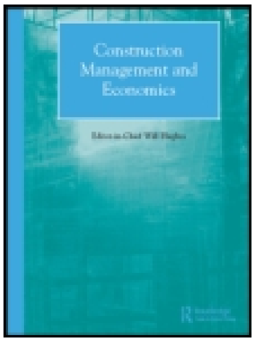

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A linear programming model for cash flow management in the Brazilian construction industry

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A linear programming model has been developed for optimal cash flow management addressing specific cash flow issues related to the construction industry. These include typical financial transactions, possible delays on payments, use of available credit lines, effect of changing interest rates, and budget constraints that often occur in the construction industry. A small size project from the Brazilian construction industry is provided as a case study, aiming at evaluating the potential benefits from using the model. Different changes to the basic structure of the model allow and establish the consistency of the results. Alternative formulations are suggested to deal with uncertainties, longer planning horizons, and multiple subcontractors and suppliers. In addition to the tangible financial earnings derived from the optimization process, the simple structure of the model, as a network flow and corresponding equations, provides much visual insight concerning the relationships between the external inputs and the variables of the problem. Optimal results have yet to be achieved in a real life situation, but a better view of whole cash flow management is provided when using the model.

Keywords: Cash flow management, financial planning, linear optimization

Introduction

Cash flow management is a complex and important problem faced by companies of different sizes, by governments and by individuals, usually requiring distinct approaches and proper tools according to the nature and complexity of the operations. It includes the management of short term investments (e.g. certificates of deposits and primary checking accounts), loans, expenses and cash income. In addition to regulatory restrictions and the rules of the financial markets, complexities also arise from the business environment in which the company operates. The construction industry is a sector where significant uncertainties arise in many aspects of the problem, including the financial and the business environments. The financial risks come from several sources, encompassing the need for intensive capital, typically occurring in many projects, possible delays on client payments, and the exposure to interest rate changes during the period between the contract closing and the end of the payment plan. Furthermore, operations planning, scheduling and control in the construction industry are relatively more distinct and complex than those in manufacturing companies (Gill, 1968), leading to difficulties in good cash flow forecasting. As a result, financial stress can occur, due to inaccurate cash forecasts and/or inadequate cash flow management (Kaka and Price, 1991). It is very difficult to convince creditors and potential lenders that these inadequacies in cash flow are only temporary. Perhaps this is one of the main reasons why the construction industry usually experiences a proportionally greater number of bankruptcies than other industries (Langford *et al.*, 1993).

Even when a firm is not facing financial stress, proper cash flow management plays a strategic role. Managers can increase firm value by managing their cash balance as buffer stocks. These buffer stocks allow

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the firm to maintain the financing of investments (projects in the construction industry) even when internally generated funds temporarily fall short. On the other hand, holding cash in liquid assets can imply two types of loss: (a) lost interest the firm might otherwise have earned on that capital, and (b) lost investments (projects) that the firm might otherwise have managed in the business that it operates. There are additional issues connected with external financing (Harford, 2000). A firm with higher cash flow variability faces a shorter expected time until the next point at which it will run out of cash. This increases the level of expected external financing costs and decreases the expected holding costs for a given level of cash reserves. Even adopting a deterministic assumption about the parameters and cash forecast, the simultaneous consideration of all aforementioned types of interdependence is feasible only by using systems management techniques supported by computers (Gill, 1968).

The relevance of cash flow management is still more evident for companies in developing countries, where the scarcity of funds emphasizes the need for efficient financial tools and management procedures. In Brazil, for example, the housing sector alone has the huge deficit of 5.5 million homes. Nevertheless, the domestic construction industry has still been developed at a moderate growth rate during the last decade, despite facing regular difficulties due to the cycles of the country's economy (Werna, 1993). The housing, building and infrastructure sectors, including all segments that integrate the supply chain, represent a total of roughly US$ 72 billion (1998) of revenues per year. It is around 15% of the country's GDP (FIESP, 1999).

This paper addresses the problem of cash flow management for a single project. The contractual arrangement treats project planning, design and construction as mutually interdependent tasks (allowing subcontractors), which are integrated by the construction manager, who acts as the client's agent. In other words, a 'construction management' contract type is assumed (Nahapiet and Nahapiet, 1985). The accounting of monthly expenses and payments has to be computed systematically, aiming at management purposes (contractor) as well as control purposes (client). This requires the cash flow management of the project on an individual basis, without fund transfers to any other project. The methodology may be applied to the cash flow management for the whole company when running multiple projects with flexible transactions among them. However, additional issues would have to be considered (Navon, 1996).

