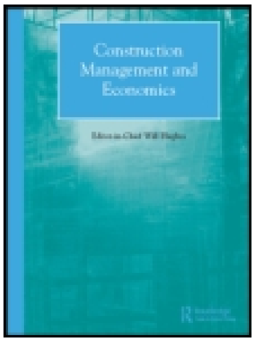

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An economic game theory model of subcontractor resource allocation behaviour

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An economic game theory model of subcontractor resource allocation behaviour

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Periodic review and adjustment of resource allocations to construction projects is critical for subcontractors to maintain profitability under traditional unit price or lump sum contracts. Project managers strive to control subcontractors in an effort to meet budgets and schedules; subcontractors often work on multiple projects simultaneously and strive independently to allocate resources to those projects where they perceive that they will bring maximum utility. An economic game theory model is proposed as a foundation for understanding the behaviour of subcontractors in allocating resources to projects. The model describes the influence of the degree of reliability of the planned schedule on subcontractors' and project managers' behaviours under traditional unit price contracting. Unreliable plans undermine efforts to promote cooperative behaviour.

Keywords: Decision making, game theory, lean construction, partnering, resource allocation, subcontracting

Introduction

In construction projects, it is common for general contractors or project managers to retain numerous specialty subcontractors to perform the majority of the work. Each construction project manager is highly focused on the project for which he/she is responsible. Subcontractors, on the other hand, perform work on multiple projects simultaneously. For subcontractors working under unit price or lump sum contracts, optimisation of their workers' productivity across multiple projects at any given time is important, because labour productivity is a direct determinant of economic success or failure. Ideally, the number of workers assigned to each project should be sufficient to perform the work available, but no more. However, the quantity of work available at each project is frequently not constant through time, whether by design or as a result of plan instability (O'Brien and Fischer, 2000).

Plan instability occurs when the work actually made ready for a subcontractor in a project is different in timing, quantity or rate from that predicted by the construction plan. The difference between actual and plan results from variability and uncertainty and is inherent in most projects (Hanna *et al.*, 1999; Tommelein *et al.*, 1999; Moselhi *et al.*, 2005; Sacks and Goldin, 2005).

Both predetermined fluctuations in work quantities and plan instability lead subcontractors to adjust resource allocations between projects from time to time in accordance with their perception of the workload that will be made available at each project.

The underlying hypothesis of this work is that there is a dissonance between the economic motives of project managers and subcontractors in most construction projects that, if not recognised and dealt with, can lead to a cycle of reduced trust and instability in production control. The high degree of waste common in most construction projects (Serpell *et al.*, 1995; Howell *et al.*, 2001; Koushki *et al.*, 2005; Love and Edwards, 2005) suggests that both parties function in a sub-optimal lose–lose equilibrium state. The notion that waste is present, and that instability and variability in production systems is one of its root causes, is central to lean thinking (Womack and Jones, 2003).

Various approaches have been proposed for dealing with this problem. Some aim to improve relationships directly by considering alternative contractual arrangements, such as partnering (Fisher and Green, 2001; Rahman and Kumaraswamy, 2004). Others aim to treat the problem at the operational level, such as the 'Last Planner' technique (Ballard, 2000). However,

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there is no clear theoretical model of subcontractors' resource allocation behaviour that can guide the development and refinement of such approaches.

This research proposes an economic model of the subcontractor resource allocation scenario using game theory. The following sections provide a background to subcontracting in construction, develop the economic formulation of the relationship, and outline the game theory model. The latter sections discuss the findings, draw conclusions, and propose specific directions for future research.

Subcontracting in construction

The importance and extent of subcontracting is familiar to all involved in construction. Its prevalence has been documented in numerous studies (Hinze and Tracey, 1994; Hsieh, 1998; Edwards, 2003). For example, the proportion of workers employed by subcontractors in the UK rose from 25% in 1983 to 45% in 1998 (Edwards, 2003). In the US, a 1998–99 study of general contractors in commercial construction found that 90.9% of the trades are subcontracted more than 75% of the time (Costantino and Pietroforte, 2002). Some of the reasons cited are increasing sophistication and specialisation of trades (which requires long-term investment in personnel and equipment), fluctuating demand for construction services (which demands agility in adjusting capacity) and transfer of liability (general contractors reduce risk).

Walsh *et al.* (2003) reported an example of large-scale residential housing construction in which subcontracting was taken to an extreme level of fragmentation and specialisation. The subcontractors are termed 'hyper-specialised', and the resulting project management strategy has been to generate fixed construction schedules with large time buffers between subcontractors. As a result, work-in-progress inventory (WIP)[1] is high and project durations are far longer than the net sum of the actual working times required.

An important characteristic of the subcontracted work environment is that multiple subcontractors each perform work on multiple projects simultaneously. Each subcontractor strives to maximise its workload so as to ensure a ready supply of available work (a backlog) at any given time for optimum resource utilisation (O'Brien, 2000). This is commonly done without any consideration for the interests of any other subcontractor (Mathews *et al.*, 2003). In projects run with 'tight' centralised control, project managers strive to control the subcontractors on their projects to maintain stability and schedule-compliance, most commonly by pushing them to perform even when conditions do not allow the work to be done efficiently or at the required level of quality (Kim and Paulson, 2003). If loose control is exercised, subcontractors perform activities subject to their individual resource constraints, rather than in compliance with the schedule, resulting in projects with long durations (Kim and Paulson, 2003). In many cases, in fact, subcontractors withhold true resource availability and scheduling information from the general contractor (Choo *et al.*, 1999).

Partnering between general contractors and subcontractors aims to avoid short-term opportunistic behaviour and encourage cooperation to achieve common goals (Rahman and Kumaraswamy, 2004). One of many formal definitions of partnering defines it as 'a management approach used by two or more organisations to achieve specific business objectives by maximising the effectiveness of each participant's resources. The approach is based on mutual objectives, an agreed method of problem resolution and an active search for continuous measurable improvements' (Bennett and Jayes, 1995). Partnering may be undertaken for a single project, or pursued as a long-term strategy. Unfortunately, partnering is often adopted by subcontractors as a short-term response to pressure from powerful clients, rather than being a fundamental cultural change (Fisher and Green, 2001). If not implemented correctly, it can have detrimental effects on subcontractors (Packham *et al.*, 2001). As Bresnen and Marshall (2000) point out, real cultural change requires an understanding of factors that dictate the basic interests of the parties involved. It is such an understanding, in the area of resource allocation, that this research seeks to explore.

