

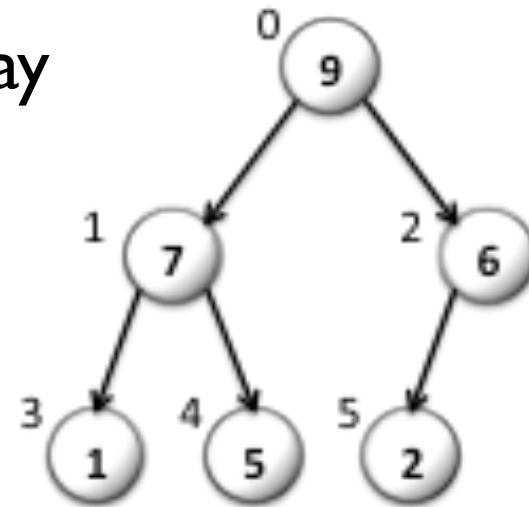
CS32 Discussion  
Section 1B  
Week 10

TA: Zhou Ren

# Heaps

- A **heap** is a
  - complete binary tree
  - every node carries a value greater than or equal to its children's (maxHeap).
  - usually implemented as an array

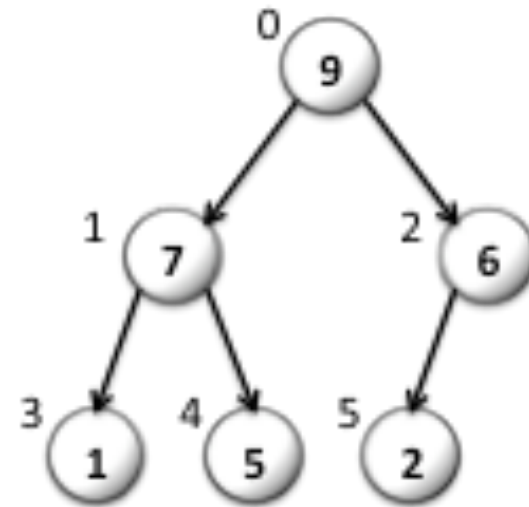
9	7	6	1	5	2
---	---	---	---	---	---



# Heaps: operations

- 3 operations for heaps
  - findMax (search)
  - insertNode (insert)
  - deleteMax (remove)

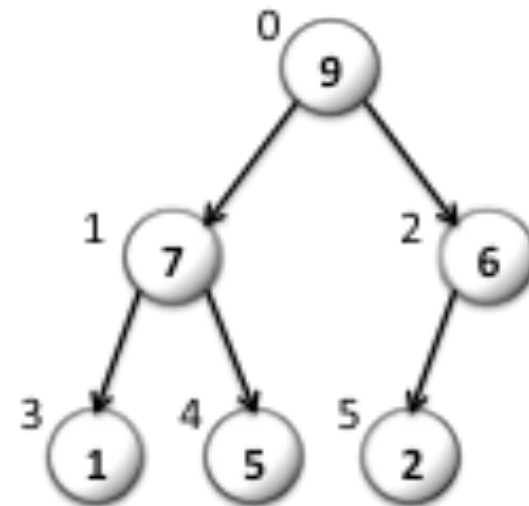
9	7	6	1	5	2
---	---	---	---	---	---



# findMax

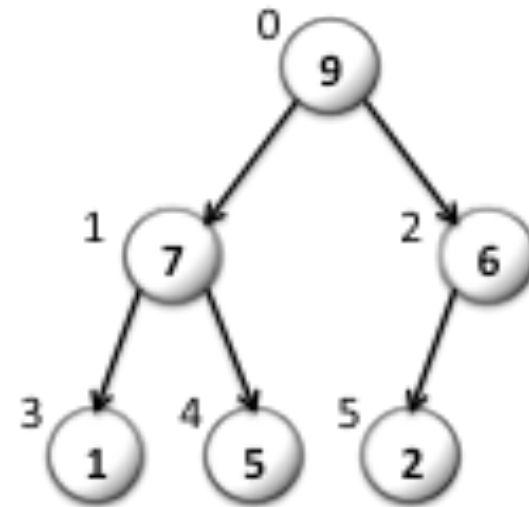
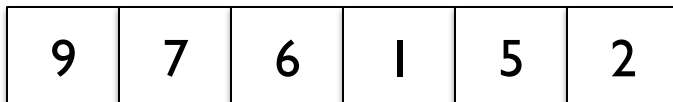
- What do you think?

