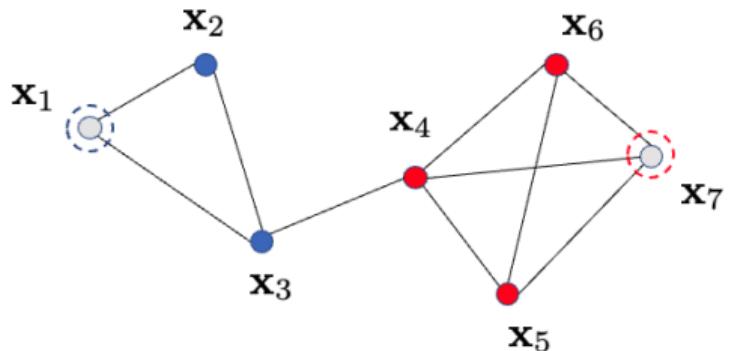
Graph-based ML leverages the network structure of the data to improve learning and processing of these data



- ▶ The structure also carries information about node labels  
⇒ Estimate labels by combining both node features and graph structure

# Graph-based Machine Learning

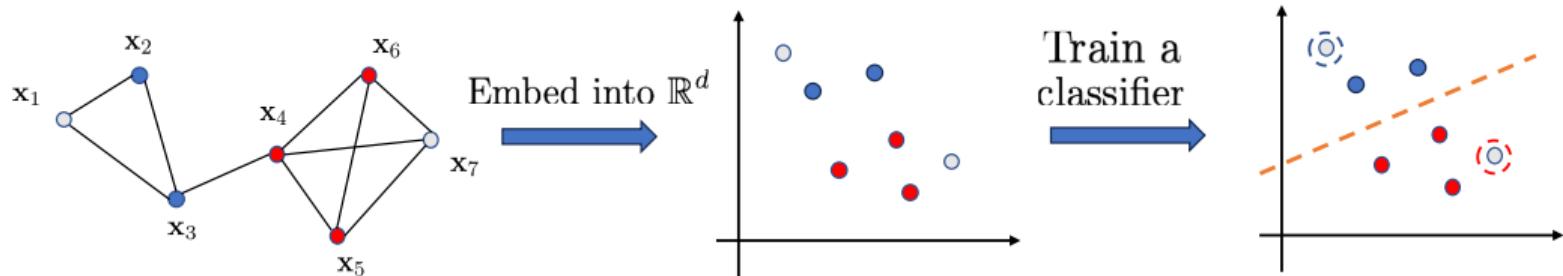
Graph-based ML leverages the network structure of the data to improve learning and processing of these data



$$\hat{y}_i = f_{\theta}(\{\mathbf{x}_j\}_{j=1}^N; \mathbf{A})$$

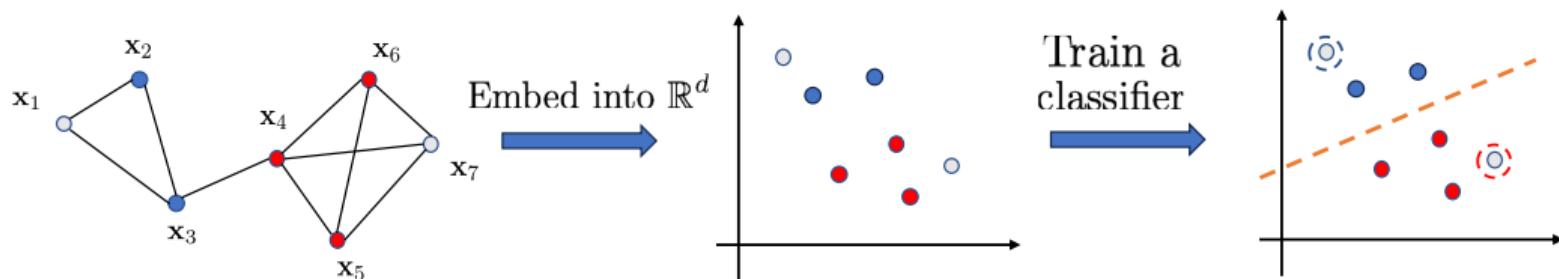
# Graph Representation Learning

- Convert raw **graph data** into a low-dimensional **vector representation**



# Graph Representation Learning

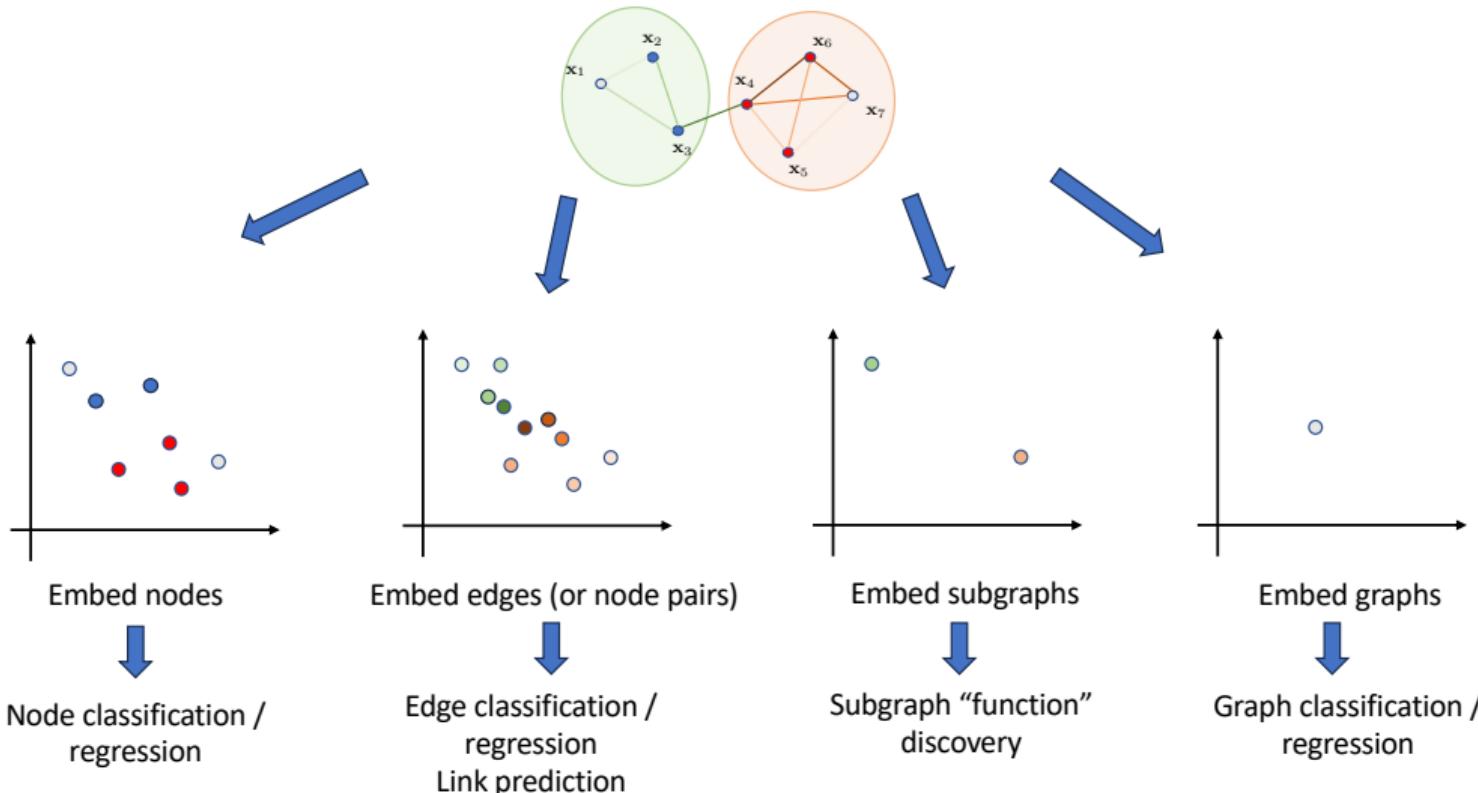
- Convert raw graph data into a low-dimensional vector representation



- Once in  $\mathbb{R}^d$ , we can apply the whole ML machinery
- Embedding can be **unsupervised**
  - ⇒ “Closeness” in the graph is preserved as “closeness” in  $\mathbb{R}^d$
- or **supervised**
  - ⇒ Trained together with the downstream classifier
- We can embed other **graph elements beyond nodes**

# Graph Representation Learning

- We can embed **nodes**, **edges**, **subgraphs**, and **whole graphs**



# Graph Representation Learning

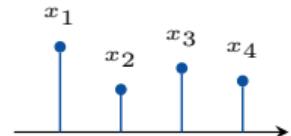
- ▶ “Shallow” embeddings (2014 - 2016): LINE, DeepWalk, node2vec
- ▶  $O(N)$  parameters are needed  $\Rightarrow$  No parameter sharing
- ▶ Inherently transductive  $\Rightarrow$  needs retraining for new nodes
- ▶ No node features  $\Rightarrow$  key in many applications

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- ▶ No node features  $\Rightarrow$  key in many applications
  
- ▶ “Deep” embeddings (2016 - present): GCN, GraphSAGE, and many others
- ▶ Graph neural networks address limitations of shallow embeddings

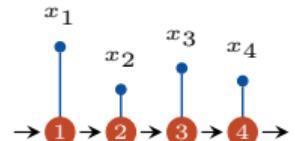
# Graph-structured data

- Discrete-time signal  $\Rightarrow$  Relation of nearby values carries information



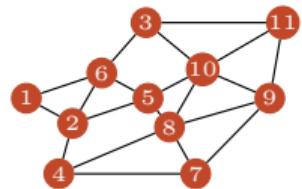
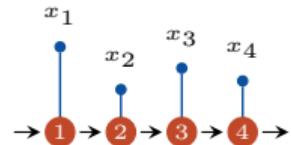
# Graph-structured data

- ▶ Discrete-time **signal**  $\Rightarrow$  Relation of **nearby values** carries information
  - $\Rightarrow$  Make the **data structure explicit**  $\Rightarrow$  Nearby elements are related
  - $\Rightarrow$  Two constitutive elements of SP: **data structure** and **signal values**



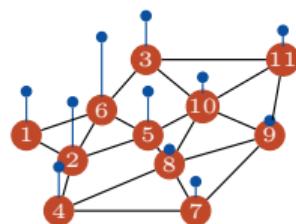
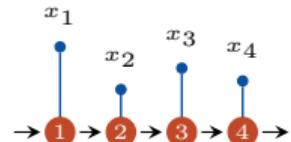
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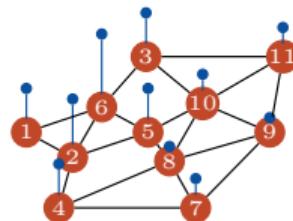
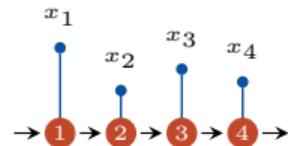
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- ▶ Matrix representation  $\Rightarrow$  Adjacency matrix  $\mathbf{A}$

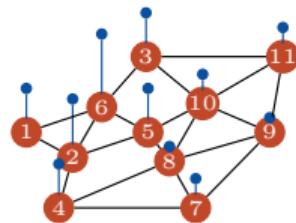
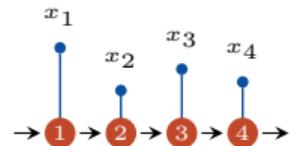
$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & \cdots \\ 1 & 0 & 0 & 0 & \cdots \\ 0 & 1 & 0 & 0 & \cdots \\ 0 & 0 & 1 & 0 & \cdots \\ 0 & 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$



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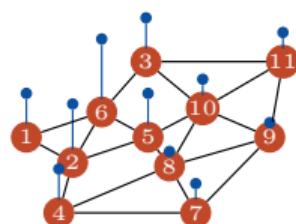
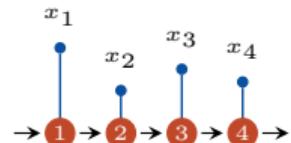
$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$



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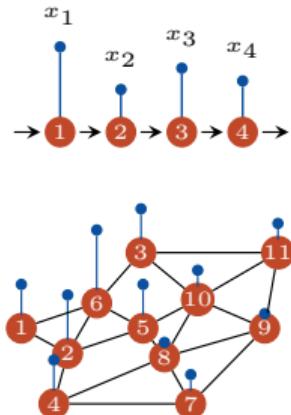
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- ▶ Matrix representation  $\Rightarrow$  Adjacency matrix  $\mathbf{A}$ , Laplacian matrix  $\mathbf{L}$

$$\mathbf{L} = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & -1 \\ 0 & -1 & 0 & 3 & 0 & 0 & -1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 4 & -1 & -1 & 0 & -1 & 0 & -1 \\ -1 & -1 & -1 & 0 & -1 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 3 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & -1 & 5 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 4 & -1 & -1 \\ 0 & 0 & -1 & 0 & -1 & 0 & 0 & -1 & -1 & 5 & -1 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 3 \end{bmatrix}$$



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  - $\Rightarrow$  Fixes ordering of the nodes  $\Rightarrow$  Permutations
  - $\Rightarrow$  Generic matrix  $\mathbf{S}$  (support matrix, graph shift operator)



Sandryhaila, Moura, "Discrete Signal Processing on Graphs", IEEE TSP, 2013

Shuman, Narang, Frossard, Ortega, Vandergheynst, "The Emerging Field of Signal Processing on Graphs", IEEE SPM, 2013

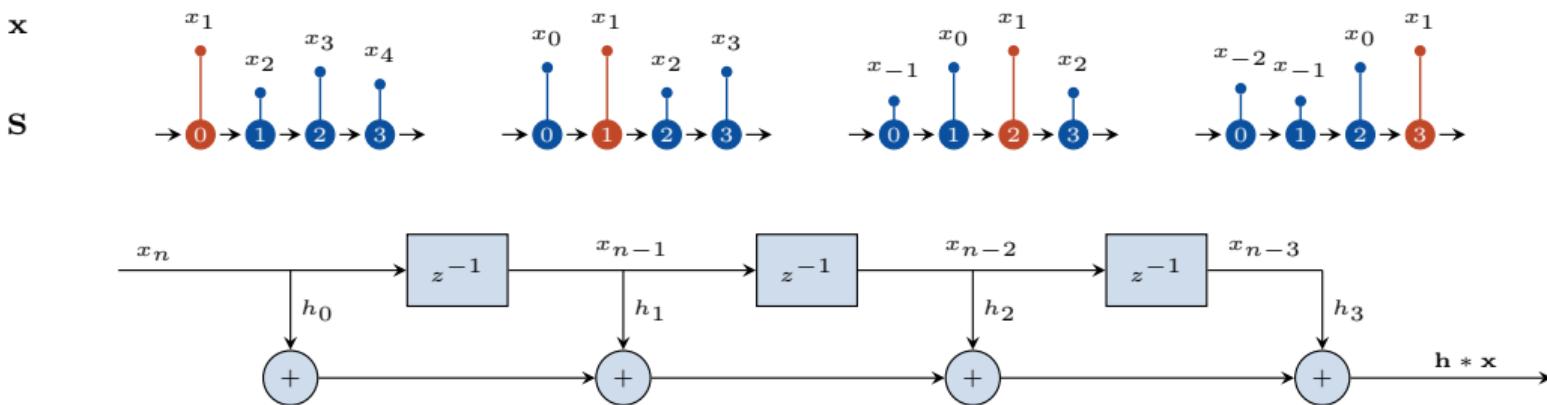
# Graph convolutions

- Graph convolution  $\Rightarrow$  Linear combination of shifted versions of the signal

$$\mathbf{x} * \mathbf{h} = \sum_{k=0}^{K-1} h_k x_{n-k}$$

$$\begin{bmatrix} \vdots & \vdots & \vdots \\ \cdots & 0 & 0 & 0 & \cdots \\ \cdots & 1 & 0 & 0 & \cdots \\ \cdots & 0 & 1 & 0 & \cdots \\ \cdots & 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots \end{bmatrix}$$

- Notion of shift  $\mathbf{S}$   $\Rightarrow$  Matrix description of graph (adjacency, Laplacian)



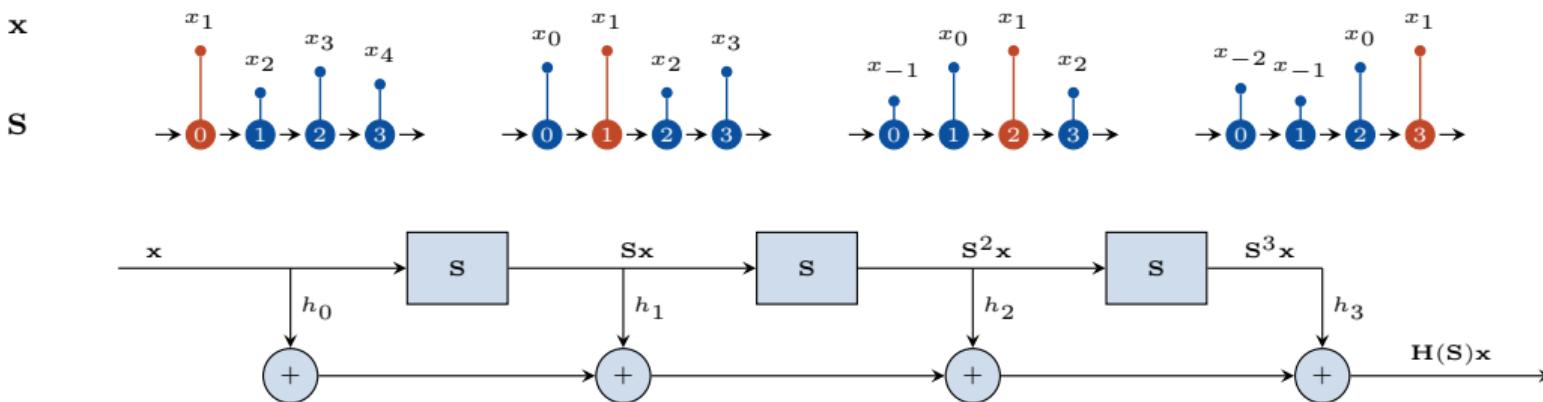
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$$\left[ \begin{array}{cccc} \vdots & \vdots & \vdots & \vdots \\ \cdots & 0 & 0 & 0 & \cdots \\ \cdots & 1 & 0 & 0 & \cdots \\ \cdots & 0 & 1 & 0 & \cdots \\ \cdots & 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots \end{array} \right] \left[ \begin{array}{c} \vdots \\ x_1 \\ x_2 \\ x_3 \\ \vdots \end{array} \right]$$



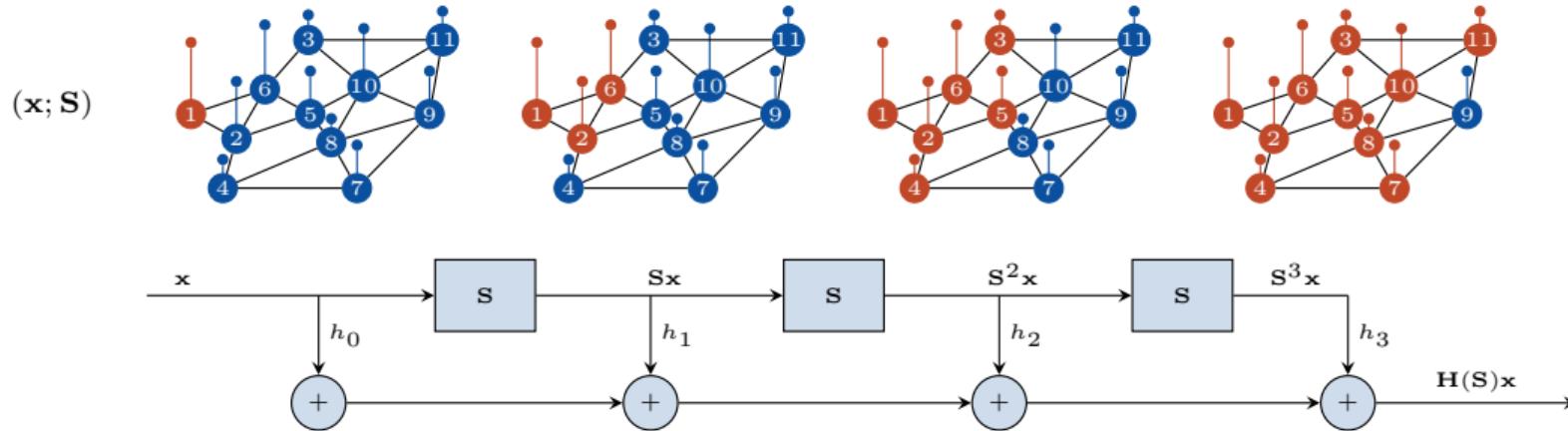
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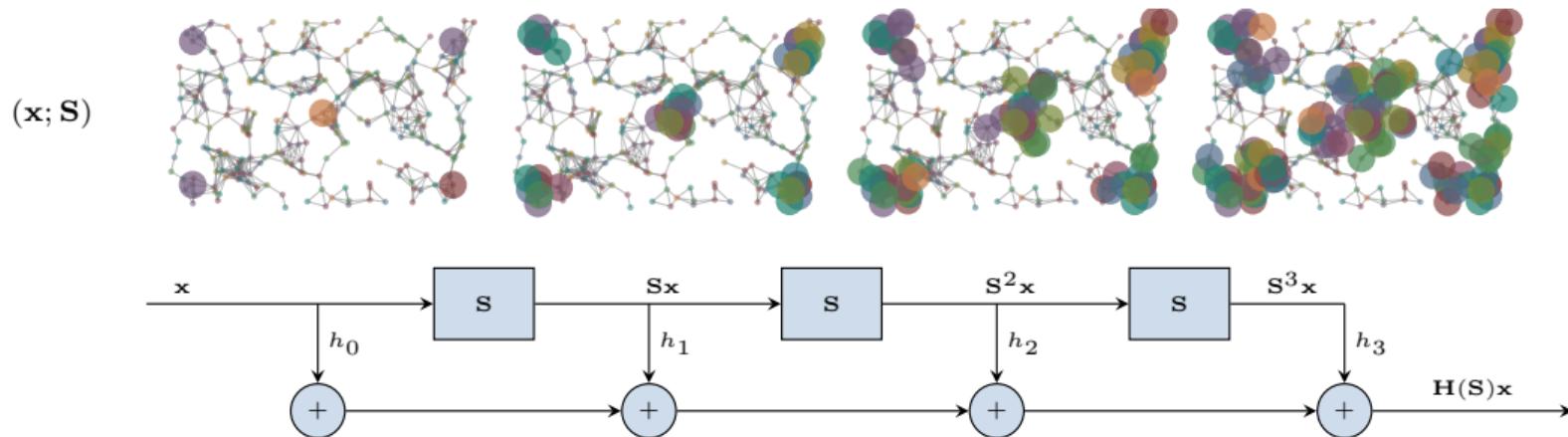


# Graph convolutions

- Graph convolution  $\Rightarrow$  Linear combination of shifted versions of the signal

$$\mathbf{x} *_{\mathbf{S}} \mathbf{h} = \sum_{k=0}^{K-1} h_k \mathbf{S}^k \mathbf{x} = \mathbf{H}(\mathbf{S})\mathbf{x}$$

- Notion of shift  $\mathbf{S}$   $\Rightarrow$  Matrix description of graph (adjacency, Laplacian)
- Linear combination of neighboring signal  $\Rightarrow$  Local operation



# Nonlinear graph signal processing

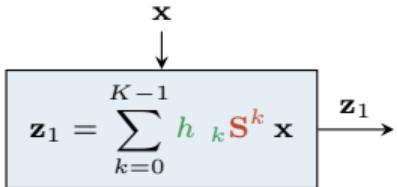
Traditional signal processing

⇒ Best linear filter that exploits **structure**

$$\min_{\{\mathbf{h}_k\}} J(\mathbf{z}_1) = \min_{\{\mathbf{h}_k\}} J(\mathbf{H}(\mathbf{S})\mathbf{x})$$

Linear models ⇒ Limited representation

⇒ Nonlinear graph signal processing



Gama, Isufi, Leus, Ribeiro, "Graphs, Convolutions and Neural Networks: From Graph Filters to Graph Neural Networks", IEEE SPM, 2020

# Nonlinear graph signal processing

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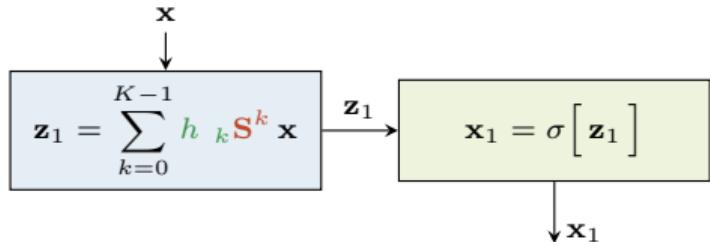
Graph perceptron ⇒ Nonlinear processing

⇒ Graph filter ⇒ Pointwise nonlinearity

⇒ Learn graph filter  $\{h_k\}$  ⇒  $\min_{\{h_k\}} J(\mathbf{x}_1)$

Basic nonlinear description of models

⇒ Increase representation power ⇒ Repeat



Gama, Isufi, Leus, Ribeiro, "Graphs, Convolutions and Neural Networks: From Graph Filters to Graph Neural Networks", IEEE SPM, 2020

# Graph convolutional networks

Cascade of  $L$  layers

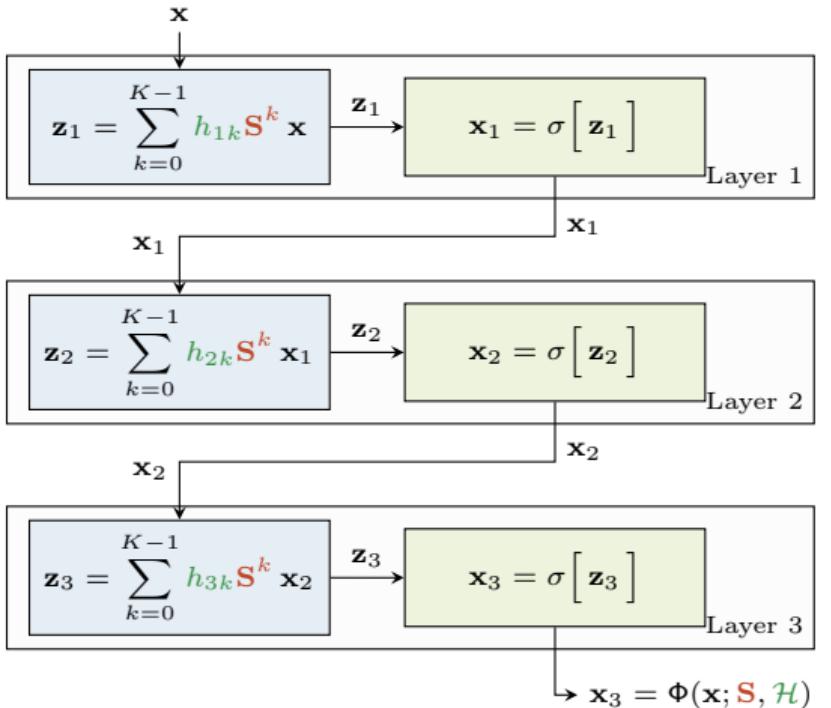
- ⇒ Graph convolutions with filters  $\mathcal{H} = \{\mathbf{h}_\ell\}$
- ⇒ Pointwise nonlinearity (activation functions)

The GCNN  $\Phi(\mathbf{x}; \mathbf{S}, \mathcal{H})$  depends on the filters  $\mathcal{H}$

- ⇒ Learn filter taps  $\mathcal{H}$  from training data
- ⇒ Also depends on the graph  $\mathbf{S}$

Nonlinear mapping  $\Phi(\mathbf{x}; \mathbf{S}, \mathcal{H})$

- ⇒ Exploit underlying graph structure  $\mathbf{S}$
- ⇒ Local information
- ⇒ Distributed implementation

