Construction Management & Economics

ISSN: 0144-6193 (Print) 1466-433X (Online) Journal homepage: https://www.tandfonline.com/loi/rcme20

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To cite this article: A. H. Boussabaine & Taha Elhag (1999) Applying fuzzy techniques to cash flow analysis, Construction Management & Economics, 17:6, 745-755, DOI: 10.1080/014461999371088

To link to this article: https://doi.org/10.1080/014461999371088

Construction Management and Economics (1999) **17**, 745–755

Applying fuzzy techniques to cash flow analysis

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Received 20 February 1998; accepted 29 October 1998

Construction managers are interested in the direction of movement of cash flow at valuation periods rather than its forecast value, and fuzzy set theory applied to decision making might help in this process. Fuzzy models are particularly suited to making decisions involving new technologies where uncertainties inherent in the situation are complex. The problem of healthy cash flow at valuation periods relates to the proper estimation of cash in and out flows and project progress. The paper presents an alternative approach to cash flow analysis for construction projects. This project is based on the assumption that cash flow at particular valuation stages of a project is ambiguous. The paper discusses the weaknesses of existing methods for cash flow and establishes the need for an alternative approach. Using an example of 30 cash flow curves, the advantage of fuzzy cash flow analysis is demonstrated. Results of the analysis are presented and discussed. The model can be used to analyse the cash flow curve of projects at any progress period to make sure it is reasonable.

Keywords: fuzzy, cash flow, valuation, fuzzy techniques, progress

Background

The magnitude and timing of future cash flows need to be estimated carefully, taking into consideration the related risks and uncertainties. The major problem that managers encounter in making financial decisions involves both the uncertainty and the ambiguity surrounding expected cash flows (Eldin, 1989). In the case of complex projects the problems of uncertainty and ambiguity assume even greater proportions because of the difficulty in predicting the impact of unexpected changes on construction progress and, consequently, on cash flows. The uncertainty and ambiguity are caused not only by project-related problems but also by economic and technological factors (Laufer and Coheca, 1990). Generally, cash flows are spread over several time periods (valuation periods) and are dominated by project-related constraints and by environmental (external) constraints over which construction managers have little control. If the cash flows are not planned at early stages (pre-contract) of the project and are spread over a number of periods, the problems of uncertainty and ambiguity become serious (Teicholz, 1994). Several methods have been used for cash flow analysis and forecasting. These include probabilistic analysis and statistical analysis (Kaka and Price, 1993; Singh and Woon, 1984; Teicholz, 1993). In probabilistic analysis, statistical measures (mean and standard deviation) of input variables are used to estimate the mean and standard deviation of the results. This approach is precluded if the factors that influence cash flow have a non-probabilistic nature. Commonly, statistical methods are used to deal with the problem of uncertainty in cash flow prospects (Teicholz, 1993). None of these approaches addresses the problem of ambiguity in the construction process because events and even probabilities associated with them are not always mutually exclusive.

A fuzzy theory approach as developed by Zadeh (1965) is useful for cash flow analysis where a cash flow at valuation periods is uncertain. The fuzzy set approach has been applied widely to help in real life decision-making processes. Fuzzy theory is finding wide popularity in various applications that include management, economics and engineering (Zadeh,

ISSN 0144–6193 print/ISSN 1466-433X online

1994). The theory was introduced by Zadeh (1965) three decades ago, but only recently has its application gained momentum. Fuzzy logic deals with uncertain or imprecise situations and decisions. Although fuzzy theory deals with imprecise information, it is based on sound quantitative mathematical theory (Chen and Hang, 1992). A variable in fuzzy logic has sets of values which are characterized by linguistic expression, such as very high cash flow, average cash flow, low cash flow, etc. These linguistic expressions are represented numerically by fuzzy sets. Every fuzzy set is characterized by a membership function, which varies from 0 to 1. Fuzzy sets have a distinct feature of allowing partial membership. A given element can be a member of a fuzzy set, with degree of membership varying from 0 (non-member) to 1 (full member), in contrast to crisp or conventional sets, where an element can either be or not be part of the set. Linguistic variables as described by Zadeh (1995) provide a means of modelling human tolerance for imprecision by encoding decision-relevant information into labels of fuzzy sets. Fuzzy technologies are an approximation that can be used to model decision processes for which mathematical precision is impossible or impractical.

The purpose of this paper is to present an alternative approach for determining (analysing) the expected value of cash flows, based on fuzzy set theory. This approach takes into account the vagueness and ambiguities inherent in the estimation of projects' cash flows at valuation stages.

Fuzziness of cash flow

A large number of methods exist for cash flow analysis. Most of these methods are based on single standard cost S curves (Singh and Woon, 1984; Kaka and Price, 1993; Teicholz, 1993). The accuracy of these models in representing the real world depends on whether the conditions assumed accurately represent the situation to be analysed (Boussabaine and Kaka, 1998). The S curve method is a very simplified approach for analysing receipts and payments at different stages of project progress. In a typical project, the cost accrual assumes the following pattern (Cooke and Jepson, 1979): (1.) during the first third of the project duration, the cost accumulates in a parabolic pattern to achieve one-quarter of the costs incurred at one-third-project duration; (2.) during the second third of the project duration, the cost accumulates in a linear fashion such that at two-thirds project duration the accumulated costs total three-quarters of the project total costs; and (3.) during the final third of the project duration, the cost accumulation is a mirror image of the first third duration, to achieve 100% cost at physical completion.

The following equations are used to model cash flow curves as defined above:

$$y = \frac{9x^2}{4} \qquad 0 '' x '' \frac{1}{3} \tag{1}$$

$$y = \frac{3x}{2} - \frac{1}{4} \qquad \frac{1}{3} '' x '' \frac{2}{3} \tag{2}$$

$$y = \frac{9x}{2} - \frac{9x^2}{4} - \frac{5}{4} \qquad \frac{2}{3} '' x '' 1 \tag{3}$$

where x is the cumulative proportion of project duration ($0 '' x '' 1$), and y is the cumulative proportion of project budget cost or value ($0 '' y '' 1$).

