Chapter 14

STORAGE AND PRICE STABILIZATION

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Contents

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Abstract

Commodity storage models, developed first within agricultural economics in the tradition of Gustafson (1958), are valuable in helping us understand how prices of storable commodity markets behave, and how they respond to policy interventions. They show that the policy-relevant dynamic effects of storage-increasing policies are quite different from comparative statics, and generally less favorable to consumers. They help us understand the implications of price controls, price supports, buffer stocks, speculative attack, and "convenience yield," and have great potential for assessing various econometric methodologies used for studying market efficiency and bias, and supply response. However, more attention should be paid to appropriate commodity market interventions in times of rapid productivity change, and in extremely depressed markets such as those of the 1930s, that influenced the course of agricultural policy in the United States over the next half-century.

Keywords

storage, buffer stock, dynamics, price stabilization

JEL classification: Q11

1. Introduction

In subsistence economies, food has a dominant share of individual consumption, and fluctuations in agricultural output are a direct threat to the health and indeed life of consumers. The success of major ancient civilizations was in many cases dependent upon effective means of ensuring stable food consumption, including irrigation systems, transportation networks, spatially diversified familial linkages, community-based consumption sharing and the selection of crop varieties with reliable yields to reduce production fluctuations or diversify their effects. For many civilizations, food storage policy has also been important for smoothing of subsistence consumption, as the biblical story of Joseph illustrates for ancient Egypt. Another example is ancient China, where the "ever-normal granary", instituted in the Han Dynasty in 54 B.C., was an important element of public policy [Liu and Fei (1979)]. Private storage has been more generally fundamental to subsistence, both for intra-year and inter-year smoothing. This is still true in some areas of the world. A recent survey of farmers in the Shaanxi Province of China showed that they store an average of over two years' supply of carryover grain (nearly a third of their assets) in their homes to buffer consumption [Park (1996)].

When market infrastructure is poor and farmers' production roughly equals their household consumption, price fluctuations are not as important to them as yield variation. But as economies develop and food consumption increases beyond subsistence, Engel's Law dictates that the food consumption share falls. Farmers become specialized market-oriented producers, or switch to other activities. As the income elasticity declines, food demand tends to become more price inelastic. Price fluctuations typically tend to be relatively severe due to inelastic demand and (short-run) supply, and substantial production risk. The direct link of yield to welfare is broken. Farmers and consumers become two distinct groups, with divergent interests in price fluctuations. Farmers recognize that inelastic demand can imply that high-yield years are not favorable to them if price is allowed to clear the market.

The collapse of commodity demand in the 1920s, the persistence of high production in the Great Depression of the 1930s, and the World War II experience with controls led to proposals for the acquisition of stocks by the government (as in the U.S.'s New Deal) or by producer cartels to "stabilize" prices, as well as to proposals for supply controls and marketing quotas. Distrust of market rationality, especially in the short run, was widespread, and stabilizing interventions were supported by economists across the philosophical spectrum including Prebisch (1950), Singer (1950), Keynes (1938, 1942), Kahn [see Palma (1994)]. Fisher (1920), and Hayek (1943) advocated a commodity reserve currency that would lead to a stable price for a basket of storable commodities.

The strategy of public buffer stock proposals was that stocks would be purchased at low prices and released later when prices rose. Evaluation of such proposals was difficult because policy had leapt ahead of analytical capacity. The positive price effects of the initial acquisition, the immediate focus of the proposals, could be confidently predicted. But the net effect of the overall program, including later release, was a task beyond the economic and computational state of the art prior to the 1950s.

The commodity demand boost associated with the onset of the Second World War made earlier buffer stock acquisitions highly beneficial *ex post* and made the benefits of such public interventions seem self-evident. Though Keynes' advocacy of International Commodity Control [Keynes (1942)] was unsuccessful, storage-based programs remained a standard feature of commodity price policies in United States agriculture and were later adopted in the European Community, and in numerous international initiatives for intervention in commodity markets, including the proposals by the United Nations Conference on Trade and Development (UNCTAD) for a New International Economic Order (NIEO) [UNCTAD (1974)]. Public storage initiatives were typically characterized as price "stabilization" schemes, although it was often questionable whether price stability dominated price level as the main objective.[1]

Since the 1950s economists have developed a greater respect for the rationality of private market participants relative to that of government bureaucrats. The modern theory of finance emphasizes the capacity of private markets to handle risks without government help.

Study of the significance for producers and consumers of disturbances, made manifest as price instability, originated within agricultural economics. But advances in modern financial economics (largely achieved outside of agricultural economics and now only slowly permeating the field) and agency theory (of which studies of sharecropping were important progenitors) have engendered an increasingly sophisticated view of the effects of agricultural price fluctuations on producers and consumers. Developments in theory and methodology, including important innovations originating within agricultural economics, have made it possible to analyze markets for storable commodities and the implications of market interventions.

