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This paper investigates the validity of the triangular distribution assumption which is commonly adopted in Monte Carlo simulations of construction costs. The study begins with an examination of the asymmetric nature of the distribution of construction costs and deduces theoretically that the triangular distribution assumption leads to an upward bias in the probability of exceeding the conventional single figure estimate for the subsystem variables and therefore the system variable. This assumption is also inconsistent with the estimators' subjective perception. An experiment has been performed to generate empirical data that test the above theoretical arguments and assesses the magnitude of the bias, if it indeed exists. Subjective estimates of the construction costs of the ten major subsystems of electrical services contracts of government clinics in Hong Kong are examined. These estimates are extracted from seven experienced estimators. The results of the analysis of the data confirm that the underlying distribution of the subsystem costs is asymmetric with a long thin tail towards the right and that the triangular distribution assumption does in fact lead to bias in the subsystem input variables and therefore the simulated output system costs. An alternative modelling approach which can reduce the bias has also been outlined in this paper.

*Keywords:* Monte Carlo simulation, probability density function, risk analysis, subjective estimates, triangular distribution.

## Introduction

Monte Carlo simulation of construction costs requires knowledge about the nature of the probability density function(s) (p.d.f.) of the underlying subsystems of the cost model. Due to the problem of availability of historical data, the p.d.f. is often estimated based on

1. subjective data (estimates given by experienced estimators),
2. *a priori* assumption about the shape of the p.d.f. of the subsystem.

The former is inevitable due to the lack of objective data and can, to a certain extent, be justified by arguing that subjective estimates extracted from estimators reflect memorized historical data. The latter is often taken for granted without any empirical or theoretical justification. The validity of such assumption is nevertheless very important as bias will be introduced if the assumed shape of the p.d.f. deviates significantly from the true p.d.f. of the subsystems.[1] This paper reports the results of empirical tests on the assumption of the triangular distribution[2] which is one of the most commonly adopted assumptions in modelling construction costs using the Monte Carlo simulation method.

[1] Another commonly adopted assumption which if found to be invalid will also lead to bias in the result is the independence assumption. The independence assumption states that the p.d.f.s of the subsystem variables are independent of each other. Chau (1993) has shown that this assumption if found to be invalid can lead to serious bias in probabilistic simulation using a linear cost model. However, this paper is confined only to the investigation of the validity of the assumption of the triangular density function.

[2] The triangular distribution refers to a density function (rather than a distribution function) that is triangular in shape and therefore should be more properly called the 'triangular density function'. However, since it is common to use the term 'triangular distribution' to refer to such a distribution, therefore this term will be used throughout the paper.

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## The nature of the p.d.f. of construction costs

Examples of previous studies on Monte Carlo simulation of construction costs adopting the assumption of the triangular distributions include studies by Wilson (1982), Raftery (1991) and Newton (1992). One reason for adopting this assumption is its ability to model the skewed nature of the p.d.f. of the subsystem costs. Other reasons for adopting the assumption are that

1. the triangular p.d.f. simplifies the computational aspect of the modelling process
2. subjective estimates of the parameters of the triangular distribution (i.e. minimum, most likely and maximum) are relatively easy to extract from estimators.

The latter is especially important as Monte Carlo simulation of construction costs often relies heavily on subjective data.

Previous empirical studies suggest that the distribution of construction costs of a building and its subsystems are skewed to the right.[3] This implies that the most likely estimate is closer to the minimum estimate than to the maximum estimate. The reason for this is that there is a theoretical lower boundary for construction costs which is determined by the minimum amount of reosurces required to construct the structure (or its components) but that a theoretical upper limit does not exist. The maximum estimate reflects the estimator's perception about the worst scenario which could mean a very high cost even though the chance of such an occurrence would be remote.[4] Based on this line of reasoning, the p.d.f. should have a long thin tail towards the right.

The above analysis also implies that the triangular distribution assumption is likely to lead to an overestimation in the probability of exceeding the most likely (i.e. the single figure) estimate. The magnitude of the bias is also a function of the estimator's perception about the maximum. Whether such bias in reality exists and/or is large enough to be of practical concern are the empirical questions which this study attempts to investigate.