The proposed model considers the typical instruments and constraints of the financial market, including investments with distinct asset returns and level of liquidity, and also available credit lines from banks. The budget constraints and payment delays that often occur in the construction industry are also taken into account. Possible alternative formulations are discussed at the end of the paper to consider uncertainties, longer planning horizons, multiple subcontractors and multiple suppliers.

Models for cash flow management

Many models have been developed for the cash flow management of companies from different industrial sectors (e.g., Golden *et al.*, 1979; Malburg, 1992). A smaller number of publications have addressed the needs of the construction industry, mainly emphasizing its central issues. The development of models for cash or cost forecasting, for example, is a central subject of concern. These models may be included in broad categories according the complexity level and main purpose. (a) There are simple solutions (e.g., Ashley and Teicholz, 1977) based on a predictable pattern of the cash flow during the construction of a building, represented as the cumulative valuation as a function of the fraction of the contract period completed (S-curves). These simpler techniques are useful tools that allow contractors to achieve a quicker cash flow forecast with reasonable accuracy, which is especially relevant during the pre-tender stage. Significant research efforts (Tucker, 1988; Kaka and Price, 1993) have been developed towards improving the accuracy and flexibility of these techniques. (b) There are detailed computer models that integrate several components (e.g., the cash flow forecast module, the project database, etc.) and allow simulation, aiming at a better representation of the ever-changing environment of a construction company (e.g. Bennett and Ormerdo, 1984; Kaka, 1996; Navon, 1996).

Another important category for cash flow modelling is cost-and-time (or cost/schedule) integration techniques (Sears, 1981; Navon, 1995). These are feasible for projects with detailed information about the available resources, building components and the associated costs and duration of the construction activities. Several optimization models have been applied in cost/schedule management. Karshenas and Haber (1990) present a typical approach using optimization. They propose a piecewise linear objective function aiming at minimization of the total project cost. It includes all project cost components, divided into two main categories: (1) resource mobilization cost and (2) resource use cost. The mobilization cost of a resource depends on the maximum number and capacity of the resource required. The resource use cost is a function of the use (e.g. hours) of the resource. Specific functions are formulated for equipment costs, labour costs, material costs, and also for the 'cost of time', which is defined as time related overhead costs and penalties/bonuses. Other contributions include the use of nonlinear optimization (Adeli and Karim, 1997), integer linear programming (Cusack, 1985) and genetic algorithms (Li and Love, 1997).

Despite the relative success of many applications, companies in the construction industry still have been facing challenging difficulties in implementing procedures for cash flow management, as recognized by the answers given in surveys among construction firms (e.g. Navon, 1996). In many cases collecting the amount of data required to achieve reliable results is very time consuming, thus requiring resources and labour that might result in a final relatively low benefit/cost ratio. Indeed, when considering sets of projects properly grouped in consistent categories in the segmented construction market, as suggested by Kaka and Price (1993), competitive companies can develop their own procedures and practices based on past experience (usually experience is a valuable resource for cost forecasting in similar new projects). For these companies, the cash flow analysis has a strategic role as a tool for decision-making, using the prevailing cost forecast estimates. This is the perspective assumed for the potential use of the proposed linear programming model. Note that this is not a new model for cash flow forecasting. Cash forecasts are used as input to the model. Once they are defined, the optimization algorithm finds an efficient way to manipulate the cash transactions over the planning horizon, aiming at achieving a greater profitability level while running the project.

Methodology

Our model assumes cash flow management on a monthly basis with compound interest rates from month to month. The user defines the planning horizon, as well as the objective function to some extent (linearity is required). Despite the deterministic nature of this version of model, an extension to a stochastic formulation is fully feasible, and recommended when uncertainties about parameters or input data are relevant. Some suggestions for these considerations are discussed later in the paper. Progressive updating (and corresponding computer runs) also are recommended wherever more accurate data or forecasts become available along the construction stage of the project.

The network in Figure 1 shows the basic components of the model. There are some pre-defined values for some variables and parameters. The pre-defined external inputs to the network are the expense forecast (EF_i), the cash income supply (IS_i) as defined by the payment plan, and the initial capital IC (front money). Pre-defined parameters include all kind of interest rates, related to earnings from deposits in assets (parameters r_1 and r_2), available credit line (r_3) and possible penalties (r_4) due to delayed payments.