These equations of cash flow representation allow precise descriptions. By contrast, the fuzzy approach to cash flow analysis extends to subjective phenomena. It includes the use of approximate reasoning by which managers model how meaningful information can be obtained from imprecise data presented to them on work progress (receipts and payment statements). Hence, in this work interim payments are dealt with as fuzzy concepts (sets) to reflect their uncertainty and ambiguity. Here fuzzy sets at any payment period introduce vagueness (with the aim of reducing complexity and increasing accuracy of representation) by eliminating the sharp boundary between different possible cash flow sums at a particular payment period. Thus, if a payment period is a member of a fuzzy set, the values of its membership function are between 0 and 1.

The uncertainty in cash flow at a particular payment period is modelled by three intervals: low cash flow, medium cash flow (the most likely) and high cash flow. The estimate of these intervals might be based on a manager's experience and judgement or on statistical analysis of the payment periods from historical data of past similar construction projects. These intervals can be changed to suit a particular project or situation using relevant experience and know-how from the past. In this study the value of these intervals is estimated as a function of the mean, standard deviation and confidence level of cash flow at any period of the project's progress. These interval values are used to construct the membership function of possible cash flow at any period of a project's progress. Therefore, the above equations might be fuzzified as flows:

$$\mu(x) = [\frac{9x^2}{4}] \qquad 0 '' x '' \frac{1}{3} \tag{4}$$

$$\mu(x) = [\frac{3x}{2} - \frac{1}{4}] \qquad \frac{1}{3} '' x '' \frac{2}{3} \tag{5}$$

$$\mu(x) = [\frac{9x}{2} - \frac{9x^2}{4} - \frac{5}{4}] \qquad \frac{2}{3} '' x '' 1 \tag{6}$$

where $\mu(x)$ is the membership function of the cumulative proportion of project cost $(0 \leq x \leq 1)$. The method used to estimate the value of $\mu(x)$ is discussed in the next section.

Data used in the study

The actual data used in this study correspond to projects that are 100% complete and carried out under the Institute of Civil Engineers Standard Conditions of Contract. All of the projects used were of a duration ranging from 1 year to 2 years, and 30 cases were acquired for this study. For each set of data the ratio of present cumulative cost against final cumulative cost was plotted against the percentage completion of the project, thus producing the sample S curves. Each of these curves was then divided into nine time periods corresponding to 10%, 20%, 30%, . . . , 90% completion. The ratio of the cumulative cost to final cumulative cost was then read off for each of the periods. Samples of these curves are shown in Figure 1. General statistical analysis of the data is shown in Table 1. Table 2 shows the correlation between the interim payment stages and it is seen that the correlation between the interim stage payments is very low. This indicates that interim payment is statistically independent. Also as can be seen from Table 2, most of the payment stages are positively correlated.

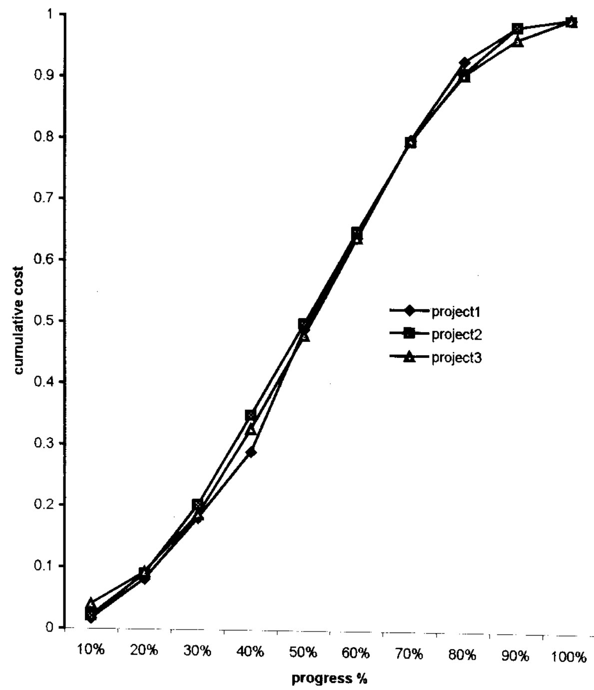

Figure 1 Sample of curves used in the study

Determination of the membership functions

The most important step in the design of fuzzy decision support systems is the determination of the membership functions of the sets. There are many guidelines on developing the membership functions for fuzzy sets (Dubois and Prade, 1980). The methods, which have been used in the past, are often heuristically based. The sets based on statistics are perhaps some of the most naturally fuzzy sets that can be used. Any measurement based on statistics must make some allowance for deviation from the value obtained by the measurement. Another advantage of statistically based membership functions is that they are naturally quantitative, that is, there is reason to believe that the membership function has a relationship to some physical property of the set. Hence, this paper uses these criteria which result in a simplified but accurate method of defining membership functions for cash flow groups of fuzzy sets (valuation

Table 1 General statistics of the data used in the study

Project progress periods	P1 (10%)	P2(20%)	P3 (30%)	P4 (40%)	P5 (50%)	P6 (60%)	P7 (70%)	P8 (80%)	P9(90%)
Minimum	0.007	0.027	0.037	0.267	0.460	0.627	0.780	0.900	0.940
Maximum	0.227	0.373	0.560	0.857	0.943	0.953	0.980	0.987	0.993
Mean value	0.041	0.111	0.234	0.433	0.635	0.764	0.874	0.943	0.980
Variance	0.003	0.008	0.014	0.022	0.025	0.016	0.005	0.001	0.000
Std. deviation	0.050	0.087	0.117	0.148	0.159	0.125	0.072	0.030	0.012
Range	0.220	0.346	0.523	0.590	0.483	0.326	0.200	0.087	0.053
Skewness	2.848	2.129	1.090	1.096	0.522	0.334	0.249	0.297	−1.436
Kurtosis	7.965	3.825	1.299	0.774	−1.071	−1.645	−1.558	−1.347	2.929
Sum	1.186	3.210	6.779	12.545	18.401	22.169	25.342	27.334	28.408
Sum of squares	0.119	0.565	1.969	6.043	12.385	17.386	22.290	25.789	27.823
Number of values	29.000	29.000	29.000	29.000	29.000	29.000	29.000	29.000	29.000
Number of missing values	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 2 Correlation matrix between the interim payment stages