This chapter does not attempt a comprehensive survey of the voluminous literature in this area. In Section 2 the focus is on the economics of markets with variable prices. Attention then turns to alternate means of stabilization, beginning in Section 3 with ideal stabilization defined as (mean-preserving) elimination of market disturbances, then introducing the importance of capitalization in assessing the effects of such stabilization in Section 4. After a brief discussion of the role of general inter-market arbitrage in Section 5, commodity storage is introduced in Section 6, which presents a formal model and outlines the numerical approach to its solution. The nature of competitive storage behavior and its implication for market dynamics are discussed in Section 7, followed by an analysis of simple market interventions in Section 8. In Section 9, the implications of market power for storage are discussed, and in Section 10 we consider the nature of (constrained) optimal public interventions. Section 11 covers extensions of the model to a spatial-temporal context and the implications for understanding the confusing literature on "convenience yield" and "backwardation". A brief review of recent tests of the

[1] See Gilbert (1996) for a postmortem on many international agreements.

model follow in Section 12, and some promising work extending the model is discussed in Section 13. Conclusions bring up the rear.

2. The welfare effects of price variability

2.1. Analysis of price stabilization

Interventions in commodity markets are typically characterized as price "stabilization" programs. Price stabilization intuitively appears to be beneficial to market participants. Yet the first formal analysis of the welfare difference between fixed and variable price by Waugh (1944) implied that price stabilization would make consumers worse off. Waugh was motivated by the problem of the effects on consumers of stochastic interruption of food shipments to wartime Britain due to enemy action. But his analysis focused on a simpler issue: Is a consumer better off consuming a commodity at a price that varies between a high price (P_{high}) and a low price (P_{low}), or consuming at the mean of these two prices, $\overline{P}$?

Using a linear demand curve with finite negative slope, Waugh showed that the Marshallian consumer surplus gained when price was (P_{low}) rather than $\overline{P}$ was greater than the consumer surplus lost when price was (P_{high}) rather than $\overline{P}$. The consumer can take advantage of the low price of a good by purchasing more of it and can reduce the effects of the high price by reducing consumption of it and purchasing more of some other goods. For mean-preserving spreads in price, Waugh's result reflects the concavity of the expenditure function in commodity price and is quite robust under risk neutrality for consumer demand with a finite negative slope. It does not depend on the linearity of demand, the accuracy of Marshallian consumer surplus, or the presence of risk as distinct from foreseeable variation. (Waugh's result can, however, be reversed in a model with random price fluctuations, if the consumer is sufficiently averse to risk.) To confirm these assertions, consider the case of an individual who has intertemporally additively separable utility, and a fixed endowment each period.

Figure 1 shows a nonlinear demand curve of an individual. If price is fixed at $\overline{P}$, and consumption is $q(\overline{P})$, the Marshallian surplus is the area under the demand curve above $\overline{P}$. If price is $P_{\text{low}} = \overline{P} - \delta$, $\delta > 0$, then consumer surplus increases by the vertically striped area. If price is instead $P_{\text{high}} = \overline{P} + \delta$, consumer surplus decreases from its value at $\overline{P}$, by the horizontally striped area. Assuming the demand curve has finite negative slope, the average of the surpluses at P_{high} and P_{low} exceeds the surplus at $\overline{P}$.

Obviously, if P_{high} and P_{low} have equal probability of 0.5, then expected surplus is higher than surplus at $\overline{P}$. If price fluctuates between P_{high} in even periods and P_{low} in odd periods, the average (undiscounted) surplus is higher under variable prices. Since the Hicksian demand through $\overline{P}$ also is negatively sloped, a similar diagram would show that the average *ex post* equivalent variation of price stabilization is negative in general.

A similar exercise with the (restricted) profit function shows that the competitive producer also gains from price variability when the marginal utility of income is constant. Convexity of the profit function (upward-sloping supply) means that average profit under price variability exceeds profit at the mean price, a result first noted by Walter Oi (1961). These results generalize to the effects on expected surplus of random price fluctuations with expected price $\overline{P}$. As long as marginal utility of income does not fall too fast as surplus increases, consumers or producers gain from such randomness of exogenous prices.

These results rightly imply that price stabilization is not necessarily beneficial for a consumer or a producer. Waugh was prompted by his own analysis to go further and ask whether a policy of price destabilization might be desirable for the economy as a whole. This is an entirely different question. In a long-delayed response to Waugh (the original was lost in the war-time mail), Samuelson (1971) pointed out that price instability is not fundamental; it must be generated by shifts in demand and/or supply. If there are no fundamental disturbances shifting demand and supply, generating instability by a public program of market intervention does not improve aggregate welfare, estimated by Marshallian measures of gains and losses that are all equally weighted.