9	7	6	1	5	2
---	---	---	---	---	---



# insertNode

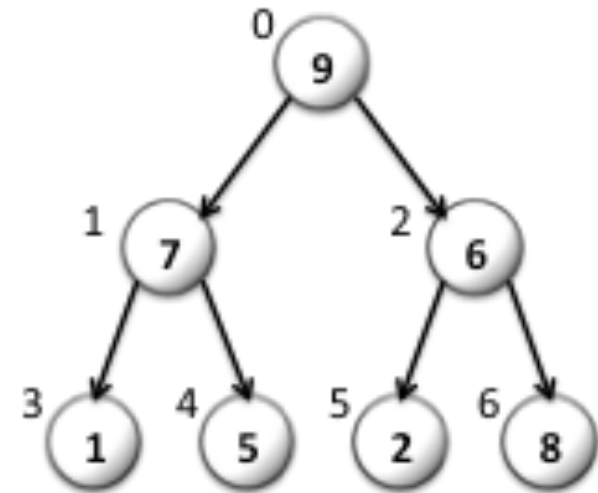
- Not so trivial
- We first add the new node and fix it



# insertNode

1. Add the new node to the tail.
2. Ask:
  - Is the new value greater than its parent?
  - If so: ??
  - Else: ??

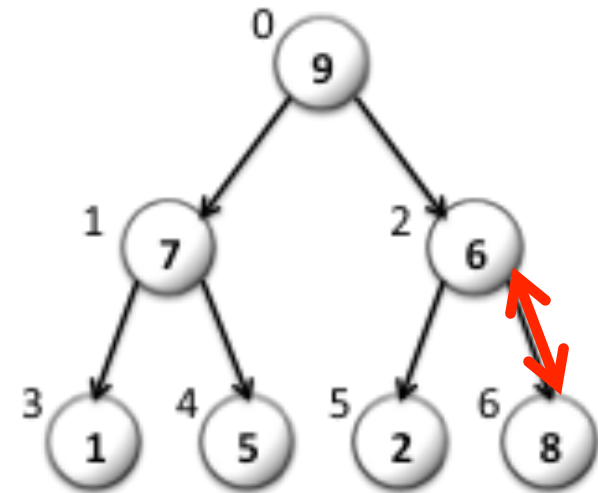
9	7	6	1	5	2	8
---	---	---	---	---	---	---



# insertNode

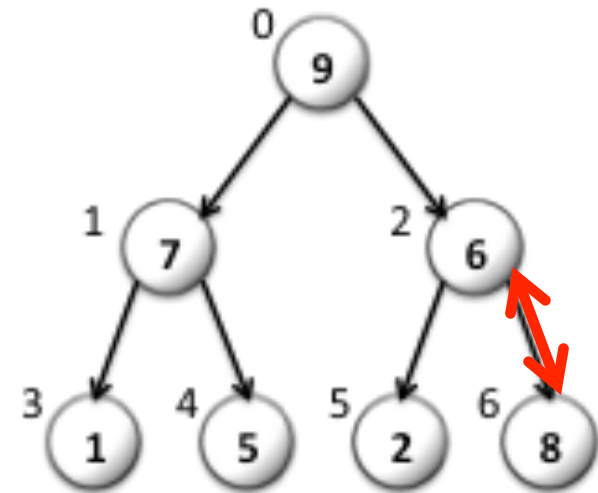
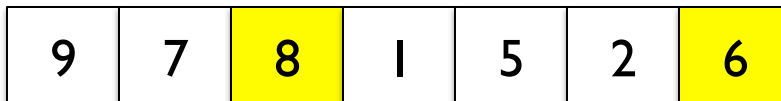
1. Add the new node to the tail.
2. Ask:
  - Is the new value greater than its parent?
  - If so: **swap**
  - Else: **done**

9	7	8	1	5	2	6
---	---	---	---	---	---	---



# insertNode

- What is the index of node i's parent in the array?