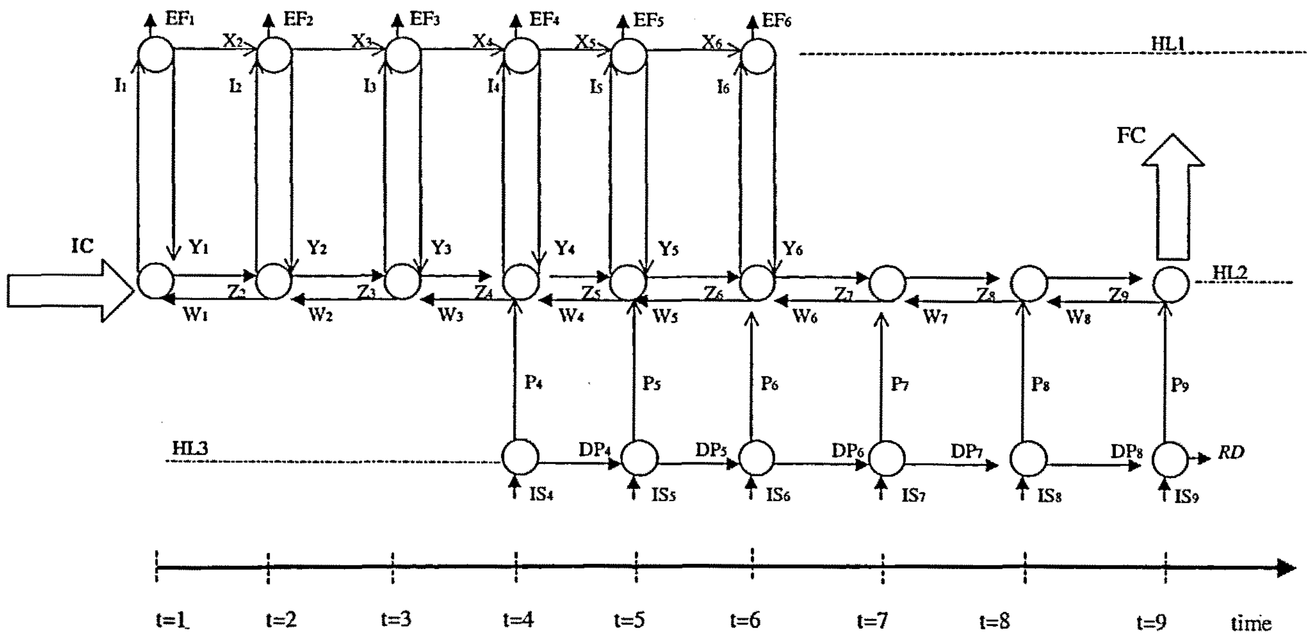

Figure 1 Network of the basic cash flow management model

Arcs and nodes of the network are associated with financial transactions. In the network shown in Figure 1, the traditional convention is not adopted for cash flow representation (expenses represented by arrows pointing along the horizontal axis and incomes represented by arrows pointing upwards). Otherwise, as in the network representation adopted in the planning of construction activities (Ahuja, 1976), a node is a particular point in time when all preceding cash flow and all immediately succeeding cash flow are computed. Horizontal arrows represent the evolution of transactions along successive time periods (nodes). The horizontal lines HL_1 and HL_2 are used to represent two assets, each one with a corresponding return and liquidity level. Asset HL_1 is a saving–checking account, which is used for the daily transactions of the project. A very small interest rate is assumed for deposits in this account. Asset HL_2 has a greater interest rate for investments (e.g. 'certificate of deposit') but requires a longer holding period, usually one month.

The computation of interest rate earnings occurring on the direct lines of the network (in Figure 1 the arrows from left to right, along HL_1 and HL_2) is explained in Figure 2(a). An input flow F at the beginning of a certain period t will be converted into an output flow X that reaches the end node of the same line, with the corresponding computation of the interest rate earning between t and $(t+1)$, as follows:

$$X = (1 + r_j)\, F \qquad (1)$$

where F is the cash flow in a line at the beginning of period t, X is the cash flow in a line at the end of period t, after computation of interest rate gains between t and $t+1$, and rj is the interest rate for deposits of asset j during the time period between t and $t+1$ for $j = 1,2$.

There are minimum cash flow balance specifications for deposits in the two lines HL_1 and HL_2, which may be set as parameters in the model, depending on compensatory balances that may be required by the bank, or even based on existing financial policies of the company.

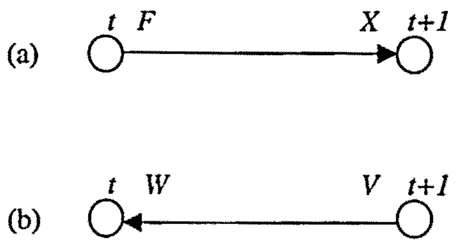

Figure 2 (a) Direct and (b) reverse arrows of a network

Along the horizontal line HL_2 there are reverse arrows connecting a node at time period $(t+1)$ to another node at time period t. These arrows represent a borrowing capability (credit line) by which an amount of cash V may be brought from the future time period $t+1$ for use at the present time t, after applying the discount factor for loans. In other words, companies can use available credit lines for short term loans. Of course, there are upper bounds to the amount of cash from these sources. Based on Figure 2(b), the prevailing interest rate for loans is applied using

$$W = V \,/\, (1 + r_i) \qquad (2)$$

where W is the loan cash flow in a line available for use at the beginning of period t, V is the loan cash flow in a line at the end of period t; r_i is the interest rate for loans, represented as reversals of HL_2 (Figure 1), during the time period between t and $t+1$, for $I = 3$.

