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Review

Systematic Review of the Time-Cost Optimization Models in Construction Management

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Abstract: The construction sector is a crucial contributor to the national and global economy. Therefore, improving the efficiency and effectiveness of construction projects can have a significant impact on gross domestic product (GDP). However, managing construction projects can be challenging due to the uncertainties and complexities involved. The three primary interrelated constraints of construction projects, namely, time, scope, and cost, require effective management to ensure successful completion. To optimize the time and cost of construction projects, various optimization models and techniques have been proposed in the literature. This paper presents a systematic review of the time-cost optimization models in construction management and proposes some future work to improve the solution of the considered problem. The review categorizes the existing models into three categories: exact models, approximate models, and hybrid algorithm models. The exact models provide optimal solutions but require a lot of computational time and may not be efficient in solving multi-objective and large-scale problems. The approximate models provide near-optimal solutions and reduce computational effort but may not be efficient in solving large-scale projects. The hybrid algorithm models combine the good properties of different algorithms to provide high-quality and efficient solutions. The purpose of this paper is accomplished through a systematic literature review following the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) guidelines. The paper analyzes the contribution, advantages, and limitations of each category and provides recommendations for future work. Based on the review, several recommendations for future work are provided, including the development of hybrid models that combine different optimization techniques, the incorporation of risk management into optimization models, and the use of advanced data analytics techniques to improve the accuracy of optimization models. Overall, this paper provides an up-to-date comprehensive review of the time-cost optimization models used in construction management and offers valuable insights for researchers and practitioners in this field. The findings of this review can be used to guide future research and improve the effectiveness of optimization models for construction projects.

Keywords: construction project scheduling; construction management; operations research; cost control; algorithms; mathematical programming

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1. Introduction

The construction industry plays a critical role in the economic growth of any country, with approximately $10 trillion being spent on construction goods and services annually [1]. The industry employs 18 million people in the European Union and contributes approximately 9% to the gross domestic product (GDP) within the European Union [2]. Because of that, construction project management is crucial as a slight improvement in this sector can have a significant positive impact on the national and global economy [2,3]. However, construction project management is complex due to the uncertainties involved in the construction processes, the complexity of construction projects, and the large number of activities involved [4]. One of the primary tasks of a project manager is to manage the trade-offs among the constraints of time, cost, and scope, based on the nature and priorities of the project, through establishing a project priority matrix [5–8].

Time-cost trade-off (TCT) problems are considered complicated combinatorial problems that require effort and careful analysis to develop and evaluate different alternative schedules in order to determine the optimum schedule that optimizes the project duration with the minimum total project cost [9,10]. Some of the most important negative risks that influence TCT include poor planning and design, poor estimation, contract modifications, increases in material prices, equipment breakdown, project location and high transportation costs, shortage of materials, labor shortage, and natural disasters [11–16]. On the other hand, early completion of projects within the budgeted cost and specified quality can lead to savings in overhead cost, a bonus for early completion, and may provide an opportunity to move resources to other projects for early commencement [17]. Thus, time-cost problems have been the focus of research since the 1960s, and various formulations have been developed to optimize these problems.

Although some researchers conducted a review study of time and cost optimization of construction projects in 2019, their study focused on the software used to improve the efficiency of optimization tools [17]. Some researchers have categorized the models into two categories, which are mathematical models and heuristics models [18–20]. The first category utilizes linear programming, integer programming, or dynamic programming to find the exact solution of the optimization problem. The second category lacks mathematical rigor and provides a good and approximate solution, but it is not guaranteed to be an optimal solution. In this paper, the same categorization was followed but a third category was added to discuss hybrid algorithm models that combine two or more algorithms. Accordingly, this paper classifies these models into three main categories according to the progression of rigor by academics. The first category is exact models, which includes solutions that are based on mathematical formulations, such as linear programming (LP), non-linear programming (NLP), or mixed programming (MP), that aim to determine the optimal solution of time-cost problems. The second category is approximate models, which includes the critical path method (CPM), the program evaluation and review technique (PERT), and metaheuristic and genetic algorithms. These models can be used to reduce the computational effort of the first category. The third category is the novel approach of hybrid algorithm models, which includes hybrid genetic algorithms, neural networks, and machine learning models.

The main aim of this research is to conduct a systematic literature review on the time-cost optimization models available in the construction industry. The study addresses the following three research questions:

1. What research has been carried out on time-cost optimization models in construction management?
2. What are the limitations and gaps of the existing time-cost optimization models in construction management?
3. What are the features of the time-cost optimization models in construction management that future research should focus on to fill the gaps in the literature?

This review paper aims to critically examine the existing time-cost optimization models in construction management and identify their limitations and gaps. The paper also proposes suggestions for future research to address the identified gaps in the literature. The insights gained from this review can help researchers and practitioners to better understand the state-of-the-art time-cost optimization models and provide direction for the development of more efficient and effective models in the future. The structure of this paper is as follows:

➢ Section 1 is an introduction about the importance of optimizing the time-cost of construction projects.
➢ Section 2 presents the methodology that is followed in this study to evaluate different models used to solve time-cost problems.

- ➢ Section 3 discusses the findings of this research and provides an extensive review of the developed models and their main contributions, strengths, and limitations.
- ➢ Section 4 provides the conclusions of the paper.
- ➢ Section 5 discusses the recommendations and future research directions.

2. Research Methodology

This section presents the methodology followed to search for related research and filter these studies before analyzing the models included in each one of them.

2.1. Materials and Methods

The methodology adopted for this research is a systematic literature review conducted according to the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) guidelines, which consist of 4 stages [21,22]. The first stage is the identification of review characteristics. This is where the scope of the research is identified. In this paper, the scope is restricted to time-cost optimization models in construction projects. After that, the second stage is the screening. This is where the databases are selected, and the search keywords are carefully chosen to yield relevant articles in the field. In this paper, the main database that was used is Scopus. As for the search keywords, they were mainly: optimization, time-cost trade-offs, scheduling, efficiency, and construction management. It is also worth mentioning that filters were applied in the screening stage to ensure that the resources collected were only published in English and over the time period 2000–2022 to get a comprehensive overview of the time-cost optimization models in construction projects.