Eccles (1981) suggested that construction subcontracting was not thoroughly adversarial, but rather that companies form long-term relationships and operate in a 'quasi-firm' model, which offers some of the advantages of a vertically integrated hierarchical organisation. However, empirical evidence from the US has shown that while the model may remain applicable to homebuilders, commercial construction contractors operate in highly competitive fashion when selecting and working with subcontractors (Costantino and Pietroforte, 2002).

The common wisdom regarding the relationship between a general contractor (GC) and its subcontractors (subs) in construction is revealed in the following:

> Undoubtedly, however, the provision having paramount importance to the success of the job is the schedule. A schedule that is unrealistic for one or more parties can be a disaster for all who plan on it. … the GC must monitor closely each day the activities of each sub and

> point out any evidence of slippage in schedule … the GC can frequently assist the sub in guidance and advice on the best use of manpower in order to meet the schedule. The more detailed the effort on the part of the GC in planning each phase of the project—and specifically the operations of each sub—the more likely it is the overall schedule will be realized. (Proctor, 1996)

But how can the GC incorporate considerations such as capacity costs of resource allocations, which are critical for subcontractors, but encompass considerations across projects (O'Brien and Fischer, 2000)? While calling for consideration of the scheduling interests of all parties at the outset, the approach leaves no room for schedule flexibility or for coping with variability once a project has begun. Only the subcontractor itself can plan its own operations to create a schedule that is economically realistic; only the subcontractor is in a position to re-evaluate its constraints from time to time as they develop in different projects running simultaneously, each of which is subject to change.

Economic analysis

Local optimisation often retards process flow (Hopp and Spearman, 1996; Womack and Jones, 2003). The following model explains the economic aspect of this tenet of lean thinking in the context of the subcontracted construction environment. The economic model begins with a statement of the basic goal from the subcontractor's point of view: to maximise profitability over some period of time T. Assuming that the subcontractor is remunerated according to the quantity of work performed in each project, which is the case for both unit price and lump sum contracts, this can be detailed as follows:

$$\max P_T : P_T = \sum_i I_i - E_F \qquad (1)$$

$$I_i = W_i U_i - W_i C_{M_i} - b(W_{P_i} - W_i) C_{M_i} - \frac{W_{P_i}}{\bar{r}} C_{S_i} - C_{O_i} \qquad (2)$$

where P_T is the subcontractor's pre-tax profit over time T, I_i is its net income from project i during any period T, E_F are its head-office fixed expenses (salaries and overheads). The terms on the right-hand side of equation (2) represent income for work performed, cost of materials actually consumed, cost of excess material, resource costs and fixed overheads: W_i is the actual work performed on project i during any period T, U_i is the contracted unit price for the works at project i, C_{Mi} is the unit cost of the materials for the works at project i, b is the waste factor for materials that remain unused at the end of any period T, W_{Pi} is the quantity of work planned in project i in period T, $\bar{r}$ is the average planned work rate, C_{Si} is the cost per unit of time for one unit of resources allocated by the subcontractor to project i (assumed constant over period T), and C_{Oi} is the management overhead cost for project i over period T.

For the sake of simplicity, it is assumed that only one type of work is performed by each subcontractor at each project, that U_i and C_{Mi} are constant for any project i, and that fixed expenses (including salaries and overheads) are constant over time period T. Under these assumptions, only the quantities of actual work performed and work planned are variable. The subcontractor's challenge in any period T is to set W_{Pi} for each project to maximise P_T, subject to uncertainty about the outcome for W_i at each project.

Tommelein *et al.* (1999) showed that variability in the rate at which work is supplied from one trade to another leads to degradation in performance of the downstream trades, and to lengthening of the project duration. An implicit assumption in that research was that average production capacity (the resources) of each trade was maintained at a constant level. However, in a subcontracted project environment, each subcontractor will periodically monitor and adjust the quantity of resources applied to each project. Determination of the correct resource level must take into account the expected amount of work that will be made available, because the maximum work that can actually be performed in period T on any project i is always the lesser of (a) the work that can be performed given the resources assigned; and (b) the work that is actually provided by the general contractor.

The maximum work that can actually be performed, W_i, is limited to the work actually made available, W_{Ai}: i.e. $W_i \leqslant W_{Ai}$. Work is only available for execution when the work area is free (preceding work teams/subcontractors have completed their work and cleared the area), the materials have been delivered, the information that controls the work is provided, and project management has signalled that work can begin. Koskela (2000) detailed seven resource flows that are prerequisite for the successful execution of a construction task. Some of the flows, such as updated design information, are not visible: in traditional push scheduling systems, the work available may be revealed to be less than apparent initially only after the work has begun, if management signals for work to start without ensuring that these conditions will be satisfied.

The quantity of work actually made available also has a second order impact on profitability, because productivity itself is a function of work quantity and space (O'Brien, 2000). The 'learning curve' effect also affects production rates (Thomas *et al.*, 1986). Here, the focus is on the subcontractors' strategy in allocating resources; for the sake of simplicity, the second order effect, learning curve, material waste and overheads will be ignored. The simplified expression for the subcontractor's net income for any single project in period T is:

$$I_i = W_i(U_i - C_{M_i}) - \frac{W_{P_i}}{\bar{r}} C_{S_i} \tag{3}$$

If the subcontractor assumes a stable, deterministic workflow (i.e. with no downtime, or in other words, the work actually made available, W_{Ai}, is equal to the work demanded by the project manager, W_{Di}) and therefore assigns resources sufficient to meet the general contractor's demand, then the resources supplied are:

$$R_i = \frac{W_{P_i}}{T\bar{r}} = \frac{W_{D_i}}{T\bar{r}} \tag{4}$$

and the resource cost over period T is fixed at:

