An Auxin Driven Model for Ovule Primordia Development

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1 Model

Auxin is a important plant hormone that determines the development pattern of plants. The local peak of auxin in plant tissues determines the origin of primordium. During the ovule primordia development, the ovule primordia usually begins at a position where the concentration of auxin is high on the placenta. This model simplifies the growth of the placenta and the auxin distribution on the placenta into a one-dimensional problem. Auxin is transported between neighboring cells by PIN1, while PIN1 is polarized depending on auxin. The auxin is almost evenly distributed with tiny perturbations at the initial moment. We also consider cells growth and division to simulate the growth of the placenta.

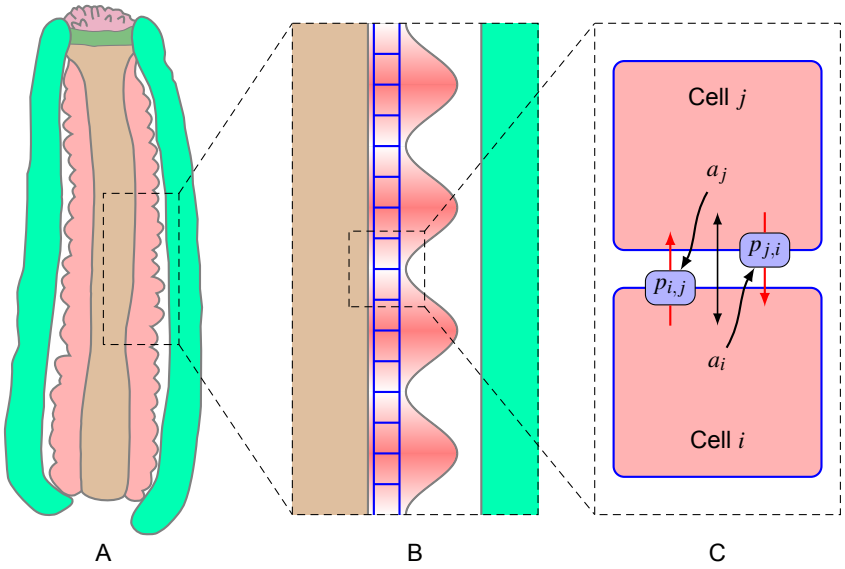

Figure 1: Schematic diagram of the model and its principle. (A) A pistil typically consists of a ovary, which contains the ovules; a stalk, arising from the ovary; and a pollen-receptive tip, the stigma. (B) The distribution of auxin on placenta is reduced to one dimension, i.e. only one layer of cells along the placenta direction is considered. (C) Auxin is transported between neighboring cells by PIN1, while PIN1 is polarized depending on auxin concentration of neighboring cells.

1.1 Auxin transport

In our model, cells are arranged in a one-dimensional space (Figure 1B). Auxin's concentrations in a cell is uniformly distributed, and it's concentrations in cell i is denoted by a_i. Auxin efflux carrier PIN1 is unevenly distributed to the cell membrane, and its density in the membrane of cell i toward neighboring cell j is denoted by $p_{i,j}$. Change of auxin concentration of cell i (a_i) is described by [1, 2, 3]

$$\frac{\mathrm{d}a_i}{\mathrm{d}t} = G_a(A - a_i) + D_a \sum_j (a_j - a_i) + \sum_j f_{i,j}, \tag{1}$$

$$f_{i,j} = E_p \left(p_{i,j} a_i - p_{j,i} a_j \right), \tag{2}$$

where cell j is a neighbor of cell i, G_a is the degradation rate, A is related to the synthesis rate, D_a is the diffusion coefficient, E_p is the efficiency of PIN1 efflux carrier, and $f_{i,j}$ ($= -f_{j,i}$) is the net flow of auxin by PIN1 from cell i to cell j, consisting of auxin efflux and influx. The first term of the right-hand side of equation 1 indicates that auxin is constantly synthesized and degraded at a constant rate. The second term indicates that auxin is transported by PIN1. The third term represents the diffusion of auxin between neighboring cells.