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9
Period 1	1								
Period 2	0.9 359 734	1							
Period 3	0.7 729 548	0.905 534	1						
Period 4	0.7 137 485	0.5 829 691	0.4 051 993	1					
Period 5	0.2 224 222	0.2 996 849	0.4 884 428	0.0 827 641	1				
Period 6	0.1 104 415	0.1 701 074	0.3 600 681	0.0 447 111	0.92 589	1			
Period 7	0.0 612 014	0.109 727	0.2 875 617	0.0 172 466	0.8 667 767	0.9 744 181	1		
Period 8	0.0 394 953	0.0 967 883	0.2 552 517	0.0 289 795	0.7 614 124	0.8 842 472	0.9 389 317	1	
Period 9	–0.033 224	0.0 017 213	0.0 571 075	0.0 066 508	0.3 076 515	0.3 642 789	0.431 804	0.6 033 338	1

periods or progress periods) which are statistically based. In this paper statistically based fuzzy cash flow sets are those whose membership function is based upon the mean and standard deviation of a defining feature of its members in the universes of discourse. A more complicated method based on the probability density function of the cash flow at each progress period can be used (Civanlar and Trussell, 1986). The problem now is how to use these statistical measures of a cash flow period of the elements of a fuzzy set to construct its membership function. In order to define a representative membership function, there are conditions which can be imposed to make the set have properties consistent with the subjective judgement of a decision-maker and the underlying statistics of the cash flow periods. From a heuristic point of view, the elements which are most likely (e.g. at mean) should have high membership values, however, the set should be as selective as possible. These sets of conditions might be quantitatively represented (Civanlar and Trussell, 1986) as follows.

1. $E\{\mu(x)\} \mid x$ is distributed according to the underlying mean and standard deviation.
2. $0 \leq \mu(x) \leq 1$ this is because an infinite scale will not be necessary to assign grade of memberships, and [0,1] is the classical interval over which the membership functions are defined.
3. $f\mu(x)^2$ should be minimized: this is because, the number of parameters in the functional representation should be as small as possible.

The fuzzy membership of x defined as, $\mu(x) \in [0, 1]$, are estimated by using the following formulae.

(a) For low cash flow

$$\mu(x) = \mid \frac{a-x}{b} \mid \qquad \text{for a} - b \text{ ″ } x \text{ ″ } a \tag{7}$$

(b) For medium cash flow

$$\mu(x) = \mid \frac{x-a+b}{b} \mid \qquad \text{for } x \text{ ″ } a \tag{8}$$

$$\mu(x) = \mid \frac{x-a-b}{b} \mid \qquad \text{for } x \geq a \tag{9}$$

(c) For high cash flow

$$\mu(x) = \mid \frac{x-a}{b} \mid \qquad \text{for a ″ } x \text{ ″ } a+b \tag{10}$$

Here a is the mean and b is the standard deviation. The above formulae indicate that there is a central member (a) for which $\mu(x)$ is greater than its peripheral members. Here b is a controlling scale factor. These parameters modify the shape and spread of the $\mu(x)$ obtained in the above equations, as shown in Figure 2. The scale values indicated on the horizontal axis represent cash flow level using mean, standard deviation and variances. The fuzziness is increased or decreased by the central prototype a, and the spread parameter b. The parameter b determines the fuzziness of peripheral members. Since the mean is an unbiased estimate for any sample set, it is an ideal choice for a. It is very convenient to express the membership function of a fuzzy subset of the real line in terms of a standard function whose parameters may be adjusted to fit a specified membership function in an approximate fashion. Figure 2 shows a graphical representation of these equations. Table 3 shows data extracted from Table 1 for developing membership functions for interim payment periods. Figure 2 shows the range of medium cash flow

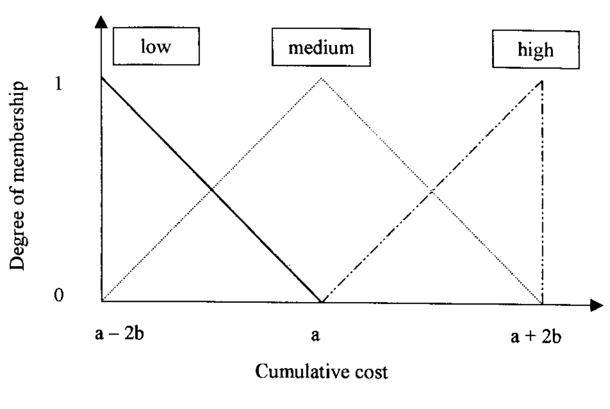

Figure 2 Membership functions of cumulative costs

Table 3 Data used for developing membership functions

Period	Mean (a)	SD (b)	$a + 2b$	$a - b$	$a - 2b$
Period 1 at 10% progress	0.041	0.050	0.141	–0.009	–0.059
Period 2 at 20% progress	0.111	0.087	0.285	0.024	–0.063
Period 3 at 30% progress	0.234	0.117	0.468	0.117	0.000
Period 4 at 40% progress	0.433	0.148	0.729	0.285	0.137
Period 5 at 50% progress	0.635	0.159	0.953	0.476	0.317
Period 6 at 60% progress	0.764	0.125	1.014	0.639	0.514
Period 7 at 70% progress	0.874	0.072	1.018	0.802	0.730
Period 8 at 80% progress	0.943	0.030	1.001	0.913	0.883
Period 9 at 90% progress	0.980	0.012	1.004	0.968	0.956

values from $(a - 2b)$ to $(a + 2b)$, with the highest degree of membership occurring at the value of $(a - 2b)$ for low cash flow, (a) for medium cash flow and at $(a + 2b)$ for high cash flow. To avoid negative cash flow at early stages of the project progress for period 1, the highest degree of membership for low cash flow occurs at the value $(a - b)$. In this respect it can be argued (assuming the distribution is normal) that 95% cash flow will lie within the limits of these membership functions. Figure 3 shows membership functions of the nine interim valuation periods used in this work.

This approach provides useful information for decision makers without imposing an inappropriate degree of precision, and avoids measurement problems associated with the derivation of a single measure of cash flow at any particular period of a project's progress.