$$E_{S_i} = R_i T C_{S_i} = W_{D_i} \frac{C_{S_i}}{\bar{r}} \tag{5}$$

where R_i is the number of units of resource allocated by the subcontractor to project i over period T and E_{Si} is the cost for the resources assigned to project i over the same period. In reality, there are practical limits to the range in which resources can be assigned, such as the minimum number required to perform work at all and the ratio of flexible to fixed resources (O'Brien, 2000). In this case, the net income for project i over period T is:

$$I_i = W_i(U_i - C_{M_i}) - W_{D_i} \frac{C_{S_i}}{\bar{r}} \tag{6}$$

and the net income per unit of work actually performed is

$$\frac{I_i}{W_i} = (U_i - C_{M_i}) - \frac{W_{D_i}}{W_i} \frac{C_{S_i}}{\bar{r}} \tag{7}$$

However, the actual net income is dependent on the availability of work:

If $W_i = W_{Ai} \leq W_{Di}$ then

$$\frac{I_i}{W_i} = (U_i - C_{M_i}) - \frac{W_{D_i}}{W_{A_i}} \frac{C_{S_i}}{\bar{r}} \tag{8}$$

Alternatively, if $W_{Ai} > W_{Di}$, then $W_i = W_{Di}$, since the resources become fully utilised, and so

$$\text{so } \frac{I_i}{W_i} = (U_i - C_{M_i}) - \frac{C_{S_i}}{\bar{r}} \tag{9}$$

Thus a theoretical upper limit is imposed on the unit profitability. This relationship is plotted in Figure 1 as the curve denoted k=1.0 (k is the ratio of resources supplied to resources demanded). The subcontractor's profitability is dependent on the ratio of the quantity of work that can actually be performed (W_{Ai}) to the quantity of work that was demanded (W_{Di}). However, the subcontractor can protect against this dependence by reducing the resources assigned to the project. In this case, the resources allocated to the project are less than what is required to meet the general contractor's demand: $R'_i \leq \frac{W_{D_i}}{T\bar{r}}$ or $R'_i = \frac{W_{P_i}}{T\bar{r}} = k\frac{W_{D_i}}{T\bar{r}} = kR_i$ where $0 < k \leqslant 1$. The total cost of the resources is now $E'_{s_i} = R'_i T C_{S_i} = k W_{D_i} \frac{C_{S_i}}{\bar{r}}$ and the net income will depend on whether the actual work made available is smaller or greater than the capacity provided:

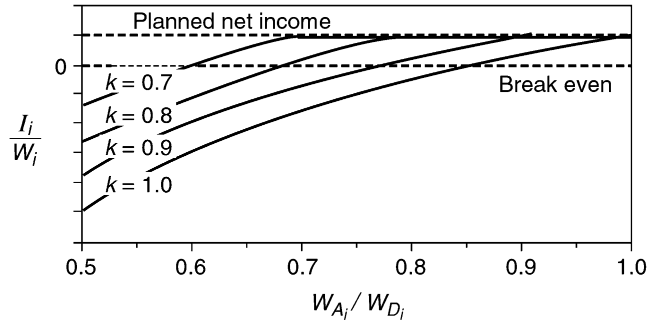

Figure 1 Typical plot of subcontractor's net income per unit of work as a function of the ratio of actual work performed to work demanded

$$\begin{aligned} &\text{If } \quad W_i = W_{Ai} \leq R'_i T\bar{r} \\ &\text{then } I_i = W_{Ai}(U_i - C_{M_i}) - k W_{D_i} \frac{C_{S_i}}{\bar{r}} \end{aligned} \tag{10}$$

$$\text{or } \quad \frac{I_i}{W_i} = (U_i - C_{M_i}) - k \frac{W_{Di}}{W_{A_i}} \frac{C_{S_i}}{\bar{r}} \tag{11}$$

$$\begin{aligned} &\text{If } \quad W_i = R'_i T\bar{r} \leq W_{Ai} \\ &\text{then } I_i = W_i(U_i - C_{M_i}) - R'_i T C_{S_i} \text{ or} \\ &\frac{I_i}{W_i} = (U_i - C_{M_i}) - \frac{R'_i C_{S_i}}{R'_i \bar{r}} = (U_i - C_{M_i}) - \frac{C_{S_i}}{\bar{r}} \end{aligned} \tag{12}$$

The latter expression is independent of W_{Ai}, which implies that the subcontractor can achieve full confidence in its level of profitability. It follows that if the subcontractor can estimate W_{Ai}, then income can be optimised by setting $W_{Pi} = W_{Ai}$, i.e. $R'_i = \frac{W_{Ai}}{T\bar{r}}$, which can be expressed as $R'_i = kR_i = k\frac{W_{Di}}{T\bar{r}} = \frac{W_{Ai}}{T\bar{r}}$, yielding $k = \frac{W_{Ai}}{W_{Di}}$.

If the subcontractor were able to assess the probability distribution for the actual work that it expects will be made available in terms of the demand stated by the general contractor, then the value k could be set according to any given desired level of confidence. Figure 2 shows a theoretical probability distribution estimated for P[W_{Ai}/W_{Di}], with the maximum probability at one, but the mean slightly less than one—representing a relatively high degree of confidence in

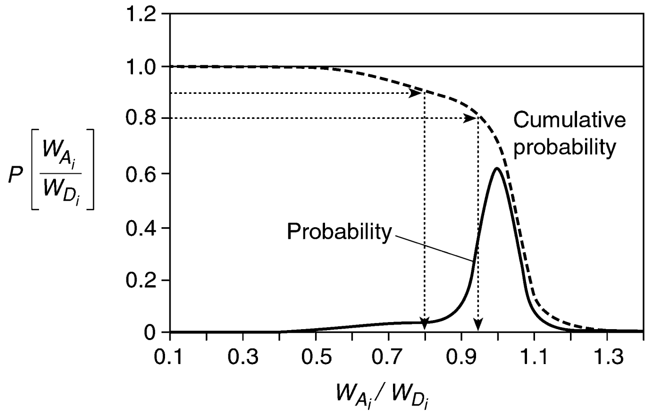

Figure 2 Subcontractor's perception of work to be made available

the general contractor's ability to maintain a schedule. If the subcontractor wishes to have an 80% probability of achieving the planned unit profitability for the work in time period T, it must set $k \approx 0.95$. For a 90% probability, $k \approx 0.80$.

