Assignment 1 for CS-6648

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1 Solve the following linear programming problem:

$$\begin{aligned} \textbf{Min } f(x_1, x_2): \ & x_1 + x_2 \\ \textbf{Subject to: } & 3x_1 - x_2 \leq 3 \\ & x_1 + 2x_2 \leq 5 \\ & x_1 + x_2 \leq 4 \\ x_1 \geq 0; x_2 \textit{ unrestricted in sign.} \quad & \textbf{(8)} \end{aligned}$$

Answer:
Let:

$$x_2 = x_2' - x_2''$$

Table 1: Formulation

x_1	x_2'	x_2''	s_1	s_2	s_3	b	r
3	-1	1	1	0	0	0	3
1	2	-2	0	1	0	0	5
1	1	-1	0	0	1	1	4
1	1	-1	0	0	0	0	0

Table 2: Find Smallest Positive Number

x_1	x_2'	x_2''	s_1	s_2	s_3	b	r	
3	-1	1	1	0	0	0	3	3
1	2	-2	0	1	0	0	5	-2.5
1	1	-1	0	0	1	1	4	-4
1	1	-1	0	0	0	0	0	

Table 3: Row Operation

x_1	x_2'	x_2''	s_1	s_2	s_3	b	r
3	-1	1	1	0	0	0	3
7	0	0	2	1	0	0	11
4	0	0	1	0	1	1	7
4	0	0	1	0	0	0	3

As the result we know:

$$\textbf{Min } f(x_1, x_2) = 3$$

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The local community college is planning to grow the biotechnology offering through new federal and state grants. An ambitious program is being planned for recruiting at least 200 students from in and out if state. They are to recruit at least 40 out of state students. They will attempt to recruit at least 30 students who are in the top 20 % of their graduating high school class. Current figures indicate that about 8 % of the applicants from instate, and 6 % of the applicants from out of state belong to this pool. They also plan to recruit at least 40 students who have AP courses in biology. The data suggests that 10 % and 15 % of in state and out of state applicants respectively, belong to this pool. They anticipate that the additional cost per student is $800 for each in state student and 1200$ for each out of state student. Find their actual enrollment needed to minimize cost and their actual cost.

Hint: Optimal Value= 324,000

As the question we can get:

$$\textbf{Min } f(x_1, x_2) : 800x_1 + 1200x_2 \tag{1}$$
$$x_1 + x_2 \geq 1200 \tag{2}$$
$$x_2 \geq 40 \tag{3}$$
$$0.08x_1 + 0.06x_2 \geq 30 \tag{4}$$
$$0.1x_1 + 0.15x_2 \geq 40 \tag{5}$$

Table 4: Formulation

x_1	x_2	s_1	s_2	s_3	s_4	a_1	a_2	a_3	a_4	b
1	1	-1	0	0	0	1	0	0	0	200
0	1	0	-1	0	0	0	1	0	0	40
0.08	0.06	0	0	-1	0	0	0	1	0	30
0.1	0.15	0	0	0	-1	0	0	0	1	40
800	1200	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	1	0

Remove the Artifical variables

Table 5: Remove First Artifical Varaible

x_1	x_2	s_1	s_2	s_3	s_4	a_1	a_2	a_3	a_4	b
1	1	-1	0	0	0	1	0	0	0	200
0	1	0	-1	0	0	0	1	0	0	40
0.08	0.06	0	0	-1	0	0	0	1	0	30
0.1	0.15	0	0	0	-1	0	0	0	1	40
800	1200	0	0	0	0	0	0	0	0	0
-1	-1	1	0	0	0	0	1	1	1	-200

Table 6: Remove Second Artifical Varaible

x_1	x_2	s_1	s_2	s_3	s_4	a_1	a_2	a_3	a_4	b
1	1	-1	0	0	0	1	0	0	0	200
0	1	0	-1	0	0	0	1	0	0	40
0.08	0.06	0	0	-1	0	0	0	1	0	30
0.1	0.15	0	0	0	-1	0	0	0	1	40
800	1200	0	0	0	0	0	0	0	0	0
-1	-2	1	1	0	0	0	0	1	1	-240

Table 7: Remove Third Artifical Varaible

x_1	x_2	s_1	s_2	s_3	s_4	a_1	a_2	a_3	a_4	b
1	1	-1	0	0	0	1	0	0	0	200
0	1	0	-1	0	0	0	1	0	0	40
0.08	0.06	0	0	-1	0	0	0	1	0	30
0.1	0.15	0	0	0	-1	0	0	0	1	40
800	1200	0	0	0	0	0	0	0	0	0
-1.08	-2.06	1	1	1	0	0	0	0	1	-270

