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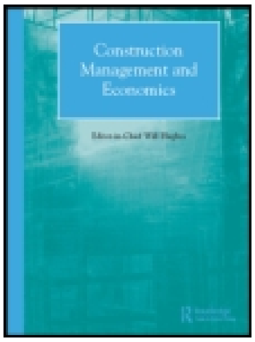

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Estimating industry-level productivity trends in the building industry from building cost and price data

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A method of measuring total-factor productivity (TFP) trends in the building industry is described in this paper. This method is an improved version of the approach described in a paper by Chau and Walker in that it requires less restrictive assumptions and is theoretically less biased while requiring only slightly more data. With small modification, the same method can be used to measure other productivity trends corresponding to other productivity concepts. The data used in measuring TFP of Hong Kong's building industry are also different from those proposed in Chau and Walker. A number of modifications have been made. These modifications have been possible both as a result of work by Chau and the increased availability of statistics in recent years.

One of the major difficulties in measuring TFP trends in the building or construction industry has been the lack of data. This is also one of the major reasons for the dearth of empirical studies in this area. Very few attempts have been made to solve the problem of data availability. Lowe recognizes the difficulties in obtaining suitable data for measuring TFP of the British construction industry. His suggestion, however, is to use capital productivity as the second-best alternative to TFP rather than solving the problem. Chau and Walker's solution is to develop a method for estimating TFP from construction cost and price data. The method is then used to measure the TFP of Hong Kong's building industry. Since construction cost and price data are usually more readily available in most countries, Chau and Walker's approach is potentially useful in these countries. There are, however, certain deficiencies in Chau and Walker's approach in that some of the underlying assumptions are relatively restrictive.

As more statistics are now available in Hong Kong, this has rendered Chau and Walker's approach a crude tool for making full use of the data. This paper presents a modified approach which requires less restrictive assumptions and can make better use of the newly available data.

Keywords: Total factor productivity, productivity, Hong Kong, translog aggregator function.

Introduction

Total-factor productivity (TFP) has been widely accepted as a much better indicator of productive efficiency than conventional labour productivity. However, very little empirical research has been carried out into TFP, particularly in the building industry mainly because of a lack of data. The purpose of this paper is to develop a framework for measuring the TFP trend of the building industry using building cost and price indices which are more readily available than the quantity data used in most TFP studies. This method is essentially an improved version of the method proposed by Chau and Walker (1988). Some of the restrictive assumptions in Chau and Walker's approach are relaxed. The modified approach presented here should give a more reliable estimate of the TFP trend of the construction industry.

The conventional approach to measuring TFP of the economy and/or industry often measures output as value-added and includes only labour and capital on the input side. Intermediate inputs (output from other industries such as construction materials, fuel etc.) are assumed to have no effect on the TFP trend. Under this assumption, it does not matter whether intermediate input is included in the TFP measurement. While this might be the case for some industries, such an assumption is unrealistic in the building/construction industry since new materials that are of better quality and easier to handle or work with are invented from time to time. Their contribution to productive efficiency cannot be ignored. The definition of TFP adopted in this paper is the same as that used by Chau and Walker in that all tangible inputs, including intermediate inputs, are included on the input side and output is measured as gross rather than value-added.

The conventional two-factor TFP trend (also referred to as value-added total factor productivity [VATFP in the rest of the paper]) and other single-factor productivity trends (such as output, either gross or value-added, per unit of labour) can also be derived using the same framework. These productivity trends are useful for comparing findings in other research, as most reported construction/building productivity trends are in these formats. No serious attempt has been made to analyse the factors shaping the TFP trend because such a discussion would require another full paper. However, a number of interesting observations, worthy of a more detailed study, have been highlighted.

Previous research

The first attempt to estimate the TFP trend of the building industry from building cost and price data was suggested by Chau and Walker (1988). Their approach is based on the assumption of constant input value shares and on an explicit input aggregator function. By ignoring the time lag between tendering and construction and using book profit as a proxy of supernormal profit, the following TFP formula is deduced for estimating the TFP trend of Hong Kong's building industry:

$$\frac{TFP_t - TFP_{t-1}}{[(TFP_t)(TFP_{t-1})]^{1/2}} = \frac{S_l(LCI_t - LCI_{t-1})}{[(LCI_t)(LCI_{t-1})]^{1/2}} + \frac{S_m(MCI_t - MCI_{t-1})}{[(MCI_t)(MCI_{t-1})]^{1/2}} + \frac{S_p(UVI_t - UVI_{t-1})}{[(UVI_t)(UVI_{t-1})]^{1/2}} + \frac{S_o(GDP_t - GDP_{t-1})}{[(GDP_t)(GDP_{t-1})]^{1/2}} - \frac{(OPI_t)(OPI_{t-1})}{[(OPI_t)(OPI_{t-1})]^{1/2}} + \frac{P_t - P_{t-1}}{[(1-P_t)(1-P_{t-1})]^{1/2}} \quad (1)$$

where TFP is the total factor productivity index of the building industry, OPI is the output price index of the building output and is approximated by the tender price index, LCI is the labour cost index, MCI is the material cost index, UVI is the unit value index of imported capital goods, GDP is the price deflator for gross domestic product, S_l, S_m, S_p and S_o are respectively the shares of labour, material, construction machinery and overhead in total cost.

What this means is that relative change in TFP (left-hand-side) is given by deducting the relative change in tender price index (the fifth term on the right-hand-side) from the sum of the relative changes in the input costs weighted by their corresponding shares in the total cost (the first four terms of the right-hand-side) plus the relative change in book (or accounting) profit margin (the last term). The intuitive interpretation is that the difference between the growths in input prices and output prices after adjustment for the change in profit margin is equal to the change in TFP.

Although such an approach solves the problem of availability of quantity data which is very common in the building industry in most countries, there are drawbacks to the approach. Firstly, because of a lack of data, the value shares of inputs are assumed to be constant over time. Although convenient, this assumption is restrictive and may not be consistent with empirical facts. As more data are now available, this assumption can be tested empirically and modified if necessary. Secondly, an explicit geometric input aggregator function is assumed in the derivation of the result. This aggregator function is somewhat arbitrary and might not be consistent with the actual underlying production structure governing the input–output relationship. Thirdly, the TFP formula is a discrete approximation of a continuous form which is derived by differentiating the input–output relation with respect to time. Such an approximation lacks a theoretical justification. The possibility of bias which might have resulted from the approximation has been also ignored. Fourthly, book profit is used as a proxy for supernormal profit, which is conceptually erroneous. Supernormal profit is a result of market imperfection while book profit is a function of factors such as interest rates, accounting/conventions and the tax system. Fifthly, tender price index is not a good proxy for the output price index due to the time lag between tendering and construction.

The modified approach presented in the later sections of this paper attempts to provide solutions to the above problems. The more conventional two-factor TFP and labour productivity can also be estimated using the same approach. These productivity measures provide the basis for comparison with results from other studies and estimation of the contribution of building materials to the growth in the conventional two-factor TFP.

The approach

Consider a single-output and multiple-input production system. Denoting quantity of output by Q_O, and quantity of the ith input as Q_{Ii}, by definition, total factor productivity T can be written as

$$T = \frac{Q_O}{g(Q_{I_1}, Q_{I_2}, \ldots, Q_{I_n})} \tag{2}$$

where $g(.)$ is an input aggregator function and is assumed to be linearly homogeneous and continuously differentiable.[1]

Total logarithmic differentiation of Equation 2 with respect to time gives

$$\frac{dQ_O}{Q_O} = \frac{dT}{T} + \sum_{i=1}^{n} \frac{\partial g}{\partial Q_{Ii}} \frac{dQ_{Ii}}{g} \tag{3}$$

where dX denotes the derivative of X with respect to t, and ∂X denotes the partial derivative of X with respect to t.