This suggests that at the start of a subcontractor's activity at any construction site, it will assign resources to the project according to its perception of the general contractor's reliability in supplying work at the rate that has been demanded of them. As the project progresses, the subcontractor is likely to revise the resource assignment in response to the actual performance of the general contractor in supplying work. However, the tendency will always be to err on the side of caution—to supply slightly less than the quantity demanded—to attempt to ensure that the resources are always fully utilised.

This economic formulation provides a limited insight into the motives of subcontractors faced with uncertainty about the quantity of work that will be made available in projects governed by unit price contracts. In the next section, the likely behaviour of a project manager and a subcontractor in an uncertain work environment are modelled using game theory.

Game theory analysis

The mathematical approach to game theory is generally attributed to von Neumann and Morgenstern (1947). The basic premises for the game theory analysis are that both the general contractor's project manager (PM) and the subcontractor's manager (SUB) behave rationally, which means that:

- the action chosen by the PM or the SUB is at least as good, according to his or her preferences, as every other available action (Fudenberg and Tirole, 1991; Osborne, 2004);
- they are both in a continuing conflict, as defined by Luce and Raiffa (1957);
- and they are both players in a non-cooperative game, which means that each is concerned only with his or her results (Gass, 1985; Osborne and Rubinstein, 1994).

The 'game' models the allocation of resources at the start of each planning period in a project. The PM sets the amount of work to be performed by the SUB in the period on the basis of the construction plan. In response, the SUB determines and supplies the resources it deems appropriate. The respective 'moves' of the players are: PM demands work, SUB allocates resources. The same sequence is repeated at the start of each ensuing period until the SUB's work on the project is complete.

The *extensive form* (Osborne and Rubinstein, 1994) is appropriate in this context because the moves are sequential and because both PM and SUB have imperfect knowledge about the outcome in terms of the work that will actually be accomplished (i.e. they cannot predict with certainty how much work will be made available by the upstream contractors, or whether design changes, material delays, weather conditions or other factors will interrupt or slow work). The extensive form can be repeated in order to examine long-term strategies that develop as the parties respond to one another's previous actions and develop a relationship over time, which may facilitate cooperative behaviour (Lazar, 2000).

The situation is modelled as shown in Figure 3. The branches of the root node of the tree represent the range of possible results in terms of the amount of work that will actually be performed by the SUB. They are expressed using the ratio of work actually performed to the work initially planned, q, where $q = W_A/W_D$. The probability of any particular value of q is described by a probability distribution, P[q], which is essentially a measure of plan reliability at the site. In Figure 3, it is simplified to four discrete values: 10% probability that 80% of the work planned for in any period will be possible, 20% that 90% will be possible, 50% chance of 100%, and 20% likelihood that the work possible will exceed that planned by 10%. The cumulative probability that at least 100% will be performed is 70%, and the weighted average of the distribution is 98%. The results of extensive empirical research (Bortolazza *et al.*, 2005) suggest that this is a medium to high level of plan reliability.

The PM's possible moves are detailed at the next level of Figure 3 using the unitless ratio d, which is the ratio of

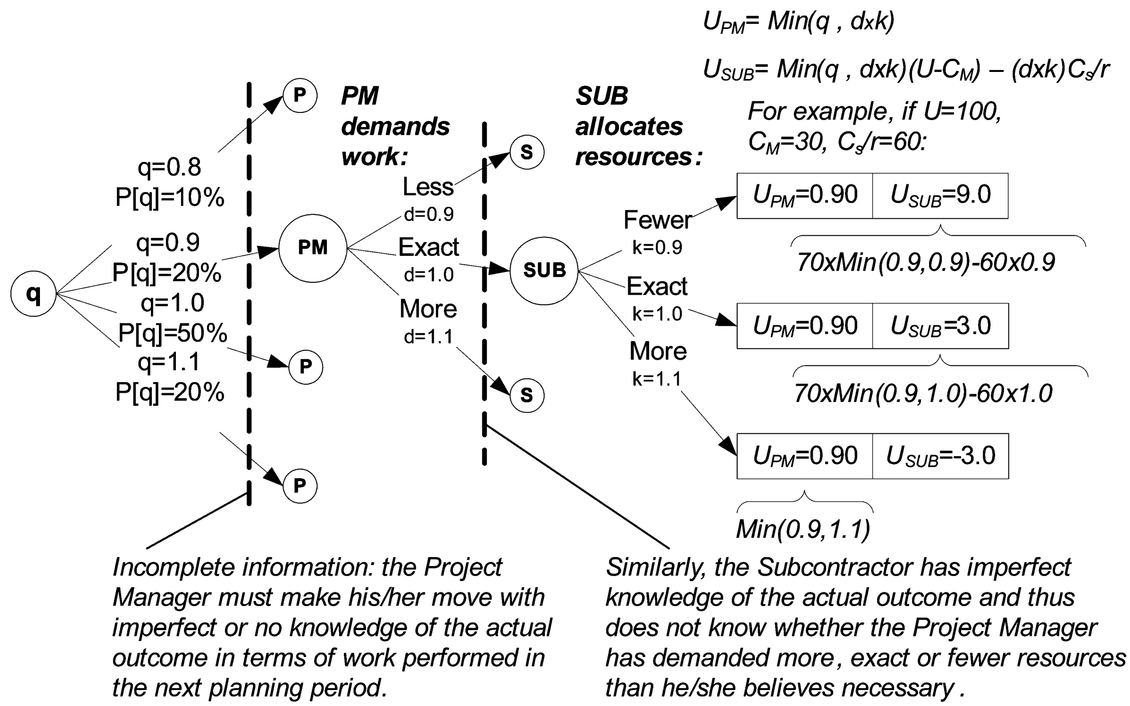

Figure 3 Extensive form game of subcontractor resource allocation

the work demanded to the work estimated to become available. In reality d is continuous, but it is modelled here by three discrete values: demand for less work than estimated (d=0.9), exactly the amount estimated (d=1) and more than estimated (d=1.1). In response to the PM's request, the SUB can then elect to allocate fewer resources than required for the work demanded (setting k=0.9, where k is the ratio of resources supplied to those demanded, as defined in the previous section), exactly the amount required (k=1) or more than demanded (k=1.1). The latter strategy reflects a situation in which the SUB has resources available, and is willing to commit them in the hope that more work than expected will in fact become available and that they would be utilised profitably.

