Assignment Two for CS-6648

Dong Zhou

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Problem to work with:

$$\begin{aligned} \textbf{Maximize: } & 24x_1 + 2x_2 + 20x_3 + 4x_4 \\ \textbf{Subject to: } & 8x_1 + 1x_2 + 5x_3 + 4x_4 \leq 9 \\ & x_i \in \{0, 1\} \textbf{ for } i = 1 \textbf{ to } 4 \end{aligned}$$

Rewrite formulars 1:

We translate [$x_i \in \{0, 1\}$ **for** $i = 1$ **to** 4] to constraints:

$$\begin{aligned} \textbf{Maximize: } & 24x_1 + 2x_2 + 20x_3 + 4x_4 \\ \textbf{Subject to: } & 8x_1 + 1x_2 + 5x_3 + 4x_4 \leq 9 \\ & x_1 \leq 1 \\ & x_2 \leq 1 \\ & x_3 \leq 1 \\ & x_4 \leq 1 \\ & x_i \geq 0 \textbf{ for } i = 1 \textbf{ to } 4 \end{aligned}$$

Rewrite formulars 2:

$$\begin{aligned} \textbf{Minimize: } & -24x_1 - 2x_2 - 20x_3 - 4x_4 \\ \textbf{Subject to: } & 8x_1 + 1x_2 + 5x_3 + 4x_4 + s_1 = 9 \\ & x_1 + s_2 = 1 \\ & x_2 + s_3 = 1 \\ & x_3 + s_4 = 1 \\ & x_4 + s_5 = 1 \\ & x_i \geq 0 \textbf{ for } i = 1 \textbf{ to } 4 \\ & s_j \geq 0 \textbf{ for } j = 1 \textbf{ to } 5 \end{aligned}$$

Steps of resolve the problem:

Step 1

x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b	r
8	1	5	4	1	0	0	0	0	9	1.125
1	0	0	0	0	1	0	0	0	1	1
0	1	0	0	0	0	1	0	0	1	0
0	0	1	0	0	0	0	1	0	1	0
0	0	0	1	0	0	0	0	1	1	0
-24	-2	-20	-4	0	0	0	0	0	0	0

x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b	r
0	1	5	4	1	-8	0	0	0	1	0.2
1	0	0	0	0	1	0	0	0	1	0
0	1	0	0	0	0	1	0	0	1	0
0	0	1	0	0	0	0	1	0	1	1
0	0	0	1	0	0	0	0	1	1	0
0	-2	-20	-4	0	24	0	0	0	24	-1.2

x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b	r
0	0.2	1	0.8	0.2	-1.6	0	0	0	0.2	-0.125
1	0	0	0	0	1	0	0	0	1	1
0	1	0	0	0	0	1	0	0	1	0
0	-0.2	0	-0.8	-0.2	1.6	0	1	0	0.8	0.5
0	0	0	1	0	0	0	0	1	1	0
0	2	0	12	4	-8	0	0	0	28	-3.5

x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	s_5	b
0	0	1	0	0	0	0	1	0	1
1	0.125	0	0.5	0.125	0	0	-0.625	0	0.5
0	1	0	0	0	0	0	0	0	1
0	-0.125	0	-0.5	-0.125	1	0	0.625	0	0.5
0	0	0	1	0	0	0	0	1	1
0	1	0	8	3	0	0	5	0	32

So we get: $x_1 = 0.5,\ x_3 = 1,\ LP(1) = 32$

Step 2: $x_1 = 0$

x_2	x_3	x_4	s_1	s_2	s_3	s_4	b	r
1	5	4	1	0	0	0	9	1.8
1	0	0	0	1	0	0	1	0
0	1	0	0	0	1	0	1	1
0	0	1	0	0	0	1	1	0
-2	-20	-4	0	0	0	0	0	0

x_2	x_3	x_4	s_1	s_2	s_3	s_4	b	r
1	0	4	1	0	-5	0	4	1
1	0	0	0	1	0	0	1	0
0	1	0	0	0	1	0	1	0
0	0	1	0	0	0	1	1	1
-2	0	-4	0	0	20	0	20	-5

