设数据集 $\mathcal{D}=\left\{(x_i,y_i)
ight\}_{i=1}^n, x_i\in\mathbb{R}^d, y_i\in\{0,1\}, heta=\{\pi,\mu_0,\Sigma_0,\mu_1,\Sigma_1\},$ 则在整个数据集上有,

$$egin{aligned} P(\mathcal{D}) &= \prod_{i=1}^n p(x_i, y_i; heta) \ &= \prod_{i=1}^n p(y_i; heta) \ p(x_i|y_i; heta) \end{aligned}$$

这里有,

$$egin{aligned} p(y_i; heta) &= \pi^{y_i}(1-\pi)^{1-y_i} \ p(x_i|y_i; heta) &= \left(\mathcal{N}(x_i|\mu_1,\Sigma_1)
ight)^{\mathbb{I}\{y_i=1\}} \cdot \left(\mathcal{N}(x_i|\mu_0,\Sigma_0)
ight)^{\mathbb{I}\{y_i=0\}} \ \mathcal{N}(x|\mu,\Sigma) &= (2\pi)^{-rac{d}{2}}det(\Sigma)^{-rac{1}{2}}expigg\{ -rac{1}{2}(x-\mu)^{ op}\Sigma^{-1}(x-\mu) igg\} \end{aligned}$$

(1) 试求解 $heta=\{\pi,\mu_0,\Sigma_0,\mu_1,\Sigma_1\}$,以优化如下的目标函数,

$$\mathop{argmax}_{ heta} \log P(\mathcal{D})$$

(2) 令

$$egin{aligned} \log rac{p(y_1|x; heta)}{p(y_0|x; heta)} &= \log rac{p(y_1; heta) \ p(x|y_1; heta)}{p(y_0; heta) \ p(x|\mu_1,\Sigma_1)} \ &= \log rac{\pi \cdot \mathcal{N}(x|\mu_1,\Sigma_1)}{(1-\pi) \cdot \mathcal{N}(x|\mu_0,\Sigma_0)} \ &\doteq w^ op x + b \quad (where \ w \in \mathbb{R}^d, b \in \mathbb{R}) \end{aligned}$$

试求解w, b, 并探究w与 $(\mu_1 - \mu_0)$ 之间的关系?

注: 可能会用到的公式

$$egin{aligned} rac{\partial \ log \ det(\Sigma)}{\partial \Sigma} &= (\Sigma^{-1})^{ op} \ & \ rac{\partial \ (x^{ op}A^{-1}x)}{\partial A} &= -(A^{-1})^{ op}xx^{ op}(A^{-1})^{ op} \end{aligned}$$

更多公式可参考这里