Fuzzy cash flow analysis

Linguistic variables as described by Zadeh (1993) provide a means of modelling human tolerance for imprecision by encoding decision relevant information

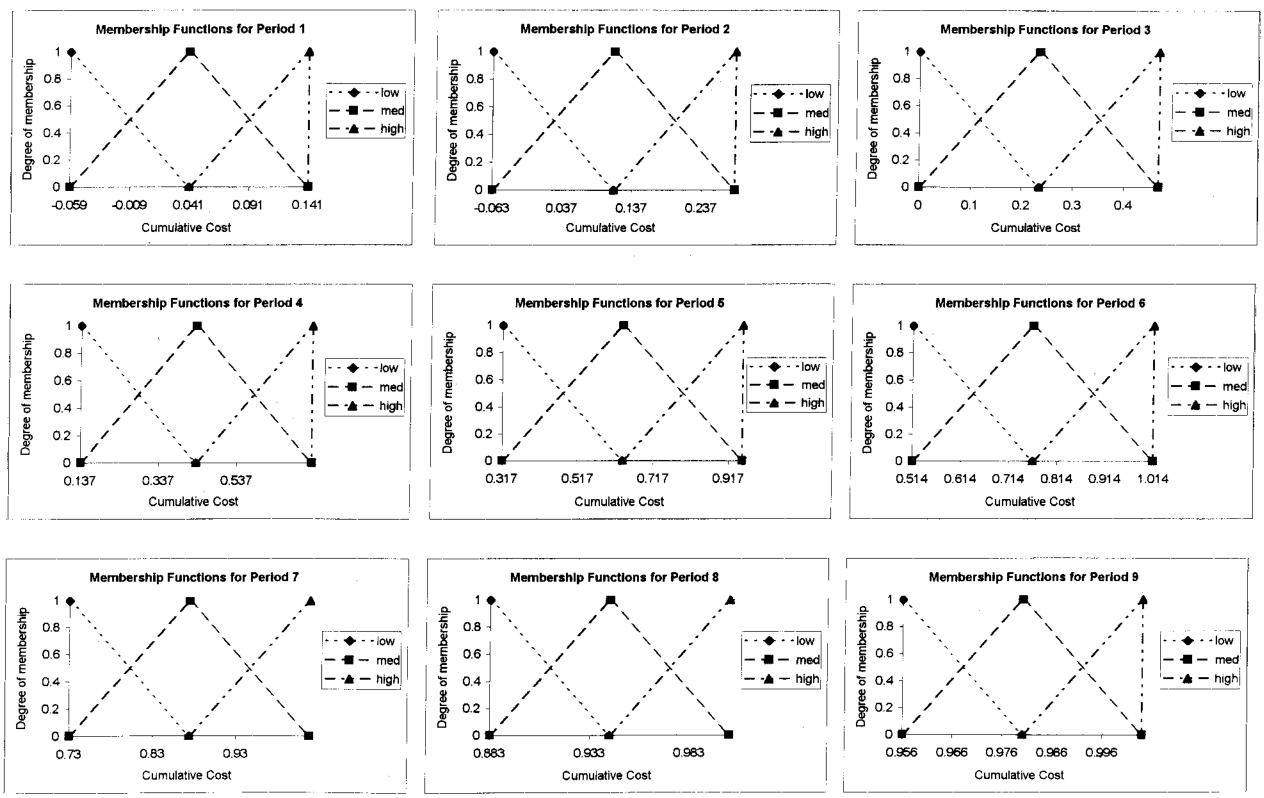

Figure 3 Membership functions of periods 1–9

into labels of fuzzy sets. Fuzzy techniques are an approximation that can be used to model decision processes for which mathematical precision is impossible or impractical. In the case of cash flow management, the shortcomings in traditional techniques for decision making under uncertainty may be addressed through the use of fuzzy systems theory, fuzzy aggregation, and fuzzy logic (Chen, 1985; Chen and Hang, 1992). Zadeh (1965) defined set intersection, union, and complement operators for fuzzy sets. For an intersection the resulting membership function was expressed as the minimum of two or more fuzzy sets, and the union of two fuzzy sets was defined as the maximum of two or more fuzzy sets. These operations are defined as follows:

$$\mu_{A \cap B}(x) = \min\{\mu_A(x), \mu_B(x)\} \tag{11}$$

$$\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\} \tag{12}$$

$$\mu_{\neg A}(x) = 1 - \mu_A(x) \tag{13}$$

The remainder of this section demonstrates the application of fuzzy aggregation techniques to assess cash flow as a means of understanding cash flow forecasts. Aggregation techniques provide appropriate handling of variables for decision making with goals and constraints.

In an analysis of risk due to earthquakes, Dong and Wong (1985) employ a fuzzy averages technique with linguistic variables to evaluate structural risk. It is this approach that the authors have chosen for assessment of cash flow at valuation periods. Recent work on this domain has developed a number of formulations. Among these formulation are fuzzy aggregation and fuzzy logic. These two techniques are used in this paper to demonstrate the application of fuzzy aggregation techniques to the problem of cash flow analysis at valuation periods. A commonly encountered problem in cash flow and decision making is the valuation of different cash flow alternatives. In a case where the value of the alternatives can be measured through fuzzy membership functions, the membership function can provide a means of evaluating trade off between cash flow alternatives. Suppose that $A_1, A_2, \ldots, A_n$ represent cash flow alternatives in a universe of alternatives $X_1, X_1, \ldots, X_n$, respectively, for achieving a given goal $\mu(y)$ (maximum cash flow) then:

$$\mu(y) = \bigvee_{x \in X_i} [\mu_{Ai}(x_1), \mu_{Ai}(x_2), \ldots \mu_{An}(x_n)] \tag{14}$$

The above equation does not provide a means for assessing the importance of each alternative against the rest of possible outcomes. Dubois and Prade (1984) developed an approach for weighting goals to provide appropriate emphasis on each alternative using the following equation:

$$\mu(y) = \sum_{i=1}^{n} W_i \mu_i(x) \qquad \text{for} \qquad \sum_i^n W_i = 1 \tag{15}$$

This equation is difficult to compute, and requires non-linear programming techniques for evaluation. This problem has been solved by Dong and Wong, (1987) by developing an approximation technique that involves α cuts (horizontal cross-sections at various levels of membership) and interval analysis. Cash flow alternatives at different valuation periods are represented by $S_1, S_2, \ldots, S_m$ and the criteria that are used in the assessment of these alternatives are $\alpha_1, \alpha_2, \ldots, \alpha_n$ (hence the name of 'α cuts'). Then for a chosen alternative S_i the relative merit criterion α_j is evaluated by a membership function $\mu_{ij}(x)$. Also, the relative importance or the degree of belief in each criterion of selection of an alternative cash flow is assessed by a weighting coefficient W_j for criterion α_j. Therefore, the cash flow alternative receives the following weighted average membership.

$$\mu_i(x) = \frac{\bullet_{j=1}^{n} W_j \mu_{ij}(x)}{\bullet_{j=1}^{n} W_j} \tag{16}$$

This equation is used to evaluate cash flow sets at different payment stages of a project. For these purpose three linguistic variables, low, medium and high, are used to evaluate cash flow. Linguistic variables provide a means of modelling human tolerance for imprecision by encoding decision-relevant information into labels of fuzzy sets (Zadeh, 1995). By evaluating cash flow in terms of linguistic approximation a multi-answer to a crisp (singleton) problem can be provided, allowing decision-makers to test different alternatives for cash flow.