x_2	x_3	x_4	s_1	s_2	s_3	s_4	b	r
0.25	0	1	0.25	0	-1.25	0	1	4
1	0	0	0	1	0	0	1	1
0	1	0	0	0	1	0	1	0
-0.25	0	0	-0.25	0	1.25	1	0	0
-1	0	0	1	0	15	0	24	-24

x_2	x_3	x_4	s_1	s_2	s_3	s_4	b
0	0	1	0.25	-0.25	-1.25	0	0.75
1	0	0	0	1	0	0	1
0	1	0	0	0	1	0	1
0	0	0	-0.25	0.25	1.25	1	0.25
0	0	0	1	1	15	0	25

So we get: $x_2 = 1,\ x_3 = 1,\ x_4 = 0.75,\ LP(2) = 25$

Step 3

$$\begin{aligned} \textbf{Because of: } & x_1 = 1 \\ \textbf{So: } & 8x_1 + 1x_2 + 5x_3 + 4x_4 \leq 9 \\ & 8 + 1x_2 + 5x_3 + 4x_4 \leq 9 \\ & 1x_2 + 5x_3 + 4x_4 \leq 1 \end{aligned}$$

x_2	x_3	x_4	s_1	s_2	s_3	s_4	b	r
1	5	4	1	0	0	0	1	0.2
1	0	0	0	1	0	0	1	0
0	1	0	0	0	1	0	1	1
0	0	1	0	0	0	1	1	0
-2	-20	-4	0	0	0	0	0	0

x_2	x_3	x_4	s_1	s_2	s_3	s_4	b
0.2	1	0.8	0.2	0	0	0	0.2
1	0	0	0	1	0	0	1
-0.2	0	-0.8	-0.2	0	1	0	0.8
0	0	1	0	0	0	1	1
2	0	12	4	0	0	0	4

So we get: $x_1 = 1,\ x_3 = 0.2,\ LP(3) = 24 * 1 + 20 * 0.2 = 24 + 4 = 28$

Step 4 $x_1 = 0,\ x_2 = 0$

x_3	x_4	s_1	s_2	s_3	b	r
5	4	1	0	0	9	1.8
1	0	0	1	0	1	1
0	1	0	0	1	1	0
-20	-4	0	0	0	0	0

x_3	x_4	s_1	s_2	s_3	b	r
0	4	1	-5	0	4	1
1	0	0	1	0	1	0
0	1	0	0	1	1	1
0	-4	0	20	0	20	-5

x_3	x_4	s_1	s_2	s_3	b
0	0	1	-5	-4	0
1	0	0	1	0	1
0	1	0	0	1	1
0	0	0	20	4	24

So we get: $x_3 = 1, x_4 = 1, LP(4) = 24$

Step 5

Because of: $x_1 = 0,\ x_2 = 1$

So: $8x_1 + 1x_2 + 5x_3 + 4x_4 \leq 9$

$0 + 1 + 5x_3 + 4x_4 \leq 9$

$5x_3 + 4x_4 \leq 8$

x_3	x_4	s_1	s_2	s_3	b	r
5	4	1	0	0	8	1.6
1	0	0	1	0	1	1
0	1	0	0	1	1	0
-20	-4	0	0	0	0	0

x_3	x_4	s_1	s_2	s_3	b	r
0	4	1	-5	0	3	0.75
1	0	0	1	0	1	0
0	1	0	0	1	1	1
0	-4	0	20	0	20	-5

x_3	x_4	s_1	s_2	s_3	b
0	1	0.25	-1.25	0	0.75
1	0	0	1	0	1
0	0	-0.25	1.25	1	0.25
0	0	1	15	0	23

So we get: $x_1 = 0,\ x_2 = 1,\ x_3 = 1,\ x_4 = 0.75,\ LP(5) = 23 + 2 = 25$

Step 6

$$\begin{aligned} \textbf{Because of: } & x_1 = 1,\ x_2 = 0 \\ \textbf{So: } & 8x_1 + 1x_2 + 5x_3 + 4x_4 \leq 9 \\ & 8 + 0 + 5x_3 + 4x_4 \leq 9 \\ & 5x_3 + 4x_4 \leq 1 \end{aligned}$$

x_3	x_4	s_1	s_2	s_3	b	r
5	4	1	0	0	1	0.2
1	0	0	1	0	1	1
0	1	0	0	1	1	0
-20	-4	0	0	0	0	0

x_3	x_4	s_1	s_2	s_3	b
1	0.8	0.2	0	0	0.2
0	-0.8	-0.2	1	0	0.8
0	1	0	0	1	1
0	12	4	0	0	4