## Approach

The method of extracting the subjective data used in this study is similar to previous studies on Monte Carlo simulation of construction costs.[5] However, in addition to estimating the minimum, most likely and maximum (which is the information collected in previous studies), estimators are also asked to estimate the probability of exceeding their most likely estimate which is used to test the triangular distribution assumption. The projects selected for this study are electrical services contracts of government clinics (ESGC). The reasons for this choice are 2-fold. First, these contracts are reasonably homogeneous. Second, the author has direct access to the cost estimators.

Ten cost components (subsystems) of ESGC contracts were identified. Seven experienced estimators were then asked to give conventional three-point estimates and the probability of exceeding the most likely estimates of the unit cost (cost per gross floor area) of the ten subsystems using a modified Delphi technique.[6] In the first round exercise, the estimators were presented with cost data for seven completed ESGC contracts (all updated to the price level at the first quarter in 1993 using an appropriate tender price index). In the second round exercise, each estimator was presented with a summary of the results of the first round estimate and they were allowed to revise their estimates. This process (second round exercise) was repeated until all the estimators declined further revisions.

The validity of assuming a triangular distribution for the subsystem unit costs can be tested by comparing the estimator's subjective estimate of the probability of exceeding the most likely estimate ($P_s$) and the implied probability of exceeding the most likely estimate (for the same subsystem) under the triangular distribution assumption ($P_t$), which is given by

$$P_t = \frac{H-M}{H-L} \quad (1)$$

where $H$, $M$ and $L$ are respectively the maximum, most likely and minimum unit cost estimates of a subsystem. If the null hypothesis Ho: $P_t - P_s = 0$, is rejected in favour of the alternative hypothesis Ha: $P_t - P_s > 0$ then our previous argument about the upward bias in the probability of exceeding the most likely estimate resulting from the triangular distribution assumption is sustainable.

If Ho is rejected, the next step is to assess the magnitude of such bias. The bias is estimated by the maximum (positive) value of $b = P_t - P_s$ such that the hypothesis Ho: $P_t - P_s = b$, is rejected in favour of Ha: $P_t - P_s > b$, at the 0.05 level of significance.

[3] See for example, Mendel (1974), Beeston (1975) and Raftery and Wilson (1982). Touran and Wiser (1992) suggest that construction cost follows a log-normal distribution which is positively skewed. If we also assume that cost and duration are correlated, then the work by AbouRizk and Halpin (1992) provides further support for positive skewness.

[4] See Chau (1993) for a more detailed theoretical discussion of the rightward skewness of construction costs.

[5] See, for example, Wilson (1982) and Newton (1992).

[6] See, for example, Linstone and Turoff (1975) for a discussion of the history, methodology and areas of application of the Delphi technique.

## Results and interpretations

Table 1 shows the subjective data extracted from the seven estimators. It is interesting to note that different estimators have used different methods of arriving at the subjective estimates. The 'rounded-off' figures, e.g. 25%, 30%, 50%, etc., represents the estimator's guesstimates which are usually not revised after the first or second round. Other more 'precise' figures such as 19%, 23%, etc., are either the results of a number of adjustments in the Delphi process or are arrived by a binary search-type method[7] or a combination of both.

It can be seen from Table 1 that the coefficient of variation of the maximum estimate is highest for all subsystems. This strongly suggests that the underlying p.d.f. of the subsystem unit cost has a long thin tail to the right (much longer than the left-hand tail). This is because a flat right-hand tail implies that a small difference in the estimator's perception about the tolerance level[8] (in absolute terms) will lead to a large difference in the maximum estimate and, hence, a high coefficient of variations as illustrated in Figure 1.

Another observation is that the most likely estimate is in general closer to the minimum than the maximum, which is indicated by $P_t > 50\%$.[9] Statistical tests show that the probability that $P_t < 50\%$ is $< 1\%$.[10] This result conforms with previous empirical studies concerning the rightward skewness of construction costs and the theoretical explanation given by Chau (1993).

There are however certain 'irregularities', i.e. cases where $P_t < 50\%$. The reason is likely to be attributable to the fact that unlike construction contracts, electrical services contracts are tendered by performance specification rather than product specification. In the case of a performance specification, the minimum cost is a function of the lowest quality acceptable to the engineer in charge which can vary considerably. Subsequent discussions with the estimators concerned confirm that estimators have very different ideas about what the lowest acceptable quality might be.

