

Image Classification using Convolution Neural Network

Zhongyu Kuang

1. Introduction

1.1 Overview

Image classification is one of the fundamental building blocks for solving visual object recognition problem in the computer vision domain. Extracting information classes according to images' visual contexts (for instance, whether this is an image of a boat or an airplane), as it seems like such a natural task for us human, however, it has proven to be a challenge for computers. With the blooming research in deep learning and the development of fast computation in the past decade, our community has achieved drastic progress in image classification problem. Ever since the annual ImageNet competition[1] first started in 2010, the top-5 error rate for classifying 1000 object categories of 150,000 images has dropped from 28% (winning result in 2010 competition) to 3.08% (winning result in 2016 competition). Prior to the success of AlexNet[2] in 2012, the popular algorithm for approaching the image classification problem is through the visual descriptors[3]: the winning team of the ImageNet competition in 2010 used descriptor coding with support vector machine to achieve the 28% top-5 error rate. In the ImageNet competition 2012, Krizhevsky et al.[2] applied deep convolutional neural network (DCNN) in the image classification problem and brought down the top-5 error rate from 26.2% to 15.3% (Figure 1 shows the winning results of ImageNet classification competition from 2012 to 2016 using DCNN based algorithms). The successful application of DCNN in visual object recognition has motivated a lot of research in applying deep learning technique to solve computer vision problems.

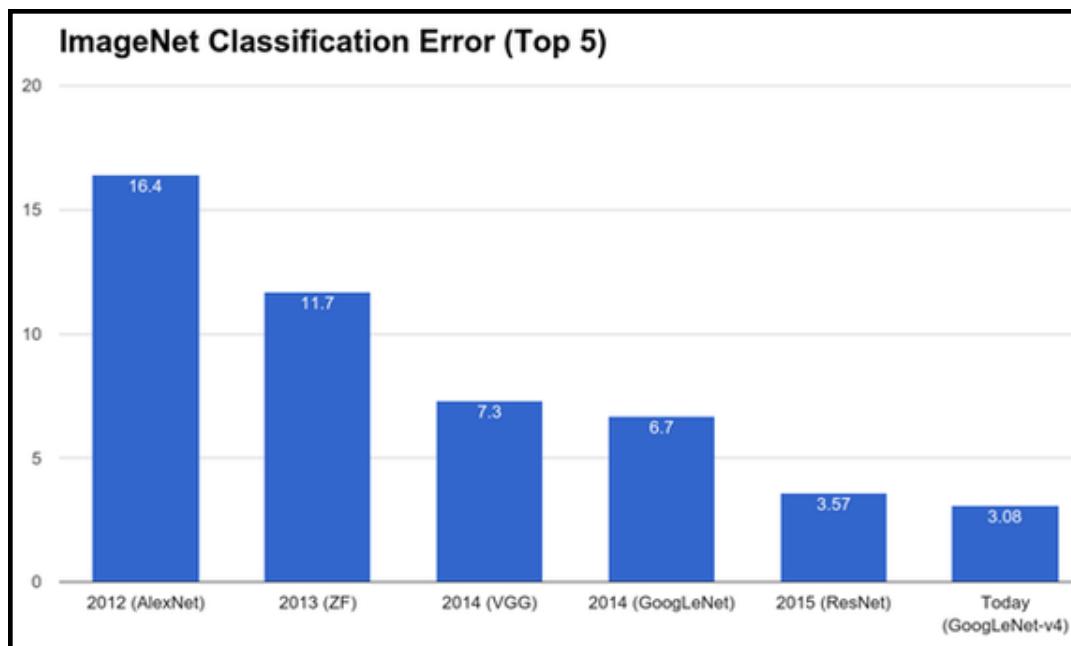


Figure 1. ImageNet competition winning team top-5 error rate

The purpose of this project is to study DCNN algorithm and apply it to solve natural image classification problem in practice. Among the various datasets available[4] for deep learning research, CIFAR-10[5] is chosen for this project for the following reasons:

- (a) CIFAR-10 has a fairly large set of samples (50,000 training images and 10,000 test images) with well-balanced object categories (10 classes total, 6000 images per class)
- (b) CIFAR-10 dataset is suitable and complex enough to employ a DCNN algorithm, however, it's simple (sample resolution: 32 pixels by 32 pixels) enough such that a less computationally expensive architecture could undertake.
- (c) CIFAR-10 has been widely used in deep learning research so that there are many benchmark results available for comparison.

1.2 Problem Statement

The objective of this project is to develop a CNN network for correctly classifying images in the CIFAR-10 dataset. Even though recent research indicates that complex connected CNN architecture has great potential in performing image classification tasks very accurately (i.e. Inception[6], ResNet[7] and GoogLeNet-v4[8]), due to the limitation of computation resources, a vanilla flavored rail-road stacking architecture is used in this project. In order to better monitor the training process, in addition to the testing images, another 10,000 images are randomly selected from the 50,000 training images to form a validation set. The category labels are marked by numbers ranging from 0~9, which each unique number corresponds to one unique category. All classes are mutually exclusive and non-overlapping. Before feeding the dataset into the designed network, one-hot encoding is applied to convert the label information into a vector format.

1.3 Metrics

The classification accuracy, where the accuracy is calculated as in equation 1, is used as the performance metric for evaluating the CNN network. The metric is chosen for its simplicity to interpret, its effectiveness for evaluation and its wide usage in similar deep learning research using the CIFAR-10 dataset. Although this is a multi-class classification problem, the reason for not using a more complex metrics (such as f1-score, precision, recall, etc.) is because they are less meaningful in the context of the general purpose of image classification.

$$\text{accuracy} = \frac{\text{number of correctly classified instances}}{\text{total number of instances}} \quad (\text{equation 1})$$

