Construction Management and Economics

ISSN: 0144-6193 (Print) 1466-433X (Online) Journal homepage: https://www.tandfonline.com/loi/rcme20

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Construction Management and Economics (April 2006) **24**, 379–389

Valuation of the minimum revenue guarantee and the option to abandon in BOT infrastructure projects

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Received 18 October 2004; accepted 24 October 2005

The real option approach is used to value the minimum revenue guarantee (MRG) and the option to abandon in Build-Operate-Transfer infrastructure projects. The option to abandon is formulated under an investment option held by the concessionaire at contract signing and to expire before construction commencement. MRG is formulated as a series of European style put options in a single option pricing model. When combined with the option to abandon in the pre-construction phase, MRG is reconstructed as a series of European style call options to develop a compound option pricing formula. The Taiwan High-Speed Rail Project is chosen as a numerical case to apply the formulas. The results show both MRG and the option to abandon can create values. When MRG and the option to abandon are combined, they will counteract each other and their values will thus be reduced. Increasing the MRG level will decrease the value of the option to abandon, and, at a certain MRG level, the option to abandon will be rendered worthless.

Keywords: BOT, infrastructure, real option, option to abandon, minimum revenue guarantee

Introduction

Build-Operate-Transfer (BOT) infrastructure projects differ significantly from more traditional construction projects in how projects are implemented during the pre-construction phase. For traditional projects, the owner is responsible for project planning, environmental assessment, property acquisition and project funding. For BOT projects, the concessionaire is usually required to undertake project development tasks. The concessionaire faces substantial project development risks. Even if the concessionaire can successfully accomplish these tasks, BOT projects often become financially not viable due to changes in external investment conditions. As a result, BOT projects are more likely to fail in the pre-construction phase than in other project phases. To reduce the concessionaire's investment risks, BOT concession contracts often grant the concessionaire an option to abandon during the pre-construction stage (Huang, 1995).

Likewise, BOT infrastructure projects usually face substantial revenue risks during the operation phase. To avoid the downside risks of revenues, the concessionaire often negotiates with the government to provide a minimum revenue guarantee (MRG). Under an MRG, the government is obligated to cover the shortfalls between a pre-specified level of MRG and operating revenues realized by the concessionaire. MRG can increase the concessionaire's willingness to invest. It can also enhance the credit worthiness of BOT projects facing high revenue risks, since the guaranteed cash inflows can provide a minimum level of debt coverage.

The presence of the abandonment option will increase the concessionaire's flexibility in investment decisions and thus increase project value. The valuation of the option to abandon can be done by real-option theories (Dixit, 1989; Trigeorgis and Mason, 1989; Trigeorgis, 1993). The use of real-option theories in BOT project evaluation is not a new idea. Ho and Liu (2002), for example, developed a real-option pricing model to value government debt guarantees in BOT projects. For traditional construction projects, the real-option approach has also been used. For example, David *et al.* (2002) show that the real-option approach can be applied in traditional project planning; Tien (2002) analysed time-to-build options in sequential construction; and Michael and Charles (2004) developed a model to evaluate strategic project deferments.

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ISSN 0144-6193 print/ISSN 1466-433X online
http://www.tandf.co.uk/journals
DOI: 10.1080/01446190500434997

The presence of MRG can also increase project value, but the valuation of MRG is still an open issue. Paddock, Siegel and Smith (1988) had developed a real option approach to value offshore petroleum leases as staged-options, which also face substantial project development risks. But the obligation to pay under an MRG is very different from that of leasing. While the lessee usually promises to make a series of fixed payments to the lessor, the undertaker of an MRG pays only when project revenues fall below a pre-specified level of MRG. The undertaker will not pay at all when the realized operating revenues are higher than the MRG level.

This paper studies the valuation of the MRG and the option to abandon in the pre-construction stage. The option to abandon is formulated under an investment option held by the concessionaire, and the expiration date of the investment option is at the targeted construction commencement date. The MRG is constructed as a series of European style put options under a single option model. When the MRG is combined with the option to abandon, they will counteract each other, and the valuation becomes more complicated. The MRG is re-constructed as a series of European style call options to form a compound option.

Taiwan High Speed Rail Project is used as a numerical case to apply the derived option pricing formulas. The results show both the MRG and the option to abandon can create substantial values under the single option settings. In the compound option model, they counteract with each other, and their values are reduced. An increase of the MRG level will decrease the value of the option to abandon, and when the MRG level is high enough, the option to abandon will become worthless.

In the following sections, this paper begins with formulating a single option pricing model for the option to abandon in the pre-construction phase. MRG is then constructed as a series of European style put options to derive a single option pricing formula. A compound option model is further developed to combine the MRG with the abandonment option under the same BOT package. The Taiwan High Speed Rail Project is used as a numerical case to apply the derived formulas, and then observations and policy implications are drawn accordingly.

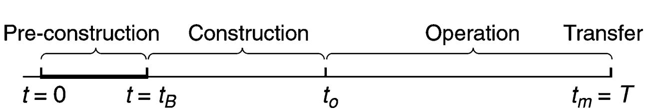

Figure 1 Typical BOT project life-cycle after contract signing

Valuating the option to abandon during the pre-construction stage

Figure 1 shows the lifecycle of a typical BOT project after concession tendering. It begins with a pre-construction phase at time $t=0$, when the concession contract is signed, and ends with project transfer at $t=T$, when the concession period expires. The concession contract often provides the concessionaire options to abandon during the lifecycle (Huang, 1995). We only focus on the pre-construction phase, since BOT projects are more likely to fail in this phase.

The concessionaire may decide to walk away at any time during the pre-construction phase, but we assume the final decision can only be made at the targeted construction commencement date, or $t=t_B$. We also assumed that once the concessionaire decides to invest, the project cannot be abandoned thereafter. Once the concessionaire decides to invest, the project will thus be completed on time and become operational at $t=t_O$. It will be operated for m years, from t_1 to t_m, and then transferred at the end of the concession, or $t=t_m=T$.

