Ada-KV: Optimizing KV Cache Eviction by Adaptive Budget Allocation for Efficient LLM Inference

Yuan Feng[1,3†] **Junlin Lv**[1,3†] **Yukun Cao**[1,3] **Xike Xie**[2,3*] **S. Kevin Zhou**[2,3]

[1]School of Computer Science, University of Science and Technology of China (USTC)
[2]School of Biomedical Engineering, USTC
[3]Data Darkness Lab, MIRACLE Center, Suzhou Institute for Advanced Research

Abstract

Large Language Models have excelled in various domains but face efficiency challenges due to the growing Key-Value (KV) cache required for long-sequence inference. Recent efforts aim to reduce KV cache size by evicting vast non-critical cache elements during runtime while preserving generation quality. However, these methods typically allocate compression budgets uniformly across all attention heads, ignoring the unique attention patterns of each head. In this paper, we establish a theoretical loss upper bound between pre- and post-eviction attention output, explaining the optimization target of prior cache eviction methods, while guiding the optimization of adaptive budget allocation. Base on this, we propose *Ada-KV*, the first head-wise adaptive budget allocation strategy. It offers plug-and-play benefits, enabling seamless integration with prior cache eviction methods. Extensive evaluations on 13 datasets from Ruler and 16 datasets from LongBench, all conducted under both question-aware and question-agnostic scenarios, demonstrate substantial quality improvements over existing methods. Our code is available at `https://github.com/FFY0/AdaKV`.

1 Introduction

Autoregressive Large Language Models (LLMs) have achieved significant success and are widely utilized across diverse natural language processing applications, including dialogue systems [1], document summarization [2], and code generation [3]. The widespread deployments of LLMs have propelled the development of their capacities to process extended sequences. For instance, GPT supports sequences up to 128K [4], Claude3 up to 200K [5], and Gemini-Pro-1.5 [6] up to 2M tokens. However, this growth in token length introduces significant challenges, particularly the rapid expansion of cache size during inference. For an 8B LLM, handling a single sequence of 2M tokens can require up to 256GB of cache, severely impacting both GPU memory efficiency and computational runtime efficiency.

In particular, the inference process, for each multi-head self-attention layer, consists of two phases: *prefilling* and *decoding*. In the prefilling phase, LLMs compute and store all Key-Value (KV) cache elements for the tokens in the input prompt. In the decoding phase, the model autoregressively uses the most recently generated token to retrieve information from the stored cache, producing the next output iteratively. The large KV cache size introduces two important efficiency challenges: First, it severely affects GPU memory efficiency, making it increasingly difficult to scale with longer inputs. Second, during decoding, increased I/O latency duo to cache access results in substantial delays, degrading runtime efficiency.

[†]Equal Contribution, [*]Corresponding Author: Xike Xie(xkxie@ustc.edu.cn)
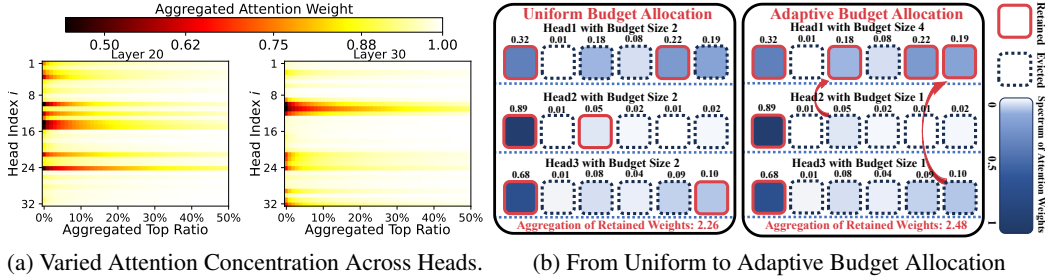
(a) Varied Attention Concentration Across Heads.　　(b) From Uniform to Adaptive Budget Allocation

Figure 1: Adaptive budget allocation accommodates varying attention concentration across heads. *Left: Analysis using Llama-3.1-8B-Instruct shows most heads retain nearly all attention weights with a small cache (e.g., top 5%), while dispersed heads require larger cache proportions. Right: Adaptive allocation, which shifts budgets from sparse to dispersed heads, increases the aggregated retained attention weights (from 2.26 to 2.48) and reduces eviction loss compared to uniform allocation.*

To address the challenges posed by large KV cache sizes, various cache eviction methods have been developed [7, 8, 9, 10, 11]. These methods constrain the cache size to a predefined budget by retaining only a subset of elements in each attention head and evicting the rest. They reduces memory usage and accelerate decoding, facilitating efficient long-sequence inference. Most of these methods rely on a Top-k selection based on attention weights, to identify and prioritize critical cache elements, deciding which to retain and which to evict.

However, existing methods uniformly allocate cache budgets across attention failing to account for the unique characteristics of each head. As shown in Figure 1a, attention heads exhibit diverse concentration patterns: some focus narrowly (or sparsely) on a small portion of the cache, while others distribute their attention more broadly [1]. This uniform allocation results in inefficiencies—either wasting cache budgets on heads with sparse concentration or incurring significant eviction losses in heads with dispersed distribution. Such imbalances degrade the trade-off between overall budget utilization and the quality of post-eviction generation. To address this issue, we analyze the impact of existing Top-k eviction methods on attention outputs and propose the first adaptive budget allocation strategy, called *Ada-KV*, as illustrated in Figure 1b. This strategy dynamically reallocates budgets from heads with sparse concentration to those with more dispersed attention, improving the quality of post-eviction generation.

Our study begins by revisiting Top-k eviction methods, demonstrating that, under specific budget allocation, these methods are equivalent to minimizing an upper bound of eviction loss between pre- and post-eviction attention outputs[2]. Based on this, we propose Ada-KV, a simple yet effective adaptive allocation strategy designed to integrate and enhance existing Top-k eviction methods in a plug-and-play manner. Guided by minimizing the theoretical upper bound of eviction loss, Ada-KV adapts to the varying concentration patterns across attention heads, significantly reducing practical eviction loss.

By integrating the Ada-KV into two state-of-the-art (SOTA) methods, SnapKV [11] and Pyramid [9, 10], we have developed two integration cases: *Ada-SnapKV* and *Ada-Pyramid* [3]. These methods are evaluated using two comprehensive benchmarks, Ruler and LongBench, which include 13 and 16 datasets, respectively. Experimental results demonstrate that Ada-KV could effectively improves performance across various budget settings under the widely studied **question-aware** compression scenario [11, 10, 8]. Furthermore, in the more challenging **question-agnostic** [12] scenario—where compression is applied without leveraging question-specific information, an important scenario that has been largely overlooked—Ada-KV demonstrates even greater advantages. The main contributions are summarized as follows.

- **Adaptive Budget Allocation.** We identify a critical limitation in current KV cache eviction methods: their uniform budget allocation overlooks the unique attention patterns of individ-

---

[1]More visualizations can be found in Appendix A.11

[2]For simplicity, we use L_1 distance to quantify eviction loss; extensions to L_p norms are possible but orthogonal to this work.

[3]The plug-and-play nature of Ada-KV enables broad applicability to existing methods and future developments, extending well beyond the two cases presented in this paper. Section 5 provides further examples of its adoption in subsequent work.

ual heads. To address this, we propose Ada-KV, the first adaptive budget allocation strategy, which enhances budget utilization across individual heads, leading to more efficient cache eviction methods.

- **Theoretical Insights.** We establish a theoretical framework for cache eviction by defining eviction loss and deriving its upper bound. This framework not only explains the optimization target of prior methods but also guides the design of Ada-KV, enabling principled and adaptive budget allocation.

- **Empirical Advances.** With efficient CUDA kernel implementations, Ada-KV offers plug-and-play compatibility, enabling seamless adoption and performance enhancements in existing methods. We evaluate Ada-KV by integrating it into SOTA cache eviction methods, and demonstrate significant improvements across 29 datasets from Ruler and LongBench, all under both question-aware and question-agnostic scenarios.

2 Related Works

2.1 Cache Eviction Methods

In the long-sequence inference, the vast scale of the KV cache elements leads to a memory-bound situation, causing significant memory burden and I/O latency [13]. Numerous studies have sought to mitigate by reducing the cache size, notably through the eviction of non-critical cache elements[4]. These methods are primarily divided into two categories: *sliding window eviction* and *Top-k eviction* methods. The sliding window eviction methods [14, 15, 16], exemplified by *StreamingLLM* [16], simply retain several initial cache elements and those within a sliding window, while evicting others. However, the undiscriminating sliding eviction of cache elements results in a significant reduction in generation quality. In contrast, Top-k eviction methods [7, 17, 18, 8, 9, 10, 11] identify and retain a selected set of k critical cache elements based on attention weights, for enhancing the post-eviction generation quality. The adaptive budget allocation presented in this paper, tailored for Top-k Eviction Methods, enhances post-eviction generation quality within the same overall budget.

In the study of Top-k eviction methods, early work like FastGen [7] searches and combines multiple strategies, such as maintaining caches of special elements, punctuation elements, recent elements, and Top-k selected elements, based on the characteristics of attention heads. H2O, as the representation [8, 17, 18], develops a Top-k based eviction scheme that leverages the query states of all tokens to identify critical cache elements. However, under unidirectional mask in LLM computations, aggregating attention weights across all query states often causes recent KV cache elements to be mistakenly evicted, degrading the quality of subsequent generations. Recent works, such as SnapKV [11] address this issue by using query states within an observation window to identify critical elements, thereby achieving SOTA performance. Subsequent methods, such as Pyramid [9, 10], further introduce budget allocation across different layers. However, existing Top-k eviction methods typically assign the overall budget uniformly across different heads. In contrast, Ada-KV, which has demonstrated superior theoretical and empirical results through adaptive budget allocation, provides a novel strategy to optimizing these methods.

2.2 Sparse Attention Methods

Sparse attention methods are conceptually related to KV cache eviction, but differ fundamentally in their approach [19, 20, 21, 22, 23, 24]. The key difference is that KV cache eviction retains only a subset of the KV cache, while sparse attention methods keep all entries but selectively utilize only a critical subset during computation. Consequently, sparse attention methods do not reduce the memory footprint of the KV cache, thus often necessitating cache offloading to CPU memory [21, 22, 23] or even disk storage [24]. Additionally, the two technique lines are, in fact, orthogonal. Future research could explore first employing KV cache eviction to compress the cache to a certain proportion and then applying sparse attention for further acceleration. Ada-KV presented in this paper leverages head-wise adaptive allocation to enhance cache eviction methods, and this approach can similarly be applied in future work to enhance dynamic sparse attention methods.

---

[4]For additional related works, see Appendix A.1

3 Methodology

We begin by providing a formal description of a multi-head self-attention layer (Section 3.1). Building on this, we theoretically revisit the foundational principles of existing Top-k eviction methods by introducing an L_1 eviction loss metric (Section 3.2). Inspired by theoretical findings, we propose a simple yet effective strategy for adaptive budget allocation, which is proven to outperform traditional uniform budget allocation (Section 3.3). We further demonstrate its compatibility with existing Top-k eviction methods (Section 3.4) and present an efficient implementation (Section 3.5).

3.1 Preliminaries

LLMs operate through an autoregressive generation process, where each step relies on the last token to predict the next one. Let $X \in \mathbb{R}^{n \times d}$ represent the embedding matrix containing all tokens in the sequence, and let $x \in \mathbb{R}^{1 \times d}$ be the embedding of the last token used as input at the current time step. Using the notation system from [25] with h attention heads, we provide a formal description of one multi-head self-attention layer. For each head $i \in [1, h]$, the transformation matrices W_i^Q, W_i^K, $W_i^V \in \mathbb{R}^{d \times d_h}$ map token embeddings to their respective Query, Key, and Value states, while the final output matrix $W_i^O \in \mathbb{R}^{d_h \times d}$ transforms the intermediate result to the output hidden states. At each time step, the previous KV cache elements of head i are initialized as: $K_i = XW_i^K, V_i = XW_i^V$. Then, the embedding of input token x is mapped to its respective Query, Key, and Value states for each head, and the previous KV cache is updated accordingly:

$$q_i = xW_i^Q, k_i = xW_i^K, v_i = xW_i^V \text{ and } K_i = Cat[K_i : k_i], V_i = Cat[V_i : v_i] \tag{1}$$

Finally, the output $y \in \mathbb{R}^{1 \times d}$ is computed using the attention weights $A_i \in \mathbb{R}^{1 \times n}$ as follows[5]:

$$y = \sum_{i \in [1,h]} A_i V_i W_i^O \text{ where } A_i = \text{softmax}(q_i K_i^T) \tag{2}$$

3.2 Theoretical Foundation: Revisiting Top-k Methods with Bounded Eviction Loss

Top-k eviction methods typically presuppose the stability of critical cache elements during future generation process, to facilitate cache eviction [8, 11, 9, 10]. SOTA methods [11, 9, 10] commonly leverage query states of a series of tokens within an observation window to calculate observed attention weights with past KV cache elements. These weights, combined with Top-k selections, approximate the identification of cache elements critical in subsequent generations.

For ease of presentation, we assume a window size of 1, implying the eviction procedure relies solely on the attention weights A between the past cache elements and the query state of the last token for the detection of critical cache elements. A set of indicator variables $\{\mathcal{I}_i \in \mathbb{R}^{1 \times n}\}$[6] represent the eviction decision, with allocated budgets $\{B_i\}$ for all heads $\{i \in [1, h]\}$, where $B = \sum_{i \in [1,h]} B_i$ is the overall budget for one attention layer:

$$\text{Eviction Decision:} \quad \mathcal{I}_i^j = \begin{cases} 1 & \text{Retain } K_i^j \text{ and } V_i^j, \\ 0 & \text{Evict } K_i^j \text{ and } V_i^j, \end{cases} \tag{3}$$

where $\mathcal{I}_i^j$ indicates whether the jth $\in [1, n]$ KV cache element in $K_i, V_i \in \mathbb{R}^{n \times d_h}$ is evicted for head i. Thus, only a budget size B_i of cache elements is retained for head i: $\sum_{j \in [1,n]} \mathcal{I}_i^j = B_i$. Then, the post-eviction output $\hat{y}$ of multi-head self-attention mechanism is:

$$\hat{y} = \sum_{i \in [1,h]} \hat{A}_i V_i W_i^O, \text{where } \hat{A}_i = \text{softmax}(-\infty \odot (\mathbf{1} - \mathcal{I}_i) + q_i K_i^T), \tag{4}$$

and $\odot$ denotes element-wise multiplication.

The degradation in generation quality after cache eviction stems from changes in the attention output. We quantify the eviction loss as the L_1 distance between the pre- and post-eviction outputs of the self-attention mechanisms:

$$L_1 \text{ Eviction Loss} = ||y - \hat{y}||_1 \tag{5}$$

---

[5]The scaling factor $\sqrt{d_h}$ is omitted for simplification.

[6]Given that the first dimension of $\mathcal{I}_i$ is 1, $\mathcal{I}_i^j$ is used to simplify the notation for $\mathcal{I}_i(1, j)$. Similarly, A_i^j follows the same notation.

Utilizing the row norm of the matrix, we derive an upper bound ϵ for the L_1 Eviction Loss in Theorem 3.1. For a detailed proof, please refer to Appendix A.8.