The analysis of market stabilization since Waugh has progressed to the extent that it has moved beyond the limitations of the pioneering literature, as discussed below. To the extent that the exposition succeeds in keeping things simple, it should make the important points seem obvious. Experience has shown that they are not necessarily obvious otherwise, even for the very best theorists in the profession who have ventured into this area [see for example Mirrlees (1988)].

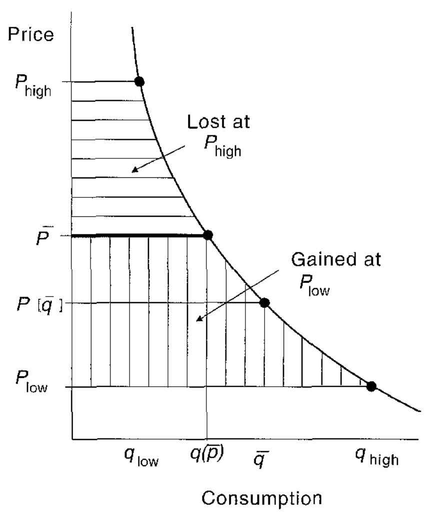

Figure 1. Consumer surplus with variable prices and quantities.

2.2. Focus shift: from prices to real variables

Studies that focus on price stabilization as the policy lever, starting with Waugh and Oi, ignore the means of achieving this objective, and indeed its feasibility. Some authors, especially in the field of international price stabilization, have noted the fact that stabilization at the arithmetic mean of price depends in general on the choice of numèraire. Indeed, Flemming et al. (1977) advocate stabilization at the geometric mean of price, based on the analytical attractiveness of numeraire-independence.

Massell (1969) moved closer to reality by considering a model in which the disturbances were explicitly related to shifts in demand or supply. Figure 2 shows one type of linear case he considered. If the supply curve alternates between curves S_1 and S_2, and the demand curve is D_0, then stabilization of price at P_0 when supply is S_2 means that producer surplus increases by area *KDCE* and consumer surplus falls by area *KBCE*. When supply is S_1, stabilization reduces producer surplus by area *JAHK*, but increases consumer surplus by area *JABK*. On average, the producer gain exceeds the consumer loss by area *BDC*.

Note that the above program is not unambiguously stabilizing. It destabilizes producer surplus and quantity produced, though it stabilizes consumer surplus. True stabilization could be achieved were a means to be found to costlessly stabilize the supply at the dashed curve S_0. Would this be beneficial as measured by net social surplus? The answer is no; on average, all parties lose. Social surplus, represented by triangle *MBF*

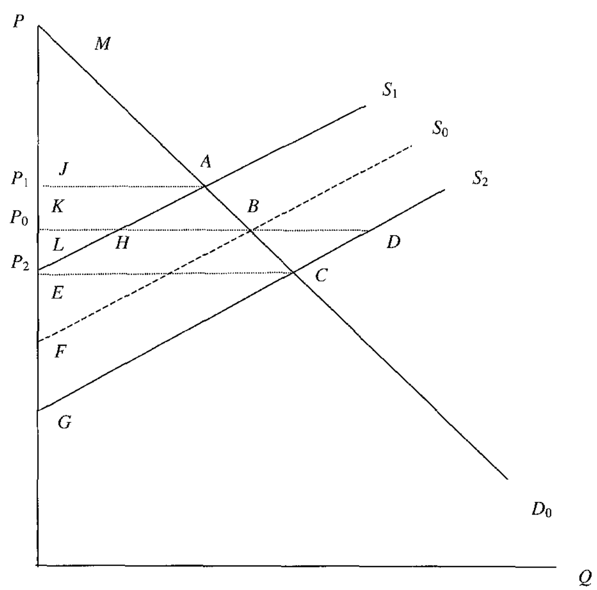

Figure 2. The effects of price stabilization when supply is variable.

under stabilization, is less than the average of triangles MAL and MCG, the values under high and low supply. Moreover, it is easy to check that both consumers and producers lose relative to the average under supply instability.

As a counterpart to Samuelson's warning against generating instability, this result shows that removing natural instability is not necessarily desirable. It is better to follow Massell's result and reallocate some supplies from the market in a state of surplus to the market in a state of shortage, if that can be achieved at a low enough cost. Similar lessons can be learned from symmetric demand shifts with fixed supply.

The Waugh-Oi-Massell analyses are a useful introduction, but their simplicity is deceptive and dangerous for policymakers. In fact, the model is not capable of addressing issues of risk, expectations, the need for policy intervention and the nature of that intervention. Even a shift from linear to nonlinear demand can reverse Massell's results.