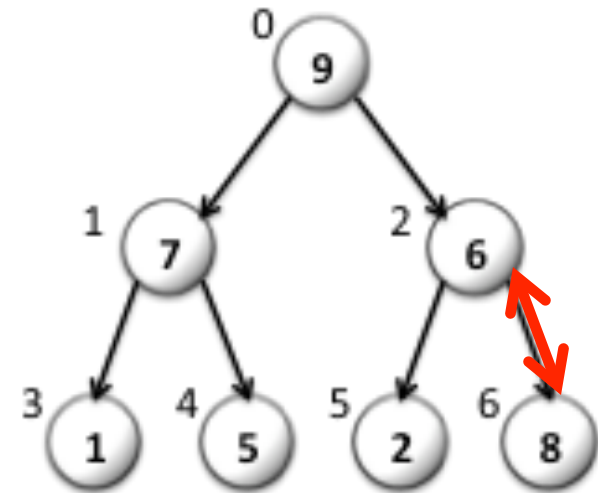




# insertNode

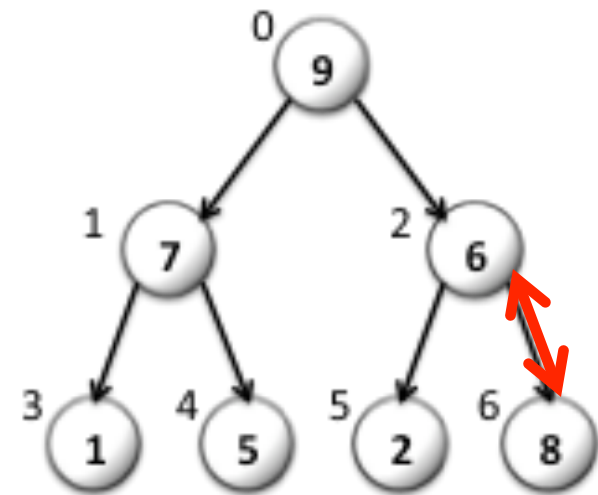
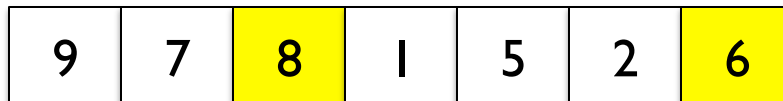
- What is the index of node i's parent in the array?
  - $\text{parent} = (i - 1) / 2$

9	7	8	1	5	2	6
---	---	---	---	---	---	---



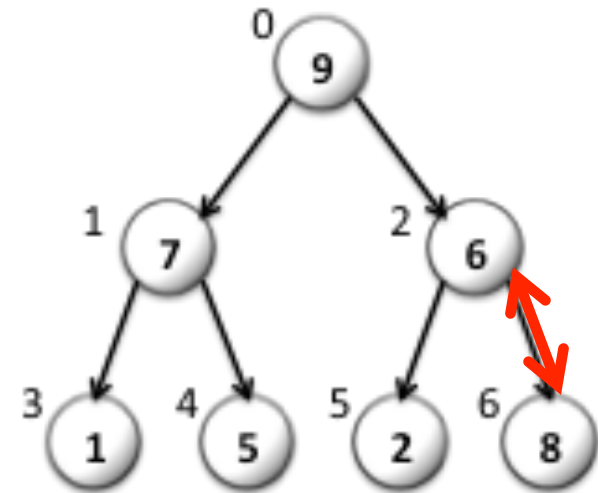
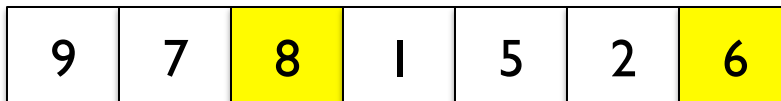
# insertNode

- Running time?