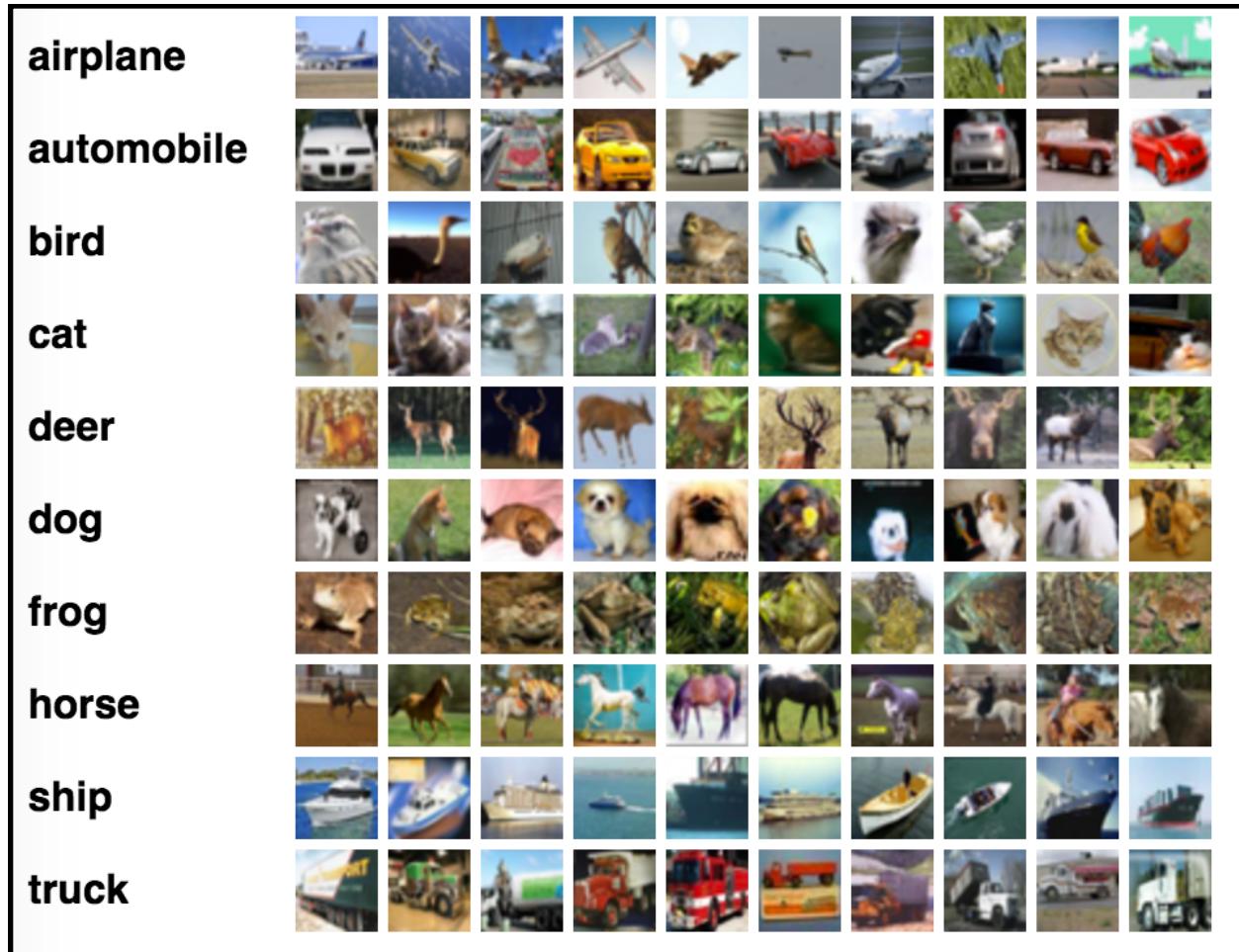
2. Analysis

2.1 Dataset Exploration and Summary

A quick summary of the CIFAR-10 dataset is provided in Table 1. As it is shown, the CIFAR-10 dataset has an equal amount of samples in each category and 10 mutually exclusive categories in total. Figure 2 lists out all of the 10 classes and provides 10 random images from each class. The CIFAR-10 dataset is divided into three subsets: a training set with 40,000 samples, a validation set with 10,000 samples, and a testing set with 10,000 samples. It is ensured that all three subsets contain equal amounts of samples of all 10 categories.

Table 1. Summary of CIFAR-10 Dataset for this project

	Training Set	Validation Set	Testing Set
sample size	40000 samples	10000 samples	10000 samples
total categories	10 categories (4000 samples per category)	10 categories (1000 samples per category)	10 categories (1000 samples per category)
sample resolution	32 pixels by 32 pixels	32 pixels by 32 pixels	32 pixels by 32 pixels
color channels	3 RGB	3 RGB	3 RGB

**Figure 2. All 10 classes of the CIFAR-10 dataset and 10 random images for each class**

2.2 Exploratory Visualization and Discussion

Each sample in the CIFAR-10 dataset is a 32-pixel by 32-pixel by 3 RGB (Red-Green-Blue) channels image which is stored as a three dimensional matrix ([32, 32, 3]) with the value at any pixel locations of any color channels ranging from 0~255. A random sample from the frog

category is plotted in Figure 3 as an example to illustrate both its raw digital representation in computer and its human-eyes-friendly image display.

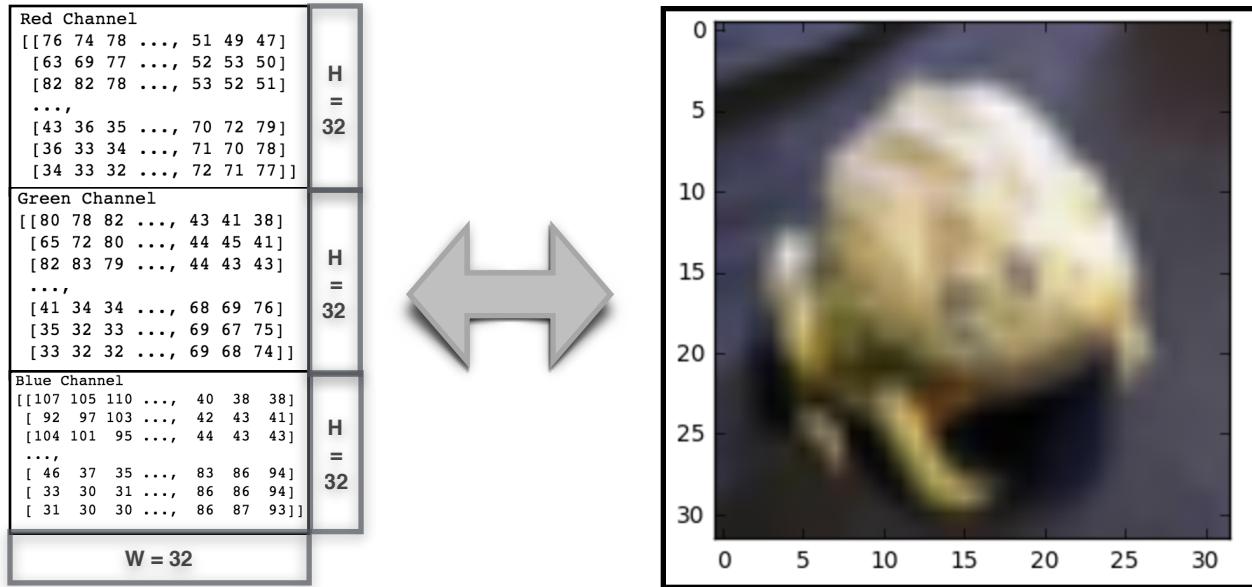


Figure 3. A random sample's raw digital representation (left) and its image display (right)