A quick glance through the data suggests that $P_t$ is in general higher than $P_s$, however statistical analysis is still required to draw such a conclusion. Two statistical tests, namely the *t*-test and the Wilcoxon signed ranks test[11] (or Wilcoxon test for short) are used to test the null hypothesis. The latter test does not require the difference $d = P_t - P_s$ be from the same normal distribution and is therefore less restrictive than the *t*-test. Both tests assumes that the *d*s (of the same subsystem) are from the same p.d.f. The differences in estimators' perceptions/assumptions about the factors (such as market conditions, contractor's bidding strategies, strength and weaknesses, etc.) affecting $P_s$s and $P_t$s may violate this assumption. This, however, should not present a major problem. First, since all the estimators are from the same organization, such differences arise mainly from the differences in the estimators' experience which can be assumed to be ironed out by the Delphi process. Second, factors affecting both $P_s$s and $P_t$s to more or less the same extent should cancel each other out and therefore differences in the assumptions about these factors will not affect the assumption that the *d*s are from the same distribution.

Even though the data are subjective in nature, the sort of statistical tests used here are still appropriate. First, on an individual level, the estimator's bias is minimized by using the Delphi technique and therefore the figures are not that arbitrary; they represent the estimator's experience which is historical data recorded in an unstructured format in the estimator's mind. Second, as can be seen in a later section of this paper, the results are very robust. This increases our confidence about the conclusion derived from the results of the tests. Provided that there is no evidence of systematic error in the data which would lead to these results, minor error in the data is acceptable. In fact, even historical 'hard facts' (especially construction cost data) cannot be 100% free from error.

The tests have been carried out for each subsystem. An overall test for the pooled data has also been conducted.[12] The results of the tests are shown in Table 2. The results of the *t*-test reject the null hypothesis at the 0.05 significance level for all subsystems as well as for the pooled data. The hypothesis is in fact rejected at the 0.01 significance level for more than half of the subsystems.

The Wilcoxon test yields similar results. All tests reject Ho at the 0.01 significance level except for the sixth and the ninth subsystem which are significant at the 0.05 level. As the Wilcoxon test requires less restrictive

[7] In the binary search process, the estimator starts with an initial estimate, e.g. 25% and then asks him/herself whether he/she feels that the figure is too high or too low. If it is too high, it is then adjusted down say to 22% (otherwise it is adjusted up). The same question is then asked again and the figure adjusted accordingly. The process goes on until the estimator is satisfied with his/her estimate (i.e. when he/she is not certain about whether the estimate is too high or too low).

[8] The tolerance level is the estimator's perception of the maximum acceptable error margin about his/her maximum estimate, i.e. the probability that it will be exceeding. As there is no theoretical maximum that can never be exceeded, all maximum estimates must be associated, often implicitly, with a tolerance level.

[9] On the other hand if $P_t < 50\%$, then the most likely is closer to the maximum than the minimum.

[10] The statistical tests were performed on the pooled data (i.e. all subsystem data pooled together). The results are the same for both the *t*-test and the less restrictive non-parametric test Wilcoxon signed rank test.

[11] See, for example, Hollander and Wolfe (1973).

[12] This test assumes that all *d*s are drawn from the same distribution, which is more restrictive than the test for individual subsystems. The results of the pooled test are, however, indicative only.

**Table 1** Probabilistic estimation of subsystem costs of electrical installation of government clinic projects in Hong Kong (in HK$ per square metre of gross floor area)