Application

The process used to compute and develop fuzzy cash flow curves is shown in Figure 4. The first stage in this process is the development of membership functions (MBF) and the definition of their linguistic variables. Linguistic variables are used to translate real values into linguistic values. The possible values of a linguistic variable are not numbers but so called 'linguistic terms'. For example, to translate the real variable 'cash flow' into a linguistic variable, three terms (low, medium and high) cash flow are defined. This stage will be followed by the development of cash flow membership functions at every valuation period and the weight membership. The membership function of cash flow periods was discussed previously. Weights can be expressed in either numeric (crisp) or linguistic (fuzzy) terms. All the weights must be defined in the same manner. This work uses linguistic weights

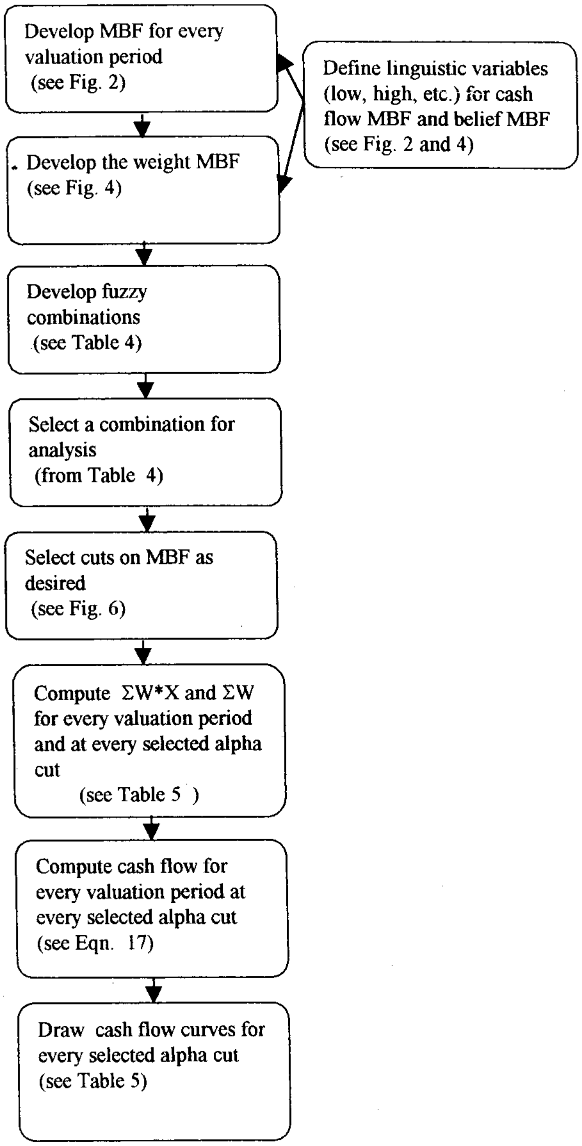

Figure 4 Fuzzy analysis flow chart

for the purpose of demonstrating the use of a fuzzy weighted average in cash flow analysis. In this study linguistic weights are expressed in terms of degree of belief that the cash flow will be low, medium or high. Figure 5 shows the membership function (fuzzy sets) for the degree of belief (weights) of the nine progress periods. Table 4 shows that there are nine possible combinations of cash flow for every valuation period. Each of these combinations might also have several alternatives depending on the degree of membership of the selected combination. Each combination represents an S curve. Table 5 shows cash flows at different project valuation periods, with their associated degree of belief (weight). These weights are assigned randomly

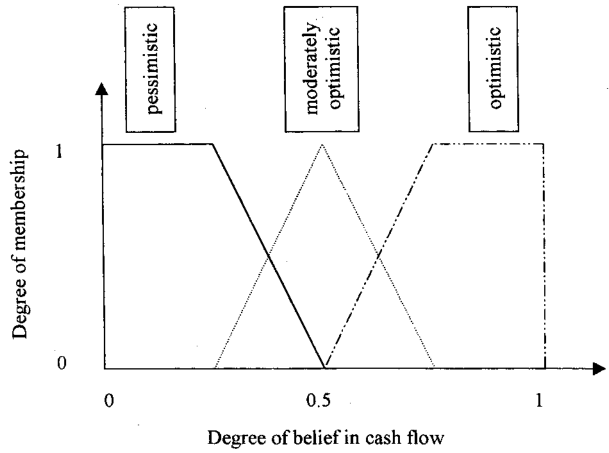

Figure 5 Membership functions of belief in cash flow

for the purpose of demonstrating the concept of fuzzy cash flow curves.

The use of the information presented in Table 4 in cash flow analysis decision-making is illustrated by the following example of a fuzzy-weighted average computation. Figure 6 illustrates the process of α cuts and demonstrates an example of fuzzy computation (in this case *WX*). Figure 4 shows a fuzzy analysis flow chart. In order to compute Equation 16, summing the weight variables derives the denominator (i.e. • *W*). When all weights have been summed, the denominator of Equation 16 is evaluated for each cash flow forecast under consideration (e.g. Table 4). In order to derive

