

## Problem A. Sum and XOR

Input file:            **standard input**  
Output file:         **standard output**  
Time limit:          2 seconds  
Memory limit:       256 megabytes

On the very usual day of spring, Daniyar is challenged with an interesting problem once again:

You need to find all possible arrays of length  $n$  where each element  $1 \leq a[i] < 2^{\text{number\_of\_bits}}, 0 \leq i < n$  with the following conditions:

- Sum of the array is equal to  $sum$ , i.e.  $a[0] + a[1] \dots + a[n - 1] = sum$ .
- XOR sum of the array is equal to  $xor\_sum$ , i.e.  $a[0] \oplus a[1] \dots \oplus a[n - 1] = xor\_sum$ .

To simplify, you are asked to find the answer modulo  $10^9 + 9$ .

### Input

A single line contains 4 integers:  $n$ ,  $sum$ ,  $xor\_sum$ , and  $number\_of\_bits$ . It's guaranteed that:

- $1 \leq n \leq 5000$
- $1 \leq sum \leq 3000$
- $1 \leq number\_of\_bits \leq 30$
- $1 \leq xor\_sum < 2^{\text{number\_of\_bits}}$

### Output

Output the answer to the problem modulo  $10^9 + 9$ .

### Examples

standard input	standard output
2 12 3 2	2
3 12 3 2	27
4 16 3 2	144

### Note

In the first sample test, there are two possible arrays:

5, 7

7, 5

## Problem B. String theory

Input file:            **standard input**  
Output file:          **standard output**  
Time limit:           **1 second**  
Memory limit:        **256 megabytes**

One of the most popular and (in a lot of physicist opinion) promising is a string theory. It states, that the multiverse is made of different dimensions but the highest is 11th dimension. Beyond 11 dimensions, the universe would become unstable and dimensions higher than 11 would collapse to an 11-dimensional universe. You are now faced the challenge in modeling String theoretical problem.

Our system has  $n$  strings numbered from 1 to  $n$  placed at the lines  $S = i$  for  $i = 1, \dots, n$  on OTP Cartesian plane, where OT represent time frames and OS represent current string of the particle. Particles moving only into the future and maybe jump between the strings expending the energy. Initially, particles only can stay in a rest without losing any energy, i.e. transition from state  $(t, p)$  to  $(t+1, p)$  costs 0 energy, and no other transitions are given within our system (yet). Note, our system does not possess backward time travelling (or assume that cost of transition back in time is infinity). Your main aim will be to calculate the minimum amount of energy particle require to travel from string  $s_1$  at the time frame  $t_1$  to the string  $s_2$  at the time frame  $t_2$ , or say that this impossible to happen.

We now describe the certain external forces that may appear: teleports and massive objects. When teleport appear at time frame  $t$  with energies costs  $(a_1, \dots, a_{n-1})$ , it allows any particle at the state  $(t, i)$  to immediately jump to the state  $(t, i+1)$  with energy loss  $a_i$  or to the state  $(t, i-1)$  with energy loss  $a_{i-1}$  (whenever the string  $i+1$  or  $i-1$  exist). If we add massive object at time frame  $t$  with energies  $(b_1, \dots, b_n)$  it affect transition between time frames  $t$  and  $t+1$  so that particle at string  $i$  require to expend  $b_i$  energy to stay at the same string (otherwise particle just disappear it a space-time). So there are 3 type of requests you must fulfil:

- 1  $t \ a_1 \dots a_{n-1}$  - add teleport at time frame  $t$  with energies  $a_i$ ;
- 2  $t \ b_1 \dots b_n$  - add massive object at time frame  $t$  with energies  $b_i$ ;
- 3  $t_1 \ s_1 \ t_2 \ s_2$  - we encountered particle travelled between the states  $(t_1, s_1) \rightarrow (t_2, s_2)$ , what is minimal possible amount of energy expended by this particle?

Note, that if you are told to add 2 teleports/massive objects in the same time frame, then later one replaces earlier one.

### Input

In the first line you are given two numbers  $n$  and  $q$  - number of strings in our system and number of requests ( $1 \leq n \leq 11, 1 \leq q \leq 10^4$ ).

In the next  $q$  lines on the  $i$ -th line you are given the description of the queries in the format described in the statement with restrictions:

- $0 \leq a_i, b_i \leq 100$  - energy costs;
- $1 \leq t \leq 5 \cdot 10^4, 1 \leq t_1 \leq t_2 \leq 5 \cdot 10^4$  - time frames;
- $1 \leq s_1, s_2 \leq n$  - string positions.