The utilities for each player are calculated at the end node of each branch of the tree. The utility for the PM is taken to be the total amount of work actually completed in the planning period. When insufficient resources are allocated (i.e. when $q \geqslant \mathrm{d}k$), the work done is constrained by the quantity of resources available, and is proportional to dk; on the other hand, when $0 \leqslant q < \mathrm{d}k$, the work done is constrained by the availability of work and is therefore directly proportional to q. Thus the utility for the PM, U_{PM}, is given by $U_{PM}=Min(q, \mathrm{d}k)$. The SUB's utility, U_{SUB}, is defined as the total income derived from the work done in the planning period, and is calculated according to equation (10). However, the work done is constrained by the resources when $q \geqslant \mathrm{d}k$. The SUB's utility (after dividing by W_D, which is constant, and substituting $q=W_A/W_D$) is therefore:

$$U_{SUB}=Min(q, \mathrm{d}k)(U-C_M)-\mathrm{d}k\frac{C_S}{\bar{r}} \tag{13}$$

For example, as shown in Figure 3, if the work made available is q=0.9, the unit price is U=100, the material cost per unit of work is C_M=30, and the unit resource cost is $C_S/\bar{r}$=60 in any unit of currency, then the utilities for the case d=1.0 and k=1.1 are U_{PM}=0.9 and $U_{SUB}=-3.0$

In extensive form games with probabilistic outcomes, the utilities are replaced by *expected utilities* to reflect the variability possible in the outcomes. In this case, since the utilities are all functions of q, the expected utility for each combination of PM and SUB strategies is the weighted average of the utilities for each possible result for q, weighted by its probability, which is given by the distribution P[q]. Thus the PM's expected utility is:

$$\begin{aligned} U_{PM}Expected &= \int_0^{\mathrm{d}k} qP[q]\mathrm{d}q+\mathrm{d}k\int_{\mathrm{d}k}^{\infty} P[q]\mathrm{d}q \\ &= \int_0^{\mathrm{d}k} qP[q]\mathrm{d}q+p\,\mathrm{d}k \end{aligned} \tag{14}$$

(where p=P[q>dk]). Similarly, since the labour cost is independent of the amount of work performed, the

SUB's expected utilities are computed as:

$$U_{SUB}Expected = \left(p\,\mathrm{d}k + \int_0^{\mathrm{d}k} qP[q]\mathrm{d}q \right)(U_i - C_{M_i}) - k\frac{C_{S_i}}{\bar{r}} \tag{15}$$

In practice, construction professionals would not estimate a continuous probability distribution, but rather use discrete values at significant intervals, expressed in the form described above and shown in Figure 3.

There are three distinct situations that can be modelled and solved using this formulation:

Case A: Neither the PM nor the SUB has any knowledge of the probability distribution of q, i.e. neither can predict how much work will be possible. This situation of imperfect information is modelled as an *information set* and denoted in Figure 3 by the dashed lines to the left of the PM's and the SUB's nodes.
Case B: The PM has perfect knowledge of q, but the SUB has none; the information set in front of the PM is removed, but that in front of the SUB remains.
Case C: Both have full knowledge of the work to be made possible, which also implies that the SUB is aware of the PM's strategy. In this form both information sets are removed.

The resulting strategies and Nash equilibria[2] for all three conditions are detailed below for the practical numerical example defined by the parameter values detailed above and in Figure 3. The relationship of the three cases and their results to real project conditions is discussed in the subsequent section.

Case A: imperfect knowledge

Calculation of the expected utilities in this form yields a strategic (normal) form game as shown in Table 1. This game has a perfect equilibrium, which is the strategy pair: PM demands more; SUB provides fewer (shown shaded grey in Table 1). Although calculated here for a specific dataset, the result can be shown analytically to be valid for a wide range of resource cost to unit price ratios (see Appendix 1). The equilibrium is insensitive to variation of the profit margin, although when the probability distribution becomes narrow (i.e. plan reliability is higher), an additional equilibrium appears at the strategy pair: PM demands exact; SUB provides exact.

Case B: PM has perfect knowledge, SUB has imperfect knowledge

The strategies available to the PM in this game must be associated with the probability distribution, since the PM now has knowledge of it. For sake of simplicity, this case was solved for a discrete distribution with three nodes: a 100% chance of completing at least 90% of the work, 70% chance of completing at least 100%, and a 20% chance of completing 110%. Thus there are now 27 PM strategies: always demand less in all three situations, demand less in the first two and exact in the third, and so on. They are listed, together with the expected utilities, in Table 2.

This case has two significant equilibria. The first occurs for the PM demanding more work than estimated in every situation and the SUB providing fewer resources. A second equilibrium, which is part of a mixed strategy, occurs when the PM demands exact work when q=0.9 or q=1.0 and more when q=1.1, and the SUB provides exact resources. Both are shaded grey in Table 2.

Case C: both have perfect knowledge

Here there are 27 strategies available for each of the PM and the SUB. A small portion of the resulting 27 × 27 matrix of utility pairs, for the most relevant strategies, is provided in Table 3. Here too there are two main equilibria. The first, similar to those in the previous cases, occurs when the PM demands more work in every case, and the SUB provides fewer resources in every case. The second occurs when exact resources are both demanded and allocated in every case. The numerical differences between them are small.