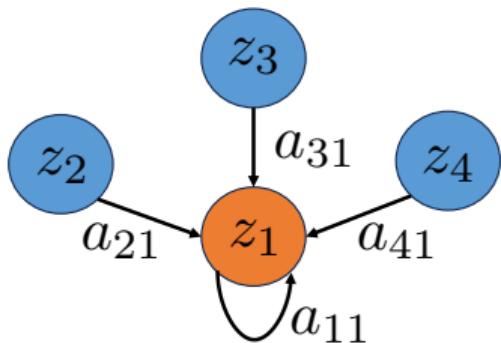


Gama. Marques. Leus. Ribeiro. "Convolutional Neural Network Architectures for Signals Supported on Graphs". IEEE TSP. 2018

# From GCNs to Message Passing Networks

- ▶ Every layer aggregates (one-hop) information and we stack several layers to increase the size of the “local” neighborhood influencing every node’s output

Graph Convolutional Network (GCN)



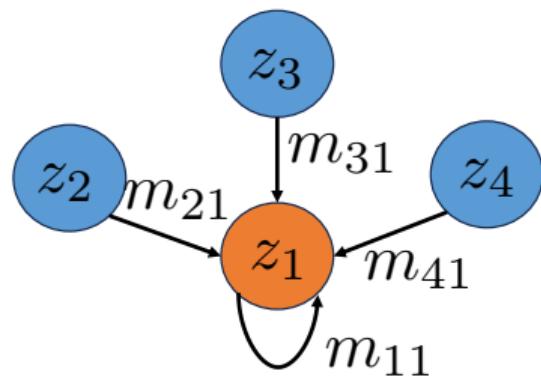
$$z_i^{(0)} = x_i$$

$$z_i^{(k+1)} = \sigma \left( \sum_{j \in (\mathcal{N}_i \cup \{i\})} w_{aj} z_j^{(k)} \right)$$

# From GCNs to Message Passing Networks

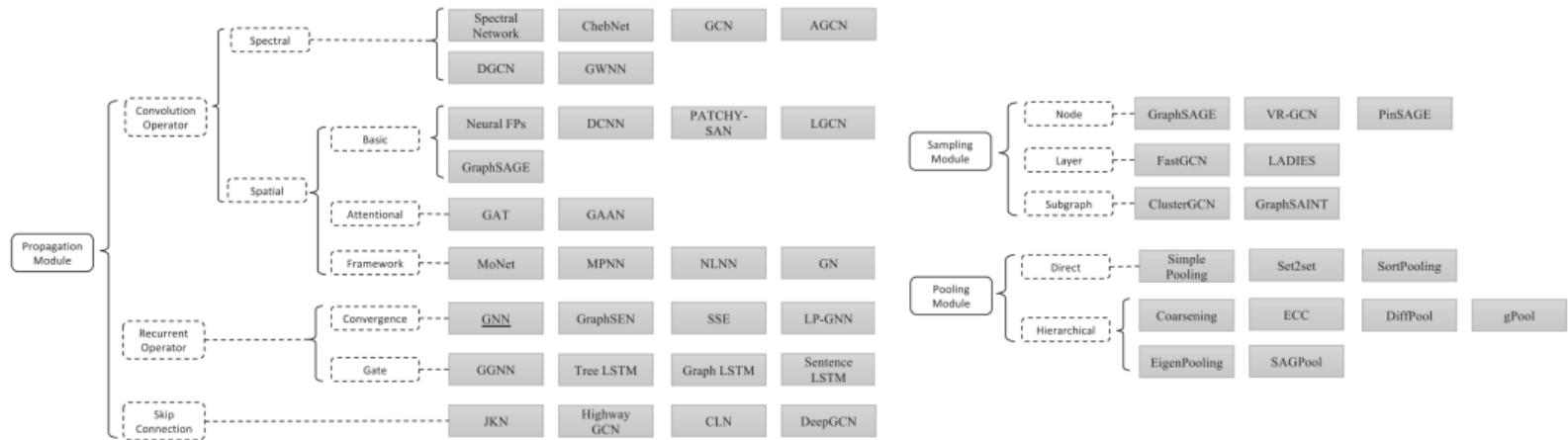
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## Message-passing Neural Network (MPNN)



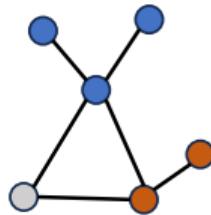
$$\begin{aligned} z_i^{(0)} &= x_i \\ m_{ji}^{(k)} &= f_e(z_i^{(k)}, z_j^{(k)}, e_{ji}) \\ z_i^{(k+1)} &= f_v \left( z_i^{(k)}, \sum_{j \in (\mathcal{N}_i \cup \{i\})} m_{ji}^{(k)} \right) \end{aligned}$$

# A Zoo of GNNs has been developed

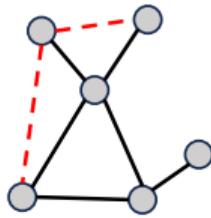


Credit: "Graph neural networks: A review of methods and applications"

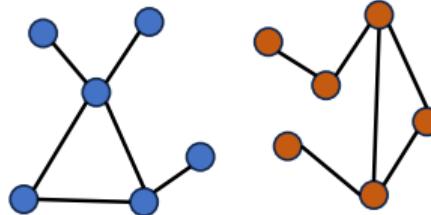
# Typical generic problems tackled with GNNs



Node classification



Link prediction

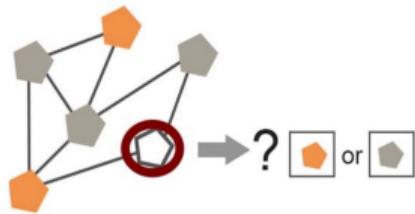


Graph classification

# Translate into Domain-Specific Problems

## Node classification

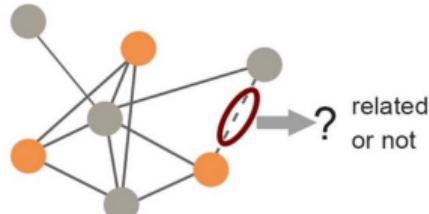
-  Active protein function
-  Inactive protein function



PPI Network

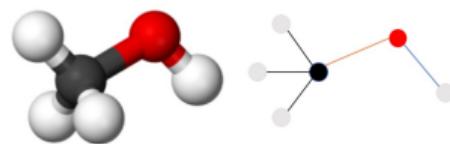
## Link Prediction

-  RNA
-  Disease



RNA-Disease Association Network

## Graph Classification

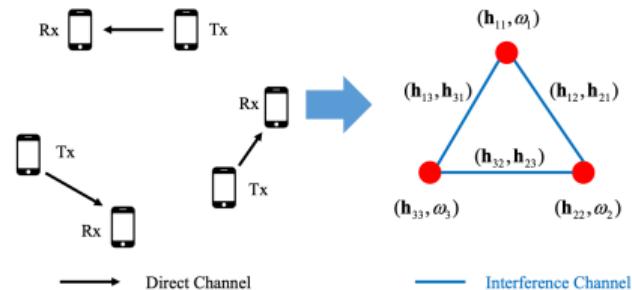


Soluble molecule or not

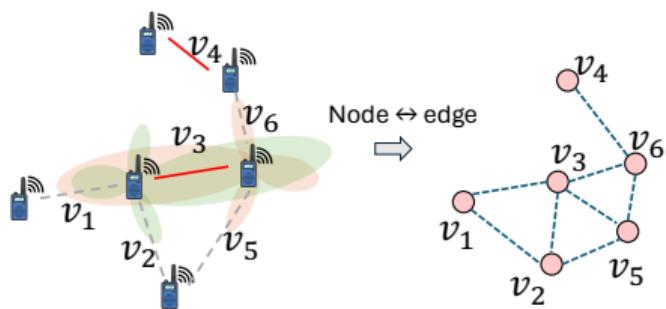
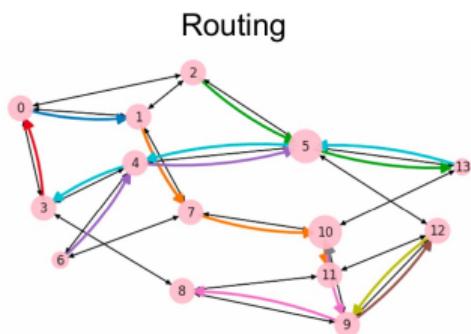
Credit: "Graph Neural Networks and their Current Applications in Bioinformatics", Zhang et al.

Credit: "Graph Neural Networks: A Review of Methods and Applications", Zhou et al.

# Graphs in Wireless Communications



Credit: "Graph-based Deep Learning for Communication Networks: A Survey", Jiang



# Why GNNs for Wireless Communications?

- ▶ Built-in **scalability**
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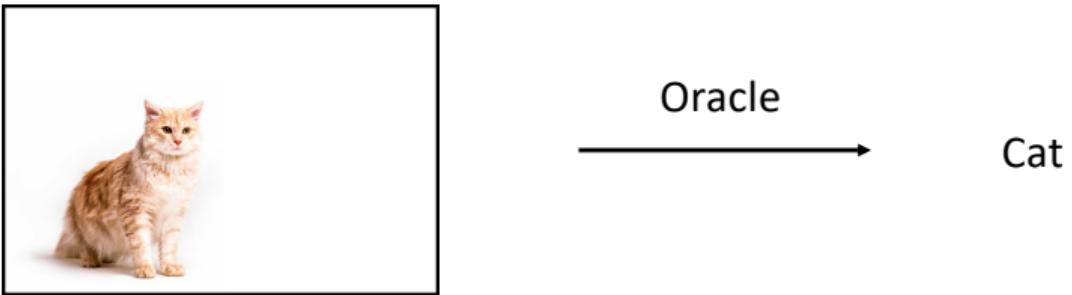
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- ▶ **Locality** plays a central role
  - ⇒ My optimal decision depends on the parts of the **network close to me**
- ▶ Exploit the correct **symmetries**
  - ⇒ **Permutation equivariance/invariance** is a natural feature of network control

# CNNs and translation invariance



# CNNs and translation invariance



Oracle

---

Cat



Oracle

---

Cat

- ▶ Architectures used for **object recognition** benefit from **translation invariance**  
    ⇒ **Convolutional Neural Networks**
- ▶ Learning in the class of function to which the oracle belongs

# GNNs and permutation equivariance

$$H \xrightarrow{\text{Oracle}} p^*$$

# GNNs and permutation equivariance

$$\Pi \mathbf{H} \Pi^\top \xrightarrow{\text{Oracle}} \Pi \mathbf{p}^*$$

$$\boldsymbol{\Pi} \mathbf{H} \boldsymbol{\Pi}^\top \xrightarrow{\text{Oracle}} \boldsymbol{\Pi} \mathbf{p}^*$$

- ▶ Architectures used for **power allocation** benefit from **permutation equivariance**  
⇒ **Graph Neural Networks**
- ▶ Learning in the class of function to which the oracle belongs

# Permutation equivariance vs. invariance

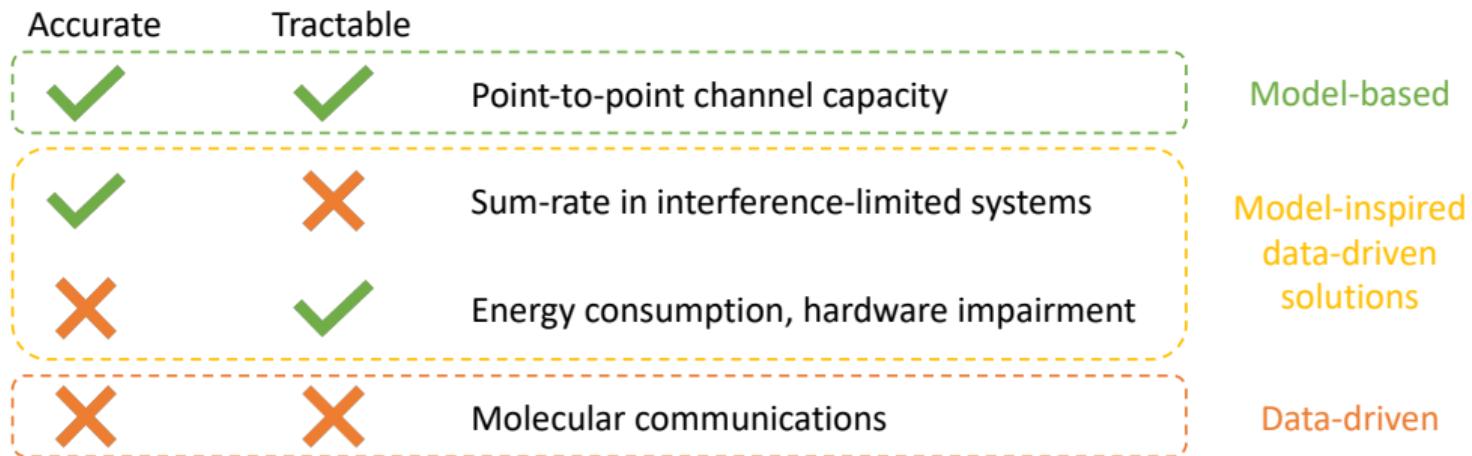
- ▶ Equivariance  $\Rightarrow f_{\theta}(\boldsymbol{\Pi}\mathbf{X}; \boldsymbol{\Pi}\mathbf{A}\boldsymbol{\Pi}^{\top}) = \boldsymbol{\Pi}f_{\theta}(\mathbf{X}; \mathbf{A})$
- ▶ Invariance  $\Rightarrow f_{\theta}(\boldsymbol{\Pi}\mathbf{X}; \boldsymbol{\Pi}\mathbf{A}\boldsymbol{\Pi}^{\top}) = f_{\theta}(\mathbf{X}; \mathbf{A})$

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- ▶ Invariance  $\Rightarrow f_{\theta}(\boldsymbol{\Pi}\mathbf{X}; \boldsymbol{\Pi}\mathbf{A}\boldsymbol{\Pi}^{\top}) = f_{\theta}(\mathbf{X}; \mathbf{A})$
- ▶ GNNs are **equivariant** at the level of the **nodes** (or **edges**) and **invariant** at the level of the **graph**
  - $\Rightarrow$  **Node** labels permute when the input is permuted
  - $\Rightarrow$  **Graph** labels are impervious to permutations
- ▶ Achievable **rates** (node-level quantity) are re-indexed with permutations
  - $\Rightarrow$  but the total **sum-rate** (graph-level quantity) is not modified

# Model-inspired Data-driven Solutions

## Theoretical model



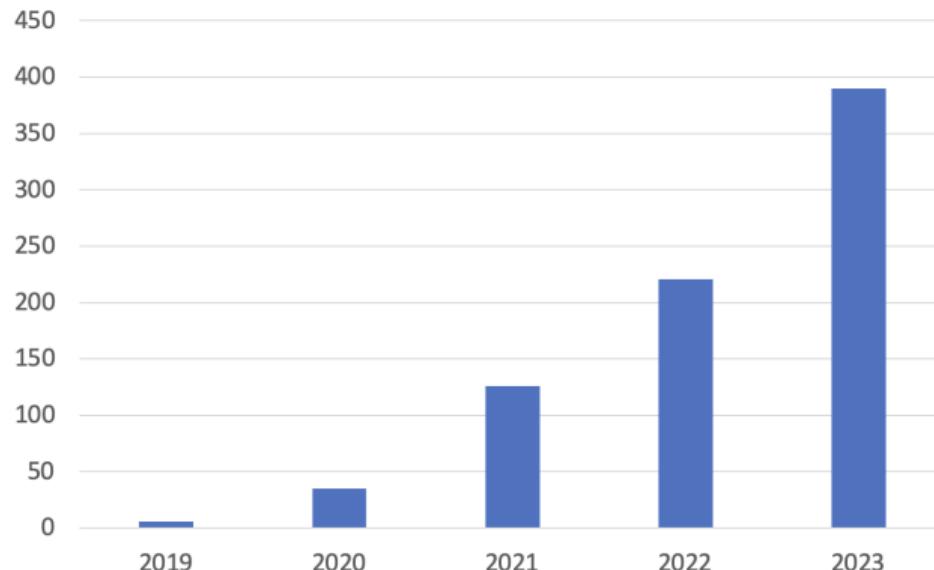
Credit: "Wireless Networks Design in the Era of Deep Learning: Model-Based, AI-Based, or Both?" Zappone et al., IEEE ToC, 2019

- ▶ Synergy between classical models and modern data-driven solutions

# Overview of GNN applications to Wireless Comms

- Very **dynamic field** ⇒ Many new papers being published

Papers "GNN" + "Wireless Network"



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  - ⇒ He et al., “An overview on the application of graph neural networks in wireless networks”, IEEE O. J. of the Comm. Soc., 2021
  - ⇒ Hu et al., “Distributed Machine Learning for Wireless Communication Networks: Techniques, Architectures, and Applications”, IEEE Comm. Surv. & Tut., 2021
  - ⇒ Shen et al., “Graph neural networks for wireless communications: From theory to practice”, IEEE Trans. Wireless Comm., 2022
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  - ⇒ Ali et al., “6G White Paper on Machine Learning in Wireless Communication Networks”, Arxiv, 2020
  - ⇒ Jiang, “Graph-based deep learning for communication networks: A survey”, Computer Comm., 2022
  - ⇒ Suárez-Varela et al., “Graph Neural Networks for Communication Networks: Context, Use Cases and Opportunities”, IEEE Network, 2023

# Overview of GNN applications to Wireless Comms

- ▶ Very dynamic field ⇒ Many new papers being published
- ▶ Several tutorials/surveys in the area
- ▶ A variety of problems have been tackled, including:
  - ⇒ Power allocation and beamforming
  - ⇒ Channel estimation
  - ⇒ Traffic prediction
  - ⇒ Spectrum allocation
  - ⇒ Cooperative caching
  - ⇒ Link scheduling
  - ⇒ Routing

## Part II: GNNs at the Physical Layer

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- ▶ Algorithms must be robust against perturbations in the network
- ▶ We consider the optimal power allocation problem
  - ⇒ Fast, efficient, and robust solution

# Overview

## ► Broad objective

- ⇒ Interference management in tactical wireless ad hoc networks
- ⇒ Network utility optimization under constraints

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- ▶ **Domain-inspired learning and reusable models**
  - ⇒ Combine classical algorithms with data-driven modules
  - ⇒ Domain knowledge with neural acceleration
- ▶ **Learning under constraints**
  - ⇒ Near-optimal solution for the unconstrained problem
  - ⇒ Flexibility of learning to operate under multiple constraints
- ▶ **Intelligent system** leverages graph structure to allocate power
  - ⇒ Requires centralized training but deployment can be distributed

# Optimal Power Allocation - SISO Case

# System Model

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  - ⇒ Encodes (time-varying) channel characteristics
  - ⇒  $H_{ji}(t)$  represents the channel from Tx  $i$  to Rx  $r(j)$  at time  $t$

$$H_{ji}(t) = H_{ji}^P H_{ji}^F(t)$$

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- ▶ Node State Information (NSI) matrix  $\mathbf{X}(t) \in \mathbb{R}^{m \times d}$ 
  - ⇒ Encodes (time-varying) node features of the Tx-Rx pair
  - ⇒ # of packets that arrived, queue length, user priority

## Problem Description

Given the CSI matrix  $\mathbf{H}(t)$ , the NSI matrix  $\mathbf{X}(t)$ , and a network utility function  $u(\mathbf{H}(t), \mathbf{X}(t), \mathbf{p}(t))$ , determine the optimal power allocation  $\mathbf{p}(t) \in \mathbb{R}_+^m$

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- ▶ Network utility function: sum rate across nodes
- ▶ Data rate at receiver  $i$  is given by (for noise variance  $\sigma^2$ )

$$c_i = \log_2 \left( 1 + \frac{|H_{ii}|^2 p_i}{\sigma^2 + \sum_{j \neq i} |H_{ij}|^2 p_j} \right)$$

- ▶ Maximize weighted sum-rate  $\sum_{i=1}^m \alpha_i c_i$ , under power constraint
- ▶ Seeking a function  $\mathbf{p}(\mathbf{H}, \mathbf{X})$  to optimize WSR

# Classical Approach

- ▶ Weighted minimum mean-square error (WMMSE) [Shi *et al.*, TSP 2011]
  - ⇒ Reformulate the optimization problem
  - ⇒ Implement block coordinate descent
  - ⇒ Leads to closed-form iteration formulas

$$\min_{\mathbf{w}, \mathbf{u}, \mathbf{v}} \sum_{i=1}^m (w_i e_i(\mathbf{H}, \mathbf{u}, \mathbf{v}) - \log w_i)$$

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- ▶ The optimal power  $p_i$  can be found as  $v_i^2$
- ▶ WMMSE is an iterative approach to solve the optimization
  - ⇒ Update  $\mathbf{u}$ ,  $\mathbf{w}$ , and  $\mathbf{v}$  at each step by block coordinate descent
  - ⇒ Stop when change between consecutive steps is small enough

# WMMSE: Update Equations

1. Initialize  $v_i = p_{\max}$
2. **repeat** (for all  $i$ )
3.      $w_i' = w_i$
4.      $u_i = \frac{H_{ii}v_i}{\sigma^2 + \sum_j H_{ji}^2 v_j^2}$
5.      $w_i = \frac{1}{1 - u_i H_{ii} v_i}$
6.      $v_i = \frac{\alpha_i u_i H_{ii} w_i}{\mu + \sum_j \alpha_j H_{ij}^2 u_j^2 w_j}$
7. **until**     $\sum_j \log w_j - \sum_j \log w_j' < \epsilon$
8.  $p_i = v_i^2$

# WMMSE: Update Equations

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- ▶ May not always converge to the global optimum
- ▶ Computationally expensive with high time complexity
- ▶ Cannot incorporate node state info
- ▶ Must be rerun for each instance of  $\mathbf{H}$

# Connectionist Approach

- ▶ Use neural networks to learn the optimal power allocation  $\mathbf{p}(\mathbf{H}, \mathbf{X})$
- ▶ GNNs are good candidates to model this allocation
  - ⇒ CSI  $\mathbf{H}$  as a **weighted adjacency** matrix of a **directed graph**
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- ▶  $\mathbf{p}(\mathbf{H}, \mathbf{X}) = \Psi(\mathbf{H}, \mathbf{X}; \Theta)$ , where  $\Psi$  is a  $K$ -layered **GNN**
  - ⇒  $\Theta$  is the set of **trainable weights**

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  - ⇒  $\Theta$  is the set of trainable weights
- ▶ Supervised Training: Learn by using WMMSE output as training signals
- ▶ Unsupervised Training: Learn using Sum-rate as the optimization objective

## REGNN

- ▶ Standard layered GNN architecture

$$\mathbf{z}_l = \text{ReLU} \left( \sum_{f=0}^{F_l} \gamma_{lf} \mathbf{H}^f \mathbf{z}_{l-1} \right) \quad \mathbf{z}_0 = \mathbf{X}, \quad \Phi(\mathbf{H}, \mathbf{X}; \boldsymbol{\gamma}) = \mathbf{z}_L$$

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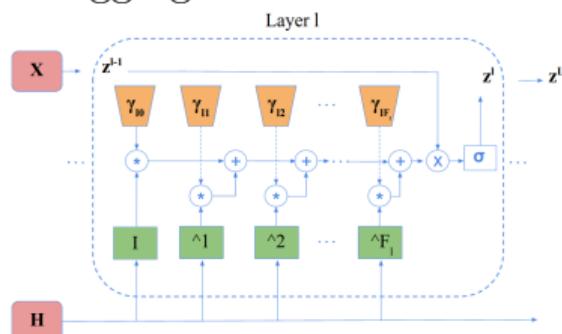
- ▶ Graph filter  $\sum_{f=0}^{F_l} \gamma_{lf} \mathbf{H}^f$  combines data within  $F_l$ -hop neighborhoods
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Eisen-Ribeiro TSP'20

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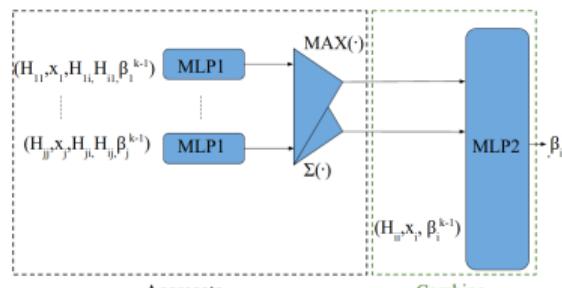
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Layer k - Node i



Shen *et al.*, Globecom'19

# Graph-based *Unfolding* Scheme

## ► Iterative algorithm

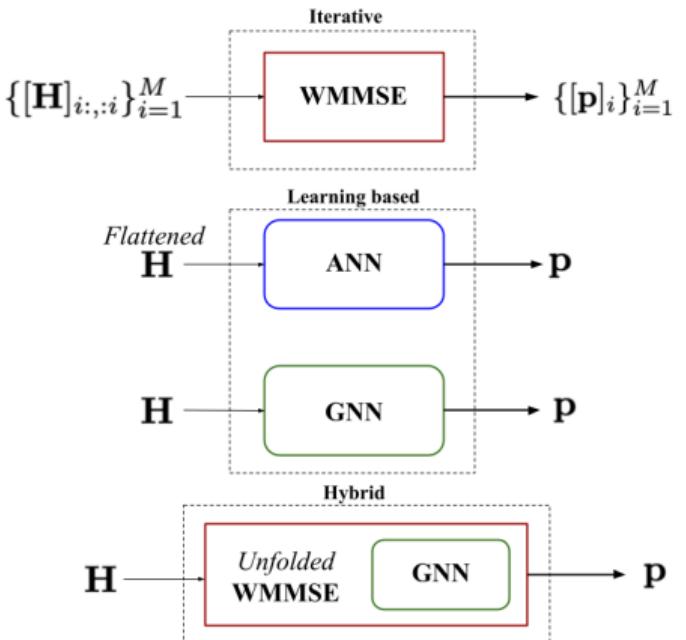
- ⇒ Near-optimal
- ⇒ Time-consuming
- ⇒ Greedy

## ► Learnable models

- ⇒ MLP ignores graph structure
- ⇒ GNN ignores domain info.