Theorem 3.1. *The L_1 eviction loss can be bounded by ϵ:*

$$L_1 \ Eviction \ Loss \leq \epsilon = 2hC - 2C \sum_{i \in [1,h]} \sum_{j \in [1,n]} \mathcal{I}_i^j A_i^j \tag{6}$$

where $C = Max \left\{ \|V_i W_i^O\|_\infty \right\}$ is a constant number, representing the max row norm .

Essentially, existing cache eviction methods are equivalent to minimizing the upper bound ϵ, as defined in Theorem 3.1. One such method, Top-k cache eviction method [8, 11, 9, 10] stands out as a prominent approach, designed to selectively retain the most critical cache elements by prioritizing those with the highest attention weights. Specifically, the Top-k eviction decision is formulated as :

$$\text{Top-}k \text{ Eviction Decision } \mathcal{I}_i^{*j} = \begin{cases} 1 \text{ if } A_i^j \in \text{Top-}k(A_i, k = B_i), \\ 0 \text{ Evict } K_i^j \text{ and } V_i^j. \end{cases}$$

This approach ensures that each head i retains only the top B_i critical cache elements based on attention weights A_i^j, evicting the rest. In Theorem 3.2, we prove that the Top-k eviction decision $\{\mathcal{I}_i^*\}$ achieves the tightest upper bound ϵ^*, establishing its effectiveness, under given budget allocation $\{B_i\}$. For a detailed proof, please refer to Appendix A.9.

Theorem 3.2. *The Top-k cache eviction $\{\mathcal{I}_i^*\}$ minimizes the upper bound ϵ of L_1 eviction loss, resulting in ϵ^*:*

$$\text{Top-}k \text{ eviction decision } \{\mathcal{I}_i^*\} = \underset{\{\mathcal{I}_i\}}{\arg\min} \ \epsilon \ and \ \epsilon^* = \underset{\{\mathcal{I}_i\}}{\min} \ \epsilon = 2hC - 2C \sum_{i \in [1,h]} \sum_{j \in [1,n]}^{A_i^j \in Top\text{-}k(A_i, k = B_i)} A_i^j \tag{7}$$

Theorems 3.1 and 3.2 establish the theoretical basis for Top-k eviction methods under any given budget allocation $\{B_i\}$. In the sequel, we show how an adaptive strategy can further minimize ϵ^*, outperforming uniform budget allocation (i.e., $\{B_i = B/h\}$).

3.3 Optimizing Top-k Methods with Adaptive Budget Allocation

Algorithm 1 Ada-KV: Adaptive Budget Allocation

Input: Total budget: B; Attention weights for head i: $\{A_i\}$
Output: Allocated budgets $\{B_i^*\}$
 1: Concatenate attention weights across heads $A = \text{Cat}(\{A_i\})$
 2: Select top B weights from A: Top-$k(A, k = B)$
 3: Count number of selected weights for each head i: $\{f_i\}$
 4: Set the allocated budgets as $\{B_i^* = f_i\}$
    Return allocated budgets $\{B_i^*\}$

Here, we propose the first adaptive budget allocation strategy for Top-k eviction methods in Algorithm 1, designed to further minimize the upper bound ϵ^* in Theorem 3.2. It begins by identifying the B largest attention weights across all heads within a single layer. Based on the frequency of selection in each head, the strategy dynamically determines the budget allocation $\{B_i^*\}$. Thereby, attention-sparse heads, where most attention weights are concentrated on only a few cache elements, are assigned a smaller budget. The saved budget is reallocated to attention-dispersed heads with more widespread concentration patterns. Thus it effectively adapts to the distinct attention patterns inherent to each head. Below, we further demonstrate the superiority of this strategy theoretically and empirically in subsequent sections.

Theoretical Advantages of Adaptive Allocation. Given the adaptive budget allocation $\{B_i^*\}$, the upper bound associated with the Top-k eviction decision $\{\mathcal{I}_i^*\}$ is expressed as ϵ^{**}:

$$\epsilon^{**} = 2hC - 2C \sum_{i \in [1,h]} \sum_{j \in [1,n]}^{A_i^j \in \text{Top-}k(A_i, B_i^*)} A_i^j \tag{8}$$

5

Algorithm 2 Ada-SnapKV/Ada-Pyramid in One Layer

Input: total budget B, tokens in observation window $X^{win} \in \mathbb{R}^{win*d}$, cache in observation window $\{K_i^{win}, V_i^{win}\}$, cache outside observation window $\{K_i, V_i\}$

Output: retained cache $\hat{K}_i, \hat{V}_i$

1: **for** $i \leftarrow 1$ to h **do**
2:    $Q_i^{win} = X^{win}W_i^Q$
3:    $\bar{A}_i = softmax(Q_i^{win}K_i^T)$
4:    $\bar{A}_i = \bar{A}_i.maxpooling(dim = 1).mean(dim = 0)$
5: **end for**
6: $B = B - winsize \times h$
7: Derive budget allocation $\{B_i^*\}$ using Algorithm 1($B, \{\bar{A}_i\}$)
8: Safeguard $\{B_i^*\} = \alpha \times \{B_i^*\} + (1 - \alpha) \times (B/h)$
9: Determine the Top-k eviction decision $\{\mathcal{I}_i^*\}$ based on $\{B_i^*\}$
10: Select $\left\{\hat{K}_i, \hat{V}_i\right\}$ from $\{K_i, V_i\}$ according to $\{\mathcal{I}_i^*\}$
11: $\left\{\hat{K}_i, \hat{V}_i\right\} = \text{Cat}(\left\{\hat{K}_i, \hat{V}_i\right\}, \{K_i^{win}, V_i^{win}\})$
   Return retained cache $\hat{K}_i, \hat{V}_i$
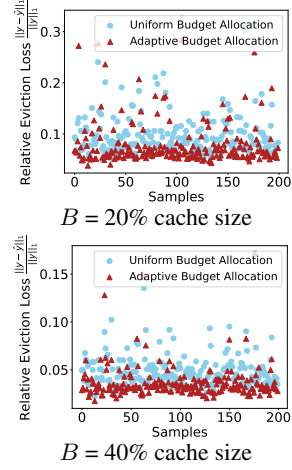
Figure 2: Adaptive allocation yields lower practical eviction losses (Llama-3.1-8B-Instruct on Qasper Dataset).

Theorem 3.3 demonstrates that the adaptive budget allocation achieves the minimum upper bound ϵ^{**} compared to any other allocation strategy, including the commonly used uniform allocation. A detailed proof is provided in Appendix A.10.

Theorem 3.3. *The adaptive budget allocation $\{B_i^*\}$, derived from Algorithm 1 achieves the minimal upper bound ϵ^{**} for loss associated with Top-k eviction strategies: $\epsilon^{**} = \min_{\{B_i\}} \epsilon^*$*

Although a lower upper bound does not strictly guarantee reduced eviction loss, it is generally a key objective in algorithm design, as optimizing a tighter bound typically leads to better results.

Empirical Evidence for Adaptive Allocation. Empirically, the adaptive budget allocation described in Algorithm 1 assigns larger budgets to dispersed heads and conservatively adjust budgets for sparse heads. This approach aligns well with the significant variation in attention concentration across heads, as shown in Figure 1a. By adjusting the head-wise budget, this strategy effectively reduces practical eviction loss. Figure 2 provides a detailed visualization, showing that adaptive budget allocation consistently reduces practical eviction loss across most samples under the same cache budget. These results demonstrate the effectiveness of the proposed method in real-world scenarios.

3.4 Integration into Existing Cache Eviction Methods

We demonstrate the Plug-and-Play compatibility of Ada-KV strategy with two SOTA methods, SnapKV and Pyramid, by integrating it into their existing Top-k eviction frameworks to create enhanced versions: Ada-SnapKV and Ada-Pyramid. Both SnapKV and Pyramid utilize tokens $X^{win} \in \mathbb{R}^{winsize*d}$ from a recent observation window (typically size 32) to identify and evict the less crucial elements in past KV cache. SnapKV excels under larger budget scenarios, while Pyramid is optimized for constrained budget conditions. This is because Pyramid uses a pyramidal budget distribution across attention layers via pre-set hyperparameters, prioritizing shallower layers. Thus, for a given layer with a total budget of B, their eviction algorithms are identical. However, both methods allocate budgets evenly across all heads within one layer.

Incorporating our adaptive allocation, as outlined in Algorithm 2, we modify these methods to better manage budget allocation at the head level. This integration occurs prior to the eviction process in each layer, where our strategy adaptively adjusts budget allocations based on the observed attention weights among heads, as shown in Line 7. Overall, they first calculate the observed attention weights $\bar{A}_i$ of past cache elements using the query states within the observation window. A max pooling layer processes these weights to preserve essential information [11], followed by a Top-k selection of past cache elements outside the observation window. These selected elements, along with others within the observation window, are retained, while the rest are evicted. Moreover, we introduce a safeguard hyper-parameter, α (defaulted to 0.2), to prevent the allocation of excessively small budgets to highly sparse heads, thereby enhancing fault tolerance for presupposed critical stability [11, 8].

6

3.5 Implementation of Computation under Adaptive Budget Allocation

Variable-length Attention with Variable-sized Cache Elements. Adaptive allocation improves cache eviction quality but introduces challenges for efficient computation due to variable-sized cache elements across attention heads. We found that the variable-length FlashAttention technique [26, 27], widely adopted in many inference frameworks for continuous batching [28], could support efficient computation under adaptive allocation. To further enable this technique, we implement a flattened cache storage layout that concatenates the caches of all attention heads within a layer into a single tensor structure. This layout, combined with a custom CUDA kernel, enables efficient cache update operations. As demonstrated in Section 4.4, these components work in synergy to maintain computational efficiency under adaptive allocation, comparable to that of conventional FlashAttention.

Compatibility with Group Query Attention. Existing SOTA LLMs like Llama [29] and Mistral [30], have employed Group Query Attention (GQA) [31] technique to reduce KV cache sizes. However, existing cache eviction methods, such as SnapKV and Pyramid, lack GQA compatibility, redundantly replicating grouped KV caches across heads. We implement a simple GQA-compatible mechanism that uses the mean attention weight within each group as the selection criterion, eliminating redundancy. This enables SnapKV, Pyramid, and their adaptive variants—Ada-SnapKV and Ada-Pyramid—to achieve significant cache size reductions, such as a 4x reduction in Llama-3.1-8B.

4 Experiments

4.1 Settings

Base Models. We employ two open-source base models: Llama-3.1-8B-Instruct [29] and Mistral-7B-instruct-v0.2 [30]. These models, widely adopted due to moderate parameter sizes and impressive performance on long-sequence tasks, both leverage the GQA [31] technique, which reduces the KV cache size to one-quarter of its original.

Datasets. We conduct evaluations using two popular benchmarks: Ruler [32] and LongBench [33]. Ruler includes 13 long-sequence tasks, primarily variations of the Needle-in-a-Haystack test, offering a range of difficulty levels for comprehensive model assessment. According to the official evaluation [32], the maximum effective length for Mistral-7B-instruct-v0.2 and Llama-3.1-8B-Instruct with the full KV Cache is 16K. Therefore, we select the 16K Ruler Benchmark as the primary benchmark for evaluation. Additionally, we also conduct evaluations using LongBench, including 16 datasets covering multiple task domains. Detailed dataset information is provided in Appendix A.5 and A.7.

Baselines. We select the *SnapKV*[11] and *Pyramid*[9, 10] as the primary baselines, given that they are SOTA methods and foundational bases for our *Ada-SnapKV* and *Ada-Pyramid* methods. All these methods implement GQA compatibility as stated in Section 3.5, reducing cache size to a quarter of previous naive implementations. Additionally, *StreamingLLM* [16] is also included as a representative of Sliding Window Eviction Methods for reference.

Parameters. Cache eviction methods can be applied whenever KV cache compression is required. Following standard practices in prior studies [11, 9, 10], we perform cache eviction after the prefilling phase of each layer for consistent comparison. In all experiments, the hyperparameter α for adaptive budget allocation is fixed at 0.2. Both *Ada-SnapKV* and *Ada-Pyramid*, as well as *SnapKV* and *Pyramid*, follow the configuration settings outlined in [11], including an observation window size of 32 and a max pooling kernel size of 7.

4.2 Ruler Benchmark

We evaluate quality scores for each method using budgets set to 20%, 40%, 60%, and 80% of the original cache size. Beyond the commonly studied **question-aware** compression scenario—where questions are known in advance—we also consider the more challenging **question-agnostic** setting [12], in which questions are revealed only after compression. This mimics more challenging scenarios, such as prompt caching [34, 35], where numerous question-independent prefix prompts need compression, or multi-turn dialogue [1], where compression occurs without future questions.

Overall Results. Figure 3 presents the average scores across 13 datasets in the Ruler Benchmark. Overall, existing Top-k eviction methods exhibit significant performance drops in the question-
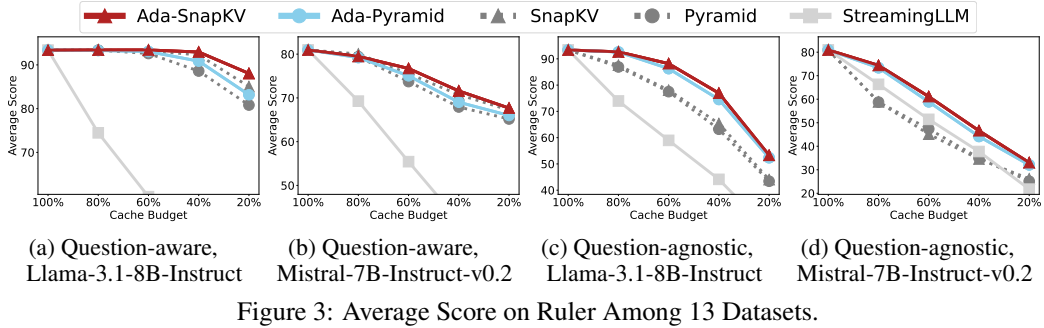
Figure 3: Average Score on Ruler Among 13 Datasets.
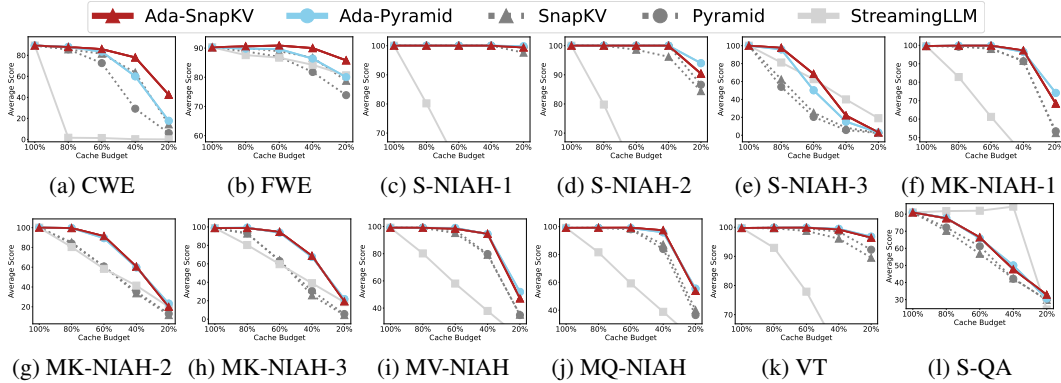
Figure 4: Subtask Analysis on Ruler (Question-agnostic, Llama-3.1-8B-Instruct).