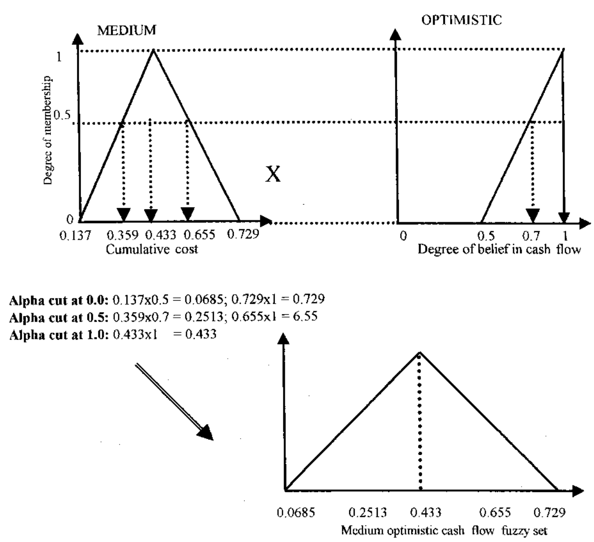

Figure 6 Example of fuzzy computation

Table 4 Possible combinations of cash flow at each valuation period

	Low cash flow (L)	Medium cash flow (M)	High cash flow (H)
Pessimistic (p)	Lp	Mp	Hp
Moderately optimistic (m)	Lm	Mm	Hm
Optimistic(o)	Lo	Mo	Ho

this, the linguistic value for each cash flow forecast (e.g. low) must be multiplied by its respective degree of belief and these products are then summed (i.e. • WX) as shown in Figures 4 and 6. This involves multiplication of the fuzzy sets representing cash flow followed by a summation of the products. This forms the numerator of Equation 16, which is then divided by the denominator. The weights of the degree of belief are summed to obtain another fuzzy set, which represents the final membership of the cash flow (Figure 5). Construction decision makers might work directly with this fuzzy set or defuzzify (it using one of the available methods for this purpose) to determine a single value that represents cash flow at a particular

Table 5 Examples of fuzzy cash flow analysis

α cuts	Lp		Mm		Ho		Y[a]
	Belief	Cash flow	Belief	Cash flow	Belief	Cash flow	
	Period 1 (10%)						
S1 0	0.5	0.041	0.6	0.059	0.3	0.041	0.048 714
S2 0.5	0.8	0.02	0.3	0.02	0.8	0.07	0.041 053
S3 1	0.9	0	0.8	0.041	0.9	0.141	0.061 423
	Period 2 (20%)						
S1 0	0.9	0.111	0.3	0.063	0.3	0.111	0.101 4
S2 0.5	0.7	0.068	0.6	0.068	0.8	0.155	0.101 1
S3 1	0.8	0.063	0.8	0.111	0.7	0.285	0.147 3
	Period 3 (30%)						
S1 0	0.5	0.234	0.7	0	0.5	0.234	0.137 65
S2 0.5	0.4	0.176	0.9	0.176	0.8	0.293	0.220 07
S3 1	0.3	0	0.6	0.234	0.4	0.468	0.252
	Period 4 (40%)						
S1 0	0.8	0.433	0.6	0.137	0.4	0.433	0.334 333
S2 0.5	0.7	0.359	0.9	0.359	0.6	0.509	0.399 909
S3 1	0.4	0.137	0.3	0.433	0.5	0.729	0.457 667
	Period 5 (50%)						
S1 0	0.4	0.635	0.6	0.317	0.9	0.635	0.534 6
S2 0.5	0.7	0.555	0.8	0.555	0.8	0.715	0.610 7
S3 1	0.5	0.317	0.4	0.635	0.7	0.953	0.674 8
	Period 6 (60%)						
S1 0	0.7	0.764	0.3	0.514	0.6	0.764	0.717 13
S2 0.5	0.6	0.702	0.7	0.702	0.7	0.827	0.745 75
S3 1	0.7	0.514	0.8	0.764	0.4	1.014	0.724 53
	Period 7 (70%)						
S1 0	0.7	0.847	0.8	0.73	0.6	0.874	0.819 143
S2 0.5	0.6	0.838	0.5	0.838	0.4	0.91	0.857 2
S3 1	0.4	0.73	0.8	0.874	0.3	1.028	0.866 4
	Period 8 (80%)						
S1 0	0.4	0.943	0.9	0.883	0.3	0.943	0.909 3
S2 0.5	0.6	0.928	0.6	0.938	0.7	0.958	0.942 2
S3 1	0.7	0.883	0.2	0.943	0.8	1.003	0.946 5
	Period 9 (90%)						
S1 0	0.8	0.98	0.2	0.956	0.7	0.98	0.977 18
S2 0.5	0.2	0.98	0.9	0.98	0.4	0.986	0.981 6
S3 1	0.6	0.956	0.5	0.98	0.6	1.004	0.98