2.3. *Nonlinearity of demand*

The assumption that demand and supply are linear is crucial for Massell's results. First, results above on efficiency can change if the supply function in some states intercepts the quantity axis (and is thus nonlinear). Second, linearity means no distinction between mean quantity and the quantity at mean price. Turning back to Figure 1, it is obvious that mean price is not the price at mean quantity $\bar{q}$, given a nonlinear demand curve. As noted by Howell (1945), fixing quantity at its mean would raise mean consumer surplus in this example, whereas stabilization at mean price reduces it, in line with Waugh's general result for the linear case. We shall see below that feasible stabilization need not preserve either mean.

2.4. *Lagged supply response*

The model assumes that the equilibrium instantly adjusts to market disturbances. There is no need for agents to form price expectations to achieve efficient resource allocation. Risk aversion does not affect efficiency at all, since the results of all decisions are certain at the time they are made. Thus the need to commit resources long before outcomes are known, a major feature of the decision processes in commodity markets, is ignored.

Recognition of a lag between commitment and realization is an essential modification that raises a number of crucial issues. First, risk aversion may affect the producer's welfare and allocative efficiency when production is lagged, as shown by Sandmo (1971). Second (and less recognized in this literature), the producer cannot be modeled as having a utility function defined on current income alone. Since she is investing, she must be involved in intertemporal arbitrage that should be considered as a means of smoothing consumption when income is variable.

2.5. *The formation of price expectations*

The need to pre-commit resources when production responds with a lag makes the formation of price expectations a relevant issue. Analysis of inefficient adaptive models of private behavior could lead to the conclusion that government interventions can be efficient essentially because they introduce the benefits of superior price forecasting. This notion was a prominent feature of the influential policy prescription of Prebisch (1950) and Singer (1950) that governments intervene to guide developing economies away from primary production via trade policy, and of the thoughts of Keynes (1942) on the benefits of commodity market intervention in the short run.

Nowadays the rational expectations hypothesis of Muth (1961) is considered the pioneering work in modeling endogenous expectations. What is remarkable is that the fundamental idea of internally consistent expectations was formulated and applied earlier, in the agricultural economics literature. This was achieved by Gustafson (1958), who modeled storage as a market-stabilizing device in his brilliant, obscurely published and unheralded USDA bulletin, completed three years prior to publication of Muth's hypothesis, and without Muth's simplifying assumption that stocks can be negative. Expectations were relevant for Gustafson [as for Williams (1936), Working and Keynes many years earlier] because he was explicitly considering intertemporal arbitrage. In the absence of serial correlation, storage is necessary to induce variation in price expectations and thus make them meaningful sources of intertemporal variation in production.

2.6. Welfare criteria

The use of Marshallian surplus measures was more analytically suspect at the time the pioneering literature was written than it is today. For consumption stabilization at the arithmetic mean $\bar{q}$, expected equivalent variation under the expected utility hypothesis is, up to a second order approximation,

$$\frac{1}{2}\bar{q}_i \frac{P_i(\bar{q}; e)}{\eta_{ii}}\left[\frac{\gamma_i}{\eta_{ii}}(\eta_{ie} - \rho) + C_i - 1\right]\Delta\sigma_q^2, \tag{1}$$

where $P_i(\bar{q}, e)$ is the inverse demand evaluated at mean quantity,

$$C_i \equiv -q_i \frac{\partial^2 P_i(\bar{q}_i, e)}{\partial P_i(\bar{q}_i, e)^2},$$

a coefficient of relative curvature of demand, and $A\sigma_{q_i}^2$ is the reduction in the square of the coefficient of variation of consumption σ_{q_i}, the standard deviation normalized by the mean [see Wright and Williams (1988a)]. η_{ii} is the price elasticity of Marshallian demand (measured at mean quantity), η_{ie} is the income elasticity, and ρ is the Pratt-Arrow coefficient of relative risk aversion. Assuming the budget share γ_i is sufficiently small, the value of C determines the sign of the expected consumer gain. For linear demand, $(C = 0)$, the consumer loses from quantity stabilization, consistent with the result of Waugh for price stabilization for general demand curves. For linear demand, expected price $\overline{P} = P(\bar{q}, e)$ and $\Delta\sigma_{q_i}^2 = \eta_{ii}^2(\bar{q}_i/P_i)^2\Delta\sigma_{P_i}^2$. Substituting these expressions in Equation (1), that equation becomes identical to that of Turnovsky et al.

(1980) for arithmetic mean-preserving price stability. But for constant elasticity demand, $C = (1 - 1/\eta_{ii})^{-1}$, if demand is inelastic, $(0 > \eta_{ii} > -1)$, the consumer gains from consumption stabilization.