Table 1 Case A: imperfect information for project manager and subcontractor

Project manager strategies \ Subcontractor strategies	Provide fewer k=0.9		Provide exact k=1.0		Provide more k=1.1	
	PM	SUB	PM	SUB	PM	SUB
Demand less d=0.9	0.809	8.0	0.890	8.3	0.953	7.3
Demand exact d=1.0	0.890	8.3	0.960	7.2	0.980	2.6
Demand more d=1.1	0.953	7.3	0.980	2.6	0.980	−4.0

Table 2 Case B: imperfect information for subcontractor

Project manager strategies \ Subcontractor strategies				Provide fewer k=0.9		Provide exact k=1.0		Provide more k=1.1	
When:	Q=0.9	Q=1.0	Q=1.1	PM	SUB	PM	SUB	PM	SUB
always demand:	Less	Less	Less	0.802	8.02	0.89	8.91	0.98	9.80
	Less	Less	Exact	0.822	8.22	0.91	9.13	1.00	10.04
	Less	Less	More	0.842	8.42	0.94	9.35	1.03	10.29
	Less	Exact	Less	0.847	8.47	0.94	9.41	0.99	6.85
	Less	Exact	Exact	0.867	8.67	0.96	9.63	1.01	7.09
	Less	Exact	More	0.887	8.87	0.99	9.85	1.03	7.34
	Less	More	Less	0.892	8.92	0.94	6.41	0.99	3.55
	Less	More	Exact	0.912	9.12	0.96	6.63	1.01	3.79
	Less	More	More	0.932	9.32	0.99	6.85	1.03	4.04
	Exact	Less	Less	0.826	8.26	0.92	9.18	0.98	8.21
	Exact	Less	Exact	0.846	8.46	0.94	9.40	1.01	8.45
	Exact	Less	More	0.866	8.66	0.96	9.62	1.03	8.69
	Exact	Exact	Less	0.871	8.71	0.97	9.68	0.99	5.26
	Exact	Exact	Exact	0.891	8.91	0.99	9.90	1.01	5.50
	Exact	Exact	More	0.911	9.11	1.01	10.12	1.04	5.74
	Exact	More	Less	0.916	9.16	0.97	6.68	0.99	1.96
	Exact	More	Exact	0.936	9.36	0.99	6.90	1.01	2.20
	Exact	More	More	0.956	9.56	1.01	7.12	1.04	2.44
	More	Less	Less	0.851	8.51	0.92	7.56	0.98	6.43
	More	Less	Exact	0.870	8.70	0.94	7.78	1.01	6.67
	More	Less	More	0.890	8.90	0.96	8.00	1.03	6.91
	More	Exact	Less	0.896	8.96	0.97	8.06	0.99	3.48
	More	Exact	Exact	0.915	9.15	0.99	8.28	1.01	3.72
	More	Exact	More	0.935	9.35	1.01	8.50	1.04	3.96
	More	More	Less	0.941	9.41	0.97	5.06	0.99	0.18
	More	More	Exact	0.960	9.60	0.99	5.28	1.01	0.42
	More	More	More	0.980	9.80	1.01	5.50	1.04	0.66

Table 3 Case C: a portion of the 27×27 matrix of expected utilities for project manager vs. subcontractor

Project manager strategies \ Subcontractor strategies			Provide Fewer, Fewer, Fewer		Provide Fewer, Exact, More		Provide Exact, Exact, Exact		Provide Exact, More, More		Provide More, More, More	
Demand:			PM	SUB	PM	SUB	PM	SUB	PM	SUB	PM	SUB
Less	Less	Less	0.794	7.94	0.882	8.82	0.882	8.82	0.949	9.49	0.970	9.70
Less	Less	Exact	0.815	8.15	0.884	7.40	0.906	9.06	0.951	8.07	0.973	8.29
Less	Less	More	0.837	8.37	0.884	5.82	0.906	7.62	0.951	6.49	0.973	6.70
Less	Exact	More	0.882	8.82	0.934	6.32	0.956	8.12	0.956	3.54	0.978	3.75
Exact	Exact	Exact	0.882	8.82	0.956	8.12	0.980	9.80	0.980	5.36	0.980	3.92
Exact	Exact	More	0.904	9.04	0.956	6.54	0.980	8.36	0.980	3.78	0.980	2.34
Exact	More	More	0.949	9.49	0.956	3.54	0.980	5.36	0.980	0.48	0.980	−0.96
More	More	More	0.970	9.70	0.978	3.75	0.980	3.92	0.980	−0.96	0.980	−2.55

Discussion

In reality, a PM's *a priori* knowledge of the work that will become available for a SUB in any given planning period will depend on his or her skill and experience and on the degree to which the flow of work in the project is stable. When time buffers between teams are short, predicting the workload with accuracy is difficult, because it is not visible and it is uncertain. Work locations are passed from team to team as work progresses, so that estimating the amount of work to be made available for any given team requires judging the work rate of the previous team and foreseeing any other constraints that may arise. Thus most PMs will function within a continuous scale of 'information confidence' that lies between cases A (perfect ignorance) and B (perfect knowledge), as depicted in Figure 4.

SUBs, on the other hand, will function somewhere along an axis between case C and the axis between cases A and B. SUBs have two principal sources of information useful for predicting the workload before making a resource allocation decision:

(1) the PM's assessment and planned demand;
(2) their own independent assessment, including (a) observation of the current situation on the site; and (b) consideration of the historical performance of the general contractor, both in the history of previous work cycles in the current project and in previous projects on which they have worked together.

Consider first the situation where plan reliability performance in the project has proved to be low. The SUB cannot make a reliable prediction, and knows that the PM's prediction is also unreliable. This is close to case A, whose solution clearly indicates that the best strategy for the SUB is to provide fewer resources than demanded, and the best strategy for the PM is to demand more resources. The SUB is essentially protecting itself against uncertainty; the PM is countering the SUB's tendency to self-protection.