| | | Estimator (1) | (2) | (3) | (4) | (5) | (6) | (7) | Mean | Coefficient of variation (%) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 LV switchboard | Maximum | 143 | 170 | 180 | 190 | 160 | 300 | 250 | 199 | 26 |
| | Likely | 87 | 124 | 120 | 140 | 100 | 110 | 100 | 112 | 15 |
| | Minimum | 50 | 55 | 50 | 60 | 60 | 50 | 65 | 56 | 10 |
| | $P_t$ (%) | 60 | 40 | 46 | 38 | 60 | 76 | 81 | 57 | 27 |
| | $P_s$ (%) | 25 | 38 | 40 | 36 | 40 | 25 | 20 | 32 | 24 |
| 2 Main and submain distribution | Maximum | 174 | 250 | 270 | 200 | 160 | 110 | 300 | 209 | 30 |
| | Likely | 111 | 112 | 150 | 130 | 110 | 75 | 120 | 115 | 18 |
| | Minimum | 56 | 85 | 60 | 80 | 60 | 60 | 70 | 67 | 16 |
| | $P_t$ (%) | 53 | 84 | 57 | 58 | 50 | 70 | 78 | 64 | 19 |
| | $P_s$ (%) | 23 | 31 | 33 | 25 | 45 | 23 | 30 | 30 | 24 |
| 3 General final circuit and equipment | Maximum | 367 | 411 | 540 | 420 | 500 | 600 | 400 | 463 | 17 |
| | Likely | 297 | 335 | 300 | 340 | 350 | 350 | 320 | 327 | 6 |
| | Minimum | 185 | 230 | 220 | 210 | 200 | 250 | 250 | 221 | 10 |
| | $P_t$ (%) | 38 | 42 | 75 | 38 | 50 | 71 | 53 | 53 | 27 |
| | $P_s$ (%) | 19 | 22 | 33 | 23 | 15 | 40 | 30 | 26 | 31 |
| 4 Special final circuit and equipment | Maximum | 43 | 62 | 50 | 80 | 50 | 72 | 70 | 61 | 21 |
| | Likely | 36 | 44 | 40 | 40 | 40 | 45 | 45 | 41 | 8 |
| | Minimum | 27 | 30 | 25 | 30 | 28 | 25 | 35 | 29 | 11 |
| | $P_t$ (%) | 44 | 56 | 40 | 80 | 45 | 57 | 71 | 56 | 24 |
| | $P_s$ (%) | 27 | 35 | 25 | 30 | 25 | 15 | 20 | 25 | 24 |
| 5 Conduit trunking system for other services | Maximum | 21 | 45 | 28 | 40 | 28 | 30 | 55 | 35 | 31 |
| | Likely | 15 | 25 | 15 | 30 | 22 | 25 | 25 | 22 | 23 |
| | Minimum | 11 | 12 | 10 | 15 | 10 | 14 | 11.5 | 12 | 15 |
| | $P_t$ (%) | 60 | 61 | 72 | 40 | 33 | 31 | 69 | 52 | 30 |
| | $P_s$ (%) | 25 | 20 | 28 | 33 | 27 | 22 | 35 | 27 | 19 |
| 6 Power supplies to airconditioning and ventilation | Maximum | 60 | 150 | 130 | 180 | 80 | 80 | 70 | 107 | 40 |
| | Likely | 41 | 71 | 60 | 60 | 45 | 50 | 40 | 52 | 20 |
| | Minimum | 12 | 20 | 11 | 20 | 15 | 12 | 20 | 16 | 25 |
| | $P_t$ (%) | 40 | 61 | 59 | 75 | 54 | 44 | 60 | 56 | 19 |
| | $P_s$ (%) | 30 | 40 | 45 | 42 | 35 | 45 | 30 | 38 | 16 |
| 7 Earthing | Maximum | 5.5 | 16 | 8 | 15 | 5 | 10 | 6 | 9 | 45 |
| | Likely | 3.5 | 7 | 5 | 6 | 3.3 | 5.5 | 4.5 | 5 | 25 |
| | Minimum | 2 | 3 | 2 | 2 | 1.5 | 2 | 2.5 | 2 | 21 |
| | $P_t$ (%) | 57 | 69 | 50 | 69 | 49 | 56 | 43 | 56 | 17 |
| | $P_s$ (%) | 30 | 37 | 40 | 51 | 42 | 30 | 40 | 39 | 18 |
| 8 Labour for fixing items supplied by government | Maximum | 3.5 | 3.3 | 4 | 4 | 7.5 | 6 | 3.5 | 5 | 32 |
| | Likely | 2.8 | 3 | 3.5 | 3 | 3 | 4 | 2.5 | 3 | 15 |
| | Minimum | 1.8 | 2.2 | 2.4 | 2 | 2.5 | 2 | 2 | 2 | 11 |
| | $P_t$ (%) | 41 | 27 | 31 | 50 | 90 | 50 | 67 | 51 | 39 |
| | $P_s$ (%) | 20 | 26 | 29 | 33 | 32 | 25 | 28 | 28 | 15 |
| 9 Testing and commissioning | Maximum | 27 | 19 | 18 | 20 | 18 | 30 | 23 | 22 | 20 |
| | Likely | 16 | 13 | 10 | 10 | 14 | 15 | 10 | 13 | 19 |
| | Minimum | 3 | 4 | 5 | 5 | 4.5 | 5 | 5 | 5 | 16 |
| | $P_t$ (%) | 46 | 40 | 62 | 67 | 30 | 60 | 72 | 54 | 27 |
| | $P_s$ (%) | 30 | 47 | 40 | 40 | 29 | 48 | 40 | 39 | 18 |
| 10 Equipotential and supplementary bonding | Maximum | 39 | 81 | 50 | 72 | 60 | 45 | 70 | 60 | 24 |
| | Likely | 31 | 47 | 35 | 40 | 35 | 35 | 30 | 36 | 15 |
| | Minimum | 22 | 17 | 15 | 15 | 15 | 20 | 20 | 18 | 15 |
| | $P_t$ (%) | 47 | 53 | 43 | 56 | 56 | 40 | 80 | 54 | 23 |
| | $P_s$ (%) | 30 | 31 | 42 | 50 | 29 | 20 | 20 | 32 | 32 |

$P_t$, probability of exceeding the most likely estimate implied by the assumption of a triangular density function; $P_s$, estimator's subjective assessment of the probability of exceeding the most likely estimate.

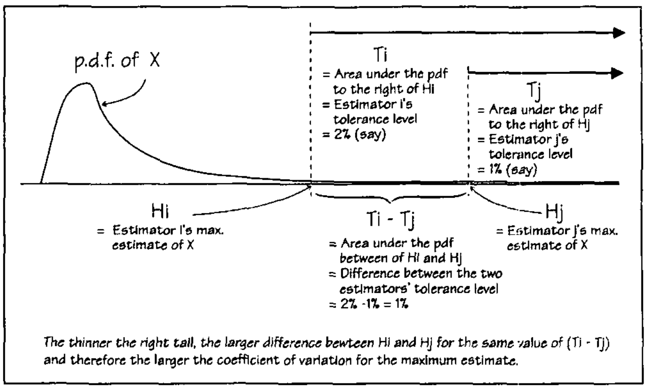


**Figure 1** Diagrammatic illustration of the high variability of the maximum estimate resulting from a long thin tail

assumptions about the underlying populations from which the data are obtained,[13] the results of the test therefore provide stronger evidence than the *t*-test for rejecting the assumption that subsystem costs are triangularly distributed.

The maximum positive values of $b$ for which Ho, $P_t - P_s = b$, is rejected in favour of Ha, $P_t - P_s > b$, at the 0.05 significance level under the *t*-test and the Wilcoxon test for each subsystem are shown respectively in columns 4 and 8 of Table 2. The results range from 3% (subsystem 9) to 21% (subsystem 2) for the *t*-test and 3% (subsystem 9) to 19% (subsystems 2 and 3) for the Wilcoxon test. The wide range is perhaps due to the small sample size. The result for the pooled data is 20%[14] (for both tests) which gives a rough indication about the order of magnitude of the bias resulting from the triangular distribution assumption.

## Modelling input distributions – an initial assessment

To formulate a solution for modelling input distributions based on subjective data is an enormous task. This is not the main purpose of this paper. This section only briefly explores alternative methods of modelling distribution and assesses the potential of these methods in solving the problems.

The major problem lies in the choice of distributions used for modelling construction costs. As construction costs have been shown to be positively skewed, symmetrical distributions are ruled out. The log-normal distribution is one of the many asymmetric distributions that has been used by Touran and Wiser (1992) to model the distribution of construction costs. Touran and Wiser (1992) claim that the log-normal distribution fits historical data better than other well-known statistical distributions such as the normal and beta p.d.f.s.[15] However, the approach may not be useful when little or no objective data is available since the parameters of the log-normal distribution are very difficult to perceive and therefore to directly estimate them subjectively is impossible. The three-point estimates are also not suitable for estimating the parameter as the log-normal distribution does not have an upper bound.

Since very little is known about the true p.d.f. of construction costs, a flexible distribution seems to be a desirable choice. The beta distribution is one of the many flexible distributions that can model positive (and negative) skewness. It has been used by AbouRizk *et al.* (1994) to model the p.d.f. of the duration of repetitive construction activities. It is also recommended by Flanagan and Stevens (1990) for modelling the p.d.f. of construction costs. Similar to the previous approach using the log-normal distribution, these studies estimate the parameters of the beta p.d.f. from objective data. Since the parameters of the beta distribution are very difficult to estimate subjectively, the beta distribution is not suitable for modelling with subjective data.

The three-point estimates are not sufficient for determining the beta distribution which is a four-parameter p.d.f. To estimate the parameters from the three-point estimates using the PERT[16] procedures[17] requires very restrictive assumptions which lack empirical justifications. Furthermore, the most likely estimates often do not coincide with the mode of the estimated beta distribution.

Since the beta distribution is a four-parameter distribution, given the three-point estimates and the $P_s$s, it is in theory possible to solve the parameters of the beta distribution from the following set of equations

$$F(L) = 0 \qquad (2)$$

$$F(M) = P_s \qquad (3)$$

[13] The Wilcoxon test does not rely on the normality assumption.

[14] Since the mean value of $P_s$ is 35–40%, the size of the bias when expressed in relation to $P_s$ is approximately 50–60% which is very high.

[15] This result is cited from an unpublished MS thesis (by E. Wiser). It is therefore difficult to comment on the result.

[16] PERT stands for Program Evaluation and Review Task which is a project management tool developed in the late 1950s by a research team of the US Navy. It is essentially a stochastic extension of the critical path method. See, for example, Battersby (1970) for a more detailed discussion on this topic.

[17] The PERT procedure approximates the mean and variance of the beta p.d.f.s of the construction duration from the subjective three-point estimates using the following formulas

$$\mu = \frac{L - 4M + H}{6}$$

$$\sigma = \frac{H - L}{6}$$

where $L$, $M$ and $H$ are, respectively, the minimum, most likely and maximum estimates and $\mu$ and $\sigma$ are the mean and standard deviation.

**Table 2** Results of statistical tests

| | | Mean $P_t - P_s$(%) | *t*-test: *t*-statistic | *t*-test: Significance level (%)[a] | *t*-test: $b^c$ (%) | Wilcoxon signed-rank test: $S+$ | Wilcoxon: Sample size | Wilcoxon: Significance level (%)[a] | Wilcoxon: $b^d$ (%) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | LV switchboard | 25 | 2.58 | <5 | 6 | 28 | 7 | <1 | 4 |
| 2 | Main and submain distribution | 34 | 5.05 | <1 | 21 | 28 | 7 | <1 | 19 |
| 3 | General final circuit and equipment | 27 | 6.70 | <1 | 19 | 28 | 7 | <1 | 19 |
| 4 | Special final circuit and equipment | 31 | 4.69 | <1 | 18 | 28 | 7 | <1 | 18 |
| 5 | Conduit/trunking system for other services | 25 | 3.65 | <1 | 12 | 28 | 7 | <1 | 8 |
| 6 | Power supplies to airconditioning and ventilation | 18 | 3.74 | <1 | 9 | 27 | 7 | <5 | 10 |
| 7 | Earthing | 18 | 3.79 | <1 | 9 | 28 | 7 | <1 | 7 |
| 8 | Labour for fixing items supplied by government | 23 | 2.85 | <5 | 7 | 28 | 7 | <1 | 9 |
| 9 | Testing and commissioning | 15 | 2.55 | <5 | 3 | 26 | 7 | <5 | 3 |
| 10 | Equipotential and supplementary bonding | 22 | 2.80 | <5 | 7 | 28 | 7 | <1 | 9 |
| | Pooled data (all observations) | 24 | 3.79 | <1 | 20 | 7.17[b] | 70 | <1 | 20 |