A third horizontal line HL_3 is shown in Figure 1 to represent the cash income supplies IS_i that are planned and specified in the contract, thus representing the contractor's earnings. The time lag between the first expense (e.g. $t = 1$ in Figure 1) and the first cash income (e.g. $t = 4$ in Figure 1) varies from contract to contract. The case shown in Figure 1 has no down payment (which would be paid before the beginning of construction) and the payments (cash income supply) are made according to the work in place. The expense flow EF_i, represented in Figure 1 as arrows pointing upwards to the nodes of HL_1, includes cost forecasts and also a fraction of the company overhead allocated to this project. An initial capital IC (front money) is required, which is assumed as available in asset HL_2 at the beginning of the planning horizon. The final capital FC is indicated at the end of last node on the same asset HL_2.

Despite the existence of a plan of payments IS, forming the flow income along HL_3; it is worth considering possible delayed payments from client. For this purpose the planned incomes IS_i along HL_3 are divided into two cash flows. The vertical arcs P_t from HL_3 go directly to HL_2 asset, thus representing cash flows derived from payments without any delay (IS_i) plus some amount of delayed payments (DP_{t-1}) due to past time periods (the component DP_{t-1} is not computed for $t = 4$ since there is no delayed payment for $t=4$). The horizontal lines (along HL_3) represent the remaining part of cash flow, that will have to paid in the future (delayed), thus requiring extra charges due to penalties, which have to be applied according to the terms and conditions of the contract. Penalties can be formulated in the same way as the computation of interest rates adopted along HL_1 and HL_2. At the end of the contract horizon (e.g. $t = 9$ in Figure 1), there will eventually be a residual amount of payment delay, which will be indicated as the variable RD.

The cash flow management problem can be formulated as: given the initial amount of capital of the company allocated for the project (front money, IC), the forecast expense flow EF_t for each time period t along time horizon T, the planned income flow supply IS_t, the interest rates for investments and loans, the applicable penalties for delayed payments, the minimum cash flow balance for investments in the assets, the upper boundary of the credit line, and the estimated flow for payment delays, the linear programming model will maximize the final capital FC available at the end of the time horizon T.

No utility function is introduced for decision-maker behaviour. As a result, risk neutral behaviour is implied, which matches with the objective function of total profits maximization. Indeed, as the initial capital is pre-defined (a fixed amount), the maximization of the final capital is almost equivalent (except by application of interest rates) to maximization of the total profit derived from running the project. Based on the case study in Figure 1, the decision variables to be found by the optimization model are: X_i, Y_i, Z_i, W_i, I_i and FC. The problem can be written in mathematical terms as follows:

$$\text{Max FC} \tag{3}$$

subject to

$$I_1 - Y_1 - EF_1 - X_2 /(1+r_1/100) = 0 \tag{4}$$

(flow balance at first node of HL_1)

$$I_i + X_i - Y_i - EF_i - X_i/(1+r_1/100) = 0 \tag{5}$$

(flow balance at intermediate nodes of HL_1)

$$I_6 + X_6 - Y_6 - EF_6 = 0 \tag{6}$$

(flow balance at last node of HL_1)

$$IC - I_1 + Y_1 - Z_2/(1+r_2/100) + W_1 = 0 \tag{7}$$

(flow balance at first node of HL_2)

$$Y_i - I_i + Z_i - Z_{i+1} / (1+r_2/100) + W_i - W_{i-1} (1 + r_3 /100) = 0 \tag{8}$$

(flow balance at second and third nodes of HL_2)

$$P_i - I_i + Y_i + Z_i - Z_{i+1} / (1+r_2/100) + W_i - W_{i-1} (1+r_3/100) = 0 \tag{9}$$