Table 1: Task Analysis for Llama-3.1-8B (Question-agnostic).

Task Domains	Full Cache	$b = 10\%$		$b = 20\%$		$b = 40\%$	
		SnapKV	Ada-SnapKV	SnapKV	Ada-SnapKV	SnapKV	Ada-SnapKV
Single-Doc. QA	43.10	23.38	**23.97**	28.78	**31.39**	35.27	**36.63**
Multi-Doc. QA	46.49	26.61	**29.06**	33.51	**34.90**	40.50	**41.36**
Summarization	28.97	21.98	**22.43**	23.82	**24.29**	26.11	**26.66**
Few-shot	69.45	57.89	**60.40**	61.95	**63.70**	65.10	**66.43**
Synthetic	53.73	**36.50**	34.88	48.19	**50.39**	**53.17**	53.00
Code	57.86	59.72	**60.91**	60.05	**61.14**	**60.49**	60.30
Ave.	49.20	36.38	**37.45**	41.29	**42.87**	45.52	**46.24**

agnostic scenario. In contrast, our Ada-SnapKV and Ada-Pyramid consistently outperform the original SnapKV and Pyramid across both question-aware and question-agnostic scenarios on the two LLMs. Using the Llama-3.1-8B-Instruct model as an example: in the simpler question-aware scenario (Figure 3a), both Ada-SnapKV and Ada-Pyramid significantly reduce quality loss under small compression budgets, such as 40% and 20% cache sizes, compared to the original SnapKV and Pyramid. In the more challenging question-agnostic scenario (Figure 3c), Ada-SnapKV and Ada-Pyramid substantially reduce quality loss across all cache budget settings. For instance, leveraging the adaptive allocation strategy, Ada-SnapKV improves SnapKV's scores at 80% and 20% cache sizes from 87.59 and 44.02 to 92.67 and 53.29, respectively.

Subtask Analysis. Figure 4 presents the results for 12 datasets using Llama-3.1-8B-Instruct in the challenging question-agnostic scenario, highlighting the improvements brought by the adaptive budget allocation strategy. More results are provided in Appendix A.2, where our method also shows significant improvements. Overall, our adaptive budget allocation strategy significantly enhances the performance of existing methods. For example, at 80% cache budget, the original SnapKV demonstrates noticeable degradation on many tasks, whereas Ada-SnapKV maintains near-lossless performance across most tasks. Particularly in difficult Needle-in-a-Haystack tasks like S-NIAH-3 and MK-NIAN-2 with 80% cache budget, Ada-SnapKV increases SnapKV's scores from 62.4 and 85.2 to 97.6 and 99.6, as shown in Figures 4e and 4g. These results emphasize the effectiveness of incorporating adaptive budget allocation for enhancement, especially in difficult tasks.

| Ada-SnapKV | Ada-Pyramid | SnapKV | Pyramid | StreamingLLM |

(a) Question-aware
Llama-3.1-8B-Instruct

(b) Question-aware
Mistral-7B-Instruct-v0.2

(c) Question-agnostic
Llama-3.1-8B-Instruct

(d) Question-agnostic
Mistral-7B-Instruct-v0.2
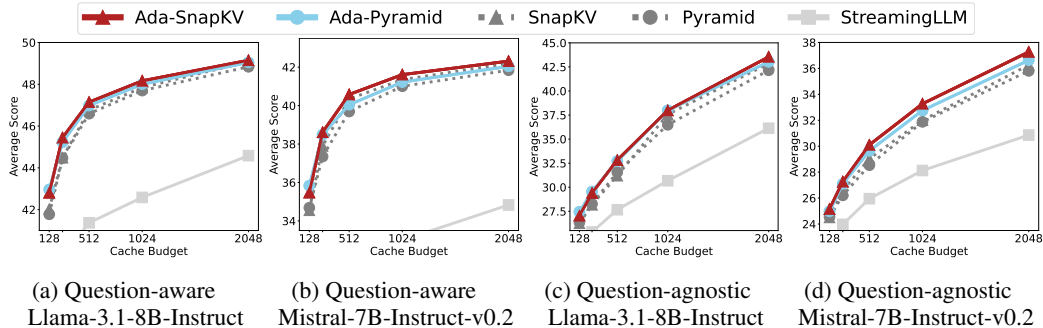
Figure 5: Average Score on LongBench among 16 Datasets under Fixed Budgets.

4.3 LongBench Benchmark

To align with previous evaluations [11, 10], we first set the average budget for each head to fixed values of $\{128, 256, 512, 1024, 2048\}$. We further extend from standard question-aware compression to question-agnostic scenarios by separating the questions in Longbench, thereby better reflecting real-world performance, as suggested by [12]. For more details, please refer to Appendix A.7.

Question-aware Compression: Figures 5a and 5b present the average scores across 16 datasets under fixed budget constraints with the question-aware compression. [7]. SnapKV and Pyramid, both utilizing Top-k selection within an observation window, achieve closely matched performance and outperform the previous sliding window eviction method, StreamingLLM. Moreover, our Ada-SnapKV and Ada-Pyramid methods consistently improve generation quality across different budgets, alternately outperforming their respective base methods and, under a fixed budget of 2048, approaching lossless performance. These consistent gains highlight the effectiveness of adaptive budget allocation.

From Question-aware to Question-agnostic Compression: Figures 5c and 5c further present the average scores with the question-agnostic compression. Our enhanced Ada-SnapKV and Ada-Pyramid methods continue to outperform the original SnapKV and Pyramid methods. However, a comparison between question-aware and question-agnostic settings reveals a notable performance drop across all methods. For example, with Llama and a 2048 budget, the average score of SnapKV decreases from 49.09 to 42.86 while transfer from question-aware to question-agnostic settings. This indicates that commonly used question-aware settings do not adequately capture the limitations of cache eviction methods in a more realistic compression scenario. We therefore suggest that future evaluations of cache eviction techniques should place greater emphasis on question-agnostic compression to improve robustness in practical applications.

More Evaluation in Question-agnostic Compression: To more accurately assess question-agnostic compression, we apply ratio-based cache budget compression, which better reflects real-world performance on Longbench's highly imbalanced sample lengths. [8] Table 1 provides a detailed breakdown of performance across task domains for Llama-3.1-8B. Overall, Code tasks are largely insensitive to cache compression, and in some cases, even see improved performance after compression. In contrast, other tasks experience significant degradation, particularly in Summarization and QA. Notably, our Ada-SnapKV effectively mitigates these losses and improves overall performance. Across 18 domain cases and three cache budgets, Ada-SnapKV delivers quality improvements in 15 domains, demonstrating its robust benefits. Table 2 further reports results on the larger Llama-3.1-70B model, showing similarly strong gains from our approach. Except for the Code domain, which remains insensitive to compression, significant improvements are observed in all other tasks.

4.4 Computation Efficiency Under Adaptive Allocation

Cache eviction methods aim to enhance the computational (memory and runtime) efficiency by selectively evicting KV cache elements. We demonstrate that, supported by our flattened cache layout and custom CUDA kernels, the computational efficiency of cache eviction under adaptive

---

[7] Detailed scores can be found in Appendix A.4

[8] As shown in Table 12 (Appendix A.7), Longbench sample lengths are highly imbalanced—for example, the Code task Lcc averages 1,235 tokens, while the QA task NarrativeQA averages 18,409. Thus, applying a fixed budget, like 2048, across all datasets may introduce bias when evaluating cache eviction across tasks.

Table 2: Task Analysis for Llama-3.1-70B (Question-agnostic).

Domain	Full Cache	$b = 20\%$		$b = 40\%$	
		SnapKV	Ada-SnapKV	SnapKV	Ada-SnapKV
Single-Doc. QA	47.15	32.54	**36.05**	39.35	**42.21**
Multi-Doc. QA	60.07	46.98	**48.45**	54.07	**55.03**
Summarization	28.69	24.40	**24.81**	26.21	**26.57**
Few-shot	72.07	66.71	**67.80**	69.29	**70.29**
Synthetic	58.25	51.25	**51.50**	56.00	**56.75**
Code	47.82	**52.44**	51.51	**51.90**	49.23
Ave.	52.25	44.95	**46.08**	48.91	**49.64**
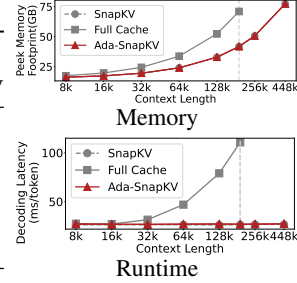
Figure 6: Efficiency (All with FlashAttention-2)

Table 3: Performance gains from applying the Ada-KV strategy to follow-up methods. All results are from the Llama-3.1-8B model on the Longbench benchmark under question-agnostic settings.

Cache	DuoAttn *Training-based*	HeadKV *Training-based*	CriticalKV w/o. Ada-KV	CriticalKV w/. Ada-KV	DefensiveKV w/o. Ada-KV	DefensiveKV w/. Ada-KV
20%	39.52	42.64	42.99	43.77	43.78	**46.68**
40%	48.17	47.23	47.29	48.00	47.76	**49.21**

allocation, based on the variable-length FlashAttention technique, matches that of traditional uniform eviction methods within the same cache budget constraints. As shown in Figure 6, with a fixed budget of 1024, Ada-SnapKV achieves peak memory usage and decoding latency comparable to the original SnapKV, and both significantly outperform the full cache case. These results demonstrate that Ada-KV strategy effectively enhance post-eviction generation quality while maintaining strong computational efficiency.

5 Broad Benefits of the Adaptive Budget Allocation Strategy

Due to its plug-and-play design, Ada-KV is broadly applicable beyond the two cases presented in this paper. As of publication, many follow-up works have adopted Ada-KV strategy, yielding a wide range of enhanced applications [36, 37, 38], especially CriticalKV [39] and DefensiveKV [40]. Beyond direct integration, concurrent methods have explored budget allocation through training-based profiling. For instance, DuoAttention [41] categorizes heads into "full attention" and "streaming" types, while HeadKV [42] enables finer-grained allocation building on our open-source implementation. We present the results of these follow-up studies in Table 3 to demonstrate the broad benefits of adaptive budget allocation strategy. When combined with CriticalKV and DefensiveKV, Ada-KV consistently improves performance, which in turn scales with the strength of the base method. Notably, even the plug-and-play CriticalKV and DefensiveKV methods, when enhanced with the Ada-KV strategy, surpass the performance of training-based approaches like DuoAttention and HeadKV. These findings demonstrate that Ada-KV retains substantial value and offers significant enhancement potential, even alongside more recent and advanced methods.

6 Conclusion

In this study, we revisit cache eviction strategies for efficient LLM inference and uncover a key overlooked factor: adaptive budget allocation across attention heads. Guided by a theoretical analysis of the loss upper bound, we propose *Ada-KV*, the first adaptive budget allocation strategy for optimizing KV cache eviction methods. To demonstrate its plug-and-play benefits, we seamlessly integrate Ada-KV into two existing SOTA methods, introducing Ada-SnapKV and Ada-Pyramid, two adaptive eviction methods. Beyond the commonly studied question-aware compression, we also evaluate these methods in the more challenging and less explored question-agnostic compression scenario. Using the Ruler and LongBench benchmarks, we demonstrate the effectiveness of adaptive budget allocation in both settings. Our results not only expose the limitations of current cache eviction strategies but also highlight the potential of adaptive allocation to enhance cache eviction.

Acknowledgments

References

[1] Zihao Yi, Jiarui Ouyang, Yuwen Liu, Tianhao Liao, Zhe Xu, and Ying Shen. A survey on recent advances in llm-based multi-turn dialogue systems. *arXiv preprint arXiv:2402.18013*, 2024.

[2] Philippe Laban, Wojciech Kryściński, Divyansh Agarwal, Alexander Richard Fabbri, Caiming Xiong, Shafiq Joty, and Chien-Sheng Wu. Summedits: measuring llm ability at factual reasoning through the lens of summarization. In *Proceedings of the 2023 Conference on Empirical Methods in Natural Language Processing*, pages 9662–9676, 2023.

[3] Qiuhan Gu. Llm-based code generation method for golang compiler testing. In *Proceedings of the 31st ACM Joint European Software Engineering Conference and Symposium on the Foundations of Software Engineering*, pages 2201–2203, 2023.

[4] Josh Achiam, Steven Adler, Sandhini Agarwal, Lama Ahmad, Ilge Akkaya, Florencia Leoni Aleman, Diogo Almeida, Janko Altenschmidt, Sam Altman, Shyamal Anadkat, et al. Gpt-4 technical report. *arXiv preprint arXiv:2303.08774*, 2023.

[5] Anthropic. The claude 3 model family: Opus, sonnet, haiku, March 2024. Accessed: 2024-07-09.

[6] Machel Reid, Nikolay Savinov, Denis Teplyashin, Dmitry Lepikhin, Timothy Lillicrap, Jean-baptiste Alayrac, Radu Soricut, Angeliki Lazaridou, Orhan Firat, Julian Schrittwieser, et al. Gemini 1.5: Unlocking multimodal understanding across millions of tokens of context. *arXiv preprint arXiv:2403.05530*, 2024.

[7] Suyu Ge, Yunan Zhang, Liyuan Liu, Minjia Zhang, Jiawei Han, and Jianfeng Gao. Model tells you what to discard: Adaptive kv cache compression for llms. *arXiv preprint arXiv:2310.01801*, 2023.

[8] Zhenyu Zhang, Ying Sheng, Tianyi Zhou, Tianlong Chen, Lianmin Zheng, Ruisi Cai, Zhao Song, Yuandong Tian, Christopher Ré, Clark Barrett, et al. H2o: Heavy-hitter oracle for efficient generative inference of large language models. *Advances in Neural Information Processing Systems*, 36, 2024.

[9] Dongjie Yang, Xiaodong Han, Yan Gao, Yao Hu, Shilin Zhang, and Hai Zhao. PyramidInfer: Pyramid KV cache compression for high-throughput LLM inference. In Lun-Wei Ku, Andre Martins, and Vivek Srikumar, editors, *Findings of the Association for Computational Linguistics ACL 2024*, pages 3258–3270, Bangkok, Thailand and virtual meeting, August 2024. Association for Computational Linguistics.

[10] Yichi Zhang, Bofei Gao, Tianyu Liu, Keming Lu, Wayne Xiong, Yue Dong, Baobao Chang, Junjie Hu, Wen Xiao, et al. Pyramidkv: Dynamic kv cache compression based on pyramidal information funneling. *arXiv preprint arXiv:2406.02069*, 2024.

[11] Yuhong Li, Yingbing Huang, Bowen Yang, Bharat Venkitesh, Acyr Locatelli, Hanchen Ye, Tianle Cai, Patrick Lewis, and Deming Chen. SnapKV: LLM knows what you are looking for before generation. In *The Thirty-eighth Annual Conference on Neural Information Processing Systems*, 2024.

[12] Maximilian Jeblick Simon Jegou. Llm kv cache compression made easy, 2024.

[13] Kefei Wang and Feng Chen. Catalyst: Optimizing cache management for large in-memory key-value systems. *Proceedings of the VLDB Endowment*, 16(13):4339–4352, 2023.