As it is shown in Figure 3, the samples in CIFAR-10 are rather low-resolution images as compared to most of the photos we see in real life nowadays. This nature inevitably leads to a better preservation of the global features (i.e. shape, sharp color contrast, etcetera) of the object and a poorer preservation of the fine features (i.e. texture, color gradient, etcetera) of the object. Therefore the CNN is expected to be better at learning the global features than the finer features. Additionally, since the deeper a convolutional layer is, the bigger area the convolutional layer can see (through projecting its receptive field back to the input image), it is expected that a relatively shallow CNN architecture with small filter size would be able to undertake the task of learning and classifying CIFAR-10. The coarse preservation of finer object features in CIFAR-10 determines that it would be a bad dataset candidate for a more demanding classification task (imagine trying to tell the golden frog in Figure 3 is from species P. terribilis or species A. zeteki). Fortunately, as it is listed out in Figure 2, all 10 categories in CIFAR-10 are pretty uniquely distinguishable from global features, as it is not too difficult to differentiate between an image of a frog and an image of a truck even though the images only have a resolution of 32-pixel by 32-pixel. Last but not the least, the low-resolution property of CIFAR-10 images lowers the computational costs for CNN.

2.3 Algorithms and Techniques

As the convolutional neural network has shown great success in image classification, it is used in this project for classifying the CIFAR-10 dataset. The basic building blocks for constructing a CNN includes convolutional layers, pooling layers, fully-connected layers, and activation layers.

As the core building block for CNN, a convolutional layer is computed by sliding certain fixed size (i.e. 3x3, 5x5, 7x7) two-dimensional filters/windows across the input images to obtain different feature maps. Suppose the input data has the dimension of [32x32x3] (input width:32, input height:32, input depth: 3) and a [3x3] filter size is chosen to construct the convolutional layer. During the forward pass, a [3x3x3] (filter width: 3, filter height: 3, filter depth: 3) filter is sliding across the width and height of the input sample and computing the dot product followed by pre-specified activation function to form the output feature map (Figure 4, adapted from [9]). During the backpropagation pass, the gradient of each neuron in the filter with respect to the final output is computed and the weight parameters are adjusted in the next step of training accordingly to reduce the total loss function of the network. The number of filters to use per convolutional layer, the size of the filter, the stride to use when slides the filter across the input and the padding choices around the border are all hyper-parameters. The design of convolutional layers enables parameter sharing and the statistical invariant of neurons by sliding filters across the height and the width dimension. Additionally, it reduces the computational costs since it has fewer parameters to train as compared to fully connected layers.

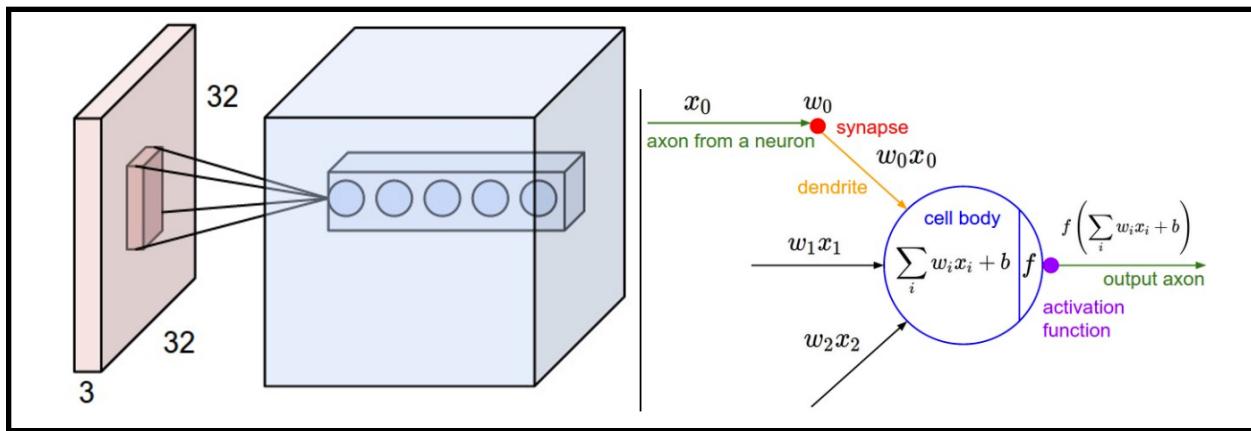


Figure 4. Diagram illustrates convolutional layers. Adapted from [9]

Pooling layers are commonly inserted in between convolutional layers to gradually reduce the spatial size of feature maps. The operation mechanism (Figure 5, adapted from [9]) is very similar to convolutional layers: with the size of filters and the stride to use for sliding procedure as hyperparameters, a pooling layer down-samples input feature maps based on the choice of specified pooling method (commonly used pooling methods: average and maximum). It is worth noting that there are no trainable parameters in pooling layers and the main purpose of pooling layers are for down sampling activated feature maps.

Different from convolutional layers, the neurons in fully-connected layers are fully connected pair-wisely to all the elements in the previous input layer and the next layer (Figure 6, adapted from [10]). Often the outputs of the last fully connected layer are fed into a classifier to transform the values into the range of 0~1, which is interpreted as the probability of the certain sample being as the certain class label. Sigmoid and soft-max have been two commonly used functions

in the last classification step, although sigmoid has gradually fallen out of favor in recent years and soft-max is becoming the default choice in this step.

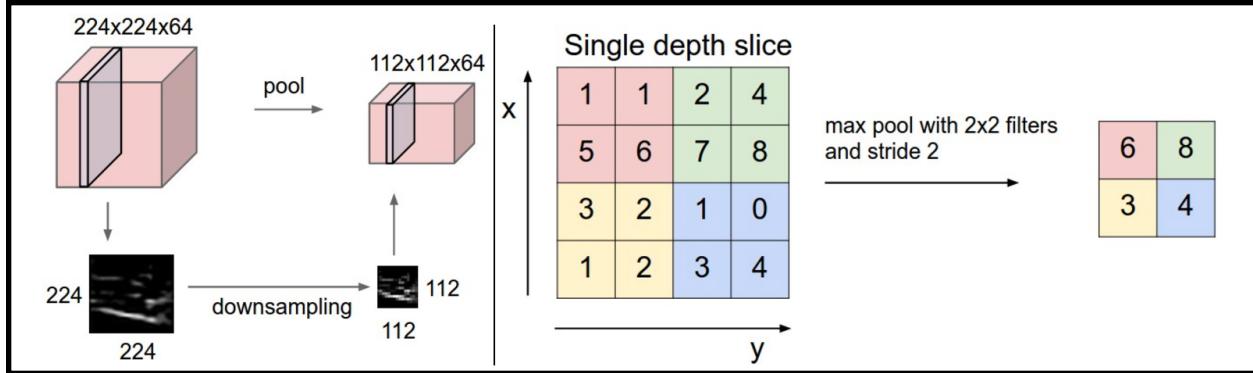


Figure 5. Diagram illustrates max pooling layers. Adapted from [9]