Based on these assumptions, the option to abandon is formulated under an investment option held by the concessionaire's at $t=0$ and to expire at $t=t_B$. Let R denote the project's operating revenues. Suppose R is a random variable, and follows a generalized Wiener process:

$$R=\left\{R_t|t\in[0,\,T]\quad \frac{dR}{R}=\alpha\,dt+\sigma\,dz=(\mu-\delta_R)dt+\sigma\,dz\right\} \quad (1)$$

Here, α is the growth rate of R. It has a variance σ^2 during a very short interval dt. δ_R is the shortfall rate between the project's discounted rate μ and the growth rate of R. The shortfall rate resembles the dividend payout rate of a stock option. The rate is the opportunity cost for keeping an option alive rather than exercising it. dz is the incremental part of the Wiener process. The expected value of dz is zero, and the variance of dz is equal to dt.

Further let I denote the project's total costs, and suppose I include both capital investment cost and operation cost. Suppose I is not stochastic in nature, and the growth rate of I is ϵ during a short interval dt. Then,

$$I=\left\{I_t|t\in[0,\,t_B]\quad \frac{dI}{I}=\varepsilon\,dt=(\mu-\delta_I)dt\right\} \quad (2)$$

Here, the shortfall rate of I is denoted as δ_I. The rate is the opportunity cost avoided by holding the option to abandon until its expiration date t_B. If the amount of I is deterministic, then ϵ is zero, and $\delta_I=\mu$.

Let F denote the value of the concessionaire's option to invest. F is a function of state variables R, I, and time

t, or $F=\{F(R, I, t) \mid t \in [0, t_B]\}$. By contingent claim analysis (Black and Scholes, 1973), F should satisfy:

$$\frac{1}{2}\frac{\partial^2 F}{\partial R^2}\sigma^2 R^2 + (r-\delta_R)R\frac{\partial F}{\partial R} + (r-\delta_I)I\frac{\partial F}{\partial I} + \frac{\partial F}{\partial t} - rF = 0 \quad (3)$$

The boundary condition of this partial differential equation (PDE) is:

$$F(R, I, t_B) = \left[\sum_{i=1}^{m}\left(R_{t_1}e^{\alpha(t_i-t_1)} - C_{t_i}\right)e^{-\mu(t_i-t_B)} - B_{t_B}\right]H(v) = [P_{t_B} - I_{t_B}]H(P_{t_B} - I_{t_B})$$

Here, $H(v)=\{1 \mid v \geqslant 0, 0 \mid v<0\}$. In this formulation, R_{t_1} is the revenue of the first operating period, r is the risk-free interest rate, and I^{t_B} is the present value of I at t_B. I^{t_B} includes both capital investment cost B^{t_B} and operation cost C^{t_B}, and P^{t_B} is the present value of total operating revenues at t_B. The boundary condition implies that the concessionaire will exercise the option to invest at t_B with strike price I^{t_B} if and only if $P^{t_B} > I^{t_B}$; otherwise, the project will be abandoned.

Following McDonald and Siegel (1986), let Z and $W(Z, t)$ be the transformation variables:

$$Z=\frac{R}{I} \qquad W(Z, t)=\frac{F}{I}$$

Accordingly, Equation 3 is simplified:

$$\frac{1}{2}\frac{\partial^2 W}{\partial Z^2}\sigma^2 Z^2 + (\delta_I - \delta_R)Z\frac{\partial W}{\partial Z} + \frac{\partial W}{\partial t} - \delta_I W = 0 \quad (4)$$

subject to

$$W(Z, t_B) = [\Phi(T)Z - 1]H(\Phi(T)Z - 1)$$

This PDE is similar to that of the original Black-Scholes model. Using the solution methods by Kutner (1988) and Hwang and Jou (1994), the price formula of the option to invest can be derived:

$$F(R, I, 0) = P^0 N(k_1) - I^0 N(k_2) \quad (5)$$

where

$$k_1 = \frac{\ln\left(P^0/I^0\right) + \left(\sigma^2/2\right)t_B}{\sigma\sqrt{t_B}}$$

$$k_2 = \frac{\ln\left(P^0/I^0\right) - \left(\sigma^2/2\right)t_B}{\sigma\sqrt{t_B}} = k_1 - \sigma\sqrt{t_B}$$

Let f denote the value of the option to abandon. The value is the difference between the value of the investment option and the project's NPV at $t=0$:

$$f = F - \left(P^0 - I^0\right) \quad (6)$$

Substituting Equation 5 into Equation 6 gives the following price formula for f:

$$f = P^0[N(k_1) - 1] - I^0[N(k_2) - 1] = I^0 N(-k_2) - P^0 N(-k_1) \quad (7)$$

This solution is similar to that of a European style put option. Here, P^0 is the present value of operating revenues, I^0 is the present value of the total investment costs at $t=0$, and $N(\cdot)$ is a cumulative normal distribution function.

Valuating minimum revenue guarantee

In a single option setting, the MRG can be constructed and valued as a series of European style put options in the operation phase. Let M_{t_i} define the MRG level specified in the concession contract, $i=1\sim m$. The downside risk of operating revenues is limited to M_{t_i}, and the project's n[th] period payoff function is shown in Figure 2. To find the pricing formula for the MRG, we decompose the project's payoff function into two parts, namely an n[th] period operating revenue and an MRG cash inflow; see Figure 3 and Figure 4 respectively. In Figure 4, the payoff from the MRG is specified as $(M_{t_n} - R_{t_n})$, which defines the shortfall below M_{t_n} undertaken by the government.