[14] Iz Beltagy, Matthew E Peters, and Arman Cohan. Longformer: The long-document transformer. *arXiv preprint arXiv:2004.05150*, 2020.

[15] Chi Han, Qifan Wang, Hao Peng, Wenhan Xiong, Yu Chen, Heng Ji, and Sinong Wang. Lm-infinite: Zero-shot extreme length generalization for large language models, 2024.

[16] Guangxuan Xiao, Yuandong Tian, Beidi Chen, Song Han, and Mike Lewis. Efficient streaming language models with attention sinks. *arXiv preprint arXiv:2309.17453*, 2023.

[17] Zichang Liu, Aditya Desai, Fangshuo Liao, Weitao Wang, Victor Xie, Zhaozhuo Xu, Anastasios Kyrillidis, and Anshumali Shrivastava. Scissorhands: Exploiting the persistence of importance hypothesis for llm kv cache compression at test time. *Advances in Neural Information Processing Systems*, 36, 2024.

[18] Siyu Ren and Kenny Q Zhu. On the efficacy of eviction policy for key-value constrained generative language model inference. *arXiv preprint arXiv:2402.06262*, 2024.

[19] Huiqiang Jiang, Yucheng Li, Chengruidong Zhang, Qianhui Wu, Xufang Luo, Surin Ahn, Zhenhua Han, Amir H Abdi, Dongsheng Li, Chin-Yew Lin, et al. Minference 1.0: Accelerating pre-filling for long-context llms via dynamic sparse attention. *Advances in Neural Information Processing Systems*, 37:52481–52515, 2024.

[20] Yucheng Li, Huiqiang Jiang, Chengruidong Zhang, Qianhui Wu, Xufang Luo, Surin Ahn, Amir H Abdi, Dongsheng Li, Jianfeng Gao, Yuqing Yang, et al. Mminference: Accelerating pre-filling for long-context vlms via modality-aware permutation sparse attention. *arXiv preprint arXiv:2504.16083*, 2025.

[21] Di Liu, Meng Chen, Baotong Lu, Huiqiang Jiang, Zhenhua Han, Qianxi Zhang, Qi Chen, Chengruidong Zhang, Bailu Ding, Kai Zhang, et al. Retrievalattention: Accelerating long-context llm inference via vector retrieval. *arXiv preprint arXiv:2409.10516*, 2024.

[22] Renze Chen, Zhuofeng Wang, Beiquan Cao, Tong Wu, Size Zheng, Xiuhong Li, Xuechao Wei, Shengen Yan, Meng Li, and Yun Liang. Arkvale: Efficient generative llm inference with recallable key-value eviction. *Advances in Neural Information Processing Systems*, 37:113134–113155, 2024.

[23] Hailin Zhang, Xiaodong Ji, Yilin Chen, Fangcheng Fu, Xupeng Miao, Xiaonan Nie, Weipeng Chen, and Bin Cui. Pqcache: Product quantization-based kvcache for long context llm inference. *Proceedings of the ACM on Management of Data*, 3(3):1–30, 2025.

[24] He Sun, Li Li, Mingjun Xiao, and Chengzhong Xu. Breaking the boundaries of long-context llm inference: Adaptive kv management on a single commodity gpu. *arXiv preprint arXiv:2506.20187*, 2025.

[25] Zichang Liu, Jue Wang, Tri Dao, Tianyi Zhou, Binhang Yuan, Zhao Song, Anshumali Shrivastava, Ce Zhang, Yuandong Tian, Christopher Re, and Beidi Chen. Deja vu: Contextual sparsity for efficient llms at inference time, 2023.

[26] Tri Dao, Dan Fu, Stefano Ermon, Atri Rudra, and Christopher Ré. Flashattention: Fast and memory-efficient exact attention with io-awareness. *Advances in Neural Information Processing Systems*, 35:16344–16359, 2022.

[27] Tri Dao. Flashattention-2: Faster attention with better parallelism and work partitioning. *arXiv preprint arXiv:2307.08691*, 2023.

[28] Woosuk Kwon, Zhuohan Li, Siyuan Zhuang, Ying Sheng, Lianmin Zheng, Cody Hao Yu, Joseph Gonzalez, Hao Zhang, and Ion Stoica. Efficient memory management for large language model serving with pagedattention. In *Proceedings of the 29th Symposium on Operating Systems Principles*, pages 611–626, 2023.

[29] Aaron Grattafiori, Abhimanyu Dubey, and Abhinav Jauhri .et al. The llama 3 herd of models, 2024.

[30] Albert Q Jiang, Alexandre Sablayrolles, Arthur Mensch, Chris Bamford, Devendra Singh Chaplot, Diego de las Casas, Florian Bressand, Gianna Lengyel, Guillaume Lample, Lucile Saulnier, et al. Mistral 7b. *arXiv preprint arXiv:2310.06825*, 2023.

[31] Joshua Ainslie, James Lee-Thorp, Michiel de Jong, Yury Zemlyanskiy, Federico Lebrón, and Sumit Sanghai. Gqa: Training generalized multi-query transformer models from multi-head checkpoints, 2023.

[32] Cheng-Ping Hsieh, Simeng Sun, Samuel Kriman, Shantanu Acharya, Dima Rekesh, Fei Jia, Yang Zhang, and Boris Ginsburg. Ruler: What's the real context size of your long-context language models? *arXiv preprint arXiv:2404.06654*, 2024.

[33] Yushi Bai, Xin Lv, Jiajie Zhang, Hongchang Lyu, Jiankai Tang, Zhidian Huang, Zhengxiao Du, Xiao Liu, Aohan Zeng, Lei Hou, et al. Longbench: A bilingual, multitask benchmark for long context understanding. *arXiv preprint arXiv:2308.14508*, 2023.

[34] In Gim, Guojun Chen, Seung-seob Lee, Nikhil Sarda, Anurag Khandelwal, and Lin Zhong. Prompt cache: Modular attention reuse for low-latency inference. *Proceedings of Machine Learning and Systems*, 6:325–338, 2024.

[35] Lianmin Zheng, Liangsheng Yin, Zhiqiang Xie, Chuyue Sun, Jeff Huang, Cody Hao Yu, Shiyi Cao, Christos Kozyrakis, Ion Stoica, Joseph E Gonzalez, et al. Sglang: Efficient execution of structured language model programs. *arXiv preprint arXiv:2312.07104*, 2024.

[36] Jang-Hyun Kim, Jinuk Kim, Sangwoo Kwon, Jae W Lee, Sangdoo Yun, and Hyun Oh Song. Kvzip: Query-agnostic kv cache compression with context reconstruction. *arXiv preprint arXiv:2505.23416*, 2025.

[37] Alessio Devoto, Maximilian Jeblick, and Simon Jégou. Expected attention: Kv cache compression by estimating attention from future queries distribution. *arXiv preprint arXiv:2510.00636*, 2025.

[38] Kevin Galim, Ethan Ewer, Wonjun Kang, Minjae Lee, Hyung Il Koo, and Kangwook Lee. Draft-based approximate inference for llms. *arXiv preprint arXiv:2506.08373*, 2025.

[39] Yuan Feng, Junlin Lv, Yukun Cao, Xike Xie, and S Kevin Zhou. Identify critical kv cache in llm inference from an output perturbation perspective. *arXiv preprint arXiv:2502.03805*, 2025.

[40] Yuan Feng, Haoyu Guo, JunLin Lv, S. Kevin Zhou, and Xike Xie. Taming the fragility of kv cache eviction in llm inference, 2025.

[41] Guangxuan Xiao, Jiaming Tang, Jingwei Zuo, Junxian Guo, Shang Yang, Haotian Tang, Yao Fu, and Song Han. Duoattention: Efficient long-context llm inference with retrieval and streaming heads. *arXiv preprint arXiv:2410.10819*, 2024.

[42] Yu Fu, Zefan Cai, Abedelkadir Asi, Wayne Xiong, Yue Dong, and Wen Xiao. Not all heads matter: A head-level kv cache compression method with integrated retrieval and reasoning. *arXiv preprint arXiv:2410.19258*, 2024.

[43] Zhewei Yao, Reza Yazdani Aminabadi, Minjia Zhang, Xiaoxia Wu, Conglong Li, and Yuxiong He. Zeroquant: Efficient and affordable post-training quantization for large-scale transformers. *Advances in Neural Information Processing Systems*, 35:27168–27183, 2022.

[44] Zirui Liu, Jiayi Yuan, Hongye Jin, Shaochen Zhong, Zhaozhuo Xu, Vladimir Braverman, Beidi Chen, and Xia Hu. Kivi: A tuning-free asymmetric 2bit quantization for kv cache. *arXiv preprint arXiv:2402.02750*, 2024.

[45] Shichen Dong, Wen Cheng, Jiayu Qin, and Wei Wang. Qaq: Quality adaptive quantization for llm kv cache. *arXiv preprint arXiv:2403.04643*, 2024.

[46] Hanshi Sun, Zhuoming Chen, Xinyu Yang, Yuandong Tian, and Beidi Chen. Triforce: Lossless acceleration of long sequence generation with hierarchical speculative decoding. *arXiv preprint arXiv:2404.11912*, 2024.

[47] Penghui Yang, Cunxiao Du, Fengzhuo Zhang, Haonan Wang, Tianyu Pang, Chao Du, and Bo An. Longspec: Long-context lossless speculative decoding with efficient drafting and verification. In *ES-FoMo III: 3rd Workshop on Efficient Systems for Foundation Models*.

[48] Yicheng Ji, Jun Zhang, Heming Xia, Jinpeng Chen, Lidan Shou, Gang Chen, and Huan Li. Specvlm: Enhancing speculative decoding of video llms via verifier-guided token pruning. *arXiv preprint arXiv:2508.16201*, 2025.

[49] Gregory Kamradt. Needle In A Haystack - pressure testing LLMs. *Github*, 2023.

[50] Simran Arora, Sabri Eyuboglu, Aman Timalsina, Isys Johnson, Michael Poli, James Zou, Atri Rudra, and Christopher Ré. Zoology: Measuring and improving recall in efficient language models. In *ICLR*, 2024.

[51] Tomáš Kočiskỳ, Jonathan Schwarz, Phil Blunsom, Chris Dyer, Karl Moritz Hermann, Gábor Melis, and Edward Grefenstette. The narrativeqa reading comprehension challenge. *Transactions of the Association for Computational Linguistics*, 6:317–328, 2018.

[52] Pradeep Dasigi, Kyle Lo, Iz Beltagy, Arman Cohan, Noah A Smith, and Matt Gardner. A dataset of information-seeking questions and answers anchored in research papers. *arXiv preprint arXiv:2105.03011*, 2021.

[53] Zhilin Yang, Peng Qi, Saizheng Zhang, Yoshua Bengio, William W Cohen, Ruslan Salakhutdinov, and Christopher D Manning. Hotpotqa: A dataset for diverse, explainable multi-hop question answering. *arXiv preprint arXiv:1809.09600*, 2018.

[54] Xanh Ho, Anh-Khoa Duong Nguyen, Saku Sugawara, and Akiko Aizawa. Constructing a multi-hop QA dataset for comprehensive evaluation of reasoning steps. In Donia Scott, Nuria Bel, and Chengqing Zong, editors, *Proceedings of the 28th International Conference on Computational Linguistics*, pages 6609–6625, Barcelona, Spain (Online), December 2020. International Committee on Computational Linguistics.

[55] Harsh Trivedi, Niranjan Balasubramanian, Tushar Khot, and Ashish Sabharwal. Musique: Multihop questions via single-hop question composition. *Transactions of the Association for Computational Linguistics*, 10:539–554, 2022.

[56] Luyang Huang, Shuyang Cao, Nikolaus Parulian, Heng Ji, and Lu Wang. Efficient attentions for long document summarization. *arXiv preprint arXiv:2104.02112*, 2021.

[57] Ming Zhong, Da Yin, Tao Yu, Ahmad Zaidi, Mutethia Mutuma, Rahul Jha, Ahmed Hassan Awadallah, Asli Celikyilmaz, Yang Liu, Xipeng Qiu, et al. Qmsum: A new benchmark for query-based multi-domain meeting summarization. *arXiv preprint arXiv:2104.05938*, 2021.

[58] Alexander R Fabbri, Irene Li, Tianwei She, Suyi Li, and Dragomir R Radev. Multi-news: A large-scale multi-document summarization dataset and abstractive hierarchical model. *arXiv preprint arXiv:1906.01749*, 2019.

[59] Mandar Joshi, Eunsol Choi, Daniel S. Weld, and Luke Zettlemoyer. Triviaqa: A large scale distantly supervised challenge dataset for reading comprehension, 2017.

[60] Bogdan Gliwa, Iwona Mochol, Maciej Biesek, and Aleksander Wawer. Samsum corpus: A human-annotated dialogue dataset for abstractive summarization. *arXiv preprint arXiv:1911.12237*, 2019.

[61] Xin Li and Dan Roth. Learning question classifiers. In *COLING 2002: The 19th International Conference on Computational Linguistics*, 2002.

[62] Daya Guo, Canwen Xu, Nan Duan, Jian Yin, and Julian McAuley. Longcoder: A long-range pre-trained language model for code completion, 2023.

[63] Tianyang Liu, Canwen Xu, and Julian McAuley. Repobench: Benchmarking repository-level code auto-completion systems, 2023.

[64] Nelson F Liu, Kevin Lin, John Hewitt, Ashwin Paranjape, Michele Bevilacqua, Fabio Petroni, and Percy Liang. Lost in the middle: How language models use long contexts. *arXiv preprint arXiv:2307.03172*, 2023.

[65] Xanh Ho, Anh-Khoa Duong Nguyen, Saku Sugawara, and Akiko Aizawa. Constructing a multi-hop qa dataset for comprehensive evaluation of reasoning steps. In *Proceedings of the 28th International Conference on Computational Linguistics*, pages 6609–6625, 2020.

[66] Mandar Joshi, Eunsol Choi, Daniel S Weld, and Luke Zettlemoyer. Triviaqa: A large scale distantly supervised challenge dataset for reading comprehension. In *Proceedings of the 55th Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pages 1601–1611, 2017.

[67] Daya Guo, Canwen Xu, Nan Duan, Jian Yin, and Julian McAuley. Longcoder: A long-range pre-trained language model for code completion. *arXiv preprint arXiv:2306.14893*, 2023.

[68] Tianyang Liu, Canwen Xu, and Julian McAuley. Repobench: Benchmarking repository-level code auto-completion systems. *arXiv preprint arXiv:2306.03091*, 2023.

A Appendix

A.1 Additional Related Works

Additional works also mitigate the challenges posed by massive KV Caches during long-sequence inference while not reducing the number of cache elements. These works are orthogonal to our work. For instance, in our implementation, we have integrated the Flash Attention [26] technique to enhance efficient computation. Similar efforts, such as Paged Attention [28], employ efficient memory management strategies to reduce I/O latency without altering the size of the KV Cache. KV cache quantization methods [43, 44, 45], reduce the size of cache by lowering the precision of individual elements. The cache eviction techniques focused in this paper also be further combined and complemented with quantization in the future. Some studies also employ speculative decoding to accelerate long-sequence generation [46, 47, 48], typically using model with a reduced KV cache to generate drafts. Future work could integrate more advanced cache eviction methods to further improve the efficiency of such approaches.