Commenting on the latter result, as derived in an example in Newbery and Stiglitz (1981), Kanbur's review article states: "The demonstration that stabilization schemes which take up existing supplies will lead to a transfer of income from producers to consumers is a striking result ..." (1984, p. 342), which he notes was anticipated in work by Harry Johnson (1976) and Michael Lipton (1970). The sensitivity of the result to demand specification has been long in the learning. The further result that the true dynamic incidence when there are "existing supplies" is much more likely to favor landowners, as well as the holders of those supplies, as noted below, is likewise taking time to permeate the literature.

In the approximation (1), the term $\gamma_i(\eta_{ie} - \rho)$ incorporates the effect of the change in the individual's marginal utility of income. For developed economies, the budget share of any given commodity is typically so low that this effect is negligible. For example, in the United States, all food has a share of expenditure of only around 10 percent (and much less at the farm gate), and the share of any one commodity is much smaller. Even in developing countries, most consumers do not consume any one commodity that has a budget share as high as 10 percent, and those that do tend to be commodity producers as well. General equilibrium considerations and the correlation of prices of different commodities then tend to be more important for accuracy than the issue of demand compensation [Wright and Williams (1988a, pp. 622–624)].

Other criteria have occasionally been chosen for evaluating market stabilization. For example, price or income stability *per se* has been advocated or adopted in many studies including Arzac and Wilkinson (1979), Cochrane (1980), Dixon and Chen (1982), and Ghosh et al. (1987). The value of price stabilizing intervention to prevent macroeconomic disruptions, a serious issue addressed in Kanbur (1984) and Timmer (1989), has proved to be difficult to analyze with any rigor in the absence of a complete and satisfactory macroeconomic model. The most severe problem seems to arise after prior interventions have left domestic food prices (and often wages) far out of line with competitive market-clearing prices. Many a less developed country (Indonesia in 1998 is a recent example) has seen riots provoked by abrupt food price increases caused by increases, mandated by the International Monetary Fund, in "stabilized" food prices.

Similarly, an exchange rate that is "fixed" to avoid macroeconomic disruption can cause severe disruption when a devaluation can no longer be avoided. In both cases, a full analysis of the policy must include the effects of its inevitable adjustment or abandonment. These will depend upon the means of stabilization. Indeed analysis of some notion of price stabilization divorced from a model including both the source of instability and the means of stabilization has no necessary relevance to any policy issue regarding stabilization policy, as the next section shows.

3. Ideal production stabilization

Market instability might arise from many sources, but it must eventually stem from fluctuations in consumer preferences or endowments or in some aspect of supply including distribution. Fluctuations in preferences are obviously important in some products like clothing, beverages and entertainment. Whether induced by advertising or reflecting stochastic underlying tastes, such instability raises challenges for welfare analysis not yet addressed in the market stabilization literature. Fluctuations in consumer endowments are not important for commodities with typically low income elasticities (that is, the vast majority of primary commodities in wealthy countries). Here we focus on production instability.

The effects of shifting instantaneous supply were considered above in a Massell-type model. In that context, instability was beneficial to both consumers and producers. Now we reconsider this issue using a modification of that pioneering model that makes risk and expectations relevant.

The demand and supply equations are

$$\begin{aligned} q(P_t) &= A - aP_t + u_t, \\ h(P_t^r) &= \left(B + bP_t^r\right)(1 + \nu_t). \end{aligned} \tag{2}$$

Consumer demand $q(P_t)$ is linear and, for simplicity, independent of income. Production, the "harvest" $h(P_t^r)$, is linear in the incentive variable P_t^r, which is effective at the time of "planting", period $t-1$, before the realizations of the independent and identically disturbed (i.i.d.) disturbances u_t and ν_t are known. The disturbance ν_t is multiplicative rather than additive; as a first approximation it seems reasonable to assume that acreage with higher mean harvest suffers proportionally higher harvest disturbances. This specification has the added bonus of straightforward decentralization: every producer can share the same proportion of the aggregate disturbance on average and at the margin. (How shares in the alternative additive aggregate disturbance should be distributed at the margin, a question crucial for incentives, is not obvious.)

