COEN 4890 HW#1 Zhou Shen

+20

2. Linear Regression & LOOCV

(1) $\underset{n\times1}{Y} = \underset{n\times m}{H}\cdot\underset{m\times1}{X}$ $\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} H_{11} & \cdots & H_{1m} \\ H_{21} & \cdots & H_{2m} \\ \vdots & & \\ H_{n1} & \cdots & H_{nm} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} \Rightarrow y_i = H_{i1}X_1 + H_{i2}X_2 + \cdots + H_{im}\cdot X_m = \sum_{j=1}^{m} A_{ij}\cdot X_j$

$\Rightarrow \hat{Y}_i = \sum_{j=1}^{m} H_{ij}\cdot X_j \Rightarrow X = Y \Rightarrow \underline{\hat{Y}_i = \sum_{j=1}^{m} H_{ij}\cdot Y_j}$ +2

(2) $SSE(Z) = \sum_{i=1}^{n}(Z_i - \hat{Z}_i)^2 = \underset{(j=i)}{(Z_i - \hat{Z}_i)^2} + \sum_{j\neq i}^{n}(Z_j - \hat{Z}_j)^2$ $Z_j = \begin{cases} Y_j, & j\neq i \\ \hat{Y}_i^{(-i)}, & j = i \end{cases}$

$= (\hat{Y}_i^{(-i)} - \hat{Z}_i)^2 + \sum_{j\neq i}^{n}(Y_j - \hat{Z}_j)^2$ +5

In order to minimize SSE for Z, ① $(\hat{Y}_i^{(-i)} - \hat{Z}_i)^2 = 0$, ② $\sum_{j\neq i}^{n}(Y_j - \hat{Z}_j)^2 = 0$

In ①, when $\hat{Z}_i = \hat{Y}_i^{(-i)}$, it is minimum, $\hat{Y}^{(-i)}$ is estimator that minimizes SSE for Z.

In ②, when $\hat{Z}_j = Y_j$, because we know "$\hat{Y}^{(-i)}$ is the estimator of Y after removing the i-th observation" from question, $\hat{Y}^{(-i)}$ also minimizes $Y_j / \hat{Z}_j$. SSE $(\hat{Z})$ satisfies SSE $(\hat{Y}_i^{(-i)})$.

(3) $\hat{Y}_i^{(-i)} = Z_i$. Similar to (1), $\hat{Y}_i = \sum_{j=1}^{m} H_{ij}\cdot X_j$ +3

$\hat{Z}_i = \sum_{j=1}^{m} H_{ij}\cdot Z_j \longrightarrow \hat{Y}_i^{(-i)} = Z_i = \sum_{j=1}^{m} H_{ij}\cdot Z_j$

(4) $\hat{Y}_i - \hat{Y}_i^{(-i)} = \sum_{j=1}^{m} H_{ij}\cdot Y_j - \sum_{j=1}^{M} H_{ij}\cdot Z_j = \underset{(j=i)}{H_{ii}\cdot Y_i} + \sum_{j\neq i}^{M} H_{ij}\cdot Y_j - \underset{(j=i)}{H_{ii}\cdot Z_i} - \sum_{j\neq i}^{M} H_{ij}\cdot Z_j$

$= H_{ii}\cdot Y_i + \sum_{j\neq i}^{m} H_{ij}\cdot Y_j - H_{ii}\cdot\hat{Y}_i^{(-i)} - \sum_{j\neq i}^{m} H_{ij}\cdot Y_j = \underline{H_{ii}\cdot Y_i - H_{ii}\cdot\hat{Y}_i^{(-i)}}$ +5

(5) $\hat{Y}_i^{(-i)} - H_{ii}\hat{Y}_i^{(-i)} = \hat{Y}_i - H_{ii}Y_i$

$\hat{Y}_i^{(-i)}(1 - H_{ii}) = \hat{Y}_i - H_{ii}Y_i \Rightarrow \hat{Y}_i^{(-i)} = \frac{\hat{Y}_i - H_{ii}Y_i}{1 - H_{ii}}$

$LOOCV = \sum_{i=1}^{n}(Y_i - \hat{Y}_i^{(-i)})^2 = \sum_{i=1}^{n}\left(Y_i - \frac{\hat{Y}_i - H_{ii}Y_i}{1 - H_{ii}}\right)^2 = \sum_{i=1}^{n}\left(\frac{Y_i - H_{ii}Y_i - \hat{Y}_i + H_{ii}Y_i}{1 - H_{ii}}\right)^2$

$= \underline{\left(\frac{Y_i - \hat{Y}_i}{1 - H_{ii}}\right)^2}$ +5