## ► Hybrid model

- ⇒ Iterations as layers
- ⇒ Embedded graph model
- ⇒ *Inherits greediness*



# Algorithm Unrolling

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  - ⇒ Supervised/Un-supervised gradient feedback
- ▶ Iterations ⇒ layers, Parameters ⇒ neural networks
- ▶ More interpretable operations, easy to follow update trajectory
- ▶ Once trained, can be used off-the-shelf ⇒ Effective for online solutions

# Proposed Method

- UWMMSE update rules at arbitrary layer  $k$

$$\textcolor{blue}{a}^{(k)} = \Psi(\mathbf{H}; \boldsymbol{\theta}_a^{(k)}), \quad \textcolor{blue}{b}^{(k)} = \Psi(\mathbf{H}; \boldsymbol{\theta}_b^{(k)}) \quad (1)$$

$$u_i^{(k)} = \frac{h_{ii}v_i^{(k-1)}}{\sigma^2 + \sum_j h_{ij}^2 v_j^{(k-1)} v_j^{(k-1)}}, \quad \forall i \quad (2)$$

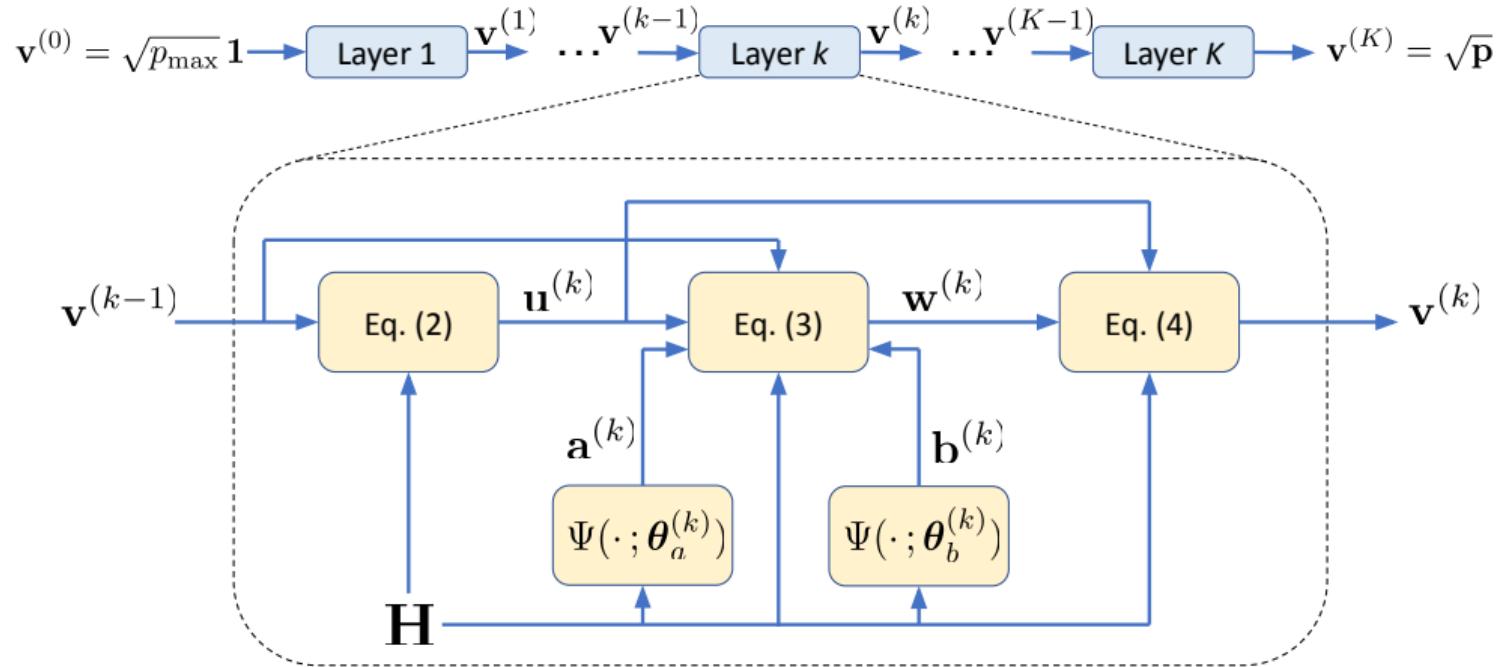
$$w_i^{(k)} = \frac{\textcolor{blue}{a}_i^{(k)}}{1 - u_i^{(k)} h_{ii} v_i^{(k-1)}} + \textcolor{blue}{b}_i^{(k)}, \quad \forall i \quad (3)$$

$$v_i^{(k)} = \alpha \left( \frac{u_i^{(k)} h_{ii} w_i^{(k)}}{\sum_j h_{ji}^2 u_j^{(k)} u_j^{(k)} w_j^{(k)}} \right), \quad \forall i \quad (4)$$

- $\textcolor{red}{v}$  and  $\textcolor{red}{u}$  as transmitter and receiver variables
- $\textcolor{green}{w}$  as a tunable parameter
- $\textcolor{blue}{a} = 1, \textcolor{blue}{b} = 0$  yields classical solution

# Block Diagram

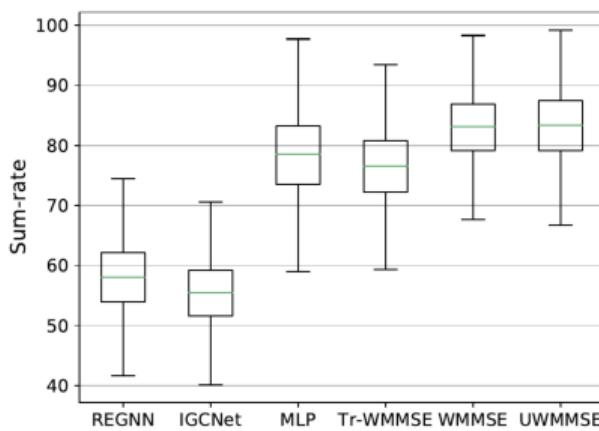
- $k^{th}$  layer of the model is shown below



# Simulation Results - 1

- ▶ Random geometric graph with  $M$  node pairs
- ▶ Path loss and Rayleigh fading
- ▶ Performance Comparison

⇒ Network size  $M = 20$ ;  
 ⇒  $K = 4, K_{\max} = 100$



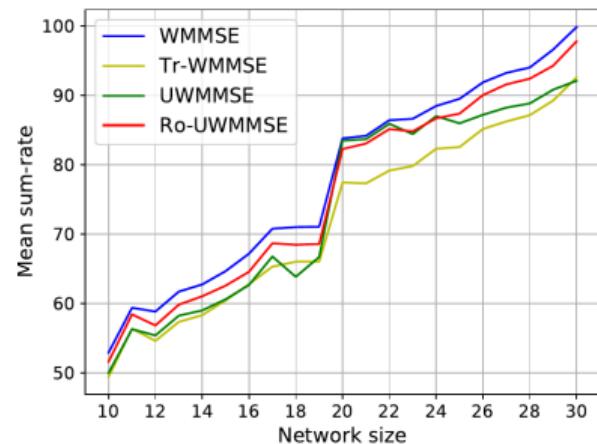
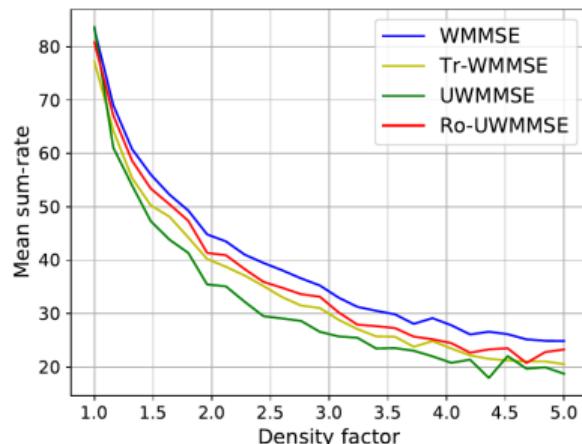
- ▶ Time Comparison

| Algorithm | Training time (m) | Test sum-rate | Test time (ms) |
|-----------|-------------------|---------------|----------------|
| WMMSE     | -                 | 82.94         | 16             |
| Tr-WMMSE  | -                 | 76.49         | 1.0            |
| MLP       | 0.5               | 78.17         | 3.2            |
| REGNN     | 15                | 57.92         | 2.5            |
| IGCNet    | 5                 | 55.30         | 3              |
| UWMMSE    | 15                | <b>83.21</b>  | <b>2.0</b>     |

WMMSE: Shi *et al.*, TSP'11, MLP: Sun *et al.*, TSP'18, REGNN: Eisen-Ribeiro TSP'20, IGCNet: Shen *et al.*, Globecom'21,  
 UWMMSE: Chowdhury *et al.*, ICASSP'21, TWC'21

## Simulation Results - 2

- ▶ Simulating dynamic network topologies
  - ⇒ Nodes in motion
  - ⇒ Insertion / Deletion of nodes
- ▶ Variation in Spatial Density
- ▶ Variation in Network Size



Chowdhury *et al.*, ICASSP'21, TWC'21

# Optimal Power Allocation & Beamforming - MIMO Case

# System Model

- ▶ Ad hoc network with  $M$  transmitter-receiver pairs (nodes)
- ▶ Transmitters have  $T$  antennas, receivers have  $R$  antennas
- ▶ Transmitter  $i$  has an associated receiver  $r(i) \forall i \in \{1, M\}$

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- ▶ Channel State Information (CSI) tensor  $\mathcal{H} \in \mathbb{R}^{M \times M \times R \times T}$ 
  - ⇒ Encodes channel characteristics
  - ⇒  $[\mathcal{H}]_{ji::} = \mathbf{H}_{ji} \in \mathbb{R}^{R \times T}$  represents a MIMO channel from  $i$  to  $r(j)$
  - ⇒ Channel between Tx-antenna  $k$  and Rx-antenna  $l$  is given by

$$[\mathbf{H}_{ji}]_{lk} = H_{jilk} = H_{jilk}^P H_{jilk}^F(t)$$

⇒ where  $H_{jilk}^P \propto \text{dist}(i, r(j))^{-k}$  for all  $l, k$  and  $H_{jilk}^F \sim \text{Rayleigh}(\alpha)$

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- ▶ Transmitter beamformer tensor  $\mathcal{V} \in \mathbb{R}^{M \times T \times d}$ 
  - ⇒  $[\mathcal{V}]_i = \mathbf{V}_i \in \mathbb{R}^{T \times d}$  transmits signal  $\mathbf{s}_i \in \mathbb{R}^d$  at node  $i$

## Problem Description

Given the CSI tensor  $\mathcal{H}$ , and a network utility function  $u(\mathcal{H}, \mathcal{V}, \mathbf{p})$ , determine the optimal power allocation  $\mathbf{p}$  and  $\mathcal{V}$

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- ▶ Power Constraint: Maximum power at each node  $\Rightarrow p_i \leq P_{\max}$
- ▶ Network utility: sum rate across nodes
- ▶ Data rate at receiver  $i$  is given by (for noise variance  $\sigma^2$ )

$$c_i(\mathcal{H}, \mathcal{V}) = \log_2 \det \left( \mathbf{I} + \mathbf{H}_{ii} \mathbf{V}_i \mathbf{V}_i^\top \mathbf{H}_{ii}^\top (\sigma^2 \mathbf{I} + \sum_{j \neq i} \mathbf{H}_{ij} \mathbf{V}_j \mathbf{V}_j^\top \mathbf{H}_{ij}^\top)^{-1} \right)$$

$$\text{where } \text{Tr} \left( \mathbf{V}_i \mathbf{V}_i^\top \right) \leq p_i$$

- ▶ Maximize weighted sum-rate  $\sum_{i=1}^M \alpha_i c_i$
- ▶ Seeking a function  $\Psi(\mathcal{H})$

# Classical Approach

- ▶ Weighted minimum mean-square error (WMMSE)
  - ⇒ Reformulate the optimization problem [Shi *et al.*, TSP 2011]
  - ⇒ Implement block coordinate descent
  - ⇒ Leads to closed-form iteration formulae

$$\min_{\mathbf{W}, \mathcal{U}, \mathcal{V}} \sum_{i=1}^M (\text{Tr}(\mathbf{W}_i \mathbf{E}_i) - \log \det \mathbf{W}_i)$$

- ▶  $\mathcal{U} \in \mathbb{R}^{M \times R \times d}$  is the **receiver beamformer** tensor
- ▶  $\mathcal{W} \in \mathbb{R}^{M \times d \times d}$  is the **node weight** tensor

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- ▶  $\mathbf{W} \in \mathbb{R}^{M \times d \times d}$  is the **node weight** tensor
- ▶ WMMSE is an iterative approach to solve the optimization
  - ⇒ Update  $\mathbf{U}$ ,  $\mathbf{W}$ , and  $\mathbf{V}$  at each step by block coordinate descent
  - ⇒ Stop if change between consecutive steps is small enough

# Graph-based and model-informed ML solution

- ▶ Use neural networks to learn the optimal power allocation  $\mathbf{p}(\mathcal{H})$
- ▶ Graph neural networks are good candidates to model this allocation
  - ⇒  $\mathbf{p}(\mathcal{H}) = \Psi(\mathcal{H}; \Theta)$ , where  $\Psi$  is an  $K$ -layered GNN
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- ▶ Exploit the right symmetries
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  - ⇒ Permutation equivariance is a natural feature for power allocation
- ▶ Model-informed solution via algorithm unfolding
- ▶ Layers in a neural architecture inspired by iterations of WMMSE
  - ⇒ More interpretable operations
  - ⇒ Easy to fall back into classical solution

# Proposed Method

- UWMMSE update rules at arbitrary layer  $k$

$$\mathbf{a}^{(k)} = \Psi(\bar{\mathcal{H}}; \theta_a), \quad \mathbf{b}^{(k)} = \Psi(\bar{\mathcal{H}}; \theta_b), \quad (2)$$

$$\mathbf{U}_i^{(k)} = \left( \sum_{j \neq i} \mathbf{H}_{ij} \mathbf{V}_j^{(k-1)} \mathbf{V}_j^{(k-1)\top} \mathbf{H}_{ij}^\top + \sigma^2 \mathbf{I} \right)^{-1} \mathbf{H}_{ii} \mathbf{V}_i^{(k-1)} \quad \forall i \quad (3)$$

$$\mathbf{W}_i^{(k)} = [\mathbf{a}^{(k)}]_i (\mathbf{I} - \mathbf{U}_i^{(k)\top} \mathbf{H}_{ii} \mathbf{V}_i^{(k-1)})^{-1} + [\mathbf{b}^{(k)}]_i \quad \forall i \quad (4)$$

$$\mathbf{V}_i^{(k)} = \beta \left( \left( \sum_{j \neq i} \mathbf{H}_{ij}^\top \mathbf{U}_j^{(k)} \mathbf{W}_j^{(k)} \mathbf{U}_j^{(k)\top} \mathbf{H}_{ij} \right)^{-1} \mathbf{H}_{ii}^\top \mathbf{U}_i^{(k)} \mathbf{W}_i^{(k)} \right) \quad \forall i \quad (5)$$

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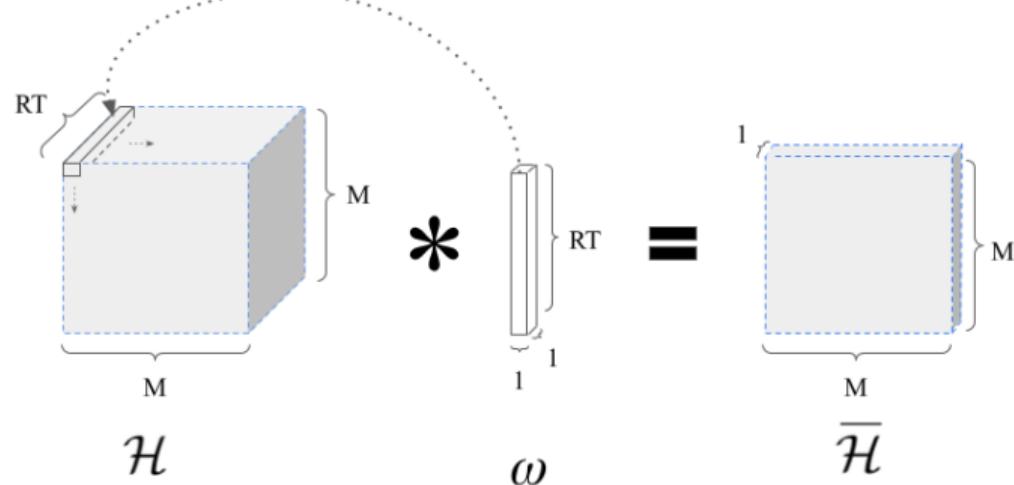
- $\beta$  is a clipper to enforce power constraint

$$\beta(\mathbf{X}) = \begin{cases} \mathbf{X}, & \text{if } \text{Tr}(\mathbf{X}\mathbf{X}^\top) \leq P_{\max}, \\ \mathbf{X} \cdot \frac{\sqrt{P_{\max}}}{\|\mathbf{X}\|_F}, & \text{otherwise,} \end{cases}$$

# Tensor Reduction

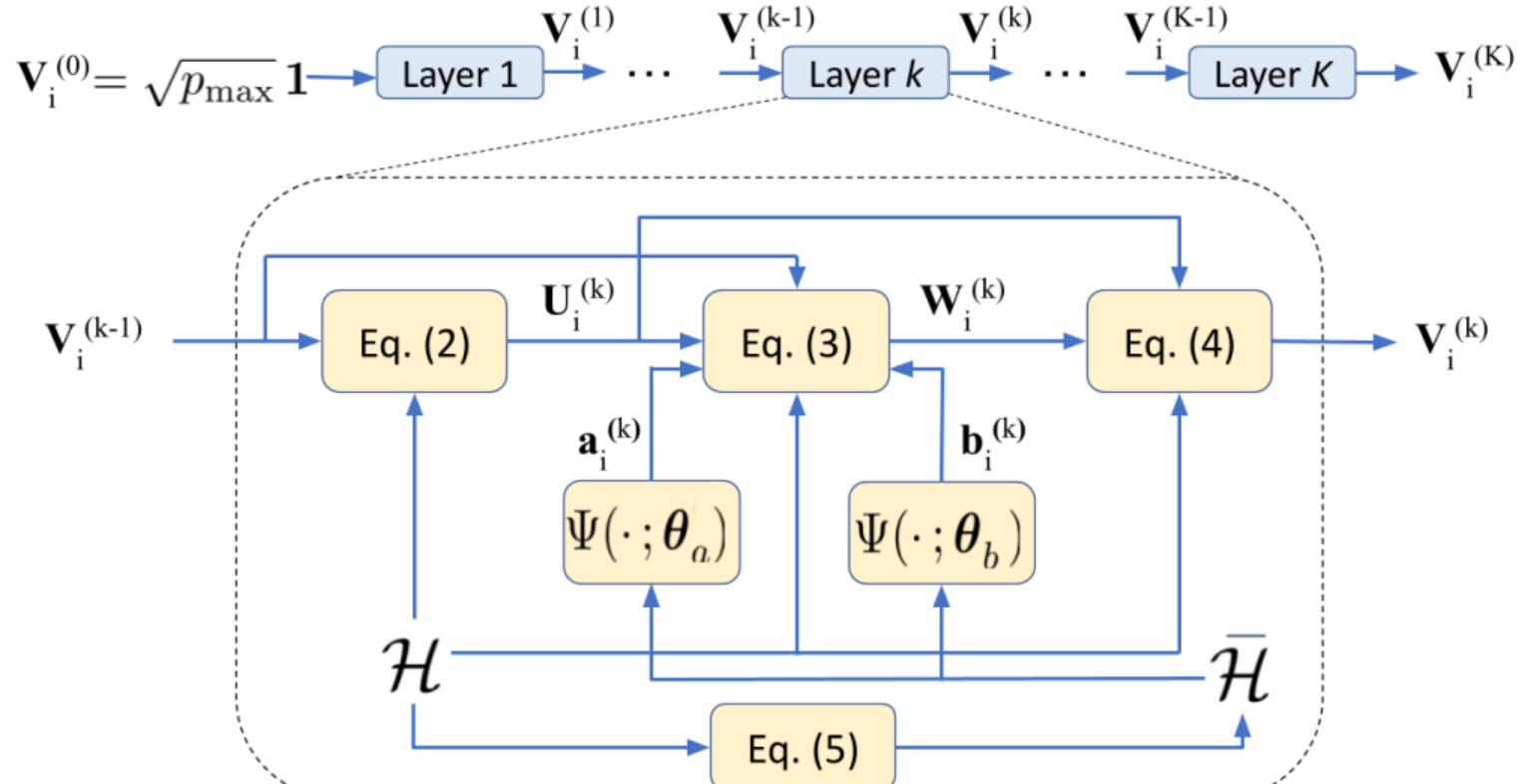
- ▶ GNN  $\Psi$  requires CSI between  $i$  and  $r(j)$  to be a **scalar**
- ▶  $\Phi(\mathcal{H}; \omega) : \mathbb{R}^{M \times M \times R \times T} \rightarrow \mathbb{R}^{M \times M \times 1}$  where  $\omega \in \mathbb{R}^{RT}$   
 $\Rightarrow$  Single-layered  $1 \times 1$  depth-wise conv with **shared weights**
- ▶ Learnable weighted combination of  $RT$  coefficients at each node

$$\bar{\mathcal{H}} = \Phi(\mathcal{H}; \omega) \quad (6)$$



# Block Diagram

- $k^{th}$  layer of the model is shown below



# Complexity Analysis

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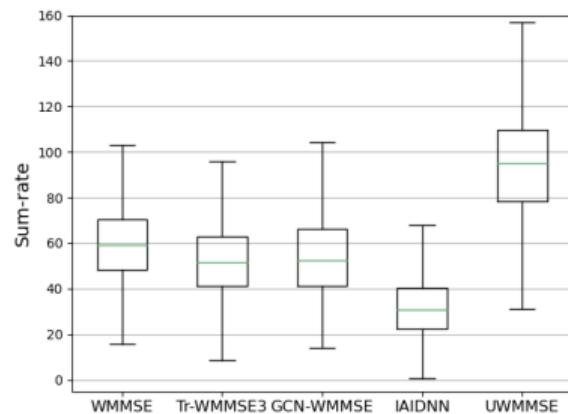
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- ▶ Each of the two **2-layered GCNs**  $\Psi$ , have  **$6h + 2$**  trainable weights  $\theta$   
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- ▶ Each of the two **2-layered GCNs**  $\Psi$ , have  **$6h + 2$**  trainable weights  $\theta$ 
  - ⇒  $h$  is the size of the hidden layer (typically  $\leq 10$ )
- ▶ Number of **trainable weights** is therefore  $12h + RT + 5$ 
  - ⇒ Independent of the number of users  $M$
- ▶ Very few trainable weights
  - ⇒ Makes model **easy** to train
  - ⇒ Likely to **generalize**

# Simulation Results - 1

- ▶ Random geometric graph with  $M$  node pairs
- ▶ Path loss & fading: Rayleigh, Rician, Network size  $M = 20$
- ▶ **Performance Comparison**
- ▶ **Time Comparison**



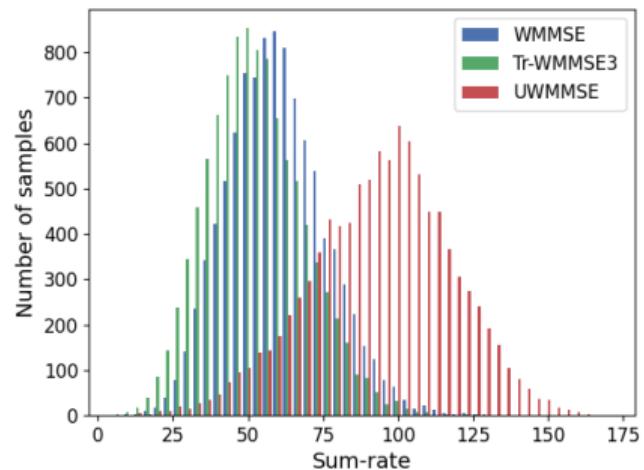
WMMSE: Shi *et al.*, TSP'11, IAIDNN: Hu *et al.* TWC'21, GCN-WMMSE: Schynol-Pesavento, JSAC'23,

UWMMSE: Chowdhury *et al.*, MILCOM'21, Asilomar'23, TWC'23

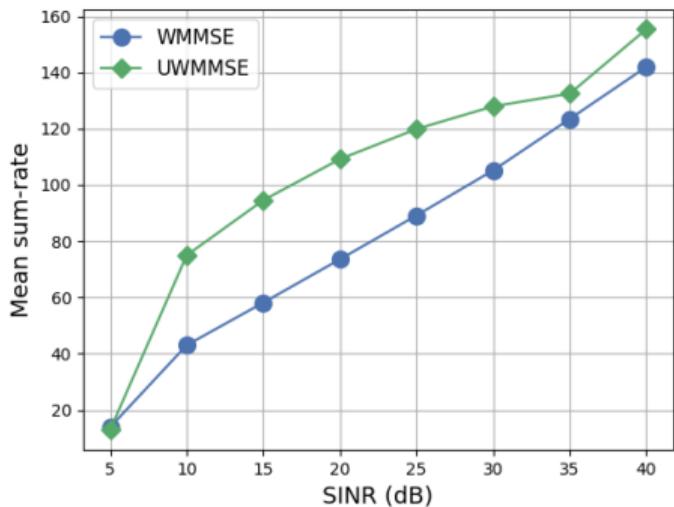
| Algorithm     | Training time (min) | Test time (sec) |
|---------------|---------------------|-----------------|
| WMMSE         | -                   | 1.305           |
| Tr-WMMSE      | -                   | 0.047           |
| IAIDNN        | ~ 10                | 0.64            |
| GCN-WMMSE     | ~ 21                | 1.365           |
| <b>UWMMSE</b> | ~ 35                | 0.054           |

# Simulation Results -2

## ► Performance Comparison



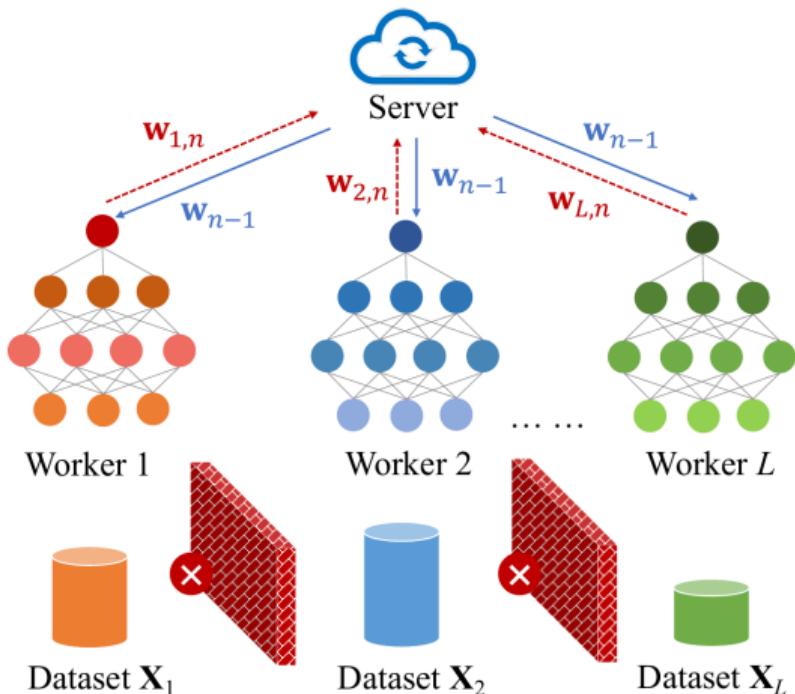
- Generalization performance  
⇒ Over SINR



Chowdhury *et al*, MILCOM'21, TWC'23, Asilomar'23

# Optimal Power Allocation - Federated Learning

# Power allocation for wireless FL



Critical step: upload local updates

How much **transmit power** should local workers use?

# Power allocation for wireless FL

Compared to the just-discussed SISO & MISO power allocation cases ...

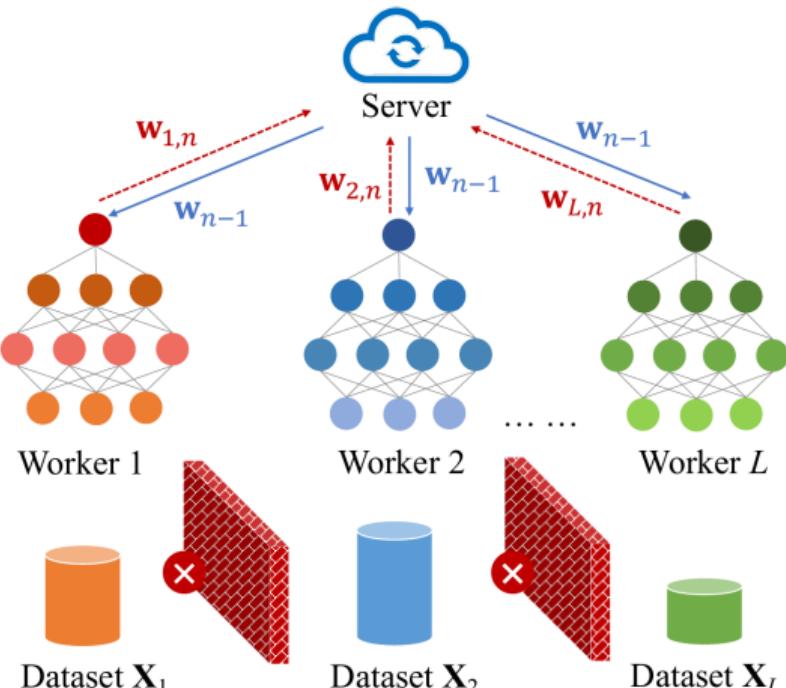


FL case is more challenging:

- ▶ Additional non-convex constraints on FL-specific requirements, e.g., delay and energy
- ▶ Ultimate goal of improving FL performance being indirect to the communication objective

# Power allocation for wireless FL

Compared to the just-discussed SISO & MISO power allocation cases ...