Let Q_{t_n} denote the value of the n[th] MRG, R for the projected operating revenues, and δ_M for the shortfall rate with the same property as δ_I. Assume R is stochastic in nature as specified in the previous section. Q_{t_n} must satisfy:

$$\frac{1}{2}\frac{\partial^2 Q_{t_n}}{\partial R^2}\sigma^2 R^2 + (r-\delta_R)R\frac{\partial Q_{t_n}}{\partial R} + (r-\delta_M)M\frac{\partial Q_{t_n}}{\partial M} + \frac{\partial Q_{t_n}}{\partial t} - rQ_{t_n} = 0 \quad (8)$$

subject to

$$Q_{t_n}(R, M, t_n) = (M_{t_n} - R_{t_n})H(M_{t_n} - R_{t_n})$$

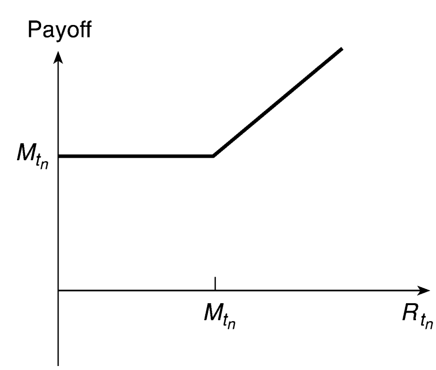

Figure 2 The n[th] period revenue payoff function

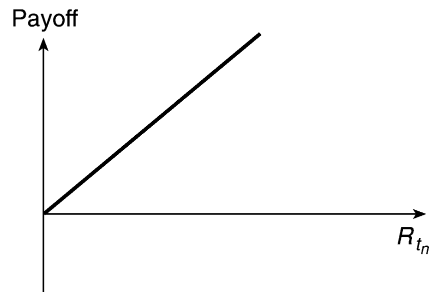

Figure 3 The n[th] period revenue payoff function from R

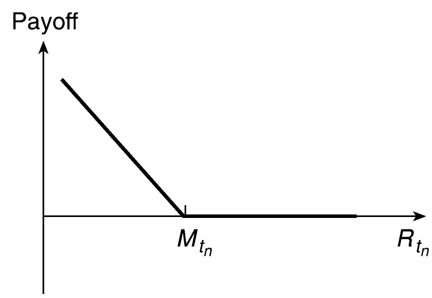

Figure 4 The n[th] period revenue payoff function from MRG

Solving by the same solution method gives the following price formula:

$$Q_{t_n}(R, M, 0) = M^0_{t_n} N(-d_2) - R^0_{t_n} N(-d_1) \tag{9}$$

where

$$d_1 = \frac{\ln\left(R^0_{t_n}/M^0_{t_n}\right) + \left(\sigma^2/2\right)t_n}{\sigma\sqrt{t_n}}$$

$$d_2 = \frac{\ln\left(R^0_{t_n}/M^0_{t_n}\right) - \left(\sigma^2/2\right)t_n}{\sigma\sqrt{t_n}} = d_1 - \sigma\sqrt{t_n}$$

Here, $R^0_{t_n}$ is the present value of R_{t_n}, and $M^0_{t_n}$ the present value of M_{t_n}. Both of them are discounted by μ at $t=0$. Let Q denote the total value of the MRG. Q is found by an aggregation of Q_{t_i}, $i=1\sim m$, or $Q=\sum_{i=1}^{m} Q_{t_i}$.

Combining MRG with the option to abandon

When the MRG and the option to abandon are combined under the same BOT package, they will interact with each other. Suppose the holding of the MRG option is dependent on the concessionaire's decision to invest at t_B. The concessionaire's option to invest at $t=0$ can be formulated as an option on option, or a compound option, under which the exercise payoff of the option to invest is related to the MRG level.

To derive the compound option pricing formula, first reconstruct Figure 2 as two different payoff functions: a constant cash flow denoted as M_{t_n} (Figure 5), and a European style call option whose value is written as A_{t_n} (Figure 6). The strike price of the call is M_{t_n}, the exercise payoff is $(R_{t_n} - M_{t_n})$, and the expiration date is at the end of t_n.

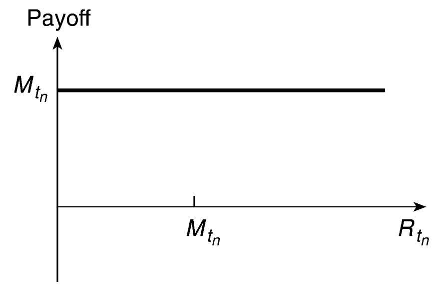

Figure 5 A constant cash flow

At $t=t_B$, the concessionaire decides whether or not to invest I^{t_B} for obtaining the combined value of M_{t_i} and A_{t_i}, i=1~m. Once the concessionaire decides to invest, the project cannot be abandoned thereafter. Thus, if the concessionaire does not abandon at $t=t_B$, s/he will obtain all M_{t_i} and A_{t_i}, i=1~m. Each call option A_{t_i} is independent, to be exercised only if the realized operating revenue at t_i is above the MRG level according to Figure 6.

Suppose F_M is the value of the option to invest at $t=0$. F_M is the aggregated value of M_{t_i} and A_{t_i}. To find F_M, first amortize the present value of I at t_B, denoted as I^{t_B}, by m portions, and let D_{t_i} denote the amortized cost at t_i, $i=1\sim m$. Under this amortization schedule, the concessionaire decides at $t=t_B$ either to invest the present value of D_{t_i} for obtaining M_{t_i} and A_{t_i} or not. The calculation of D_{t_i} is presented in the next section.

Then, let's start with the n[th] period of the concession contract. When the concessionaire decides to invest at $t=t_B$, the present value of the n[th] amortized cost $D^{t_B}_{t_n}$ is invested for M_{t_n} and A_{t_n}. We treat the contingent MRG cash inflows as cost-reducing factors for option valuation, and the amount invested for the n[th] period can thus be rewritten as $(D^{t_B}_{t_n} - M_{t_n} e^{-\mu(t_n - t_B)})$, with a corresponding exercise payoff A_{t_n}. Here, M_{t_n} is discounted by μ at t_B.

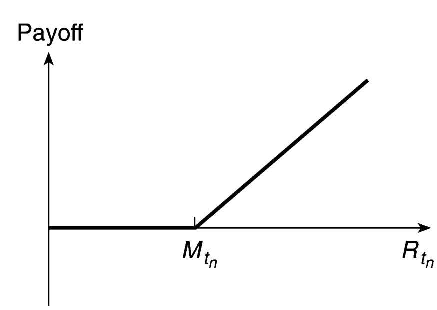

Figure 6 A European style call option

Let F_{t_n} denote the value of the option to invest for the n[th] operating period. F_{t_n} is the function of state variables, R_{t_n}, D_{t_n}, and time t: $F_{t_n}=\{F(R,D,t)|t\in[0,t_B]\}$. Accordingly, the strike price of F_{t_n} is $(D_{t_n}^{t_B}-M_{t_n}e^{-\mu(t_n-t_B)})$. By contingent claims analysis, F_{t_n} must satisfy:

$$\frac{1}{2}\frac{\partial^2 F_{t_n}}{\partial R^2}\sigma^2R^2+(r-\delta_R)R\frac{\partial F_{t_n}}{\partial R}+(r-\delta_D)D\frac{\partial F_{t_n}}{\partial D}+\frac{\partial F_{t_n}}{\partial t}-rF_{t_n}=0 \tag{10}$$

