2020.3.21

March 2020

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1 Problem 1

At a nursery, 2006 babies sit in a circle. Suddenly, each baby randomly pokes either the baby to its left or to its right. What is the expected value of the number of unpoked babies?

2 Problem 2

One day, a bishop and a knight were on squares in the same row of an infinite chessboard, when a huge meteor storm occurred, placing a meteor in each square on the chessboard independently and randomly with probability p. Neither the bishop nor the knight were hit, but their movement may have been obstructed by the meteors. For what value of p is the expected number of valid squares that the bishop can move to (in one move) equal to the expected number of squares that the knight can move to (in one move)?

3 Problem 3

Richard has a four infinitely large piles of coins: a pile of pennies, a pile of nickels, a pile of dimes, and a pile of quarters. He chooses one pile at random and takes one coin from that pile. Richard then repeats this process until the sum of the values of the coins he has taken is an integer number of dollars. What is the expected value of this final sum of money, in cents?

4 Problem 4

Tom has a scientific calculator. Unfortunately, all keys are broken except for one row: $1, 2, 3, +$ and $-$. Tom presses a sequence of 5 random keystrokes; at each stroke, each key is equally likely to be pressed. The calculator then evaluates the entire expression, yielding a result of E. Find the expected value of E. (Note: Negative numbers are permitted, so $13 - 22$ gives $E = 9$. Any excess operators are parsed as signs, so $-2 - +3$ gives $E = 5$ and $- + -31$ gives $E = 31$. Trailing operators are discarded, so $2 + + - +$ gives $E = 2$. A string consisting only of operators, such as $- + + - +$, gives $E = 0$.)

5 Problem 5

The permanent of a square matrix $A = (a_{ij})$ of size n with row sums $r_i = a_{i1} + \cdots + a_{in}$ for $i = 1, \ldots, n$ can be estimated by

$$perA \leq \prod_{i=1}^{n} (r_i!)^{\frac{1}{r_i}}$$

We now give the difinition of *a permanent*. Let $A = (\mathrm{a}_1, \dots, \mathrm{a}_n)$ be an $n \times n$ matrix with columns $\mathrm{a}_j = (a_{1j}, \dots, a_{nj})^T$. Then $per A$, the permanent of A, is defined by

$$per A := \sum_{\pi \in S_n} a_{1\pi(1)} \dots a_{n\pi(n)}$$

So the permanent is defined in the same way as the determinant but without the signs depending on whether the permutation π is even or odd.