A.2 Detail results of Ruler Evaluation

Figure 7 complements the missing result for the multi-hop QA subtask in Figure 4 due to the space limitation. Figures 9 8 10 further provide a comprehensive overview of the performance of Ada-SnapKV and Ada-Pyramid across all 16K Ruler subtasks on two LLMs in both question-agnostic and question-aware scenarios. The results demonstrate that regardless of the scenario (question-agnostic or question-aware), model (LLama or Mistral), or cache size budgets, Ada-SnapKV and Ada-Pyramid outperform the original SnapKV and Pyramid approaches in most tasks. This highlights the broad applicability and effectiveness of the adaptive allocation strategy.
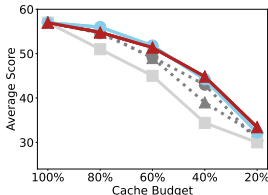
Figure 7: M-QA Subtask Analysis on Ruler for Question-agnostic Scenario (Llama3.1-8B-Instruct)

A.3 Robustness Analysis of Safeguard α

Rather than fine-tuning the safeguard parameter, α, for each model or budget—a process that would introduce considerable complexity—we opted for a fixed value. As shown in Table [Table Number], we conducted a robustness analysis of α using Mistral-7B on the LongBench benchmark. The results indicate that a smaller α allows for more aggressive budget allocation, improving performance under limited budgets. Conversely, a larger α performs slightly better with higher budgets. Based on this trade-off, we selected a balanced value for $\alpha = 0.2$ in this work.

Table 4: Robustness Analysis of α (Question-aware)

Longbench, Mistral-7B	Budget 128	Budget 256	Budget 512	Budget 1024
Ada-SnapKV $\alpha = 0$	**35.69**	**38.9**	40.53	41.44
Ada-SnapKV $\alpha = 0.2$	35.48	38.61	40.58	41.61
Ada-SnapKV $\alpha = 0.5$	35.2	38.46	**40.66**	**41.62**

A.4 Detail results of LongBench Evaluation

Tables 5, 6, 8 and 7 present the detailed evaluation results of different methods across 16 datasets on LongBench for two models. As shown, the Ada-SnapKV and Ada-Pyramid methods, with our adaptive allocation enhancement, achieve consistent improvements in overall generation quality across different budgets and models.
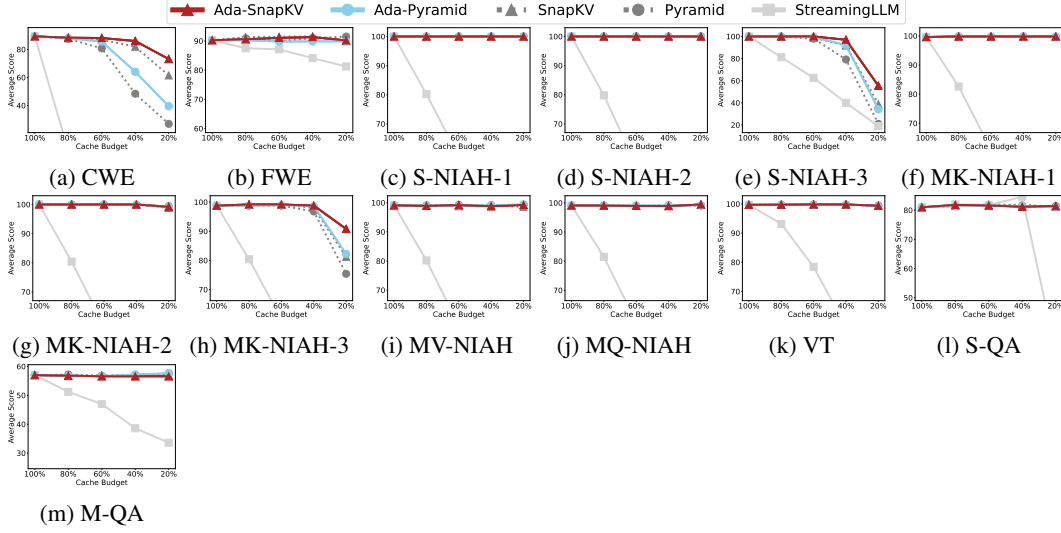
Figure 8: Subtask Analysis on Ruler (Question-aware, Llama3.1-8B-Instruct).
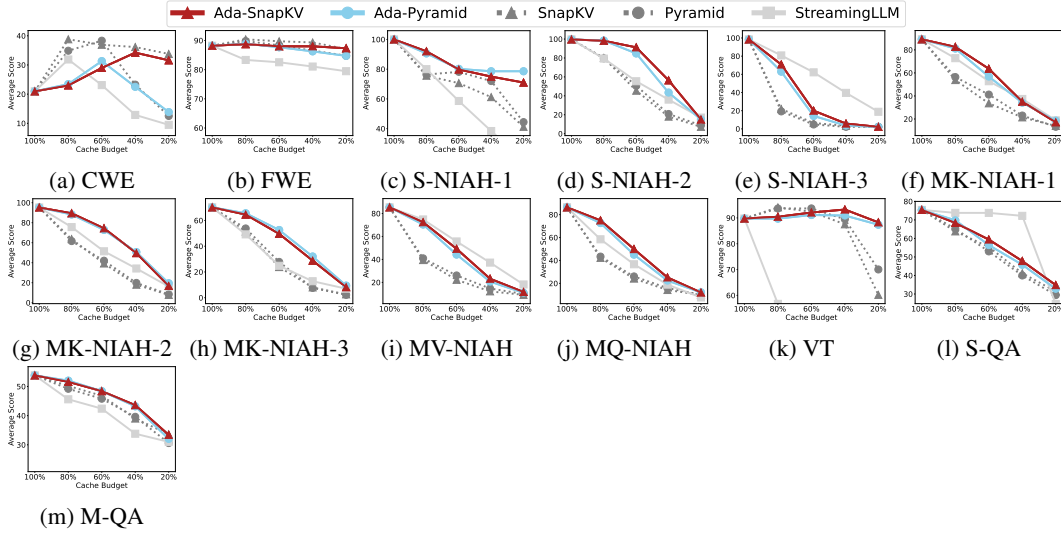
Figure 9: Subtask Analysis on Ruler (Question-agnostic, Mistral-7B-Instruct-v0.2).

A.5 Detailed Information of Ruler Benchmark

Below is a brief overview of the different tasks in Ruler. For detailed examples, please refer to Tables 9 10 and 11. We adhere to the input prompt format from KVPress [12], dividing the input into context and question segments. The question segment is highlighted in green, while other colors represent the context segment. In question-aware compression, both the context and question segments are input into the model and compressed. For question-agnostic compression, where future questions are unpredictable, only the context segment is input for compression. After compression, the question segment is input for answer generation.

- **Single NIAH (S-NIAH):** A basic information retrieval task. In this scenario, a keyword sentence, referred to as the "needle," is embedded within a lengthy text, called the "haystack." The goal is to retrieve the "needle" from the context. The "haystack" may consist of repetitive, noisy sentences or essays by Paul Graham [49].

- **Multi-keys NIAH (MK-NIAH):** Multiple "needles" are inserted into the "haystack," but the task requires retrieving only one of them.

17
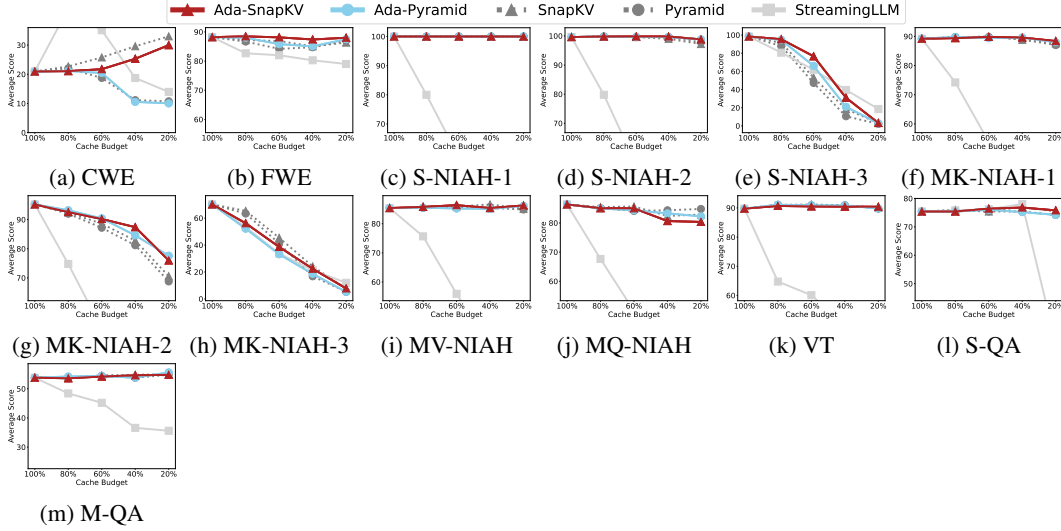
Figure 10: Subtask Analysis on Ruler (Question-aware, Mistral-7B-Instruct-v0.2).

- **Multi-values NIAH (MV-NIAH):** Multiple "needles" in the form of key-value pairs are placed within the "haystack." The task is to retrieve all values associated with a given key.
- **Multi-queries NIAH (MQ-NIAH):** Multiple "needle" in the form of key-value pairs are inserted into the "haystack". All "needles" corresponding to the keys specified in the queries need to be retrieved. **(author?)** [50]
- **Variable Tracking (VT):** A series of variable binding statements (e.g., $X2 = X1, X3 = X2, ...$) are inserted at various positions within a long text to simulate the recognition of entities and relationships. The task objective is to return all variable names that point to the same value V.
- **Common words extraction task (CWE):** The input sequence is constructed by sampling words from a discrete uniform distribution within a pre-defined (synthetic) word list. Words that appear most frequently are termed "common words". The model is required to identify all common words. The number of common words remains constant while the number of uncommon words increases with the length of the sequence.
- **Frequent words extraction task (FWE):** The input sequence is constructed by sampling words from a Zeta distribution within a pre-defined (synthetic) word list. The model need to return the top-K frequent words in the context.
- **Single-Hop QA (S-QA)** Answer questions based on passages. The context is extended to simulate long-context input by adding distracting information(golden paragraphs). And answers come from a single piece of evidence.
- **Multi-Hop QA (M-QA)** Answer questions based on passages. For Multi-hop QA, which requires integrating information from multiple sources.

A.6 Limitations

This paper, for the first time, introduces a head-wise allocation strategy for KV cache compression, supported by both theoretical analysis and empirical validation. However, this head-wise allocation is still limited to within single layer. In future work, we plan to extend the head-wise allocation mechanism across layers and further develop corresponding theoretical analysis to demonstrate advantages.

A.7 Detailed Information of LongBench Benchmark

LongBench [33] spans multiple task domains with an average length of 6711, including single-document QA [51, 52], multi-document QA [53, 54, 55], summarization [56, 57, 58], few-shot learning [59, 60, 61], synthetic tasks [33], and code generation [62, 63].

- **Single-Doc QA:** Single-document QA requires models to obtain the answer from a single piece of source. The test samples are derived from diverse datasets including NarrativeQA [51], qasper [52], and MultiFieldQA [64], encompassing a range of documents such as legal files, governmental reports, encyclopedia, and academic papers.

- **Multi-Doc QA:** Multi-document QA requires models to extract and combine information from several documents to obtain the answer. The test samples are built from three Wikipedia-based multi-hop QA datasets: HotpotQA [53], 2WikiMultihopQA [65] and MuSiQue [55].

- **Summarization:** Summarization task requires a comprehensive understanding of the context. The test samples are drawn from the GovReport dataset [56], and QMSum [57]. Additionally, the MultiNews dataset originates from a multi-document summarization corpus detailed in [58]

- **Few-shot Learning:** Few-shot Learning task have integrated classification, summarization, and reading comprehension tasks to maintain task diversity. For classification, the TREC dataset [61] is included. The SAMSum dataset [60], which comprises messenger-style conversations with human-annotated summaries, has been incorporated for summarization tasks. Additionally, TriviaQA [66], a dataset of question-answer pairs, is utilized for reading comprehension tasks.

- **Synthetic Task:** Synthetic Tasks are designed to assess the model's ability in handling specific scenarios and patterns. Two synthetic tasks, PassageRetrieval-en and PassageCount, are included in LongBench. PassageRetrieval-en is derived from English Wikipedia. 30 passages is randomly selected and use GPT-3.5-Turbo to summarize one of them. The task challenges the model to identify the original paragraph that matches the generated summary. PassageCount is designed to be more challenging, this task requires the model to leverage the entire context to solve it. Several passages are randomly pickedfrom English Wikipedia, randomly duplicate each paragraph multiple times, and then shuffle the paragraphs. The task is to determine the number of unique passages within the provided set.

- **Code Completion:** Code Completion task is to assist users by completing code based on previous code input and context. The LCC dataset, derived from the original Long Code Completion dataset [67]. The RepoBench-P dataset [68] is designed to aggregating information from code across files. Model is required to predict the next line of code.

Table 12 provides a comprehensive description of information pertaining to 16 datasets in LongBench. In the question-agnostic scenarios, we separate the questions from the context, with the question portions highlighted in Tables 13, 14, 15, 16, 17, and 18.

A.8 Proof of Theorem 3.1

Theorem A.1. *The L_1 eviction loss can be bounded by ϵ:*

$$L_1 \text{ Eviction Loss} \leq \epsilon = 2hC - 2C \sum_{i \in [1,h]} \sum_{j \in [1,n]} \mathcal{I}_i^j A_i^j \tag{9}$$

where $C = Max\left\{\|V_i W_i^O\|_\infty\right\}$ is a constant number, representing the max row norm among all matrices.