The third stage is the eligibility analysis. This was first done using abstract review, where the abstracts were read and evaluated to decide whether or not the paper fits the scope of the literature review. If it did not fit, it was eliminated right away. If the abstract did fit the scope, then the full paper was read to summarize the main highlights and contributions and also to collect more eligible documents and references from the bibliography of the paper in a snowball, backward referencing approach. The fourth stage of PRISMA deals with data synthesis and analysis. In order to do that, the papers collected were first classified according to the date and place of publication as well as the affiliation of the authors, which was considered the first step in the data analysis process. The selected papers were then categorized into three classes: exact models, approximate models, and hybrid models. This is very useful as it helps in providing an overview of the time-cost optimization models in construction management, highlighting the limitations of the existing models in the literature and identifying the critical features that future research should focus on when developing time-cost optimization models in construction management [21,22].

2.2. Descriptive Analysis

The keywords "Time-Cost" AND "Optimization" AND "Construction management" OR "Efficiency" were entered into the Scopus database to search within the title, abstract, and keywords of the documents. A total of 198 articles were collected from the Scopus database search. After that, these articles were screened based on abstract and title, and 74 papers were excluded. The remaining 124 papers were evaluated through full article assessment, and 33 were excluded as they were outside the scope of this research. Finally, 91 papers were included in the final comprehensive review of time-cost optimization models in construction, as shown in Figure 1. The results were then analyzed to collect information about the number of papers published in this field over the years. Figure 2 illustrates that in general there is a positive increasing trend from 2000 to 2022 in the number of papers published, which is justified by the attention that the literature has been giving to this field over the years. Furthermore, the results also showed that China is the leading country in the number of papers published in this field, followed by the United States (Figure 3).

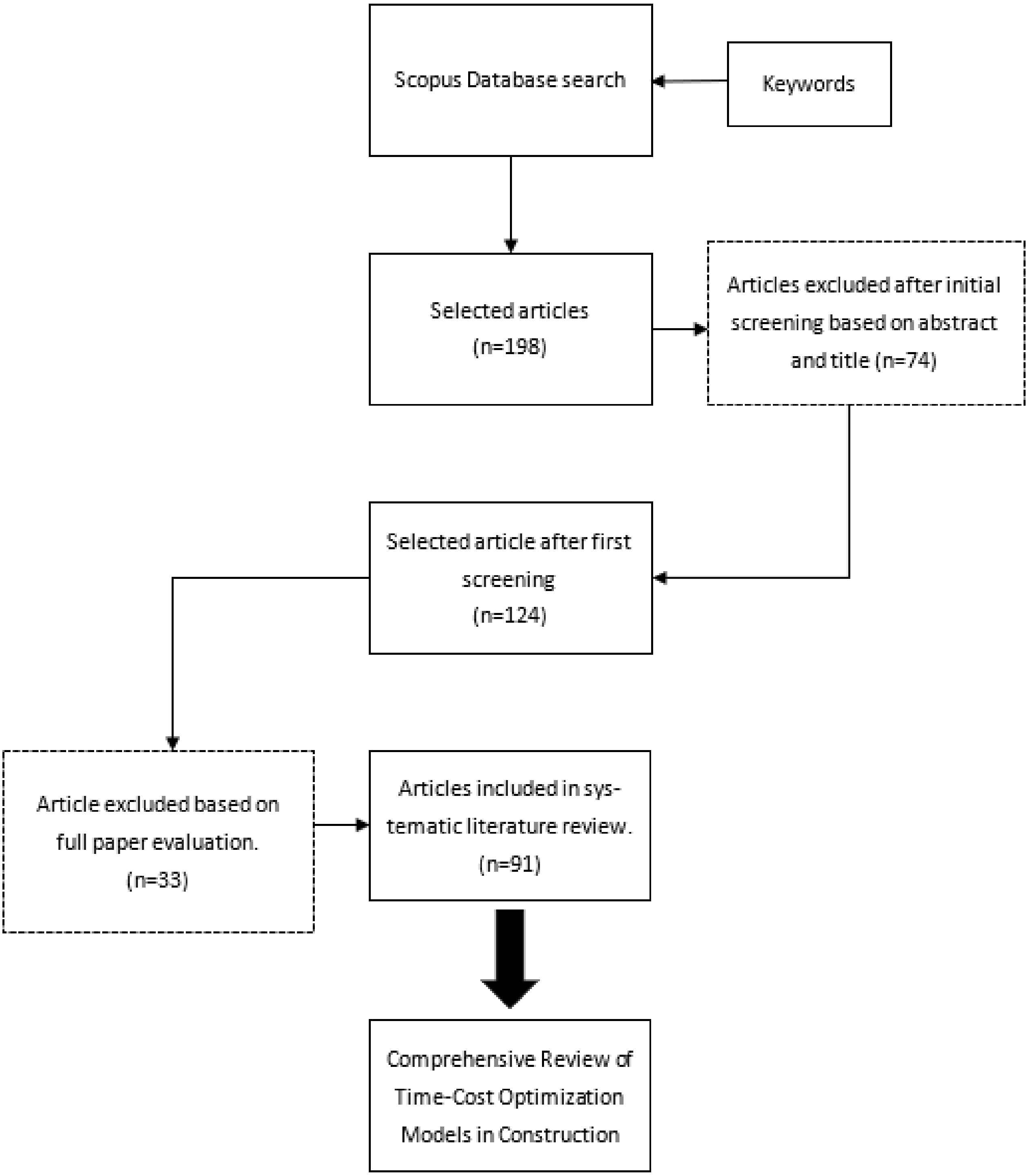

Figure 1. Systematic review steps.