Under the assumptions of Hicks Neutral Technical Change[2] and constant return to scale, partial differentiation of Equation 2 with respect to Q_{Ii} gives

$$T \frac{\partial g}{\partial Q_{Ii}} = \frac{\partial Q_O}{\partial Q_{Ii}} \tag{4}$$

or

$$\frac{\partial g}{\partial Q_{Ii}} \cdot \frac{1}{g} = \frac{\partial Q_O}{\partial Q_{Ii}} \cdot \frac{1}{Q_O} \tag{5}$$

therefore, Equation 3 can be written as

$$\frac{dQ_O}{Q_O} = \frac{dT}{T} + \sum_{i=1}^{n} \frac{\partial Q_O}{\partial Q_{Ii}} \cdot \frac{dQ_{Ii}}{Q_O} \tag{6}$$

The first-order conditions for producer equilibrium in a competitive market are

$$\frac{\partial Q_O}{\partial Q_{Ii}} = \frac{P_{Ii}}{P_O} \quad \forall i \tag{7}$$

where P_O is the price of output and P_{Ii} is the price of the ith input. Substituting Equation 7 into Equation 6 gives

$$\frac{dT}{T} = \frac{dQ_O}{Q_O} - \sum_{i=1}^{n} \frac{P_{Ii}}{P_O} \cdot \frac{Q_{Ii}}{Q_{Ii}} \cdot \frac{dQ_{Ii}}{Q_O} \tag{8}$$

but

$$s_i = \frac{P_{Ii} Q_{Ii}}{P_O Q_O} \tag{9}$$

which is the value share of the ith input, hence

$$\frac{dT}{T} = \frac{dQ_O}{Q_O} - \sum_{i=1}^{n} s_i \frac{dQ_{Ii}}{Q_{Ii}} \tag{10}$$

Assuming that supernormal profit does not exist at industry level,[3] total differentiation of the following identity with respect to time

$$P_O Q_O \equiv \sum_{i=1}^{n} P_{Ii} Q_{Ii} \tag{11}$$

yields

$$\frac{dQ_O}{Q_O} - \sum_{i=1}^{n} s_i \frac{dQ_{Ii}}{Q_{Ii}} = \sum_{i=1}^{n} s_i \frac{dP_{Ii}}{P_{Ii}} - \frac{dP_O}{P_O} \tag{12}$$

Thus Equation 10 can also be written as

$$\frac{dT}{T} = \sum_{i=1}^{n} s_i \frac{dP_{Ii}}{P_{Ii}} - \frac{dP_O}{P_O} \tag{13}$$[4]

The TFP index derived from Equation 13 is a special type of index devised by Divisia (1926). This is known as the Divisia index which can be defined as a sum of infinitesimal rates of change in the price or quantity of components weighted by the corresponding value share of each component. The properties of the Divisia index have been extensively researched; examples include the work of Richter (1966), Hulten (1973) and Diewert (1976).

The Divisia index is known to have many desirable properties which are not shown by the more conventional Laspeyres and Paasche indexes. These include conformation to Fisher's (1922) factor-reversal rule, and the reproductive property, i.e. the Divisia index of Divisia indexes is a Divisia index of the components.[5] This property is particularly attractive for analysis at industry or national/regional levels where aggregated variables are used.

However, the Divisia index is not free of problems. The value of a Divisia index is found by integrating Equation 13 and is therefore a line integral over a time interval which depends on the path over which the integration is taken. This gives rise to the problem of cycling, i.e. the value of the Divisia index is not uniquely determined by the values of its components. This problem can be avoided only if the index is path independent. Hulten (1973) has identified the following three necessary and sufficient conditions for path independence:

(a) there exists a continuously differentiable aggregator function F defined on the set of values of components;
(b) F is linearly homogeneous in all the components;
(c) the existence of normalized value shares for all aggregate components that are observable and unique up to a scalar multiplication.

In the derivation of Equation 13 above, conditions (a) and (b) are assumed. Condition (c) is implied by the conditions of producer equilibrium. Thus no additional assumption needs to be made to ensure path independence.

Since data are available over discrete time intervals, it is necessary to find a discrete approximation of Equation 13 for empirical research. For this purpose, the following discrete form, which was first suggested by Tornqvist (1936), is used in this research.

$$\Delta(\ln T)_t = \sum_{i=1}^{n} \bar{s}_{it}\, \Delta(\ln P_{Ii})_t - \Delta(\ln P_O)_t \quad (14)$$

where the Δ sign denotes the difference in the value of the corresponding variable between two successive periods, e.g.

$$\Delta(\ln T)_t = \ln T_t - \ln T_{t-1} \quad (15)$$

and the bar sign denotes the average of the same two successive periods, i.e.

$$\bar{s}_{it} = \frac{s_{it} + s_{it-1}}{2} \quad (16)$$

The discrete approximation is 'natural' since Equation 13 can also be written as

$$\mathrm{d} \ln T = \sum_{i=1}^{n} s_i \,\mathrm{d} \ln P_{Ii} - \mathrm{d} \ln P_O \quad (17)$$

which is the same as Equation 14 when the time interval between two successful periods approaches zero.

The discrete approximation suggested above is superior to the discrete approximation suggested by Chau and Walker, since the underlying production function is much less restrictive. The Tornqvist discrete formula has been shown by Diewert (1976) to be consistent with (or 'exact for' using Diewert's terminology) a linearly homogeneous transcendental logarithmic (or Translog in short) aggregator function. The Translog aggregator function was introduced by Christensen *et al.* (1973) and is defined as

$$\ln Q_O = \ln a + a \ln T + \sum_{i=1}^{n} a_i \ln Q_{Ii} + \tfrac{1}{2} b (\ln T)^2$$

$$+ \tfrac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij} \ln Q_{Ii} \ln Q_{Ij} + \sum_{i=1}^{n} b_i \ln Q_{Ii} \ln T \quad (18)$$

where a, b, a_i, b_i, b_{ij} are parameters and $b_{ij} = b_{ji}$. If constant returns to scale and Hicks Neutral Technical Change hold, then Equation 18 can be simplified to

$$\ln Q_O = \ln T + \ln a + \sum_{i=1}^{n} a_i \ln Q_{Ii}$$

$$+ \tfrac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij} \ln Q_{Ii} \ln Q_{ij} \quad (19)$$

Based on Taylor's series approximation, Christensen *et al.* have shown that the Translog aggregator function is a second-order local approximation to any linearly homogeneous, twice-differentiable function. Therefore the aggregator function is very flexible within the limit of second-order approximation. Since Equation 14 is consistent with the Translog function, which is very flexible, it does not impose serious restrictions on the functional form of the aggregator function.

Value-added total factor productivity

Value-added TFP (VATFP) is defined as

$$T_A = \frac{Q_A}{h(Q_K, Q_L)} \quad (20)$$

where Q_A is value added in real terms, Q_K and Q_L are quantity of capital and labour respectively and $h(.)$ is an aggregator function.