Here, D is assumed to have the same property as I, as stated above. If the amount of D is fixed, then the shortfall rate of D is equal to that of I, which in turn is equal to the discount rate. Since F_{t_n} resembles a European style call option, Equation 10 must satisfy the following boundary condition:

$$\begin{aligned}F_{t_n}(R,D,t_B)&=\left[A_{t_n}-\left(D_{t_n}^{t_B}-M_{t_n}e^{-(t_n-t_B)}\right)\right]H\\&\quad\left[A_{t_n}-\left(D_{t_n}^{t_B}-M_{t_n}e^{-(t_n-t_B)}\right)\right]\\&=\left[A_{t_n}-D_{t_n}^{t_B}(1-\rho)\right]H\left[A_{t_n}-D_{t_n}^{t_B}(1-\rho)\right]\end{aligned} \tag{11}$$

Here, $\rho=M_{t_n}e^{-\mu(t_n-t_B)}\Big/D_{t_n}^{t_B}$, and $H(\cdot)$ is the same unit step function as defined before. This boundary condition implies that, if $F_{t_n}>0$ at t_B, then the amount $D_{t_n}^{t_B}(1-\rho)$ is invested for keeping the European style call option A_{t_n} alive. A_{t_n} is a function of state variables R, M, and time t: $F_{t_n}=\{A(R,M,t)|t\in[0,t_n]\}$.

Further let $R_{t_B}^{*t_n}$ be the critical asset price, so that $F_{t_n}(R,D,t_B)=0$. If $R_{t_B}^{t_n}>R_{t_B}^{*t_n}$, then the concessionaire will decide to invest at t_B. By contingent claims analysis, the value of A_{t_n} must satisfy:

$$\frac{1}{2}\frac{\partial^2 A_{t_n}}{\partial R^2}\sigma^2R^2+(r-\delta_R)R\frac{\partial A_{t_n}}{\partial R}+(r-\delta_M)M\frac{\partial A_{t_n}}{\partial M}+\frac{\partial A_{t_n}}{\partial t}-rA_{t_n}=0 \tag{12}$$

subject to

$$A_{t_n}(R,M,t_n)=(R_{t_n}-M_{t_n})H(R_{t_n}-M_{t_n})$$

Here, since the MRG level is specified, M has the same property as I. This boundary condition implies that M_{t_n} will be invested for R_{t_n} if and only if $A_{t_n}>0$ at t_n. Since $M_{t_n}e^{-\mu(t_n-t_B)}=\rho D_{t_n}^{t_B}$, by chain rule,

$$M\frac{\partial A_{t_n}}{\partial M}=M\frac{\partial A_{t_n}}{\partial D}\frac{\partial D}{\partial M}=\rho De^{\mu(t_n-t_B)}\frac{\partial A_{t_n}}{\partial D}\frac{e^{-\mu(t_n-t_B)}}{\rho}=D\frac{\partial A_{tn}}{\partial D} \tag{13}$$

Substituting Equation 13 into Equation 12 gives:

$$\frac{1}{2}\frac{\partial^2 A_{t_n}}{\partial R^2}\sigma^2R^2+(r-\delta_R)R\frac{\partial A_{t_n}}{\partial R}+(r-\delta_D)D\frac{\partial A_{t_n}}{\partial D}+\frac{\partial A_{t_n}}{\partial t}-rA_{t_n}=0 \tag{14}$$

subject to

$$A_{t_n}(R,D,t_n)=\left(R_{t_n}-D_{t_n}^{t_B}\rho e^{\mu(t_n-t_B)}\right)H\left[R_{t_n}-D_{t_n}^{t_B}\rho e^{\mu(t_n-t_B)}\right]$$

Furthermore, let ξ, $X(\xi,t)$, and $Y(\xi,t)$ be the following transformation variables:

$$\xi=\frac{R}{D}\qquad X(\xi,t)=\frac{F_{t_n}}{D}\qquad Y(\xi,t)=\frac{A_{t_n}}{D}$$

Accordingly, Equation 10 is simplified:

$$\frac{1}{2}\frac{\partial^2 X}{\partial \xi^2}\sigma^2Z^2+(\delta_D-\delta_R)\xi\frac{\partial X}{\partial \xi}+\frac{\partial X}{\partial t}-\delta_D X=0 \tag{15}$$

subject to

$$X(\xi,t_B)=[Y-(1-\rho)]H[Y-(1-\rho)]$$

and Equation 14 simplified as:

$$\frac{1}{2}\frac{\partial^2 Y}{\partial \xi^2}\sigma^2Z^2+(\delta_D-\delta_R)\xi\frac{\partial Y}{\partial \xi}+\frac{\partial Y}{\partial t}-\delta_D Y=0 \tag{16}$$

subject to

$$Y(\xi,t_n)=\left[\xi-\rho e^{\mu(t_n-t_B)}\right]H\left[\xi-\rho e^{\mu(t_n-t_B)}\right]$$

Equations 15 and 16 are the joint PDE of the compound option. Following the solution method by Geske (1979) gives the price formula for F_{t_n} at t=0 as a closed-form solution:

$$\begin{aligned}F_{t_n}(R,D,0)&=R_{t_n}^0B\left(a_{1_{t_n}},b_{1_{t_n}};\sqrt{t_B/t_n}\right)\\&\quad-M_{t_n}^0B\left(a_{2_{t_n}},b_{2_{t_n}};\sqrt{t_B/t_n}\right)\\&\quad-\left(D_{t_n}^0-M_{t_n}^0\right)N\left(a_{2_{t_n}}\right)\end{aligned} \tag{17}$$

where

$$a_{1_{t_n}}=\frac{\ln\left(R_{t_n}^0\Big/R_{t_n}^{*0}\right)+\left(\sigma^2\big/2\right)t_B}{\sigma\sqrt{t_B}}$$

$$a_{2_{t_n}}=\frac{\ln\left(R_{t_n}^0\Big/R_{t_n}^{*0}\right)-\left(\sigma^2\big/2\right)t_B}{\sigma\sqrt{t_B}}=a_{1_{t_n}}-\sigma\sqrt{t_B}$$

$$b_{1_{t_n}}=\frac{\ln\left(R_{t_n}^0\Big/M_{t_n}^0\right)+\left(\sigma^2\big/2\right)t_n}{\sigma\sqrt{t_n}}$$

$$b_{2_{t_n}}=\frac{\ln\left(R_{t_n}^0\Big/M_{t_n}^0\right)-\left(\sigma^2\big/2\right)t_n}{\sigma\sqrt{t_n}}=b_{1_{t_n}}-\sigma\sqrt{t_n}$$