(flow balance at 4th, 5th and 6th nodes of HL_2)

$$P_i + Z_i - Zi+1 / (1+r2/100) + Wi - Wi-1 (1+r3/100) = 0 \tag{10}$$

(flow balance at 7th and 8th nodes of HL_2)

$$P_9 + Z_9 - W_8 (1+r_3/100) - FC = 0 \tag{11}$$

(flow balance at last node of HL_2)

$$IS_4 - P_4 - DP_4 / (1+ r_4) = 0 \tag{12}$$

(flow balance at first node of HL_3)

$$IS_i + DP_{i-1} - P_i - DP_i / (1+ r_4) = 0 \tag{13}$$

(flow balance at 2nd, 3rd, 4th and 5th nodes of HL_3)

$$IS9 + DP8 - P9 - RD = 0 \tag{14}$$

(flow balance at last node of HL_3)

$$X_j >= X_{min} \text{ for } j = 2\text{–}6 \tag{15}$$

(minimum cash balance required for investments in asset of HL_1)

$$Z_j >= Z_{min} \text{ for } j = 2\text{–}9 \tag{16}$$

(minimum cash balance required for investments in asset of direct line HL_2)

$$W_j <= W_{max} \text{ for } j = 1\text{–}8 \tag{17}$$

(upper bound on credit line for money borrowed: reverse line of HL_2)

where IC is the initial capital available for the contract (*front money*), FC is the final capital, at the end of the time horizon, EF_i is the total cash expenses estimated for the period i, IS_i is the total income supplies planned for the period i,

P_i is the total effective payment received in period i, DP_i is the total of amount of cash related to delayed payments occurred in period i, eventually to be paid in future time periods, I_i is the cash flow converted from asset HL_2 to asset HL_1 at time period i, Y_i is the cash flow converted from asset HL_1 to asset HL_2 at time period i, X_i is the inventory of asset HL_1 at the beginning of time period i, before completion of conversion transactions at node i, Z_i is the inventory of asset HL_2 at the beginning of time period i, before completion of the conversion transactions at node i, W_I is the cash credit available at the beginning of time period i, r_1 is the interest rate per period provided by asset HL_1, r_2 is the interest rate per period provided by asset HL_2, r_3 is the interest rate per period on money borrowed (lower HL3 lines), and r_4 is the interest rate per period to be applied on delayed payments, as penalties, according to the terms of the contract.

The representation of the cash flow as a network instead of the traditional way usually adopted in cash diagram analysis is a result of the need to consider a greater number of variables than just cash income and cash expenses (Figure 1). In addition to the schematic representation it is important to make clear the conceptual differences among: (a) traditional cash diagram analysis as represented in a diagram of financial events versus time (e.g. Collier and Halperin, 1984); (b) cash flow forecasting models; and (c) the optimal cash flow management, as proposed. In the traditional cash diagram analysis, the main concern is the computation of the money value over a timespan and under different hypotheses (e.g. interest rates). Even exhibiting distinct aspects (e.g. present value of a cash stream, computation of internal rate of return, etc.) the traditional cash diagram analysis is not a decision-making tool, at least from a systems analysis perspective. Of course, it is an analytical and consistent tool for 'what if' analysis, to be taken into account in any cash transaction. As described before in the literature review, a significantly higher status and useful potential should be assigned to models based on a predictable pattern of cash flow during construction of a building, represented as the cumulative valuation as a function of the fraction of the contract period completed (S-curves). More accurate cash forecasts are fundamental input to any model.

Nevertheless, the proposed model is a more complete management tool since: (1) a broader set of cash transactions is allowed and explicitly formulated, including bank deposits and credit; (2) the cash valuation over time is also properly considered by computation of compound interest earnings (deposits) or financial costs (loans); (3) the overall interconnection among external inputs (expense forecasts and cash income stream), pre-defined parameters (e.g. interest rates) and the decision variables are formulated explicitly in the corresponding equations (4–17); and (4) an optimal solution is found according to certain relevant criteria (e.g. maximum final capital, as adopted in the case study).

The proposed model (equations 3–17) can be used as a tool for providing very interesting and relevant answers, such as: (a) the effect of interest rates (r_1, r_2, r_3 and r_4) on the objective function; (b) the effect of the upper bound (W_{max}) of the credit line on the final solution; (c) the amount of initial capital (IC) necessary to guarantee reliable cash flow management for the project as a whole, along the planning horizon; and (d) the maximum sustainable level of delayed payments. Some of these questions are illustrated in the case study to be discussed in the following section.

Case study

The methodology described above is applied to data collected from a small-scale project (private 10 floor building for housing) located in the city of Campinas, State of Sao Paulo, Brazil. All data, contractual arrangements and interest rates adopted in this case study correspond to information based on interviews undertaken with practitioners, and from government reports during 1999. The following monthly interest rates were adopted: $r_1 = 0.2\%$, $r_2 = 1.5\%$, $r_3 = 5\%$ and $r_4 = 10\%$. The initial capital (*front money*) is IC = \$ 2 000 000 and the following remaining parameters were adopted: Z_{min} = \$ 50 000; X_{min} = \$ 30 000 and W_{max} =\$ 200 000. A basic structure of monthly expenses and also a payment plan were assumed, based on the original planning for this project, as shown in Table 1.