Ideal stabilization is defined as fixing of ν_t at 0, by some means not explicitly modeled. Random sources of disturbance such as weather are perhaps stabilized (for example, by irrigation), or the effects are removed by choosing a technology (such as crop varieties resistant to droughts and floods) insensitive to the disturbances. Storage is assumed away, so market clearing implies

$$q(P_t) = h\left(P_t^r\right) \quad \text{for all } t.$$

In period t, the representative competitive producer i has output h_t and revenue $r_t^i = P_t h_t^i$. Substitution for P_t from the demand equation (2) yields

$$r^i = \frac{1}{a}\left[A - \left(B + bP_t^r\right)(1 + \nu_t) + u_t\right]\left[\bar{h}^i(1 + \nu_t)\right],$$

where $\bar{h}_t^i = E_{t-1}(h_t^i)$, planned production of producer i. In period $t-1$, when production commitments are made for period t, expected revenue is

$$E_{t-1}(\pi_t^i) = \frac{1}{a}\left[A - \left(B + bP_t^r\right)\left(1 + \sigma_v^2\right)\right]\bar{h}_t^i.$$

Notice that the competitive producer recognizes the correlation of his own output with aggregate output and hence with price. The marginal incentive for the risk-neutral producer [what Newbery (1990, p. 1045) calls the "action certainty equivalent price, the price which yields the same choice of inputs in the absence of risk as that chosen under risk"] is defined as

$$\begin{aligned} P_t^r &= \frac{\partial E_{t-1}(\pi_t^i)}{\partial \bar{h}_t^i} \\ &= \frac{1}{a}\left[A - \left(B + bP_t^r\right)\left(1 + \sigma_v^2\right)\right] \\ &= \frac{A - B(1+\sigma_v^2)}{a + b(1+\sigma_v^2)}. \end{aligned}$$

This is true for all t; planned output is constant in this model. Note that P_t^r differs from expected price:

$$\begin{aligned} E_{t-1}(P_t) &= \frac{1}{a}\left[A - B - bP_t^r\right] \\ &= \frac{1}{a}\left[A - \frac{aB - bA}{a + b(1+\sigma_v^2)}\right]. \end{aligned}$$

Ideal stabilization fixes σ_v^2 at zero. Then the producer incentive is the expected price

$$P^S = \frac{A - B}{a + b}.$$

This is also the producer incentive and expected price when the output disturbance is additive.

For this linear model, if $\sigma_v^2 > 0$,

$$P^r < P^s < E_{t-1}(P_t).$$

So ideal stabilization reduces expected price but increases the producer incentive in this model. With responsive supply ($b > 0$), expected output increases. Both producers and consumers gain if (one-period lagged) supply is more elastic than demand, in contrast to the linear Massell model [Wright (1979, p. 1026)]. (In contrast to the Massell case illustrated in Figure 2, the instantaneous inverse supply function is nonlinear; horizontal starting at zero, turning vertical at actual production.) The total social welfare change tends to be less negative than in the additive-disturbance case.

When demand is nonlinear in this model, the distributional results can be reversed. Using second-order approximations, one can show they depend on the ratio of the demand to the supply elasticity and on the demand curvature parameter C, which is important for the effect of quantity stabilization as noted above. This model can be generalized to the case of multiple producing regions with different i.i.d. disturbances feeding a single consumer market. Stabilization of production in one region can be interpreted as stabilization of excess demand in the others. If those other regions have stable output, they generally lose from ideal stabilization of a competing supply region [Wright (1979, p. 1029)].

Since the curvature of consumption demand is very difficult to measure empirically, the distributional effects of ideal stabilization are difficult to establish. Indeed the main general lesson from this model is that the distributional results are sensitive to the specification, even when risk aversion and problems of consumer heterogeneity are ignored. Signs may be reversed by a change from additive to multiplicative shocks, by a change in relative demand curvature, in the timing and extent of supply response, or in the source of the disturbance. The sign of the change in mean consumption and the extent of efficiency effects also depend on the interaction of supply responsiveness, demand curvature, and the nature of the disturbances.

At this point, it is useful to become a little more precise about the incidence of "producer gains" in the context of commodity production. In admitting lagged supply response, the reality of intertemporal allocation of productive resources by producers has already been recognized. And upward-sloped (lagged) supply implies the existence of factor rents. It is crucial to recognize that the expected stream of rents is then capitalized into the price of fixed and quasi-fixed factors if they have competitive markets.

4. Capitalization

The literature on market stabilization from its beginnings through the classic work of Gardner (1979) and Newbery and Stiglitz (1981) assesses the effects of stabilizing interventions using comparative statics. If the change under consideration is fully realized from one period to the next, as in the model of ideal stabilization discussed above, comparative statics, in the form of comparison of an equilibrium with and without ideal stabilization, accurately indicates the changes in prices and quantities in the first period in which production is stabilized. But the comparative statics effects on surpluses do not indicate how gains and losses are distributed.