FL case is more challenging:

- ▶ Additional non-convex constraints on FL-specific requirements, e.g., delay and energy  
⇒ Primal-dual (PD) algorithm enhanced by graph learning
- ▶ Ultimate goal of improving FL performance being indirect to the communication objective  
⇒ Local data heterogeneity

# Problem formulation

Determine the power allocation policy  $p^* : \mathbb{R}^{L \times L} \rightarrow \mathbb{R}^L$  that solves the following optimization problem<sup>1</sup>, subject to bounds on **transmission rate**, **energy efficiency**, and **power**

$$\begin{aligned}
 p^* &= \underset{p}{\operatorname{argmax}} \ g(\mathbb{E}_{\mathbf{H} \sim \mathcal{H}} [\text{PSR}(\mathbf{p}, \mathbf{H})]), \\
 \text{s.t. } &\textcolor{blue}{r_{0,i}} \leq \mathbb{E}_{\mathbf{H} \sim \mathcal{H}} [R_i(\mathbf{p}, \mathbf{H}) \mid \textcolor{red}{p_i > 0}], \\
 &\textcolor{green}{e_{0,i}} \leq \mathbb{E}_{\mathbf{H} \sim \mathcal{H}} \left[ \frac{R_i(\mathbf{p}, \mathbf{H})}{p_i + P_{c,i}} \mid \textcolor{red}{p_i > 0} \right], \forall i, \\
 &\mathbf{p} = p(\mathbf{H}) \in [0, P_{\max}], \quad \forall \mathbf{H},
 \end{aligned}$$

---

<sup>1</sup>PSR: Packet success rate,  $\text{PSR} = \exp(-m/\text{SINR})$

# Problem formulation

**Parameterize** the power policy with learnable parameters  $\Theta$ , st  $p_\psi(\mathbf{H}) = \Psi(\mathbf{H}; \Theta)$ , and restate P1 ...

---

$$p^* = \underset{p}{\operatorname{argmax}} \ g(\mathbb{E}_{\mathbf{H} \sim \mathcal{H}} [\text{PSR}(\mathbf{p}, \mathbf{H})])$$

$$\begin{aligned} \text{s.t. } r_{0,i} &\leq \mathbb{E}_{\mathbf{H} \sim \mathcal{H}} [R_i(\mathbf{p}, \mathbf{H}) \mid p_i > 0], \\ e_{0,i} &\leq \mathbb{E}_{\mathbf{H} \sim \mathcal{H}} \left[ \frac{R_i(\mathbf{p}, \mathbf{H})}{p_i + P_{c,i}} \mid p_i > 0 \right], \forall i, \\ \mathbf{p} &= p(\mathbf{H}) \in [0, P_{\max}], \quad \forall \mathbf{H}, \end{aligned}$$

...in a manner that is amenable to a **Primal-Dual (PD) solution:**

---

$$P_\psi^* = \max_{\Theta, \mathbf{y}, \mathbf{r}, \mathbf{e}} \ g(\mathbf{y}), \quad (\text{P2})$$

$$\text{s.t. } \mathbf{y} \leq \mathbb{E}[\text{PSR}(\mathbf{p}_\psi, \mathbf{H})],$$

$$r_i \leq \mathbb{E}[R_i(\mathbf{p}_\psi, \mathbf{H}) \mid p_{\psi,i} > 0],$$

$$e_i \leq \mathbb{E} \left[ \frac{R_i(\mathbf{p}_\psi, \mathbf{H})}{p_{\psi,i} + P_c} \mid p_{\psi,i} > 0 \right],$$

$$r_i \in [r_{0,i}, +\infty),$$

$$e_i \in [e_{0,i}, +\infty), \quad \forall i,$$

$$\mathbf{p}_\psi = p_\psi(\mathbf{H}) \in [0, P_{\max}], \quad \forall \mathbf{H}.$$

# PD learning

- ▶ (P1) has a **zero duality gap**.
- ▶ (P2)'s duality gap depends on the **expressiveness of  $\Psi$** .

The Lagrangian of (P2)

$$\mathcal{L}_\psi(\boldsymbol{\Theta}, \mathbf{y}, \mathbf{r}, \mathbf{e}, \boldsymbol{\lambda}_y, \boldsymbol{\lambda}_r, \boldsymbol{\lambda}_e) = g(\mathbf{y}) + \boldsymbol{\lambda}_y^\top (\mathbb{E}[f_y] - \mathbf{y}) + \boldsymbol{\lambda}_r^\top (\mathbb{E}_c[f_r] - \mathbf{r}) + \boldsymbol{\lambda}_e^\top (\mathbb{E}_c[f_e] - \mathbf{e})$$

motivates iterative gradient updates to:

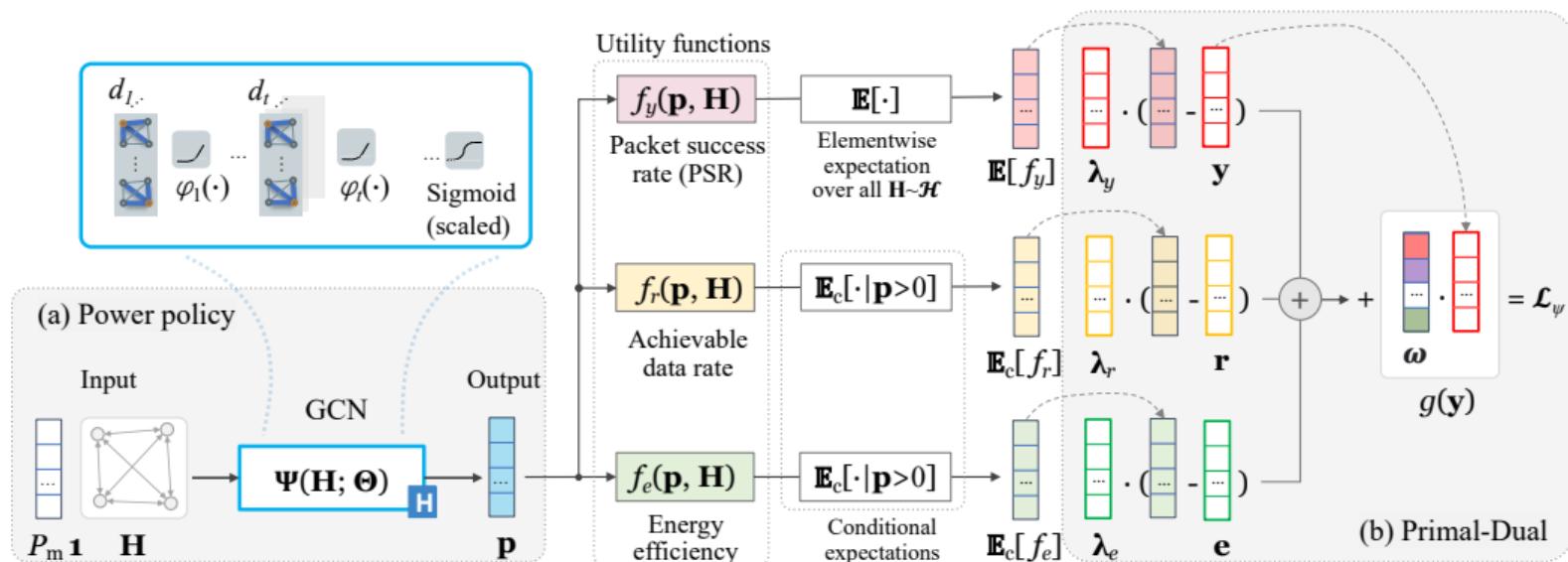
1. Learnable parameters  $\boldsymbol{\Theta}$ ;
2. Primal variables  $\mathbf{y}$ ,  $\mathbf{r}$ , and  $\mathbf{e}$ ;
3. Dual variables  $\boldsymbol{\lambda}_y$ ,  $\boldsymbol{\lambda}_r$ , and  $\boldsymbol{\lambda}_e$ .

Choosing GCN as  $\Psi$  constitutes our primal-dual graph convolutional (PDG) power network.

# Proposed solution (before FL)

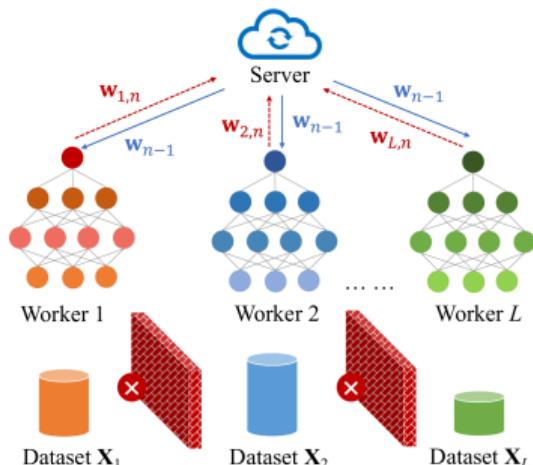
Two-stage solution with **two separate learning models**.

1. Before FL, train a **power allocation policy model** (see below).
2. During FL, apply the policy model to upload **local FL models** in each FL iteration that updates the **global FL model**.



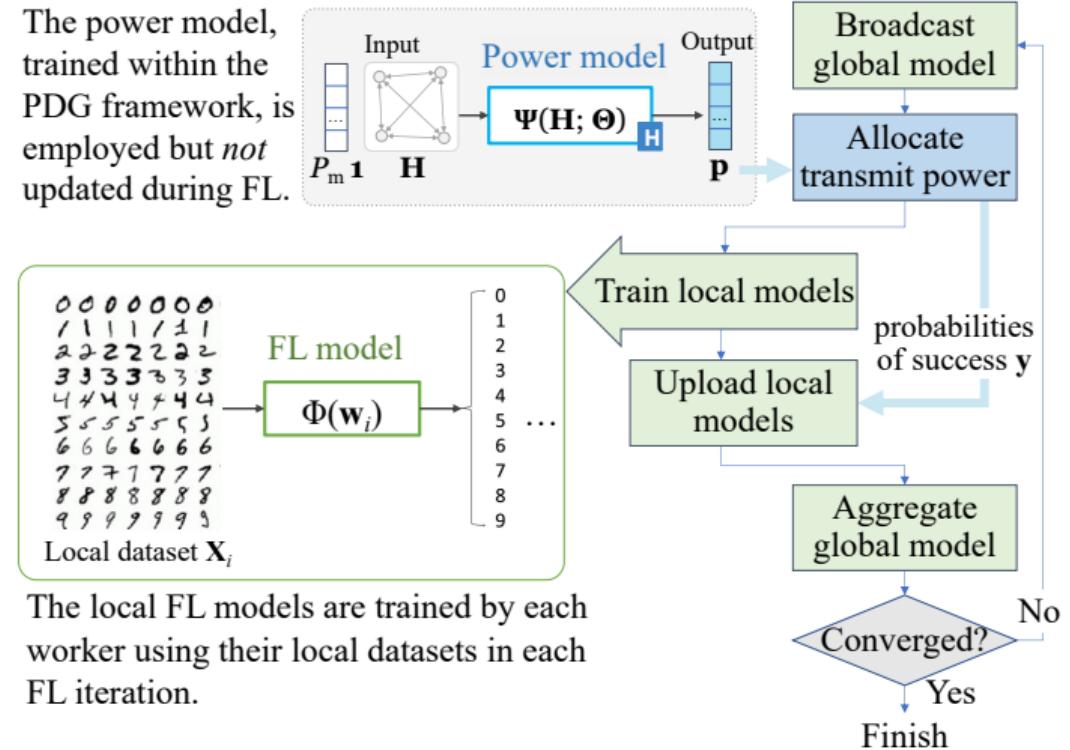
# Proposed solution (during FL)

FL system:



FL pipeline with power allocation policy:

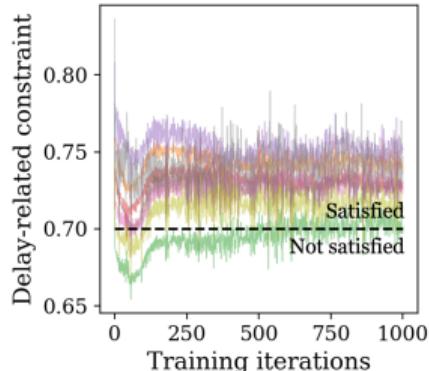
The power model, trained within the PDG framework, is employed but *not* updated during FL.



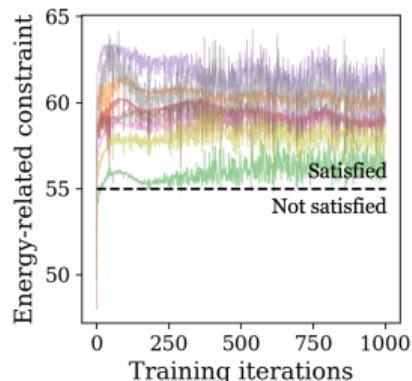
# PD learning curves for the power policy

Learning curves<sup>2</sup> of PDG demonstrate convergence to

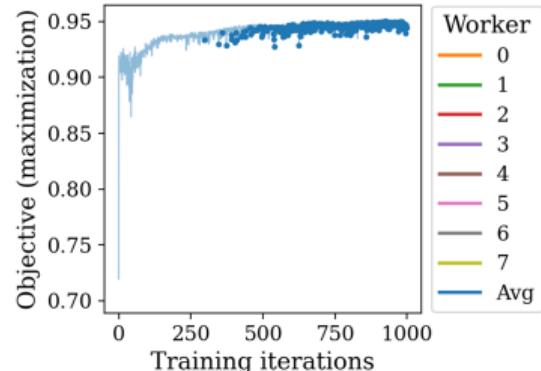
- (a) delay constraint,      (b) energy constraint, and      (c) objective PSR.



(a)



(b)



(c)

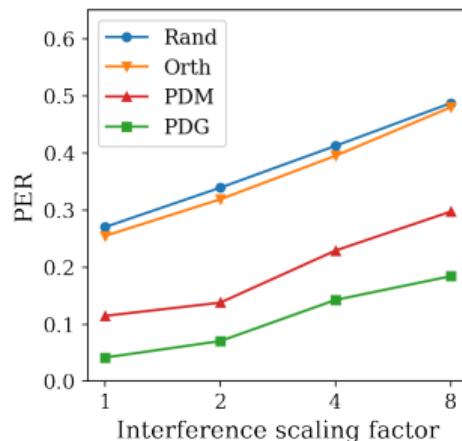
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<sup>2</sup>Constraint constants  $r_0$  and  $e_0$  are annotated as dashed lines in (a) and (b). Larger markers in (c) are where all workers satisfy both constraints.

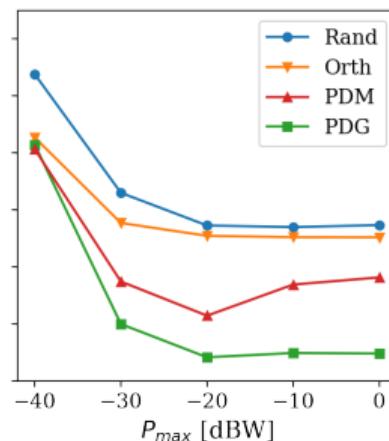
# Communication proxy

Performance comparison (**system-level transmission error rate**) of PDG to other power allocation methods under different network configs:

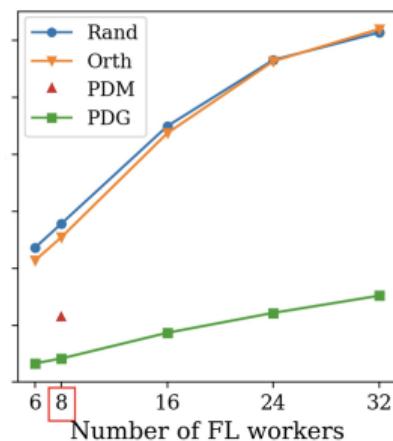
(a) Interference strength



(b) Max power value



(c) Network size



PDG ensures **more accurate transmissions** than the topology- agnostic learning-based PDM and other rule-based power methods.

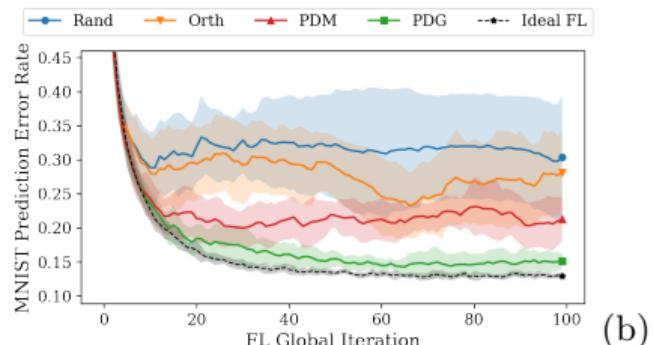
Orth: Chen *et al.*, TWC'20

# FL performance on I.I.D. data

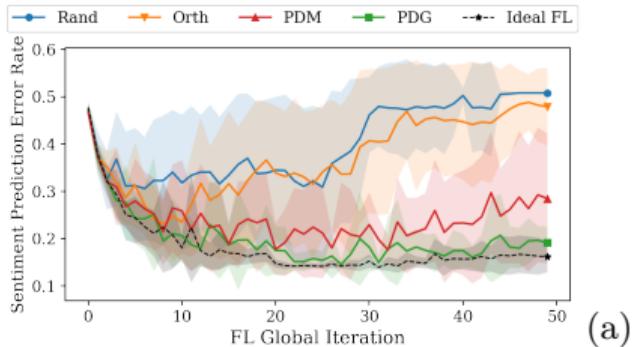
Tests on FL benckmark tasks: (a) NLP: IMDb sentiment classification, (b) MNIST digit classification, and (c) regresison: Air quality prediction.

Figures show **global FL** validation errors vs completed FL iterations.

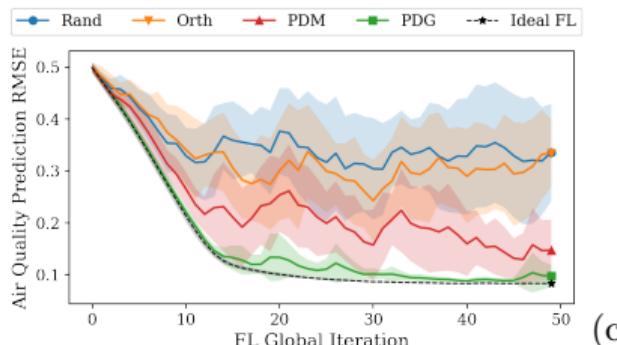
PDG consistently results in the **best FL performance**, close to ideal.



(b)



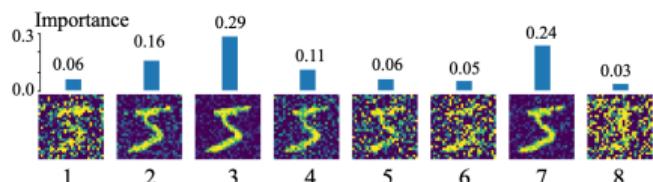
(a)



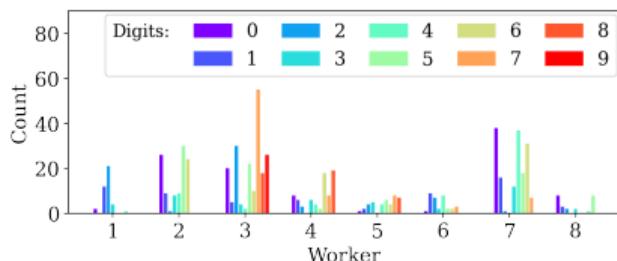
(c)

## Non-I.I.D. case

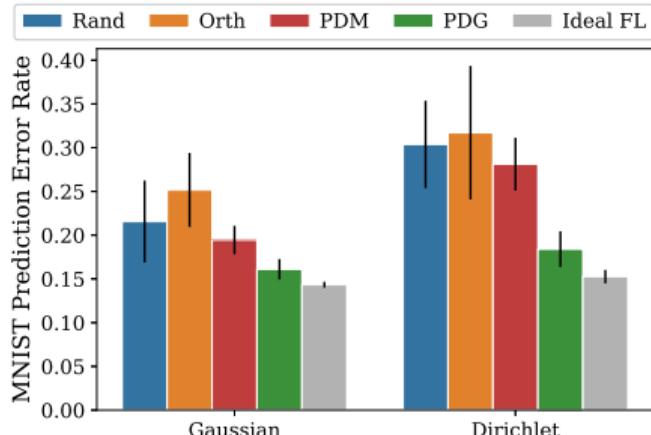
Local datasets may have different types, degrees, or patterns of random noise specific to the device or local environment.



(a) Heterogeneous AWGN. Bars denote corresponding worker weights that reflect local data quality.



(b) Skewed label histograms drawn from a Dirichlet distribution.

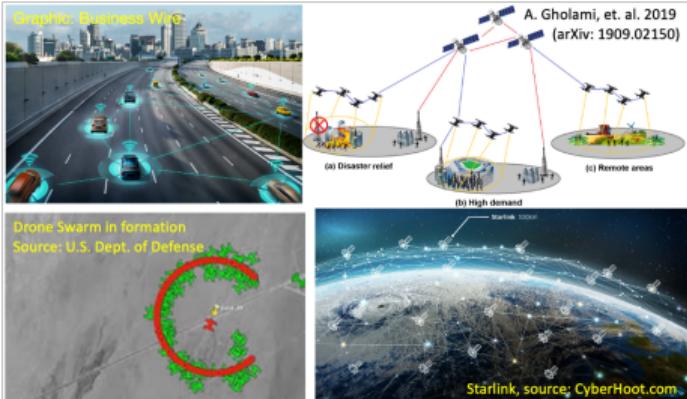
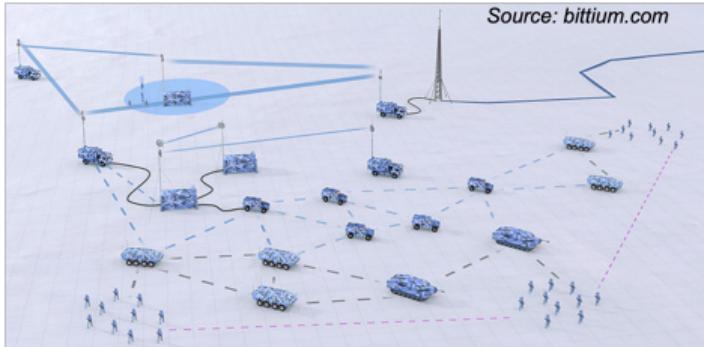


(c) Federated MNIST classification performance averaged across 5 random realizations. Results are shown for both Gaussian (noisy data) and Dirichlet (imbalanced labels) non-i.i.d. scenarios.

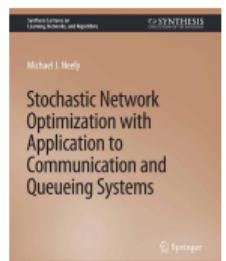
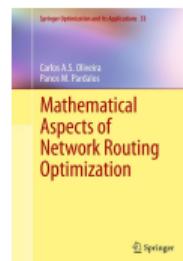
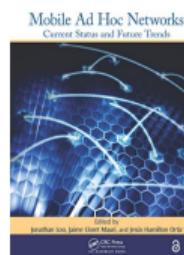
Boning Li *et al.* ICASSP'22, TWC'23

## Part III: Graph-based ML for Wireless Networking

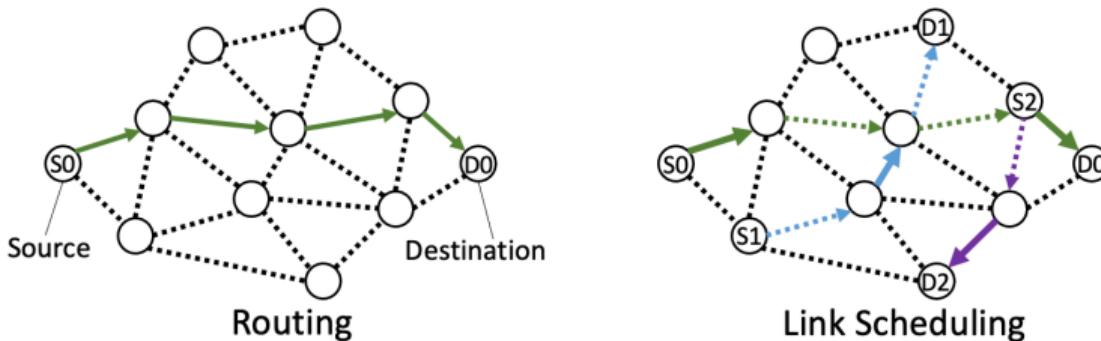
# Introduction: wireless multihop networks



- ▶ User devices self-organization
- ▶ Infrastructure-less communications
  - ⇒ Military and disaster relief
- ▶ Emerging applications
  - ⇒ wireless backhaul, satellite constellation
  - ⇒ Traffic offloading (D2D, IoT)

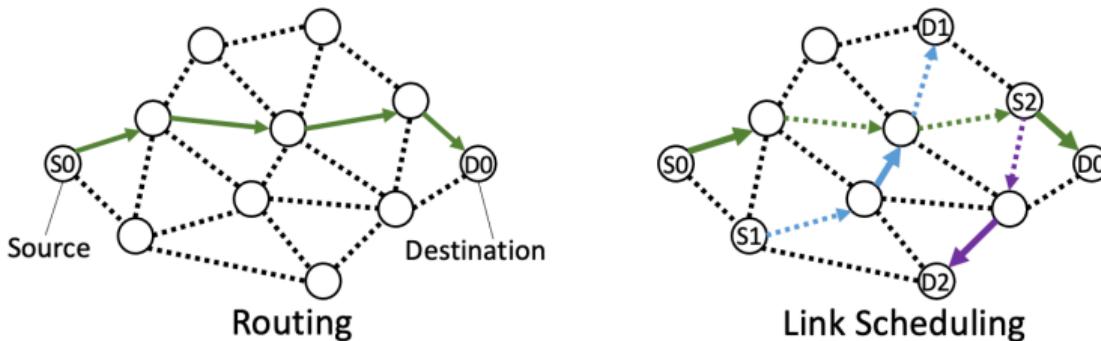


# Fundamental wireless networking tasks



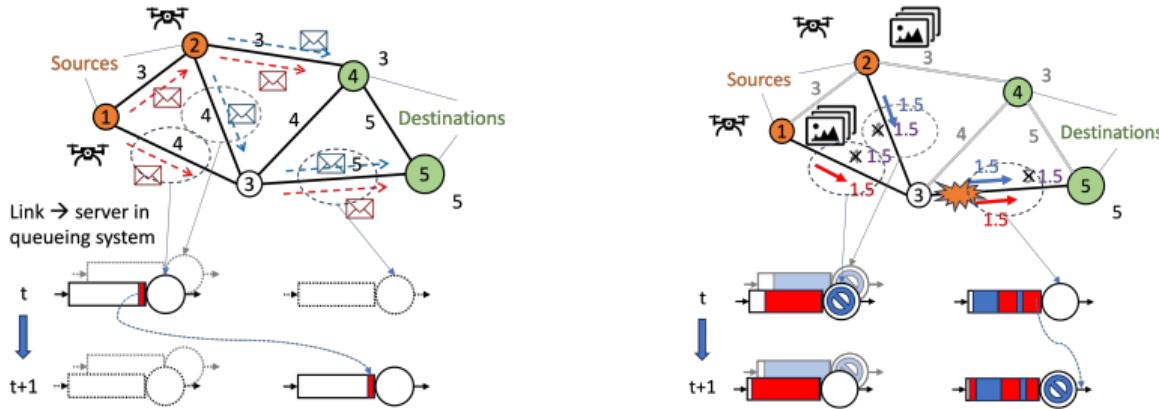
- ▶ **Routing:** send packets from source(s) to destination(s) through relay nodes
  - ⇒ Path finding: 1-to-1 (unicast), 1-to-many: multi-cast, broadcast
  - ⇒ Orchestration: cluster head election, virtual backbone establishment
- ▶ **Link scheduling:** decide which links to be activated in each time slot
  - ⇒ MaxWeight scheduling, carrier sensing multiple access (CSMA)
- ▶ **Combinatorial & discrete nature**
- ▶ **Distributed solutions** are preferred (our focus)

# Fundamental wireless networking tasks



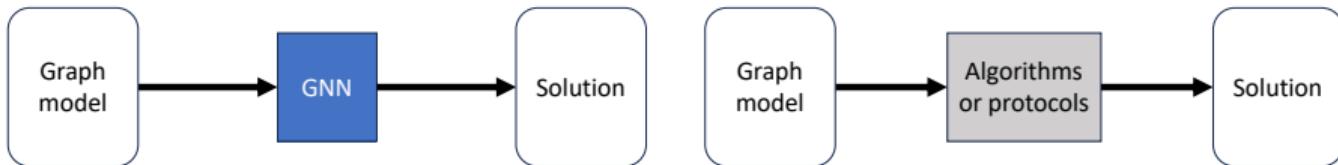
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  - ⇒ MaxWeight scheduling, carrier sensing multiple access (CSMA)
- ▶ **Combinatorial & discrete nature**
- ▶ **Distributed solutions** are preferred (our focus)
- ▶ **Performance analysis:** latency, jitter, throughput, packet drops ...