Proof. Consider the softmax function as:

$$softmax(x)^j = \frac{exp(x^j)}{\sum_j exp(x^j)} \tag{10}$$

Thus, the attention weight after eviction procedure is:

$$\hat{A}_i = \text{softmax}(-\infty \odot (\mathbf{1} - \mathcal{I}_i) + s_i) \text{ where } s_i = q_i K_i^T \tag{11}$$

$$\hat{A}_i^j = \frac{exp(s_i^j - \infty \odot (1 - \mathcal{I}_i^j))}{\sum_j exp(s_i^j - \infty \odot (1 - \mathcal{I}_i^j))} \tag{12}$$

$$= \frac{\mathcal{I}_i^j exp(s_i^j)}{\sum_j \mathcal{I}_i^j exp(s_i^j)} = \frac{\mathcal{I}_i^j exp(s_i^j)}{\sum_j exp(s_i^j)} \frac{\sum_j exp(s_i^j)}{\sum_j \mathcal{I}_i^j exp(s_i^j)} \tag{13}$$

Table 5: Detailed results of Llama-3.1-8B-Instruct on LongBench.(Question-aware)

	Single-Doc. QA			Multi-Doc. QA			Summarization			Few-shotLearning			Synthetic		Code		Ave. Score
	NrtvQA	Qasper	MF-en	HotpotQA	2WikiMQA	Musique	GovReport	QMSum	MultiNews	TREC	TriviaQA	SAMSum	PCount	PRe	Lcc	RB-P	
Full Cache	30.22	45.37	55.80	55.97	45.00	31.26	35.12	25.38	27.20	72.50	91.64	43.57	9.41	99.50	62.88	56.43	49.20
B=128																	
SLM	22.24	20.87	31.72	44.02	37.55	24.54	18.76	21.09	18.48	40.50	84.41	38.82	8.00	**99.50**	57.02	47.29	38.43
Pyramid	25.70	24.69	47.74	52.87	40.57	27.23	20.02	22.38	19.74	44.50	88.81	40.30	7.22	**99.50**	57.25	49.90	41.78
SnapKV	25.54	24.45	48.03	**53.31**	40.75	28.19	20.13	22.36	19.55	45.50	89.20	40.62	6.97	**99.50**	58.45	49.90	42.03
Ada-Pyramid	**27.07**	**25.61**	49.30	53.02	41.29	27.83	**20.70**	23.18	**20.38**	**51.50**	**90.76**	40.62	6.92	99.00	**59.30**	50.88	**42.96**
Ada-SnapKV	24.90	24.41	**49.95**	53.15	**41.73**	**28.55**	20.54	**23.21**	20.28	50.50	89.49	**40.71**	**7.45**	99.00	58.74	**52.40**	42.81
B=256																	
SLM	22.71	23.79	31.80	43.43	36.55	25.55	21.29	20.68	20.67	46.00	87.11	40.82	7.20	99.50	59.89	49.19	39.76
Pyramid	25.53	33.15	51.44	**55.03**	42.42	28.62	22.57	23.37	22.33	56.50	91.19	**41.28**	6.97	99.50	60.36	51.18	44.47
SnapKV	26.02	32.49	51.62	54.40	**42.77**	28.94	22.83	23.54	22.55	53.50	91.10	40.95	7.48	99.50	60.67	53.39	44.48
Ada-Pyramid	25.12	**35.06**	52.28	54.66	41.89	28.76	23.14	23.36	22.67	63.00	90.72	41.21	7.75	99.50	61.47	53.09	45.23
Ada-SnapKV	**26.11**	33.39	51.44	54.94	42.15	**29.54**	23.01	**23.85**	**22.88**	**63.50**	**91.57**	40.94	**8.00**	99.50	**61.95**	**54.33**	**45.44**
B=512																	
SLM	25.51	25.78	34.19	45.01	35.91	24.93	23.61	21.26	23.57	57.50	87.86	41.44	6.98	96.50	60.85	51.02	41.37
Pyramid	28.71	39.89	52.86	54.00	**44.20**	31.22	24.74	23.73	24.28	66.00	91.07	41.42	**8.39**	99.50	61.99	53.44	46.59
SnapKV	**29.22**	40.01	**53.15**	54.47	43.63	**31.32**	25.04	23.77	24.19	64.00	92.05	41.57	8.01	99.50	63.21	55.05	46.76
Ada-Pyramid	28.04	**40.63**	53.03	**54.71**	43.39	30.26	25.35	24.12	24.61	69.00	91.79	**42.55**	7.95	99.50	62.28	54.49	46.98
Ada-SnapKV	29.07	40.16	52.44	53.90	43.05	31.10	**25.75**	**24.39**	**24.85**	69.00	**92.34**	42.05	7.98	99.50	**63.43**	**55.32**	**47.15**
B=1024																	
SLM	24.97	30.22	37.06	46.57	39.14	25.24	26.01	21.08	25.72	63.50	88.87	42.28	6.98	89.00	61.30	53.40	42.58
Pyramid	**29.62**	43.66	54.10	**55.06**	44.22	31.30	27.27	24.30	25.68	68.50	91.27	41.96	7.73	99.50	63.13	55.85	47.70
SnapKV	29.28	43.64	**54.34**	54.24	**44.34**	31.52	27.80	24.39	25.45	69.00	**91.72**	42.50	7.80	99.50	62.99	56.45	47.84
Ada-Pyramid	28.76	**44.57**	53.73	54.89	44.15	**31.97**	27.75	**25.26**	25.84	70.50	91.62	42.37	7.67	99.50	62.96	**56.52**	48.00
Ada-SnapKV	29.23	44.09	53.82	54.80	44.01	31.40	**28.86**	24.73	**26.04**	**72.50**	91.72	**42.56**	7.82	99.50	63.22	56.33	**48.16**
B=2048																	
SLM	26.25	35.81	38.78	48.71	41.78	25.01	28.48	22.13	26.55	67.50	90.86	42.21	9.41	87.50	64.80	57.56	44.58
Pyramid	29.81	45.13	54.88	55.74	44.36	**31.71**	30.28	24.83	26.64	71.00	91.35	42.42	**10.43**	99.50	64.66	58.64	48.84
SnapKV	29.75	45.18	55.06	**56.09**	44.82	31.42	31.13	24.92	26.93	72.00	**91.65**	42.81	9.99	99.50	64.82	**59.30**	49.09
Ada-Pyramid	**30.88**	44.64	54.94	55.59	45.00	31.04	30.30	**25.14**	26.93	72.00	91.64	**43.25**	10.09	99.50	64.60	59.20	49.05
Ada-SnapKV	30.64	**45.23**	**55.46**	55.55	**45.26**	31.14	**31.28**	24.89	26.72	**72.50**	91.64	42.75	9.68	99.50	**64.88**	59.25	**49.15**

Table 6: Detailed results of Mistral-7B-Instruct-v0.2 on LongBench. (Question-aware)

	Single-Doc. QA			Multi-Doc. QA			Summarization			Few-shotLearning			Synthetic		Code		Ave. Score
	NrtvQA	Qasper	MF-en	HotpotQA	2WikiMQA	Musique	GovReport	QMSum	MultiNews	TREC	TriviaQA	SAMSum	PCount	PRe	Lcc	RB-P	
Full Cache	26.74	32.84	50.00	43.45	27.77	18.49	32.91	24.64	26.99	71.00	86.23	43.32	2.94	86.31	57.39	54.32	42.83
B=128																	
SLM	18.02	13.32	27.41	30.49	21.81	11.85	15.49	19.42	17.84	44.00	80.74	37.35	**3.57**	22.93	49.07	43.74	28.57
Pyramid	20.78	19.76	42.92	36.31	22.37	13.91	18.84	21.84	20.42	46.50	**84.55**	**40.23**	2.79	65.46	51.49	46.83	34.69
SnapKV	20.98	19.05	**44.63**	35.31	22.91	13.95	18.76	21.36	20.31	45.00	83.83	39.52	3.23	64.53	52.19	47.30	34.55
Ada-Pyramid	**21.18**	**20.23**	44.54	**37.97**	22.85	14.54	19.10	21.91	**20.84**	52.00	84.15	39.49	2.87	**71.81**	**52.34**	47.50	**35.83**
Ada-SnapKV	20.10	20.14	44.20	37.32	**23.90**	**15.29**	**19.15**	**22.27**	20.71	50.00	84.20	39.64	3.09	67.11	52.26	**48.25**	35.48
B=256																	
SLM	19.08	15.30	28.27	31.87	22.26	11.37	18.08	19.37	19.99	51.00	80.92	39.62	**3.57**	15.90	51.74	45.22	29.60
Pyramid	20.39	22.63	46.67	38.64	23.73	15.82	21.34	22.30	22.01	57.50	83.63	40.49	2.99	75.84	53.56	50.03	37.35
SnapKV	20.90	21.95	47.49	**39.62**	24.89	15.04	21.46	22.80	22.62	57.50	84.86	40.51	**3.76**	77.95	54.08	50.98	37.90
Ada-Pyramid	**22.21**	23.26	46.92	39.04	**25.01**	**16.99**	21.43	**23.07**	22.28	62.50	85.70	**41.11**	2.71	78.86	54.26	50.49	38.49
Ada-SnapKV	20.97	**23.71**	**47.52**	38.48	24.77	15.76	**21.89**	22.94	**22.70**	**63.50**	**86.25**	40.75	2.46	**80.24**	**54.42**	**51.46**	**38.61**
B=512																	
SLM	21.12	15.99	30.82	30.39	22.32	10.92	21.43	19.98	22.88	61.50	82.11	41.89	3.21	17.32	53.43	47.05	31.40
Pyramid	21.81	24.33	48.67	39.31	25.14	17.51	23.22	23.16	23.87	66.00	85.20	41.96	3.08	85.71	55.10	50.98	39.69
SnapKV	**24.29**	**27.83**	**49.00**	39.63	25.26	17.63	**23.53**	23.36	24.44	65.00	86.18	42.01	**3.27**	85.29	56.10	52.73	40.35
Ada-Pyramid	22.92	26.37	47.66	39.49	25.52	**18.50**	23.43	**23.53**	23.87	66.50	85.42	41.98	2.59	85.49	55.14	52.29	40.04
Ada-SnapKV	23.62	27.34	48.70	**39.81**	**26.42**	17.36	23.42	23.26	**24.50**	**67.50**	**86.46**	**42.04**	3.04	**86.64**	**56.11**	**53.05**	**40.58**
B=1024																	
SLM	22.15	18.64	31.03	32.94	22.45	11.93	23.89	20.60	25.48	64.00	84.71	41.59	3.49	25.33	53.72	49.19	33.00
Pyramid	24.12	29.44	48.83	40.30	25.94	19.42	25.29	23.52	25.77	68.00	**86.30**	41.62	2.84	86.07	56.06	52.57	41.01
SnapKV	24.10	**30.11**	48.97	40.97	26.89	18.12	**25.93**	23.75	26.02	67.50	86.25	42.33	2.94	87.23	**57.07**	53.43	41.35
Ada-Pyramid	24.82	28.94	48.47	**41.44**	26.58	**19.75**	25.08	**23.84**	25.24	68.00	85.80	42.44	**3.51**	85.68	56.45	52.72	41.22
Ada-SnapKV	**25.11**	29.98	**49.27**	40.62	**27.05**	18.54	25.85	23.46	**26.08**	**68.50**	**86.30**	**42.77**	2.92	**88.27**	56.88	**54.16**	**41.61**
B=2048																	
SLM	22.63	22.68	35.44	33.66	22.92	13.76	26.96	20.80	26.71	66.00	85.94	42.11	2.40	25.33	57.09	52.82	34.83
Pyramid	25.68	31.39	**49.25**	41.01	27.06	**19.35**	27.52	23.46	26.52	70.00	86.30	42.65	2.72	85.93	56.96	53.63	41.84
SnapKV	25.79	32.54	49.18	42.09	27.31	18.91	28.50	24.01	26.83	70.00	**86.46**	42.86	**2.95**	86.56	57.24	53.89	42.20
Ada-Pyramid	26.41	31.74	49.08	40.95	**27.79**	18.89	27.54	23.88	26.78	**70.50**	86.30	**43.44**	2.61	85.93	57.22	53.56	42.04
Ada-SnapKV	**26.60**	**32.68**	49.10	**42.65**	27.27	18.97	**28.60**	**24.17**	**26.86**	70.00	86.30	43.12	2.45	**86.81**	**57.33**	**54.11**	42.31

Given $A_i = \mathrm{softmax}(s_i)$ where $s_i = q_i K_i^T$, we can get $A_i^j = \frac{exp(s_i^j)}{\sum_j exp(s_i^j)}$.

$$\hat{A}_i^j = \mathcal{I}_i^j A_i^j \frac{\sum_j exp(s_i^j)}{\sum_j \mathcal{I}_i^j exp(s_i^j)} = \frac{\mathcal{I}_i^j A_i^j}{||A_i \odot \mathcal{I}_i||_1} \tag{14}$$

Table 7: Detailed results of Llama-3.1-8B-Instruct on LongBench. (Question-agnostic)

| | Single-Doc. QA | | | Multi-Doc. QA | | | Summarization | | | Few-shot Learning | | | Synthetic | | Code | | Ave. |
	NrtvQA	Qasper	MF-en	HotpotQA	2WikiMQA	Musique	GovReport	QMSum	MultiNews	TREC	TriviaQA	SAMSum	PCount	PRe	Lcc	RB-P	Score
Full Cache	30.22	45.37	55.80	55.97	45.00	31.26	35.12	25.38	27.20	72.50	91.64	43.57	9.41	99.50	62.88	56.43	49.20
B=128																	
SLM	13.57	11.48	24.47	34.25	25.35	11.31	20.06	17.39	17.80	30.50	50.14	35.33	3.00	5.50	15.50	60.25	23.49
Pyramid	**13.91**	13.05	**18.71**	27.43	14.36	6.35	17.93	17.25	19.52	21.00	89.09	38.03	1.11	5.00	61.93	57.00	26.35
SnapKV	12.23	11.88	17.93	25.32	12.41	8.20	17.88	16.84	19.44	22.00	90.97	37.83	2.50	5.50	61.62	**57.54**	26.26
Ada-Pyramid	13.30	**13.78**	17.92	**31.35**	**15.90**	7.65	**19.20**	**17.48**	19.83	24.00	90.67	**38.23**	4.10	**6.00**	**63.40**	56.38	**27.45**
Ada-SnapKV	13.52	12.79	18.30	28.94	14.13	**9.35**	19.04	17.26	**19.94**	**25.00**	**91.44**	37.96	1.54	4.00	63.34	56.39	27.06
B=256																	
SLM	14.25	12.92	28.75	31.80	19.19	11.39	22.86	17.26	22.73	44.00	52.80	38.77	4.00	9.00	16.11	59.79	25.35
Pyramid	13.57	17.36	19.29	30.95	18.10	8.21	20.30	17.65	20.90	26.00	90.94	40.02	4.73	4.50	64.71	55.06	28.27
SnapKV	14.82	16.32	18.44	29.53	14.25	**9.26**	20.32	17.56	21.70	28.50	91.49	40.15	4.12	5.00	64.60	54.78	28.18
Ada-Pyramid	14.39	**18.66**	**20.79**	32.07	**20.08**	6.52	20.84	**18.20**	21.51	31.00	91.39	41.26	**7.10**	**7.00**	65.87	**56.06**	**29.55**
Ada-SnapKV	**15.19**	15.97	20.05	**35.08**	16.31	8.32	**21.31**	17.99	**22.11**	**33.00**	**91.68**	**41.28**	5.93	5.50	**66.14**	54.76	29.41
B=512																	
SLM	14.47	16.86	34.18	31.88	18.76	11.80	26.01	18.20	25.10	53.00	60.04	41.23	4.00	9.50	19.74	57.87	27.66
Pyramid	15.67	21.59	23.56	35.63	24.36	9.77	22.18	19.05	23.25	34.00	91.11	42.44	4.61	**17.50**	65.87	54.41	31.56
SnapKV	15.22	22.90	23.41	33.00	22.54	8.91	22.63	18.31	23.50	35.00	91.39	42.83	4.50	15.50	**66.79**	53.75	31.20
Ada-Pyramid	16.05	**25.46**	**25.75**	37.04	**26.72**	**10.65**	22.80	**19.75**	23.28	39.50	91.65	42.64	5.75	15.00	65.55	**55.73**	32.71
Ada-SnapKV	**17.72**	24.22	25.38	**38.00**	23.47	9.25	**23.55**	18.91	**23.82**	**43.00**	**92.14**	**42.81**	**6.34**	15.00	66.12	55.32	**32.82**
B=1024																	
SLM	13.36	21.55	41.70	32.80	22.63	12.88	28.40	19.36	25.93	62.00	75.54	41.76	2.00	11.00	22.91	57.05	30.68
Pyramid	16.67	29.11	28.74	40.13	27.89	13.43	24.63	20.36	25.01	43.00	**91.86**	43.48	6.78	52.50	65.34	54.87	36.49
SnapKV	19.57	**31.67**	31.52	**42.04**	27.89	12.94	25.31	19.66	25.12	48.50	91.37	42.83	5.99	**56.00**	**66.56**	53.97	37.56
Ada-Pyramid	**20.04**	31.37	**32.77**	40.91	**30.40**	14.35	24.97	**21.36**	25.37	46.50	91.61	43.78	**9.13**	54.50	65.35	55.27	37.98
Ada-SnapKV	19.74	30.47	31.42	40.82	28.98	**16.07**	25.35	20.57	**25.57**	**53.50**	91.76	**43.81**	4.63	53.50	65.75	**55.90**	**37.99**
B=2048																	
SLM	16.86	31.83	48.22	33.59	29.36	15.74	30.11	20.89	26.68	65.50	92.49	44.12	5.25	21.00	39.75	56.66	36.13
Pyramid	21.16	36.66	37.56	43.28	36.25	19.72	27.90	21.73	26.35	53.00	92.36	44.61	6.94	**89.50**	64.23	53.11	42.15
SnapKV	21.24	39.86	38.21	45.96	35.36	**21.20**	**28.68**	21.49	26.50	58.00	**92.36**	44.06	6.31	88.50	**64.26**	53.73	42.86
Ada-Pyramid	20.75	39.12	**40.18**	44.49	**41.17**	18.94	27.90	**22.06**	26.71	57.50	91.86	44.35	**10.30**	84.50	63.72	**54.81**	43.02
Ada-SnapKV	**22.42**	**40.26**	40.13	**49.60**	38.23	19.70	28.49	21.76	**26.77**	**61.50**	92.35	44.04	8.27	85.00	64.11	54.09	**43.55**