### Output

For each query of the third type output single number - the answer for the query.

## Example

standard input	standard output
3 12	10
2 1 1 10 3	-1
3 1 2 2 2	2
3 1 2 2 3	10
1 1 2 1	12
3 1 1 1 2	6
3 1 2 3 2	3
3 1 1 2 2	5
3 1 1 2 3	4
1 2 2 2	
3 1 1 1 3	
3 1 1 2 3	
3 1 3 2 1	

## Problem C. Binary matching

Input file:            **standard input**  
Output file:          **standard output**  
Time limit:           1 second  
Memory limit:        256 megabytes

You are given two binary trees and your task is to find their largest rooted isomorphic subtrees.

Two trees are called *isomorphic* if one of them can be obtained from other by a series of flips, i.e. by swapping left and right children of a number of nodes.

*Subtree* is a tree that was obtained from the original one by continuously deleting some, possibly zero, leaf nodes.

### Input

The first line contains the only positive integer  $n$  ( $1 \leq n \leq 1000$ ) — number of nodes.

Each of the following  $(n - 1)$  contains a description of the tree edges, represented as two positive integers  $u, v$  ( $1 \leq u, v \leq 1000$ )

Then follows a description of the second tree in the same format.

In both trees node with index 1 is a root node.

### Output

Output a single positive integer - number of vertices in the largest rooted isomorphic subtree of given trees.

### Example

standard input	standard output
5 1 2 2 3 3 4 3 5 6 1 2 1 3 3 4 4 5 4 6	5

## Problem D. Box packing

Input file:            **standard input**  
Output file:          **standard output**  
Time limit:           1 second  
Memory limit:        256 megabytes

Zharaskhan is working hard in Facegle. But after we announced KBTU Open 2021 spring, he decided to come back to Almaty and help us in preparation. Traditionally, when people come abroad, they bring gifts. So Zharaskhan prepared  $n$  boxes,  $i$ -th one is of size  $a_i \times b_i$  (the height doesn't matter), to make  $n$  people happy.

It turns out, air transport regulations do not allow more than  $k$  boxes per person for the flight. What Zharaskhan came up with, is that he can put box into another boxes (items are then placed in the most deep box) if the sizes fit. For example, if there are 3 boxes  $2 \times 2$ ,  $2 \times 4$  and  $5 \times 5$ , he can put first one into the second, and then second (which already contain first box) inside the third one. For safety of the boxes, Zharaskhan cannot put two separate boxes within one box (but can put box containing box within another box that doesn't contain box). Pretty much like russian doll Matryoshka.

Help Zharaskhan to make as many people happy as possible, knowing the air transport regulations.

### Input

At the first line there are two integers  $n$  and  $k$  ( $1 \leq n \leq 10^5, 1 \leq k \leq 100$ ).

At the next  $n$  lines,  $i$ -th line contain two space separated integers  $a_i$  and  $b_i$  ( $1 \leq a_i, b_i \leq 10^9$ ).

### Output

Output single integer - maximal number of boxes Zharaskhan can take with him, packing everything within not more than  $k$  boxes.

### Examples

standard input	standard output
4 1 2 2 4 2 3 4 5 5	3
4 2 2 2 4 2 3 4 5 5	4

## Problem E. Alimzhan loves ACM

Input file:            **standard input**  
Output file:          **standard output**  
Time limit:           1 second  
Memory limit:        256 megabytes

Alimzhan is desperately trying not to come up with another combinatorics problem. He knows that competitive programmers love arrays and binary numbers. He created a function  $f_a(m)$  for a number  $m(0 \leq m \leq 2^n - 1)$  and an array  $a_0, a_1, \dots, a_{n-1}$ .  $f_a(m) = \sum_{0 \leq i \leq k-1} a_{b_i}$ , where  $b_0, b_1, \dots, b_{k-1}$  are indexes of ones in the binary representation of  $m$ .

For example,  $f_a(5) = a_0 + a_2$  and  $f_a(11) = a_0 + a_1 + a_3$ .

Suddenly, Alimzhan's inner voice started shouting: "Count number of such  $m$ -s so that  $f_a(m+1) > f_a(m)$  !!!". To make this problem a little bit more ACM alike Alimzhan asks you to answer to  $q$  queries.