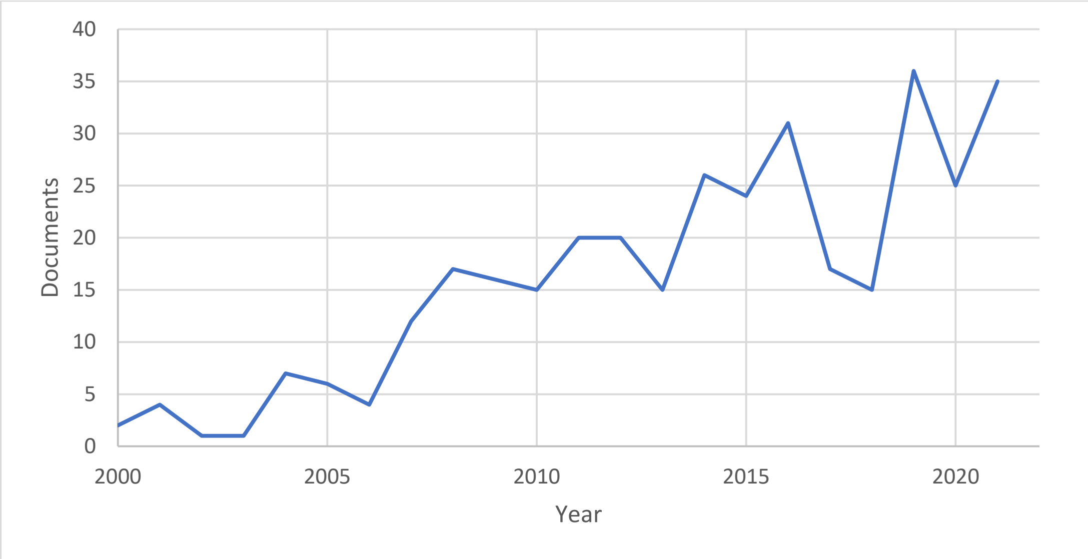

Figure 2. Number of documents published per year (extracted from Scopus).

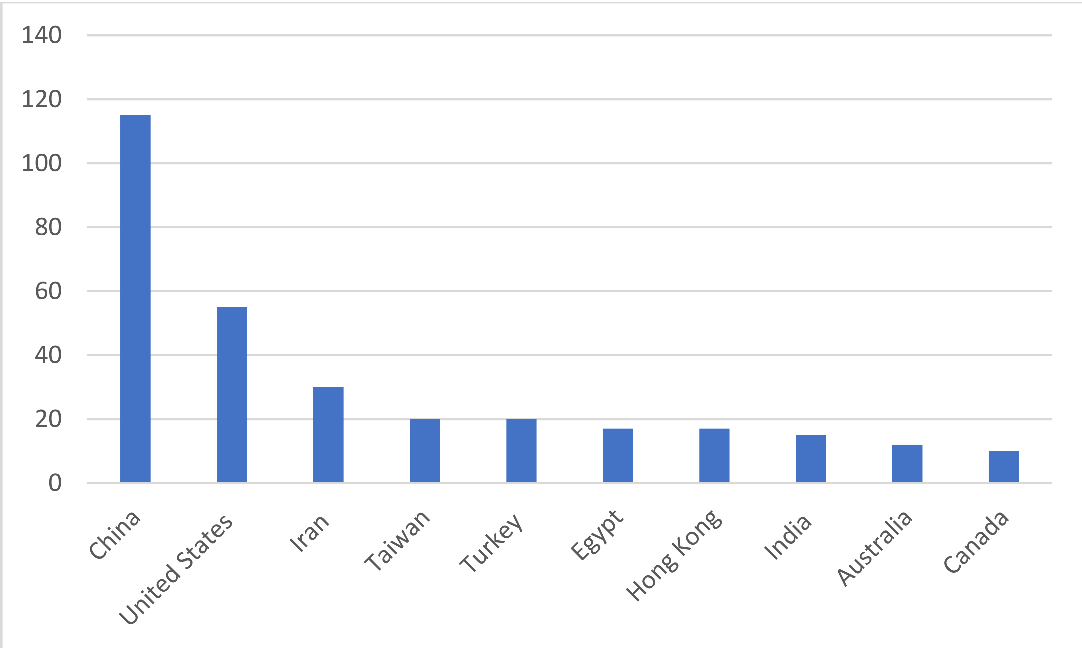

Figure 3. Number of documents per country (extracted from Scopus).

3. Findings

This section provides an overview of some of the developed models that are used to optimize time-cost problems. In addition, the features, the advantages, and the weaknesses of these models are evaluated.

3.1. Exact Models: Linear Programming

Traditionally, the exact mathematical models that were used to solve time-cost optimization problems were mainly based on LP models [23]. In fact, the earliest reference to activity crashing dates back to 1961, when Fulkerson used linear programming to compute the least cost curve for a project [24]. Similarly, Islam [25] used an LP model to minimize the cost of crashing the total project time, subject to crash time, unfolding the network, and project completion constraints. The model helped to reach optimality with flexibility provided by sensitivity analysis in a controlled computational effort and cost. Additionally, Karmaker and Halder used a linear programming approach to crash the activities of a construction project. The objective function was to minimize the total cost of crashing activities, taking into account the maximum reduction constraints, start time constraints, and project duration constraints. When the developed optimization model was applied, it was able to reduce the time by 17% while only increasing the cost by 3.73% [26]. However, these models had their own deficiencies such as compressing unnecessary activities and computing an inaccurate earliest start time of the activities if they were not on the critical path [27]. As time progressed, many studies attempted to challenge the linearity of the relationship between time and cost, and more complex models were developed to overcome the deficiencies of previous models, such as linear integer, mixed integer, non-linear, and dynamic programming models. Chassiakos and Sakellaropoulos [28] developed a linear integer programming model to generate all time-cost combination alternatives for a construction project with generalized activity constraints to generate the optimal time-cost curve at minimum project cost. The authors concluded that the results were very accurate, but this comes at the expense of a large amount of computational time. Other researchers have used dynamic programming models to decompose and schedule the project network into several subnetworks in order to reduce the computational effort [29].