In this solution, $B(\cdot)$ is a two-dimensional cumulative bivariate normal distribution function with a correlation coefficient $\sqrt{t_B/t_n}$ for overlapping Brownian increments. $R^{*0}_{t_n}$ is the present value of $R^{*t_B}_{t_n}$ at $t=0$. The calculation of the critical asset price $R^{*0}_{t_n}$ is related to the amortized cost $D^{0}_{t_n}$, as presented in the next section. Let F_{t_i} denote the value of the option to invest for the i$^{\text{th}}$ period. F_M is found as an aggregation of F_{t_i}, $i=1\sim m$, or $F_M=\sum_{i=1}^{m}F_{t_i}$.

Now, we can derive price formulas for the MRG and the option to abandon. First let Q_f denote the value of the MRG at $t=0$. Q_f is given by:

$$Q_f=\sum_{i=1}^{m}\left[F_{t_i}(R,D,0)\Big|_{M_{t_i}}-F_{t_i}(R,D,0)\Big|_{M_{t_i}=0}\right]=F_M-F \quad (18)$$

Here, $F_{t_i}(R,D,0)\Big|_{M_{t_i}=0}$ is the value of the option to invest at $t=0$ for the i$^{\text{th}}$ period underlying assets without the MRG. Note that, in the pricing Equation 17, if $M_{t_n}=0$, $R^{*0}_{t_n}$ is equal to $D^{0}_{t_n}$, and $F_{t_i}(R,D,0)\Big|_{M_{t_i}=0}$ is given by:

$$F_{t_i}(R,D,0)\Big|_{M_{t_i}=0}=R^0_{t_i}N\left(a_{1_{t_i}}\right)-D^0_{t_i}N\left(a_{2_{t_i}}\right) \quad (19)$$

This solution is the similar to Equation 5.

Then, let f_M denote the value of the option to abandon at $t=0$. Since the holding of the MRG is dependent on the concessionaire's decision to invest at t_B, f_M is calculated by subtracting the project's NPV and the value of the MRG from the value of the compound option at $t=0$; that is,

$$f_M=\sum_{i=1}^{m}F_{t_i}-\left[\left(P^0-I^0\right)+\sum_{i=1}^{m}Q_{t_i}\right] \quad (20)$$

Calculating the amortized cost $D^{t_B}_{t_n}$ and the critical asset price $R^{*t_B}_{t_n}$

The foregoing compound option is formulated under the condition that the decomposed m investment options are all exercised together at $t=t_B$ or never, and the MRG is constructed as a series of independent options, which will be exercised only if the realized operating revenues fall below the pre-specified MRG level. In other words, once the concessionaire decides to invest at $t=t_B$, the following boundary conditions should be satisfied:

$$\begin{aligned}
&F_{t_1}(R,D,t_B)=A_{t_1}-\left(D^{t_B}_{t_1}-M^{t_B}_{t_1}\right)=0\\
&F_{t_2}(R,D,t_B)=A_{t_2}-\left(D^{t_B}_{t_2}-M^{t_B}_{t_2}\right)=0\\
&\vdots\\
&F_{t_i}(R,D,t_B)=A_{t_i}-\left(D^{t_B}_{t_i}-M^{t_B}_{t_i}\right)=0\\
&\vdots\\
&F_{t_m}(R,D,t_B)=A_{t_m}-\left(D^{t_B}_{t_m}-M^{t_B}_{t_m}\right)=0\\
&\sum_{i=1}^{m}D^{t_B}_{t_i}=I^{t_B}
\end{aligned} \quad (21)$$

Here, the value of A_{ti} at t_B can be found by the solution methods from Kutner (1988) and Hwang and Jou (1994):

$$A_{t_i}(R,D,t_B)=R^{t_B}_{t_i}N\left(k_{1_{t_i}}\right)-M^{t_B}_{t_i}N\left(k_{2_{t_i}}\right) \quad (22)$$

where

$$k_{1_{t_i}}=\frac{\ln\left(R^{t_B}_{t_i}\big/M^{t_B}_{t_i}\right)+\left(\sigma^2/2\right)(t_i-t_B)}{\sigma\sqrt{(t_i-t_B)}}$$

$$k_{2_{t_i}}=\frac{\ln\left(R^{t_B}_{t_i}\big/R^{t_B}_{t_i}\right)-\left(\sigma^2/2\right)(t_i-t_B)}{\sigma\sqrt{(t_i-t_B)}}=k_{1_{t_i}}-\sigma\sqrt{(t_i-t_B)}$$

Substituting A_{t_i} into Equation 21 gives the following (m+1) equations:

$$\begin{aligned}
&R^{*t_B}_{t_1}N\left(k_{1_{t_1}}\right)-M^{t_B}_{t_1}N\left(k_{2_{t_1}}\right)-\left(D^{t_B}_{t_1}-M^{t_B}_{t_1}\right)=0\\
&R^{*t_B}_{t_1}e^{(-\mu+\alpha)(t_2-t_1)}N\left(k_{1_{t_2}}\right)-M^{t_B}_{t_2}N\left(k_{2_{t_2}}\right)-\left(D^{t_B}_{t_2}-M^{t_B}_{t_2}\right)\\
&=0\\
&\vdots\\
&R^{*t_B}_{t_i}e^{(-\mu+\alpha)(t_i-t_1)}N\left(k_{1_{t_i}}\right)-M^{t_B}_{t_i}N\left(k_{2_{t_i}}\right)-\left(D^{t_B}_{t_i}-M^{t_B}_{t_i}\right)\\
&=0\\
&\vdots\\
&R^{*t_B}_{t_m}e^{(-\mu+\alpha)(t_m-t_1)}N\left(k_{1_{t_m}}\right)-M^{t_B}_{t_m}N\left(k_{2_{t_m}}\right)-\left(D^{t_B}_{t_m}-M^{t_B}_{t_m}\right)\\
&=0\\
&\sum_{i=1}^{m}D^{t_B}_{t_i}=I^{t_B}
\end{aligned} \quad (23)$$

