

# Assignment One for CS-6648

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July 9, 2015

## 1 Question 1

### 1.1 Question

Solve the following linear programming problem:

$$\begin{aligned} \text{Min } f(x_1, x_2) : & x_1 + x_2 \\ \text{Subject to: } & 3x_1 - x_2 \leq 3 \\ & x_1 + 2x_2 \leq 5 \\ & x_1 + x_2 \leq 4 \\ & x_1 \geq 0; x_2 \text{ unrestricted in sign.} \end{aligned} \quad (8)$$

### 1.2 Answer

Answer:

Because of  $x_2$  is unrestricted, so  $x_2$  may be negative, so we can let:

$$x_2 = x'_2 - x''_2$$

Table 1: Describe the formulas by Table

$x_1$	$x'_2$	$x''_2$	$s_1$	$s_2$	$s_3$	b	r
3	-1	1	1	0	0	0	3
1	2	-2	0	1	0	0	5
1	1	-1	0	0	1	1	4
1	1	-1	0	0	0	0	0

We can find the most negative line is  $x''_2$ , so we do  $r/x''_2$

Table 2:  $r/x_2''$ 

$x_1$	$x_2'$	$x_2''$	$s_1$	$s_2$	$s_3$	b	r	
3	-1	1	1	0	0	0	3	3
1	2	-2	0	1	0	0	5	-2.5
1	1	-1	0	0	1	1	4	-4
1	1	-1	0	0	0	0	0	

We can find the Smallest Positive Number is 3.

Table 3: Row Operation

$x_1$	$x_2'$	$x_2''$	$s_1$	$s_2$	$s_3$	b	r
3	-1	1	1	0	0	0	3
7	0	0	2	1	0	0	11
4	0	0	1	0	1	1	7
4	0	0	1	0	0	0	3

As the result we know:

$$\text{Min } f(x_1, x_2) = 3$$

## 2 Question 2

### 2.1 Question

The local community college is planning to grow the biotechnology offering through new federal and state grants. An ambitious program is being planned for recruiting at least 200 students from in and out of state. They are to recruit at least 40 out of state students. They will attempt to recruit at least 30 students who are in the top 20 % of their graduating high school class. Current figures indicate that about 8 % of the applicants from instate, and 6 % of the applicants from out of state belong to this pool. They also plan to recruit at least 40 students who have AP courses in biology. The data suggests that 10 % and 15 % of in state and out of state applicants respectively, belong to this pool. They anticipate that the additional cost per student is 800 *foreach instate student and 1200* for each out of state student. Find their actual enrollment needed to minimize cost and their actual cost.

Hint: Optimal Value= 324,000

### 2.2 Answer

We can formulize the Question to:

$$\text{Max } f(x_1, x_2) : 800x_1 + 100x_2$$

$$x_1 + x_2 \geq 1200$$

$$x_2 \geq 40$$

$$0.08x_1 + 0.06x_2 \geq 30$$

$$0.1x_1 + 0.15x_2 \geq 40$$

Table 4: Use table describe the formulas

$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	$s_4$	$a_1$	$a_2$	$a_3$	$a_4$	b
1	1	-1	0	0	0	1	0	0	0	200
0	1	0	-1	0	0	0	1	0	0	40
0.08	0.06	0	0	-1	0	0	0	1	0	30
0.1	0.15	0	0	0	-1	0	0	0	1	40
800	1200	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	1	0

Stage 1: Remove the Artifical variables one by one

Table 5: Remove the first artifical variable

$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	$s_4$	$a_1$	$a_2$	$a_3$	$a_4$	b
1	1	-1	0	0	0	1	0	0	0	200
0	1	0	-1	0	0	0	1	0	0	40
0.08	0.06	0	0	-1	0	0	0	1	0	30
0.1	0.15	0	0	0	-1	0	0	0	1	40
800	1200	0	0	0	0	0	0	0	0	0
-1	-1	1	0	0	0	0	1	1	1	-200

Table 6: Remove the second artifical variable

$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	$s_4$	$a_1$	$a_2$	$a_3$	$a_4$	b
1	1	-1	0	0	0	1	0	0	0	200
0	1	0	-1	0	0	0	1	0	0	40
0.08	0.06	0	0	-1	0	0	0	1	0	30
0.1	0.15	0	0	0	-1	0	0	0	1	40
800	1200	0	0	0	0	0	0	0	0	0
-1	-2	1	1	0	0	0	0	1	1	-240