3.2. Exact Models: Non-Linear Programming

Al-Haj and El-Sayegh [30] presented a non-linear integer programming model to solve time-cost optimization problems by taking into account the effect of total float loss. The authors claimed that such a model can provide managers with more flexibility and new trade-offs between time and cost, thereby increasing the success rate of construction projects. Moreover, Tatar et al. [31] utilized a mixed integer programming model with a delay penalty for discrete time-cost optimization in a total of 600 problem instances with 1000 activities, and the majority of the instances were solved to optimality. Among the generated instances, there were large-scale instances, which reflect real-life construction projects. A follow-up study by Moussourakis and Haksever presented a flexible mixed integer programming model to solve the time-cost trade-off problem in construction [32]. The model is flexible as it explores different possibilities and answers "what if" questions, which is very helpful to the decision maker. Similarly, in an attempt to reflect more real-life characteristics of construction projects, Ammar [33] presented a non-linear programming time-cost optimization model that takes into account discounted cashflows. The author concluded that the model guarantees an optimum solution while using a precise discrete activity time-cost relationship and considering the value of money over time.

Klansek and Psunder [34] used a non-linear programming algorithm, where a continuous total project cost function was subjected to activity precedence relationship constraints, as well as activity duration and project duration constraints. The model yielded the optimal start times, durations, and direct costs of the project activities, which provides invaluable insight to contractors before tender submission and proved to surpass methods such as CPM and PERT analysis. Other researchers who also implemented non-linear models include Diaby et al. [35], who incorporated negative exponential curves, as well as Goh and Hall [36], who used convex piecewise linear functions. The composite objective function of Goh and Hall was to minimize the total project cost through three cost components: contract penalty, overhead cost component, and sum of crashing cost of each activity. The first cost component was modeled as a piecewise linear non-decreasing convex function, while the second component is linear, and the third component was assumed to be individually piecewise linear, non-decreasing, and convex [36]. Moreover, in a recent study by Ballesteros-Pérez et al., two non-linear models were proposed that assumed either collaborative or non-collaborative resources to allow for both discrete and continuous as well as deterministic and stochastic configurations [37]. Despite the wide use of exact mathematical methods to optimize time-cost problems and their obvious strengths in guaranteeing optimality, they are not very efficient in solving multi-objective time-cost optimization problems. These models may also not be compatible with very large-scale problems and discontinuous decision space [38]. That is why approximate, metaheuristic, and evolutionary models have started gaining more attention in the literature of this field [39].

3.3. Approximate Models

Usually, reducing the time for the project or some of the critical activities in the project is associated with an increase in the direct cost. However, there are some circumstances or reasons that require a reduction in time, for instance, when the imposed completion date of the project cannot be met with the normal duration of the activities. Thus, one or more of the activities in the critical path must be compressed, which will increase its direct cost. The total cost of the project consists of direct cost (cost of materials, labor, equipment, etc.) and indirect cost (overhead cost of supervision, administration, etc.). The direct cost is associated with the duration of each activity, so it increases as the project time is reduced from its planned time. However, the indirect cost is directly related to the project's duration, and it is not associated with any activity. Thus, it will decrease as the duration of the project is reduced. Thus, the project manager faces a TCT problem in which a decision has to be made: Is the reduction in project time worth the additional cost? Which activity or activities should be shortened? And to what extent should it be shortened? As this is a complicated problem and involves many uncertainties about the durations and cost of the activities, approximate models can be used to give near-optimal solutions of time and cost problems [28].

3.3.1. Network Analysis Methods

CPM and PERT are widely used techniques in planning and scheduling large and complex projects [40,41]. They provide graphical representation of the activities that constitute the projects and the associated estimated cost and duration of each activity [42,43]. Therefore, they can be used to allocate resources and manage activities in a way that can optimize the cost and the time of the project [43]. CPM is a tool that can be used to determine critical activities and their durations. In this process, the total cost of the project is computed before compressing the activities and then compared with the cost of reducing the project's time. In addition, it provides a systematic approach to making decisions on the activities to be compressed, since the cost of shortening any activity is not the same for all activities. Hence, the decision can be made based on a rational creation, which is to minimize the additional cost incurred from shortening the project duration by a certain amount of time. Huang et al. [44] utilized CPM to develop a time-cost model for construction projects that aims to find activities in which their direct cost slope δ_d (the ratio of the change in direct cost to the change in the activity's duration) is less than or equal to the slope of indirect cost δ_i. Based on the values of the two slopes, if the duration of an activity is compressed by Δt days, that means the total cost of the project is either decreased or will not change after the activity is compressed by a certain amount Δt in the compressible range $(\delta_d - \delta_i)\ \Delta t \leq 0$. In the same way, Siemens exploited the CPM tool to develop a systematic approach to optimizing the total cost of the project for various possible project completion dates called the Siemens approximation method (SAM) [45]. The advantage of this method is that it is relatively simple and systematic. Accordingly, it can be used for hand computation unlike other methods that require computer programming and complex computation. However, it does not guarantee an optimal solution. In addition, SAM assumes that the cost/day of reducing the completion time is a convex function so that the activity time-cost trade-off curves can be approximated with piecewise linear curves. The algorithm starts by developing the project network and then identifying all possible paths to complete the project. Then, the expected completion time of the project can be calculated based on the longest path (critical path), which is compared with the desired completion time. Then, the algorithm determines the cost slope of the activities in this path, and the activity or activities with the lowest cost slope are selected to be compressed, similar to the procedure followed by Huang et al. [44].

PERT is another project management planning tool that was developed in 1958 by the US Navy Special Projects Office to manage the US Navy's Polaris nuclear submarine project [46]. PERT is commonly used in conjunction with CPM to estimate the duration of the project. However, it assumes that activities' duration follows a statistical distribution (Beta distribution) [47]. Unlike CPM, which provides one time estimation and one cost estimation, PERT utilizes three time estimates for each activity (optimistic, expected, and pessimistic) [48]. Thus, it accommodates the uncertainty in project scheduling as it creates three different estimates of the project duration, and based on that a normal distribution for the project duration can be created [8]. In other words, CPM provides deterministic estimates of the activities' durations, while PERT provides probabilistic estimates [49]. Hussein and Habib [50] used PERT in conjunction with CPM to optimize the time and cost of completing the construction of a university project in Iraq. PERT was used to determine the earliest start and finish times of each activity or stage of the project in addition to the latest start and finish times. Then, CPM was used to determine the critical path of the project. Similarly, Harjanto et al. [51] implemented CPM and PERT analysis to optimize the duration of a hospital project in Bogor District, Indonesia at the lowest possible cost. PERT analysis was used to compute the three time estimates for each activity. The optimization process resulted in a reduction of 31 days in the project duration with an insignificant increase in the cost of about 0.25%.