These equations have $(m+1)$ unknowns, namely $R^{*t_B}_{t_1}$ and $D^{t_B}_{t_i}$ $i=1\sim m$. The first m equations can be summed up as:

$$R^{*t_B}_{t_1}\sum_{i=1}^{m}e^{(-\mu+\alpha)(t_i-t_1)}N\left(k_{1_{t_i}}\right)+\sum_{i=1}^{m}M^{t_B}_{t_i}\left(1-N\left(k_{2_{t_i}}\right)\right)-I^{t_B}=0 \quad (24)$$

$N(\cdot)$ is the one-dimensional cumulative normal distribution function. Since the values of k_1 and k_2 are dependent on $R_{t_1}^{*t_B}$, Equation 24 is nonlinear, and thus $R_{t_1}^{*t_B}$ can only be found numerically. Once $R_{t_1}^{*t_B}$ is found, the amortized costs $D_{t_i}^{t_B}$ can be calculated by Equation 23, the critical asset prices $R_{t_i}^{*t_B}$ can be calculated by $e^{(-\mu+\alpha)(t_i-t_1)}R_{t_1}^{*t_B}$, and F_{t_i} can be obtained by Equation 18.

Now, let's examine the special case when the MRG is absent from the foregoing formulation. First, if $M=0$, then $k_1\to\infty$, $k_2\to\infty$, $N(\cdot)=1$, and Equation 24 can be simplified as:

$$R_{t_1}^{*t_B}\sum_{i=1}^{m} e^{(-\mu+\alpha)(t_i-t_1)}-I^{t_B}=P^{t_B}-I^{t_B}=0 \tag{25}$$

Here, the calculation of $R_{t_1}^{*t_B}$ is straightforward, and the amortized costs $D_{t_i}^{t_B}$ are equal to their corresponding critical asset values $R_{t_1}^{*t_B}$, which can further be found by:

$$\begin{bmatrix} R_{t_1}^{*0} \\ R_{t_2}^{*0} \\ \vdots \\ R_{t_i}^{*0} \\ \vdots \\ R_{t_m}^{*0} \end{bmatrix} = \begin{bmatrix} D_{t_1}^{0} \\ D_{t_2}^{0} \\ \vdots \\ D_{t_i}^{0} \\ \vdots \\ D_{t_m}^{0} \end{bmatrix} = \frac{I^0}{\sum\limits_{i=1}^{m} e^{(-\mu+\alpha)(t_i-t_1)}} \begin{bmatrix} 1 \\ e^{(-\mu+\alpha)(t_2-t_1)} \\ \vdots \\ e^{(-\mu+\alpha)(t_i-t_1)} \\ \vdots \\ e^{(-\mu+\alpha)(t_m-t_1)} \end{bmatrix} \tag{26}$$

In Equation 17, if $M=0$, then $b_{1_{t_i}}\to\infty$, $b_{2_{t_i}}\to\infty$, and the two-dimensional cumulative bivariate normal distribution function can be written as:

$$B\left(a_{1_{t_i}},\infty;\sqrt{t_B/t_i}\right)=N\left(a_{1_{t_i}}\right)$$
$$B\left(a_{2_{t_i}},\infty;\sqrt{t_B/t_i}\right)=N\left(a_{2_{t_i}}\right)$$

Equation 17 can thus be simplified as Equation 19. Since $R_{t_i}^{*t_B}=R_{t_1}^{*t_B}e^{(-\mu+\alpha)(t_i-t_1)}$, $a_{1_{t_i}}$ and $a_{2_{t_i}}$ can be calculated by:

$$a_{1_{t_1}}=a_{1_{t_2}}=\ldots=a_{1_{t_i}}=\ldots=a_{1_{t_m}},$$
$$a_{1_{t_i}}=\frac{ln\left(R_{t_i}^0/D_{t_i}^0\right)+\left(\sigma^2/2\right)t_B}{\sigma\sqrt{t_B}}=\frac{ln\left(P^0/I^0\right)+\left(\sigma^2/2\right)t_B}{\sigma\sqrt{t_B}}=k_1$$

and

$$a_{2_{t_1}}=a_{2_{t_2}}=\ldots=a_{2_{t_i}}=\ldots=a_{2_{t_m}},$$
$$a_{2_{t_i}}=\frac{ln\left(R_{t_i}^0/D_{t_i}^0\right)-\left(\sigma^2/2\right)t_B}{\sigma\sqrt{t_B}}=\frac{ln\left(P^0/I^0\right)-\left(\sigma^2/2\right)t_B}{\sigma\sqrt{t_B}}=k_2$$

As a result, the price formula is given by:

$$\begin{aligned}\sum_{i=1}^{m}F_{t_i}(R,D,0)|_{M=0}&=\sum_{i=1}^{m}R_{t_i}^0N\left(a_{1_{t_i}}\right)-\sum_{i=1}^{m}D_{t_i}^0N\left(a_{2_{t_i}}\right)\\&=N(k_1)\sum_{i=1}^{m}R_{t_i}^0-N(k_2)\sum_{i=1}^{m}D_{t_i}^0\\&=P^0N(k_1)-I^0N(k_2)\end{aligned} \tag{27}$$

This result is the same as Equation 5. In fact, when the MRG is not provided, the compound and the single option pricing formulas will always lead to the same valuation so long as the amortization schedule satisfies the following conditions:

$$\begin{aligned}\frac{R_{t_1}^0}{D_{t_1}^0}=\frac{R_{t_2}^0}{D_{t_2}^0}&=\ldots=\frac{R_{t_i}^0}{D_{t_i}^0}=\ldots\\&=\frac{R_{t_{m-1}}^0}{D_{t_{m-1}}^0}=\frac{R_{t_m}^0}{D_{t_m}^0}=\frac{\sum\limits_{i=1}^{m}R_{t_i}^0}{\sum\limits_{i=1}^{m}D_{t_i}^0}=\frac{P^0}{I^0}\end{aligned} \tag{28}$$