If in period $t-1$ it becomes known that ideal stabilization (deterministic production) will be permanently in effect from period t, there will be an immediate response in the prices of fixed factors like land and quasi-fixed factors such as machines and human capital. Figure 3 shows a case in which land is the only fixed factor, its market is competitive, and the interest rate is constant at r. If ideal stabilization is announced at

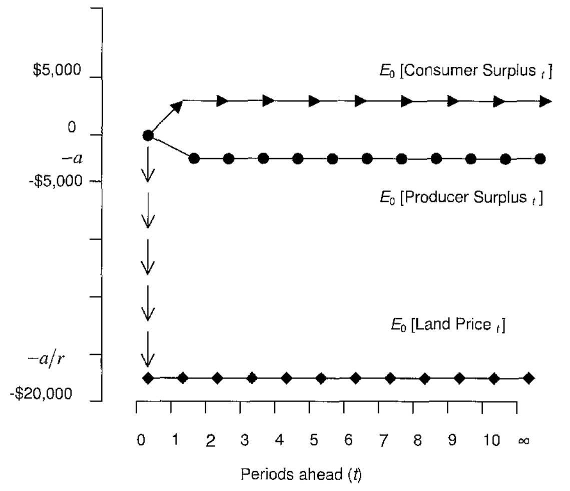

Figure 3. The welfare effects of ideal stabilization if announced the period before the stabilization operates.

period 0 and takes effect in period 1, the expected streams of consumer and producer surplus take permanent jumps to their new steady state levels in period 1.[2] Assuming demand is sufficiently convex, the jumps for consumer surplus and producer surplus are positive and negative as illustrated. But the figure does not imply that producers beyond period 1 are expected to be worse off. The incidence of the drop in the path of producer surplus occurs entirely in period 0, when the land price, the discounted present value of the expected path of land rents starting in period 1, falls by $1/r$ times the change in producer surplus to be realized first only in the next year. Producers who buy land after period 0 neither gain nor lose from ideal stabilization regardless of the effects on producer surplus. (Consumers might also find that at least some of the change in the flow of consumer surplus ends up as a jump in the costs of complementary durable consumption goods such as housing.)

The main lessons of this elementary illustration are simple but crucial: the incidence of agricultural policy change is dynamic and quite distinct, in general, from changes in current or steady-state surplus flows. But these insights are virtually ignored in current discussions of subsidized crop insurance and other means of assisting farmers via risk reduction. To assess the welfare implications of the rich dynamics in prices, quantities,

[2] If savings and the interest rate are endogenous, the path of land price and the interest rate will in general jump to new dynamic paths which, if stable and unique, converge on a new, steady-state level. In an overlapping generations model, Chamley and Wright (1987) show that only the initial jump is directly relevant for incidence on the initial landholders (the older generation). The incidence may be greater or less than in the Ricardian case considered here. In a modern open economy, induced effects of domestic agricultural policy on the cost of capital are unlikely to be very significant.

and surplus flows associated with changes in market stability, it is useful to become comfortable with the dynamics of capitalization introduced into the market stabilization literature by Wright and Williams (1984a) and explored in a policy context by Miranda and Helmberger (1988) and Wright and Williams (1988b).

5. Handling instability: from ideal stabilization to arbitrage

Most of the literature in this area that has moved beyond addressing disembodied "price stabilization" takes the source of instability, such as a stochastic shift in supply or demand, as given. It addresses activities that reduce the induced variation in some variable (for example, price) by increasing the variation in others. The simplest approach, chosen by Massell and others, and used extensively in several important papers by Newbery and Stiglitz incorporated in their classic book [Newbery and Stiglitz (1981, Chapters 17, 18, 21)], is to view stabilization as an arbitrage that moves supplies from a market in a low-price state to another in a high-price state. Newbery and Stiglitz identify the welfare effects on market participants, including the arbitrageur, of a small amount of "stabilizing" arbitrage. They distinguish these effects from those of a small amount of destabilization of a price stabilized by arbitrage. Though the transfer mechanism is sometimes described as "storage" or a "buffer stock", such features fundamental to intertemporal exchange as the unidirectionality of time, discounting, and non-negativity of stocks, are generally ignored.

Indeed the obvious empirical analog of such a mechanism is not storage but trade in the stylized form of costless contemporaneous spatial arbitrage between markets, addressed in Samuelson (1957). As for ideal stabilization, the welfare effects are heavily parameter-dependent.

Newbery and Stiglitz (1981, Chapter 23; 1984) explicitly embrace the spatial interpretation of stabilization in their discussion of the possibility that trade can be Pareto-inferior. If individual price variability exactly offsets the destabilizing influence of output fluctuations on revenue, price fluctuations are a part of natural revenue insurance. Stabilization of price via trade is a Pareto-inferior policy if producers are risk-averse, given that the fundamental disturbances remain, and revenue insurance is unavailable.

For consumers, the stabilizing role of spatial arbitrage in the form of internal and international trade and assistance has been crucial over the past several centuries. Reductions in transport costs and trade barriers have greatly reduced the harm done by regional supply shortfalls. Worldwide supply variation cannot be removed by trade, but it can be mitigated by intertemporal arbitrage in the form of storage (in association with supply response), to which we now turn.