3.3.2. Evolutionary Algorithms

For construction projects, especially large projects, there are different alternatives of possible durations and costs for the activities of the projects. Therefore, finding the optimal solution among these combinations can be difficult and time-consuming. CPM- and PERT-based methods are not efficient in solving large-scale projects with hundreds of activities, where all possible paths cannot be easily enumerated. Therefore, alternative algorithms are needed to search for optimal or near-optimal solutions for large-scale problems in a reasonable computational time. Heuristics and metaheuristics are other approximate methods that can be used to find a good and feasible solution in a reasonable computing time for large-scale complex problems [52]. A genetic algorithm (GA) is a metaheuristic search algorithm that is inspired by the natural evolution of the biological world and the survival of the strong [26,27]. GAs mimic the natural selection process to search in the decision space for near-optimal solutions [53,54]. Hence, they are utilized as a search and planning tool for project scheduling problems that are NP-hard problems [19,41]. GAs can provide good enough and fast enough solutions without mathematical rigor, and because of that, GAs have been widely utilized for construction project optimization [55–57]. Haque and Hasin [58] used a GA as a searching tool to solve the time-cost optimization problem of projects under uncertain conditions and fuzzy time periods. The fuzziness and vagueness involved in construction projects were taken into consideration when developing the model. The authors used α-cut to represent risk levels. The α-cut levels incorporate the confidence of the experts or decision makers. The proposed model can help decision makers to find the optimal combination of activities' durations at different risk levels with a minimum total project cost and to ensure that the completion of the project is within the planned time.

The objective function is subjected to two constraints. The first constraint is that the project duration cannot exceed the permissible maximum project duration, and the second constraint is to restrict the duration of each activity between the maximum and minimum durations. Likewise, Feng et al. [20] developed a GA to solve time-cost problems of construction projects. According to Feng et al., optimizing the cost and time of the project is achieved by making a sequence of decisions such as selecting appropriate resources, proper methods, and equipment to perform each activity. This sequence of decisions was represented in the developed GA as a string, and the phases of the GA were carried out to search for the optimal time and cost of the project by selecting the optimal sequence of decisions. The performance of the developed model was tested on an 18-activity CPM network. Based on the precedence relationships between the activities and the associated time and cost of each activity, there were about 5 billion possible combinations. The developed GA model was able to find a solution by searching among 20,000 possible combinations only. Although it was a small fraction of the total search space, this solution was near to the optimal solution obtained from exhaustive enumeration. However, the developed model was designed to handle only finish-to-start relationships between the activities, it did not consider the other types of relationships (start-to-start, start-to-finish, and finish-to-finish) between the activities. Moreover, the model was formulated on the assumption that resources are not limited, which contradicts the reality that resources are limited and the required resources to compress any activity may exceed the available resources.

Although GAs provide efficient, fast, and powerful search methods, they have some limitations and drawbacks. The efficiency of a GA depends on the parameters used such as fitness function, selection mechanism, mutation, and crossover operators, and it may not converge to the optimal solution.

3.3.3. Swarm Intelligence (SI) Algorithms

Another category of metaheuristic optimization algorithms has evolved, which is called swarm intelligence (SI) algorithms. Such algorithms can search for an optimal solution of computationally sophisticated combinatorial problems. This category of search algorithms was inspired by the social behavior of biological populations in nature [59]. Ant colony optimization (ACO) is a metaheuristic approach that falls within the SI category that is used to solve combinatorial problems [59]. ACO was developed by Colorni et al. [60], and it was inspired by the behaviors of ants as when they search for food, they tend to search for the shortest path to the food sources from their nest [61,62]. Ng and Zhang [39] used an extension of the ACO algorithm called ant colony system (ACS) to establish a model to solve multi-objective time-cost optimization problems of construction projects. The ACS algorithm consists of four elements: construct solutions, selection probability, update pheromone rule, and stopping criterion. In the first element, solutions are constructed randomly, which represents the path (i.e., the order of graph edges that it will follow) that an ant travels from the first to the last activity to finish the project, with each edge having a pheromone level. Based on this path, the total duration and cost of the project can be calculated and evaluated through a fitness function using the modified adaptive weight approach (MAWA). MAWA integrates the time and cost of the project into a single objective function to find the minimum combination of them. In the second element, the selection probability is determined by the pheromone level assigned to each edge. In the third stage, the trail is memorized, and the pheromone level of each edge is updated. The fourth stage determines the stopping criterion of the algorithm. The stopping criterion is when a certain number of generations has been reached, a specific central processing time has been reached, or the quality of the generated solutions remains unchanged for several consecutive iterations. However, the ACO model is very sensitive to the parameters selected such as the number of ants in each iteration, weighting factors, and pheromone levels. These parameters can greatly impact the quality of the solution and the convergence speed of the algorithm. Moreover, Abdallah et al. [43] utilized CPM and PERT networks to develop deterministic and probabilistic networks of the project's activities to account for uncertainty in construction projects. Then, ACO was implemented to solve path minimization of these networks. Similar to the network that contains all possible paths that the ants may follow from their nest to the food location, CPM and PERT networks can imitate this network, where the start node of the project is equivalent to the ants' nest and the end of the project is equivalent to the food location. The authors considered a deterministic network (CPM) and a probabilistic network (PERT), and ACO was implemented in the two networks to determine the shortest path. First, a random amount of pheromone trails is distributed randomly on all possible paths in the network. This step is called the pheromone initialization step. Then, the first ant is placed on the start node, and it walks through every possible path from the start node to the end node, which is called local search. The length of each path is computed depending on the pheromone traces left on each line between two nodes. The pheromone released by the ants is a function of the length of the path, where pheromone concentration is inversely proportional to the length. Thus, the shortest path will have the highest pheromone concentration. For each step, the length is recorded in a tabu list. After each trial, the algorithm checks whether a better solution has been obtained and updates the tabu list with the optimal solution (the shortest path). The benefits of using this method are that it can be used to solve both deterministic and probabilistic networks and it can be used for complicated or large projects. The main limitations and drawbacks of the ACO model are that it is not suitable for large-scale problems because the individual ants will move randomly, and the time complexity of the model will increase. In addition, ACO models include many parameters such as pheromone parameters and the number of ants, and the initial values of these parameters must be selected properly. Improper selection of these parameters may result in failure to obtain the optimal solution, stagnation around a local optimum, or poor convergence [63]. Moreover, the convergence of the ACO model is guaranteed, but the convergence time is uncertain [64].