A numerical case

The data

The Taiwan High Speed Rail BOT Project is now used as a numerical case to apply the foregoing pricing formulas. The Project's detailed financial data and projections can be obtained from the authors. As summarized in Table 1, the Project's total concession period is 36 years, the present value of the total capital investment cost is NT$ 242 422 M, and the present value of the operating costs is NT$ 101 429 M. The

Table 1 Data and assumptions

The concession period (yrs)	Pre-construction	1	Estimated parameters	Expected growth rate α	0.06
	Construction	5		Growth rate volatility σ	0.3
	Operation	30		Discount rate μ	0.12
Investment costs (NT$ million)	Pre-construction cost $\mathcal{J}^0$:	3400	Annual level of MRG, M_{t_i}, i=1~30	The corresponding annual operating cost, C_{t_i}	
	Construction cost B^0:	242 422			
	Operating cost C^0:	101 429			

variation of the Project's revenues is small, so the Generalized Wiener process can be applied. The average growth rate is thus estimated at 6.02%, and we use 6% as an approximation. The volatility of Taiwan's stock market price varies from 0.2 to 0.4, and we use 0.3 as the volatility of the growth rate. Finally, the Project's discount rate is estimated at 12% by the financial consultant.

For the purpose of this application, we suppose the Project has an MRG, and the annual MRG levels are equal to the corresponding annual operating costs. In addition, although the construction commencement date is not specified in the concession contract, suppose the pre-construction period of the project is one year after the contract signing date. Table 1 includes a pre-construction cost and its present value is denoted as $\mathcal{J}^0$. The cost is required to keep the Project alive until $t=t_B$. It will reduce the Project's NPV, or $NPV=P^0-I^0-\mathcal{J}^0$, and affect the value of the investment option. Therefore, the price formula for F should be adjusted. When the MRG is not available, the price formula is as the following:

$$F=P_0N(k_1)-I_0N(k_2)-\mathcal{J}_0 \tag{29}$$

When the MRG is available, the price formula for F_M is adjusted as:

$$F_M=\sum_{i=1}^{m}\left[\begin{array}{l} R^0_{t_n}B\left(a_{1_{t_n}}, b_{1_{t_n}};\ \sqrt{t_B/t_n}\right) \\ -M^0_{t_n}B\left(a_{2_{t_n}}, b_{2_{t_n}};\ \sqrt{t_B/t_n}\right) \\ -\left(D^0_{t_n}-M^0_{t_n}\right)N\left(a_{2_{t_n}}\right)-\mathcal{J}^0/m \end{array}\right] \tag{30}$$
$$=\sum_{i=1}^{m}F_{t_i}-\mathcal{J}^0$$

Results and interpretations

The results of this numerical case are shown in Table 2, from which we have the following observations:

(a) The Project's NPV is NT$139,508 M, so the Project is financially feasible.

(b) The option to abandon creates values. The value of the option, or f, is NT$7436 M, calculated by either Equations 5 or 28. By Equation 5, for example, the shortfall rate is calculated at 6% from the growth rate and the discount rate. $N(k_1)=0.9047$, $N(k_2)=0.8434$, and F=NT$ 146 944 M. F is greater than the Project's NPV, and the amount of f is obtained by subtracting the NPV from it.

(c) The MRG also creates values. The option value of the MRG, or Q, is $7716 M, calculated by Equation 9.

(d) When the options are combined, the foregoing valuations change. The value of the option to invest with the MRG, or F_M, is NT$152 897 M, calculated by the adjusted compound option pricing formula in Equation 29. The MRG value or Q_f is NT$5953 M, calculated by subtracting F from F_M by Equation 19. And the value of the option to abandon is $5673 M, calculated by subtracting the NPV and the value of the MRG from F_M by Equation 21.

(e) As a result, when the MRG and the option to abandon are combined, their values are reduced. The values disappeared are called joint value, which is equal to NT$1763 M in this numerical case. This result is consistent with Trigeorgis' observation. When options intended to control downside risks are exercised at the same time, they will counteract each other, and thus reduce their own values (Trigeorgis, 1993). There are two counteracting forces in our compound option. On one hand, the value of the MRG cannot be realized if the option to abandon is exercised at t_B. So long as there is a chance that the project will be abandoned, the value of the MRG will not be fully realized. On the other, the presence of the MRG will increase the value of the underlying assets of the option to abandon, and thus reduce the value of the option itself.

(f) Although the option values are reduced, the total value created by the compound option is still substantial. The value is NT$13 389 M, calculated by subtracting the joint value from the original values of the MRG and the option to abandon in the single option models.

Table 2 Results (unit: million of NT$)

Year (i)	R_{t_i}	C_{t_i}	M_{t_i}	$N(\text{-}d_2)$	$N(\text{-}d_1)$	Q_{t_i}
1	75 879	12 740	12 740	0.0321	0.0041	43
2	81 125	13 586	13 586	0.0463	0.0057	64
3	86 744	14 630	14 630	0.0633	0.0076	91
4	92 685	15 568	15 568	0.0798	0.0093	116
5	99 155	16 537	16 537	0.0964	0.0108	140
6	105 999	17 353	17 353	0.1109	0.0119	158
7	113 303	18 582	18 582	0.1291	0.0135	183
8	120 457	19 942	19 942	0.1489	0.0153	211
9	128 184	21 144	21 144	0.166	0.0165	230
10	136 347	22 229	22 229	0.181	0.0174	243
11	145 027	23 599	23 599	0.1978	0.0185	259
12	154 256	24 969	24 969	0.2135	0.0194	270
13	164 069	26 540	26 540	0.2299	0.0203	283
14	174 501	28 763	28 763	0.2505	0.0220	306
15	185 594	30 434	30 434	0.2651	0.0226	311
16	197 386	32 090	32 090	0.2785	0.0230	313
17	209 924	34 503	34 503	0.2961	0.0242	326
18	222 578	36 379	36 379	0.3094	0.0246	325
19	235 987	38 600	38 600	0.3238	0.0252	327
20	250 196	41 005	41 005	0.3382	0.0258	328
21	265 253	44 198	44 198	0.3556	0.0269	337
22	281 206	46 728	46 728	0.3681	0.0272	332
23	298 110	49 493	49 493	0.3807	0.0275	328
24	316 018	52 620	52 620	0.3938	0.0279	325
25	334 993	56 671	56 671	0.4096	0.0288	329
26	355 096	59 865	59 865	0.4205	0.0289	321
27	376 393	63 756	63 756	0.4331	0.0292	316
28	398 341	67 404	67 404	0.4439	0.0294	308
29	416 926	71 081	71 081	0.4565	0.0298	300
30	434 757	75 295	75 295	0.4705	0.0305	295