The difference between VATFP and TFP is that in the former approach, intermediate inputs are subtracted from both the input and output side.

VATFP is one of the most popular definitions used in many productivity studies at the aggregate level. Using the VATFP concept, all production systems are simplified to a two-input (namely labour and capital) and single-output system; this greatly simplifies the problem of estimation of the parameters characterizing the production structure (e.g. elasticities of substitution) and reduces the amount of data required.

Assuming a linearly homogeneous aggregator function, Hicks Neutral Technical Change and producer equilibrium in the two-input single-output production system, differentiation of Equation 20 with respect to time, following manipulations similar to those of Equations 2–10 gives

$$\frac{\mathrm{d}T_A}{T_A} = \frac{\mathrm{d}Q_A}{Q_A} - \left(S'_K \frac{\mathrm{d}Q_K}{Q_K} + S'_L \frac{\mathrm{d}Q_L}{Q_L} \right) \quad (21)$$

where S'_K and S'_L are respectively the value share of capital and labour in the value-added output measure, i.e.

$$S'_K = \frac{V_K}{V_A} \quad (22)$$

and

$$S'_L = \frac{V_L}{V_A} \quad (23)$$

where V_K, V_L and V_A are the market values of capital, labour and added value respectively.

Since only price data are available, the following equivalent form of Equation 21 can be obtained by undertaking similar manipulations to those in Equations 11–13.

$$\frac{\mathrm{d}T_A}{T_A} = \left(S'_K \frac{\mathrm{d}P_K}{P_K} + S'_L \frac{\mathrm{d}P_L}{P_L} \right) - \frac{\mathrm{d}P_A}{P_A} \quad (24)$$

where P_A is the price deflator of added value. The discrete approximation of Equation 24 for empirical study is

$$\Delta(\ln T_A)_t = \bar{s}'_{K_t}\,\Delta(\ln P_K)_t + \bar{s}'_{L_t}\,\Delta(\ln P_L)_t - \Delta(\ln P_A)_t \tag{25}$$

By definition,

$$V_O = V_K + V_L + V_m \tag{26}$$

and

$$V_A = V_O - V_m \tag{27}$$

where V_O and V_m are the market values of gross output and intermediate inputs respectively. Therefore

$$S'_K = \frac{S_K}{1 - S_m} \tag{28}$$

and

$$S'_L = \frac{S_L}{1 - S_m} \tag{29}$$

where S_m is the value share of intermediate inputs.

The most commonly used method for measuring value added in real terms is the double deflation procedure, i.e.

$$\frac{V_A}{P_A} = \frac{V_O}{P_O} - \frac{V_m}{P_m} \tag{30}$$

where V_m and P_m are the value and price of intermediate input respectively.

Since

$$V_O = \frac{V_A}{1 - S_m} = \frac{V_m}{S_m} \tag{31}$$

Equation 30 can be divided by V_O to give

$$\frac{1 - S_m}{P_A} = \frac{1}{P_O} - \frac{S_m}{P_m} \tag{32}$$

If there is more than one intermediate input, Equation 32 becomes

$$\frac{1 - \sum_{i=1}^{n} S_{m_i}}{P_A} = \frac{1}{P_O} - \sum_{i=1}^{n} \frac{S_{m_i}}{P_{m_i}} \tag{33}$$

where S_{m_i} and P_{m_i} are the value share and price index of the ith intermediate input. The implicit price deflator of value added in terms of value shares, gross output price and price of intermediate inputs can be written:

$$P_A = \frac{1 - \sum_{i=1}^{n} S_{m_i}}{\frac{1}{P_O} - \sum_{i=1}^{n} \frac{S_{m_i}}{P_{m_i}}} \tag{34}$$

Substituting Equations 28, 29 and 34 into Equation 25, the VATFP can be calculated using the same set of data as used for estimating TFP.

Single-factor productivity

It is also possible to calculate the partial factor (single-factor) productivity from the price trends. The partial-factor productivity (PFP) of input i is defined as

$$Z_i = \frac{Q_O}{Q_{Ii}} \tag{35}$$

Differentiating Equation 35 logarithmically with respect to time, we have

$$\frac{dZ_i}{Z_i} = \frac{dQ_O}{Q_O} - \frac{dQ_{Ii}}{Q_{Ii}} \tag{36}$$

Differentiating

$$Q_{Ii}P_{Ii} = s_i P_O Q_O \tag{37}$$

logarithmically with respect to time, we have

$$\frac{dQ_O}{Q_O} - \frac{dQ_{Ii}}{Q_{Ii}} = \frac{dP_{Ii}}{P_{Ii}} - \frac{dP_O}{P_O} - \frac{ds_i}{s_i} \tag{38}$$

Substituting the above into Equation 36, we have

$$\frac{dZ_i}{Z_i} = \frac{dP_{Ii}}{P_{Ii}} - \frac{dP_O}{P_O} - \frac{ds_i}{s_i} \tag{39}$$

The corresponding discrete approximation for empirical study is

$$\Delta(\ln Z_i)_t = \Delta(\ln P_{Ii})_t - \Delta(\ln P_O)_t - \Delta(\ln s_i)_t \tag{40}$$

Value-added single-factor productivity

The single-factor productivity of input i when output is measured as value added is

$$Z_{Ai} = \frac{Q_A}{Q_I} \tag{41}$$

where Q_A is value added in real terms and Q_I is the quantity of the ith input which is either capital or labour. Differentiating Equation 41 logarithmically with respect to time gives

$$\frac{dZ_{Ai}}{Z_{Ai}} = \frac{dQ_A}{Q_A} - \frac{dQ_{Ii}}{Q_{Ii}} \tag{42}$$

Differentiating

$$Q_{Ii}P_{Ii} = S_{Ai}P_A Q_A \tag{43}$$

logarithmically gives

$$\frac{dQ_A}{Q_A} - \frac{dQ_{Ii}}{Q_{Ii}} = \frac{dP_{Ii}}{P_{Ii}} - \frac{dP_A}{P_A} - \frac{dS_{Ai}}{S_{Ai}} \tag{44}$$

where S_{Ai} is the share of input i in value added. Substituting the above into Equation 41, we have

$$\frac{dZ_{Ai}}{Z_{Ai}} = \frac{dP_{1i}}{P_{1i}} - \frac{dP_A}{P_A} - \frac{dS_{Ai}}{S_{Ai}} \quad (45)$$

The corresponding discrete approximation for empirical study is therefore

$$\Delta(\ln Z_{Ai})_t = \Delta(\ln P_{1i})_t - \Delta(\ln P_A)_t - \Delta(\ln S_{Ai})_t \quad (46)$$

Data

To use Equation 14 to estimate the TFP trend of Hong Kong's building industry, the trend of building cost (input price) and price (output price) data and the value shares of different types of inputs are required.

Four types of physical inputs are identified, i.e. labour, material, capital goods (which includes construction machinery and real estate, e.g. offices) and other miscellaneous expenses. The price indexes of Labour and Material are Labour Cost Index (LCI) and Material Cost Index (MCI). A detailed description of the LCI and MCI can be found in Chau and Walker (1988).

The price index for miscellaneous expenses is the implicit price deflator for consumption expenses (PCE) constructed by the Census and Statistics Department.[6] The other data, namely capital price index and output price index are not readily available and must be derived from available data. The methods of deriving these data are presented in the following sections.

Capital price index

The two major categories of capital goods consumed by the building industry are construction plant and machinery and office space.