In construction projects, resources are often limited. Hence, the development of construction projects is constrained by resource supply. Therefore, resource management and optimization are essential because they directly affect the cost and duration of the project [65]. If the resource demand is higher than available resources, this might increase the project's duration or cost, if extra resources are purchased. ACO has been used extensively to solve resource leveling problems (RLPs) in construction projects [66–70]. The ACO model checks the required resources for each activity and then identifies non-critical activities in the project and their float days to determine the optimal shift of these activities to solve the resource overrun problem. This can reduce the resource demand without delaying the project or increasing its cost.

Particle swarm optimization (PSO) is a computational metaheuristic algorithm that also falls within the SI category [59]. PSO was developed in 1995 [71,72], and it mimics the social behavior of a school of fish or a flock of birds that moves in a swarm [73]. The members of the flock share their experiences and discoveries with all other members of the group to find their food sources [74]. Thus, PSO follows the same behavior to optimize the problem by iteratively improving a population of possible solutions, called particles, with respect to a given degree of quality [75]. The problem is solved by moving these particles in the search space based on a mathematical formula of the particles' positions and velocities [76,77]. The movement of each particle is influenced by its local best-known position, but it is also guided toward the best recognized positions in the search space. Then, these positions are updated to better positions found by other particles [76,77]. Several researchers have utilized PSO to solve TCT problems [78–80]. Yang [18] used PSO to optimize time-cost trade-off problems by using a multi-attribute utility function in terms of the cost, the time, and the quality of the project as the evaluation function. As mentioned previously, the project consists of a set of activities, and each activity has a normal completion time and a crash completion time. The normal completion time of the activity i, T_n (i), is associated with normal cost, C_n (i), and normal quality, Q_n (i). The crash completion time of the activity i, T_c (i), is associated with crash cost, C_c (i), and normal quality, Q_c (i). The direct cost of the activity is inversely related to its duration, while the quality of the activity is proportional to its duration. The objective of the developed model is to make a trade-off among time, cost, and quality of the project. Thus, a model of multi-attribute utility function was developed that searches for the best combination that provides the minimum cost and time with good quality. There are several approaches to evaluate the overall quality of project Q. For example, it can be calculated by the quality-weighted average method, where higher weight coefficients ω are allocated to the activities that have higher impact on the whole project. The objective function of the model is expressed as follows:

$$u(T,C,Q) = \omega_T\, u(T) + \omega_C\, u(C) + \omega_Q\, u(Q) \tag{1}$$

where $u(T)$, $u(C)$, and $u(Q)$ are the univariate utility functions of time, cost, and quality respectively. ω_T, ω_c, and ω_Q are the corresponding weight coefficients, respectively. The objective function is subjected to four constraints.

The first constraint is to make sure that the earliest starting time of the first activity is zero.

$$E(1) = 0 \tag{2}$$

where $E(i)$ is the earliest time of activity i.

The three other constraints are to limit the completion time, the cost, and the quality of each activity between the normal and crash values. After formulating the objective function and the constraints, PSO was used as a search algorithm to find the optimal combination in the search space. A case study of a construction project was selected to test the proposed algorithm. The results showed that the proposed algorithm can provide the global optimum and avoid local optima while maintaining a high convergence speed. However, in this algorithm, the authors assumed that both the quality and the cost of the activity are linear functions of the activity's completion time, which is not always true. Likewise, Zhang and Li [81] utilized the mechanism of PSO to search for optimal solutions for TCT problems for a set of construction methods with the associated durations and costs of the construction activities. The performance of the proposed algorithm was evaluated in terms of the degree of convergence, the speed of the convergence, and the diversity using three quantitative metrics. These metrics of the developed algorithm were compared with the metrics of two other algorithms. The proposed algorithm achieved superior performance compared with the other algorithms with regard to convergence degree, diversity, and speed of convergence. Similarly, Lin et al. [82] developed two PSO-based models to find the optimal solution of shortest duration for construction projects with limited resources. The main disadvantage of the PSO model is the premature convergence to local optimum instead of global optimum [83–85].

3.4. Hybrid Algorithm Models

Interest in hybrid algorithms to solve optimization problems has grown significantly in recent years as they exploit the benefits of different algorithms and ultimately provide high-quality solutions [86]. Among the famous hybrid genetic algorithm models that have been adopted in the literature to optimize TCT in construction projects is the combination of genetic algorithms with simulated annealing as it enables an improved hill-climbing ability and has a better fining capability [87,88]. In fact, Sonmez and Bettemir [89] presented a hybrid strategy for a time-cost trade-off problem that incorporated genetic algorithm (GA), simulated annealing (SA), and quantum simulated annealing (QSA) techniques. SA is a stochastic search algorithm that uses temperature to determine the probability of moving from one state to another. Meanwhile in QSA, the tunneling field strength determines the distance between the current and neighboring states [90]. The results showed that the local search capability was enhanced due to the quantum simulated annealing techniques, and the adopted hybrid strategy significantly improved the convergence of the genetic algorithm [89]. In fact, when the authors compared the results of the hybrid algorithm with GAs, they concluded that, on average, the genetic algorithm has 4.63% deviation from the optimal solution, while the hybrid algorithm only has 1.12% deviation. Alavipour and Arditi [91] proposed a hybrid algorithm that combines both a genetic algorithm and linear programming for time-cost trade-off, which considered several financing alternatives rather than the conventional one line of credit for contractors. The results concluded that an optimum combination of different financing alternatives leads to an optimum financing schedule, which increases the negotiating power of the contractors in front of the lender.

Another hybrid genetic algorithm time-cost optimization model applied a combination of GA and dynamic programming. In a study by Ezeldin and Soliman [92], they proposed this hybrid technique to solve time-cost trade-off problems of project schedules with repetitive non-serial subprojects. The authors first generated a mathematical model to explore the factors that affect cost and duration relationships at activity and project levels.