The optimal results from the previous input data (Table 1) are presented in Table 2. This will be named case A-1. From data of Table 2, we can see a minimum investment in asset HL_1 (variable X), only enough to fulfil the minimum requirement at the end of each month (X_{min} = \$ 30 000). On the other hand, significant investments were made on asset HL_2 due to greater interest rate ($r_2 > r_1$). The credit line was not used ($W_i = 0$ for i =1–8) and no delayed payment occurred since there was no assumption requiring any minimum amount of it. The 'present value of the net profit' (PVNF) for this contract is \$ 308 771. It is evaluated by the difference between cash stream of incomes (including the final capital, FC) and cash outflow (monthly expenses, initial capital and residual delayed payments when existing), discounted in time (di, discount factor), as shown below. The interest rate r_2 = 1.5% was applied for the calculation of the present value.

$$\text{PVNF} = d_9\ (\text{FC} - \text{RD}) - \text{IC} - \sum_{i=1}^{6} di\,EFi + \sum_{i=4}^{9} di\,ISi \tag{18}$$

where

$$di = 1/(1 + r)^{i-1} \quad \text{(discount factor)} \tag{19}$$

A second case study (A-2) differs from case A-1 only in the initial capital. A lower initial capital of IC = \$ 1 365 500 was assumed in case A-2. The optimal solution for case A-2 is shown in Table 3. In case A-2 the credit line was necessary and active from the 4th to the 6th months, since the total of the initial

Table 1 Data for the case study

Month	Monthly expenses		Monthly Incomes, IS_i (\$)
	\$	% of the total	
1	600 000	20	–
2	300 000	10	–
3	300 000	10	–
4	750 000	25	500 000
5	600 000	20	550 000
6	450 000	15	600 000
7	–	–	600 000
8	–	–	550 000
9	–	–	500 000
Total	3 000 000	100	3 300 000

Table 2 Optimal cash flow solution for input data of Table 1 (case A-1)

Month	*I* ($)	*X*($)	*Y*($)	*Z*($)	*W*($)	DP($)	FC($)
1	629 940	–	0	–	0	–	–
2	299 940	30 000	0	1 390 924	0	–	–
3	299 940	30 000	0	1 107 598	0	–	–
4	749 940	30 000	0	819 957	0	0	–
5	599 940	30 000	0	578 698	0	0	–
6	420 000	30 000	0	536 810	0	0	–
7	–	–	–	727 726	0	0	–
8	–	–	–	1 347 945	0	0	–
9	–	–	–	1 926 848	–	0	2 426 000

Table 3 Optimal cash flow solution for input data of Table 1 (case A-2)

Month	*I* ($)	*X*($)	*Y*($)	*Z*($)	*W*($)	DP($)	FC($)
1	629 940	–	0	–	0	–	–
2	299 940	30 000	0	746 761	0	–	–
3	299 940	30 000	0	453 626	0	–	–
4	749 940	30 000	0	156 026	143 164	0	–
5	599 940	30 000	0	50 000	199 512	0	–
6	420 000	30 000	0	50 000	28 738	0	–
7	–	–	–	50 000	0	0	–
8	–	–	–	629 265	0	0	–
9	–	–	–	1 197 223	–	0	1 697 223

capital IC=$ 1 365 000 plus the income supply at t=4 (IS_4 = $ 500 000) is not great enough to cover the expenses during the first 4 months ($ 1 950 000) plus the minimum cash flow balance required at HL_1 ($ 30 000) and HL_2 ($ 50 000). As a result of using the credit line (and paying the interest rate for it), the 'present value of the net profit' (PVNF) calculated according equation (19) decreased to $ 295 575.

It is worth highlighting that the problem can be formulated using different objective functions from that of equation (3). For example, the expression of equation (18) could be easily assumed as objective function. Nevertheless, a maximum limit for IC becomes necessary otherwise the problem is unbounded (as the financial investments in the assets are profitable, a higher IC means more increase in net profit). Of course, the initial capital is also limited in practice.

Aiming at providing a better appreciation of the potential of the model for such analysis, the impact on the 'present value of net profit' (PVNF) has been evaluated for changing levels of initial capital IC. The model was run for each value of IC and the corresponding optimal values were calculated. After computing PVNT, the internal rate of return (IRR, annual basis) was also calculated. These values are shown in Table 4, where a comparison is also made with case B-0. Case B-0 has no optimization procedure but only the earnings computation of the available deposits along HL_2 due to the greater interest rate (r_2=1.5% per month). More than the absolute value of the profitability measures, it is interesting to compare the benefits of using the model. In addition to tangible financial benefits (e.g. case A-1 versus case B-0), the model represented in the network (Figure 1) and corresponding equations gives insight concerning the relationships among the variables in the problem, since a better view of the whole process is provided.