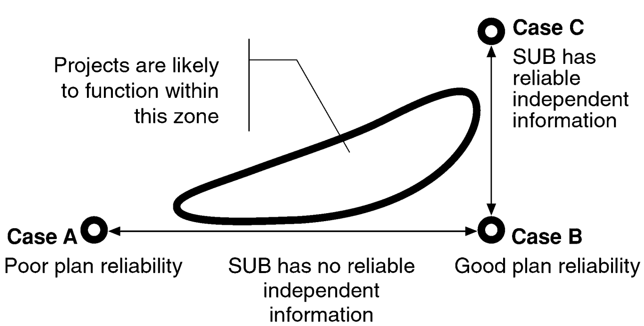

Figure 4 Ranges of PM and SUB function for subcontractor resource allocation

Next, consider the situation where the PM is competent and can better predict what will actually take place on site, and previous experience has led the SUB to respect the PM's capability, but the SUB is unable to make an accurate independent assessment. This is close to case B, because the SUB must decide to what extent to trust the PM's work demand, given that some plan uncertainty remains, and that, knowing this, the PM may have exaggerated the work demand. Case B's game theory solution shows that the only pure equilibrium is for the PM indeed to exaggerate the demand and for the SUB to consistently provide fewer resources.

Unlike for Case A, however, in this case there are additional equilibria which result in better results for both PM and SUB (see Table 2). Unfortunately, they are not pure equilibria, and therefore require cooperation to achieve and maintain. As can be seen from Table 2, once the SUB begins providing resources exactly as demanded (collaborative behaviour), the PM may be tempted to demand more (non-collaborative behaviour), since in periods where more work does indeed became available, he or she would derive greater benefit if more resources were allocated. In future rounds the SUB would be likely to respond in kind (non-collaborative behaviour), resulting in the pure equilibrium, which is sub-optimal for both parties. It is in these situations that partnering agreements can contribute, although only if they engender collaborative behaviour by recognising the divergent incentives; the implications of this result for partnering and lean construction are discussed below.

Finally, consider the situation where the SUB has a very reliable assessment of the project state; this approaches case C (see Figure 4). An obvious situation in practice where this can occur is when a backlog of work has accumulated between the preceding subcontractor and the SUB, making the work available plainly visible. The solution of the game shows that essentially the same two equilibria appear. Where trust is present, the parties are likely to demand and provide exactly the resources required. However, in a less cooperative mode, an equilibrium in which more work is demanded and fewer resources are provided in every case can be stable. Note that the utilities for both situations are close in value: from an operational standpoint, as long as the SUB has reliable information, which of the equilibria is achieved over time may make little difference.

Evaluating remedies

The economic formulation and game theory model enable evaluation of different approaches that the PM and SUB can adopt to achieve short-term collaborative behaviour within the confines of unit price contracts.

The PM, for example, may elect to impose fines or offer rewards for allocation of resources exactly as planned. Imposition of a fine can be modelled by adding a conditional term C_F, the cost of fines, to equations (9) and (13). The SUB's utilities can be recalculated and the new situation modelled. For case A, using the values U=100, C_M=30, and $C_S/\bar{r}$=60 as above, and assuming the fine is applied when fewer resources are provided than demanded (k<1) and the amount of work provided turned out to exceed the capacity of the resources (q>k), the minimum fine needed to move the pure equilibrium for the SUB from 'provide fewer' to 'provide exact' is C_F=6.8 (i.e. 6.8% of contract value for any planning period). For case B, the minimal fine is C_F=4.3. Thus the less the plan is reliable, the greater the fines needed to change behaviour. An alternative positive approach that could be investigated is compensation of subcontractors for wasted capacity when they supply resources exactly as demanded and the work available proves to be less than expected.

The SUB, on the other hand, achieves best results when plan reliability is high. The easiest way for a SUB to achieve this is to allow a buffer of ready work to accumulate between it and the preceding work team, which can be done by delaying the start of work at a site or by providing fewer resources over a number of periods. In this way the SUB positions itself closer to case C. This phenomenon is common in practice and has been reported in the research literature (Thomas *et al.*, 2004). In effect, in the absence of any explicit pull mechanism, the subcontractor increases its confidence in the supply of work by simply waiting for inventory of work in progress to accumulate, i.e. for $q=W_{Ai}/W_{Di}$ to rise.

Implications for partnering and lean construction

Partnering aims to achieve optimal exploitation of the resources of all participants through contractual arrangements. According to Lazar (2000), maintenance of long-term cooperative behaviour (of the kind contemplated in Axelrod's game theory research) requires more than intent—person to person relationships that span beyond individual projects are necessary, and the partners must have not only the will but also the ability to fulfil the commitments made. Packham *et al.* (2001) cite a case study in which a general contractor reverted to non-collaborative behaviour despite a partnering arrangement, with the result that the mechanical and electrical subcontractor in question did the same.

Unfortunately, neither PMs nor SUBs have full control over all of the conditions needed to make resource requests consistently reliable, which means that cases B and C are ideal and cannot be achieved completely in practice. Real projects can be assumed to function in the zone outlined in Figure 4.

A lean approach to moving a construction project to function closer to case C is to improve plan reliability by improving the production system itself. Reducing uncertainty and variability should enable movement to equilibria with higher values of k and higher outcome utilities. The 'Last Planner System' (Ballard, 2000), which enhances plan reliability by employing the concept of pull flow,[3] appears to work in this way.

Miller *et al.* (2002) argue that harmonisation between contractors and subcontractors is a prerequisite for lean construction. While mutual cooperation and partnering arrangements can undoubtedly enhance construction performance, the economic model suggests that it should be possible to engender behaviour that enhances workflow stability at the project level through effective production management.

Model limitations

Although the model explains the economically motivated behaviour of project managers and subcontractors, it only incorporates capacity utilisation considerations, it does not account for the fact that the subcontractor's decisions are made across multiple projects simultaneously, and it does not account for second-order effects. It is valid for larger firms where recurring personal relationships are less likely, that operate under contracts governed by unit price or lump sum contracts, and where SUBs are selected by bid price (i.e. where the 'shadow of the future' can be considered to be weak).