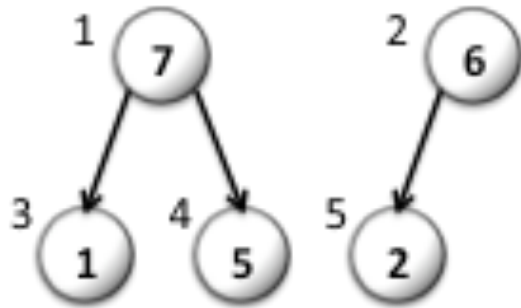
# insertNode

- Running time?
  - proportional to the height of the tree:  **$O(\log n)$**

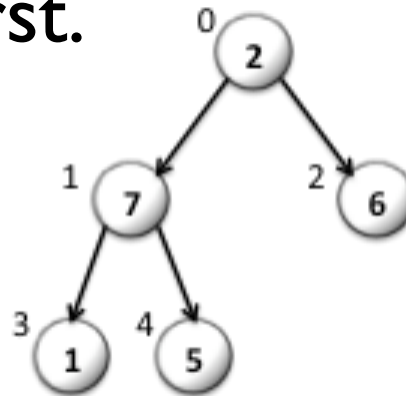


# deleteMax

- Again, take the action first and fix it.

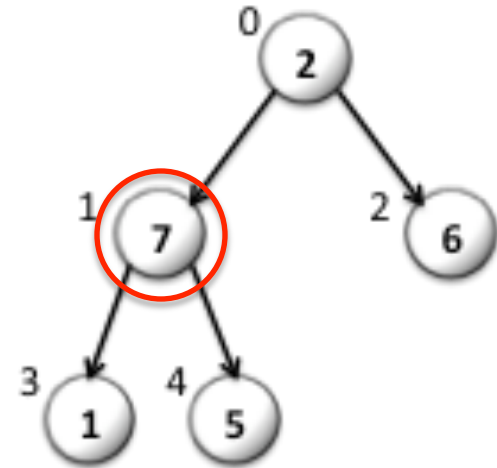


- Fill in the void first.



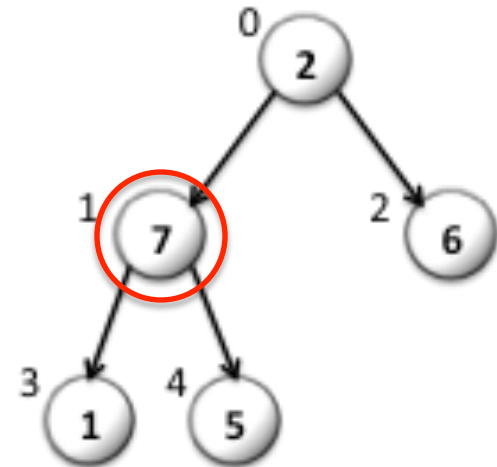
# deleteMax

- Now compare the values of the two children, take the greater of the two (why?), and swap.
- What are the indices of
  - Left child:
  - Right child:of the node  $i$ ?



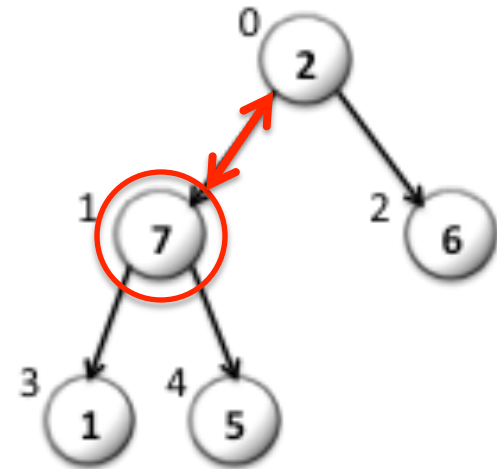
# deleteMax

- Now compare the values of the two children, take the greater of the two (why?), and swap.
- What are the indices of
  - Left child:  $2 * i + 1$
  - Right child:  $2 * i + 2$of the node  $i$ ?



# deleteMax

- Now compare the values of the two children, take the greater of the two (why?), and swap.
- What are the indices of
  - Left child:  $2 * i + 1$
  - Right child:  $2 * i + 2$of the node  $i$ ?



# Heapsort

- Can you use a heap to sort a set of elements?



# Heapsort

- Can you use a heap to sort a set of elements?
  - Insert all elements into a heap
  - Extract the maximum element from the heap one by one

# Heapsort

- Can you use a heap to sort a set of elements?
  - Insert all elements into a heap
  - Extract the maximum element from the heap one by one
- Running time?