So we get: $x_1 = 1,\ x_2 = 0,\ x_3 = 0.2,\ LP(6) = 24 * 1 + 0 + 0.2 * 20 = 24 + 4 = 28$

Step 7

$$\begin{aligned} \textbf{Because of: } & x_1 = 1,\ x_2 = 1 \\ \textbf{So: } & 8x_1 + 1x_2 + 5x_3 + 4x_4 \leq 9 \\ & 8 + 1 + 5x_3 + 4x_4 \leq 9 \\ & 5x_3 + 4x_4 \leq 0 \\ \textbf{We know: } & x_i \geq 0 \textbf{ for } i = 1 \textbf{ to } 4 \\ \textbf{So: } & x_3 = 0,\ x_4 = 0 \\ \textbf{We can get: } & 24x_1 + 2x_2 + 20x_3 + 4x + 4 = 24 + 2 + 0 + 0 = 26 \end{aligned}$$

Step 8

$$\begin{aligned}\textbf{Because of: } & x_1 = 0,\ x_2 = 1,\ x_3 = 0 \\ \textbf{So: } & 8x_1 + 1x_2 + 5x_3 + 4x_4 \leq 9 \\ & 0 + 1 + 0 + 4x_4 \leq 9 \\ & 4x_4 \leq 8 \\ & x_4 \leq 2\end{aligned}$$

x_4	s_1	s_2	b	r
1	1	0	2	2
1	0	1	1	1
-4	0	0	0	0

x_4	s_1	s_2	r
0	1	-1	1
1	0	1	1
0	0	4	4

So we get:

$$x_1 = 0, x_2 = 1, x_3 = 0, x_4 = 1$$

$$LP(8) = 24 * 0 + 2 * 1 + 20 * 0 + 4 * 1 = 6$$

Step 9

$$\begin{aligned}\textbf{Because of: } & x_1 = 0,\ x_2 = 1,\ x_3 = 1 \\ \textbf{So: } & 8x_1 + 1x_2 + 5x_3 + 4x_4 \leq 9 \\ & 0 + 1 + 5 + 4x_4 \leq 9 \\ & 4x_4 \leq 3\end{aligned}$$

x_4	s_1	s_2	b	r
4	1	0	3	0.75
1	0	1	1	1
-4	0	0	0	0

x_4	s_1	s_2	b
1	0.25	0	0.75
0	-0.25	1	0.25
0	1	0	3

So we get:

$$x_1 = 0, x_2 = 1, x_3 = 1, x_4 = 0.75$$
$$LP(8) = 24 * 0 + 2 * 1 + 20 * 1 + 4 * 0.75 = 25$$

Step 10

Because of: $x_1 = 1,\ x_2 = 0,\ x_3 = 0$
So: $8x_1 + 1x_2 + 5x_3 + 4x_4 \leq 9$
$8 + 0 + 0 + 4x_4 \leq 9$
$4x_4 \leq 1$

x_4	s_1	s_2	b	r
4	1	0	1	0.25
1	0	1	1	1
-4	0	0	0	0

x_4	s_1	s_2	b
1	0.25	0	0.25
0	-0.25	1	0.75
0	1	0	1

So we get:

$$x_1 = 1, x_2 = 0, x_3 = 0, x_4 = 0.25$$
$$LP(10) = 24 * 1 + 2 * 0 + 20 * 0 + 4 * 0.25 = 25$$

Step 11

Because of: $x_1 = 1,\ x_2 = 0,\ x_3 = 1$
So: $8x_1 + 1x_2 + 5x_3 + 4x_4 \leq 9$
$8 + 0 + 5 + 4x_4 \leq 9$
$13 + 4x_4 \leq 9$
So it is not feasible.

In Step 7 we get the best solution: 26