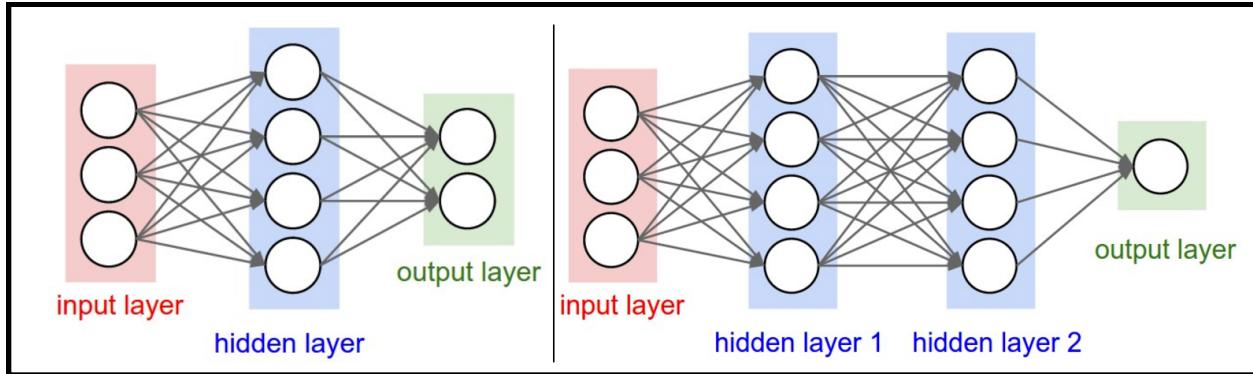


Figure 6. Diagram illustrates fully-connected layers. Adapted from [10]

The neural network is known for owning a high capacity for representing a wide range of non-linear spaces. As both the convolutional layers and the fully-connected layers are all performing linear transformations, the non-linearity in the neural network is introduced by the activation layers. The inspiration of the activation layers comes from biological neurons, which receive input signal but do not fire up unless the input signal surpasses a certain threshold. Similar to the pooling layers, adding the activation layers does not add in trainable parameters to the model. As the activation layers are inserted right after each convolutional layer and each fully-connected layer, they apply a specified non-linear function to the output of convolutional layers and fully-connected layers. Sigmoid, tanh, and ReLu are three popular choices for the activation function (Figure 7). Although practical experiences indicate that ReLu function out-performs the other two options with an extra gain of cheap computation costs (Figure 8).

Besides the fundamental building blocks discussed above, a technique called Dropout[11] is often applied to fully-connected layers for regularization. With a specified keep probability as a hyperparameter to tune, each neuron has $(1 - \text{keep probability})$ that it'd be turned off during the training phase (Figure 9, adapted from [11] Figure 1). While during the testing phase, the results

of all “thinned” networks would be averaged to come up with the final classification. The dropout technique is proven to be computationally cheap and effective in practice. It prevents deep neural networks from overfitting and can be thought of as a kind of ensemble techniques.

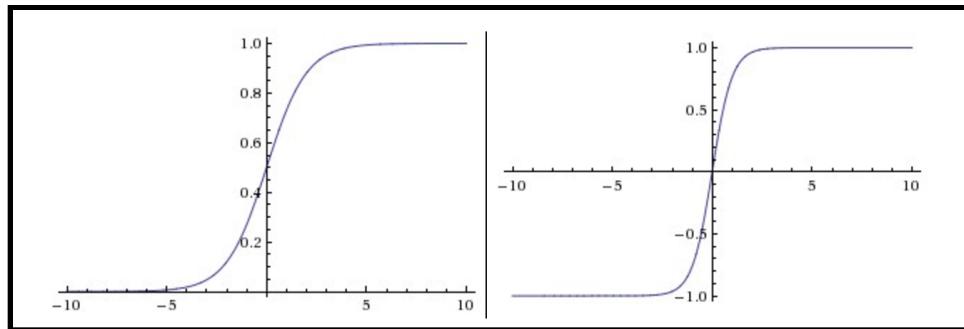


Figure 7. Activation functions. Left: Sigmoid. Right: tanh. Adapted from [10]

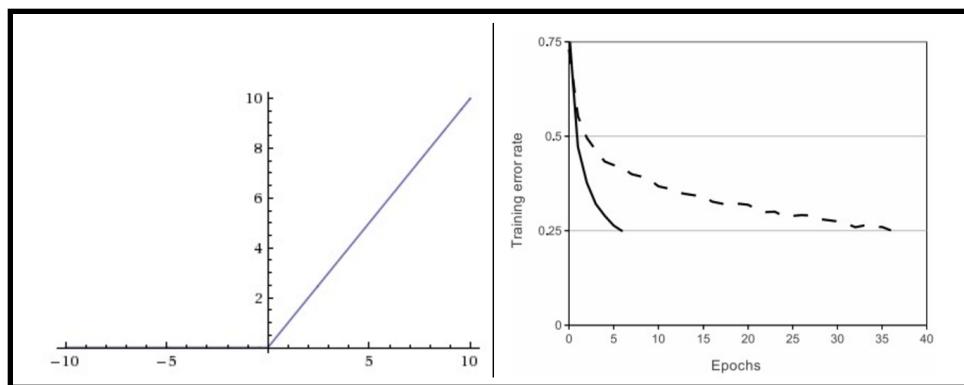


Figure 8. Left: ReLu function. Right: ReLu converges faster than tanh. Adapted from [10]