Table 4 Comparative results for different cases

Case	IC ($)	FC ($)	PVNP ($)	IRR(annual)
B-0	2 000 000	2 300 000	196 167	6.08%
A-1	2 000 000	2 426 848	308 771	9.63%
A-2	1 365 500	1 697 223	295 575	10.60%
A-3	3 000 000	3 555 370	310 572	8.02%
A-4	10 000 000	11 429 289	300 338	3.52%
A-5	20 000 000	22 714 513	318 357	2.09%

Potential applications to large scale problems: computational aspects and extensions

Actually, the optimal results of the case study have not been implemented in the company. Despite using input data of a real world case, the project selected for test in the case study is relatively small, aiming at easy understanding of the methodology and the corresponding optimal results, although several alternative formulations could be accommodated by using the same linear programming–modelling tool, and some of them are discussed next. The corresponding impact in terms of computational burden is also evaluated.

As mentioned before, the connection between project scheduling/planning and the cash flow management is provided for the model by external inputs (cash flow expenses and cash flow supply income). Of course this is a simplified form that allows focusing on the financial and economic transactions from a perspective outside the project. A greater level of integration can be achieved through the decomposition of those model components that had been treated as aggregate ones before. The cash expense flow, for example, could be divided into several flows, each one detailing the payments due to a specific subcontractor or supplier. The same procedure adopted to represent payment delays from clients (HL_3 in Figure 1) could also be adopted to represent possible delayed payments to subcontractors and suppliers.

Figure 3, for example, illustrates the procedure based on an assumption that there are expenses with two subcontractors only (no other expense). Compared with Figure 1, only the upper portion of the network is being considered, since HL_1, HL_2 and HL_3 remain unchanged. The original expense flow EF_i is now spent to pay subcontractor A (monthly cash amount SCA_i) and subcontractor B (monthly cash amount of SCB_i) through the expense flows SCA_i and SCB_i. Thus, there will be one new horizontal line for each subcontractor in the new network. Each horizontal arrow in these lines represents the amount of delayed payments to the corresponding subcontractor in that month (e.g. DPB_3: the amount of delayed payment to subcontractor B coming from the third month to cover expenses at the 4th month).

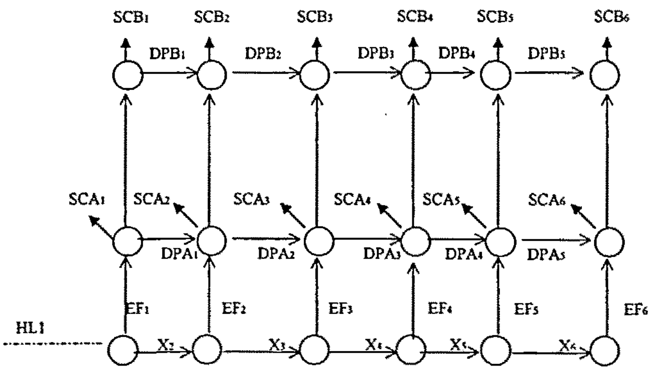

Figure 3 Decomposition of the expense flow (Ef_i) into separate payments to subcontractors

Another formulation could adopt the weekly basis for computation of the project expenses, although the income supply (payments from the client) might be unchanged (monthly basis). Introducing additional intermediate nodes in horizontal line HL_1, with corresponding arrows between two successive nodes, could easily represent this option.

A final relevant issue to be discussed is the effect of uncertainties on parameters or external inputs to the model. A possible stochastic tool to be adopted to deal with uncertainties is the scenario optimization approach (Birge and Louveaux, 1997). Since the time of the first applications, this methodology has gradually evolved to assume a very significant role among the existing tools for optimization under uncertainty (Dembo, 1999). We let a scenario s be a particular realization of a stochastic process. The whole set S of scenarios is used as a broad representation of the possible outcomes that arise from the multiple realizations of that stochastic process. Each scenario s could be defined by a particular realization of two or more variables (or parameters), subject to randomness.