The price index of capital inputs (KPI) is therefore constructed as a discrete Divisia index of the construction machinery rental index and the office rental index with their average value shares as weights.

The rental index of construction plant and machinery is estimated from the capital prices of construction plant and machinery[7] and its average rate of return (see Appendix 1).

Output price index

The lack of a true official output price index for building works in Hong Kong[8] is one of the major problems in applying any theoretical framework for estimating industry-level productivity trends in Hong Kong's building industry. Chau and Walker's solution to this problem is to use tender price index as a proxy for output price index. This approach, however, relies on the assumption that production periods (contract durations) are short enough to be ignored in the productivity calculation. This assumption may be acceptable in certain industries but is certainly not realistic in building construction. As a result, a new output price index for building works has to be constructed, taking into account the long construction periods of building works.

The index used in this research is constructed from the private sector output price index (PRO) and the public sector output price index (PUO) which are in turn constructed from other available statistics. Since private sector building contracts in Hong Kong are firm-price contracts (i.e. fluctuation clauses are not included in the conditions of contracts), output price at any time t is largely determined by the tender price levels before t.[9] The PRO can therefore be constructed as a weighted moving average of previous tender prices. The weight for TPI at time $t-s$ represents the value of work tendered at $t-s$ but completed at time t as a proportion of total value of work completed at time t. The weights are determined empirically, based on the following information:

(a) value of work tendered in each quarter;
(b) distribution of contract durations of value of work tendered;
(c) the rate of progress of a typical construction project.

A more detailed description of the derivation of the index can be found in Chau (1990).

The PUO is also constructed in a similar way except that it is modified by the LMI since fluctuation clauses are normally included in government contracts with project durations greater than 12 months.

The final output price deflator for all building works (OPI) is calculated as a weighted discrete Divisia index of the public and private sector output price indices, as described above. The weights are calculated as the proportion of total expenditure in building works in each sector averaged over two successive periods.

Value shares

The value share of individual inputs after 1979 can be calculated from the data collected in the annual survey of building, construction and real estate sectors conducted by the Census and Statistics Department. Value shares before 1979 and those of 1980 have to be estimated indirectly. A similar survey was not conducted in 1980 because of insufficient manpower to cope with the workload in consolidating the first benchmark survey conducted in 1979.

The constant value share assumption

In reality, value shares may change for different reasons.

Following the assumptions made earlier, i.e. Hicks Neutral Technical Change, constant returns to scale and producer equilibrium, changes in value shares of inputs can only be caused by changes in their relative prices. Intuitively, changes in relative prices can result in the substitution of the dearer input by the relatively cheaper input. The effect of price changes on values shares thus depends on how easily one input can be substituted for/by another input. Hicks (1932) proposed a very useful means of measuring the ease by which inputs can be substituted for each other in a two-input production system, namely the elasticity of substitution. It is defined as

$$e_{12} = -\frac{\mathrm{d}\ \ln(Q_{I_1}/Q_{I_2})}{\mathrm{d}\ \ln(m_1/m_2)} \tag{47}$$

where Q_{I_1} and Q_{I_2} are the quantities of inputs 1 and 2, and m_1 and m_2 are the marginal products of inputs 1 and 2. Under the assumption of constant returns to scale and Hicks Neutral technical change, e_{12} is independent of the level of output and technology. Under producer equilibrium, m_1/m_2 is equal to the ratio of the prices of the two inputs. Therefore, e_{12} can also be defined as the negative[10] value of the ratio of relative change in their quantity proportion to the relative change in their price ratio.

In other words, the 'ease of substitution' between two inputs is measured by the sensitivity of relative change in input proportions to relative change in their price ratio or relative prices. The numerical value of e_{12} ranges from zero to infinity. The larger the value of e_{12} (more sensitive) the easier the substitution.[11] When $e_{12}=0$, substitution is impossible and the input proportion is not affected by any change in relative prices. It can be seen that e_{12} as defined by Equation 47 is symmetrical, i.e.

$$e_{12} = e_{21} \tag{48}$$

Another way of expressing e_{12} is in terms of their cross-price elasticities. Standard transformations in the theory of production[12] show that

$$e_{12} = \frac{E_{21}}{s_2} \tag{49}$$

where E_{21} is the cross-elasticity of demand for input 1 with respect to the price of input 2 and is defined as

$$E_{21} = -\frac{\mathrm{d}\ \ln(Q_{12})}{\mathrm{d}\ \ln(P_{11})} \tag{50}$$

In the two-input case, both E_{21} and E_{12} must be positive.

Generalization of the definition of elasticity of substitution for more then two inputs is, however, not straightforward. Different generalizations have been proposed by Robinson (1933), Allen (1938), McFadden (1963), Samuelson (1968) and Hicks (1970) all of which reduced to the Hicks definition in the two-input case. We have adopted Allen's definition in this research simply because it is most widely used and easiest to understand. Allen has defined the partial elasticity of substitution (AES) between two inputs in a multi-input production system as

$$e_{ij} = \left(\frac{M_{ij}}{|\mathbf{M}|}\right) \cdot \frac{1}{Q_{Ii}Q_{Ij}} \cdot \sum_{i=1}^{n} Q_{Ii}m_i \tag{51}$$

where Q_{Ii} and m_i denote respectively the quantity and marginal product of input i, M_{ij} is the cofactor of m_{ij} in $\mathbf{M}$ which is the matrix

$$\mathbf{M} = \begin{pmatrix} 0 & \cdots & m_j & \cdots & m_n \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ m_i & \cdots & m_{ij} & \cdots & m_{in} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ m_n & \cdots & m_{nj} & \cdots & m_{nn} \end{pmatrix} \tag{52}$$

m_{ij} is the partial derivative of m_i with respect to Q_{ij} and $|\mathbf{M}|$ is the determinant of $\mathbf{M}$. Allen has shown that under constant returns to scale

$$e_{ij} = \frac{E_{ij}}{s_j} \tag{53}$$

where E_{ij} is the cross-price elasticity of demand as defined for the two-factor case. It can easily be seen that Equation 52 reduces to Equation 47 in the two-input case. Allen has also shown that

$$\sum_{j=1}^{n} s_j e_{ij} = 0 \tag{54}$$

The economic interpretation of AES (i.e. e_{ij}) is similar to that of Hicks's definition except that more than two inputs are involved. It describes changes in input proportions when the price of one input is allowed to vary and the prices of all other inputs are kept constant. Inputs i and j are said to be substitutes or complements depending on whether e_{ij} is positive or negative.

To see how value shares are affected by the ease of substitution between input pairs (i.e. the value of e_{ij}), the following identity is differentiated logarithmically with respect to time

$$P_{Ii}Q_{Ii} \equiv s_i V_O \tag{55}$$

where P_{Ii} and Q_{Ii} are the quantity and price of input i, s_i is its value share and V_O is the total value of output. By making use of Equations 53 and 54 and after tedious manipulation (see Appendix 2), we have

$$\frac{\mathrm{d}\ \ln(s_i)}{\mathrm{d}t} = \left(\frac{\mathrm{d}\ \ln(P_{Ii})}{\mathrm{d}t} - \sum_{j=1}^{n} s_j \frac{\mathrm{d}\ \ln(P_{Ii})}{\mathrm{d}t}\right) + \sum_{j=1}^{n} s_j e_{ij} \frac{\mathrm{d}\ \ln(P_{Ij})}{\mathrm{d}t} \tag{56}$$

assuming Hicks Neutral Technical Change and constant returns to scale. It can be seen from Equation 56 that changes in value shares depend on changes in relative prices (expression inside the parentheses) and/or elasticity of substitution between input pairs (the last expression).