Then, a genetic algorithm was used to locate optimum and near-optimum solutions from the hyperplane developed by the mathematical code. Finally, the dynamic programming model searched the vicinity of each of the near-optimum solutions found by the genetic algorithm to converge on the global optimum [92]. This type of hybridization provides effective optimal solutions to real-life construction problems. Additionally, in an attempt to account for labor and equipment allocation during time-cost optimization in construction projects, Shen et al. [93] implemented a hybrid algorithm that combined a genetic algorithm and the Cobb–Douglas production function (CDPF). The function relates technology to labor input and capital input to compute the total production. Indeed, CDPF is a feasible tool that has very significant features that can explain the origin of crashing cost under diverse scenarios. The results indicated that in all the ten cases where the hybrid algorithm was implemented, the outcome was the same as the optimum solution, which confirms the efficiency of the model.

A novel hybrid genetic algorithm that has recently been gaining attention in the literature of time-cost optimization in construction projects is the hybridization of PSO and GA. In fact, Albayark [94] developed this hybrid algorithm for resource-constrained construction projects. The author concluded that this hybrid strategy is very helpful in global optimization as it exploits the social component of PSO and the local search capability of genetic algorithms. Similar results were found by Ashuri and Tavakolan [95], who stated that a fuzzy-enabled GA-PSO hybrid strategy had a faster processing time for optimizing TCT problems in construction management than other existing algorithms. On a different note, Albayrak and Özdemir [96] hybridized a firefly algorithm with PSO to optimize time-cost trade-off in construction projects. The authors had two objective functions: minimize total project cost and minimize total project duration with constraints such as non-violation of precedence constraints, one mode per activity, initial project start time, and maximum project completion time. The outcomes were proven to be shorter and more economical than other metaheuristic algorithms [96].

Moghayedi and Windap [97] proposed a novel method for modeling uncertainty in construction projects. They identified three sources of uncertainties, namely, variability, correlation, and disruptive events, that can affect the time and cost of highway projects. To address these uncertainties, they developed a hybrid intelligent tool based on a neuro-fuzzy inference system (NFIS), which combines neural networks and fuzzy logic. The proposed tool was able to effectively model the different sources of uncertainty, leading to improved project planning and decision making. The advantages of the algorithm include more accurate cost and time estimations, better risk management of cost and time overruns, and enhanced project performance. The potential limitations and weaknesses of the proposed hybrid intelligent tool for modeling uncertainty in construction projects include the need for more sensitivity analysis to validate its effectiveness in capturing the impact of various sources of uncertainties on estimating the cost and duration of infrastructure projects. Additionally, the tool may require significant data preparation efforts and may not be easily applicable to smaller-scale projects due to the complexity of the model. Furthermore, the accuracy of the model's predictions depends on the quality and quantity of available data, and the model may require frequent updates as new data becomes available. Moreover, the tool relies heavily on historical data, which may not always be available or accurate.

4. Conclusions

The optimization of time-cost trade-off problems in construction projects is a critical task for project managers to ensure successful completion within the available budget and scheduled time. However, this is a complex process as it involves comparing and evaluating alternative schedules to determine the optimum one with the lowest total cost, despite significant uncertainty in the time-cost trade-off. To report the state of the art on time-cost optimization models in construction projects, a systematic literature review was conducted according to PRISMA guidelines, which consist of four stages. This paper presents a comprehensive overview of time-cost optimization models used in construction projects, which were classified into three categories: exact models, approximate models, and hybrid models. Exact models generate an exact optimal time-cost solution but are computationally expensive and inefficient in solving multi-objective and large-scale problems. Approximate models provide near-optimal solutions and reduce computational effort but may not be able to solve large-scale projects. The third category, hybrid models, showed great promise by combining the strengths of different algorithms to yield high-quality and efficient solutions. The review identified knowledge gaps, such as improving the deficiencies of some models that were too simplistic and using larger test examples that mimic real-life construction challenges. Hybrid algorithm models have great potential due to their ability to handle large-scale problems with discontinuous decision spaces, which traditional exact models struggle to solve efficiently. Hybrid algorithms can provide near-optimal solutions with reduced computational effort compared with approximate models. Moreover, hybrid algorithms offer flexibility in selecting and combining different algorithms, depending on the specific problem and objectives, allowing for a customized approach and potentially better results compared with using a single algorithm. Therefore, it is recommended that researchers invest more effort into comparing the performance of different hybrid algorithm models against each other to better understand which algorithm combinations yield the highest-quality solutions. This paper has contributed to the existing knowledge of time-cost optimization models in construction projects by categorizing these models and identifying knowledge gaps. The emphasis on the advantages of hybrid algorithm models could lead to the development of more efficient and effective models in the future. Table 1 summarizes the aforementioned categories of time-cost optimization models.

Table 1. Time-cost optimization models in construction projects.

Category	Model	Author(s)	Main Contributions	Limitations and Future Research Opportunities
Exact models	Linear programming	Islam, 2011 [25] Karmaker and Halder, 2017 [26]	Optimal solutions can be achieved with flexibility from sensitivity analysis at a controlled computational time	The assumption of linearity between time and cost remains unchallenged
	Linear integer programming	Chassiakos and Sakellaropoulos, 2005 [28]	All time-cost alternatives for activities with generalized precedence relationships were generated with very accurate results	Requires a lot of computational effort
	Non-linear integer programming	Al-Haj and El-Sayegh, 2015 [30]	The impact of total float loss is accounted for, which increases the success rate of construction projects	Not very efficient in solving multi-objective time-cost optimization problems
	Non-linear programming	Ammar, 2011 [33] Klansek and Psunder, 2008 [34]	Real-life characteristics are considered such as discounted cashflows and continuous objective function subjected to general activity precedence relationships	Incompatible with very large-scale problems and discontinuous decision space
	Mixed integer programming	Tatar et al., 1997 [31] Moussourakis and Haksever, 2004 [32]	Discrete time-cost optimization models with delay penalties and very large instances that reflect real-life characteristics	Some test samples were too small, and some assumptions rendered some of the models simplistic and not representative of real-life construction projects