You have  $q$  pairs of numbers  $l_i, r_i(0 \leq l_i \leq r_i \leq n - 1)$ . For each query  $i$ , consider an array  $c_i = (a_{l_i}, a_{l_i+1}, \dots, a_{r_i})$ . Count the number of  $m(0 \leq m \leq 2^{r-l+1} - 1)$ , such that  $f_c(m+1) > f_c(m)$ .

### Input

The first line contains an integer  $n(1 \leq n \leq 2 \cdot 10^5)$  – length of an array  $a$ .

Second line contains  $n$  integers  $a_0, a_1, \dots, a_{n-1}(-10^9 \leq a_i \leq 10^9)$  - elements of an array  $a$ .

Third line contains an integer  $q$  – the number of queries.

The next  $q$  contain pairs of integers  $l_i, r_i(0 \leq l_i \leq r_i \leq n - 1)$ .

### Output

For each query, print one integer – the answer to the query, in separate lines.

### Example

standard input	standard output
5	26
1 2 2 6 3	3
3	2
0 4	
2 3	
3 4	

## Problem F. Data compression

Input file:            **standard input**  
Output file:         **standard output**  
Time limit:          1 second  
Memory limit:       256 megabytes

Nurbakyt started getting tired at his developer job and decided that he needs a hobby. Computer games are too brutal and embroidery is too expensive. He started producing music, but everything sounded like "Darude - Sandstorm".

After weeks of struggles, he finally found something he could stick to. Nurbakyt started participating in algorithmic trading tournaments. He uses machine learning to build predictive models.

Usually, Nurba's dataset looks like an array of  $n$  number. Array is too large to execute any deep learning algorithm. Nurbakyt decided that he needs to compress his array of number. While browsing the forums, he found a compression algorithm known as "nonsensical compression". Algorithm splits an array into several segments and replaces the segment with a pair of two highest numbers in the segment. User named ElonMusk228 suggests that the statistical error in compression depends on the sum of these two numbers for each segment. Nurbakyt wants to test this "nonsensical compression" algorithm, so he needs a program that splits the array minimizing the sum of products of two largest numbers in each segment.

Note that to compress an array each segment should contain at least two number.

### Input

The first line contains one integer  $n$  ( $2 \leq n \leq 10^6$ ).

Second line contains  $n$  numbers  $a_1, a_2, \dots, a_n$  ( $1 \leq a_i \leq 10^6$ ) - the Nurbakyt's array.

### Output

Print one integer – minimum sum of products of pairs after compression.

### Examples

standard input	standard output
4 8 2 10 1	26
3 1 2 4	8

## Problem G. Alimzhan's Triplets

Input file:            **standard input**  
Output file:         **standard output**  
Time limit:          1 second  
Memory limit:       256 megabytes

Zharaskhan has a permutation that consist of  $n$  distinct integers, where  $0 \leq a[i] < n$ . But as he dislikes his friend Alimzhan, he wants to transform his array by deleting a minimum number of elements from his array so long there is no *Alimzhan's Triplets* in it.

What is *Alimzhan's Triplet*, you may wonder. Well, Alimzhan describes it as any three consecutive elements in the array that are either ascending or descending, i.e.  $a[i] < a[i+1] < a[i+2]$  or  $a[i] > a[i+1] > a[i+2]$  for some  $i$ .

Zharaskhan is very busy making money for Mark over at Facebook, so he asked you, the participant of this very KBTU Open, to find the answer.

### Input

The first line contains a single integer  $n$ . The next line contains  $n$  integers describing the array  $a[i]$ , which is Zharaskhan's array.

- $1 \leq n \leq 2 * 10^5$
- $0 \leq a[i] < n$  and all elements are distinct

### Output

Output a single number – the minimum number of elements required to remove from Zharaskhan's array to make it free of *Alimzhan's Triplets*.

### Examples

standard input	standard output
4 0 2 3 1	1
5 0 2 1 4 3	0
6 3 0 5 2 1 4	1

### Note

In the first sample test, there is only a single *Alimzhan's Triplet*:  $[0, 2, 3]$ . Thus, deleting any of these 3 elements is enough.



## Problem H. Fun Problem

Input file:            **standard input**  
Output file:          **standard output**  
Time limit:           **1 second**  
Memory limit:        **256 megabytes**

Paradoxically, there are no hints in this statement.