Change of PIN1 density ($p_{i,j}$) is described by

$$\frac{\mathrm{d}p_{i,j}}{\mathrm{d}t} = G_p \left(np \frac{\varphi_0(a_j)}{\sum_j \varphi_0(a_j)} - p_{i,j} \right) \tag{3}$$

where G_p is the degradation rate, n is the number of neighboring cells, p is a constant related to PIN1 density, and $\varphi_0(a_j)$ is the regulatory function for PIN1 polarization (Figure 1C). PIN1 is localized to cell membrane depending on the auxin concentration of neighboring cells and is degraded at a constant rate. The total PIN1 amount of cell i, $P_i = \sum_j p_{i,j}$, satisfies

$$\frac{\mathrm{d}p_i}{\mathrm{d}t} = G_p(np - P_i). \tag{4}$$

The above equation indicate that the stable equilibrium of p_i is np. Thus, equilibria of a_i and $p_{i,j}$ are given respectively by

$$a_{\mathrm{eq}} = A, \quad p_{\mathrm{eq}} = p. \tag{5}$$

When G_p is sufficiently large, $p_{i,j}$ quickly approaches equilibrium:

$$p_{i,j} = np \frac{\varphi_0(a_j)}{\sum_j \varphi_0(a_j)} \tag{6}$$

To simplify the model, we take $\varphi_0(a_j) = a_j$ and equations (1-3) can be simplified to

$$\frac{\mathrm{d}a_i}{\mathrm{d}t} = G_a(A - a_i) + D_a \sum_j (a_j - a_i) + E_p \sum_j (p_{i,j} a_i - p_{j,i} a_j), \tag{7}$$

$$p_{i,j} = np \frac{a_j}{\sum_j a_j}. \tag{8}$$

In this paper, the auxin-regulated ovule development is simplified to the one-dimensional periodic boundary problem. Therefore, $j = i \pm 1$ and $n = 2$ in Equations (7) and (8).

1.2 Cells growth and division

The growth of placenta is a macroscopic manifestation of cell growth and division. To simulate the growth of the placenta, we need consider cell growth and division. Since our model is one-dimensional, we only consider the growth and division of cells in the length of the placenta. Cell growth satisfies the following equation

$$\frac{\mathrm{d}\ell_i}{\mathrm{d}t} = r\ell_i, \tag{9}$$

where ℓ_i is the length of the cell. r is the rate of growth of the cells, usually associated with auxin levels in the cells.

As the cells grow, the length of the cell, ℓ_i, continues to grow. When the cell length ℓ_i is greater than a division threshold ℓ_{div}, the cell divides into two equal-length daughter cells, both of which share the auxin of the mother cell:

$$\ell_{\text{daughter1}} = \ell_{\text{daughter2}} = \frac{1}{2}\ell_{\text{mother}}, \qquad a_{\text{daughter1}} = a_{\text{daughter2}} = \frac{1}{2}a_{\text{mother}} \tag{10}$$

2 Implementation

The model was implemented in a MATLAB program. The differential equations (7) and (9) are solved by the difference method. At the initial time, the simulation system consists of 50 cells of length 1, each with a auxin level of $a = 0.95 + 0.03(1 - i/50)^2 + 0.02\theta$, where θ is a random number uniformly distributed from 0 to 1. This will result in a slightly higher initial auxin level at the base than at the top. We used the following parameters: simulation time step $\Delta t = 5 \times 10^{-4}$, $D_a = 0.5$, PIN1 density constant $p = 1$, efficiency of PIN1 efflux carrier $E_p = 1$, auxin degradation rate $G_a = 0.1$, cell length threshold $\ell_{\text{div}} = 4$, cell growth rate $r = 0.01$.

3 Result

In order to make the spatial variation of the auxin concentration smoother, we performed cubic spline interpolation on the simulation results.

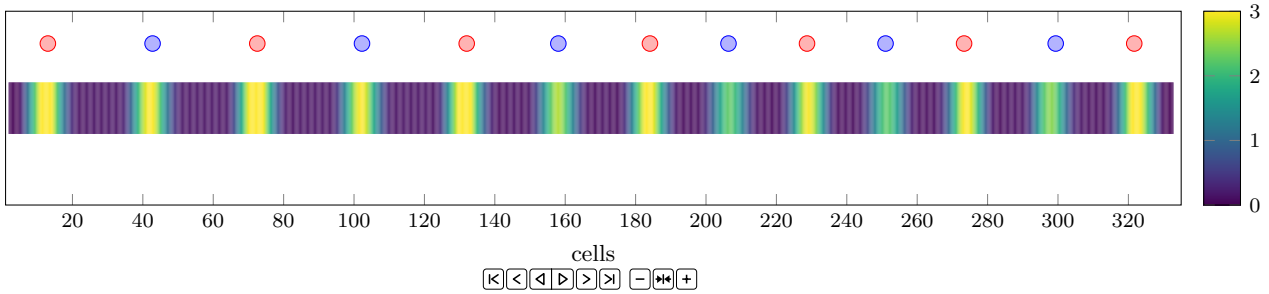

Figure 2: Simulated animation (you may need adobe reader to play them).