Table 8: Detailed results of Mistral-7B-Instruct-v0.2 on LongBench. (Question-agnostic)

| | Single-Doc. QA | | | Multi-Doc. QA | | | Summarization | | | Few-shot Learning | | | Synthetic | | Code | | Ave. |
	NrtvQA	Qasper	MF-en	HotpotQA	2WikiMQA	Musique	GovReport	QMSum	MultiNews	TREC	TriviaQA	SAMSum	PCount	PRe	Lcc	RB-P	Score
Full Cache	26.74	32.84	50.00	43.45	27.77	18.49	32.91	24.64	26.99	71.00	86.23	43.32	2.94	86.31	57.39	54.32	42.83
B=128																	
SLM	7.62	7.49	26.12	18.97	14.44	7.26	19.46	17.29	19.46	26.50	73.27	36.65	1.50	4.50	14.58	59.10	22.14
Pyramid	10.69	8.37	20.65	18.34	13.78	**5.99**	17.32	17.71	**19.20**	22.50	82.69	37.48	3.25	**3.00**	53.63	58.13	24.55
SnapKV	10.51	9.52	21.28	18.76	13.85	5.57	17.68	18.05	18.18	21.50	82.52	36.61	**3.74**	2.50	52.84	59.04	24.51
Ada-Pyramid	**12.53**	**9.61**	20.94	18.54	14.38	4.90	17.54	**18.15**	19.02	22.00	**84.44**	**37.72**	3.67	1.33	**55.49**	59.28	24.97
Ada-SnapKV	11.13	9.33	**21.78**	**19.41**	**14.96**	5.95	**17.82**	17.39	19.15	**24.50**	83.71	37.33	3.73	1.50	54.91	**59.92**	**25.16**
B=256																	
SLM	8.37	7.47	28.82	21.42	15.30	6.98	21.74	18.23	22.92	38.50	74.46	37.77	1.25	3.75	17.64	58.95	23.97
Pyramid	9.86	**11.24**	21.86	20.71	13.40	6.58	20.07	18.21	21.04	26.00	85.73	39.34	3.11	**8.68**	57.18	56.31	26.21
SnapKV	10.99	10.94	23.27	21.67	13.87	6.42	20.16	18.26	21.54	28.50	85.72	39.21	2.56	6.50	57.74	58.10	26.59
Ada-Pyramid	**11.01**	9.98	22.14	22.26	**15.42**	7.17	19.78	**18.74**	21.27	31.00	**87.30**	39.32	3.42	8.43	58.18	58.08	27.09
Ada-SnapKV	10.45	10.47	**23.67**	**22.45**	13.48	**7.29**	**20.58**	18.21	21.26	**32.00**	87.05	**39.74**	**3.66**	8.13	**58.99**	**58.44**	**27.26**
B=512																	
SLM	9.54	9.05	32.27	21.64	15.17	8.76	24.89	18.33	25.39	49.00	75.51	39.40	1.50	5.75	21.43	57.64	25.95
Pyramid	11.72	12.41	23.61	23.77	14.36	6.61	21.92	19.09	23.25	37.50	87.18	40.61	3.40	14.29	59.68	57.17	28.54
SnapKV	11.01	**13.37**	24.86	25.33	13.39	6.37	22.98	19.00	**23.48**	41.50	86.94	40.05	**3.83**	14.97	60.18	57.33	29.04
Ada-Pyramid	11.51	12.59	24.90	23.29	**15.62**	7.10	21.78	**19.41**	23.13	45.00	87.42	40.96	3.37	**19.10**	60.65	**58.46**	29.64
Ada-SnapKV	**13.07**	12.48	**26.69**	**26.33**	14.12	**7.74**	**23.02**	19.13	23.43	**47.00**	**87.81**	**41.05**	3.77	17.83	**60.95**	57.20	**30.10**
B=1024																	
SLM	11.05	14.15	38.89	23.79	18.25	9.71	27.20	20.25	26.20	52.50	76.96	41.79	0.00	8.50	24.24	56.58	28.13
Pyramid	12.22	16.32	28.01	26.55	**15.58**	7.59	23.91	**20.04**	25.38	46.00	87.41	41.37	3.74	34.29	63.44	**58.19**	31.88
SnapKV	13.43	16.30	30.57	27.04	14.88	**8.48**	**24.97**	19.71	**25.64**	50.50	87.32	41.19	2.87	28.21	63.55	57.38	32.00
Ada-Pyramid	12.34	17.09	30.24	27.13	14.01	8.26	23.60	20.03	25.01	52.50	87.26	**42.75**	3.96	39.33	62.92	57.67	32.76
Ada-SnapKV	**13.86**	**18.39**	**31.07**	**29.03**	14.92	8.24	24.82	19.86	25.59	**55.50**	**87.81**	41.98	3.04	35.96	**64.32**	57.95	**33.27**
B=2048																	
SLM	11.80	20.40	43.35	25.01	18.21	9.80	29.40	21.40	26.73	59.00	83.06	43.26	0.38	15.75	30.11	56.05	30.86
Pyramid	14.04	21.19	36.29	27.07	15.74	10.21	25.62	20.71	26.25	52.50	87.31	43.53	4.05	65.42	64.97	58.04	35.81
SnapKV	**15.91**	21.64	35.48	29.94	16.34	11.49	**27.29**	20.67	**26.78**	56.00	87.46	42.19	**4.93**	61.83	65.34	56.88	36.26
Ada-Pyramid	15.44	21.48	36.86	29.43	**16.88**	10.75	25.52	20.74	26.60	54.50	87.17	43.05	2.89	**70.96**	65.15	**58.66**	36.63
Ada-SnapKV	15.84	**22.24**	**38.67**	**32.03**	16.54	**12.68**	26.35	**20.93**	26.65	**57.50**	87.49	**43.55**	4.66	68.67	64.24	58.25	**37.27**

Then we can obtain:

$$\hat{A}_i = \frac{A_i \odot \mathcal{I}_i}{||A_i \odot \mathcal{I}_i||_1} \tag{15}$$

Thus:

$$\hat{y} = \sum_{i \in [1,h]} \frac{A_i \odot \mathcal{I}_i}{||A_i \odot \mathcal{I}_i||_1} V_i W_i^O \tag{16}$$

By calculating the L1 distance between their outputs, we can obtain

$$||y - \hat{y}||_1 = || \sum_{i \in [1,h]} (\mathbf{1} - \frac{\mathcal{I}_i}{||A_i \odot \mathcal{I}_i||_1}) \odot A_i V_i W_i^O ||_1 \tag{17}$$

$$\leq \sum_{i \in [1,h]} ||(\mathbf{1} - \frac{\mathcal{I}_i}{||A_i \odot \mathcal{I}_i||_1}) \odot A_i V_i W_i^O ||_1 \tag{18}$$

$$\leq \sum_{i \in [1,h]} ||(\mathbf{1} - \frac{\mathcal{I}_i}{||A_i \odot \mathcal{I}_i||_1}) \odot A_i ||_1 ||V_i W_i^O||_\infty \tag{19}$$

$$\leq C \sum_{i \in [1,h]} ||(\mathbf{1} - \frac{\mathcal{I}_i}{||A_i \odot \mathcal{I}_i||_1}) \odot A_i ||_1 \tag{20}$$

$$where \; C = Max \left\{ ||V_i W_i^O||_\infty \right\}$$

By expanding A_i, we can further simplify the expression.

$$\text{Let } ||A_i \odot \mathcal{I}_i||_1 \text{ as } F_i \in (0,1) \tag{21}$$

$$||y - \hat{y}||_1 \leq C \sum_{i \in [1,h]} ||(\mathbf{1} - \frac{\mathcal{I}_i}{||A_i \odot \mathcal{I}_i||_1}) \odot A_i ||_1 \tag{22}$$

$$= C \sum_{i \in [1,h]} \sum_{j \in [1,n]} \frac{|F_i - \mathcal{I}_i^j| A_i^j}{F_i} \tag{23}$$

Considering

$$\mathcal{I}_i^j = \begin{cases} 1 & \text{if } K_i^j \text{ and } V_i^j \text{ are retained} \\ 0 & \text{otherwise, evict } K_i^j \text{ and } V_i^j \end{cases} \; and \; \sum_{j \in [1,n]} A_i^j = 1 \tag{24}$$

$$||y - \hat{y}||_1 \leq C \sum_{i \in [1,h]} \overset{if \mathcal{I}_i^j = 0}{\sum_{j \in [1,n]}} A_i^j + C \sum_{i \in [1,h]} \frac{(1 - F_i)}{F_i} \overset{if \mathcal{I}_i^j = 1}{\sum_{j \in [1,n]}} A_i^j \tag{25}$$

$$\text{Due to } F_i = ||A_i \odot \mathcal{I}_i||_1 = \sum_{j \in [1,n]} \mathcal{I}_i^j A_i^j = \overset{if \mathcal{I}_i^j = 1}{\sum_{j \in [1,n]}} A_i^j$$

$$||y - \hat{y}||_1 \leq C \sum_{i \in [1,h]} \overset{if \mathcal{I}_i^j = 0}{\sum_{j \in [1,n]}} A_i^j + C \sum_{i \in [1,h]} (1 - \overset{if \mathcal{I}_i^j = 1}{\sum_{j \in [1,n]}} A_i^j) \tag{26}$$

$$= 2C \sum_{i \in [1,h]} \overset{if \mathcal{I}_i^j = 0}{\sum_{j \in [1,n]}} A_i^j \tag{27}$$

$$= 2C \sum_{i \in [1,h]} \sum_{j \in [1,n]} (1 - \mathcal{I}_i^j) A_i^j \tag{28}$$

$$= 2hC - 2C \sum_{i \in [1,h]} \sum_{j \in [1,n]} \mathcal{I}_i^j A_i^j \tag{29}$$

Finally,

$$L_1 \text{ Eviction Loss} \leq \epsilon = 2hC - 2C \sum_{i \in [1,h]} \sum_{j \in [1,n]} \mathcal{I}_i^j A_i^j \tag{30}$$

$$\square$$

A.9 Proof of Theorem 3.2

Theorem A.2. *The Top-k cache eviction $\{\mathcal{I}_i^*\}$ minimizes the upper bound ϵ of L_1 eviction loss:*

$$\text{Top-k eviction decision } \{\mathcal{I}_i^*\} = \arg\min_{\{\mathcal{I}_i\}} \epsilon, \tag{31}$$

$$\epsilon^* = \min_{\{\mathcal{I}_i\}} \epsilon = 2hC - 2C \sum_{i \in [1,h]} \overset{A_i^j \in \text{Top-k}(A_i, B_i)}{\sum_{j \in [1,n]}} A_i^j \tag{32}$$

Proof. Given the budget allocation results $\{B_i\}$, the objective of minimizing ϵ can be decomposed into $i \in [1, h]$ independent subproblems, each expressed as follows:

$$\arg\max_{\mathcal{I}_i} \sum_{j \in [1,n]} \mathcal{I}_i^j A_i^j, \quad \text{s.t.} \quad \sum_{j \in [1,n]} \mathcal{I}_i^j = B_i \tag{33}$$

The Top-k cache eviction maximizes the h independent subproblems, thereby minimizing the upper bound ϵ. Formally:

$$\mathcal{I}_i^* = \arg\max_{\mathcal{I}_i} \sum_{j \in [1,n]} \mathcal{I}_i^j A_i^j \tag{34}$$

The optimal solution satisfies:

$$\min_{\mathcal{I}_i} \sum_{j \in [1,n]} \mathcal{I}_i^j A_i^j = \sum_{j \in [1,n]} \mathcal{I}_i^{*j} A_i^j = \overset{A_i^j \in \text{Top-k}(A_i, B_i)}{\sum_{j \in [1,n]}} A_i^j \tag{35}$$

$\square$

A.10 Proof of Theorem 3.3

Theorem A.3. *The adaptive budget allocation $\{B_i^*\}$ derived from Algorithm 1 achieves the minimal upper bound ϵ^{**} for loss associated with Top-k eviction methods:*

$$\epsilon^{**} = \min_{\{B_i\}} \epsilon^* \tag{36}$$

Proof. From line 3 of Algorithm 1, which adaptively allocates budgets $\{B_i^*\}$ based on the Top-k indices $T = \text{Top-k}(A, B)$.indices, it follows that the resulting Top-k eviction leads to an upper bound ϵ^{**}:

$$\epsilon^{**} = 2hC - 2C \sum_{i \in [1,h]} \overset{A_i^j \in \text{Top-k}(A_i, B_i^*)}{\sum_{j \in [1,n]}} A_i^j \tag{37}$$

$$= 2hC - 2C \sum_{i \in [1,h]} \overset{A_i^j \in \text{Top-k}(A, B)}{\sum_{j \in [1,n]}} A_i^j \tag{38}$$

Given a fixed overall budget, i.e., $\sum_{i \in [1,h]} B_i = \sum_{i \in [1,h]} B_i^*$, we can derive that $\epsilon^{**} \leq \epsilon^*$ for any budget allocation results $\{B_i\}$. This is because, in the upper bound, the term $\sum_{i \in [1,h]} \sum_{j \in [1,n]}^{A_i^j \in \text{Top-k}(A,B)} A_i^j \geq \sum_{i \in [1,h]} \sum_{j \in [1,n]}^{A_i^j \in \text{Top-k}(A_i, B_i)} A_i^j$. This result can also be understood as a global optimal solution always outperforming a local optimal solution. $\square$