Policy implications

Both the MRG and the option to abandon are valuable policy tools as we have shown, but the government should check if they will produce intended policy effects when combined in the same BOT package. For example, Table 3 shows if the pre-specified level of the MRG is increased by 250%, then the value of the option to abandon will decrease to NT$10 M, which is no longer substantial compared to the Project's NPV.

In general, the investment value of BOT projects increases with the MRG level, but the increase of the MRG will decrease the value of the option to abandon. In terms of risk allocation, the higher the MRG level is, the more the down-side risk is transferred to the government, and the less likely the concessionaire will exercise the option to abandon. When both the MRG and the option to abandon are proposed at the same time, the government should check if the proposed MRG level will render the option to abandon worthless.

In practice, the MRG may be valuable from an investor's perspective, but it requires substantial budgetary commitments. The benefits and costs of the commitments should be justified, and the pricing formulas can be used as valuation tools. When the project is not bankable due to high revenue risks, the lender may ask the government to provide MRG for a minimum level of debt coverage. In this case, MRG can also be justified by credit enhancement benefits to the concessionaire, such as reduced financing costs. However, to know how much the financing costs can be saved, MRG should be valued from lender's perspective. This appears to be another open issue.

Whenever there are budgetary constraints, the option to abandon is a preferable, more easily justified policy choice; especially it can only be used in the pre-construction phase. If the option can be used during construction and operation, abandonment will cause greater disruptions and disturbances, and this will make the justification more difficult.

Conclusions

Two single option pricing models were first developed for the valuation of the MRG and the option to

Table 2 Continued

Year (i)	$R^{*0}_{t_i}$	$D^0_{t_i}$	$B(a_2, b_2)$	$B(a_1, b_1)$	$N(a_2)$	F_{t_i}
1	22 433	22 516	0.8473	0.9185	0.8669	10 675
2	21 272	21 392	0.8367	0.9172	0.8669	10 103
3	20 174	20 337	0.8236	0.9157	0.8669	9560
4	19 118	19 319	0.8104	0.9143	0.8669	9042
5	18 140	18 375	0.7970	0.9130	0.8669	8564
6	17 199	17 457	0.7849	0.9121	0.8669	8109
7	16 305	16 597	0.7698	0.9107	0.8669	7674
8	15 374	15 702	0.7532	0.9092	0.8669	7221
9	14 511	14 860	0.7388	0.9080	0.8669	6805
10	13 689	14 050	0.7260	0.9073	0.8669	6414
11	12 914	13 292	0.7116	0.9063	0.8669	6043
12	12 183	12 571	0.6981	0.9054	0.8669	5695
13	11 493	11 892	0.6840	0.9045	0.8669	5367
14	10 841	11 266	0.6663	0.9031	0.8669	5053
15	10 226	10 653	0.6537	0.9025	0.8669	4764
16	9646	10 070	0.6420	0.9021	0.8669	4491
17	9099	9534	0.6267	0.9010	0.8669	4231
18	8557	8986	0.6150	0.9006	0.8669	3977
19	8046	8473	0.6024	0.9001	0.8669	3738
20	7566	7991	0.5899	0.8995	0.8669	3512
21	7114	7546	0.5746	0.8985	0.8669	3299
22	6689	7112	0.5636	0.8982	0.8669	3101
23	6289	6703	0.5526	0.8979	0.8669	2914
24	5913	6320	0.5410	0.8975	0.8669	2739
25	5560	5968	0.5272	0.8967	0.8669	2573
26	5227	5622	0.5175	0.8966	0.8669	2418
27	4914	5301	0.5064	0.8962	0.8669	2273
28	4612	4987	0.4968	0.8961	0.8669	2133
29	4282	4645	0.4858	0.8957	0.8669	1979
30	3960	4314	0.4734	0.8951	0.8669	1829

abandon in the pre-construction phase. The MRG was further combined with the option to abandon to develop a compound option model.

The Taiwan High-Speed Rail Project was used as a numerical case to apply the derived option pricing formulas. The results indicated that both MRG and the option to abandon could create substantial values. When the MRG and the option to abandon were combined, they counteracted each other and their values were reduced. If the level of the MRG were high enough, the option to abandon would be rendered valueless.

MRG involves substantial budgetary commitments, and its benefits and costs should be carefully justified. The option to abandon is a preferable policy choice under budgetary constraints, and its justification is more straightforward.

Overall, the application of the real option approach in BOT project evaluation looks promising. The option pricing formulas developed in this paper can be used as valuation tools for the foregoing justifications as well as extensions of traditional project evaluation approaches, such as the discounted cash flow model. But the presented formulas are limited in scope. They do not

Table 3 Comparative static

Change of the MRG level	NPV	F_M	Q_f	f_M	Joint value
0% (no MRG)	139 508	146,944 (=F)	0	7436 (=f)	0
50%	139 508	148 267	1323	6992	444
100%	139 508	152 897	5953	5673	1763
150%	139 508	161 233	14 290	4132	3304
250%	139 508	187 708	40 715	883	6602
350%	139 508	227 037	80 093	10	7426

consider the options to abandon during construction and operation. In addition, if the MRG is to be used as a credit enhancement tool, it should also be valued from the lender's perspective. Future researches are required to tackle these problems.

Acknowledgements

The authors wish to thank the National Science Council for research grant under contract number NSC 93-2211-E-009-043.

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