# Why wireless networking is so challenging?



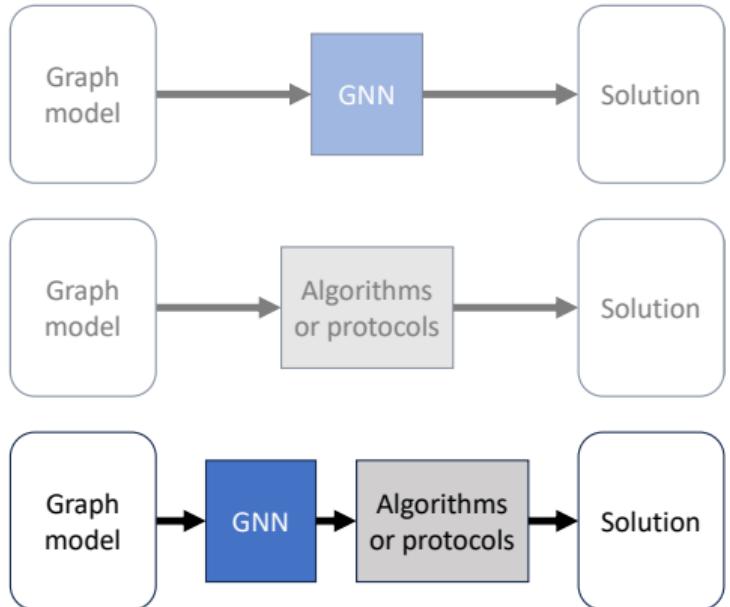
- ▶ Model: queueing networks subject to **conflict constraints** → no analytical model
- ▶ Instantaneous link rates fluctuate due to **channel fading**
- ▶ Changing network topology due to **mobility**
- ▶ Link capacity **coupled** with **routing & scheduling decisions** and input **flow rates**

# GNNs for networking: opportunities and challenges

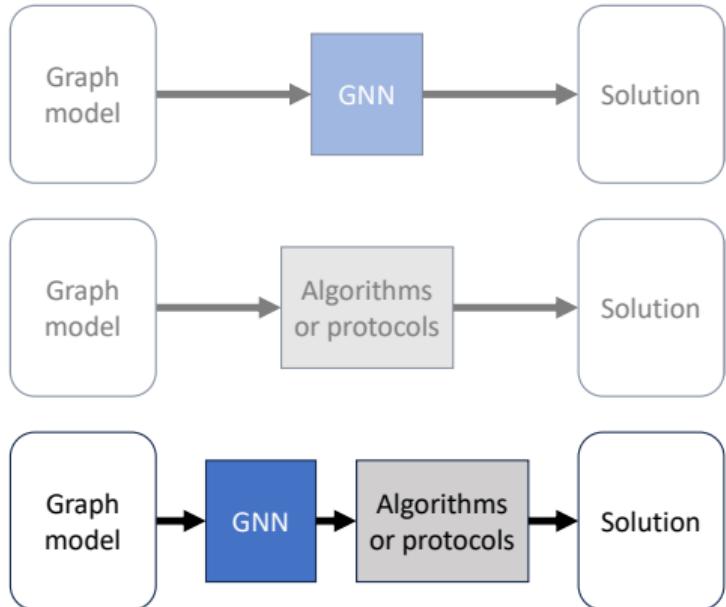


|   | GNNs          | Algorithms |
|---|---------------|------------|
| Transparency, interpretability explainability                                       | poor          | good       |
| Theoretical bound & guarantees  | poor          | good       |
| <b>Natural aspect of networks</b>   | opportunities | limited    |
| Representation of network structure   | yes           | limited    |
| Learning from graph-structured data   | yes           | no         |
| Inductive bias: permutation equivariance  | yes           | yes        |
| <b>Engineering aspect of networks</b>   | challenges    | easy       |
| Formulate networking tasks as link prediction, node classification, graph embedding | challenging   | —          |
| Domain knowledge: observe rules, constraints  | hard          | easy       |
| Discrete decision-making  | hard          | easy       |
| Data labeling for supervised learning   | hard          | —          |
| <b>Overall limitation</b>   | functionality | optimality |

## Part III Overview



## Part III Overview



Graph-based machine learning for

- ▶ Max-Weight link scheduling
- ▶ Repetitive combinatorial optimization
- ▶ Conflict-aware packet routing
- ▶ Rapid network simulation
- ▶ Summary & future applications

Let's start with a particular networking task

## Link Scheduling with Graph Neural Networks <sup>34</sup>

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<sup>3</sup>Z. Zhao, G. Verma, C. Rao, A. Swami and S. Segarra, "Distributed Scheduling Using Graph Neural Networks," IEEE ICASSP 2021, pp. 4720-4724

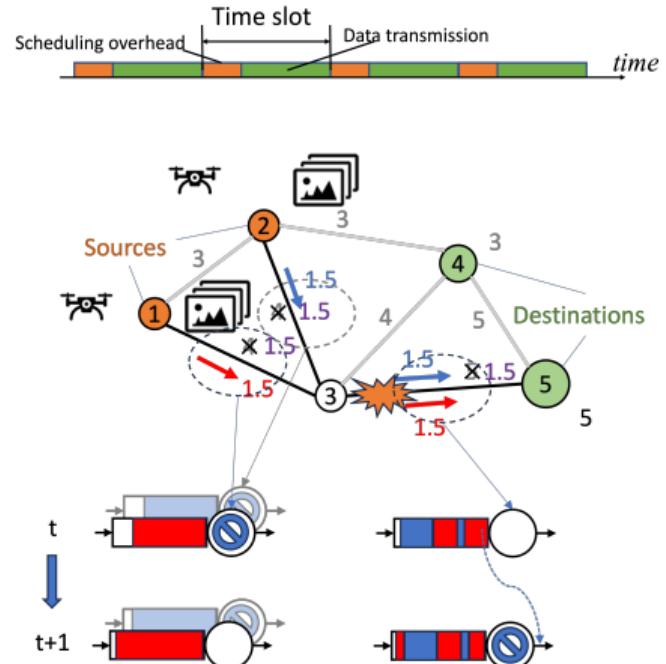
<sup>4</sup>Z. Zhao, G. Verma, C. Rao, A. Swami and S. Segarra, "Link Scheduling Using Graph Neural Networks," in IEEE Trans. on Wireless Comms., vol. 22, no. 6, pp. 3997-4012, June 2023

# Link scheduling

Decide **when** and **which** links to be activated

## Medium Access Control (MAC)

- ▶ Synchronized, time-slotted system
- ▶ Orthogonal multiple access
  - ▶ A resource block is exclusively assigned to an active link
  - ▶ In spatial, temporal, frequency, or code domains
- ▶ Bidirectional link → Undirected edge



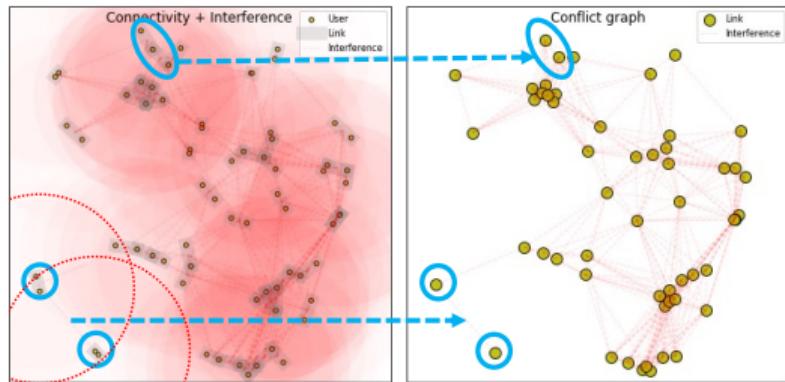
# Link scheduling: graph modeling

Conflict graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$

- ▶ Vertex  $v \in \mathcal{V} \rightarrow$  wireless link
- ▶ Edge  $e \in \mathcal{E} \rightarrow$  conflict relationship between two wireless links that
  - ⇒ share the same device (interface)
  - ⇒ interfere each other if both activated

Vertex weights  $\mathbf{u} \in \mathbb{R}_+^{|\mathcal{V}|} = [u(v)|v \in \mathcal{V}]$

- ▶  $u(v)$ : utility of activating wireless link  $v$



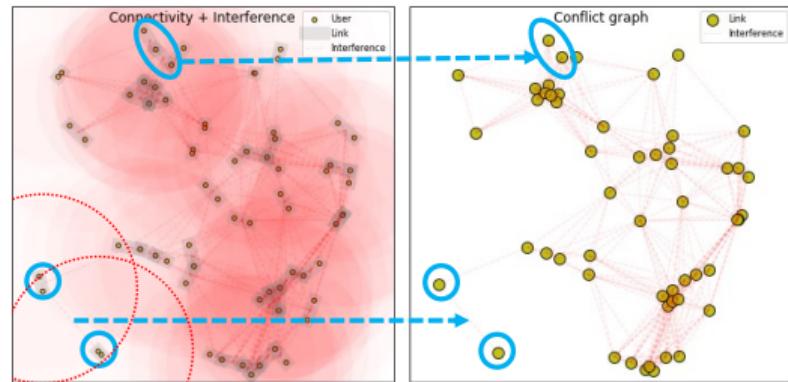
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- ▶  $u(v)$ : utility of activating wireless link  $v$



Utility function  $u : \mathcal{V} \rightarrow \mathbb{R}_+$

- ▶ E.g.,  $u(v) = q(v)l(v)$ ,  $u(v) = \min\{q(v), l(v)\}$  for throughput maximization
  - ⇒ Queue length  $q(v)$ , link rate  $l(v)$
  - ⇒ Vector form  $\mathbf{u} = \mathbf{q} \odot \mathbf{l}$ ,  $\mathbf{u}_v = u(v)$ ,  $\mathbf{q}_v = q(v)$ ,  $\mathbf{l}_v = l(v)$

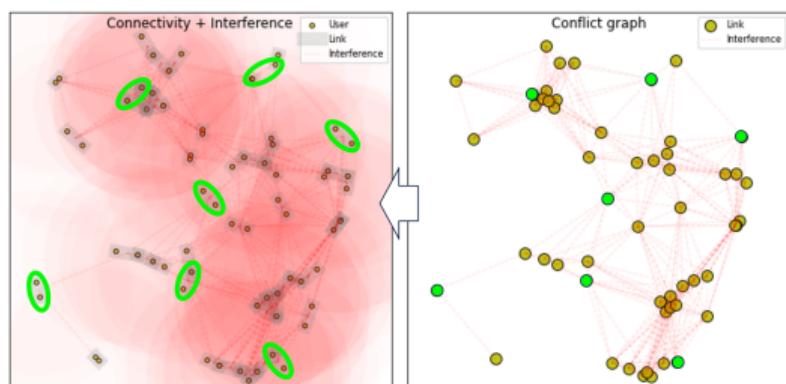
# Max-Weight scheduling: MWIS formulation

Maximum weighted independent set (MWIS)

Consider a **conflict graph**  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ , where  $\mathcal{V}$  and  $\mathcal{E}$  describe all the links and their conflict relationships in the wireless network, respectively, and a **utility function**  $u : \mathcal{V} \rightarrow \mathbb{R}_+$ . The optimal schedule is given by

$$\mathbf{v}^* = \underset{\mathbf{v} \subseteq \{0,1\}^{|\mathcal{V}|}}{\operatorname{argmax}} \quad \mathbf{v}^\top \mathbf{u} \quad (7a)$$

$$\text{s.t. } \mathbf{v}_i + \mathbf{v}_j \leq 1, \forall (i, j) \in \mathcal{E}. \quad (7b)$$



- ▶ MWIS problem is **NP-hard**
- ▶ Fast & distributed heuristics in practice

# Distributed local greedy solver (LGS), $\mathcal{O}(\log |\mathcal{V}|)$

LGS<sup>5</sup> denoted as function  $\hat{\mathbf{v}}_{Greedy} = h(\mathcal{G}, \mathbf{u})$ , inspired by Ruby's algorithm<sup>6</sup>

- ▶ All links initialized as undecided  $\mathbf{v} = -1$
- ▶ Link  $i$  is scheduled ( $\mathbf{v}_i = 1$ ) if its utility exceeds all neighbors

$$u(i) > \max_{j \in \mathcal{N}(i)} u(j)$$

- ▶ Link  $i$  is muted ( $\mathbf{v}_i = 0$ ) if one of its neighbors is scheduled
- ▶ Undecided nodes enter next iteration until all nodes are decided

## An illustrative example

Local greedy solver (LGS)



<sup>5</sup>C. Joo and N. B. Shroff, "Local greedy approximation for scheduling in multihop wireless networks," IEEE Trans. on Mobile Computing, vol. 11, no. 3, pp. 414–426, 2012

<sup>6</sup>M. Luby. "A simple parallel algorithm for the maximal independent set problem." SIAM journal on computing 15.4 (1986): 1036-1053.

# Why not just let a GNN directly output solution?

Graph neural networks (GNNs)

- ▶ **Distributed & fast** execution
- ▶ Generalize to different topologies
- ▶ Unable to encode **relational constraints** in COPs, e.g.,  $\mathbf{v}_i + \mathbf{v}_j \leq 1 , \forall (v_i, v_j) \in \mathcal{E}$ .

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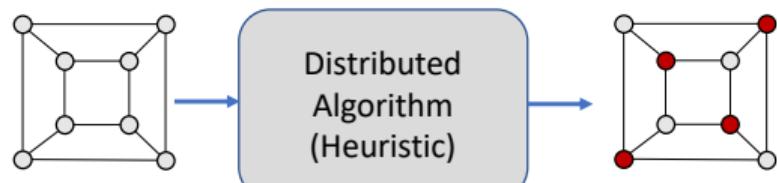
Graph convolutional layer (local form)

$$\mathbf{x}_{e^*}^l = \sigma_l \left( \mathbf{x}_{e^*}^{l-1} \Theta_0^l + \left[ \mathbf{x}_{e^*}^{l-1} - \sum_{u \in \mathcal{N}_{\mathcal{G}^c}(e)} \frac{\mathbf{x}_{u^*}^{l-1}}{\sqrt{d(e)d(u)}} \right] \Theta_1^l \right)$$



Identical input → identical prediction

Constraint violations 

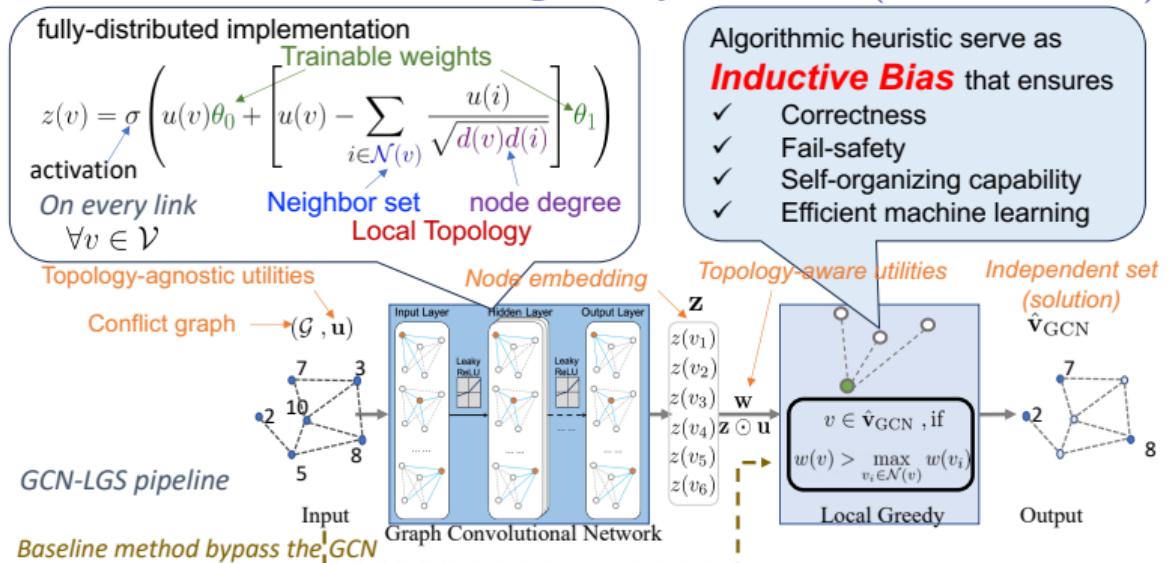


Tie-breaking rules



**Example:** MWIS problem on a regular graph, where every node has identical weight

# GCN-enhanced local greedy solver (GCN-LGS)



Function notations:

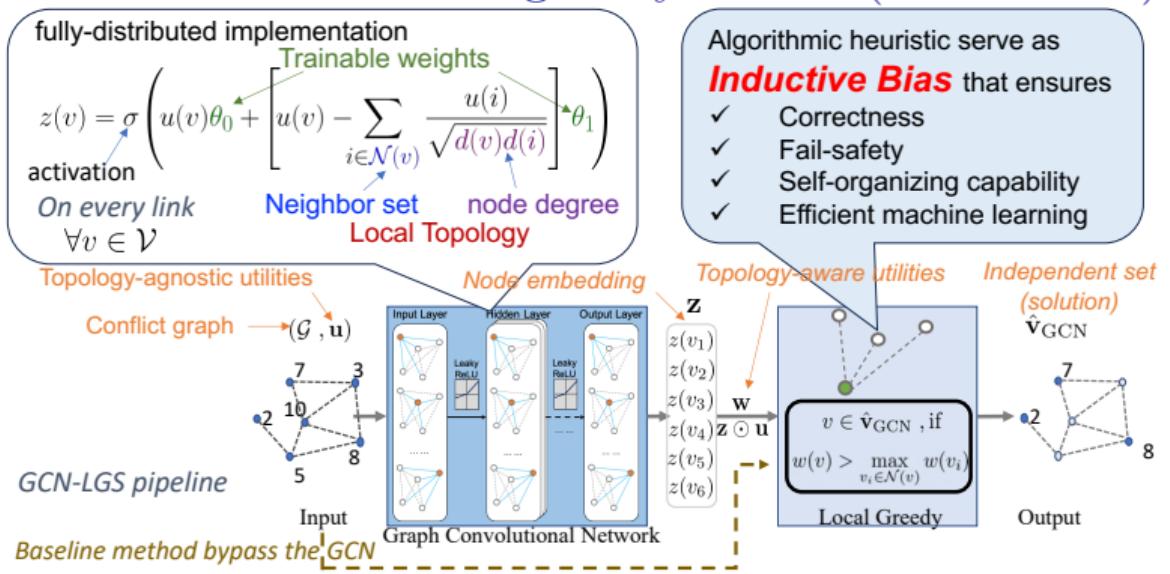
LGS:  $\hat{\mathbf{v}}_{\text{Greedy}} = h(\mathcal{G}, \mathbf{u})$

GCN-LGS:  $\hat{\mathbf{v}} = h(\mathcal{G}, \mathbf{z} \odot \mathbf{u})$

GCN:  $\mathbf{z} = \Psi_{\mathcal{G}}(\mathbf{S}; \omega)$ ,

$\omega$ : trainable parameters

# GCN-enhanced local greedy solver (GCN-LGS)

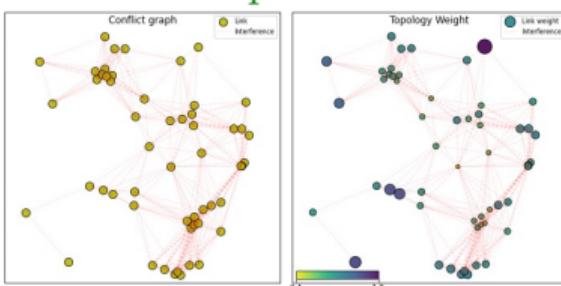


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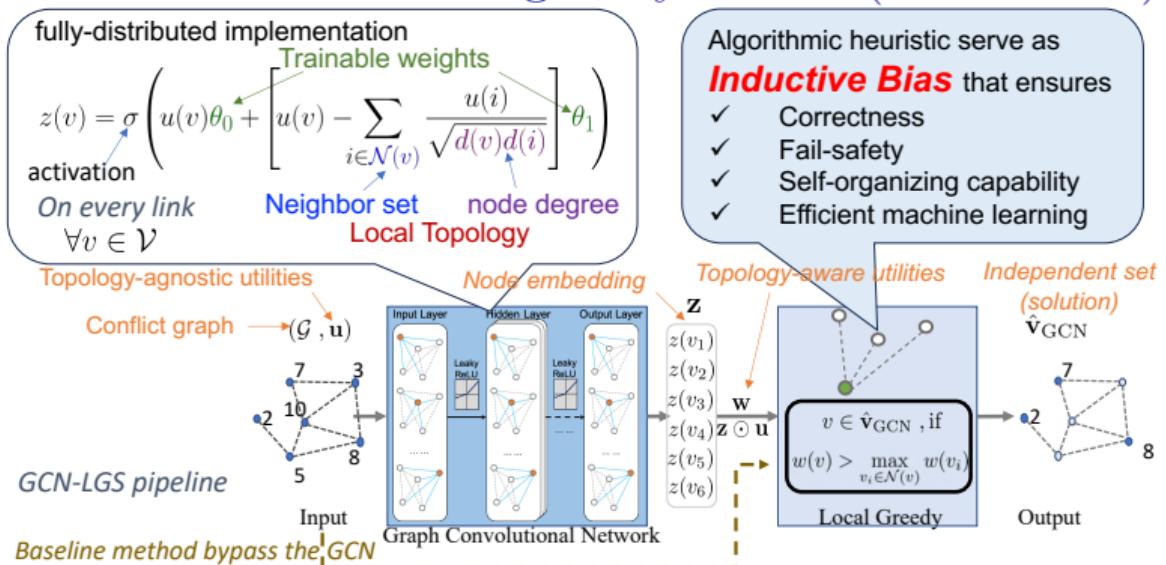
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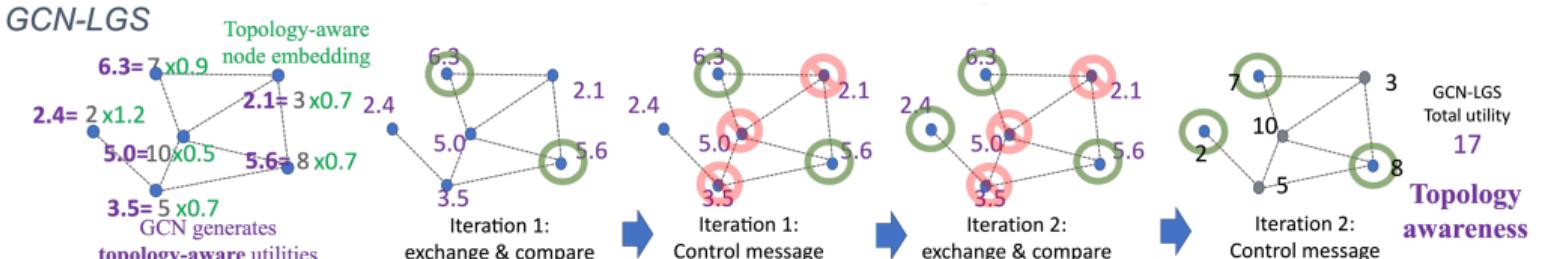
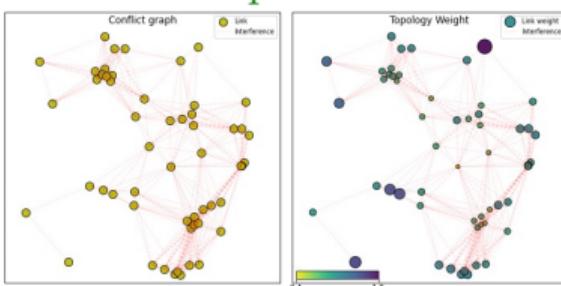


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## Why reinforcement learning?

- ▶ Avoid **data labeling** –  
MWIS problem is  
NP-hard
- ▶ Treats **non-differentiable**  
LGS as part of the  
environment

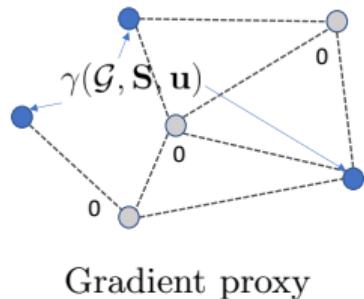
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<sup>7</sup>Z. Zhao, G. Verma, C. Rao, A. Swami and S. Segarra, "Link Scheduling Using Graph Neural Networks," in IEEE Trans. on Wireless Comms., vol. 22, no. 6, pp. 3997-4012, June 2023

# GCN Training: customized deterministic policy gradient<sup>7</sup>

Why reinforcement learning?

- ▶ Avoid **data labeling** – MWIS problem is NP-hard
- ▶ Treats **non-differentiable** LGS as part of the environment



$$\omega^* = \operatorname{argmax}_{\omega} J(\omega) \quad (8a)$$

$$\text{s.t. } J(\omega) = \mathbb{E}_{(\mathcal{G}, \mathbf{S}, \mathbf{u}) \sim \Omega} [\gamma(\mathcal{G}, \mathbf{u}, \mathbf{z})] , \quad (8b)$$

$$\gamma(\mathcal{G}, \mathbf{u}, \mathbf{z}) = \frac{\hat{\mathbf{v}}^\top \mathbf{u}}{\hat{\mathbf{v}}_{Greedy}^\top \mathbf{u}} , \quad (8c)$$

$$\hat{\mathbf{v}}_{Greedy} = h(\mathcal{G}, \mathbf{u}) , \quad (8d)$$

$$\hat{\mathbf{v}} = h(\mathcal{G}, \mathbf{z} \odot \mathbf{u}) , \quad (8e)$$

$$\mathbf{z} = \Psi_{\mathcal{G}}(\mathbf{S}; \omega) . \quad (8f)$$

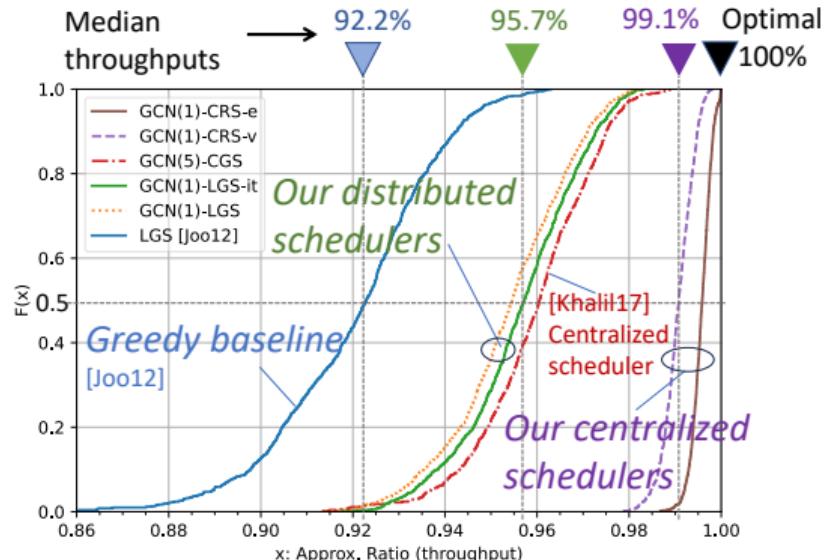
- ▶ Draw random network state  $(\mathcal{G}, \mathbf{S}, \mathbf{u}) \sim \Omega$
- ▶  $\widehat{\nabla J(\omega)} = \gamma(\mathcal{G}, \mathbf{u}, \mathbf{z}) \nabla \Psi_{\mathcal{G}}(\mathbf{S}; \omega) \hat{\mathbf{v}}$ , gradient
- ▶ Weight update  $\omega = \omega + \alpha \widehat{\nabla J(\omega)}$ , learning rate

<sup>7</sup>Z. Zhao, G. Verma, C. Rao, A. Swami and S. Segarra, "Link Scheduling Using Graph Neural Networks," in IEEE Trans. on Wireless Comms., vol. 22, no. 6, pp. 3997-4012, June 2023

# Numerical results: throughput maximization<sup>8</sup>

## Key Takeaways for GCN-LGS

- ▶ Close nearly half optimality gap
- ▶ Reusable GCN model
  - ⇒ Inner loop of LGS: GCN-LGS-it
  - ⇒ Centralized rollout search (CRS)
- ▶ Lightweight GCN: 2 trainable weights
- ▶ Failsafe: can fallback to vanilla LGS for basic functionality if GCN went crazy
- ▶ Low complexity:  $\mathcal{O}(\log |\mathcal{V}|)$



- ▶ 100 nodes, 40 ~ 60 links
- ▶ Utility function  $u(v) = \min [r(v), q(v)]$
- ▶ Flooding traffic
- ▶ 100 graphs × 10 instances × 200 time steps

<sup>8</sup>Z. Zhao, G. Verma, C. Rao, A. Swami and S. Segarra, "Link Scheduling Using Graph Neural Networks," in IEEE Trans. on Wireless Comms., vol. 22, no. 6, pp. 3997-4012, June 2023

How to generalize GCN-LGS to broader networking tasks?