Figure 2 and 3 shows the spatiotemporal distribution of the auxin concentration after interpolation and the length of the placenta. The simulation results show that under the action of PIN, the initial uniform distribution of auxin spontaneously forms the first periodic high points. As the cells grow and divide, the entire tissue grows longer. Subsequently, the second batch of auxin concentrations high points emerged.

References

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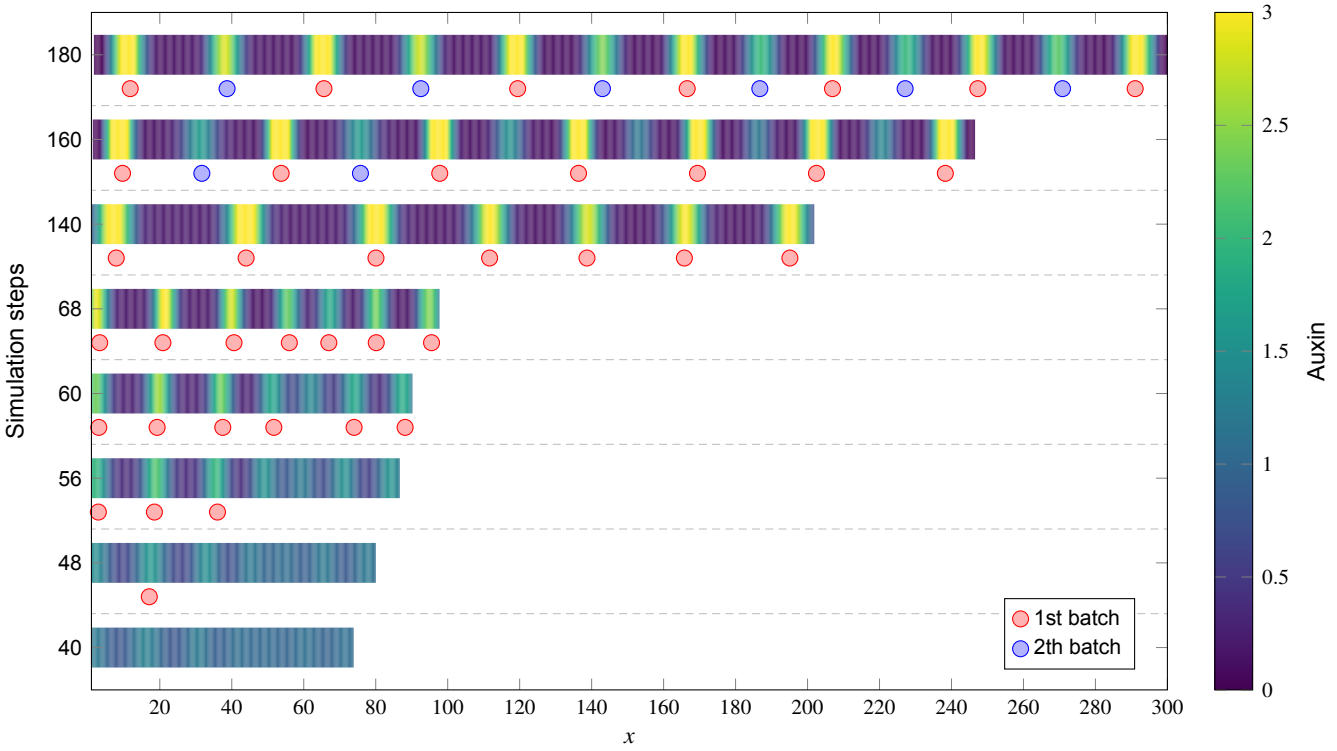

Figure 3: Placenta length and auxin concentration distribution at different simulation time steps. The red dot indicates where the first batch of ovule, and the blue dot indicates where the second batch ovule.

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[3] Hironori Fujita and Masayoshi Kawaguchi. Spatial regularity control of phyllotaxis pattern generated by the mutual interaction between auxin and pin1. *PLoS computational biology*, 14(4):e1006065, 2018.