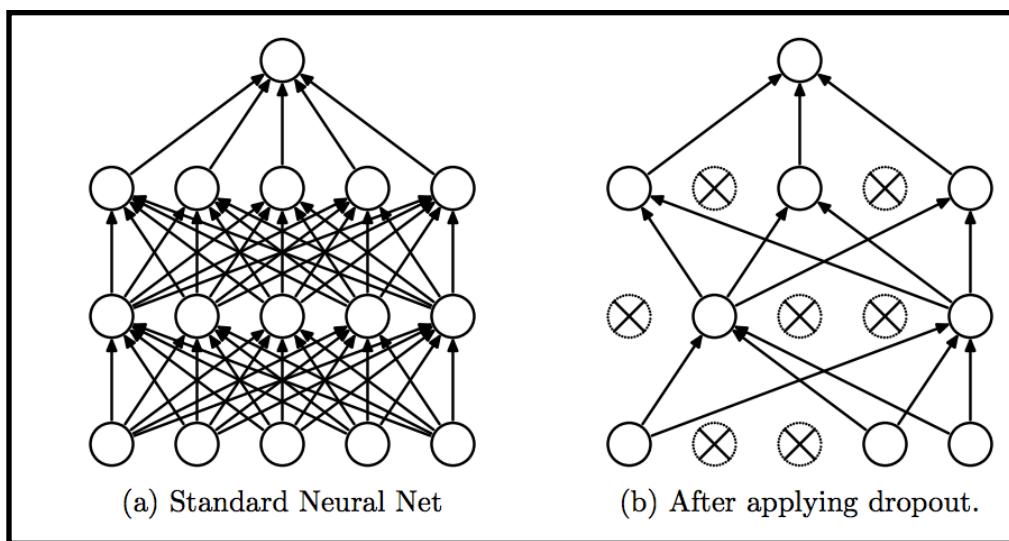


Figure 9. Left: without dropout. Right: with dropout. Adapted from [11]

2.4 Benchmark

As vast amounts of efforts are devoted into CNN researches aiming at further improving performance and reducing costs through creative design of new techniques, CIFAR-10 dataset, along with a few other datasets (i.e. MNIST, SVHN, CIFAR-100), is commonly used as one of the standard datasets for measuring the effectiveness of new deep learning techniques. Hence, there is a wide range of test set accuracy scores being reported in different studies [12] with the state of art performance being 96.53%[13]. Given the simplicity of a vanilla CNN architecture, the limitation of computational resources, and the main aim of this project is for practice and learning instead of creating the best record, the test set accuracy score of 75.86% reported by McDonnell and Vladusich[14] obtained through their Fast-Learning Shallow CNN (FLS-CNN) architecture is used as a comparison for this project. As it is shown in Figure 10, the core design of the FLS-CNN also uses a simple vanilla rail-road stacking flavor to connect different layers. The overall similarity between the FLS-CNN and the CNN architecture used in this project (details see section 3.2 and Figure 13) makes it a good benchmark comparison candidate.

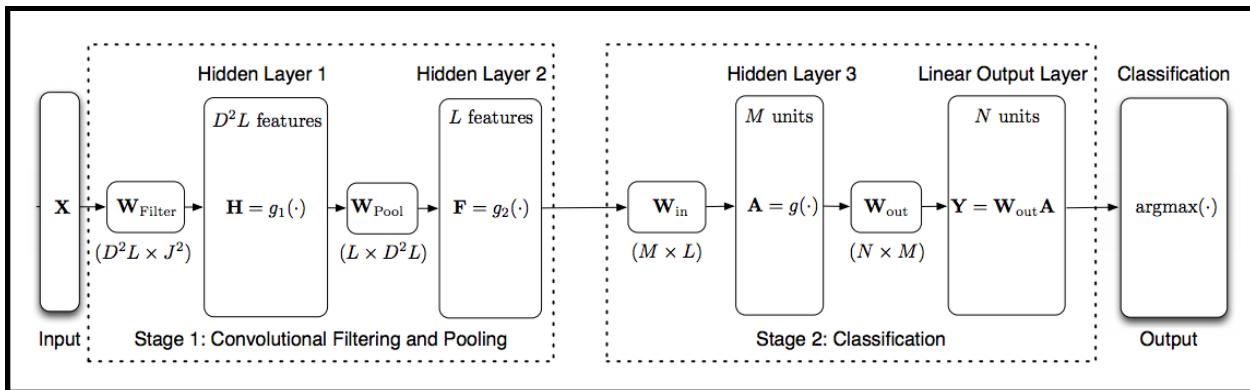


Figure 10. The Fast-Learning Shallow CNN architecture adapted from [14] Figure 1

3. Methodology

3.1 Data Preprocessing

As past research indicates that whitening input data and data augmentation are beneficial for training neural networks, several data preprocessing techniques are explored and among which a few are applied to this project.

(a) Data augmentation technique: random flipping. For each sample in the training set, there is 50% of the chance that it augmented by flipping upside down and 50% of the chance it is augmented through mirror flipping (Figure 11). Hence the training set size is doubled to 80,000 before training.

(b) Data augmentation technique: Principle Component Analysis (PCA) based color jittering[2]. As the technique proposed and used in AlexNet[2], it is explored in this project (Figure 12). However due to lack of fast implementation, eventually this data augmentation technique is not applied to the full dataset.

(c) All the data (training set, validation set, testing set) has been centered around zero means. No scaling is performed on each element since the scales of all values are in the same range (0~255)

(d) One-hot encoding is applied to convert the numerical labels into a vector format.

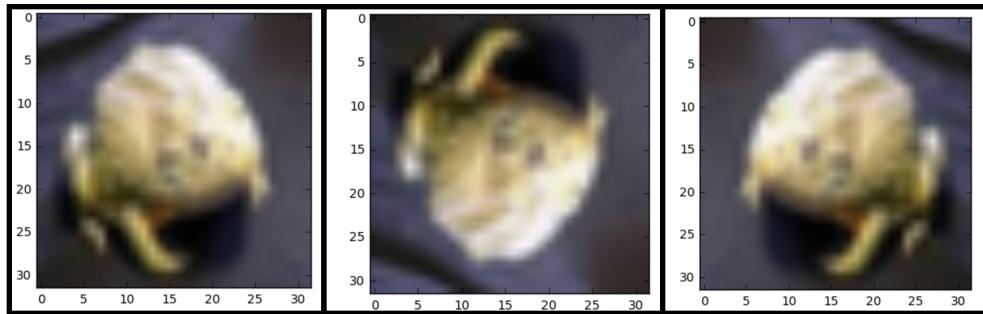


Figure 11. Data Augment: random flipping. Left: Original sample. Middle: 50% probability of up side down flip. Right: 50% probability of mirror flip

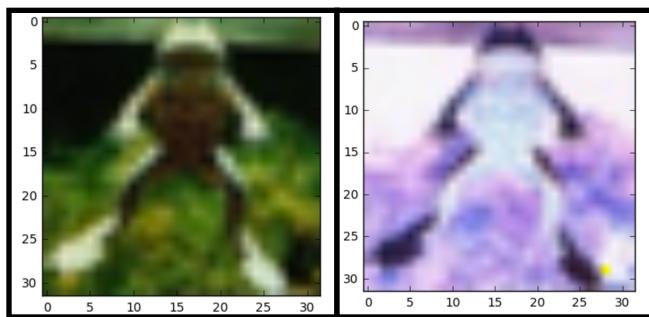


Figure 12. Data Augment: PCA color jitter. Left: Original sample. Right: PCA color jittered

3.2 The Design and Implementation of a Shallow CNN Architecture

A shallow CNN is constructed using the building blocks and techniques discussed in section 2.3. A diagram of the designed shallow CNN architecture is provided in Figure 13. Some of the selected parameter choices pertaining to the convolutional and pooling layers are provided in Table 2. ReLu activations are used after each convolutional layer (before max pooling layer) and after each fully-connected layer. The implementation of the designed CNN is carried out employing python TensorFlow framework (the explicit code can be found at [author's GitHub CNNplayground Repository](#)).