Table 1. *Cont.*

Category	Model	Author(s)	Main Contributions	Limitations and Future Research Opportunities
Approximate models	CPM	Huang et al., 2008 [44] Siemens, 1971 [45]	Utilizes CPM to find the minimum cost of the project associated with the shortest duration without exceeding the planned budget	It provides one time estimation and one cost estimation, which does not account for uncertainty in construction projects. In addition, for bigger projects, CPM may be complicated and critical path is not clear.
	Combination of PERT and CPM	Hussein and Habib, 2019 [50] Harjanto et al., 2019 [51]	Usually, PERT is used in conjunction with CPM to accommodate the uncertainty in project scheduling as it provides probabilistic estimates for duration and cost	They are not suitable for large–scale projects with hundreds of activities where all possible paths cannot be easily enumerated
	Genetic algorithm	Haque and Hasin, 2012 [58] Feng et al., 1997 [20]	Helps decision makers to find the optimal combination of activities' durations at different risk levels with a minimum total project cost and to ensure that the completion of the project is within the planned time. It provides efficient, fast, and powerful search methods.	The efficiency of GAs depends on the parameters used such as fitness function, selection mechanism, mutation, and crossover operators, and it may not converge to the optimal solution
	Ant colony optimization (ACO)	Ng and Zhang, 2008 [39] Duraiswamy and Selvam, 2022 [66] Xiong and Kuang, 2006 [67] Garmsiri and Abassi, 2012 [68] Savitri, 2020 [69] Sun et al., 2021 [70]	ACO-based algorithms were developed to optimize the total project duration and cost simultaneously. In addition, ACO-based models were used to minimize the project's duration with limited resources.	It includes many parameters. Improper selection of the initial values of these parameters can significantly lower the efficiency of finding the optimal solution.
	Particle swarm optimization (PSO)	Yang, 2009 [18] Elbeltagi et al., 2016 [78] Zhang et al., 2014 [79] Wang and Feng, 2008 [80] Zhang and Li, 2010 [81] Lin et al., 2022 [82]	PSO provides fast and efficient search solutions to find the best solution for TCT problems. It is a multi-objective optimization algorithm that compares a set of alternative construction methods and the associated cost and duration and finds the optimal method among them. Additionally, a PSO-based model was used to find the optimal solution of shortest duration of construction projects with limited resources.	There is a chance that the model will be trapped in a local optimum instead of finding the global optimum

Table 1. *Cont.*

Category	Model	Author(s)	Main Contributions	Limitations and Future Research Opportunities
Hybrid algorithm models	Genetic algorithm with linear programming	Alavipour and Arditi, 2018 [91]	Several financial alternatives can be considered for an optimum financing schedule that increases the negotiating power of contractors	Very promising as it exploits the good properties of different algorithms, which ultimately leads to high-quality and efficient solutions
	Genetic algorithm with simulated annealing	Sonmez and Bettmir, 2012 [89]	Hybrid strategy improved convergence as local search capability was enhanced by QSA	• Very promising as it exploits the good properties of different algorithms, which ultimately leads to high-quality and efficient solutions • More research is required to understand better how the hybridization of different algorithms affect the final outcomes • Comparison of different hybrid algorithm models against each other is required to better understand which algorithm combinations yield the highest-quality solutions
	Genetic algorithm with dynamic programming	Ezeldin and Soliman, 2009 [92]	Dynamic programming searches the vicinity of the near-optimal solutions generated by GA to reach global optimum	
	Genetic algorithm with Cobb–Douglas function	Shen et al., 2016 [93]	The origin of the crashing costs from labor and equipment can be quantitively explained	
	Genetic algorithm with particle swarm optimization	Albayark, 2020 [94] Ashuri and Tavakolan, 2012 [95]	Social component of PSO and local search capability of GA are combined for better global optimization	
	Neuro-fuzzy inference system (NFIS)	Moghayedi and Windap, 2022 [97]	A hybrid intelligent tool proposed to model three different sources of uncertainty. The results showed that this tool can improve accuracy of cost and time estimations and reduce the risk of cost and time overruns in highway projects.	• Need for more sensitivity analysis to validate its effectiveness in capturing the impact of various sources of uncertainties • Significant data preparation efforts may be required • May not be easily applicable to smaller-scale projects due to the complexity of the model • The accuracy of the model's predictions depends on the quality and quantity of available data

5. Recommendations and Future Research Direction

Time-cost optimization models have been developed to minimize costs and maximize profits in construction projects by allocating resources such as labor, materials, and equipment. However, there are still several research gaps and potential areas for future research in this field. Firstly, the majority of current time-cost optimization models are based on simplified assumptions that do not fully capture the complexity and uncertainties of real-world construction projects. Therefore, there is a need to incorporate more realistic and complex constraints into the models. Secondly, construction projects often involve multiple objectives, such as minimizing costs, meeting deadlines, and ensuring quality. Thus, there is a need for time-cost optimization models that can handle multiple objectives and trade-offs between them. Thirdly, many time-cost optimization models are computationally intensive and may not be practical for very large or complex projects. Hence, there is a need to improve the scalability and computational efficiency of the models. Fourthly, construction projects often involve changes and uncertainties that can affect the allocation of resources. Future research could focus on developing models that can handle dynamic environments and adapt to uncertainties in the construction process. Additionally, there are many other types of models that are used in construction management, such as cost estimation models, risk assessment models, and scheduling models. Future research could focus on integrating time-cost optimization models with these other types of models to improve decision-making processes. Finally, future research could explore the use of different types of fuzzy logic functions, such as type-2 fuzzy logic functions, to approximate the uncertainties involved in construction projects and further improve the accuracy and effectiveness of project time-cost optimization models. As an example, recent studies have successfully utilized type-2 fuzzy logic functions in various fields, such as control systems and chaos theory. For instance, Sabzalian et al. [98] developed an observer-based type-2 fuzzy control that was proposed for chaotic systems. In another study, robust control of a class of induction motors was achieved using rough type-2 fuzzy neural networks [99]. These innovative approaches could serve as a guide for future research seeking to improve the accuracy and effectiveness of time-cost optimization models for construction projects.

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