

Assignment 1

Yunfei Yan 1424307

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Question:

Maximize : $24x_1 + 2x_2 + 20x_3 + 4x_4$

Subject to : $8x_1 + x_2 + 5x_3 + 4x_4 \leq 9$

$0 \leq x_j \leq 1$ for $j = 1, 2, 3, 4$

Answer:

Node 1:

x1	x2	x3	x4	s1	s2	s3	s4	s5	b	r
8	1	5	4	1	0	0	0	0	9	1.125
1	0	0	0	0	1	0	0	0	1	1
0	1	0	0	0	0	1	0	0	1	0
0	0	1	0	0	0	0	1	0	1	0
0	0	0	1	0	0	0	0	0	1	0
-24	-2	-20	-4	0	0	0	0	0	0	

x1	x2	x3	x4	s1	s2	s3	s4	s5	b	r
1	0.125	0.625	0.5	0.125	0	0	0	0	1.125	1.8
0	-0.125	-0.625	-0.5	-0.125	1	0	0	0	-0.125	0.2
0	1	0	0	0	0	1	0	0	1	0
0	0	1	0	0	0	0	1	0	1	1
0	0	0	1	0	0	0	0	1	1	0
0	1	-5	8	3	0	0	0	0	27	

x1	x2	x3	x4	s1	s2	s3	s4	s5	b	r
1	0	0	0	0	1	0	0	0	1	1
0	0.2	1	0.8	0.2	-1.6	0	0	0	0.2	-0.125
0	1	0	0	0	0	0	0	0	1	
0	-0.2	0	-0.8	-0.2	1.6	0	1	0	0.8	0.5
0	0	0	1	0	0	0	0	0	1	
0	2	0	12	4	-8	0	0	0	28	

x1	x2	x3	x4	s1	s2	s3	s4	s5	b
1	0.125	0	0.5	0.125	0	0	-0.625	0	0.5
0	0	1	0	0	0	0	1	0	1
0	1	0	0	0	0	0	0	0	1
0	-0.125	0	-0.5	-0.125	1	0	0.625	0	0.5
0	0	0	1	0	0	0	0	0	1
0	1	0	8	3	0	0	5	0	32

$$x_1 = 0.5, x_3 = 1, LP(1) = 32$$

Node 2:

x1=0	x2	x3	x4	s1	s2	s3	s4	b	r
	1	5	4	1	0	0	0	9	1.8
	1	0	0	0	1	0	0	1	0
	0	1	0	0	0	1	0	1	1
	0	0	1	0	0	0	1	1	0
	-2	-20	-4	0	0	0	0	0	

x1=0	x2	x3	x4	s1	s2	s3	s4	b	r
	1	0	4	1	0	-5	0	4	1
	1	0	0	0	1	0	0	1	0
	0	1	0	0	0	1	0	1	0
	0	0	1	0	0	0	1	1	1
	-2	0	-4	0	0	20	0	20	

x1=0	x2	x3	x4	s1	s2	s3	s4	b	r
	0.25	0	1	0.25	0	-1.25	0	1	4
	1	0	0	0	1	0	0	1	1
	0	1	0	0	0	1	0	1	0
	-0.25	0	0	-0.25	0	1.25	1	0	0
	-1	0	0	1	0	15	0	24	

x1=0	x2	x3	x4	s1	s2	s3	s4	b	r
	0	0	1	0.25	-0.25	-1.25	0	0.75	4
	1	0	0	0	1	0	0	1	1
	0	1	0	0	0	1	0	1	0
	0	0	0	-0.25	0.25	1.25	1	0.25	0
	0	0	0	1	1	15	0	25	

$$x_2 = 1, x_3 = 1, x_4 = 0.75, LP(2) = 25$$

Node 3:

x1=1	x2	x3	x4	s1	s2	s3	s4	b	r
	1	5	4	1	0	0	0	1	0.2
	1	0	0	0	1	0	0	1	0
	0	1	0	0	0	1	0	1	1
	0	0	1	0	0	0	1	1	0
	-2	-20	-4	0	0	0	0	0	