### Examples

standard input	standard output
41	30
7	2021
43	Online
1	Subscribe

## Problem I. Script

Input file:            **standard input**  
Output file:         **standard output**  
Time limit:          1 second  
Memory limit:       256 megabytes

Temirulan is an advanced programmer and often uses various scripts to do routine tasks. Sometimes instead of writing new scripts, his scripts are just sequences of scripts he wrote before.

Currently, Temirulan is working with NOOB(Network Optimized Object Base) online machine which accepts two types of requests:

1. Upload a script file and place it at the end of the machine's buffer for one dollar.
2. Copy the subsequent scripts in the machine's buffer and place them at the end of the buffer with no fee.

For simplicity, let's represent the sequence of scripts that Temirulan wants to execute with the string  $s$  of lowercase English letters. Each symbol represents a script file. Find the minimum amount of money Temirulan has to pay to run his script on the machine.

### Input

Input contains one string  $s$  ( $1 \leq |s| \leq 50$ ) - the sequence of scripts.

### Output

Print the minimum amount of money Temirulan has to pay to run his script.

### Example

standard input	standard output
abcbcd	4

### Note

In the sample test, Temirulan first uploads scripts a, b, c, paying 3 dollars. Then puts "abc" copy at the end. Finally, he pays one dollar for d script.

## Problem J. Dilemma

Input file:            **standard input**  
Output file:           **standard output**  
Time limit:            2 seconds  
Memory limit:         256 megabytes

Askar and Damir love teaching students and kids. Recently, they found the birthplace of many genius kids, so they've decided to teach them. They were given the map of the city with  $4n$  schools,  $2n$  girls only schools and  $2n$  boys only schools. The map of the city represented as a Cartesian plane and coincidentally no two schools were on the same line on the map (this is part of the city's geniusity magic). Now elders want to split the city by the line, so that there is an equal number of boys only schools and girls only schools from both sides of this line. Since  $n$  could be very large, they instructing this task to you.

### Input

In the first line of the input given only number  $n$  ( $1 \leq 4n \leq 10^6$ ).

Following  $4n$  lines are coordinates of schools, each in separate line. First half is  $2n$  boys only schools, then  $2n$  girls only schools, where  $i$ -th line contain two space separated integers  $x_i, y_i$  ( $-10^9 \leq x_i, y_i \leq 10^9$ ) for  $i = 1 \dots, 4n$ .

### Output

If there is no such line output 'No'.

If there is such line, print 'Yes' without quotes and in the next 3 lines output 3 real numbers  $A, B, C$  that determine line by equation  $Ax + By + C = 0$ . This line must splits  $4n$  schools equally for both type of schools.

Checker splits points into two groups with respect to this line:  $\{(x, y) : Ax + By + C > 10^{-6}\}$  and its complement.

### Example

standard input	standard output
2	Yes
1 4	0.000000000
4 5	-1.000000000
5 1	3.500000000
6 3	
3 1	
3 4	
6 2	
7 4	

## Problem K. Avengers

Input file:            **standard input**  
Output file:          **standard output**  
Time limit:           **1 second**  
Memory limit:        **256 megabytes**

NurlashKo, Nurbakyt, and Zhora are members of the last ninja clan fighting against even emperor Ren's wild reign. After devastating defeat in an open battle, they decided split their army into three camps and wage a guerrilla war.

One Emperor Ren's ridiculous reforms allows to pass roads between cities only in one direction. Also, he chose the allowed directions of the roads in such way, so that it's impossible to start and return to the same city after passing several roads.

Right now, the clan is deciding where to place their camps. Emperor Ren's army makes regular raids inspecting some path. If Army crushes all three of the camps during their raid, clan wouldn't be able to regroup and would loose the war. Help the clan to choose three cities, so that there is no path that passes through all three of these cities.

### Input

First line contains two numbers  $n, m$  ( $1 \leq n, m \leq 10^6$ ) – number of cities and roads in the Empire.

Next  $m$  contain pairs of numbers  $v_i, u_i$  ( $1 \leq v_i, u_i \leq n$ ), describing the directed road from  $v_i$  to  $u_i$ .

### Output

Print three numbers, indexes of cities where the clan should place their camps. If there are such three cities print  $-1$ . If there are more than 2 answers, print any of them.

### Examples

standard input	standard output
3 2 1 2 2 3	-1
3 2 1 2 1 3	2 3 1