Table 7: Remove third artificial variable

$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	$s_4$	$a_1$	$a_2$	$a_3$	$a_4$	b
1	1	-1	0	0	0	1	0	0	0	200
0	1	0	-1	0	0	0	1	0	0	40
0.08	0.06	0	0	-1	0	0	0	1	0	30
0.1	0.15	0	0	0	-1	0	0	0	1	40
800	1200	0	0	0	0	0	0	0	0	0
-1.08	-2.06	1	1	1	0	0	0	0	1	-270

Table 8: Remove forth artificial variable

$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	$s_4$	$a_1$	$a_2$	$a_3$	$a_4$	b
1	1	-1	0	0	0	1	0	0	0	200
0	1	0	-1	0	0	0	1	0	0	40
0.08	0.06	0	0	-1	0	0	0	1	0	30
0.1	0.15	0	0	0	-1	0	0	0	1	40
800	1200	0	0	0	0	0	0	0	0	0
-1.18	-2.21	1	1	1	0	0	0	0	1	-310

Stage 2: Row operation

We can get the smallest col is  $x_2$ , so we  $b/x_2$ :

Table 9:  $b/x_2$ 

$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	$s_4$	$a_1$	$a_2$	$a_3$	$a_4$	b	r
1	1	-1	0	0	0	1	0	0	0	200	200
0	1	0	-1	0	0	0	1	0	0	40	40
0.08	0.06	0	0	-1	0	0	0	1	0	30	500
0.1	0.15	0	0	0	-1	0	0	0	1	40	667
800	1200	0	0	0	0	0	0	0	0	0	0
-1.18	-2.21	1	1	1	-1	0	0	0	0	-310	932

We can find pivot is  $x_2$  where  $r = 40$ .

Table 10: Use pivot  $x_2$  where  $r = 40$  to Row operation

$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	$s_4$	$a_1$	$a_2$	$a_3$	$a_4$	b	r
1	0	-1	1	0	0	1	-1	0	0	160	160
0	1	0	-1	0	0	0	1	0	0	40	-40
0.08	0	0	0.06	-1	0	0	-0.06	1	0	27.6	460
0.1	0	0	0.15	0	-1	0	-0.15	0	1	34	667
800	0	0	1200	0	0	0	-1200	0	0	-48000	-40
-1.18	0	1	-1.21	1	-1	0	2.21	0	0	-221.6	959

$r/s_1$  we found 160 is smallest positive number

Table 11:  $r/s_1$

$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	$s_4$	$a_1$	$a_2$	$a_3$	$a_4$	b	r
1	0	-1	1	0	0	1	-1	0	0	160	160
1	1	-1	0	0	0	1	0	0	0	200	-200
0.02	0	0.06	0	-1	0	-0.06	0	1	0	18	300
-0.05	0	0.15	0	0	-1	-0.15	0	0	1	10	667
-400	0	1200	0	0	0	-1200	0	0	0	-240000	-200
0.03	0	-0.21	0	1	-1	1.21	1	0	0	-28.6	333

Table 12: Row operation

$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	$s_4$	$a_1$	$a_2$	$a_3$	$a_4$	b	r
0.67	0	0	1	0	-6.7	0	-1	0	6.7	222.67	-34
0.67	1	0	0	0	-6.7	0	0	0	6.7	266.67	-40
0.04	0	0	0	-1	0.4	0	0	1	-0.4	14	35
-0.33	0	1	0	0	-6.7	-1	0	0	6.7	66.67	-10
0	0	0	0	0	8000	0	0	0	-8000	-320000	-40
-0.04	0	0	0	1	-0.4	1	1	0	1.4	-14	35

Table 13: Row operation

$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	$s_4$	$a_1$	$a_2$	$a_3$	$a_4$	b
1.33	0	0	1	-16.67	0	0	-1	16.67	0	460
1.33	1	0	0	-16.67	0	0	0	16.67	0	500
0.1	0	0	0	-2.5	1	0	0	2.5	-1	35
0.33	0	1	0	-16.67	0	-1	0	16.67	0	300
-800	0	0	0	2000	0	0	0	-20000	0	-600000
0	0	0	0	0	0	1	1	1	1	0