Labour productivity is but one of many factors that may be assumed to influence subcontractors' resource allocation decisions. In a preliminary pilot survey undertaken in the framework of ongoing research in this regard, 29 senior managers of leading subcontractors were asked to rank the factors they considered most relevant (Harel, 2005). The factors, in order of highest to lowest priority, were (1) capacity utilisation/productivity; (2) workflow stability; (3) contractual commitments; (4) level of financial exposure; and (5) cash flow. Factors such as the promise of future work were ranked very low, suggesting that the 'shadow of the future' (Lazar, 2000) needed to support long-term cooperative behaviour, may be short.

Despite excluding these factors, the model nevertheless appears to provide important insights. An explanation can be found in two aspects of decision theory: (a) in situations of uncertainty, where it is impractical to calculate all of the parameters and outcomes, decision makers rely on simplified heuristics (Tversky and Kanhneman, 1974) that focus on the most intuitive and accessible parameters; and (b) decision makers are generally more sensitive to potential losses than to gains (Thaler *et al.*, 1997). The model represents a simplified heuristic, focuses on the most important criteria, and highlights the potential for loss.

Second-order effects arise because plan reliability is in itself dependent, among other things, on the resource allocation behaviour of the subcontractors. Failure to provide the resources demanded can delay the work of successive work teams, which reduces plan reliability, thus impacting in turn on their resource allocation decisions. Conversely, when all of the subcontractors on a project consistently provide exactly the resources demanded, the project manager's ability to achieve plan reliability is enhanced, reinforcing subcontractors' motivation to collaborate.

Conclusions

The primary goal of this work is to seek a theoretical model of conflict between general contractors and subcontractors with regard to resource allocation, its relationship to plan reliability, and its impact on workflow stability. Construction subcontracting is more complex than the simplified economic model and game theory analysis. Nevertheless, the model accurately reflects the adversarial nature of the relationship in regard to resource allocation, showing that lack of faith between project managers and subcontractors is an expected result of the conditions created by traditional unit price or lump sum contractual arrangements, particularly when work plans are unreliable. The model explains how sub-optimal conditions can be stable, how local optimisation works against flow, and how unit price contracting hides the waste of sub-optimal productivity within the price paid per unit of work completed. It represents a first step towards developing a theoretical model of subcontracting in construction, which should aid identification and analysis of ways to align subcontractors' inherent interests with behaviour that enhances flow in construction projects.

Partnering is intended to improve performance by aligning long-term interests and increasing trust between project participants. The model suggests that partnering can only reach its full potential if the economic risk of low productivity due to plan instability can be shared equitably, in a way that makes it in the short-term interests of all parties actively to foster collaborative behaviour and achieve stable workflows. Sharing production-planning information with subcontractors openly can reduce uncertainty and help avoid defensive behaviour. One way to achieve this is to involve them directly in setting realistic production goals; this is an aspect tackled by lean construction techniques.

Empirical research is needed to establish whether long-term cooperation arrangements do in fact increase subcontractors' willingness to allocate resources exactly as demanded, even when short-term losses occur. Such research would attempt to measure the strength of the 'shadow of the future' in shaping behaviour.

Much of the research and implementation of lean construction has been carried out within the conceptual boundary of a single project or a single value stream. However, construction functions at a multi-project, multi-subcontractor level. A better and more complete theoretical model, which can account for multiple projects and multiple subcontractors, is needed. It should consider the social, character, organisational and other aspects of subcontractors that dictate their behaviour as well as the economic and financial aspects; and it should recognise that different projects have different environments (strategic, physical and contractual) that determine local levels of productivity.

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Notes

1. Work-in-progress inventory (WIP) is the total amount of unfinished goods present in a production system at any given time.
2. In game theory, a Nash equilibrium can be defined as 'an optimal collective strategy in a game involving two or more players, where no player has anything to gain by changing only his or her own strategy. If each player has chosen a strategy and no player can benefit by changing his or her strategy while the other players keep theirs unchanged, then the current set of strategy choices and the corresponding payoffs constitute a Nash equilibrium' (Wikipedia, 2005).
3. Pull flow is a method for assigning work in production systems in which products are processed according to actual demand from the end of a system rather than being pushed through the system according to predetermined forecasts of demand.

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Appendix 1

Analytical proof of the range of validity for Case A

The subcontractor's strategy 'Provide fewer resources' will have greater expected utility than 'Provide exact resources' if the following condition (based on the formula for U_{SUB}—equation (15)) holds true:

$$\left(p_k k + \int_0^k qP[q]\mathrm{d}q\right)(U_i - C_{M_i}) - k\frac{C_{S_i}}{\bar{r}}$$

$$\geq \left(p_1 + \int_0^1 qP[q]\mathrm{d}q\right)(U_i - C_{M_i}) - \frac{C_{S_i}}{\bar{r}}$$

where $k<1$ (provide fewer resources) on the left and $k=1$ (provide exact resources on the right), $p_k=P[q\leqslant k]$ and $p_1=P[q\leqslant 1]$, both of which can be estimated for any given project. The condition reduces to:

$$\left(p_k k - p_1 + \int_k^1 qP[q]\mathrm{d}q\right)(U - C_M) + \frac{C_S}{\bar{r}}(1-k) \geq 0$$

Over the small range of $P[q]$ between k and 1, $\int_k^1 qP[q]\mathrm{d}q \approx \frac{1}{2}(k+1)(p_1 - p_k)$. Thus the condition is: $\left(p_k k - p_1 + \frac{1}{2}(k+1)(p_1 - p_k)\right)(U - C_M) + \frac{C_S}{\bar{r}}(1-k) \geq 0$, which reduces to: $\dfrac{C_S/\bar{r}}{(U - C_M)} \geq \dfrac{(p_k + p_1)}{2}$

For the numerical example, where $U=100$, $C_M=30$, and $C_S/\bar{r}=60$, $p_1=0.3$ (70% probability of providing 100% of the work planned) and for $k=0.9$, $p_k=0.1$, the left hand side equates to 60/70=0.86. The limiting condition on the right-hand side is (0.1+0.3)/2=0.2. Thus the condition would hold true for all values of the labour cost until it were reduced to 14, which is just 14% of the unit price.