Table 2. Selected Hyper-parameter Settings for Convolutional Layers and Pooling Layers

	Filter/Window size	Stride	Padding	Number of Filters
Convolutional Layer 1	5 x 5	1	SAME	64 or 16
Convolutional Layer 2	3 x 3	1	SAME	64 or 16
Convolutional Layer 3	5 x 5	1	SAME	64 or 16
Max Pooling Layers	2 x 2	2	SAME	N/A

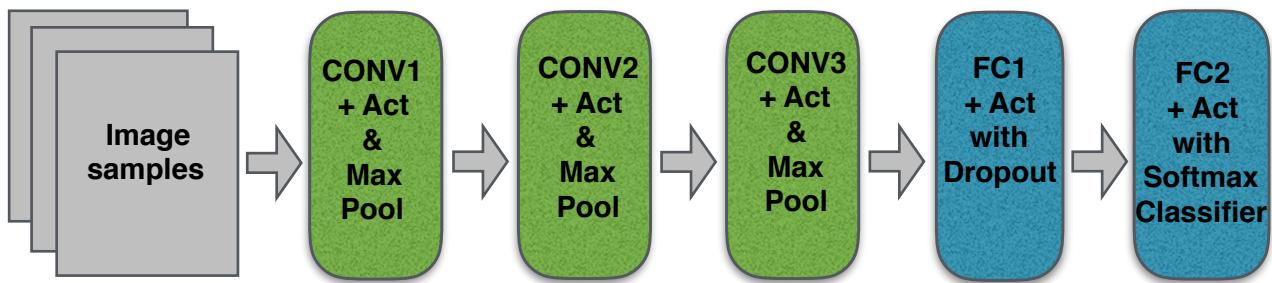


Figure 13. Designed Shallow CNN architecture (Act stands for activation layers)

Although there were no significant complications happened during the actual implementation process of the designed CNN architecture, it is worth mention that proper parameter initialization is found to be crucial for model learning. During the early stage of the model implementation and testing, the model is suspected to suffer from gradient vanishing problem caused by bad initialization as the model appears to be not learning at all. As TensorBoard is proven to be a wonderful model diagnosis and debugging tool, even though it was not used in this project due to late discovery, it is expected to be heavily used in future projects.

3.3 Hyperparameters Tuning and Refinement

The designed CNN architecture in Figure 13 is initially trained on random sets of small samples (64 samples) with a wide range of hyperparameter settings and validated on random sets of small samples (32 samples). The small sets of instances are sampled from the whole training set using bootstrap sampling. The choice of hyperparameter settings is first tuned using a random search approach through the log space range of the hyper-parameters, then refined into a narrower range based on the performances measured during the random search through the hyper-parameter spaces. A few training experiments with hyper-parameters picked from the refined ranges/choices are then used to train the full dataset on an AWS 16GB RAM CUP server. The coarse, refined and the further refined hyperparameter ranges/choices are shown in Table 3. The results using different hyper-parameter settings picked out from Table 2 and Table 3 are demonstrated, compared and analyzed in section 4.

Table 3. Random Search through Hyperparameter Spaces

Hyperparameters	Coarse ranges/choices	Refined ranges/choices	Further refined choices
weights initialization	(1) Xavier Initializer [15] (2) ReLu Xavier Initializer [16] (3) Random normal initializer with zero mean and standard deviation ranges from $10^{(-5)} \sim 10^{(-1)}$	Random normal initializer with zero mean and a standard deviation range of 0.01~0.02	Random normal initializer with zero mean and a standard deviation of 0.015
biases initializer	(1) constant zeros; (2)constant ones	constant ones	constant ones
dropout keep probability	0.1 ~ 0.8	0.3 ~ 0.5	0.5
initial learning rate	$10^{(-6)} \sim 10^{(-1)}$	0.0001 ~ 0.0007	0.0003 or 0.0004
optimizer	(1) Stochastic Gradient Descent (2) AdamOptimizer[17]	AdamOptimizer	AdamOptimizer

4. Results - Model Evaluation, Validation and Justification

The results achieved through employing the CNN architecture designed in Figure 13 with slightly different hyperparameters settings picked out from Table 2 and Table 3 are shown in Table 4 and Figure 14.

As shown in Table 4 and Figure 14, Model 2 only uses 16 filters per convolutional layer and achieves the lowest test set accuracy of 67.94% after 3000 training steps. This indicates that the learning power with a less complex CNN model is weaker as compared to a CNN model used a larger number of convolutional filters. On the other hand, with the filter size per convolutional layer (3 convolutional layers total) decreases by a factor of 4, the runtime for one training step decreases by a factor of 3.6.

Without data augmentation, Model 1 produces a test set accuracy of 73.00%, while with data augmentation, the test set accuracy trained by model 3 increases 0.40% as compared to Model 1. It is also worth noticing that in Figure 14, the gap between training accuracy and validation accuracy in the top left subplot (Model 1) starts to appear after 600 training steps. However, in the middle left subplot (Model 3) there is no noticeable gap between training and validation. This indicates that data augmentation has a mild effect in improving model performance and by increasing the complexity and variance of training samples it delays overfitting.

Among the all six CNN models, Model 6 produces the highest test set accuracy score of 74.74% after 2500 training steps and a dropout keep probability of 0.30, Model 5 produces the second highest of 74.27% after 2500 training steps, and Model 4 produces the third highest score of 73.51% after 1700 training steps. In the bottom left subplot (Model 5) in Figure 14, a small gap between the training and the validation line emerges after about 1500 training steps and grows gradually. In the contrast, in the middle right subplot (Model 4) in Figure 14, with a smaller dropout keep probability, there is no apparent sign of overfitting after 1500 steps of training. This suggests that the setting of dropout keep probability may be too high in Model 5.

In summary, Model 6 achieves the highest test set accuracy score and hence is selected as the final model. The achieved test accuracy score of 74.74% is slightly lower than the FLS-CNN benchmark score of 75.86%. Perhaps there are potentials for further improvement upon this performance (more details are discussed in the following conclusion section). In addition to that, the analysis of all five model performances indicates that:

- (a) Data augmentation helps increase the model performance and delays the emergence of overfitting;
- (b) Complex CNN models are computationally more expensive to train, however, it has bigger learning power than simple CNN models;
- (c) Bigger dropout rate helps regularize model training and prevents overfitting.