a Y = • WX/• W where W is the degree of belief and X is cash flow

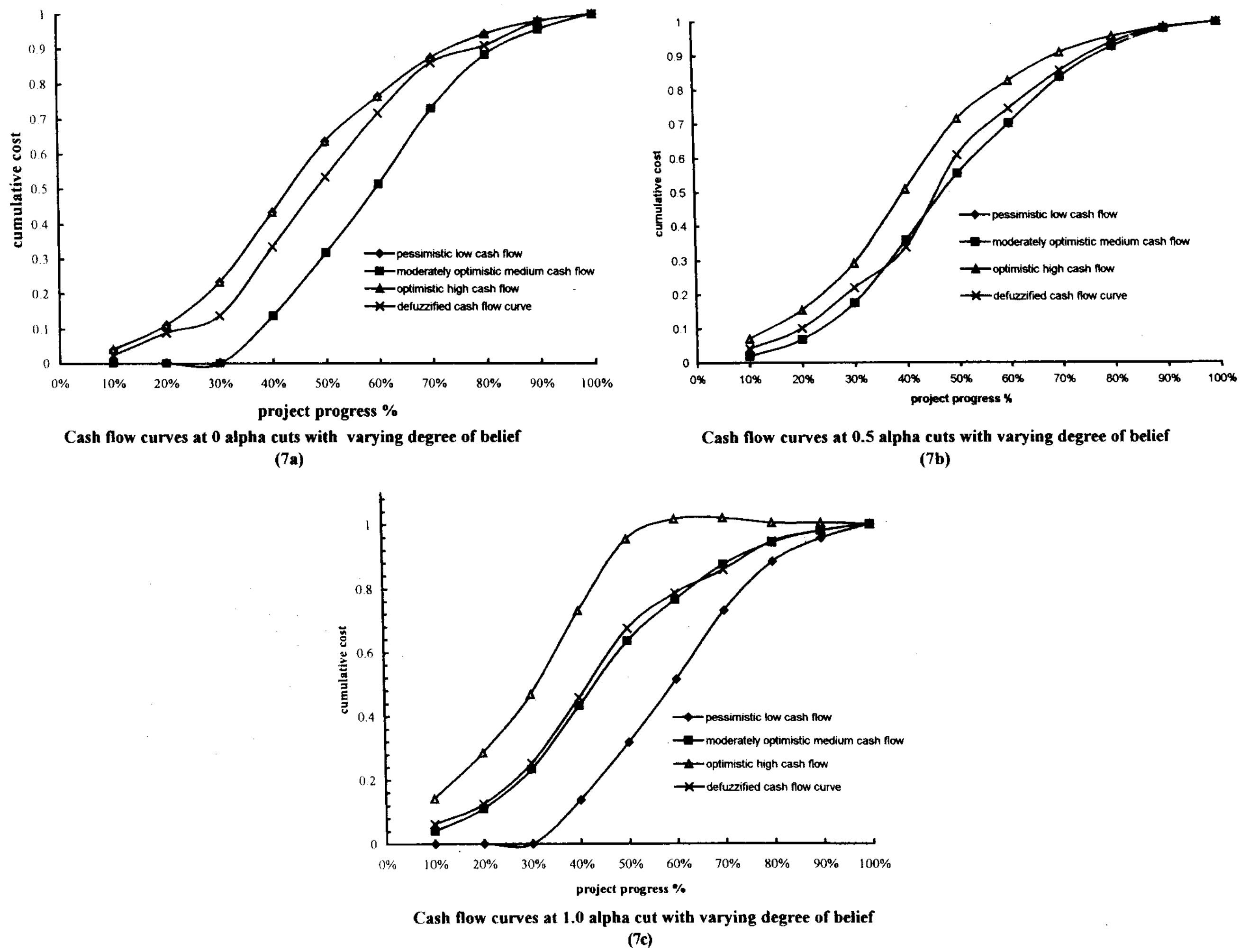

Figure 7 Fuzzy cash flow curves

period of project progress. Alpha cuts at various levels of membership levels of 0.0, 0.5 and 1 are used here, although α cuts may be taken at other intervals, depending on the level of precision desired.

Example

As an example, the cash flow for period 3 (30% progress) is assumed uncertain and cannot be expressed as a precise number. In such cases, several cash flow alternatives may exist for a given scenario with varying degrees of belief in them. Low pessimistic (Lp), moderate optimistic (Mm), and high optimistic (Ho) cash flow combination are selected from Table 4 to illustrate the computation of the expected cash flow. The corresponding cash flow curves for the combination shown in Table 4 will be fuzzy sets:

(1) at 0 membership

$$S_0 = 0.5 \mid 0.234 + 0.7 \mid 0.468 + 0.5 \mid 0.234$$

(2) at 0.5 membership

$$S_{0.5} = 0.4 \mid 0.117 + 0.9 \mid 0.117 + 0.8 \mid 0.411$$

(3) at 1 membership

$$S_1 = 0.3 \mid 0 + 0.61 \mid 0.234 + 0.4 \mid 0.468$$

There are two ways to defuzzify the set and obtain the expected cash flow at any valuation period. With the first method, competing alternatives are ignored and construction manager picks the alternative in which he believes the most. For example, the cash flow with the highest membership function (belief) in the S_0 curve is $0.7 \mid 0.468$. In the second approach the average of the whole set of competing alternatives (Lp, Mm, Ho) is calculated. This requires the calculation of the centre of gravity of the cash flow for predicted amounts of (low, medium and high) cash

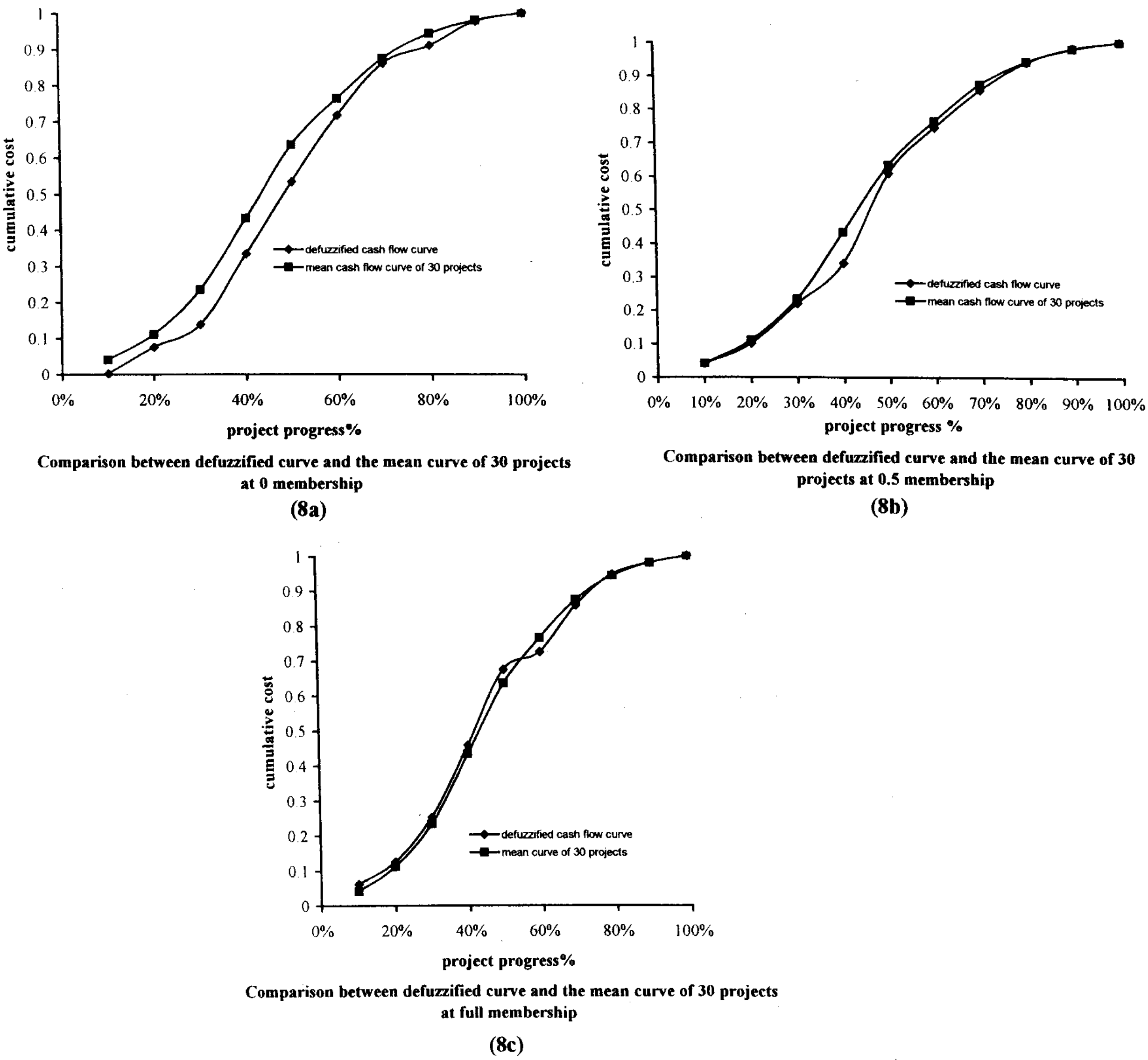

Figure 8 Comparison between defuzzified and statistical curves

flow. Equation 16 is used for this purpose. The expected cash flow for this period is