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<sup>9</sup>Z. Zhao, A. Swami, S. Segarra, "Graph-based Deterministic Policy Gradient for Repetitive Combinatorial Optimization Problems," ICLR 2023

How to generalize GCN-LGS to broader networking tasks?

# Graph-based deterministic policy gradient (GDPG-Twin) for repetitive combinatorial optimization problems (R-COPs)<sup>9</sup>

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<sup>9</sup>Z. Zhao, A. Swami, S. Segarra, "Graph-based Deterministic Policy Gradient for Repetitive Combinatorial Optimization Problems," ICLR 2023

# Combinatorial Optimization Problem (COP)

Typical formulation

$$\mathbf{x}^* = \arg \min_{\mathbf{x}} \mathbf{c}^\top \mathbf{x} \quad \text{COP}$$

s.t. *Discrete constraint on nodes or edges*  
*Constraints defined on Graph,  
Hypergraph, or Simplicial Complex*

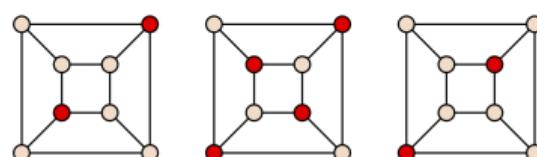
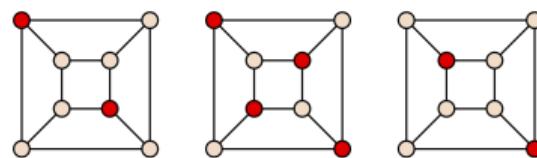
Characters

- ▶ Input: a graph with cost vector  $\mathbf{c}$
- ▶ Decision variables  $\mathbf{x}$ 
  - ⇒ Discrete (integer) constraints
  - ⇒ Relational constraints
- ▶ Minimize total cost
- ▶ Non-convex, often **NP-hard!**

Maximum Weighted Independent Set

$$\mathbf{v}^* = \operatorname{argmax}_{\mathbf{v} \subseteq \{0,1\}^{|\mathcal{V}|}} \mathbf{u}^\top \mathbf{v} \quad (9a)$$

$$\text{s.t. } \mathbf{v}_i + \mathbf{v}_j \leq 1, \forall (i,j) \in \mathcal{E}. \quad (9b)$$



Source: Wikipedia – Maximal independent set

# Repetitive Combinatorial Optimization Problem (R-COP)



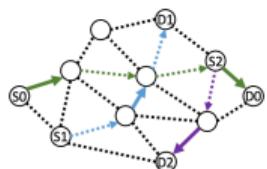
- ▶ Graph-based Markov decision process
  - ⇒ Network state as a weighted graph  $(\mathcal{V}(t), \mathcal{E}(t), \mathbf{c}(t))$
  - ⇒ Network state of  $t + 1$  depends on decisions  $\mathbf{x}(t)$
  - ⇒ Decision  $\mathbf{x}(t)$  found by solving a COP on  $(\mathcal{V}(t), \mathcal{E}(t), \mathbf{c}(t))$
  - ⇒ Cost vector  $\mathbf{c}(t)$  changes rapidly compared to topology  $(\mathcal{V}(t), \mathcal{E}(t))$

# Repetitive Combinatorial Optimization Problem (R-COP)



- ▶ Graph-based Markov decision process
  - ⇒ Network state as a weighted graph  $(\mathcal{V}(t), \mathcal{E}(t), \mathbf{c}(t))$
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  - ⇒ Cost vector  $\mathbf{c}(t)$  changes rapidly compared to topology  $(\mathcal{V}(t), \mathcal{E}(t))$
- ▶ Many applications

Routing & Scheduling in  
communication networks



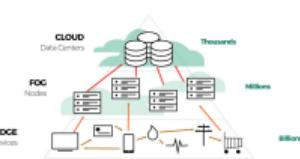
Multi-object tracking in  
computer vision



Vehicle routing problems in  
distribution networks

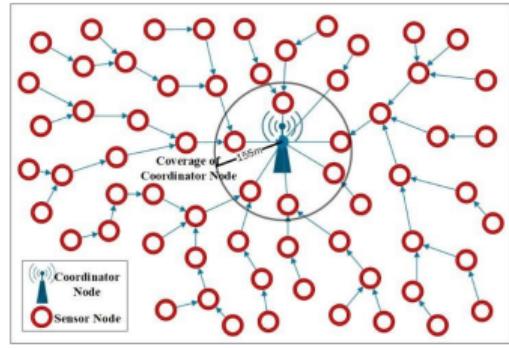
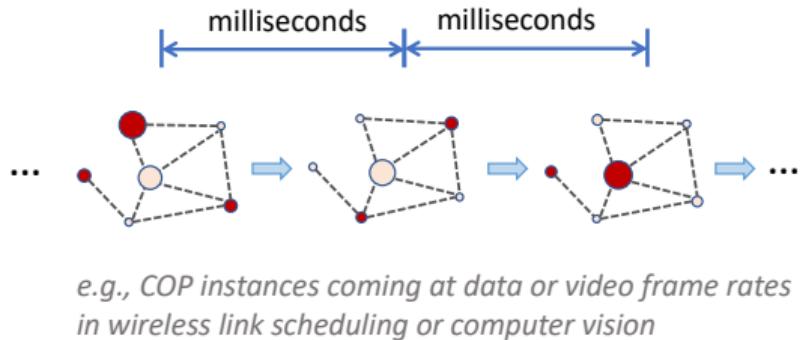


Resource allocation & job scheduling in  
cloud, fog, edge computing



- ▶ Practical restrictions: **limited runtime** and/or **distributed execution**

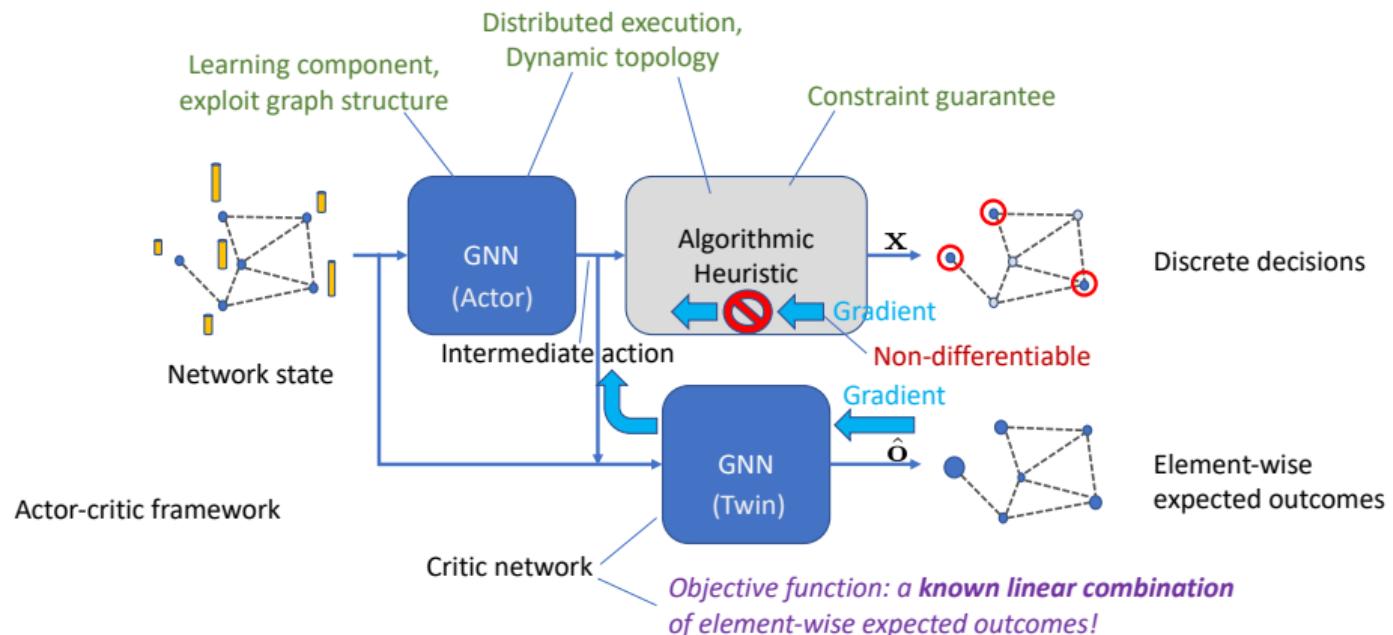
# Restrictions on runtime and distributed execution



Source: (D. Ari , M. Çibuk and F. Ağgün , 2017)

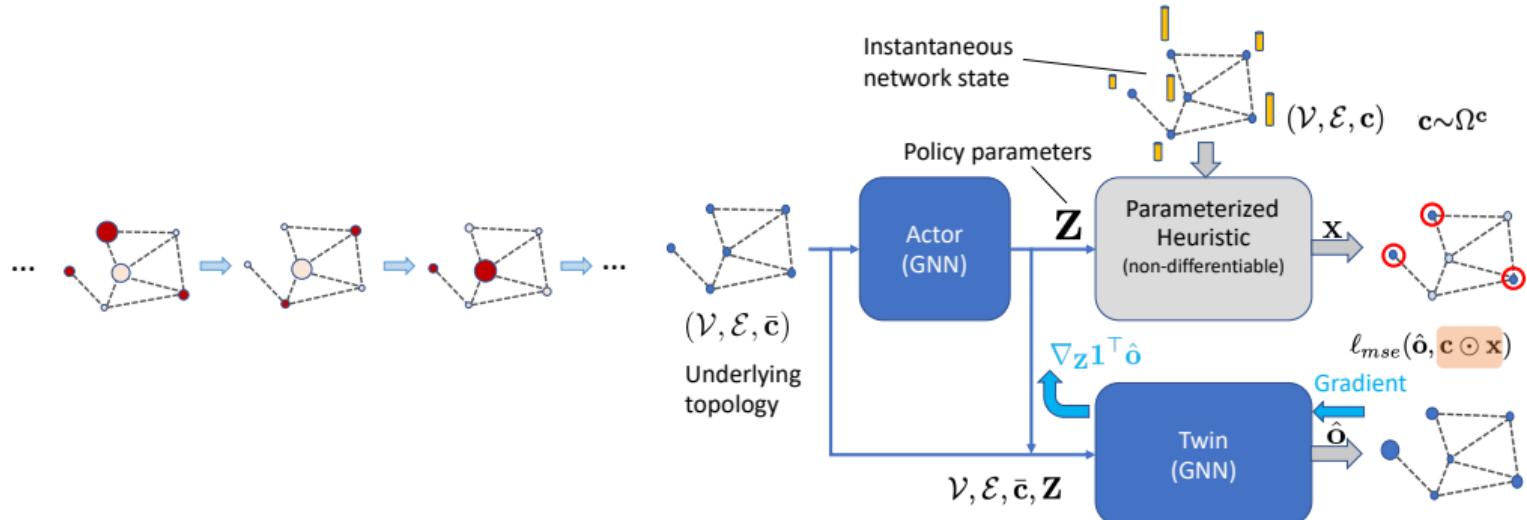
- ▶ Centralized COP solver
  - ⇒ High communication overhead to collect full network state to a server
  - ⇒ High computational complexity, scale up quickly by network size
  - ⇒ Single point of failure
- ▶ Distributed COP solver for **scalability** and **robustness**
  - ⇒ Fast & robust execution using only neighborhood information (exchange)

# GDPG-Twin: a general actor-critic framework for R-COP



- ▶ Actor GNN exploits graph structure
- ▶ Algorithmic heuristic guarantee correctness (relational constraints)
- ▶ Twin GNN bridges the non-differentiability gap of algorithmic heuristic

# Independent R-COP



- ▶ Goal: reduce **optimality gap** with minimal overhead
  - ⇒ Optimize each COP instance individually, **ignore inter-state dependency**
- ▶ GNN encodes the underlying topology, embeddings reused for many time steps
- ▶ Expected element-wise outcome  $\hat{\mathbf{o}} \approx \mathbf{o} = \mathbb{E}(\mathbf{c} \odot \mathbf{x})$
- ▶ Gradient on intermediate action  $\nabla_{\mathbf{Z}} \mathbf{1}^\top \hat{\mathbf{o}} \approx \nabla_{\mathbf{Z}} \mathbb{E}(\mathbf{c}^\top \mathbf{x})$

# Independent R-MWIS

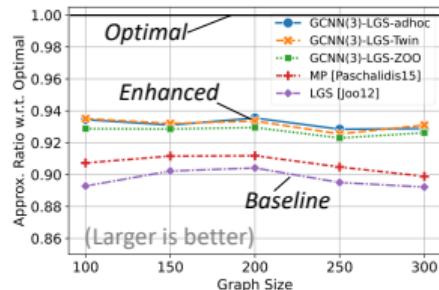


Figure 1: Approximation ratios (Larger is better) of the vanilla and GCNN-enhanced distributed heuristics for MWIS problem (max), w.r.t. the optimal solver.

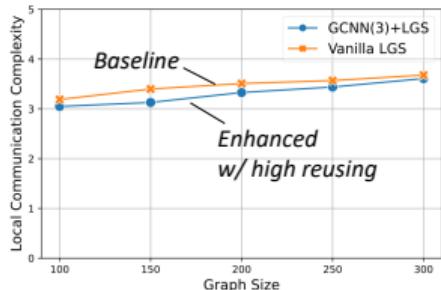


Figure 3: Average local communication complexity of GCNN-enhanced and vanilla LGS-MWIS solvers per instance, in rounds, excluding the GCNN ( $N = \infty$ ).

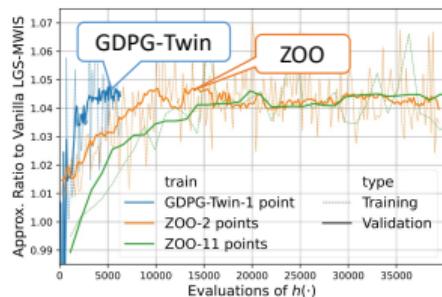


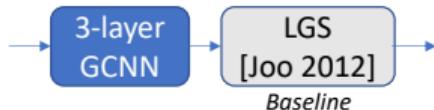
Figure 8: Performance trajectories of GCNN-enhanced LGS-MWIS trained by GPGD-Twin and ZOOs with 2-point and 11-point gradient estimations. Larger is better. GPGD-Twin needs fewer evaluations of  $h(\cdot)$ .

## Approximation ratio

## Execution local complexity

## Training complexity

Benchmark: ZOO (zeroth-order optimization)



- ▶ Tested on 500 random graphs from Erdős–Rényi model
- ▶ Baseline: LGS<sup>10</sup>, Benchmark: Zeroth-order optimization (ZOO)

<sup>10</sup>C. Joo and N. B. Shroff, "Local Greedy Approximation for Scheduling in Multihop Wireless Networks," in IEEE Trans. on Mobile Computing, vol. 11, no. 3, pp. 414-426, March 2012.

# Generalize to more Independent R-COPs

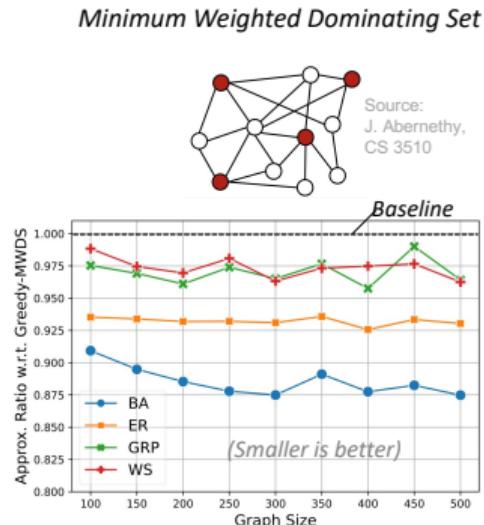


Figure 2: Approximation ratio (Smaller is better) of the GCNN-enhanced w.r.t. the vanilla Greedy-MWDS for MWDS problem (min) on 4 sets of random graphs.

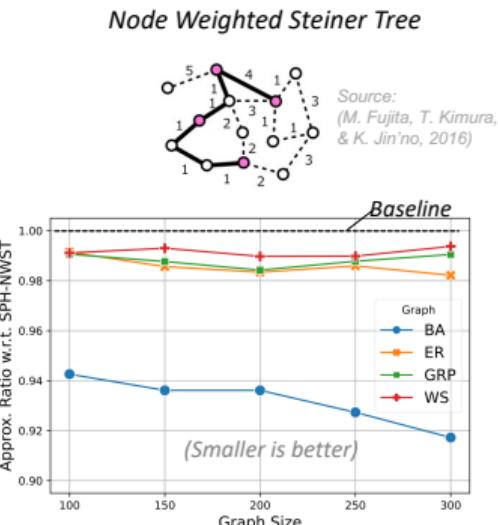
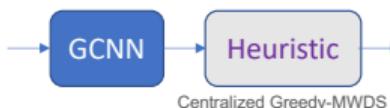


Figure 5: Approximation ratio (Smaller is better) of the GCNN-enhanced w.r.t. vanilla K-SPH-NWST for NWST problem on 4 sets of random graphs. NWST is a minimization (min) problem.

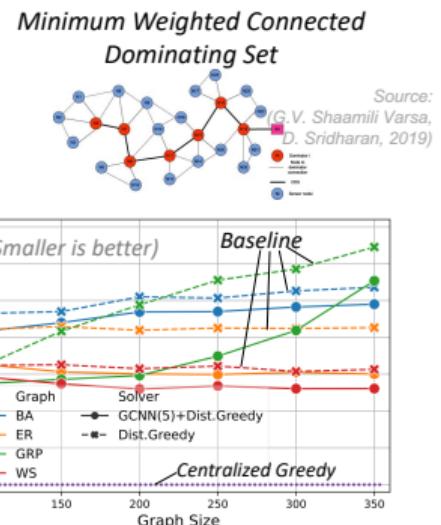
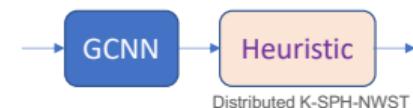
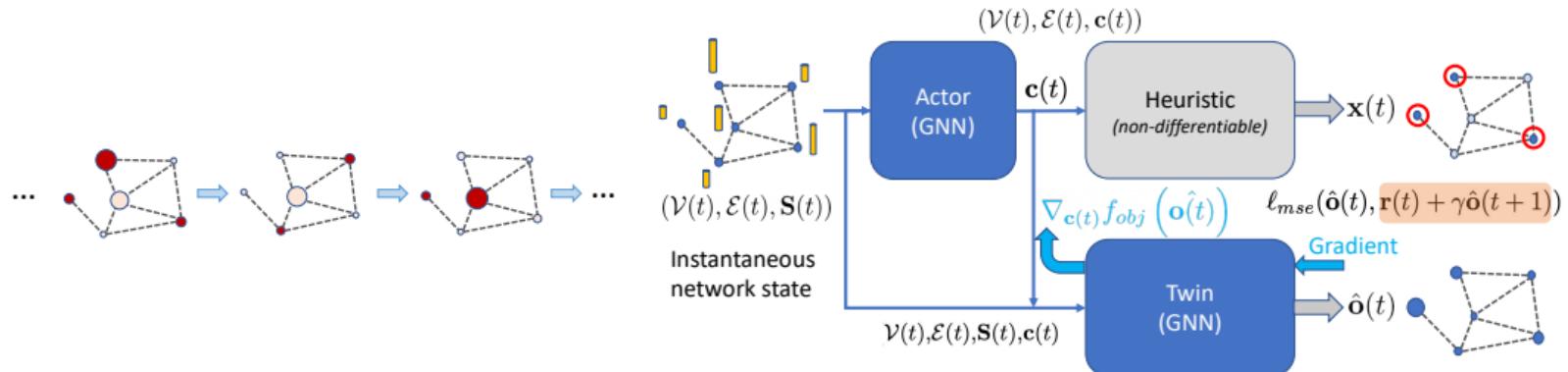


Figure 6: Approximation ratios (Smaller is better) of the vanilla and GCNN-enhanced distributed heuristics w.r.t. a centralized heuristic for MWCDS problem on 4 sets of random graphs. MWCDS is a min. problem.

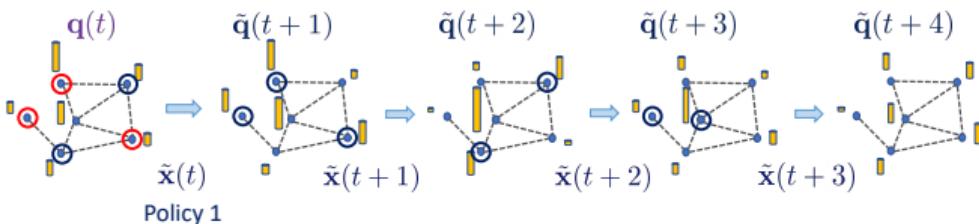


# R-COP in graph-based Markov decision process



- ▶ Goal: optimize **long-term** system-level objective  
⇒ Inter-state dependency MUST be considered
- ▶ GNN encodes network state  $(\mathcal{V}(t), \mathcal{E}(t), \mathbf{S}(t))$  into cost vector  $\mathbf{c}(t)$  in each time step  
⇒ Consider future element-wise rewards
- ▶ Expected element-wise outcome  $\hat{\mathbf{o}}(t) \approx \mathbf{o}(t) = \mathbb{E} [\mathbf{r}(t) + \gamma \hat{\mathbf{o}}(t+1)]$
- ▶ Gradient on intermediate action  $\nabla_{\mathbf{c}(t)} f_{obj} (\hat{\mathbf{o}}(t))$ ,  $f_{obj}$  is a linear function

# Delay-oriented link scheduling



$$\omega^* = \underset{\omega}{\operatorname{argmax}} \mathbb{E}_{i \in \mathcal{V}, t \leq T} [\mathbf{q}_i(t)] \quad (10a)$$

$$\text{s.t. } \mathbf{q}(t+1) = \mathbf{q}(t) + \mathbf{a}(t) - \mathbf{x}(t) \odot \min(\mathbf{l}(t), \mathbf{q}(t)), \quad (10b)$$

$$\mathbf{x}(t) = h(\mathcal{V}, \mathcal{E}, \mathbf{q}(t), \mathbf{l}(t), \mathbf{a}(t); \omega). \quad (10c)$$

$\mathbf{l}(t)$  link rates,  $\mathbf{q}(t)$  queue lengths,  $\mathbf{a}(t)$  new packet arrivals

- ▶ The ML pipeline is supposed to improve delay on centralized networks
- ▶ GDPG-Twin can do the same job as *ad-hoc RL scheme*<sup>a</sup> at  $\frac{1}{5}$  computational cost

<sup>a</sup>Z. Zhao, G. Verma, A. Swami and S. Segarra, "Delay-Oriented Distributed Scheduling Using Graph Neural Networks," IEEE ICASSP 2022, pp. 8902-8906

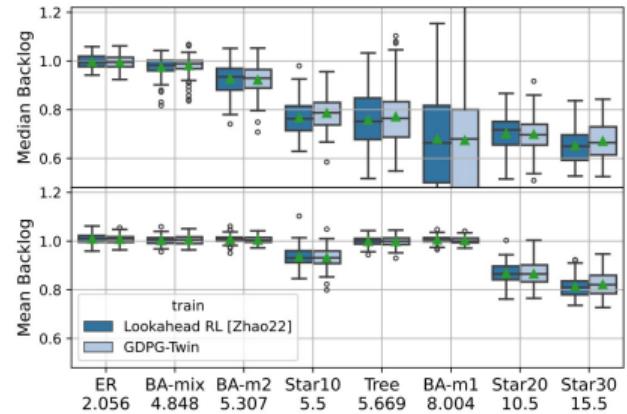
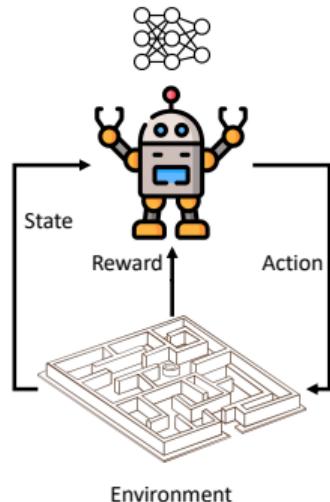


Figure 7: GDPG-Twin achieves similar network-wide mean and medium backlogs (smaller is better) of lookahead RL (Zhao et al., 2022b) in training a distributed link scheduler, using only  $\frac{1}{5}$  evaluations of  $h(\cdot)$  of it.