Table 4. Six CNN model Performances with their Hyper-Parameter Settings

CNN Model I.D.	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
No. of Filters per CONV	64	16	64	64	64	64
Data Augmentation	No	Yes	Yes	Yes	Yes	Yes
Initial Learning Rate	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004
Dropout Keep Probability	0.50	0.50	0.50	0.33	0.5	0.30
Weight Initializer std dev	0.015	0.015	0.015	0.015	0.015	0.015
Time cost per training step	72 secs	20 secs	77 secs	72 secs	72 secs	72 secs
Total training steps	1200	3000	1200	1700	2500	2500
Final Training Accuracy	0.8633	0.6719	0.7623	0.7695	0.8809	0.7695
Final Validation Accuracy	0.7395	0.6809	0.7334	0.7421	0.7465	0.7499
Test Accuracy	0.7300	0.6794	0.7340	0.7351	0.7427	0.7474

5. Conclusion

In this project, a simple vanilla shallow CNN network is designed and trained using the CIFAR-10 dataset. The final selected CNN model is able to achieve an accuracy score of 74.74% on the test set after 2500 steps of training. This performance is slightly lower than the benchmark score produced by the FLS-CNN model[14] and fairly far away from the state of art performance score of 96.53%[13]. A deeper network usually has higher potential in resulting a better performance than a shower network. Given that the architecture of the designed CNN model in this project has one more convolutional layer than the FLS-CNN model[14], the author believes that there is potential for further improve the performance of Model 6 to beat the accuracy score of 75.86% obtained by the FLS-CNN model[14].

In Figure 14 and Table 4, comparing Model 5 and Model 6, it is shown that increasing dropout regularization helps diminish the appearing gap between the training loss and the validation loss and leads to a small increase in performance. Hence perhaps further enhancing regularization (dropout and L2 regularization) would possibly help further improve model performance. Given that 2500 training-step is a rather small training-step setting, increasing model training steps is another simple approach worth to try out for improving performance. Additionally, batch

normalization[18] is another technique known for helping network converging faster, preventing overfitting and improving performance. Perhaps implementing batch normalization is another approach for further improve model performance in the next step. (As a matter of fact, the author has attempted to insert batch normalization layers into the designed CNN architecture in this project. However, unfortunately, there appears to be some bug in the implementation which leads to strange training performances and will need more time to resolve) Lastly, ensemble predictions of multiple trained models are known to be an effective way to increase performance, it is worth exploring if enough computation resources are available.

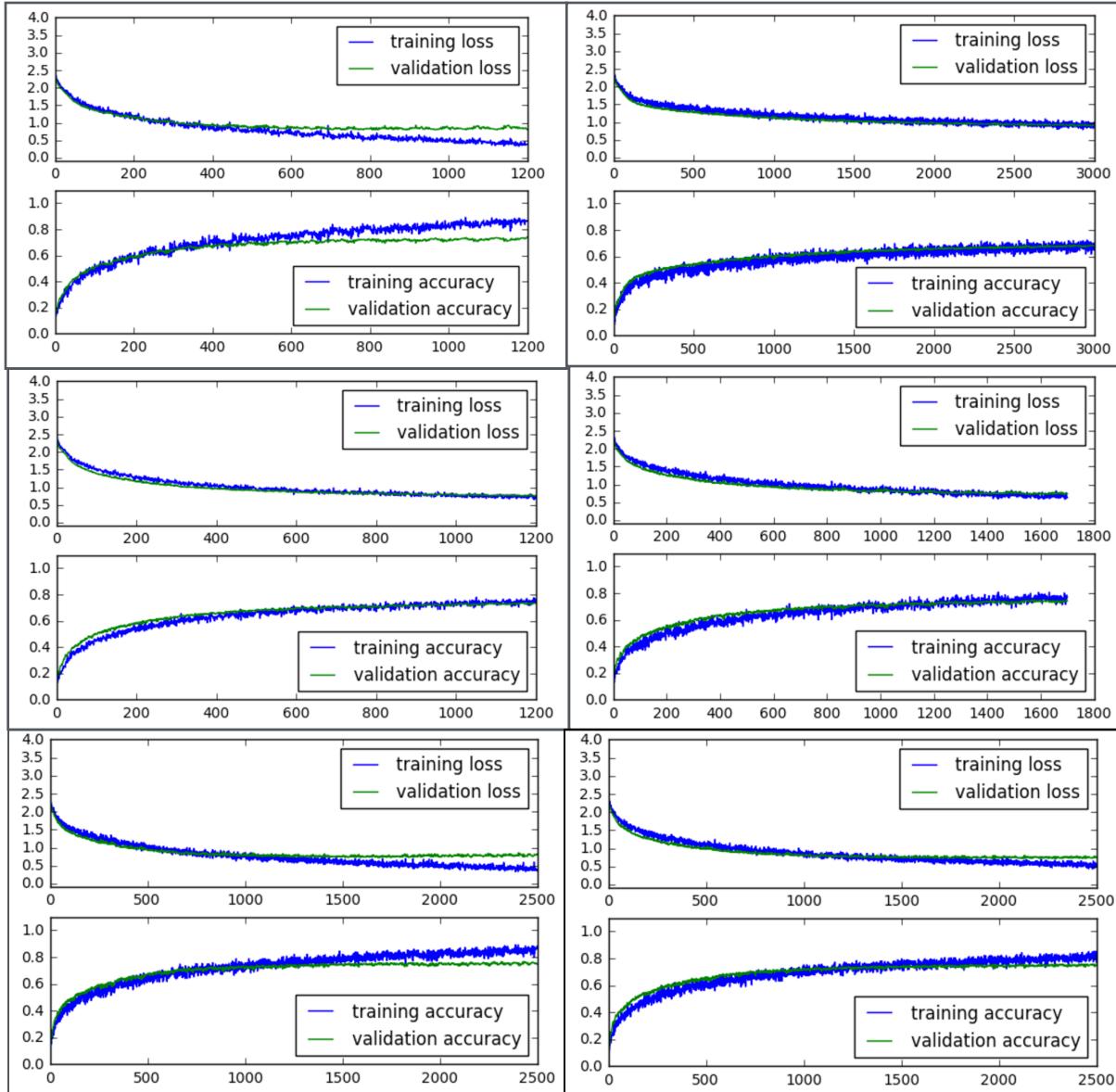


Figure 16. Six CNN model performances trained with different hyper-parameters. Top Left: Model 1. Top Right: Model 2. Middle Left: Model 3. Middle Right: Model 4. Bottom Left: Model 5. Bottom Right: Model 6. (Model ID maps to Table 4)