$$S_0 = \frac{0.5 \times 0.234 + 0.7 \times 0 + 0.5 \times 0.234}{0.5 + 0.7 + 0.5} = 0.1376 \qquad (17)$$

The same process can be applied for computing cash flow at any level of membership function. The rest of the computation is summarized in Table 5. Figure 7 compares the cash flow curves for different scenarios at different levels of membership. These figures show clearly that the defuzzified cash flow curve is a good compromise between pessimistic, moderate and high cash flow alternatives. However, the shape of this curve is affected greatly by the ambiguity of the cash flow and the degree of belief (weight) assigned to each valuation period. Figure 8 shows a comparison between the mean curve of the 30 projects and defuzzified curves at 0, 0.5 and 1 membership. Note that the 0.5 membership curve is very close to the mean curve. This is just a coincidence since the shape of the fuzzy curve is dependent on the amount of cash flow as well as the degree of belief in this cash flow. Figure 9 shows examples of typical anticipated cash flow curves. Observe how the low confidence in cash flow at 40% of project progress for 0.5 membership affects the cash flow. Also, cash flow at 70% progress for full membership is affected.

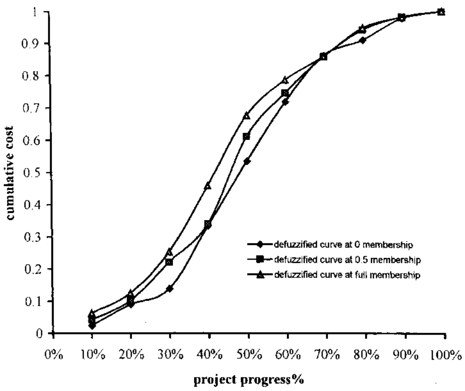

Figure 9 Defuzzified cash flow curves

The usefulness and practical implication of this work are discussed in detail in Boussabaine and Kaka (1998). This model provides a relatively neat and compact means of taking into account many linguistic possibilities (ambiguities) in cash flow analysis. However, the variables involved must be independent. It is impossible to satisfy this restriction every time. Therefore, when the variables involved are interactive, a hybrid model may be used that involves the method described here and fuzzy logic which allows for relationships among variables. Further research will explore this and extract rules of thumb from the cash flow curves presented in this study. Then this information will be used to develop and train a neuro-fuzzy model for cash flow forecasting.

Conclusion

This paper demonstrates how fuzzy sets can be used to describe ambiguous terms that often are encountered in cash flow analysis at any valuation period of a project's progress. An alternative to the traditional method of analysing expected values of cash flows at different project stages is presented. The approach is based on the assumption that cash flows at different stages of the project progress are ambiguous. The cash flow curves of 30 projects were analysed and the expected cash flow curves were obtained using fuzzy averaging techniques. Defuzzified cash flow curves were found to be a good compromise between different cash flow scenarios. However, the input variables must be independent. It is impossible to satisfy this restriction everytime. Therefore, further research will examine the interaction between the input variables.

References

Boussabaine A.H. and Kaka, A. (1998) A neural networks approach for cost-flow forecasting. *Construction Management and Economics Journal*, **16**, 471–9.

Chen, S.H. (1985) Ranking fuzzy numbers with maximising set and minimising set. *Fuzzy Sets Systems*, **17**(2), 113–29.

Chen, S.J. and Hang, C.L. (1992) *Fuzzy Multiple Attribute Decision-Making: Methods and Applications*, Springer-Verlag, Berlin.

Civanlar, M.R. and Trussell, H.J. (1986) Constructing membership functions using statistical data. *Fuzzy Sets and Systems*, **18**, 1–13.

Cooke, B. and Jepson, W.B. (1979) *Cost and Financial Control for Construction Firms*, Macmillan, London.

Dong, W. and Wong, F. (1985) Fuzzy computations and risk in decision analysis. *Civil Engineering Systems*, **2**, 201–8.

Dong, W. and Wong, F. (1987) Fuzzy weighted averages and implementation of the extension principle. *Fuzzy Sets and Systems*, **21**, 183–99.

Dubois, D. and Prade, H. (1980) *Fuzzy Sets and Systems: Theory and Application*, Academic Press, New York.

Dubois, D. and Prade, H. (1984) A review of fuzzy set aggregation connectives. *Information Science*, **36**, 85–121.

Eldin, N. (1989) Cost control systems for PMT use. *Transactions of the AACE*, F3.1–F3.5.

Kaka, A.P. and Price, A.D.F. (1993) Modelling standard cost commitment curves for contractors' cash flow forecasting. *Construction Management and Economics*, **11**, 271–83.

Laufer, A. and Coheca, D. (1990) Factors affecting construction planning outcomes. *Journal of Construction Engineering and Management*, **116**(6), 135–56.

Singh, S. and Woon, P.W. (1984) Cash flow trends for high-rise building projects, organising and managing construction, in *Proceedings of the 4th International Symposium on Organisation and Management of Construction*, University of Waterloo, Canada.

Teicholz, P. (1993) Forecasting final cost and budget of construction projects. *Journal of Computing in Civil Engineering, ASCE*, 7(4), 511–29.

Zadeh, L.A. (1965) Fuzzy sets. *Information and Control*, **8**, 338–53.

Zadeh, L.A. (1994) Fuzzy logic, neural networks, and soft computing. *Communications of the ACM*, **37**(3), 77–84.

Zadeh, L.A. (1995) The theory of a linguistic variable and its application to approximate reasoning. *Information Science*, **8**, 199–249.