Assignment One for CS-6648

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1 Question 1

1.1 Question

Solve the following linear programming problem:

$$\begin{aligned} \textbf{Min } f(x_1, x_2): \ & x_1 + x_2 \\ \textbf{Subject to: } & 3x_1 - x_2 \leq 3 \\ & x_1 + 2x_2 \leq 5 \\ & x_1 + x_2 \leq 4 \\ & x_1 \geq 0; x_2 \textit{ unrestricted in sign.} \end{aligned}$$

1.2 Answer

Answer:

Because of x_2 is unrestricted, x_2 may be negative, so we can let:

$$x_2 = x_2' - x_2''$$

Table 1: Describe the formulas by Table

x_1	x_2'	x_2''	s_1	s_2	s_3	b	r
3	-1	1	1	0	0	0	3
1	2	-2	0	1	0	0	5
1	1	-1	0	0	1	1	4
1	1	-1	0	0	0	0	0

We can find the most negative line is x_2'', so we do r/x_2''

Table 2: r/x_2''

x_1	x_2'	x_2''	s_1	s_2	s_3	b	r	
3	-1	1	1	0	0	0	3	3
1	2	-2	0	1	0	0	5	-2.5
1	1	-1	0	0	1	1	4	-4
1	1	-1	0	0	0	0	0	

We can find the Smallest Positive Number is 3.

Table 3: Row Operation

x_1	x_2'	x_2''	s_1	s_2	s_3	b	r
3	-1	1	1	0	0	0	3
7	0	0	2	1	0	0	11
4	0	0	1	0	1	1	7
4	0	0	1	0	0	0	3

As the result we know:

$$\textbf{Min } f(x_1, x_2) = 3$$

2 Question 2

2.1 Quesion

The local community college is planning to grow the biotechnology offering through new federal and state grants. An ambitious program is being planned for recruiting at least 200 students from in and out if state. They are to recruit at least 40 out of state students. They will attempt to recruit at least 30 students who are in the top 20 % of their graduating high school class. Current figures indicate that about 8 % of the applicants from instate, and 6 % of the applicants from out of state belong to this pool. They also plan to recruit at least 40 students who have AP courses in biology. The data suggests that 10 % and 15 % of in state and out of state applicants respectively, belong to this pool. They anticipate that the additional cost per student is $800 for each in state student and 1200$ for each out of state student. Find their actual enrollment needed to minimize cost and their actual cost.

Hint: Optimal Value= 324,000

2.2 Answer

We can formulize the Question to:

$$\begin{aligned}
\textbf{Min } f(x_1, x_2) : 800x_1 + 1200x_2 \\
x_1 + x_2 \geq 1200 \\
x_2 \geq 40 \\
0.08x_1 + 0.06x_2 \geq 30 \\
0.1x_1 + 0.15x_2 \geq 40
\end{aligned}$$

Table 4: Use table describe the formulas

x_1	x_2	s_1	s_2	s_3	s_4	a_1	a_2	a_3	a_4	b
1	1	-1	0	0	0	1	0	0	0	200
0	1	0	-1	0	0	0	1	0	0	40
0.08	0.06	0	0	-1	0	0	0	1	0	30
0.1	0.15	0	0	0	-1	0	0	0	1	40
800	1200	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	1	0

Stage 1: Remove the Artifical variables one by one

Table 5: Remove the first artifical varaible

x_1	x_2	s_1	s_2	s_3	s_4	a_1	a_2	a_3	a_4	b
1	1	-1	0	0	0	1	0	0	0	200
0	1	0	-1	0	0	0	1	0	0	40
0.08	0.06	0	0	-1	0	0	0	1	0	30
0.1	0.15	0	0	0	-1	0	0	0	1	40
800	1200	0	0	0	0	0	0	0	0	0
-1	-1	1	0	0	0	0	1	1	1	-200

Table 6: Remove the second artifical varaible

x_1	x_2	s_1	s_2	s_3	s_4	a_1	a_2	a_3	a_4	b
1	1	-1	0	0	0	1	0	0	0	200
0	1	0	-1	0	0	0	1	0	0	40
0.08	0.06	0	0	-1	0	0	0	1	0	30
0.1	0.15	0	0	0	-1	0	0	0	1	40
800	1200	0	0	0	0	0	0	0	0	0
-1	-2	1	1	0	0	0	0	1	1	-240