6. Reflection

As neural networks are also statistical models, although often omitted, the induction (from observations to generalization) that a deep learning model is trying to perform is highly sensitive to the distribution of the data. While practical experiences indicate that data augmentation improves performance by reinforcing certain classification irrelevant statistical invariance and increasing training sample size, a “careless” mistake made during this project demonstrates that although deep learning neural network can effectively learn the deliberately inserted statistical invariances from manually introduced variances, it is also very sensitive to the sample distribution of data. Figure 15 is a performance record after the “careless” mistake was made. Initially, the augmented data was simply concatenated after the original training samples. During training, whenever the mini-batch switches from feeding the network with the original training samples to the augmented training samples, the training loss experiences a sudden jump up and the training accuracy experiences a sudden performance decline. This sudden dip of model performance demonstrates that the CNN model is very sensitive to its mini-batch data’s distribution change. However, as it is shown in Figure 15, whenever after the sudden jump happens, the loss does still converge and the performance does also improve gradually, this implies that the model is able to adjust itself to learn a new induction rule to perform the classification task by purely observing the new sample distribution characteristics (of the augmented data). Eventually, this “careless” mistake was easily corrected by randomly shuffle the concatenated training set (original training data + augmented copy of training data) and then the repeatedly appeared performance dips in Figure 15 disappeared (as shown in Figure 14).

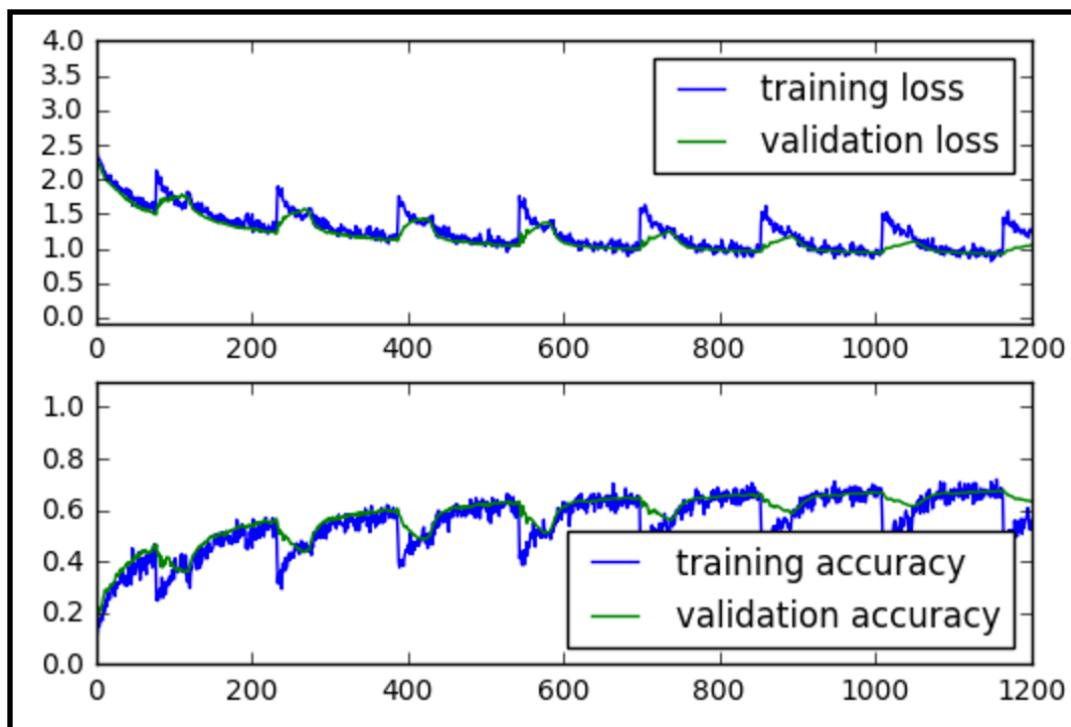


Figure 15. Performance record before shuffling original training data and augmented data

7. Reference

- [1] <http://image-net.org/>
- [2] Krizhevsky, A., Sutskever, I. and Hinton, G.E., 2012. Imagenet classification with deep convolutional neural networks. In Advances in neural information processing systems (pp. 1097-1105).
- [3] Lin, Y., Lv, F., Zhu, S., Yang, M., Cour, T., Yu, K., Cao, L. and Huang, T., 2011, June. Large-scale image classification: fast feature extraction and svm training. In Computer Vision and Pattern Recognition (CVPR), 2011 IEEE Conference on (pp. 1689-1696). IEEE.
- [4] <http://deeplearning.net/datasets/>
- [5] <http://www.cs.utoronto.ca/~kriz/cifar.html>
- [6] Szegedy, C., Liu, W., Jia, Y., Sermanet, P., Reed, S., Anguelov, D., Erhan, D., Vanhoucke, V. and Rabinovich, A., 2015. Going deeper with convolutions. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (pp. 1-9).
- [7] He, K., Zhang, X., Ren, S. and Sun, J., 2015. Deep residual learning for image recognition. arXiv preprint arXiv:1512.03385.
- [8] Szegedy, C., Ioffe, S. and Vanhoucke, V., 2016. Inception-v4, inception-resnet and the impact of residual connections on learning. arXiv preprint arXiv:1602.07261.
- [9] <http://cs231n.github.io/convolutional-networks/>
- [10] <http://cs231n.github.io/neural-networks-1/>
- [11] Srivastava, N., Hinton, G.E., Krizhevsky, A., Sutskever, I. and Salakhutdinov, R., 2014. Dropout: a simple way to prevent neural networks from overfitting. Journal of Machine Learning Research, 15(1), pp.1929-1958.
- [12] [Collected CIFAR-10 Benchmark Results](#)
- [13] Graham, B., 2014. Fractional max-pooling. arXiv preprint arXiv:1412.6071.
- [14] McDonnell, M.D. and Vladusich, T., 2015, July. Enhanced image classification with a fast-learning shallow convolutional neural network. In 2015 International Joint Conference on Neural Networks (IJCNN) (pp. 1-7). IEEE.
- [15] Glorot, X. and Bengio, Y., 2010, May. Understanding the difficulty of training deep feedforward neural networks. In Aistats (Vol. 9, pp. 249-256).
- [16] He, K., Zhang, X., Ren, S. and Sun, J., 2015. Delving deep into rectifiers: Surpassing human-level performance on imagenet classification. In Proceedings of the IEEE International Conference on Computer Vision (pp. 1026-1034).
- [17] Kingma, D. and Ba, J., 2014. Adam: A method for stochastic optimization. arXiv preprint arXiv:1412.6980.
- [18] Ioffe, S. and Szegedy, C., 2015. Batch normalization: Accelerating deep network training by reducing internal covariate shift. arXiv preprint arXiv:1502.03167.