A.11 Detailed Visualization of Head Concentration

Figure 11 supplements Figure 1a in the main paper by presenting the visualization results across all layers. It can be observed that in all layers, different heads exhibit significant variations in attention concentration. This indicates that the adaptive allocation strategy has great potential to reduce the eviction loss in practice.
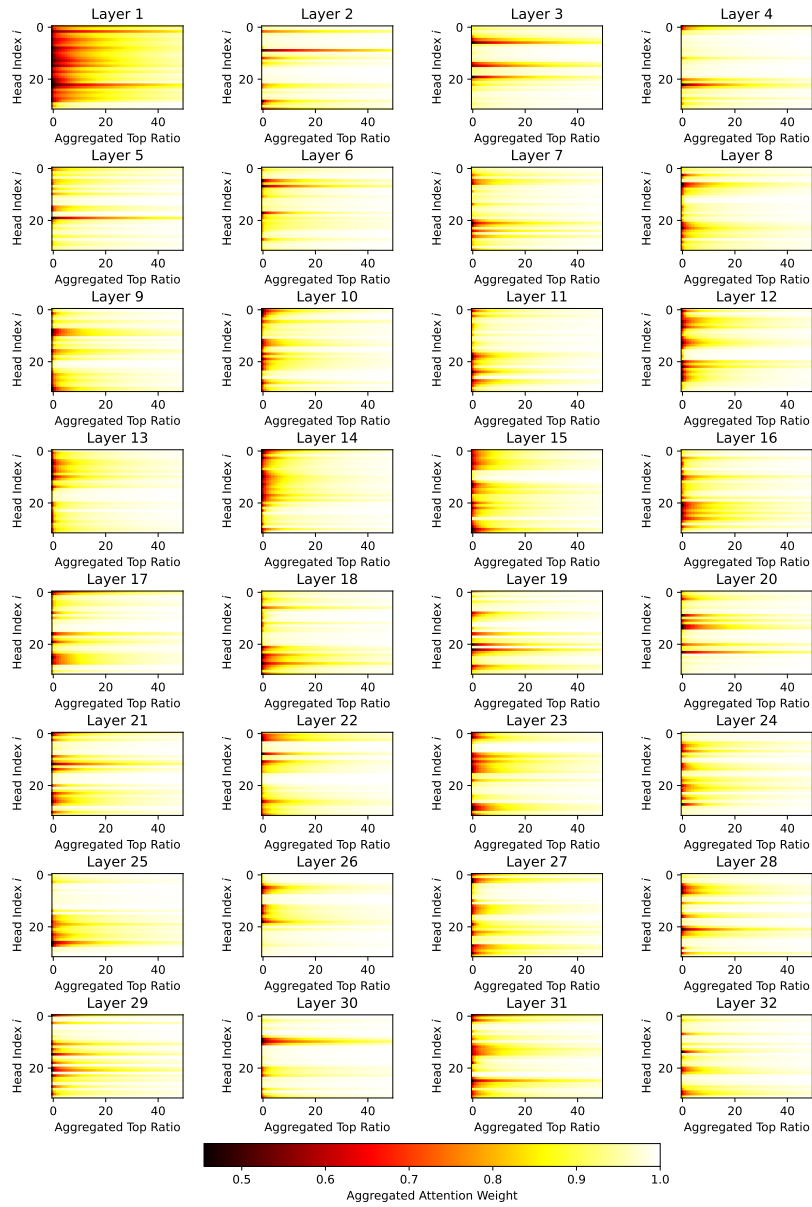
Figure 11: Visualization of Heads' Concentrations

Table 9: S-NIAH and MK-NIAH templates in Ruler Benchmark. [32]

Single NIAH Subtask-1 (S-NIAH-1)	**Task Template:** Some special magic numbers are hidden within the following text. Make sure to memorize it. I will quiz you about the numbers afterwards. The grass is green. The sky is blue. The sun is yellow. Here we go. There and back again. ...... One of the special magic numbers for {word} is: {number}. ...... What is the special magic number for {word} mentioned in the provided text? The special magic number for {word} mentioned in the provided text is
Single NIAH Subtask-2 (S-NIAH-2)	**Task Template:** Some special magic numbers are hidden within the following text. Make sure to memorize it. I will quiz you about the numbers afterwards. Paul Graham Essays. ...... One of the special magic numbers for {word} is: {number}. ...... What is the special magic number for {word} mentioned in the provided text? The special magic number for {word} mentioned in the provided text is
Single NIAH Subtask-3 (S-NIAH-3)	**Task Template:** Some special magic words are hidden within the following text. Make sure to memorize it. I will quiz you about the words afterwards. Paul Graham Essays. ...... One of the special magic words for {word} is: {word}. ...... What is the special magic word for {word} mentioned in the provided text? The special magic word for {word} mentioned in the provided text is
Multi-keys NIAH Subtask-1 (MK-NIAH-1)	**Task Template:** Some special magic numbers are hidden within the following text. Make sure to memorize it. I will quiz you about the numbers afterwards. Paul Graham Essays. ...... One of the special magic numbers for {word-1} is: {number-1}. .... One of the special magic numbers for {word-2} is: {number-2}. .... One of the special magic numbers for {word-3} is: {number-3}. .... One of the special magic numbers for {word-4} is: {number-4}. ...... What is the special magic number for {word-4} mentioned in the provided text? The special magic number for {word-4} mentioned in the provided text is
Multi-keys NIAH Subtask-2 (MK-NIAH-2)	**Task Template:** Some special magic numbers are hidden within the following text. Make sure to memorize it. I will quiz you about the numbers afterwards. One of the special magic numbers for {word-1} is: {number-1}. One of the special magic numbers for {word-2} is: {number-2}. ...... One of the special magic numbers for {word-x} is: {number-x}. ...... One of the special magic numbers for {word-n-1} is: {number-n-1}. One of the special magic numbers for {word-n} is: {number-n}. What is the special magic number for {word-x} mentioned in the provided text? The special magic number for {word-x} mentioned in the provided text is
Multi-keys NIAH Subtask-3 (MK-NIAH-3)	**Task Template:** Some special magic uuids are hidden within the following text. Make sure to memorize it. I will quiz you about the uuids afterwards. One of the special magic uuids for {uuid-1} is: {uuid-1}. One of the special magic uuids for {uuid-2} is: {uuid-2}. ...... One of the special magic uuids for {uuid-x} is: {uuid-x}. ...... One of the special magic uuids for {uuid-n-1} is: {uuid-n-1}. One of the special magic uuids for {uuid-n} is: {uuid-n}. What is the special magic number for {uuid-x} mentioned in the provided text? The special magic number for {uuid-x} mentioned in the provided text is

Table 10: MV-NIAH, MQ-NIAH, VT, CWE, and FWE templates in Ruler Benchmark. [32]

Multi-values NIAH (MV-NIAH)	**Task Template:** Some special magic numbers are hidden within the following text. Make sure to memorize it. I will quiz you about the numbers afterwards. Paul Graham Essays. ...... One of the special magic numbers for {word} is: {number-1}. .... One of the special magic numbers for {word} is: {number-2}. .... One of the special magic numbers for {word} is: {number-3}. .... One of the special magic numbers for {word} is: {number-4}. ...... What are all the special magic numbers for {word} mentioned in the provided text? The special magic numbers for {word} mentioned in the provided text are
Multi-queries NIAH (MQ-NIAH)	**Task Template:** Some special magic numbers are hidden within the following text. Make sure to memorize it. I will quiz you about the numbers afterwards. Paul Graham Essays. ...... One of the special magic numbers for {word-1} is: {number-1}. .... One of the special magic numbers for {word-2} is: {number-2}. .... One of the special magic numbers for {word-3} is: {number-3}. .... One of the special magic numbers for {word-4} is: {number-4}. ...... What are all the special magic numbers for {word-1}, {word-2}, {word-3}, and {word-4} mentioned in the provided text? The special magic numbers for {word-1}, {word-2}, {word-3}, and {word-4} mentioned in the provided text are
Variable Tracking (VT)	**Task Template:** {one task example} Memorize and track the chain(s) of variable assignment hidden in the following text. The grass is green. The sky is blue. The sun is yellow. Here we go. There and back again. ...... VAR {X1} = {number} .... VAR {X2} = {X1} .... VAR {X3} = {X2} .... VAR {X4} = {X3} .... VAR {X5} = {X4} ...... Question: Find all variables that are assigned the value {number} in the text above. Answer: According to the chain(s) of variable assignment in the text above, 5 variables are assigned the value {number}, they are:
Common Words Extraction (CWE)	**Task Template:** {one task example} Below is a numbered list of words. In these words, some appear more often than others. Memorize the ones that appear most often. 1. word-a 2. word-b 3. word-c 4. word-a 5. word-d 6. word-a 7. word-e 8. word-f ...... Question: What are the 10 most common words in the above list? Answer: The top 10 words that appear most often in the list are:
Frequent Words Extraction (FWE)	**Task Template:** Read the following coded text and track the frequency of each coded word. Find the three most frequently appeared coded words. ... ... word-a ... word-b ... ... word-c ... word-a ... word-d word-e ... word-a ... ... word-f ... ... word-g ... word-h ... word-a ... word-i ...... Question: Do not provide any explanation. Please ignore the dots '....'. What are the three most frequently appeared words in the above coded text? Answer: According to the coded text above, the three most frequently appeared words are:

Table 11: QA templates in Ruler Benchmark. [32]

	Task Template: Answer the question based on the given documents. Only give me the answer and do not output any other words. The following are given documents. Document 1: {document-1} ...... Document x: {document-x} ...... Document n: {document-n} Answer the question based on the given documents. Only give me the answer and do not output any other words. Question: {question} Answer:
Single Hop QA (S-QA)	
Multi Hop QA (M-QA)	**Task Template:** Answer the question based on the given documents. Only give me the answer and do not output any other words. The following are given documents. Document 1: {document-1} ...... Document x: {document-x} ...... Document y: {document-y} ...... Document n: {document-n} Answer the question based on the given documents. Only give me the answer and do not output any other words. Question: {question} Answer:

Table 12: Details of 16 Datasets in LongBench

Label	Task	Task Type	Eval metric	Avg len	Language	Sample Num
NrtvQA	NarrativeQA	Single-Doc. QA	F1	18,409	EN	200
Qasper	Qasper	Single-Doc. QA	F1	3,619	EN	200
MF-en	MultiFieldQA-en	Single-Doc. QA	F1	4,559	EN	150
HotpotQA	HotpotQA	Multi-Doc. QA	F1	9,151	EN	200
2WikiMQA	2WikiMultihopQA	Multi-Doc. QA	F1	4,887	EN	200
Musique	MuSiQue	Multi-Doc. QA	F1	11,214	EN	200
GovReport	GovReport	Summarization	Rouge-L	8,734	EN	200
QMSum	QMSum	Summarization	Rouge-L	10,614	EN	200
MultiNews	MultiNews	Summarization	Rouge-L	2,113	EN	200
TREC	TREC	Few-shotLearning	Accuracy	5,177	EN	200
TriviaQA	TriviaQA	Few-shotLearning	F1	8,209	EN	200
SAMSum	SAMSum	Few-shotLearning	Rouge-L	6,258	EN	200
PCount	PassageCount	Synthetic	Accuracy	11,141	EN	200
PRe	PassageRetrieval-en	Synthetic	Accuracy	9,289	EN	200
Lcc	LCC	Code	Edit Sim	1,235	Python/C#/Java	500
RB-P	RepoBench-P	Code	Edit Sim	4,206	Python/Java	500

Table 13: LongBench templates. Single-Doc. QA Tasks.

NarrativeQA	**Task Template:** You are given a story, which can be either a novel or a movie script, and a question. Answer the question asconcisely as you can, using a single phrase if possible. Do not provide any explanation. Story: {context} Now, answer the question based on the story asconcisely as you can, using a single phrase if possible. Do not provide any explanation. Question: {question}
Qasper	**Task Template:** You are given a scientific article and a question. Answer the question as concisely as you can, using a single phrase or sentence if possible. If the question cannot be answered based on the information in the article, write "unanswerable". If the question is a yes/no question, answer "yes", "no", or "unanswerable". Do not provide any explanation. Article: {context} Answer the question based on the above article as concisely as you can, using a single phrase or sentence if possible. If the question cannot be answered based on the information in the article, write "unanswerable". If the question is a yes/no question, answer "yes", "no", or "unanswerable". Do not provide any explanation. Question: {question}
MultifieldQA EN	**Task Template:** Read the following text and answer briefly. {context} Now, answer the following question based on the above text, only give me the answer and do not output any other words. Question: {question}

Table 14: LongBench templates. Multi-Doc. QA Tasks.

HotpotQA	**Task Template:** Answer the question based on the given passages. Only give me the answer and do not output any other words. The following are given passages. {context} Answer the question based on the given passages. Only give me the answer and do not output any other words. Question: {question}
2WikimQA	**Task Template:** Answer the question based on the given passages. Only give me the answer and do not output any other words. The following are given passages. {context} Answer the question based on the given passages. Only give me the answer and do not output any other words. Question: {question}
Musique	**Task Template:** Answer the question based on the given passages. Only give me the answer and do not output any other words. The following are given passages. {context} Answer the question based on the given passages. Only give me the answer and do not output any other words. Question: {question}

Table 15: LongBench templates. Summarization Tasks.

Gov Report	**Task Template:** You are given a report by a government agency. Write a one-page summary of the report. Report: {context} Now, write a one-page summary of the report.
QMSum	**Task Template:** You are given a meeting transcript and a query containing a question or instruction. Answer the query in one or more sentences. Transcript: {context} Now, answer the query based on the above meeting transcript in one or more sentences. Query: {question}
Multi News	**Task Template:** You are given several news passages. Write a one-page summary of all news. News: {context} Now, write a one-page summary of all the news.

Table 16: LongBench templates. Few-shot Learning Tasks.

TREC	**Task Template:** Please determine the type of the question below. Here are some examples of questions. {context} {question}
TriviaQA	**Task Template:** Answer the question based on the given passage. Only give me the answer and do not output any other words. The following are some examples. {context} {question}
SAMSum	**Task Template:** Summarize the dialogue into a few short sentences. The following are some examples. {context} {question}

Table 17: LongBench templates. Synthetic Tasks.

Passage Count	**Task Template:** There are some paragraphs below sourced from Wikipedia. Some of them may be duplicates. Please carefully read these paragraphs and determine how many unique paragraphs there are after removing duplicates. In other words, how many non-repeating paragraphs are there in total? {context} Please enter the final count of unique paragraphs after removing duplicates. The output format should only contain the number, such as 1, 2, 3, and so on.
Passage Retrieval EN	**Task Template:** Here are 30 paragraphs from Wikipedia, along with an abstract. Please determine which paragraph the abstract is from. {context} The following is an abstract. {question} Please enter the number of the paragraph that the abstract is from. The answer format must be like "Paragraph 1", "Paragraph 2", etc.

Table 18: LongBench templates. Code Tasks.

Lcc	**Task Template:** Please complete the code given below. {context} Next line of code:
Repobench-P	**Task Template:** Please complete the code given below. {context} {question} Next line of code: