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Simulation of expenditure patterns of construction projects

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The existing models for project budgeting and forecasting are either alienating to the user, or involve a laborious process of preparation. This is mainly due to the structure of the model, which itself is determined by the approach adopted in developing the model.

In this paper, a new approach to model development based on the analysis and examination of the shape of the pattern of project expenditure is described and contrasted with the approaches that have been adopted so far. Also, the procedure for the generation of a model based on the new approach to model development is outlined.

This is an attempt to overcome many of the shortcomings associated with the current models, by building on the many advantageous features of these models. It is anticipated that the approach adopted and the accompanying mathematical model can facilitate the provision of a balanced combination of ease of application, user involvement and user comprehension.

Keywords: Project financial management, cashflow forecasting, mathematical modelling, growth curves.

Overall objectives

This paper reflects part of a broader research programme recently completed by the author. The objectives underlined below relate to the overall research programme. It consists of exploring the nature of construction projects to establish the factors which govern the pattern of expenditure.

The process involves identification of the major factors which affect the pattern of project expenditure and separate one project from another, hence establishing categories of project expenditure profile. Following quantification of the variables which describe the categories, the quantified variables are then translated to the parameters of the mathematical model. Hence the characteristics of the categories can be interpreted in terms of the parameters of the mathematical model. The categorization is based on observational and statistical analysis of 480 construction projects of varying type and characteristics.

However, it is not the aim of this paper to discuss the nature of the categories of project expen-diture, nor is it intended to describe the means and process by which the categories are quantified.

Introduction

Basically, modelling is a conversion process which attempts to imitate the real situation for a given input. A mathematical model is one which has representation in the form of a mathematical expression.

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By its nature, incurred expenditure over the project period falls into the category of growth curves, which in cumulative terms assumes the familiar 'S'-shaped curve. Such curves can be generated by means of a variety of models.

There has been a predilection for use of mathematical modelling for budgeting, forecasting and monitoring construction project expenditure for the past quarter of a century. During this period, three generations of model development have taken place in the UK.* The need to advance the knowledge for practical use of such models has long been recognized by both academics and practitioners. The current practice is flawed by various shortcomings. The traditional method of project expenditure budgeting is time-consuming, costly and subject to cumulative error. Furthermore, it is a prerequisite that the project programme must be established before a budget can be determined. The alternative approach, as well as having a limited range of application and lacking flexibility, often provides no means for the user to comprehend the logic of prediction, hence being alienating to the user. In instances where the user is required to provide input, an extensive knowledge, experience and information is demanded from the user.

It is envisaged that one way by which the current models can be analysed, in an attempt to overcome some of their shortcomings, is to study and analyse their structure, with the view of classifying the approaches adopted in developing the model.

Initially, in this paper, a classification of the current models has been carried out with a fresh perspective. Here, attention is given to the nature of the approaches to the development of the model, with specific reference to the means by which the dependent variables of the project are related to the mathematical expression. In other words, before a model can simulate an expenditure pattern in its analogue, a link between the variables of the project and the parameters of the mathematical expression must be established. The establishment of this link relies on the assumption that the attributes of a construction project expenditure are contained within the pertinent raw data. Based on this argument, a new approach to model development has been proposed and described.

In the latter part of the paper, the procedures involved in the development of a unique mathematical model, the configuration of which is compatible with the defined structure of the new approach to model development, has been described.

Approaches to model development

Based on the virtue of the proposed approach, the current models were analysed and, as a result, three classes of approach to establishing the aforementioned link between the data and the mathematical model have been identified.†

In-built parameter criteria

The term 'in-built parameter' criteria is intended to imply that the model is the product of the nature of the data and the criteria by which the parameters of the model are related to the

* The three generations mentioned have been identified for building cost modelling (Raftery, 1986; Brandon and Newton, 1986), but it can be argued that the same is applicable to the models for project budgeting and forecasting.
† The models tested are described in detail by Khosrowshahi (1986). These models include the DHSS model, Peer (1982), the models of Berny, Howes, Preece and Bains (four models), and that of Keller-Singh, Cusack and Bennett. The latter is not included in the classification.

characteristics contained in the data are built into the model itself. This method has been the most popular approach for developing a mathematical model. It involves the use of statistical techniques (normally regression analysis) which, in effect, identify the significant variables and interpret them accordingly. This method is sometimes referred to as the 'black box' method, which is criticized for it. This is because it tends to deprive the user from appreciating the logic of the exercise, as no opportunity exists for the user's interactive participation.

Independent parameter criteria

This approach assumes little or no link between the data and the parameters of the mathematical model prior to the development of the model. Hence, the process of identifying the criteria for establishing the aforesaid link, commences after the model has been developed.

Also, in this approach, the parameters of the mathematical model have no real meaning. However, the extreme dependency of the parameters on the data, characterized by the first approach, is eliminated, and the model is developed and used independently from the data. It is due to this total separation of the model from the subject, that there is a need to establish an artificial relationship between the two. This relies on the use of a large quantity of data; however, there can be little guarantee that the model has the mathematical adequacy to reflect all the conditions required of it.

No parameters criteria

The result of this approach is the product of an extensive investigation of the specifications of each individual project. It relies, significantly, on the elemental arrangement of the cost attributes. Therefore, it consists of laboriously identifying all the cost constituents (elemental or based on arbitrary cost centres), their quantity and rate, and the order of the construction sequence of these cost components. Therefore, there exists no criterion other than the information contained within the bill of quantities and the schedule of work.

In contrast with the 'black box' type of approach, the major advantage of this method is its meaningfulness and high degree of user comprehension, whereby the user can fully appreciate the logic of estimating and forecasting.

The proposed approach

The new approach suggested in this paper and adopted in my research is based on the assertion that the 'shape' of the expenditure profile of a construction project, expresses the characteristics associated with physical properties of the project. Some of these characteristics are common to all construction projects.

Basically, the pattern of expenditure can be expressed in terms of a shape, which can also be generated using a mathematical model. It will be shown that the 'shape' can be expressed in terms of certain independent variates to which both the project-dependent variables and the parameters of the model can be related. This implies that the mathematical model which is required to be adopted should, on the one hand, be of an independent nature and, on the other, be capable of being related to both the general and specific characteristics of the construction project expenditure pattern.

It is believed that this approach facilitates, to some extent, the combination of advantageous features of the earlier approaches. Therefore, it has the data-related nature of the first approach, parameter independence of the second approach and, to a degree, the logical input feature of the third approach. Furthermore, the parameters have real, rather than artificial meaning.

Figure 1 reflects the refining process, before the historical data relating to construction project expenditure are transformed into categories of expenditure profile (Khosrowshahi, 1985). The stages of the data processing include normalization of the raw data and smoothing of the normalized data (data for 480 projects of several types and categories have been collected and processed: Khosrowshahi, 1985). Figure 2 displays a normalized and smoothed curve of an office project, and illustrates that during this process the major variables relating to the project are classified, coded and recorded. This has been carried out for approximately 480 projects.

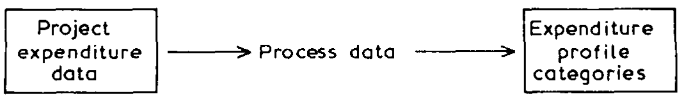


Fig. 1. Processing of project expenditure data.

Figure 3 shows that for a given parametric specification, an expenditure profile can be generated by the mathematical model. The process of identification of the criteria is to establish a link between Figs 1 and 3, which is shown in Fig. 4.

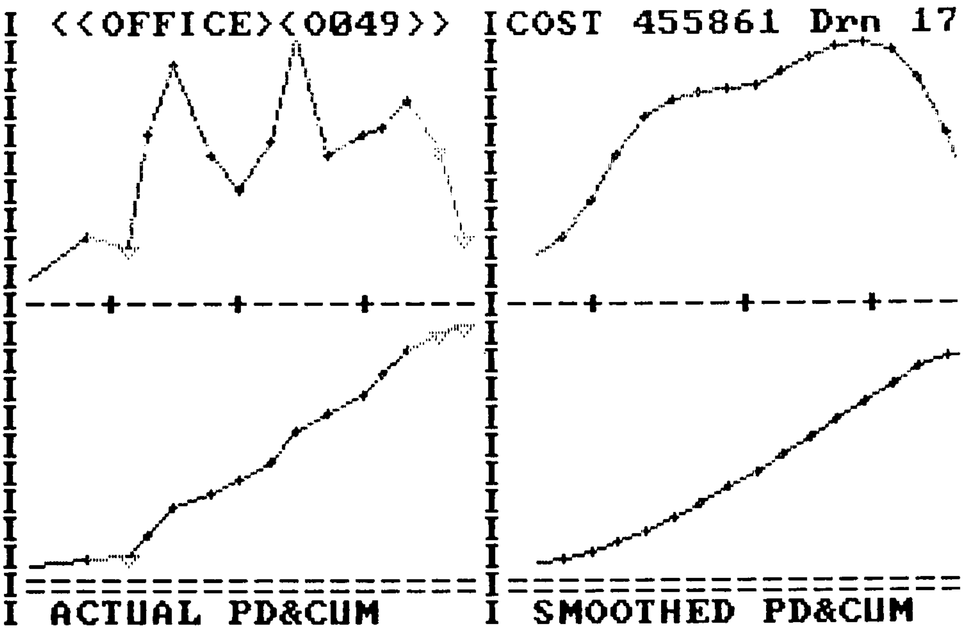
The criteria

In order to establish the criteria, the general and the specific characteristics of construction project expenditure should be identified and expressed in a quantified form.

General characteristics (properties)

The general properties of project expenditure patterns are those which are common for all construction projects. It is due to the presence of these general characteristics that some mathematical models, despite their limitations, produce results with practical uses. The general properties include the following:

1. *Non-presence of negative values.*
2. *Periodic values are discrete.* The 'S' curve is a continuous curve and represents the cumulative expenditure at any point in the project life. The periodic values representation consists of a set of discrete points forming a pattern which resembles the rate of change of expenditure, i.e. the differentiated form of the cumulative 'S' curve.
3. *Initial and end values.* It is assumed that at time zero (one period before the start of the project), the value is zero and no payment is made. Similarly, the end value (one period after final completion) is assumed to be zero. The final completion is considered as the absolute end of the project and the payments after that are normalized and dissolved in the flow of project expenditure prior to final completion.



PROJECT SPEC. ═══> S:M:O:—.—. NC. N

COST × 1000: =>	455 K	DURATION (M) =>	17
START DATE : =>	OC 83	PERIOD NO : =>	14
SOURCE DATA =>	QH	FLUCTUATION: =>	F
TIME FACTOR =>	1.02	LOCATION : =>	0.9
Mangt. CODE =>	T	Struc. CODE : =>	SC
Hor. ACCESS =>	G	Vert. ACCESS: =>	P
GROUND Cond =>	G	NO OF STOREY=>	2
COMPLEXITY : ═══>		OF CONST'N : =>	S
: ═══>		Build. TYPE : =>	41

SLOPES = Initial: (1.00) End : (2.00)

	NO:	TIME% (1) (2)	COST% (1)(2)	CUMCOST% (1) (2)
PEAKS: =>	(1)	76, 0	9, 0	77, 0
TROUGH =>	(0)	0' 0	0, 0	0, 0

SUPPRESSION ═══> TOTAL 0 LARGEST at 0

= % TO TIME =>	0	0
= % CUMCOST=>	—	0

G = good F = fair P = poor S = simple C = complex
F = fair B = brick S = steel C = concrete

Fig. 2. Normalized and smoothed project expenditure pattern and description.

Fig. 3. Simulation using a mathematical model.

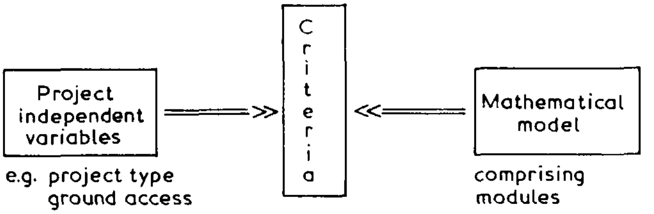


Fig. 4. Relationships between the independent variables and the mathematical model.

4. *Two-phased monotonic nature of the periodic pattern.* It is accepted universally and is also evident from the data that the cumulative expenditure profile of a construction project can be expressed by a growth curve. The latter, when transformed into a periodic pattern, consists mainly of two parts, separated at the point of inflection. The first part represents the growth and consists of the curve moving away from its lower bound, and in the second part (i.e. the decay part) the curve moves towards the upper bound. This indicates that the pattern of expenditure, as well as being smooth, is mainly formed by a two-phased monotonic curve. The first is a monotonic-increasing phase and the second is a monotonic-decreasing phase.

Specific characteristics (properties)

Through observation of the data and the shape of the periodic expenditure profile, certain characteristics were identified as being significant in distinguishing one category of project from another. Further, this conclusion is validated through statistical analysis of the data by carrying out statistical methods such as principal component analysis. The specific characteristics are:

1. The co-ordinates of the peak period on the graph representing the periodic expenditure *vs* time.
2. The position and intensity of any possible distortions which cause troughs and minor peaks on the periodic expenditure pattern.
3. The measure of the overall rate of growth commencing from time zero to the peak point.
4. The measure of the overall rate of decay commencing from the peak point to the end of the project.

The mathematical model

Due to the nature and structure of the current models, a fundamental inadequacy exists in the ability of their mathematical function to satisfy the identified conditions. Furthermore, in view of the complexity involved in incorporating all conditions into one expression, the problem has to be subdivided into smaller modules, each of which satisfies a particular task in simulating the overall expenditure curve. These modules are listed below, and are discussed in more detail later.

1. *Control module.* As explained earlier, an analysis of the data reveals that as far as the periodic pattern of expenditure is concerned, behind all fluctuations in the monthly values lies an underlying pattern which reflects the major general properties of the pattern. It is intended that these properties are generated and controlled by the control module. This consists of a single smoothed, continuous, two-phased monotonic curve, which also satisfies the requirements relating to the co-ordinates of the peak point (xp and yp) on the periodic value *vs* time curve. The control module also meets the boundary conditions specified by the initial and end values as well as the total project cost.
2. *Distortion module.* This consists of one or more curves, facilitating simulation of interferences over the underlying trend. The latter tends to be distorted as a result of the application of such events.
3. *Kurtosis module.* This is made up of two independent curves, the role of which is to make adjustments for the initial and end slopes of the curve, and to facilitate reflection of the measure of concavity/convexity of the first and the last parts of the curve.

Control module

The role of the control module is to generate the underlying pattern of expenditure, configured mainly by the position of the peak period on both the time axis and the value axis, as well as complying with the boundary conditions. It maintains control over these properties, in the face of application of other conditions.

In order to avoid a possible re-invention of the wheel, a comprehensive investigation was carried out to test the existing mathematical expressions which have the potential of possessing the properties of a growth curve. The expressions examined consist of sigmoids (Stone, 1980), including both continuous and discrete probability distribution functions (Hastings, 1975). They include the following expressions: logistic, log-logistic, log-normal, exponential, Rayleigh, Weibull, gamma, Gompers, binomial, Poisson, etc. (Khosrowshahi, 1986).

As a result of testing, it was established that none of the expressions satisfied the stated requirements. Nevertheless, this study proved informative and has led to further developments. A common problem associated with all existing expressions was that none could satisfy the conditions relating to the peak point. In other words, no two parameters/expressions can be identified as having independent control over the positions of xp and yp. For the majority of cases, it appeared that this was due to the structure of the expression.

Therefore, by this stage, it was inevitable that a new mathematical expression, based on the conditions set by the aforementioned criteria, was required.

Mathematical model development

Two separate attempts were made from first principles to construct the required mathematical expression for the control module. The first is based on a polynomial equation and the second on an exponential function. The exponential expression was developed as an alternative to the polynomial function, for the latter lacks the necessary flexibility, and poses many restrictions.

Polynomial

Two polynomial expressions were developed and tested. One which is developed from first principles is described here. The equation for the periodic curve $f(x)$ expressing values as a proportion of the total value at any point in time x (ratio of period to total number of periods n) can be expressed in terms of the following polynomial equation:

$$f(x)=a_0+a_1x^1+a_2x^2+a_3x^3+\cdots+a_tx^t$$

where coefficients $a_0, a_1, a_2 \ldots a_t$, are real numbers.

Properties of the expression

$$f(0)=0 \xrightarrow{\text{gives}} a_0=0 \qquad \text{initial value is zero}$$

$$f(1)=0 \longrightarrow a_1+a_2+a_3+a_4+\cdots=0 \qquad \text{end value is zero}$$

Let R be the time of the peak point. But at peak point $f'(x)=0$:

$$f'(R)=0 \longrightarrow a_1+2a_2R^1+3a_3R^2+4a_4R^3+\cdots+ta_tR^{t-1}=0 \qquad (R=Xp/n)$$

Let (Q) be the value of the peak point. But at peak point $f(x)=Q$;

$$f(R)=Q \longrightarrow a_1R+a_2R+a_3R+a_4R+\cdots+a_tR=Q \qquad (Q=Yp/C)$$

Also, for the total value condition (total discrete values sum to unity):

$$\operatorname*{sum}_{x=0}^{x=1} f(x)=1$$

Therefore:

$$a_1K+a_2L+a_3M+a_4N+\cdots=1$$

where

$$K=n!/n, \quad L=(n)^2!/n^2, \quad M=(n)^3!/n^3, \quad N=(n)^4!/n^4, \ldots$$

The solution to the simultaneous equation constructed from the above equations determined that the polynomial equation is of the 4th degree. The expressions for the coefficients were calculated to be as follows.

$$a_4 = \text{top}/\text{bot}$$

where

$$\begin{aligned}\text{top} = {} & R^3(K-L)+2R^2(L-K)+(K-L)+3Q(L-K^2)+(Q/R)(L^2+ML-MK+LK) \\ & +2Q(-K^2-ML+MK+LK)\end{aligned}$$

$$\begin{aligned}\text{bot} = {} & R(-R^4(L-K^2)+2R^3(K^2-LK+LM-MK) \\ & +R^2(3L^2-K^2-2LK-LN+KN-3ML+3KM)\ +2R(-2L^2+LN-KN+LK) \\ & +(NK+ML-MK-LN)\end{aligned}$$

$$a_3 = (1-2R)-a_4(4R^3(L-K)+2R(K-N))+(N-L)/3LR^2(L-K)+2R(K-M)+(M-L)$$

$$a_2 = 1-a_3(M-K)+a_4(N-K)/L-K$$

$$a_1 = -(a_2+a_3+a_4)$$

The polynomial expression satisfies all but two conditions; namely, the nullification of negative values and the generation of the monotonic curve. Judging by the nature of polynomial equations, this was not unexpected. However, it was foreseen that a practical range for the value of the parameter R and Q can be identified, within which a viable solution can be obtained. This range does exist (see Fig. 5) and was found for the values of Q to be between 0.13 and 0.18, which satisfies a range for R from 0.3 to 0.7. However, in practice, this expression is too limited for use. Not only is the practical range of application too narrow, but as the total number of periods (n) changes, this range varies. The latter may be solved by identifying the logic of these changes. The former, however, cannot be remedied, for it is evident from the data that the required ranges, given below, are in relative terms broad:

for Q (Yp/C) from 0.04 to 0.22 with mean 0.1

for R (Xp/n) from 0.21 to 0.83 with mean 0.496

Exponential

The alternative approach to polynomial equations is the exponential function (Rektorys, 1969). An exponential function typically assumes a process for situations where the rate of growth is proportional to the state of growth and where each value can be expressed as a constant percentage of the neighbouring value. Therefore, exponential equations tend to comply with the laws of growth, and can produce monotonic, smooth and continuous curves with positive values.

Several *ad hoc* attempts were made before a trend was identified, whereby a step-by-step procedure towards development of the desired expression was made possible. A study of the expressions similar to the exponential and Rayleigh probability density function (PDF) revealed that for an expression to satisfy the peak point conditions, there should exist at least

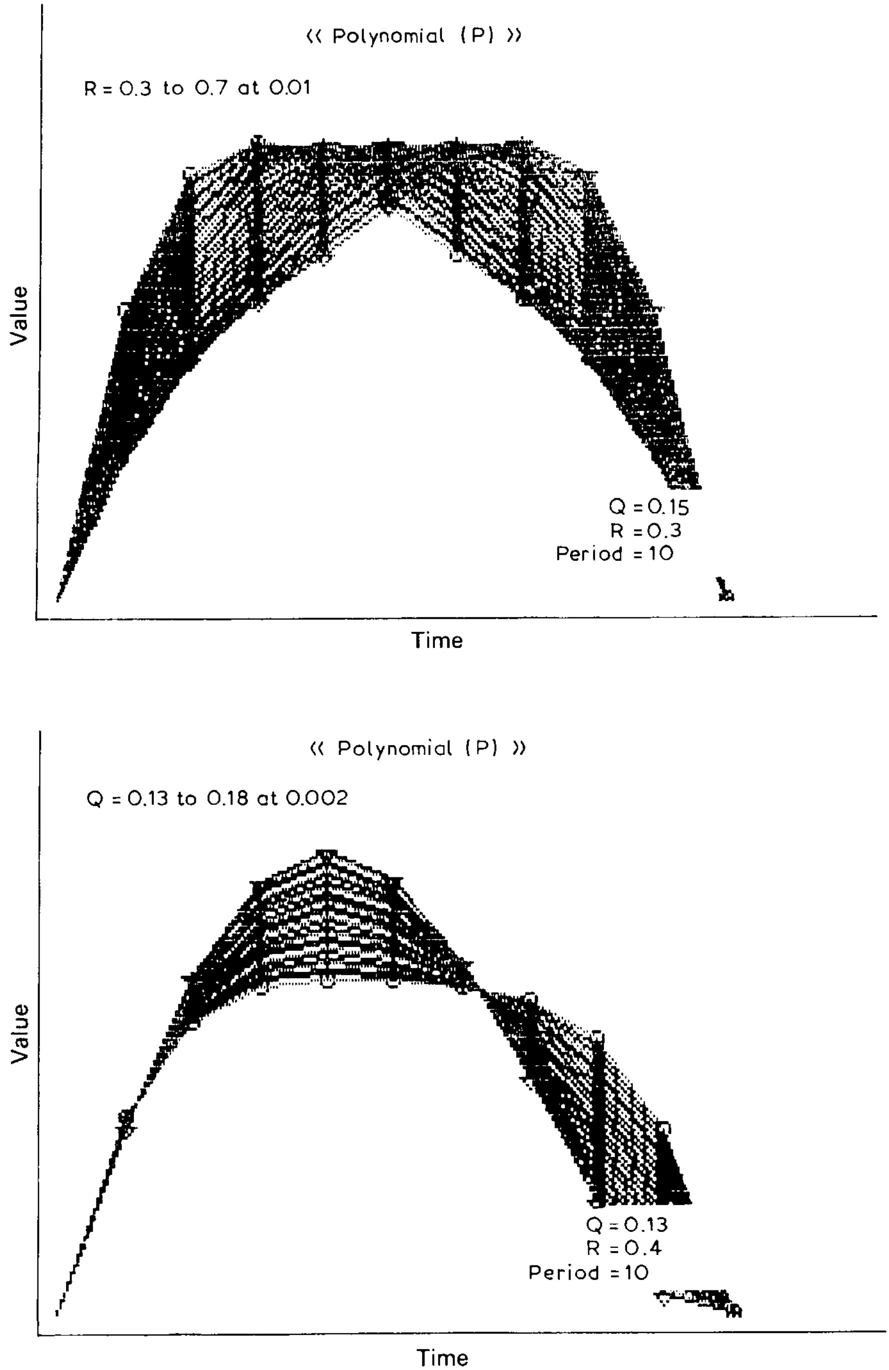


Fig. 5. Polynomial expression: Workable range for parameters of the model. Q, Peak value; R, peak time.

two parameters in the expression whereby the expressions representing xp and yp can be independent of one another:

$$\text{exponential PDF} \longrightarrow y = 1/a\, e^{-x/a} \qquad \text{no turning point}$$

$$\text{Rayleigh PDF} \longrightarrow y = ax\, e^{-a(x)^2/2} \qquad xp = 1/\sqrt{a}, \quad yp = \sqrt{a}$$

Both exponential and Rayleigh PDFs can be deduced mathematically from a more general expression, namely Weibull PDF. The latter has two parameters. The equation is:

$$y = dx^{d-1}(1/a^d)e^{-(x/a)^d}$$

where d is denoted as a shape parameter and a is denoted as a scale parameter.

The analysis of Weibull PDF indicates that with some verification, independent expressions for the evaluation of xp and yp can be established. However, there is a need for an additional requirement to meet the condition relating to the final cost of the project, i.e. the total sum of all discrete periodic values must be equal to the final cost of the project (unity in the case of the expression). To tackle this problem, it is necessary to increase the number of parameters to three. The introduction of the third parameter increases the complexity of the expression. It was envisaged that production of a model for simulation of periodic, rather than cumulative, expenditure pattern would reduce the complexity. This is primarily because of the condition relating to the total value.

Also at this stage, further adjustments were required in order to cater for the conditions relating to the initial and the end values both being zero, i.e.:

$$x = 0 \longrightarrow y = 0 \qquad (x = X/n) \qquad \text{(A)}$$

$$x = 1 \longrightarrow y = 0 \qquad (y = Y/C) \qquad \text{(B)}$$

where Y is the actual periodic value at a given period X.

The equation $y = e^{ax^b} - 1$ was evolved as a simplified general form of the required equation which complies with condition A. In order to fulfil condition B, a modification/expansion of the equation was necessary. This expansion was achieved by the introduction of the additional term $(1-x)$:

$$y = C(e^{ax^b(1-x)} - 1)$$

where C is the total value.

The aforementioned third parameter d is thus allocated to the new term as a power. Therefore, the new equation, i.e. the final version, is:

$$y = C(e^{ax^b(1-x)^d} - 1) \tag{1}$$

Tests show that the influence of the term $(1-x)^d$ on the overall sensitivity of the curve is relatively milder than that by the x^b term. Therefore, the parameter d was considered more appropriate for the total value adjustment.

Peak point co-ordinate conditions

Having satisfied the conditions relating to the initial and the end values, the crucial requirement was then to test the new expression's ability to produce independently a controllable parameter/expression with necessary flexibility and sensitivity to evaluate xp and yp. This task was accomplished and the following expressions were derived:

$$a = dR/(1-R)$$

$$b = \log(1+Q)/[R^a(1-R)^d]$$

Here, a and b are the parameters of the expression. For given values of d, R and Q, the values of a and b can be calculated.

Therefore, equation (1) is capable of satisfying all but one condition required of the control module. This is demonstrated in Fig. 6, which illustrates that while the curves are all smoothed and continuous and $f(0)=0$ and $f(1)=0$, full control over the exact positions of both xp and yp is achieved, which is implemented by R and Q respectively. The missing part relates to the total value condition.

Total value adjustment

It was concluded from the examination of the expression that the final adjustment for the total project value cannot be defined directly by means of a mathematical expression, partly due to the fact that exponential equations of the form

$$y = \exp(m^m)$$

cannot be integrated with respect to m by methods of the fundamental theorem of calculus, and only through numerical methods problems of this nature can be tackled. Therefore, the identification of the best value for the parameter d, whereby the summation of all discrete points become unity, was based on the iteration technique.

The iteration process was initially based on the Newton Raphson method, but due to some practical difficulties encountered, an improvised computerized iteration technique was devised. The technique consists of two major stages. Stage 1 relies on the proven assertion that the relationship between d and project total sum can be roughly expressed by a polynomial function. Therefore, initially, five pairs of known values are used to construct the polynomial equation. The latter is then used to find the approximate value of d. In stage 2, an effective iteration process is carried out to evaluate the exact value of d.

Distortion module

The underlying pattern of expenditure was described as consisting of a two-phased increasing and decreasing monotonic curve. Assuming that the rate of expenditure also represents the rate of production, should an unexpected change appear in the rate of production, the curve is disturbed. Depending on the intensity of the disturbance, a minor peak and a trough may be formed. Observing the pattern of expenditure relating to the 480 projects reveals that while there exists an underlying pattern, frequently that pattern contains minor peaks and/or troughs.

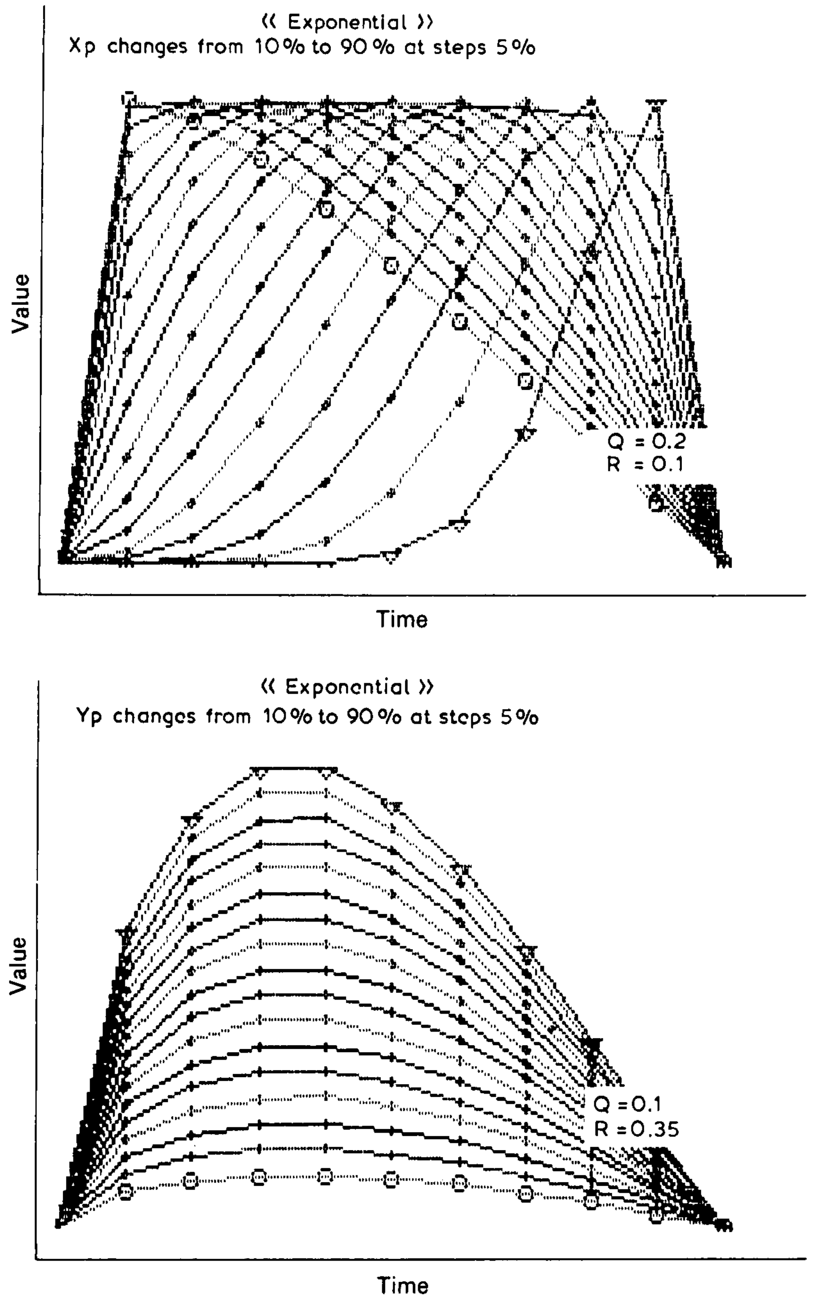


Fig. 6. Exponential expression. Q, Peak value; R, peak time.

Examination of the data indicates that the distortions can be classified into broad and local distortions (see Fig. 7). The former most often consists of those, the span of which runs between two major peak points on the periodic values curve $f(x)$. The locally applied distortions, many of which appear at the vicinity of the Christmas holidays, tend to exert

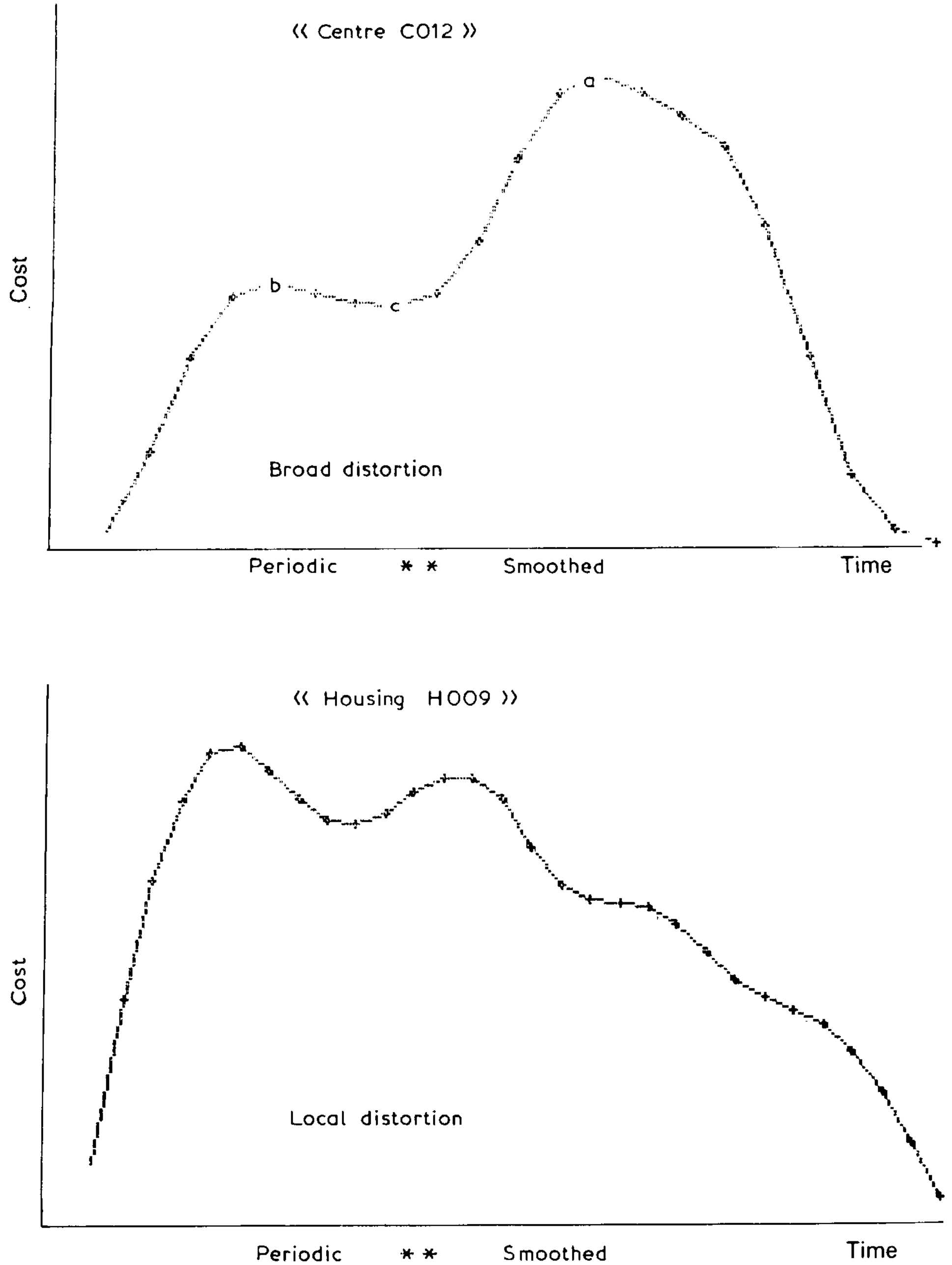


Fig. 7. Broad and local distortions.

local influence on the curve and in quantified terms they can be classified into only a few categories.

Both types of distortions are measured from the data in the same way, and are simulated on the overall curve in the same way. The measurement is based on identifying the total value required to restore the curve back to its pre-distorted state. The adjustments for the distortions are made by means of locally superimposed curves starting at a certain period and lasting for a certain duration with a given intensity. The latter is based on the calculation of the total value to be compensated due to the application of the distortion.

The measurement of the properties of the distortions from the data is aimed at producing a database of variables relating to the identified distortion. It consists of recording, from the normalized and smoothed data, the central point of application of the distortion (i.e. the co-ordinates of the trough), the duration of application (i.e. the number of periods between two successive peaks) and the intensity of the distortion. The first two are identified and recorded directly from the data. The calculation of the intensity, however, involves a complex process of constructing an exponential curve (a, b, c), the origin and end point of which are the co-ordinates of the two peaks (a and b), and its only point of inflection is the trough (c) enclosed between the two peaks (Fig. 7).

The simulation of the distortion curve on the overall curve is carried out by superimposing a 4th degree polynomial curve over the control module. The polynomial curve is applied at a given point, spanning over a given period. The total sum of discrete points on the curve is equal to the intensity of the distortion.

Depending on the nature of the distortion and its effect on the rate of production, the distortion can be accelerating or retarding. The former expresses an increase in production rate, whereas the latter indicates otherwise. Figure 8 illustrates the effect of distortion module curves 1 and 2 on the control module curve 3.

Kurtosis module

The measure of peakedness of a curve is known as kurtosis. It can be meso-kurtic, platy-kurtic or lepto-kurtic for moderate, flat or peaked curves respectively. Figure 9a contrasts between a meso-kurtic and a lepto-kurtic curve.

For a given total project cost, the general increase and decrease in the rate of expenditure, in the first and second parts of the control module, are determined by the peak value (yp), the number of periods (n) and the peak value period (xp). However, for a given xp, yp, n and total project cost, the acceleration of the initial portion of the curve towards the peak and the retardation of the end portion of the curve towards the end of the project, vary from project to project. These variations are measured from the data and recorded with the intention that they are classified so that they can be simulated and superimposed over the overall curve.

The measurement for the initial portion of the curve consists of calculating the slope of the periodic expenditure curve at the origin (i.e. time zero: $x=0$). This slope is measured by constructing a quadratic equation $f(x)$, using as many points as possible on that portion of the curve, based on least-square polynomial fit. The slope is then calculated by setting $x=0$ in the equation of the derivative of the $f(x)$.

As well as the initial slope, the concavity (or convexity) of the portion has been measured which is referred to as the intensity of expenditure. This intensity is the total sum of values from the origin to the peak point, measured as a proportion of total project value. Regarding the end portion of the curve, the same process is carried out in the reverse order.

