Pricing Behavior of an Equity-Linked Structured Product

You are invited to explore the pricing behavior of an equity-linked structured product

"24-month callable dual accrual cash or share security on Wal-Mart Stores, Inc and Intel Corp"

launched by Merill Lynch in 2008. The product description is outlined below:

Issue size:	10,000,000 warrants
Minimum subscription:	100,000 warrants
Notional Amount:	USD 1 per warrant
Issue Price:	100% of the Notional Amount
Valuation Date:	Feb. 11, 2006
Maturity Date:	Feb. 19, 2008

The two underlying stocks are

	Reference price	*Exercise price*
Wal-Mart Stores Inc.	USD 45.48	USD 39.5676
Intel Corp	USD 20.77	USD 18.0699

where Exercise Price = 87% x Reference Price. The Reference Price is taken to be the closing price of the stock on the valuation date. Note that the two stocks have been chosen such that the price processes of them are expected to exhibit minimal correlation.

1. Payoff structure:
   Full par payment or delivery of the "worst performing" stock on the maturity date.

   - If the settlement prices of BOTH the underlying stocks are higher than or equal to the respective Exercise Price, then each warrant holder receives 100% of the notional amount per warrant held.

   - If either one of the settlement prices is lower than the respective exercise price, then each holder receives per warrant physical delivery of a number of the "Worst performing" stock equal to

     Notional amount / Exercise Price

     of the worse performing stock.

Terminal payoff at time T
$= \min(1, \min(S_1(T) / S_{1,exer}, S_2(T) / S_{2,exer}))$
$= 1 - \max(1 - \min(S_1(T) / S_{1,exer}, S_2(T) / S_{2,exer}), 0)$,

where $S_{1,exer}$ and $S_{2,exer}$ are the Exercise Price of Stock 1 and Stock 2, respectively. That is, the investor shorts a put on the minimum of the two stocks.

This is a contingent forced conversion which occurs when either one of the two share prices declines. This is just the opposite to that of a convertible bond where the holder of a convertible bond chooses to convert the bond par into shares only when the share price appreciates above certain threshold value.

Query: Would the chance of occurrence of contingent forced conversion become higher or otherwise when the correlation between the price processes of the two stocks becomes closer to zero?

2. Additional coupon (accrual feature)
   The warrant pays out a fixed 4.075% coupon for the first quarter (that is, 16.3% per annum). Afterwards, unless the warrant has been called, over each observation period (3-month period), the holder receives

   $$4.075\% / 252 \text{ of notional amount}$$

   when the closing prices of BOTH the Underlying Stocks are at or above the respective Exercise Price. Here, 252 is the number of trading days per year.

   This is like an accrual note with the underlying index being the minimum of the two share prices. The accrual feature can be viewed as a series of daily binary options, and the warrant pays at the nth time step

   $$4.075\% / 252 \times \text{notional amount}$$

   when

   $$\min(S_1(n,i) / S_{1,exer}, S_2(n,j) / S_{2,exer}) \geq 1,$$

   where $S_1(n,i)$ is the price of Stock 1 at n time steps from initiation and i up moves, and $S_2(n,j)$ is the price of Stock 2 at n time steps from initiation and j up moves.

3. Issuer's Call:
   On any of the Observation Date over each 3-month Observation Period, provided that BOTH underlying stocks are greater than or equal to the reference prices, the issuer can call by paying 100% of the Notional Amount (together with the coupons accrued in the last 3-month Observation period). The Observation Dates are set to be at the end of each 3-month period over the life of the warrant. For convenience, we assume that the Coupon Dates and the Observation Dates coincide. Similar to a Bermudan option, the optimal calling policy adopted by the issuer is determined as part of the solution procedure.

To incorporate the call feature, at each lattice node on an Observation Date (excluding the maturity date), where $\min(S_1(n,i) / S_{1,ref}, S_2(n,j) / S_{2,ref}) \geq 1$, we apply the dynamic programming procedure: $\min(W_{cont}, K + \textit{accrual coupon})$, where W_{cont} is the continuation value of the warrant, K is the call price (taken to be 100% of the Notional Amount) and accrual coupon is the coupon amount over the 3-month period prior to the Observation Date.

Comments on the nature of the product

warrant = bond (series of binary options due to the accrual feature of coupons)
– European put on minimum of two uncorrelated stocks
– issuer's calling right (Bermudan call option with multiple call dates)

- The investor believes that the prices of BOTH underlying shares at maturity will remain at a level above or equal to their respective Exercise Prices, earning an enhanced yield.
- Note that $\min(W_{cont}, K) = W_{cont} - \max(W_{cont} - K, 0)$. This "call" right given to the issuer is like a Bermudan call option with strike price equal to the call price. The call price is 100% of the Notional.
- The coupons received depend on the realized paths of BOTH underlying stocks according to the rule of accrual. The coupons can be visualized as a series of binary options, paying coupon or otherwise on each date that is contingent on

$$\min(S_1(n,i) / S_{1,exer}, S_2(n,j) / S_{2,exer}) \geq 1.$$

Sources of risks faced by the investor

1. Market risks – stochastic movement of the prices of the underlying shares
2. Interest rate risk – the present value of the bond component, including par plus coupons.
3. Issuer's call risk.
4. Counterparty risk – default of Merrill Lynch (more noticeable after the event of Lehman Brothers' minibonds)
5. Liquidity risk – will not be listed on any securities exchange and do not expect a trading market with only Merrill Lynch as a possible buyer.