The expression inside the parentheses on the right-hand-side is the deviation of relative change in the price of input i from the average of relative changes in the prices of all inputs weighted by their value shares.

The intuitive interpretation of this expression is that, other things being equal (including input proportions), the larger the magnitude of the price decrease/increase of an input relative to the others, the higher the decrease/increase in its value share. This is because when input proportions remain unchanged, an increase in the price of an input relative to other inputs will result in an increase in its value share. Since input proportions are kept constant, this effect does not take into account the possibility of substitution between input pairs.

Changes in relative prices of inputs may result in the substitution of the relatively cheaper inputs for the relatively dearer inputs. This effect is represented by the expression on the right-hand side of Equation 56 which involves the e_{ij} as the major parameters.

When all inputs are substitutes rather than complements, the substitution effect and pure price effect will work in opposite directions in terms of their effects on value shares. When the price of an input decreases relative to the other inputs, the pure price effect implies that its value share will decrease. However, if substitution is possible, under the assumption of cost minimization more of the input will be used (relative to other inputs) and thus its value share may increase.

When inputs are close substitutes for each other (i.e. the values of the AESs are very large), the substitution effect dominates. At the extreme end of this (all AESs approach infinity), a small decrease in the price of an input will raise its value share to 100% since cost minimization implies that only the cheapest input will be used when inputs are perfect substitutes.

It is also interesting to see that, somewhere in between the two extreme cases, the pure price effect will just be offset by the substitution effect, in which case the value shares of inputs will remain constant, irrespective of any change in prices. To see the implied value of the AESs in this case, Equation 56 is rearranged to yield (see Appendix 3 for the derivation):

$$\frac{\mathrm{d}\ln(s_i)}{\mathrm{d}t}=\sum_{j\neq 1}^{n} s_i s_j(e_{ij}-1)\left(\frac{\mathrm{d}\ln(P_{1j})}{\mathrm{d}t}-\frac{\mathrm{d}\ln(P_{1i})}{\mathrm{d}t}\right) \quad (57)$$

Constant value shares implies that

$$\frac{\mathrm{d}\ln(s_i)}{\mathrm{d}t}=0 \quad (58)$$

Therefore

$$e_{ij}=1 \qquad \forall i\neq j \quad (59)$$

ensures that all value shares are constant irrespective of price changes.[13] If the AESs are constant over time irrespective of any changes in relative prices, then the assumption of constant value share is the same as assuming all AESs equal unity. This is the constant value share assumption suggested by Chau and Walker (1988). Such an assumption of unitary elasticity of substitution is very restrictive, since the AESs may not be constant over time irrespective of price ratios and, even if they are, they may not necessarly be equal to unity.

In theory, even if we can assume that all AESs are constant over time it is still impossible to estimate all the AESs without imposing any restriction on them. This is because Equation 57 is a system of m equations, which represents an m-input production system. However, as the left-hand side of the m equations sum to zero, one equation must be a linear combination of the others, therefore there are $(m-1)$ independent equations. Since e_{ij} equals e_{ji}, there are $(m)(m-1)/2$ unknowns (AESs). When the number of inputs (m) is greater than two, there is no unique solution for each AES.

Depending on the restrictions imposed on the AESs, a large number of time series and/or cross-sectional data in general is required to estimate all AESs statistically. The less restrictive the assumption about the AESs, the greater the amount of data required to estimate them. As such data are not available in sufficiently large quantities, the extent to which the unitary partial elasticities can be relaxed is therefore limited.

Constant and equal elasticity of substitution

Within the limited sets of available data, an attempt has been made to estimate the value shares under assumptions which are less restrictive than the unitary AES assumption. This approach assumes that all the AESs are constant and equal but not necessarily equal to one, i.e.

$$e_{ij}=e \qquad \forall i\neq j \quad (60)$$

so that Equation 59 becomes

$$\frac{\mathrm{d}\ln(s_i)}{\mathrm{d}t}=\sum_{j\neq i}^{n} s_i s_j(e-1)\left(\frac{\mathrm{d}\ln(P_{1j})}{\mathrm{d}t}-\frac{\mathrm{d}\ln(P_{1i})}{\mathrm{d}t}\right) \quad (61)$$

which can be approximated by

$$\Delta[\ln(s_i)]_t=\sum_{j\neq i}^{n} s_{i_{t-1}}s_{j_{t-1}}(e-1)\,(\Delta[\ln(P_{1j})]_t-\Delta[\ln(P_{1i})]_t) \quad (62)$$

where Δ denotes the difference between two successive periods.

Given the value shares of a particular (base) year, a time series of value shares can be simulated using Equation 62 for different assumed values of e. To determine which value of e is the best approximation to reality, the simulated value shares are compared with the observed value shares which are available since 1979. The closer the simulated value shares to the observations, the better the approximation.

For a given value of e, the closeness between the simulated and the observed value shares is measured by the sum of the squares of their differences,[14] which is denoted by D_e. The smaller the value of D_e the more realistic the assumed value of e. The values of D_e are calculated for values of e between 0 and 1 in 0.1 intervals. The results are summarized in Fig. 1. which shows the observed relationship between D_e and e.

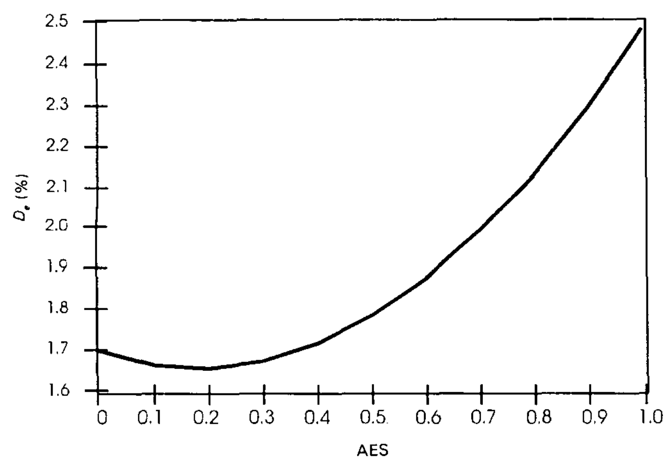


Figure 1 Simulated D_e values at different AES values

It can be seen that the value of e is likely to be closer to zero than unity. The estimated value of e is approximately equal to 0.2 since the value of D_e is lowest at this value of e.

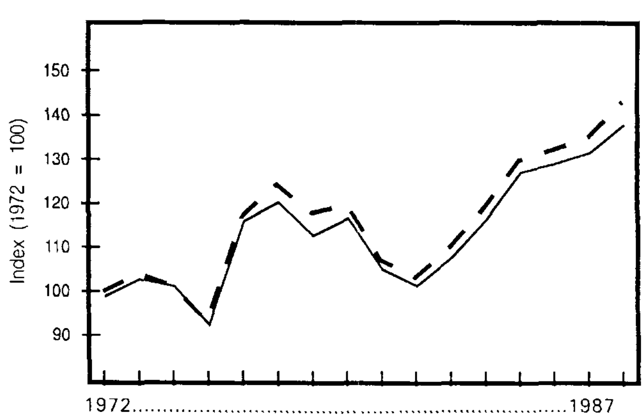
When e is equal to unity, D_e is the deviation of the observed value shares from their averages over the years. The results suggest that the ease of substitution between input pairs is likely to be over-estimated under the assumption of constant value shares.

The productivity trends

Figure 2 shows the TFP and LP trends of Hong Kong's building industry over the period 1972–1987 for AES = 0.2. The LP and TFP trends are very similar, which is a result of the low substitution possibility between labour and other inputs. Although the trends fluctuate considerably over the observed period, the estimated average annual compound growth rate is about 2% p.a. for both trends, indicating a general upward-sloping tendency. It is perhaps interesting to investigate the sources of such growth, as research and development, which is often the major source of productivity advance, has been almost non-existent in Hong Kong's building industry.

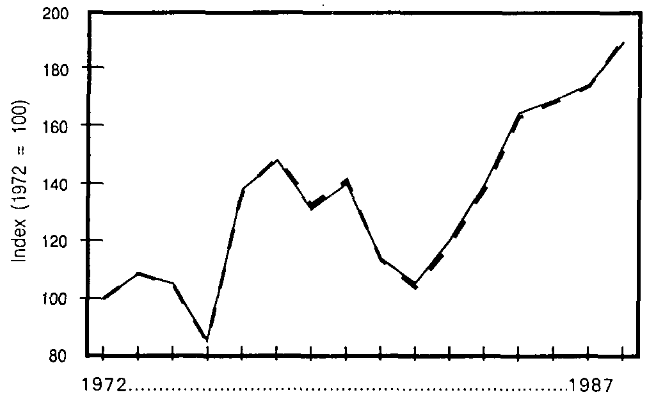
The cyclical nature of the trend suggests that the TFP trend is under the influence of certain changing economic forces. The identification of these forces and the way they affect the TFP trend also requires more detailed investigation.

In terms of international comparison, the VATFP trend is more useful as in most productivity studies construction output is measured in terms of value added. The VATFP[15] trend and the VALP[16] trend in Hong Kong's building industry are shown in Fig. 3. Similarity of the two trends is a result of the low substitution possibility between capital and labour. The cyclical pattern of the VATFP trend is very similar to that of the TFP trend although the former grows at a much higher rate. The average annual compound growth rate of VATFP is about 4.3%, which is very high compared with those of other developed countries. The average annual growth rate in VATFP in these countries ranges from 0.2% in the UK to 2.2% in Japan in the 1950s and 1960s, during which period these economies



Year	LP	TFP
72	100	100
73	104	104
74	101	102
75	93	94
76	117	117
77	124	121
78	118	114
79	120	118
80	107	106
81	103	102
82	110	109
83	119	118
84	130	128
85	132	130
86	135	133
87	143	139

Figure 2 TFP and LP of Hong Kong's construction industry. – – –, LP (average growth rate = 2.42% p.a.); ——, TFP (average growth rate = 2.23% p.a.)



Year	VALP	VATFP
72	100	100
73	109	109
74	105	105
75	85	85
76	137	138
77	149	148
78	132	131
79	141	140
80	114	114
81	104	105
82	118	120
83	138	140
84	163	164
85	168	169
86	174	174
87	190	189

Figure 3 VATFP and VALP of Hong Kong's construction industry. – – –, VALP (average growth rate = 4.38% p.a.); ——, VATFP (average growth rate = 4.34% p.a.)

experienced fast economic and productivity growth. More recent studies show that the growth in construction productivity in many developed countries has been declining since the 1970s.[17] The comparison is summarized in Table 1.

These figures, however, have to be interpreted with care since the definition, method of measurement and the data used are not exactly comparable with the present study. Nevertheless, they do give us an approximate idea, by international standards, of productivity growth in Hong Kong's building industry.

The growth rate of the VATFP trend is about twice that of the TFP trend. This suggests that intermediate inputs (mainly construction materials) have played an important role in the growth in the productive efficiency of Hong Kong's building industry. Technological diffusion from the manufacturing industry to the building industry in the form of new building materials that are, for example, more durable or easier to work with could be one of the major contributing factors in the high growth rate of VATFP, although more empirical evidence is required to support this.

The actual mechanism and magnitude of the impact of product invention and innovation in the manufacturing industry on construction productivity could be another interesting area for a more in-depth study.

Conclusion

A method of estimating the various productivity trends (including the conventional two-factor VATFP and TFP embracing all tangible inputs as well as other single-factor productivity measures) has been presented in this paper. The method requires only cost and price data and the relative value shares of inputs. These data are in general more readily available than the corresponding quantity data in the building/construction industry. Compared with Chau and Walker's approach, the present method is less restrictive. It does not rely on the constant value share assumption which has been shown in this paper to be unrealistic. The underlying production function is also much more flexible. The output price deflator used in this research should reflect the trend of output prices better than the tender price index used in Chau and Walker's study since the latter does not take into account the time lag between tendering and construction.

Table 1 International comparison of average growth rates in productivity in the building/construction industry

	USA	Canada	UK	Japan	Germany	Denmark	HK
VATFP (%)	1.0[a]	0.6[b]	0.2[b]	2.2[b]	–	<0[c]	4.3[e]
Period	1948–76	1949–60	1953–59	1951–59		1966–87	1972–84
VALP (%)	2.0[b]	1.3[b]	0.8[b]	4.0[b]	2.97[d]	<0[c]	4.4[e]
Period	1948–76	1949–60	1953–59	1951–59	1968–71	1966–87	1972–84

VATFP: Total-factor productivity (output measured in value added).
VALP: Labour productivity (output measured in value added).
Sources: [a] Kendrick and Grossman (1980).
[b] Domar *et al.* (1964).
[c] Pedersen (1990).
[d] Smith (1982).
[e] Author's estimate.

Results using Hong Kong data suggest that the value shares of inputs are not constant over time. The pattern of change of the value shares in response to change in relative input prices indicates a relatively low substitution possibility between inputs. This means that shortage in any one input can lead to a significant decline in TFP growth as such shortage cannot be easily compensated by using more of other inputs, as least not in the short term.

The estimated growth rates in Hong Kong's TFP and VATFP trends are approximately 2% and 4% respectively over the period 1972–1987. This suggests that about 50% of the growth in VATFP, which is the conventional two-factor VATFP, is attributable to technological diffusion from other industries through the use of new building materials. The 4% growth rate in VATFP is also high when compared with the construction industries in other developed countries during the 1960s and 1970s when these economies experienced strong growth in productivity.

Previous research in TFP suggests that research and development is a major contributor to growth in TFP. There has been, however, very little research and development in Hong Kong's building industry during the period of strong growth in TFP, which is an interesting area for a more in-depth study.