Table 8: Remove Forth Artifical Varaible

x_1	x_2	s_1	s_2	s_3	s_4	a_1	a_2	a_3	a_4	b
1	1	-1	0	0	0	1	0	0	0	200
0	1	0	-1	0	0	0	1	0	0	40
0.08	0.06	0	0	-1	0	0	0	1	0	30
0.1	0.15	0	0	0	-1	0	0	0	1	40
800	1200	0	0	0	0	0	0	0	0	0
-1.18	-2.21	1	1	1	0	0	0	0	1	-310

Row operation

Table 9: Find Smallest positive result

x_1	x_2	s_1	s_2	s_3	s_4	a_1	a_2	a_3	a_4	b	r
1	1	-1	0	0	0	1	0	0	0	200	200
0	1	0	-1	0	0	0	1	0	0	40	40
0.08	0.06	0	0	-1	0	0	0	1	0	30	500
0.1	0.15	0	0	0	-1	0	0	0	1	40	667
800	1200	0	0	0	0	0	0	0	0	0	0
-1.18	-2.21	1	1	1	-1	0	0	0	0	1	932

Table 10: Row operation

x_1	x_2	s_1	s_2	s_3	s_4	a_1	a_2	a_3	a_4	b	r
1	1	-1	0	0	0	1	0	0	0	200	200
0	1	0	-1	0	0	0	1	0	0	40	40
0.08	0.06	0	0	-1	0	0	0	1	0	30	500
0.1	0.15	0	0	0	-1	0	0	0	1	40	667
800	1200	0	0	0	0	0	0	0	0	0	0
-1.18	-2.21	1	1	1	-1	0	0	0	0	1	932

Table 11: Row operation

x_1	x_2	s_1	s_2	s_3	s_4	a_1	a_2	a_3	a_4	b	r
1	0	-1	1	0	0	1	-1	0	0	160	160
0	1	0	-1	0	0	0	1	0	0	40	-40
0.08	0	0	0.06	-1	0	0	-0.06	1	0	27.6	460
0.1	0	0	0.15	0	-1	0	-0.15	0	1	34	667
800	0	0	1200	0	0	0	-1200	0	0	-48000	-40
-1.18	0	1	-1.21	1	-1	0	2.21	0	0	-221.6	959

Table 12: Row operation

x_1	x_2	s_1	s_2	s_3	s_4	a_1	a_2	a_3	a_4	b	r
1	0	-1	1	0	0	1	-1	0	0	160	160
1	1	-1	0	0	0	1	0	0	0	200	-200
0.02	0	0.06	0	-1	0	-0.06	0	1	0	18	300
-0.05	0	0.15	0	0	-1	-0.15	0	0	1	10	667
-400	0	1200	0	0	0	-1200	0	0	0	-240000	-200
0.03	0	-0.21	0	1	-1	1.21	1	0	0	-28.6	333

Table 13: Row operation

x_1	x_2	s_1	s_2	s_3	s_4	a_1	a_2	a_3	a_4	b	r
0.67	0	0	1	0	-6.7	0	-1	0	6.7	222.67	-34
0.67	1	0	0	0	-6.7	0	0	0	6.7	266.67	-40
0.04	0	0	0	-1	0.4	0	0	1	-0.4	14	35
-0.33	0	1	0	0	-6.7	-1	0	0	6.7	66.67	-10
0	0	0	0	0	8000	0	0	0	-8000	-320000	-40
-0.04	0	0	0	1	-0.4	1	1	0	1.4	-14	35

Table 14: Row operation

x_1	x_2	s_1	s_2	s_3	s_4	a_1	a_2	a_3	a_4	b	r
1.33	0	0	1	-16.67	0	0	-1	16.67	0	460	345
1.33	1	0	0	-16.67	0	0	0	16.67	0	500	375
0.1	0	0	0	-2.5	1	0	0	2.5	-1	35	350
0.33	0	1	0	-16.67	0	-1	0	16.67	0	300	900
-800	0	0	0	20000	0	0	0	-20000	1	-60000	750
-0.00	0	0	0	0	0	1	1	1	0	0	0

Table 15: Row operation

x_1	x_2	s_1	s_2	s_3	s_4	a_1	a_2	a_3	a_4	b
1	0	0	0.75	-12.5	0	0	-0.75	12.5	0	345
0	1	0	-1	0	0	0	1	0	0	40
0	0	0	-0.075	-1.25	1	0	0075	1.25	-1	0.499
0	0	1	-0.25	-12.5	0	-1	025	12.5	0	185
0	0	0	600	10000	0	0	-600	-10000	0	-324000
0	0	0	0	0	0	1	1	1	1	0

$x_1 = 345$, $x_2 = 40$, **Min** $f(x_1, x_2) = 800 * 345 + 1200 * 40 = 320000$