# Heapsort

- Can you use a heap to sort a set of elements?
  - Insert all elements into a heap
  - Extract the maximum element from the heap one by one
- Running time?

Inserting  $n$  items:  $n \times O(\log n) = O(n \log n)$

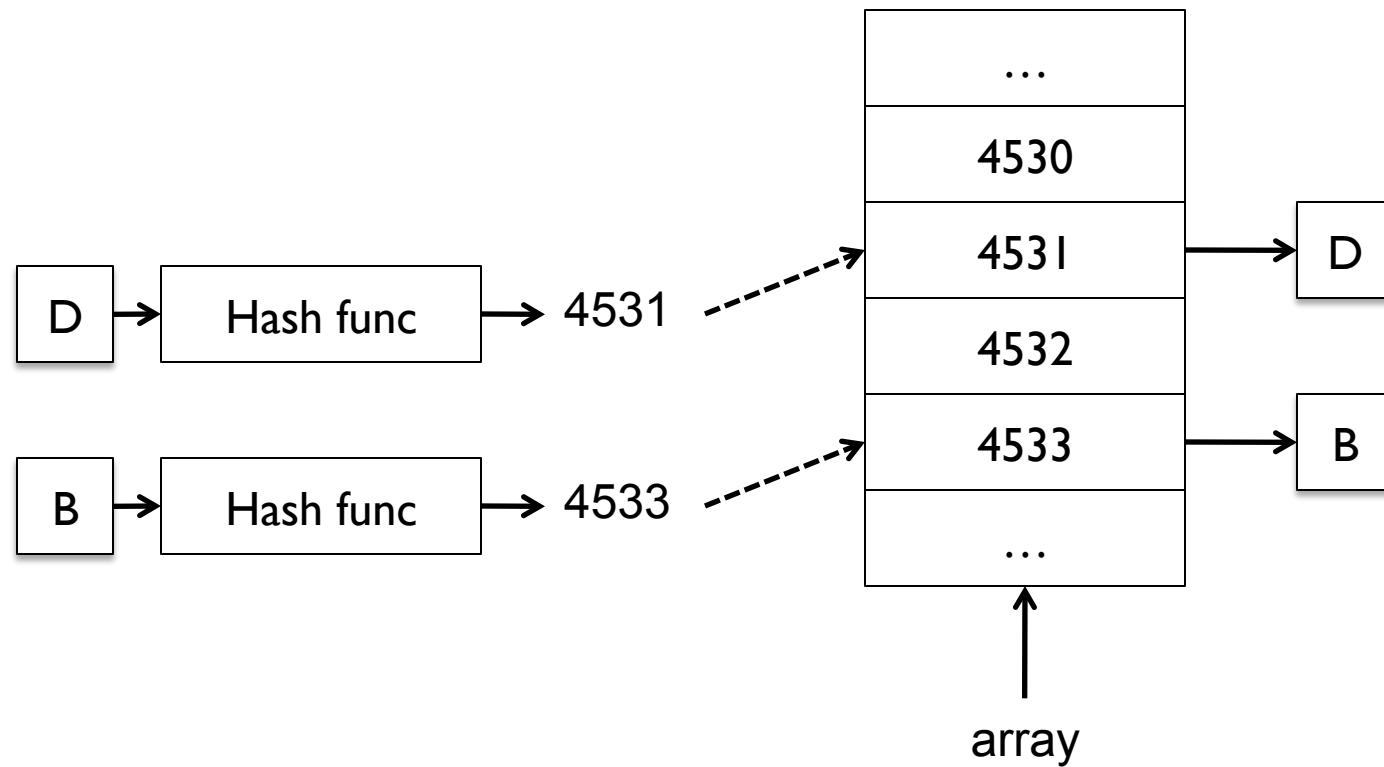
Extracting  $n$  items:  $n \times O(\log n) = O(n \log n)$

$O(n \log n) + O(n \log n) = \mathbf{O(n \log n)}$

# In-place Heapsort

- Heapsort is an in-place sorting algorithm – you don't need an auxiliary structure for the sorting operation.
- Let us try using a **maxHeap** to sort the elements in an array in the **increasing order**.

# Hash Table



# Hash Table