Notes

1. In Chau and Walker's approach an explicit geometric aggregator is assumed, i.e. $g(.) = \prod_{i=1}^{n} Q_{1i}^{S_i}$ where s_i is the value share of the ith input and is assumed to be constant over time. The approach presented here is much less restrictive since it does not assume an explicit functional form for the input aggregator function.
2. Under this assumption, input proportions remain unchanged if there is no change in relative prices.
3. The zero-profit assumption does not appear in Chau and Walker's approach. However, such an assumption does not add any restriction, in practical terms, to the measurement of TFP for three reasons. Firstly, supernormal profit is very difficult, if not impossible, to measure. Secondly, there is no barrier to entry into the building industry. This means that the building industry is competitive for small- to medium-scale projects. For larger-scale projects, market competition is ensured by the Hong Kong Government's policy towards overseas contractors. Consistent with the free-trade policy, the Hong Kong Government has never imposed any restrictions discriminating against overseas contractors. This is evident by the increasing number of overseas contractors in Hong Kong. At the end of 1991, there were 84 building contractors on the Government's List of Contractors (Hong Kong Government Gazette, 1992) who were allowed to bid for large building construction projects (contract sum over US$4 million). Of the 84 building contractors, 29 are classified as overseas contractors (List II Contractors). In fact, many of the list I Contractors also came from overseas originally, but since they have established their Hong Kong subsidiaries here for a long time they are now regarded as 'local contractors'. These companies include Dragges et Travaux Publics, Nishimatsu Construction Co. Ltd, Kumagai Gumi (Hong Kong) Ltd, Leighton Contractor (Asia) Ltd, just to name a few. Thirdly, in the absence of a better indicator, book profit is used by Chau and Walker (1988) as a proxy for supernormal profit. The resultant TFP trend, after adjustment for changes in book profit margin, was not dissimilar to the unadjusted trend. Since the difference is small, it is therefore not worthwhile, in this case, to adjust the TFP trend for possible changes in profit margin using a dubious proxy.
4. See Chau and Walker (1988) for an intuitive interpretation of Equation 13.
5. As a result of the reproductive property, relative change in output price in Equation 13 can be replaced by a Divisia index of price of outputs in the case of a multi-output production system.
6. For a more detailed description of the index see, for example, Census and Statistics Department (1992).
7. Similar to Chau and Walker's approach, the UVI (unit Value Price Index of imported capital goods) is used as a proxy of the capital price of construction plant and machinery since almost all of them are imported.
8. The official price deflator for deflating construction outputs is the Labour and Material Cost Index which is an input price (or factor price) input rather than output price index. Also see Chau (1990) for a more detailed discussion.
9. Since the output price is the price paid by the client and this price is largely determined by the unit rates in the accepted tender which determines the tender price index, there is a definite relationship between output price index and tender price index. However such relationship is not a simple time-lagged relationship, i.e. the output price index at time t does not equal the tender price index at time $t-s$, since projects tendered at any time t comprise projects with different contract sums and durations. See Chau (1990) for a more detailed discussion.
10. The increase in the relative price of an input will

normally result in a decrease in its quantity ratio; the negative sign is simply used to make the value of e_{12} positive.

11. When two inputs are close substitutes, i.e. one can be easily substituted by the other, a small change in their price ratio (i.e. when price of one increases relative to the other), under the assumption of cost minimization, will cause producers to use more of the relatively cheaper inputs and less of the relatively dearer inputs resulting in a substantial change in the quantity ratio of the two inputs. The value of elasticity of substitution which is defined in Equation 47 as the negative value of the ratio of change in quantity ratio to the change in the price ratio must be very large. On the other hand, if two inputs cannot be substituted for each other, the value of elasticity of substitution is zero since the quantity ratio is unchanged irrespective of any price changes.
12. See, for example, Allen (1938).
13. The same result has also been derived by both Sato and Koizumi (1973) and Samuelson (1973) using different approaches.
14. Since the simulated value shares differ slightly when different years' observed value shares are used as base year value shares, the sum of the squares of differences for a given value of *e* is calculated as the averages of the sum of the squares of differences using different years' value shares as base year value shares.
15. Total Factor Productivity trend with output measured in value added. This is the same as the conventional two-factor total-factor productivity.
16. Labour productivity with output measured in value added.
17. See, for example, Allen (1985).

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Appendix 1

The price of capital input (or rental price) can be defined as total return (rent paid) for the use of capital divided by the quantity of capital (see, for example, Kendrick and Sato (1963)), i.e.

$$P_K = \frac{T_C}{K_R} \tag{A1-1}$$

where T_C is the total cost for using K_R units of capital. This is analogous to the definition of the price of labour which is equal to wage payment (return for labour) per man-day (quantity of labour). Let K_P be the purchase price of the capital, then the total initial capital cost is

$$K_V = K_R \cdot K_P \quad \text{(A1-2)}$$

By definition, the current dollar rate of return of capital is

$$R = \frac{T_C}{K_V} \quad \text{(A1-3)}$$

Therefore, Equation A1-1 can be written as

$$P_K = \frac{R \cdot K_R \cdot K_P}{K_R} = R \cdot K_P \quad \text{(A1-4)}$$

Thus the price deflator of capital input (or the rental index) is equal to an index of its purchase price times its current dollar rate of return.

The trend in the purchase prices of plant and machinery is estimated by the Unit Value Index of imported capital goods (UVI) constructed by the Census and Statistics Department. As a price index for plant and machinery used in the building industry is not available, the UVI is the best information we can get for estimating the trend in the purchase prices of all construction plant and machinery (since they are all imported). The average rate of return of plant and machinery is estimated by the smoothed trend in interest rates. A smoothed trend is used since it is unlikely that the rate of return on plant machinery will respond immediately to a fluctuation in interest rates.

Appendix 2

Logarithmically differentiating

$$s_i = \frac{P_{Ii} Q_{Ii}}{V_I} \quad \text{(A2-1)}$$

with respect to time gives

$$\frac{d\ln(s_i)}{dt} = \frac{d\ln(P_{Ii})}{dt} + \frac{d\ln(Q_{Ii})}{dt} - \frac{1}{V_I}\frac{d\ln(V_I)}{dt} \quad \text{(A2-2)}$$

where V_I is the total market value of inputs. Since

$$V_I = \sum_{j=1}^{n} P_{Ij} Q_{Ij} \quad \text{(A2-3)}$$

therefore

$$\frac{d\ln(s_i)}{dt} = \frac{d\ln(P_{Ii})}{dt} + \frac{d\ln(Q_{Ii})}{dt} - \frac{1}{V_I}\sum_{j=1}^{n}\frac{d(P_{Ij}Q_{Ij})}{dt} \quad \text{(A2-4)}$$

or

$$\frac{d\ln(s_i)}{dt} = \frac{d\ln(P_{Ii})}{dt} + \frac{d\ln(Q_{Ii})}{dt} - \sum_{j=1}^{n}\frac{P_{Ij}Q_{Ij}}{V_I}\left(\frac{d\ln(P_{Ij})}{dt} + \frac{d\ln(Q_{Ij})}{dt}\right) \quad \text{(A2-5)}$$

By definition

$$\frac{P_{Ij}Q_{Ij}}{V_I} = s_j \quad \text{(A2-6)}$$

and

$$\sum_{j=1}^{n} s_j = 1 \quad \text{(A2-7)}$$

Thus Equation A2-5 can be written as

$$\frac{d\ln(s_i)}{dt} = \left(\frac{d\ln(P_{Ii})}{dt} - \sum_{j=1}^{n} s_j \frac{d\ln(P_{Ij})}{dt}\right) + \frac{d\ln(Q_{Ii})}{dt}\sum_{j=1}^{n} s_j - \sum_{j=1}^{n} s_j \frac{d\ln(Q_{Ii})}{dt} \quad \text{(A2-8)}$$

or

$$\frac{d\ln(s_i)}{dt} = \left(\frac{d\ln(P_{Ii})}{dt} - \sum_{j=1}^{n} s_j \frac{d\ln(P_{Ij})}{dt}\right) + \sum_{j=1}^{n} s_j \frac{d\ln(Q_{Ii}/Q_{Ij})}{dt} \quad \text{(A2-9)}$$