Table 7: Remove third artifical varaible

x_1	x_2	s_1	s_2	s_3	s_4	a_1	a_2	a_3	a_4	b
1	1	-1	0	0	0	1	0	0	0	200
0	1	0	-1	0	0	0	1	0	0	40
0.08	0.06	0	0	-1	0	0	0	1	0	30
0.1	0.15	0	0	0	-1	0	0	0	1	40
800	1200	0	0	0	0	0	0	0	0	0
-1.08	-2.06	1	1	1	0	0	0	0	1	-270

Table 8: Remove forth artifical varaible

x_1	x_2	s_1	s_2	s_3	s_4	a_1	a_2	a_3	a_4	b
1	1	-1	0	0	0	1	0	0	0	200
0	1	0	-1	0	0	0	1	0	0	40
0.08	0.06	0	0	-1	0	0	0	1	0	30
0.1	0.15	0	0	0	-1	0	0	0	1	40
800	1200	0	0	0	0	0	0	0	0	0
-1.18	-2.21	1	1	1	1	0	0	0	0	-310

Stage 2: Row operation, We can eliminate artifical variables now

Table 9: Find the most negative number

x_1	x_2	s_1	s_2	s_3	s_4	b
1	1	-1	0	0	0	200
0	1	0	-1	0	0	40
0.08	0.06	0	0	-1	0	30
0.1	0.15	0	0	0	-1	40
800	1200	0	0	0	0	0
-1.18	-2.21	1	1	1	1	-310

We can get the smallest column is x_2, so we b/x_2:

Table 10: b/x_2

x_1	x_2	s_1	s_2	s_3	s_4	b	r
1	1	-1	0	0	0	200	200
0	1	0	-1	0	0	40	40
0.08	0.06	0	0	-1	0	30	500
0.1	0.15	0	0	0	-1	40	266.666...
800	1200	0	0	0	0	0	0
-1.18	-2.21	1	1	1	-1	-310	140.271...

We can find pivot is x_2 where $r = 40$.

Table 11: Use pivot x_2 where $r = 40$ to Row operation

x_1	x_2	s_1	s_2	s_3	s_4	b	r
1	0	-1	1	0	0	160	160
0	1	0	-1	0	0	40	-40
0.08	0	0	0.06	-1	0	27.6	460
0.1	0	0	0.15	0	-1	34	266.666...
800	0	0	1200	0	0	-48000	-40
-1.18	0	1	-1.21	1	-1	-221.6	183.140...

r/s_1 we found 160 is smallest positive number

Table 12: b/s_1

x_1	x_2	s_1	s_2	s_3	s_4	b	r
1	0	-1	1	0	0	160	160
1	1	-1	0	0	0	200	-200
0.02	0	0.06	0	-1	0	18	300
-0.05	0	0.15	0	0	-1	10	66.666...
-400	0	1200	0	0	0	-240000	-200
0.03	0	-0.21	0	1	1	-28	133.333...

Table 13: Row operation

x_1	x_2	s_1	s_2	s_3	s_4	b	r
0.67	0	0	1	0	-6.7	222.67	-34
0.67	1	0	0	0	-6.7	266.67	-40
0.04	0	0	0	-1	0.4	14	35
-0.33	0	1	0	0	-6.7	66.67	-10
0	0	0	0	0	8000	-320000	-40
-0.04	0	0	0	1	-0.4	-14	35

Table 14: Row operation

x_1	x_2	s_1	s_2	s_3	s_4	b	r
1.33	0	0	1	-16.67	0	460	345
1.33	1	0	0	-16.67	0	500	375
0.1	0	0	0	-2.5	1	35	350
0.33	0	1	0	-16.67	0	300	900
-800	0	0	0	20000	0	-600000	750
-0.00	0	0	0	0	0	0	0

Table 15: Row operation

x_1	x_2	s_1	s_2	s_3	s_4	b
1	0	0	0.75	-12.5	0	345
0	1	0	-1	0	0	40
0	0	0	-0.075	-1.25	1	0.50
0	0	1	-0.25	-12.5	0	185
0	0	0	600	10000	0	-32400
0	0	0	0	0	0	0

As a result, we get $x_1 = 345$, $x_2 = 40$
Min $f(x_1, x_2) = 800 * x_1 + 1200 * x_2 = 32000$