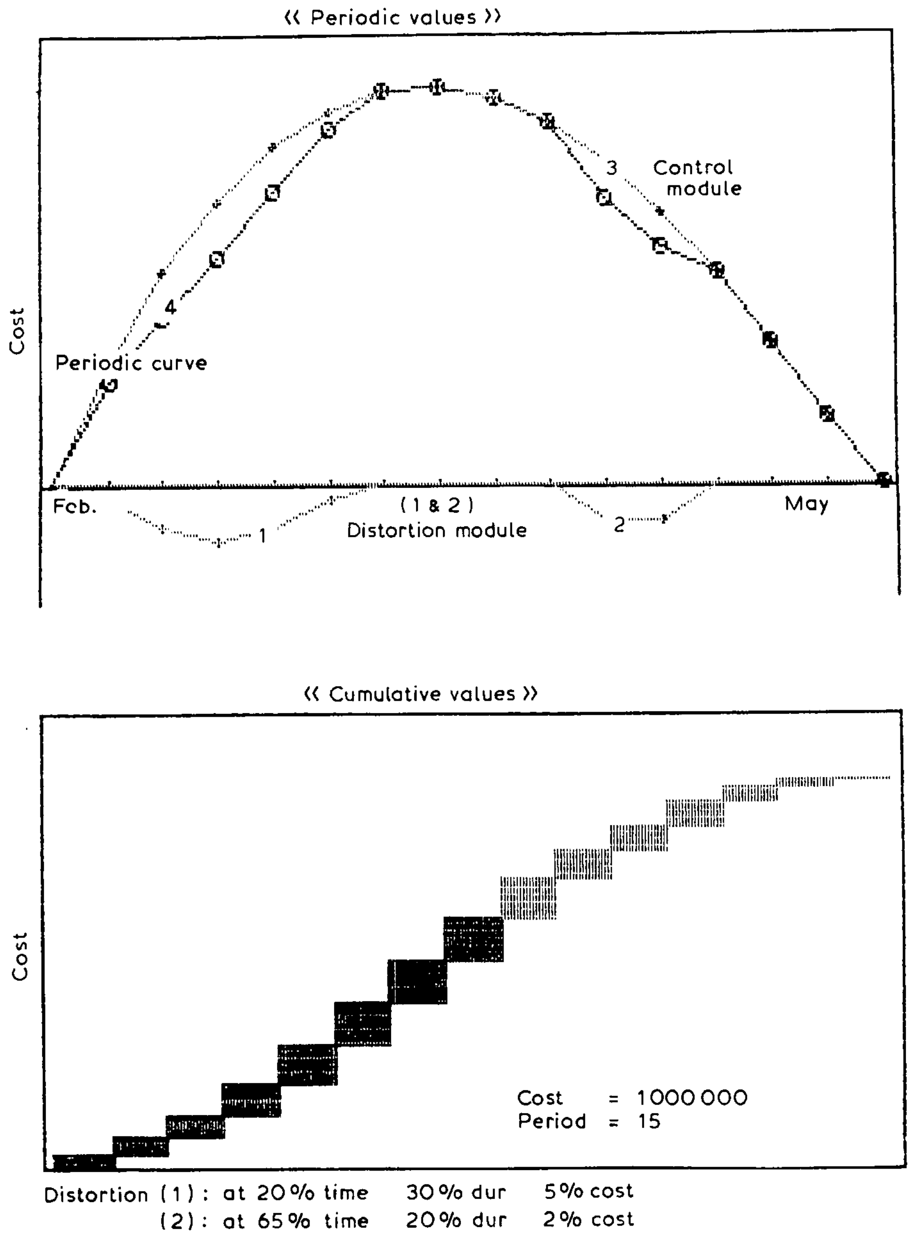


Fig. 8. The effect of the distortion module on the overall curve.

Figure 9b illustrates that the reflection of the kurtosis model on the overall curve is by superimposition of a 4th degree polynomial curve which has the following properties:

1. The initial slope of the curve and the total sum of the discrete points on the curve are controllable.

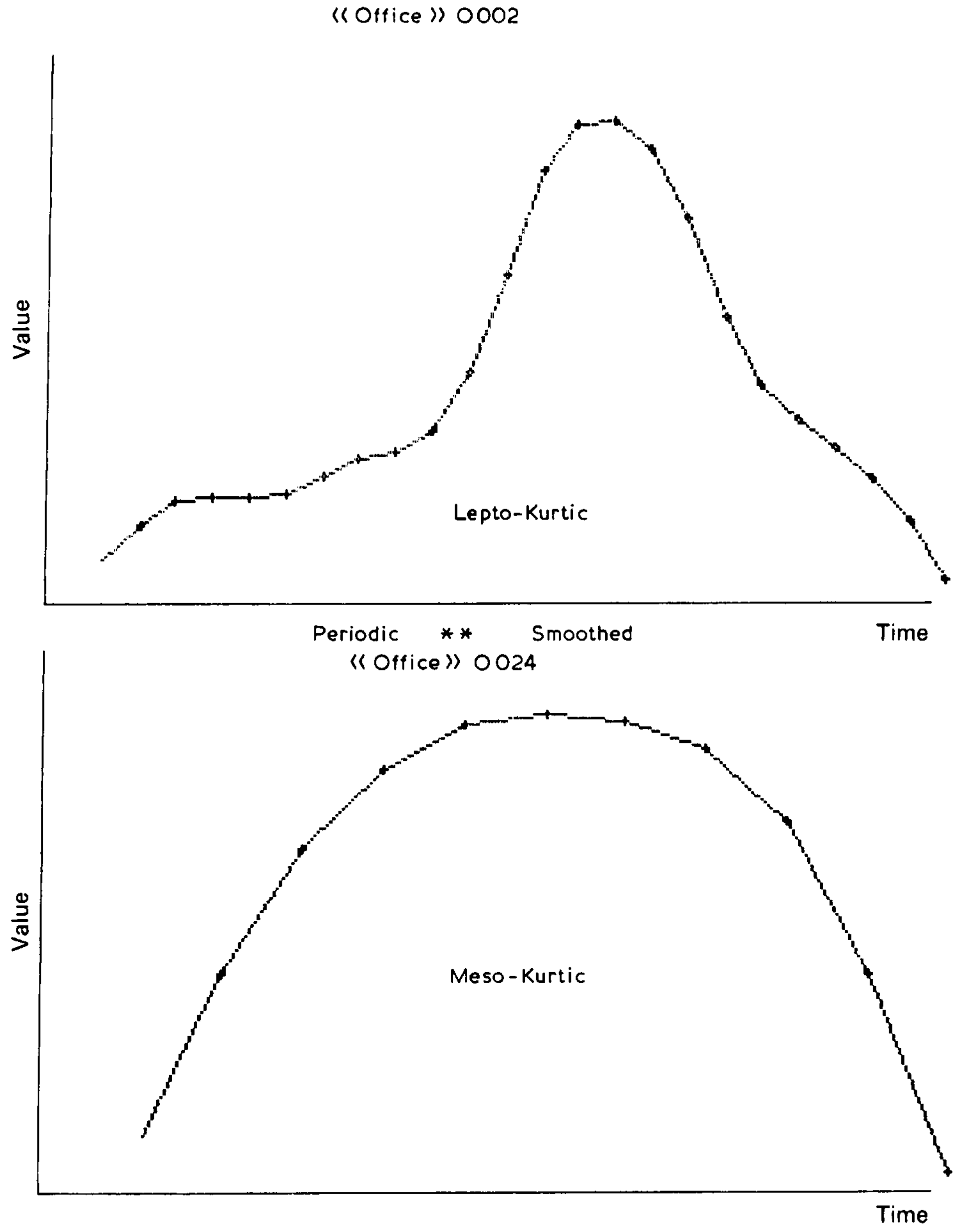


Fig. 9a. Comparison of lepto-kurtic and meso-kurtic curves.