Work elements in this computer assignment

1. Construct the two-state lattice tree algorithm for pricing this two-state option product (warrant), taking into consideration of (i) terminal payoff structure, (ii) accrual feature of the coupons, (iii) issuer's call right. Constant interest rate and zero default risk of the issuer are assumed. In your report, you are required to summarize the special considerations that have been taken to incorporate the issuer's call, accrual coupons and terminal payoff structure in your scheme (hints are given below).

2. Examine the variation of the warrant's price with respect to the following parameters:

(i) correlation coefficient between the two underlying stock price processes,
(ii) volatility of the stock prices,
(iii) level of the riskless interest rate.

Plot the warrant price against each of the above parameters. Give your comments on the pricing behavior of the warrant. Do the computed results coincide with your financial intuition?

3. Compute the value of the embedded issuer's call right by finding the difference of the warrant's price with and without the call right. How does the value of the call right change with varying model parameters?

Hints on the construction of the two-state trinomial scheme

1. We take the number of trading days in a year to be 252 so that one quarter of a year is 63 days. To avoid excessive computational time, it suffices to take each time step to be one day so that the 2-year term of the warrant corresponds to total number of time steps equal to 504.

2. When n = 63, 126, …, 441, the holder is entitled to receive the accrual coupon. Also, the issuer may call on these dates. Note that the coupon will always be received by the holder even upon calling by the issuer. That is, the total cash amount received by the holder upon calling is the notional plus the accrual coupons. The actual amount of the accrual coupons is dependent on the realization of the stock price processes during the Observation Period, which is the time interval between this coupon date and the last coupon date.

   Recall that we are using the backward induction procedure, where we have to enforce the issuer's call policy $\min(W_{cont}, K + \textit{accrual coupon})$ on the call (Observation) date (provided that the condition on call is met). Note that backward marching of the 3-month coupon collection period has yet to be performed, so the coupon amount received is not exactly known.

   In the backward induction procedure, we perform calculations up to the time immediately right after an Observation (call) Date. Conditional on $\min(S_1 - (n,i) / S_{1,exer}, S_2(n,j) / S_{2,exer}) > 1$, the dynamic programming procedure is applied across an Observation Date by simply taking the minimum of the warrant value right after the Observation Date and K = notional value = 1. The warrant value decreases by the coupon received when we move forward in time across the Observation Date. Therefore, it suffices to find the minimum among the warrant value right after the call date and K, which is equivalent to compare the warrant value before the call date and K + accrual coupon. With this observation, it is lucky that we do not need to consider various possible values of accrual coupon on the Observation (call) date.

3. Without the burden of considering different possible values of coupon received on the call date, the two-state lattice tree algorithm can be simplified as follows:

$$W(n,i,j) = [p_{uu}\, W(n+1,i+1,j+1) + p_{ud}\, W(n+1,i+1,j-1) + p_{du}\, W(n+1,i-1,j+1) + p_{dd}\, W(n+1,i-1,j-1) + p_{00}\, W(n+1,i,j)] / R + \text{PV (coupon)}\, \mathbf{1}\{\min(S_1(n,i) / S_{1,exer}, S_2(n,j) / S_{2,exer}) \geq 1\}.$$

It suffices to note that the warrant value is increased due to collection of coupon to be recorded at the current time level n but collected on the next Observation Date, conditional on $\{\min(S_1(n,i) / S_{1,exer}, S_2(n,j) / S_{2,exer}) \geq 1\}$. To find the present value PV (coupon), the coupon value has to be discounted between the current time level n and the next Observation Date. Since each time step corresponds to one business day, so the coupon dollar amount over one time step is 0.04075 / 252. In the first quarter, the coupon dollar per day is 4 x 0.04075 / 252 and without the condition on the stock prices.

4. The maturity date corresponds to $n = 504$. The terminal payoff is the par minus the put on the minimum of the two stock prices, plus the last coupon accrued between $n = 442$ and $n = 504$. In a similar manner, the contribution to the warrant value due to the last coupon collected on the maturity date can be effectively captured by the extra coupon term in the lattice tree algorithm listed above.

5. As there is no intermediate knock-out (barrier feature), the use of the two-dimensional trinomial feature is acceptable. If otherwise, special precautions are required to implement the boundary conditions along the 4 sides of the computational domain in order to incorporate the knock-out feature.

6. You have the freedom to choose the stepwidth for the two state variables ln S_1 and ln S_2. Be careful that you choose the computational domain which spans sufficiently large positive and negative values for both state variables. Recall that the domain of definition of the continuous model is the whole infinite (ln S_1 - ln S_2) plane. The error associated with the truncation of the domain can be quite significant if the span of the computational domain is not sufficient.