Under the assumption of constant returns and Hicks Neutral Technical Change, factor proportions can only be affected by price changes. The last term of Equation A2-9 can thus be written as

$$\sum_{j=1}^{n} s_j \sum_{k=1}^{n} \frac{\partial\ln(Q_{Ii}/Q_{Ij})}{\partial\ln(P_{Ik})} \cdot \frac{d\ln(P_{Ik})}{dt} \quad \text{(A2-10)}$$

which is equal to

$$\sum_{j=1}^{n} s_j \sum_{k=1}^{n} \left(\frac{\partial\ln(Q_{Ii})}{\partial\ln(P_{Ik})} - \frac{\partial\ln(Q_{Ij})}{\partial\ln(P_{Ik})}\right) \cdot \frac{d\ln(P_{Ik})}{dt} \quad \text{(A2-11)}$$

Allen (1938) has shown that

$$\frac{\partial\ln(Q_{Ii})}{\partial\ln(P_{Ik})} = E_{jk} = e_{jk} \cdot s_k \quad \text{(A2-12)}$$

Substituting into (A2-10) gives

$$\sum_{j=1}^{n} s_j \sum_{k=1}^{n} (e_{ik} - e_{jk}) s_k \cdot \frac{d\ln(P_{Ik})}{dt} \quad \text{(A2-13)}$$

which is equal to

$$\sum_{j=1}^{n} s_j \sum_{k=1}^{n} e_{ik} s_k \frac{d\ln(P_{Ik})}{dt} - \sum_{j=1}^{n} s_j \sum_{k=1}^{n} e_{jk} s_k \frac{d\ln(P_{Ik})}{dt} \quad \text{(A2-14)}$$

Re-arranging the terms and switching the summation sign gives

$$\sum_{k=1}^{n} e_{ik} s_k \frac{d\ln(P_{Ik})}{dt} \sum_{j=1}^{n} s_j - \sum_{k=1}^{n} s_k \frac{d\ln(P_{Ik})}{dt} \sum_{j=1}^{n} s_j e_{jk} \quad \text{(A2-15)}$$

Allen has also shown that

$$\sum_{j=1}^{n} s_j e_{ji} = 0 \quad \text{(A2-16)}$$

but e_{ij} is symmetrical, i.e.

$$\sum_{j=1}^{n} s_j e_{ij} = 0 \quad \text{(A2-17)}$$

The last term of Equation A2-15 vanishes and since

$$\sum_{j=1}^{n} s_j = 1 \tag{A2-18}$$

we have

$$\sum_{j=1}^{n} s_j \frac{\mathrm{d}\ln(Q_{1i}/Q_{1j})}{\mathrm{d}t} = \sum_{k=1}^{n} e_{1k} s_k \frac{\mathrm{d}\ln P_{1k}}{\mathrm{d}t} \tag{A2-19}$$

To maintain consistency in notation, j is used as the index of summation in the right-hand side, i.e.

$$\sum_{j=1}^{n} s_j \frac{\mathrm{d}\ln(Q_{1i}/Q_{1j})}{\mathrm{d}t} = \sum_{j=1}^{n} e_{ij} s_j \frac{\mathrm{d}\ln P_{1j}}{\mathrm{d}t} \tag{A2-20}$$

Substituting Equation A2-20 back into Equation A2-9, we get

$$\frac{\mathrm{d}\ln(s_i)}{\mathrm{d}t} = \left(\frac{\mathrm{d}\ln(P_{1i})}{\mathrm{d}t} - \sum_{j=1}^{n} s_j \frac{\mathrm{d}\ln(P_{1j})}{\mathrm{d}t}\right) + \sum_{j=1}^{n} s_j e_{ij} \frac{\mathrm{d}\ln(P_{1j})}{\mathrm{d}t} \tag{A2-21}$$

Appendix 3

The assumption of constant value shares implies that

$$\frac{\mathrm{d}\ln s_i}{\mathrm{d}t} = 0 \tag{A3-1}$$

From the result obtained in Appendix 2, we have

$$\left(\frac{\mathrm{d}\ln(P_{1i})}{\mathrm{d}t} - \sum_{j=1}^{n} s_j \frac{\mathrm{d}\ln(P_{1j})}{\mathrm{d}t}\right) + \sum_{j=1}^{n} s_j e_{ij} \frac{\mathrm{d}\ln(P_{1j})}{\mathrm{d}t} = 0 \tag{A3-2}$$

Extracting the ith item from both summations, Equation A3-2 becomes

$$\frac{\mathrm{d}\ln(P_{1i})}{\mathrm{d}t} - \left(\sum_{j\neq i}^{n} s_j \frac{\mathrm{d}\ln(P_{1j})}{\mathrm{d}t} + s_i \frac{\mathrm{d}\ln(P_{1i})}{\mathrm{d}t}\right) + \left(\sum_{j\neq i}^{n} s_j e_{ij} \frac{\mathrm{d}\ln(P_{1j})}{\mathrm{d}t} + s_i e_{ii} \frac{\mathrm{d}\ln(P_{1i})}{\mathrm{d}t}\right) = 0 \tag{A3-3}$$

From Equation A2-16, we know that

$$s_i e_{ii} = -\sum_{j\neq i}^{n} s_j e_{ij} \tag{A3-4}$$

which when substituted into Equation A3-3 gives

$$\left(\frac{\mathrm{d}\ln(P_{1i})}{\mathrm{d}t} - s_i \frac{\mathrm{d}\ln(P_{1i})}{\mathrm{d}t}\right) + \left(\sum_{j\neq i}^{n} s_j e_{ij} \frac{\mathrm{d}\ln(P_{1j})}{\mathrm{d}t} - \sum_{j\neq i}^{n} s_j \frac{\mathrm{d}\ln(P_{1j})}{\mathrm{d}t}\right) - \sum_{j\neq i}^{n} s_j e_{ij} \frac{\mathrm{d}\ln(P_{1i})}{\mathrm{d}t} = 0 \tag{A3-5}$$

or

$$\frac{\mathrm{d}\ln(P_{1i})}{\mathrm{d}t}(1-s_i) + \sum_{j\neq i}^{n} s_j(e_{ij-1}) \frac{\mathrm{d}\ln(P_{1j})}{\mathrm{d}t} - \sum_{j\neq i}^{n} s_j e_{ij} \frac{\mathrm{d}\ln(P_{1i})}{\mathrm{d}t} = 0 \tag{A3-6}$$

Since

$$\sum_{j=1}^{n} s_j = 1 \tag{A3-7}$$

that is

$$\sum_{j\neq i}^{n} s_j = 1 - s_i \tag{A3-8}$$

thus

$$\frac{\mathrm{d}\ln(P_{1i})}{\mathrm{d}t} \sum_{j\neq i}^{n} s_j(1-e_{ij}) + \sum_{j\neq i}^{n} s_j(e_{ij}-1) \frac{\mathrm{d}\ln(P_{1j})}{\mathrm{d}t} = 0 \tag{A3-9}$$

Multiplying by s_i and rearranging, we get

$$\sum_{j\neq i}^{n} s_i s_j (e_{ij}-1)\left(\frac{\mathrm{d}\ln(P_{1j})}{\mathrm{d}t} - \frac{\mathrm{d}\ln(P_{1i})}{\mathrm{d}t}\right) = 0 \tag{A3-10}$$