$S+$, the sum of ranks associated with positive $(P_t - P_s)$s.
[a] Level of significance at which Ho, $P_t - P_s = 0$, is rejected in favour of Ha, $P_t - P_s > 0$.
[b] $Z$ value for large sample approximation.
[c] Maximum value of $P_t - P_s$ which still rejects Ho at significance level 0.05.
[d] Maximum value of $P_t - P_s$ which still rejects Ho at significance level 0.05.

$$F(H) = 1 \tag{4}$$

$$f'(M) = 0 \tag{5}$$

where $F(x)$ is the distribution function, $f'(x)$ is the derivative of $f(x)$ (which is the p.d.f.) with respect to $x$ and $L$, $M$ and $H$ are respectively the minimum, most likely and maximum estimates. The last equation follows from the fact that $M$ is the turning point of the p.d.f.[18] This method is superior to the PERT procedure since the most likely estimate coincides with the mode of the estimated p.d.f. Moreover, the estimated p.d.f. is also consistent with the estimator's perception about the risk exposure of the most likely estimate. The obvious problem of the approach is the tedious and sophisticated computation involved. Another problem is that a solution may not exist or even if it does exist, the estimated p.d.f. may not be unimodal.

It is also important to note that the method need not be confined to the estimation of the beta distribution. In theory, the p.d.f. to be estimated can be any flexible four-parameter distribution. Therefore, despite the difficulties mentioned above, the author believes that this approach has the potential to provide a theoretical solution to the problem although the approach may, at the present stage, be too complicated for practitioners to implement.

[18] We also want $f(M)$ to be the only maxima of $f(x)$ since this guarantees that $f(x)$ is a unimodal p.d.f. which corresponds to our present knowledge about the p.d.f. of construction costs. Therefore, the number of solutions to Equation 5 and the sign of the second derivative of $f(x)$ should also be checked.

## Conclusion

The validity of the assumption of the triangular distribution for subsystem unit costs in Monte Carlo simulation has never been seriously investigated. There are two reasons for this. First, the assumption is convenient and makes life much easier. If refuted, it would be very difficult to find another equally convenient but valid substitute. The second reason, which the author believes to be more important, is the contention that any possible error that may be introduced by adopting the assumption is insignificant compared with the error inherent in the subjective data which is by no means precise. Obviously the first reason cannot be sustained, as a convenient but invalid assumption that can lead to bias in the results cannot be justified. The second reason can only be sustained if the magnitude of the error resulting from the assumption is small and/or non-systematic (i.e. neither biased upward nor downward).

However, the results from this empirical study show that the error introduced by the assumption is in fact systematic. It will therefore result in an upward bias in the probability of exceeding the most likely estimate.[19] The magnitude of such a bias is, on average, approximately 20 percentage points (or 50%–60% of $P_s$) which is by no means small, the implication being that the simulation result under such an assumption is no longer reliable.[20] The triangular distribution assumption is therefore invalid and should not be used in its present form.

Although the results reported in this paper have been generated using electrical services subcontracts of government clinics, the author believes that the results can be generalized to all construction projects on two grounds. First, ESGCs are part of the whole construction process which also involves site operations. They both share many similarities such as method of procurement, contract structure, organization of production and method of price determination, etc., which partly determine the nature of the distribution of their construction costs. Second, the major difference between a building construction contract and an ESGC contract is that the latter is priced based on performance specification rather than production specification. The performance specification will lead to a longer tail to the left (i.e. reduces the rightward skewness) due to varying assumptions about the lowest acceptable quality made by different estimators. This will reduce the rightward skewness of the cost distribution and, thus, in turn reduces the possible bias that is brought about by the triangular distribution assumption.[21] Therefore, if results from ESGC data show that a bias exists, the bias will be even larger for other types of construction work.

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[19] This is the same as the risk exposure of the single figure estimate in Newton (1992). It is also a useful and important indicator for decision making in practice.

[20] The bias in the simulation result will be even larger than the bias in the individual subsystem under the independence assumption if this assumption is in fact invalid. See Chau (1993) for a more detailed discussion.

[21] The upward bias in the probability of exceeding the most likely estimate under the triangular distribution assumption is a consequence of the asymmetric nature (i.e. a comparative much longer right-hand tail) of the underlying distribution. The longer the left-hand tail, the more symmetrical the distribution becomes and therefore the less the resulting bias.

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