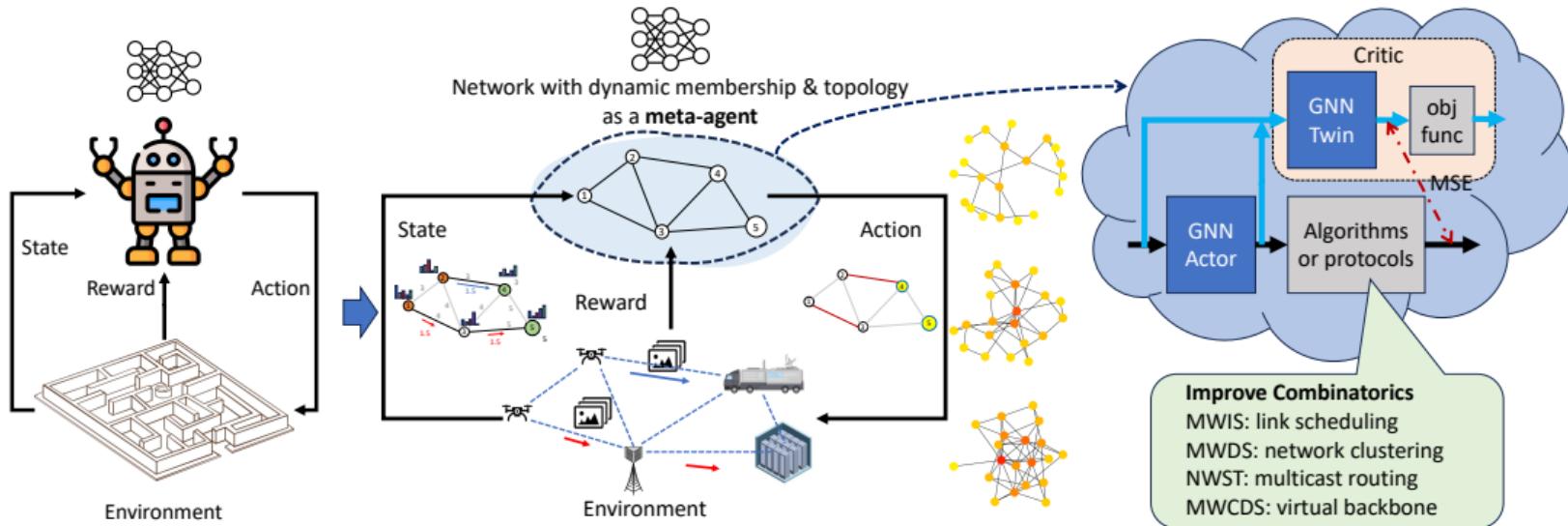
# Recap on GPG-Twin for R-COPs

- ▶ Single-agent reinforcement learning for: **scalar** action & reward, state in **regular** domain

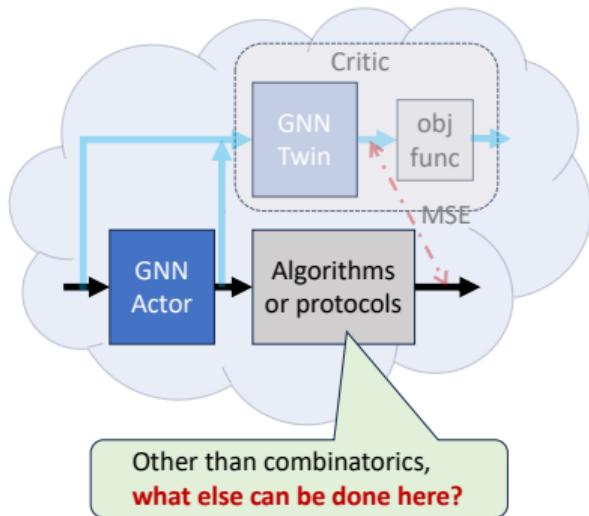


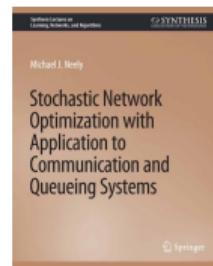
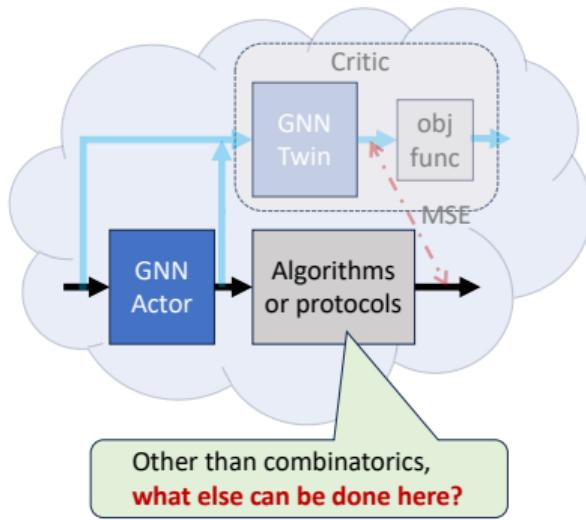
# Recap on GDPG-Twin for R-COPs

- ▶ Single-agent reinforcement learning for: **scalar** action & reward, state in **regular** domain



- ▶ GPG-Twin as a general reinforcement learning framework for distributed networks
  - ⇒ High-dimensional **parallel** action, reward, & state in **irregular** (graph) domain
  - ⇒ Generalize to dynamic graphs thanks to shared core model in GNN
  - ⇒ Follow engineered rules, leveraging domain knowledge





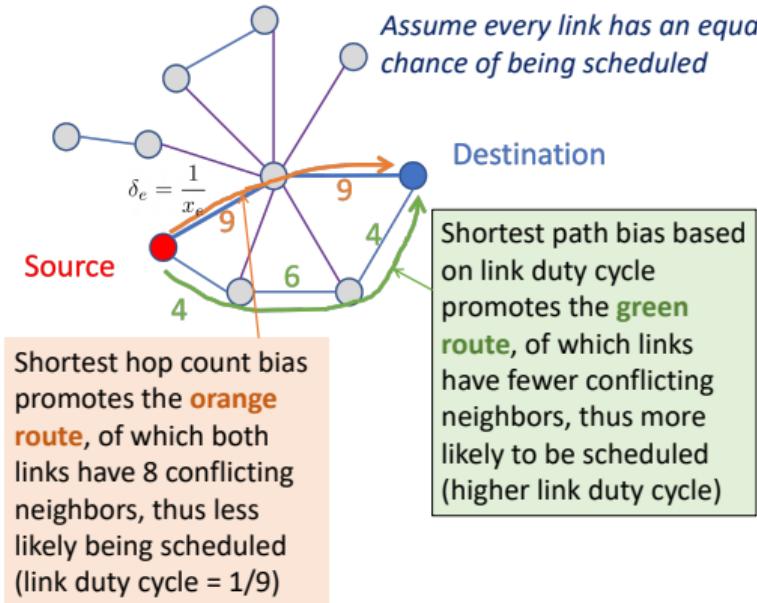
## Encode Network Context into Backpressure Routing<sup>abc</sup>

<sup>a</sup>Z. Zhao, B. Radojicic, G. Verma, A. Swami and S. Segarra, "Delay-Aware Backpressure Routing Using Graph Neural Networks," IEEE ICASSP 2023, pp. 1-5

<sup>b</sup>Z. Zhao, G. Verma, A. Swami and S. Segarra, "Enhanced Backpressure Routing Using Wireless Link Features," IEEE CAMSAP, 2023, pp. 271-275

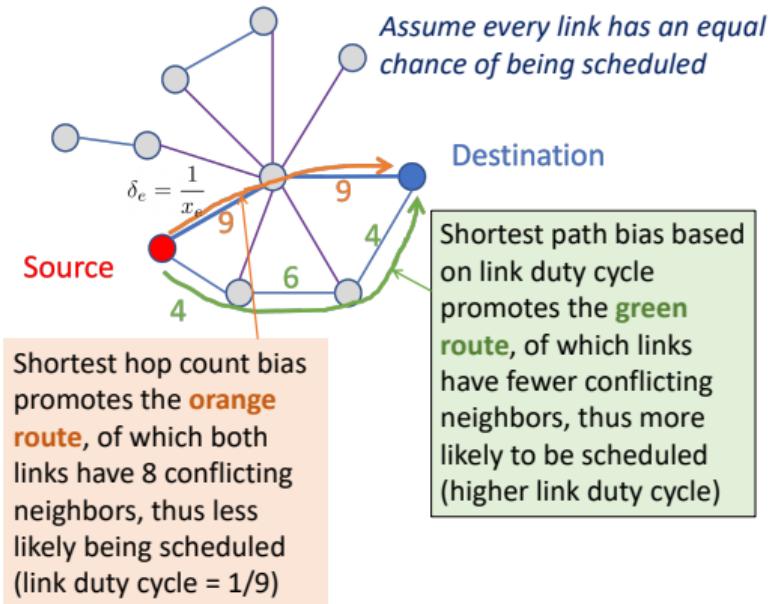
<sup>c</sup>Z. Zhao, B. Radojićić, G. Verma, A. Swami, S. Segarra, Biased Backpressure Routing Using Link Features and Graph Neural Networks, submitted to IEEE Trans. on Machine Learning In Comms. and Netw.

# From hop distance to conflict-aware shortest path



**Insight:** In wireless networks, links should not be treated equally since they introduce different latencies depending on local conflict topology.

# From hop distance to conflict-aware shortest path



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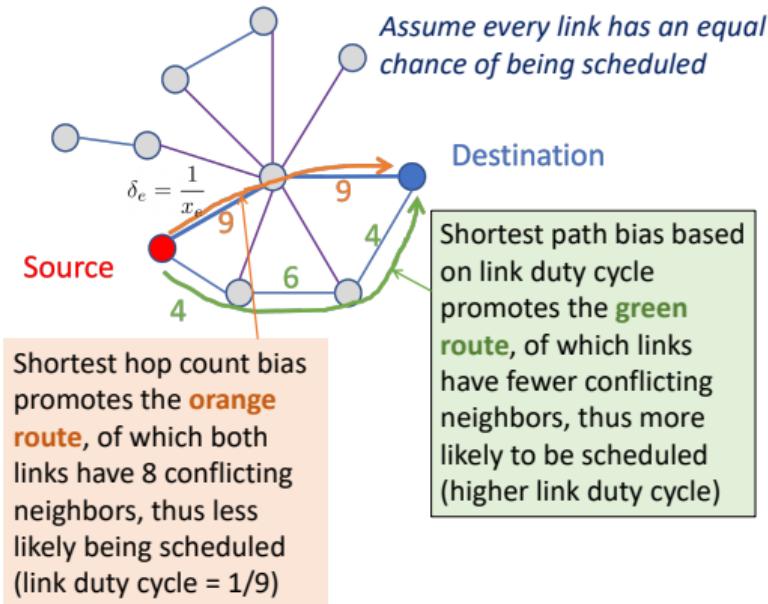
## Graph modeling

- ▶ Connectivity graph  $\mathcal{G}^n = (\mathcal{V}, \mathcal{E})$
- ▶ Conflict graph  $\mathcal{G}^c = (\mathcal{E}, \mathcal{C})$

Per-hop distance  $\delta_e, e \in \mathcal{E}$

- ▶ Shortest hop distance,  $\delta_e = 1$
- ▶ With link duty cycle  $0 < x_e < 1$ 
  - ⇒ Definition 1:  $\delta_e = 1/x_e$
  - ⇒ Definition 2:  $\delta_e = \frac{\bar{l}}{x_e l_e}$
- ▶ Link duty cycle predicted by GCNN  
 $\mathbf{x} = \Psi_{\mathcal{G}^c}(\mathbf{S}; \omega)$

# From hop distance to conflict-aware shortest path



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## Graph modeling

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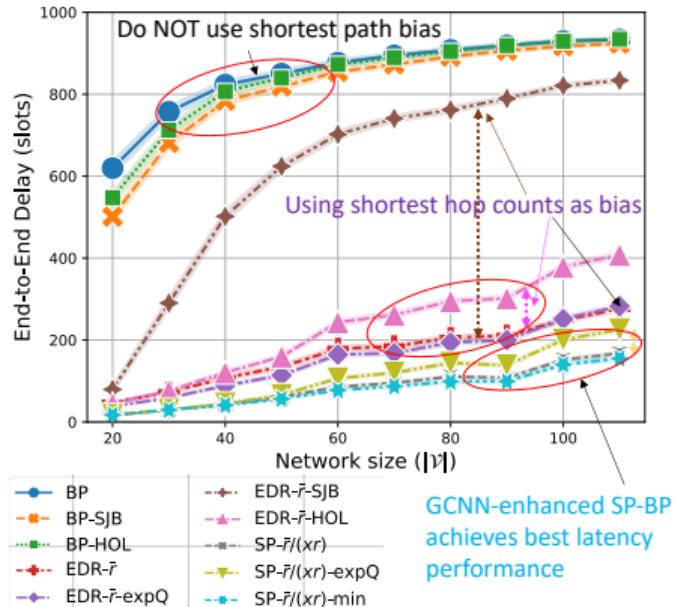
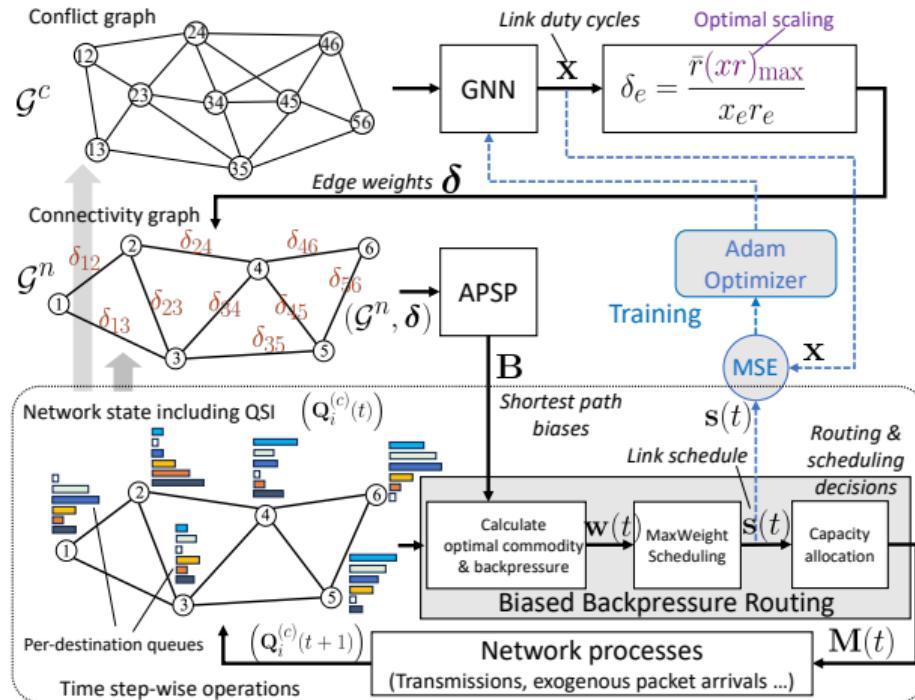
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Graph convolutional layer (local form)

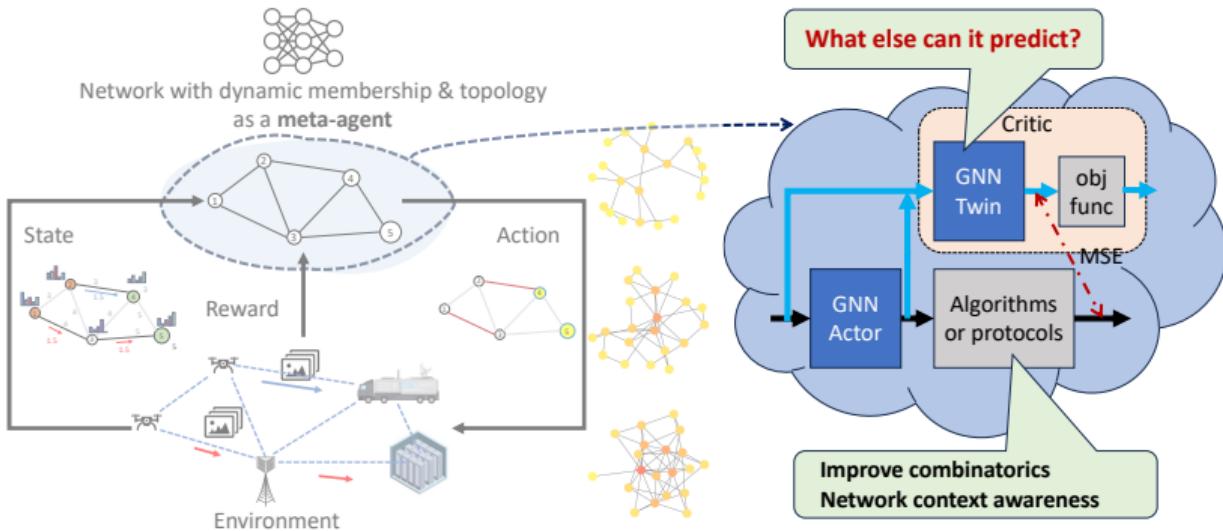
$$\mathbf{X}_{e*}^l = \sigma_l \left( \mathbf{X}_{e*}^{l-1} \Theta_0^l + \left[ \mathbf{X}_{e*}^{l-1} - \sum_{u \in \mathcal{N}_{\mathcal{G}^c}(e)} \frac{\mathbf{X}_{u*}^{l-1}}{\sqrt{d(e)d(u)}} \right] \Theta_1^l \right)$$

# Conflict-aware shortest path for Backpressure routing<sup>11</sup>



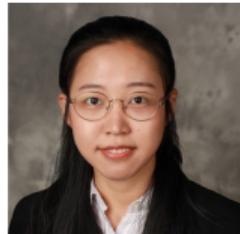
SJB: Sojourn time of all packets in the queue [L. Hai, TVT, 2018]  
 HOL: Sojourn time of head-of-line packet in the queue [B. Ji, ToN, 2012]  
 EDR: Enhanced Dynamic Routing [M. Neely, JSAC, 2005]

<sup>11</sup> Z. Zhao, B. Radojičić, G. Verma, A. Swami, S. Segarra, Biased Backpressure Routing Using Link Features and Graph Neural Networks, submitted to IEEE Trans. on Machine Learning In Comms. and Netw., under review.



New approach to network simulation & optimization

## Network Digital Twin for Fast KPI prediction<sup>12</sup>

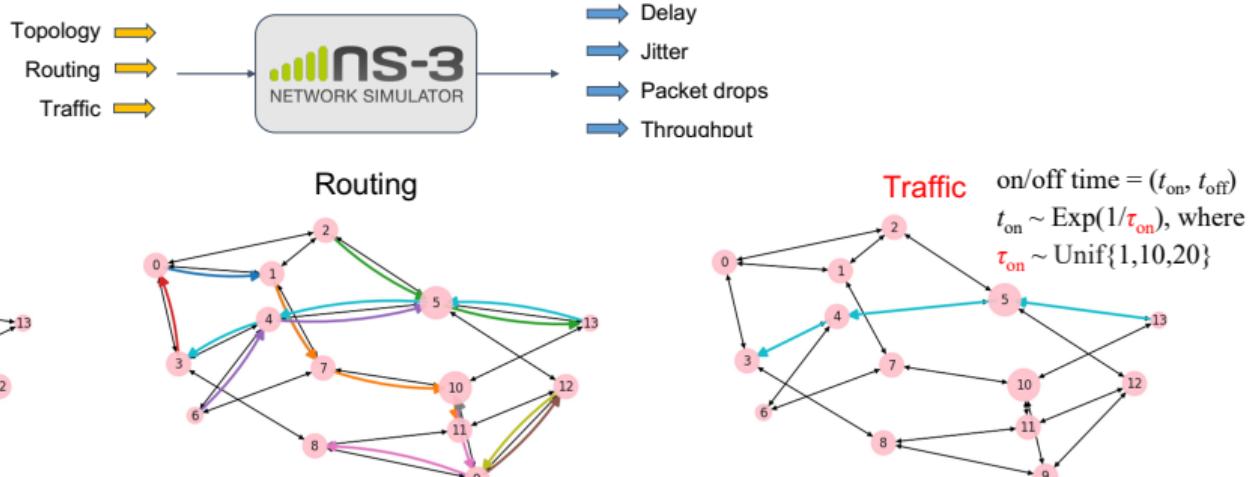


Credit: Boning Li

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<sup>12</sup>B. Li, T. Efimov, A. Kumar, J. Cortes, G. Verma, A. Swami, and S. Segarra. "Learnable Digital Twin for Efficient Wireless Network Evaluation." In IEEE MILCOM, pp. 661-666., 2023.

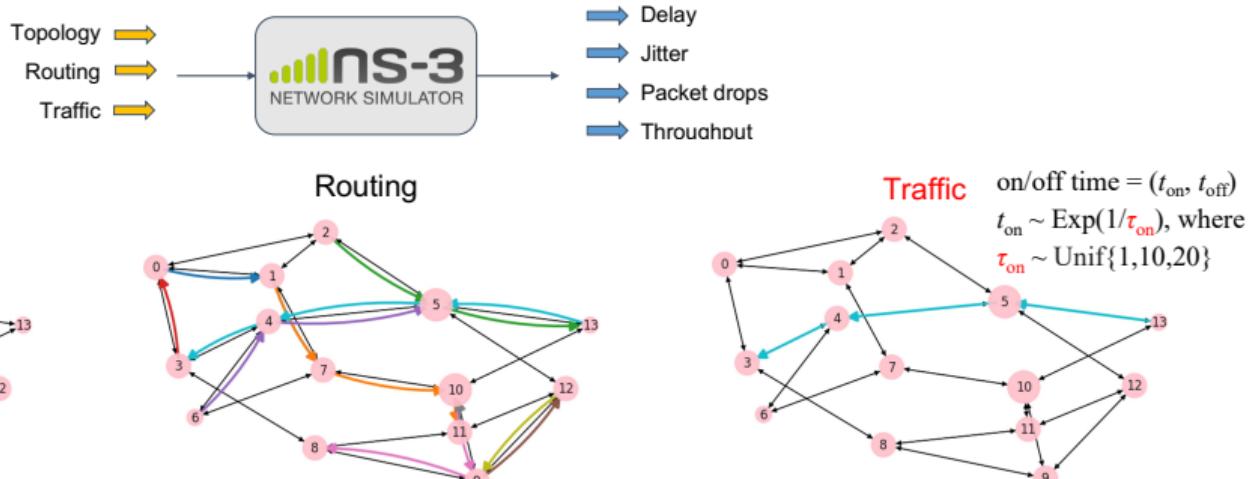
# Introduction of network simulators



Example inputs: NSFNet (14 nodes, 42 links, 10 flows/paths)

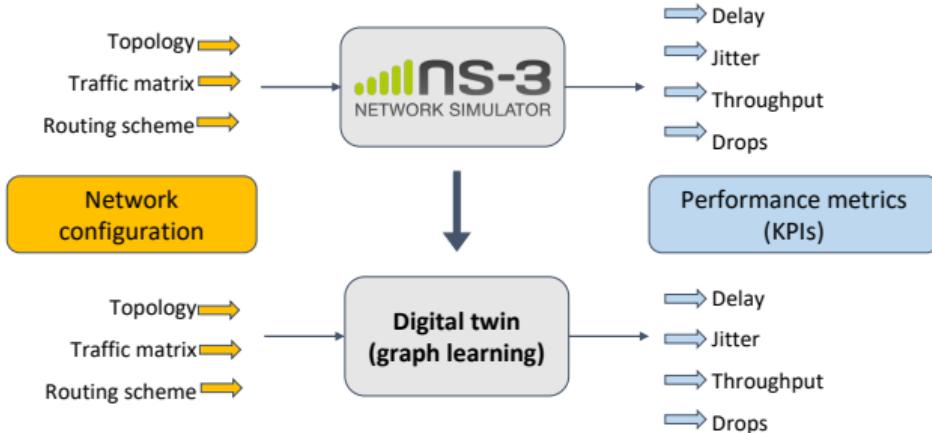
- ▶ Each **flow** corresponds to a set of KPIs (key performance indicators)
  - ⇒ Guide the design, evaluation, and optimization of networks & protocols

# Introduction of network simulators



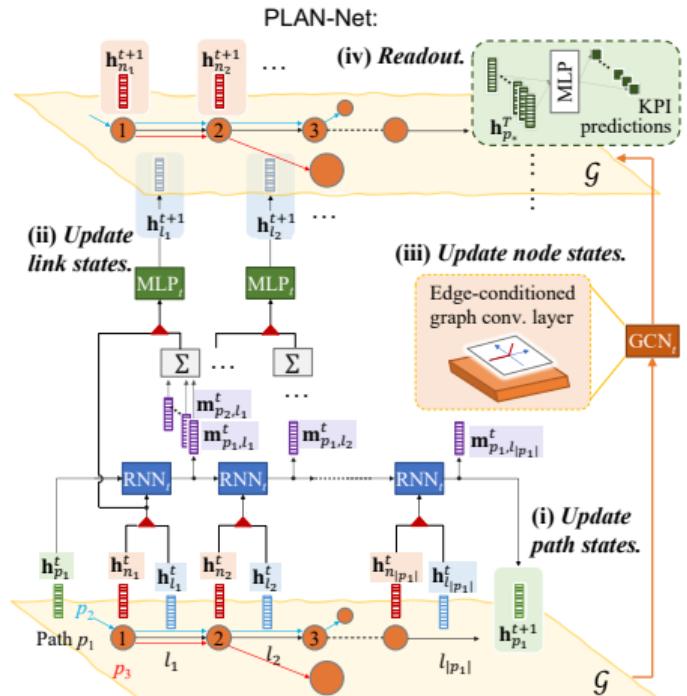
- ▶ Each **flow** corresponds to a set of KPIs (key performance indicators)
  - ⇒ Guide the design, evaluation, and optimization of networks & protocols
- ▶ Network simulator emulates every step in the network protocols and wireless channels
  - ⇒ Very **slow**, difficult to **scale up**

# What network digital twin can do?



- ▶ Fast KPI prediction and differentiable process
- ▶ Digital twin of network simulators
  - ⇒ Predict KPIs rapidly (fast execution)
  - ⇒ Enable iterative optimization (fast execution)
  - ⇒ Training machine learning-based network solutions (differentiability)

# Message-passing architecture of GNNs (PLAN-Net)



**Algorithm 1** PLAN-Net algorithm.

**Input:** Graph  $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ , routing list  $\mathcal{R}$

**Initialize:**  $\rho_p, c_l, d_n > 0$

1:  $\mathbf{h}_p^0 \leftarrow [\rho_p, 0, \dots, 0]^\top, \forall p \in \mathcal{P}$

2:  $\mathbf{h}_l^0 \leftarrow [c_l, 0, \dots, 0]^\top, \forall l \in \mathcal{L}$

3:  $\mathbf{h}_n^0 \leftarrow [d_n, 0, \dots, 0]^\top, \forall n \in \mathcal{N}$

4: **for**  $t = 0, 1, \dots, T - 1$  **do**

5:   **(i) Update path states.**

6:   **for** every path  $p$  in  $\mathcal{P}$  **do**

7:     **for** every link  $l$  in  $p$  **do**

8:        $\mathbf{h}_p^t \leftarrow \text{RNN}_t(\mathbf{h}_p^t, \text{cat}[\mathbf{h}_l^t, \mathbf{h}_n^t])$ , where  $n = \text{src}(l)$   
 $\triangleright n$  is the source node of  $l$

9:        $\mathbf{m}_{p,l}^t \leftarrow \mathbf{h}_p^t$

10:     **end for**

11:      $\mathbf{h}_p^{t+1} \leftarrow \mathbf{h}_p^t$

12:   **end for**

13:   **end for**

14:   **(ii) Update link states.**

15:   **for** every link  $l$  in  $\mathcal{L}$  **do**

16:      $\mathbf{h}_l^{t+1} \leftarrow \text{MLP}_t(\text{cat}[\mathbf{h}_l^t, \mathbf{h}_n^t, \text{agg}\{\mathbf{m}_{p,l}^t \mid l \in p\}])$   
 $\triangleright p$  is all paths that contain  $l$

17:   **end for**

18:   **(iii) Update node states.**

19:   **for** every node  $n$  in  $\mathcal{G}$  **do**

20:      $\mathbf{h}_n^{t+1} \leftarrow \text{GCN}_t(\text{cat}[\mathbf{h}_n^t, \text{agg}\{\mathbf{h}_l^t \mid l = L^+(n)\}]; \mathcal{G})$   
 $\triangleright l$  is all links out of  $n$

21:   **end for**

22:   **end for**

23:   **end for**

24: **end for**

25: **(iv) Readout.**

26:  $\mathbf{y} = \text{MLP}(\mathbf{h}_p^T)$

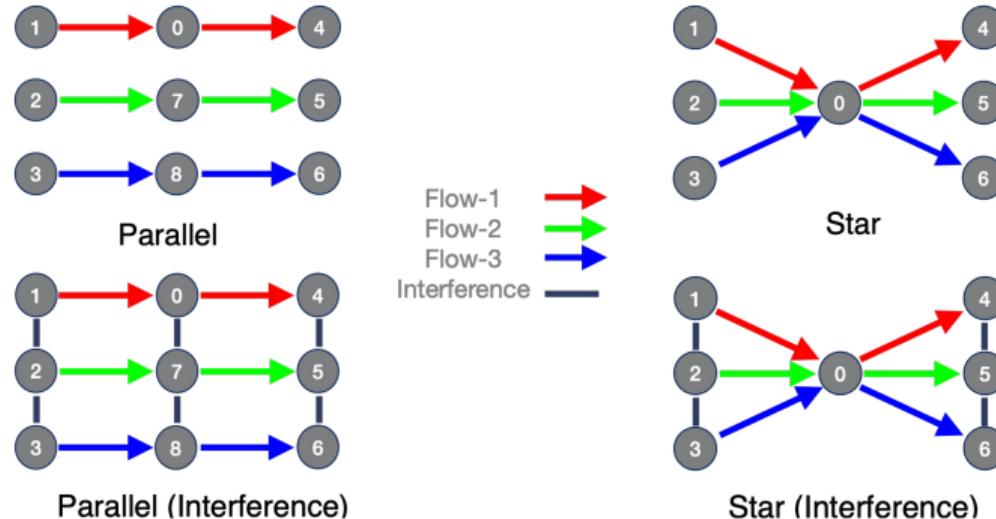
RNN

MLP

GCN

# PLAN-Net (Path, Link, And Node)

- ▶ PLAN-Net improves existing RouteNet<sup>13</sup> for wired networks
- ▶ Leverage node embeddings to distinguish different interference topologies