The interest rate r_2 for deposits in the account of HL_2 (Figure 1) and the interest rate r_3 for loans can be selected as random parameters, for example. Both parameters are time dependent. Some samples of $r_2(t)$ and $r_3(t)$ series can be extracted from historical observations, for $t = 1,T$ (T: horizon planning). Thus, each 'scenario' is compounded by a combination of a particular realization of $r_2(t)$ with a particular realization of $r_3(t)$. Each scenario will generate a set of constraints to the optimization problem where the combined realization of parameters is introduced in equations 4–17. The multiple scenarios are simultaneously submitted to the linear programming formulation. The resulting optimal solution is a 'robust' solution since it has considered distinct combinations of parameters, some of them providing the worst financial stress to the cash flow management (e.g. high interest rates for loans with low interest rates for deposits). Of course, knowing the scenarios that are being used in a stochastic model is essential. As mentioned by Dembo (1999), scenarios are easy for non-technical managers to define and understand. Their definition allows evaluating whether or not a scenario makes sense or is reasonable. However, there is a progressive computational burden as the number of scenarios is increased.

Based on the different assumptions discussed before, an assessment of the corresponding problem size is undertaken and this is shown in Table 5, aiming at possible extensions of the methodology to real world projects, for the different formulations and levels of complexity. The problem size under the deterministic (only one possible scenario) and the stochastic (100 possible scenarios) assumptions are shown.

Data presented in Table 5 give some idea about the computational requirements for running large scale problems. Despite the large size of some formulations (e.g. problem P5), this scale of problem does not offer major difficulties when using modern linear programming solvers. For example, AMPL (a modelling language for mathematical programming) commercial software was selected to run the linear programming model for the case study. This is powerful computational software with robust numerical properties and the capability to solve large optimization problems (Fourer *et al.*, 1993). A typical run of small size problems like the one studied here (47 variables and 42 constraints) requires only about 0.1 second on a PC Pentium III. From the authors' experience, on a slightly larger problem (2100 variables and 870 constraints) the required runtime was around 40 seconds (PC Pentium III). With a medium size problem (262 387 variables and 180 009 constraints) the run time was around 5 hours and 23 minutes (on a Silicon Graphics station, Unix environment). This powerful capability is one of the reasons for selecting the linear programming formulation. Moreover, the wide range of commercial software packages for linear programming available in the market at low prices (e.g. the solver of Excel, Microsoft) enables any construction company to have this tool in its office.

Of course, even considering these powerful extensions the model is still a limited representation of the complex real world decision-making process in the construction management environment. In a broader sense, the optimal results are still incomplete, since the problem formulation has not included the connection and trade-off functions between project duration and costs. In other words, a broader formulation should consider distinct alternatives of project duration and the corresponding cost, aiming at achieving global optimization. Conversely, the formulation proposed herein is a good decision making tool when the project duration is defined mainly from a consideration of other external factors (e.g. marketing and sales opportunities, technological constraints, etc.), and thus not subject to optimization.

Table 5 Dimension of the optimization problem for different formulations

Case	Horizon planning (months)	Time interval for expenses	Number of sub-contractors	Number of suppliers	Dimension of the problem	
					Number of variables	Number of constraints
P1-case study	9	month	aggregate	aggregate	47 (deterministic) 4 700 (stochastic)	42 (deterministic) 4 200 (stochastic)
P2	9	week	aggregate	aggregate	73 (deterministic) 7 300 (deterministic)	69 (deterministic) 6 900 (stochastic)
P3	9	month	20	20	267 (deterministic) 26 700 (stochastic)	282 (deterministic) 28 200 (stochastic)
P4	36	month	aggregate	aggregate	233 (deterministic) 23 300 (stochastic)	168 (deterministic) 16 800 (stochastic)
P5	36	month	20	20	2873 (deterministic) 287 300 (stochastic)	2680 (deterministic) 268 000 (stochastic)

Conclusions

A linear programming model is presented aiming at providing cash flow management for projects in the construction industry. The model assumes cash flow management on monthly basis with compound interest rates from one month to the next.

The proposed model considers typical banking instruments and the constraints of the financial market, including investments with distinct asset returns and levels of liquidity, and also the available credit lines from banks. Moreover, there are budget constraints and delayed payments that often occur in the construction industry. Despite the deterministic assumption adopted in this version of model, an extension to a stochastic formulation is suggested, and recommended when the uncertainties about parameters or input data are relevant.

A small size case study has been developed. Different changes to the basic structure of the model revealed the consistency of the results. In addition to the direct financial gains, the simple structure of the model represented in the network (Figure 1) and the corresponding equations allow more visual insight concerning the relationship between the external inputs and the variables in the problem. A better view of whole cash flow management is provided when using the model.

The computational requirements for the inclusion of uncertainties, a longer planning horizon, multiple subcontractors and suppliers are compatible with the capability of the commercial solvers available in the area of linear optimization. Moreover, the wide range of commercial linear programming software available in the market at low prices enables any construction company to have this tool in its office.

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