x1=1	x2	x3	x4	s1	s2	s3	s4	b	r
	0.2	1	0.8	0.2	0	0	0	0.2	0.2
	1	0	0	0	1	0	0	1	0
	-0.2	0	-0.8	-0.2	0	1	0	0.8	1
	0	0	1	0	0	0	1	1	0
	2	0	12	4	0	0	0	4	

$$x_1 = 1, x_3 = 0.2, LP(3) = 4 + 24 = 28$$

Node 4:

x1=0	x2=0	x3	x4	s1	s2	s3	b	r
		5	4	1	0	0	9	1.8
		1	0	0	1	0	1	1
		0	1	0	0	1	1	0
		-20	-4	0	0	0	0	

x1=0	x2=0	x3	x4	s1	s2	s3	b	r
		0	4	1	-5	0	4	1
		1	0	0	1	0	1	0
		0	1	0	0	1	1	1
		0	-4	0	20	0	20	

x1=0	x2=0	x3	x4	s1	s2	s3	b
		0	0	1	-5	-4	0
		1	0	0	1	0	1
		0	1	0	0	1	1
		0	0	0	20	4	24

$$x_3 = 1, x_4 = 1, LP(4) = 24$$

Node 5:

x1=0	x2=1	x3	x4	s1	s2	s3	b	r
		5	4	1	0	0	8	1.6
		1	0	0	0	0	1	1
		0	1	0	0	1	1	0
		-20	-4	0	0	0	0	

x1=0	x2=1	x3	x4	s1	s2	s3	b	r
		0	4	1	0	0	3	0.75
		1	0	0	1	0	1	0
		0	1	0	0	1	1	1
		0	-4	0	0	0	20	

x1=0	x2=1	x3	x4	s1	s2	s3	b
		0	1	0.25	0	0	0.75
		1	0	0	1	0	1
		0	0	-0.25	0	1	0.25
		0	0	1	0	0	23

$$x_2 = 1, x_3 = 1, x_4 = 0.75, LP(5) = 23 + 2 = 25$$

Node 6:

x1=1	x2=0	x3	x4	s1	s2	s3	b	r
		5	4	1	0	0	1	0.2
		1	0	0	1	0	1	1
		0	1	0	0	1	1	0
		-20	-4	0	0	0	0	

x1=1	x2=0	x3	x4	s1	s2	s3	b
		1	0.8	0.2	0	0	0.2
		0	-0.8	-0.2	1	0	0.8
		0	1	0	0	1	1
		0	12	4	0	0	4

$$x_1 = 1, x_3 = 0.2, LP(6) = 4 + 24 = 28$$

$$\text{Node 7: } x_1 = 1, x_2 = 1, LP(7) = 26$$

Node 8:

x1=0	x2=1	x3=0	x4	s1	s2	b	r
			4	1	0	8	2
			1	0	1	1	1
			-4	0	0	0	

x1=0	x2=1	x3=0	x4	s1	s2	b
			0	1	-4	4
			1	0	1	1
			0	0	4	4

$$x_2 = 1, x_4 = 1, LP(8) = 4 + 2 = 6$$

Node 9:

x1=0	x2=1	x3=1	x4	s1	s2	b	r
			4	1	0	3	0.75
			1	0	1	1	1
			-4	0	0	0	

x1=0	x2=1	x3=1	x4	s1	s2	b
			1	0.25	0	0.75
			0	-0.25	1	0.25
			0	1	0	3

$$x_2 = 1, x_3 = 1, x_4 = 0.75, LP(9) = 3 + 2 + 20 = 25$$

Node 10:

x1=1	x2=0	x3=0	x4	s1	s2	b	r
			4	1	0	1	0.25
			1	0	1	1	1
			-4	0	0	0	

x1=1	x2=0	x3=0	x4	s1	s2	b
			1	0.25	0	0.25
			0	-0.25	1	0.75
			0	1	0	1

$$x_1 = 1, x_4 = 0.25, LP(10) = 1 + 24 = 25$$

Node 11: There is no feasible solution when $x_1 = 1, x_2 = 0, x_3 = 1$

The solution for IP is: $x_1 = 1, x_2 = 1, x_3 = 0, x_4 = 0$
 So Max $24x_1 + 2x_2 + 20x_3 + 4x_4 = 26$

Thanks