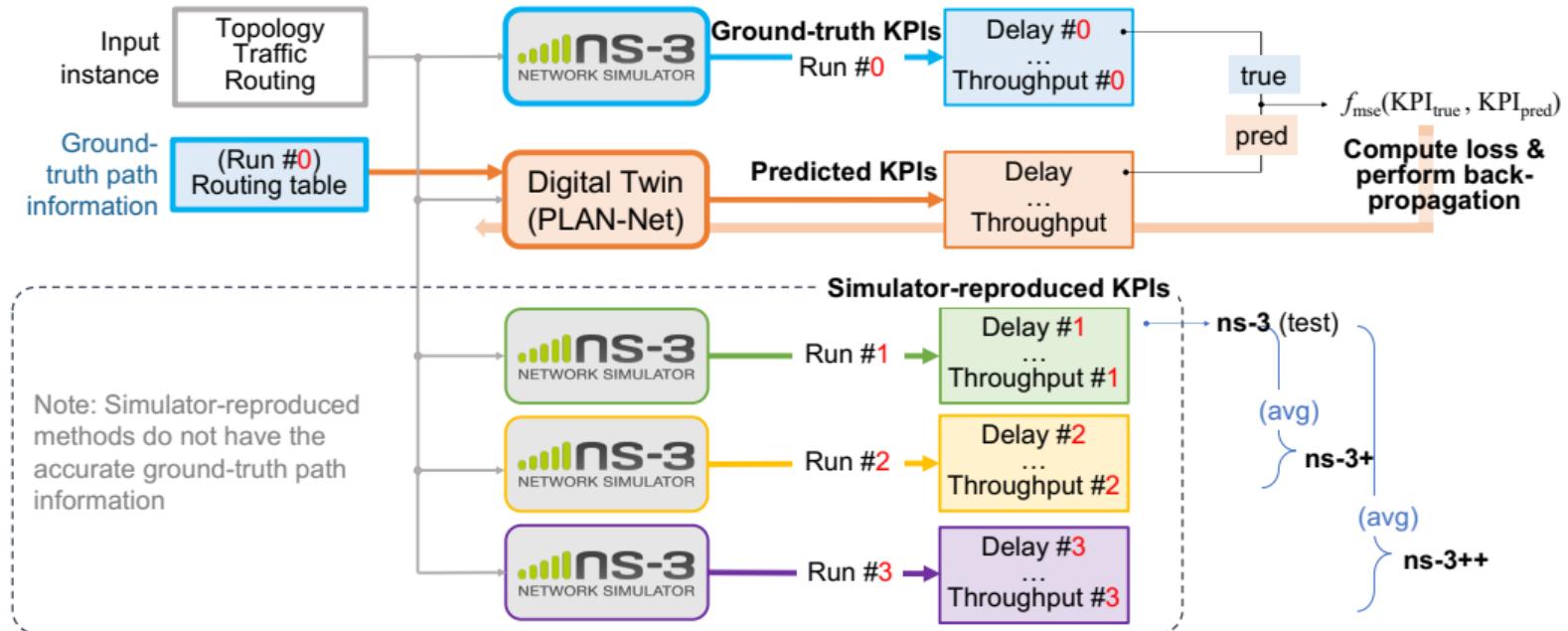



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<sup>13</sup>K. Rusek, et al., "RouteNet: Leveraging graph neural networks for network modeling and optimization in SDN," IEEE J. Sel. Areas Commun., vol. 38, no. 10, pp. 2260–2270, 2020.

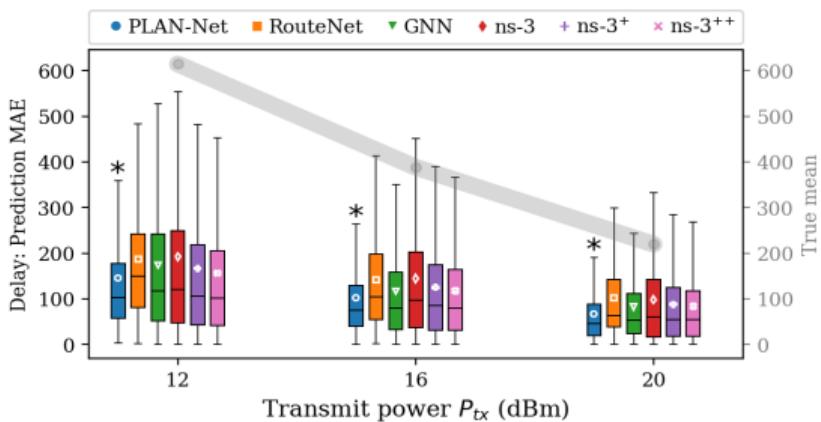
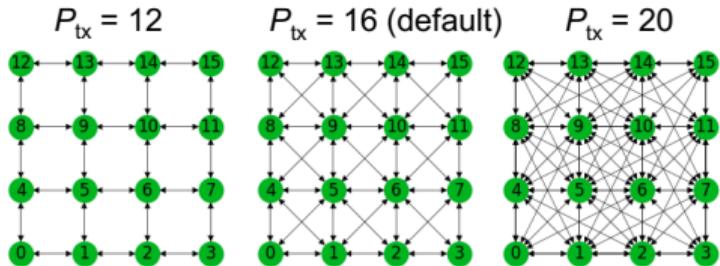
# Training and evaluation

- ▶ Supervised training, using ns-3 single-run output as training labels
- ▶ Performance evaluated by mean absolute error (MAE)

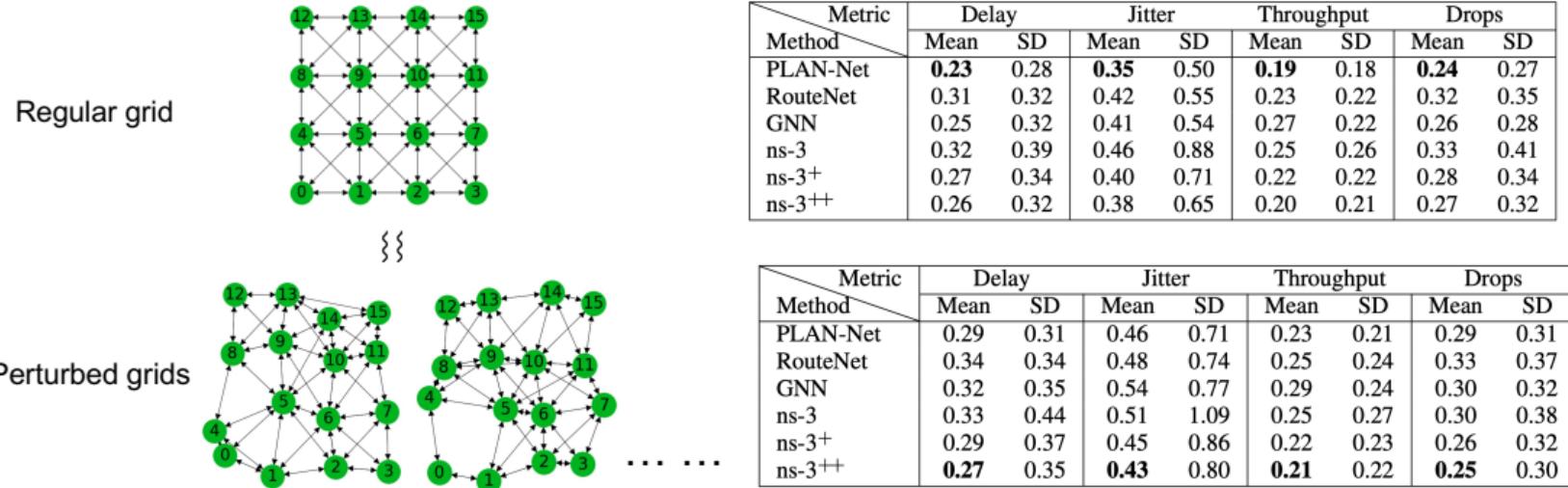


# Numerical results: wireless networks of grid topology

- ▶ Alter transmit power to test for different levels of interference
- ▶ PLAN-Net achieves the lowest MAE
- ▶ Generalize to different network topologies



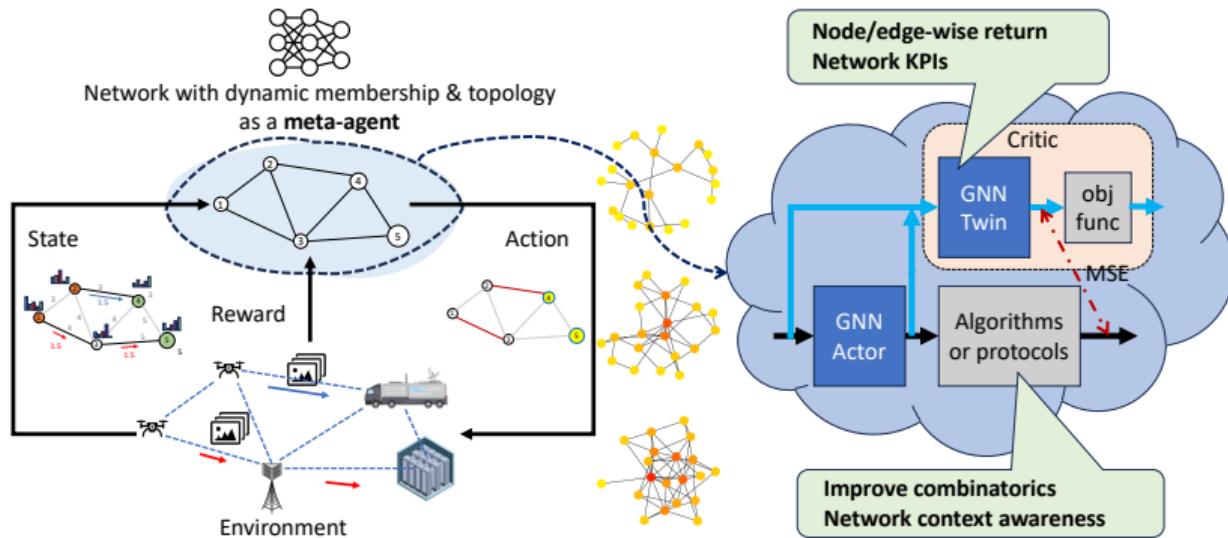
# Numerical results: regular vs perturbed grid topologies



## Key take-aways

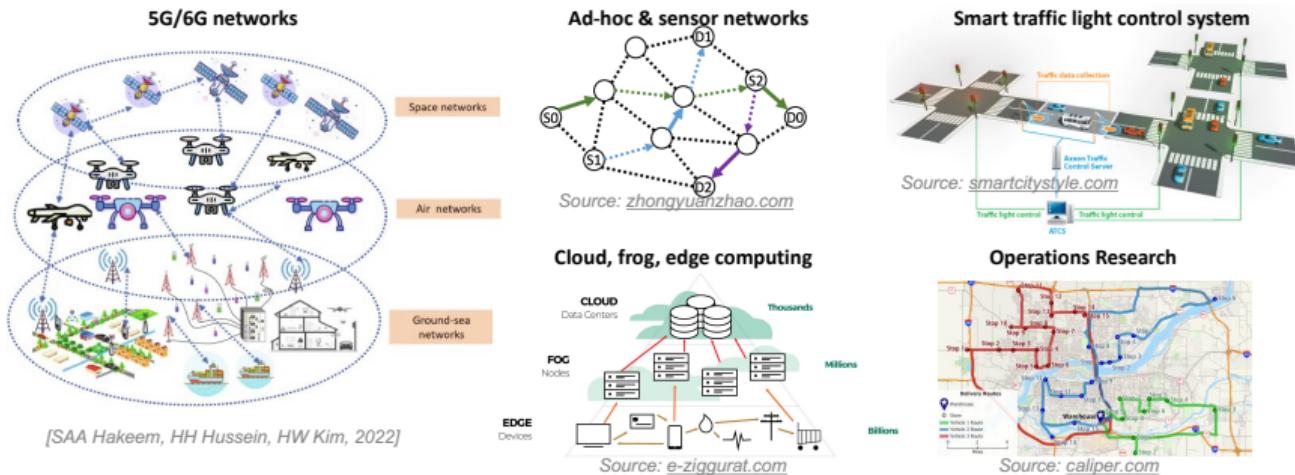
- ▶ PLAN-Net is more accurate than a single-run of **ns-3**
- ▶ PLAN-Net can generalize to random perturbation of network topology
- ▶ PLAN-Net runs **1000x faster** than **ns-3**, e.g., 100s → 0.01-0.1 s

# Summary & future work



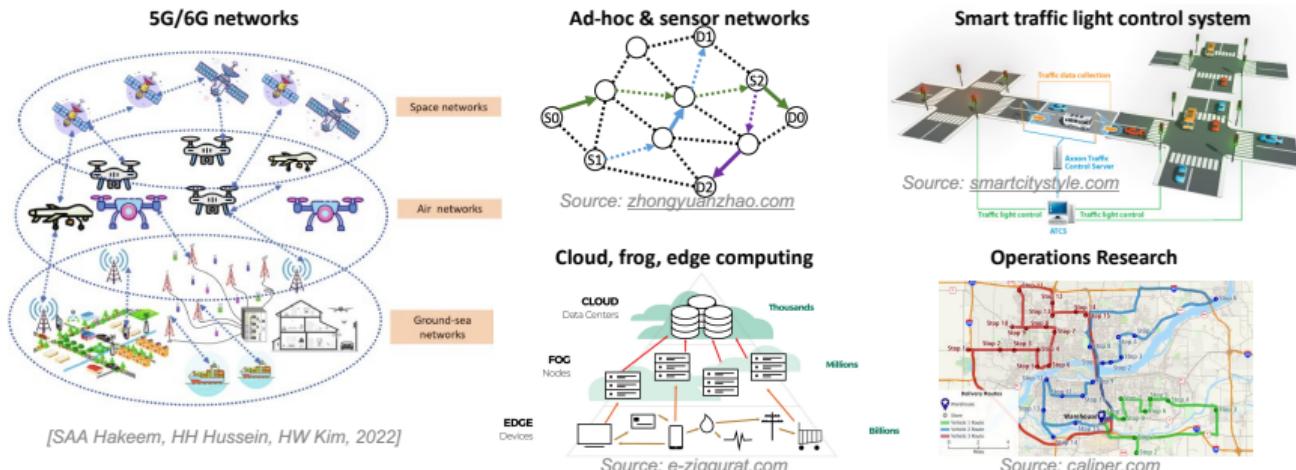
- ▶ Graph-based ML: **permutation invariant**, **distributed** (scalable), **no data labeling**
  - ⇒ Networks are dynamic and parallel & networking tasks are often discrete
- ▶ Hybrid ML pipelines → graph learning + domain knowledge = performance boost
- ▶ Digital twin → fast KPI prediction and action critic = new optimization tools

# Potential applications of graph-based ML in large-scale networked systems



<sup>14</sup>Z. Zhao, J. Perazzzone, G. Verma and S. Segarra, "Congestion-Aware Distributed Task Offloading in Wireless Multi-Hop Networks Using Graph Neural Networks," IEEE ICASSP, 2024, pp. 8951-8955.

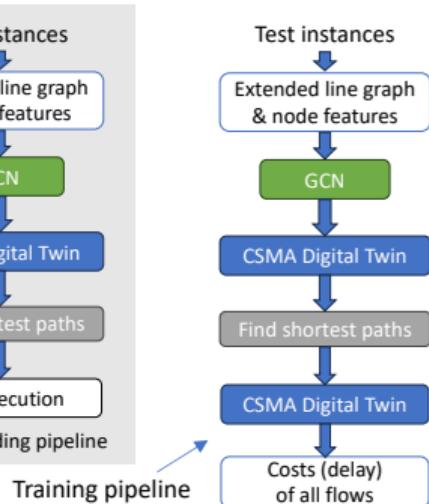
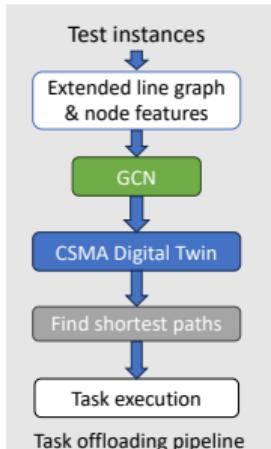
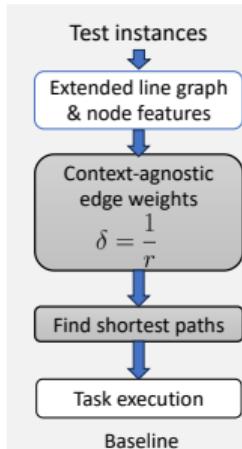
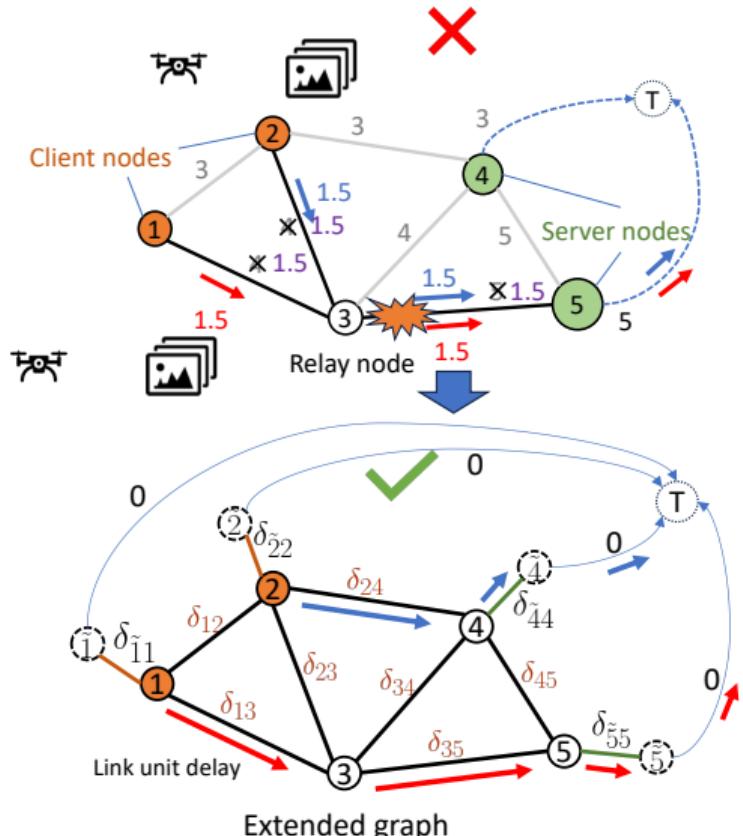
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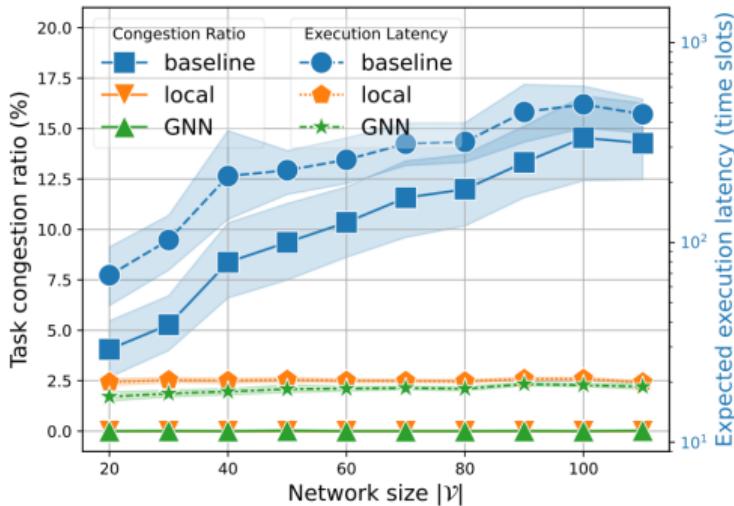
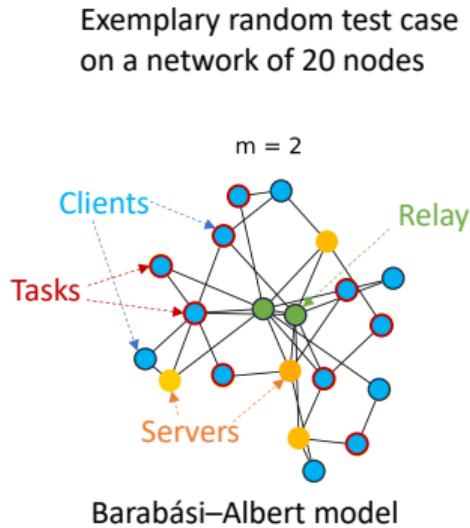
## Edge computing & AI in wireless multihop networks<sup>14</sup>

<sup>14</sup>Z. Zhao, J. Perazzzone, G. Verma and S. Segarra, "Congestion-Aware Distributed Task Offloading in Wireless Multi-Hop Networks Using Graph Neural Networks," IEEE ICASSP, 2024, pp. 8951-8955.

# Congestion-aware distributed task offloading



# Congestion mitigation in distributed task offloading<sup>15</sup>



If a task is congested, its execution latency > 1000 time slots

**Local:** all clients can execute their own tasks without congestion

**GNN:** some tasks offloaded to remote servers without congestion, reducing average execution latency compared to the local policy

**Baseline:** 4%~15% congestion ratio, and high average execution latency (500)

<sup>15</sup>Z. Zhao, J. Perazzzone, G. Verma and S. Segarra, "Congestion-Aware Distributed Task Offloading in Wireless Multi-Hop Networks Using Graph Neural Networks," IEEE ICASSP, 2024, pp. 8951-8955.

## Conclusions and Future Directions

# Going back to our Key Takeaways

- ▶ What are graph neural networks (GNNs)?
- ▶ Why are GNNs well suited to tackle problems in wireless communications?
- ▶ How have GNNs been applied to specific problems?
- ▶ What are open problems/challenges to which you can contribute?

## Going back to our Key Takeaways

- ▶ Class of **parametric, layered, non-linear** functions that incorporate information both from **features** and **graph structure**
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- ▶ **Scalability, distributed** implementation, and **permutation equivariance/invariance**
- ▶ We covered **power allocation & beamforming, link scheduling, and routing** problems
- ▶ Hopefully, the technical discussion have triggered some thoughts. We will also discuss **open directions** now

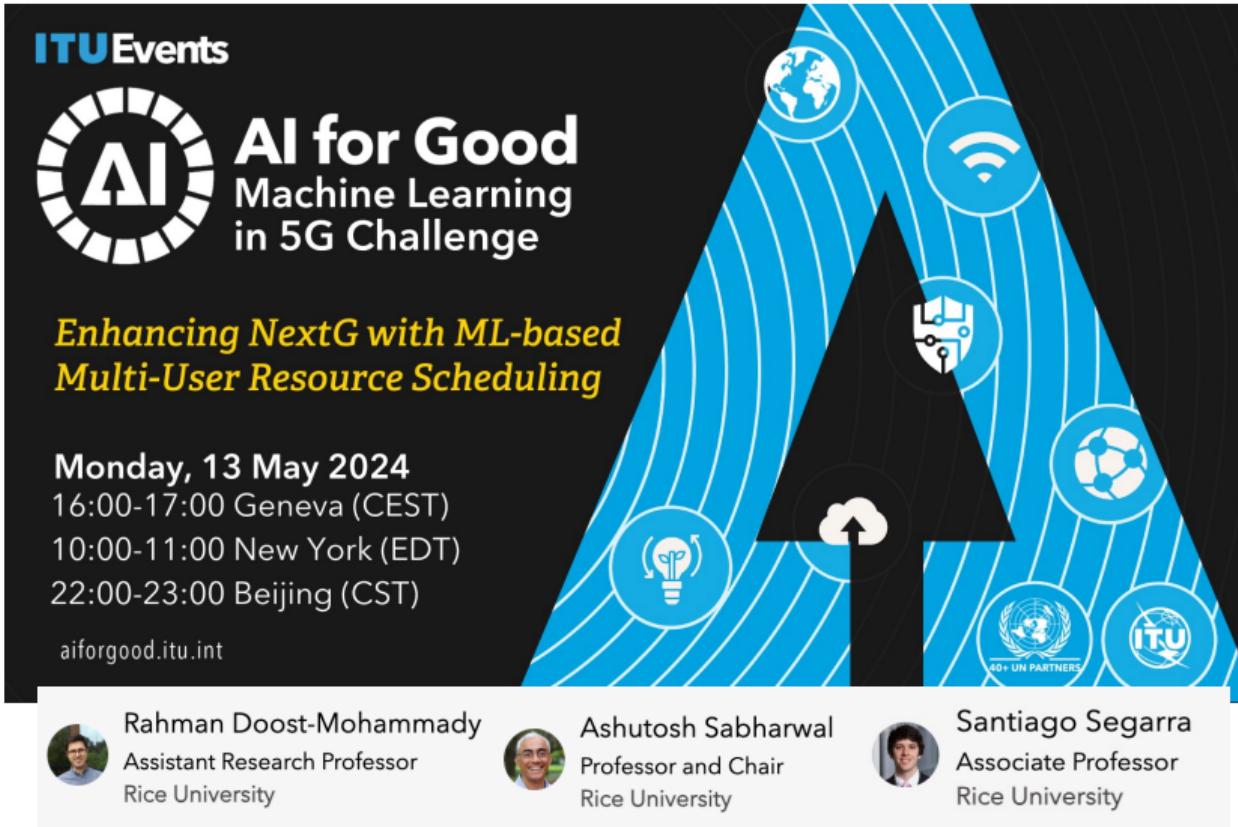
# Open and Future Directions

- The ‘easy’ one  $\Rightarrow$  Applications to other problems in wireless (and beyond)

# Open and Future Directions

- ▶ The ‘easy’ one ⇒ Applications to other problems in wireless (and beyond)
- ▶ Implementation in real wireless networks
  - ⇒ Fading, inexact channel info, packet drops, adversarial/malfunctioning nodes
  - ⇒ Specific protocols for message passing implementation

# Working with Real Data



The slide features a dark background with a large blue triangular graphic on the right side. Inside the triangle are several white icons: a globe, a Wi-Fi signal, a soccer ball, a cloud with an upward arrow, a lightbulb, and two circular logos for the UN and ITU. The text on the slide is as follows:

**ITUEvents**

**AI for Good**  
Machine Learning  
in 5G Challenge

*Enhancing NextG with ML-based  
Multi-User Resource Scheduling*

**Monday, 13 May 2024**  
16:00-17:00 Geneva (CEST)  
10:00-11:00 New York (EDT)  
22:00-23:00 Beijing (CST)

[aiforgood.itu.int](http://aiforgood.itu.int)

**Rahman Doost-Mohammady**  
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Rice University

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Professor and Chair  
Rice University

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Associate Professor  
Rice University

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  - ⇒ Data augmentation and large training datasets

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  - ⇒ New challenges in distributed optimization
  - ⇒ Key for implementation in real systems
- ▶ Combination with **generative AI**
  - ⇒ Data augmentation and large training datasets
- ▶ **Privacy-preserving** message passing in GNNs
- ▶ **Uncertainty** and implementation in critical infrastructure

# Thank you

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