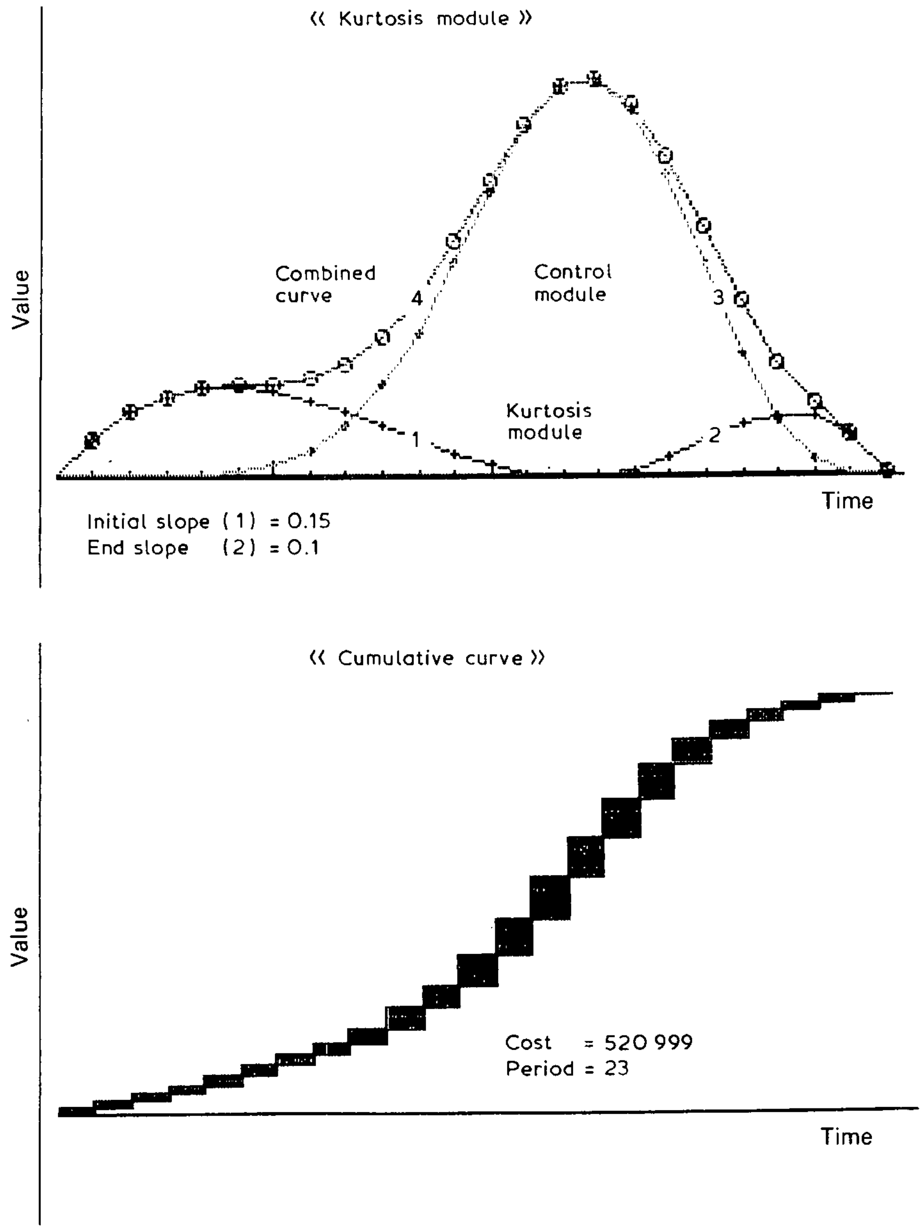


Fig. 9b. The effect of the kurtosis module on the overall curve.

2. The duration of its application is from the origin to the peak point for the initial portion, and from the peak point to the end of the project for the end portion.

The model

So far, the structure of the model for the simulation of the expenditure pattern of construction projects has been described, and the modules constituting the mathematical expression are introduced.

The product of the research and the subsequent analysis have been converted into a highly professional and sophisticated computer-based forecasting system, namely TASC (The Advanced 'S' Curve), equipped with graphic facilities. The program operates in two modes: manual and automatic (Khosrowshahi, 1987). In the manual mode, the user anticipates the value of the criteria variables with which the values of the parameters of the mathematical model are calculated, whereas in the automatic mode, the user defines the project in terms of its variables, such as project type, operation ground condition, access, and so on. Through analysis of the data, models have been developed which facilitate interpretation of the project variables in terms of parameters of the mathematical expression.

The proposed system has the advantage of having a greater degree of user appreciation, without compromising the simplicity of use. In either mode, the user can investigate the effect of various combinations of possibilities and their effect on the expenditure pattern.

As regards the performance of the forecast, it is envisaged that the model has the advantage of improved accuracy. In the manual mode, the level of accuracy is mainly determined by the experience of the user. In the automatic mode, however, the assessment of the predictive performance of the model requires extensive testing. The latter should be based on a sound evaluation method carried out in a systematic way. However, this is not the objective set in this paper, and requires a separate investigation, the product of which shall be the subject of another paper. It is envisaged that the best performance is achieved when the manual mode and the automatic mode are combined.

Conclusion

This paper describes a method of simulating patterns of construction project expenditure, based on a new approach to model development. The methodology facilitates quantification and translation of variables relating to specific project categories, defined in terms of the parameters comprising the mathematical expression. This is facilitated by establishing a set of criteria based on the shape of the periodic expenditure profile, to which both the data and the mathematical expression can be related.

A major advantage of this approach is that the model is independent of the data. Hence, new parameters can be established for a new category of project, and amendments of the existing parameters can be carried out independently of the mathematical expression.

Moreover, the whole system has been fully computerized with graphics facilities to give the user a much improved understanding of the process logic governing the basis for pro ject expenditure curve generation. Also, a means has been established by which ratification of the relationships between the variables of the construction project can take place.

References

Brandon, P.S. and Newton, S. (1986). Improving the forecast. *Chartered Quantity Surveyor*, May, pp. 24–7.

Hastings, N.A. (1975). *Statistical Distributions*. New York.

Khosrowshahi, F. (1985). *Database Value/Time Related Project Information for Construction Projects*. Interim Report, South Bank Polytechnic, August.

Khosrowshahi, F. (1986). *Database Value/Time Related Project Information for Construction Projects*. Interim Report II, South Bank Polytechnic.

Khosrowshahi, F. (1987). An interactive computer-based simulation for budgeting and forecast of construction project expenditure pattern using a new model *Computer-aided Learning in Construction*, AHDSP, September.

Raftery, J. (1986). Models of building cost: How good are they? *Seminar in RICS-QS Division, Research and Development Forum*, 18 September.

Rektorys, K. (1969). *Survey of Applicable Mathematics*.

Stone, R. (1980). Sigmoids. *Bulletin of Applied Statistics*, 7 (1), 59–119.