6. The nature of commodity storage

A broad definition of storage could encompass any activity that transforms a commodity available at a given point in time into a similar commodity available later. Examples abound in nature.[3] Human storage activities include money management, inventory management at the firm level, water storage in reservoirs and cisterns, conservation of gases such as methane (natural gas) or helium below or above ground, management of forests, fisheries and mineral deposits, and preservation of information using various media. A common feature of all storage activity is that stocks are constrained to be non-negative. If current stocks are zero, it is impossible at the margin to "borrow from the future".

Though conservation of inputs such as soil, water, and genetic resources is important in agriculture, the focus of this chapter is on storage of non-perishable agricultural commodities such as the major grains, responsible for most of the world's food supply. An important stylized feature of these commodities (and of most minerals) is that the marginal cost of storage per period, including physical protection, insurance, and spoilage, increases only slowly if at all with the size of the total stock and may approach a finite upper bound; the assumption of constant unit costs is supported by Paul (1970).

In contrast, water storage in a reservoir may have highly nonlinear marginal cost, approximating zero up to full capacity, then increasing without bound. The implications for intraseasonal smoothing of price when water is a nonstochastic input have been elucidated by Pyatt (1978), in an excellent introduction to the economics of storage.

The fact that their supply is usually seasonal distinguishes major storable agricultural commodities and water from minerals. In modeling, choice of seasonal or higher frequency for time intervals is appropriate when the arrival of significant information within the harvest year engenders a dynamic intra-period response. Such information may be, for example, related to the evolution of the prospects for the next harvest or other relevant matters such as the harvest of competing crops grown in the other hemisphere. But important features of carryover storage associated with the non-negativity constraint can be studied at an annual frequency. Accordingly, agricultural commodity storage is often studied in models with annual time intervals.

Formal models of market-stabilizing storage also have focused on market aggregates. Transaction costs associated with adding or removing stocks are assumed negligible. These two features distinguish formal commodity storage models from the (S, s) model of firm inventory management [Scarf (1959)] in which ordering costs are dominant, and natural resource problems in which extraction costs are prominently featured. The literature on inventory management, which combines a firm-oriented perspective with a macroeconomic focus, is surveyed elsewhere [see for example Blinder and Maccini (1991)] and will not be discussed further here.

The arbitrage possible via storage is more restricted than that attainable via costless spatial arbitrage. The transfers are unidirectional; negative storage is not feasible. Furthermore even if storage itself is costless, discounting makes the transfer costly, and also makes the sequential ordering of welfare effects crucial to incidence. Finally, there is no obvious endpoint to the chain of potential intertemporal transfers.

[3] The economics of foraging and food storage by animals is addressed by Salant et al. (1995).

The value of storage today depends on its expected value tomorrow, and so on to infinity. It seems one needs to know the answer for tomorrow before solving for the problem today. The first satisfactory solution to this conundrum did not appear until the 1950s in the pioneering work of Gustafson (1958). Important subsequent analytical models included Samuelson (1971), who addressed the optimality of competitive storage and showed that prices follow a nonlinear first-order Markov process, Scheinkman and Schechtman (1983), and Deaton and Laroque (1992).

6.1. A simple competitive model of storage

To facilitate further discussion, consider a competitive partial equilibrium model for a single storable consumption commodity. In general, we follow the notation of Scheinkman and Schechtman (1983). In recognition of a single exogenously determined annual harvest season, time is discrete. All agents have rational expectations. Production is subject to a market-wide exogenous multiplicative disturbance $\omega_t \in K \equiv [\underline{\omega}, \overline{\omega}]$, $0 = \underline{\omega} < \overline{\omega}$, such as a common realization of weather in period t. Let the history of disturbance from period 0 be $\omega^t = (\omega_0, \dots, \omega_t)$. Then $\omega_t \in K^{t+1} \subset R_+^{t+1}$, where K^{t+1} is the Cartesian product of $t+1$ replicates of K, one for each period in the history of observed realizations, ω^t. A one-period lag exists between the representative competitive producer's choice of effort for "planting", $\lambda_t \in [0, \bar{\lambda}]$ and output, the "harvest" $h^t = \lambda_t \omega_{t+1}$. The cost of effort is $g : [0, \bar{\lambda}] \to \overline{R}_+$, with marginal cost $g' : [0, \bar{\lambda}] \to \overline{R}_+$, where $\overline{R}_+$ is the extended positive real half-line, with

$$g(0) = 0, \quad g'(\bar{\lambda}) = \infty, \quad \text{and} \quad g''(\lambda) > 0 \quad \text{for all } \lambda \geqslant 0.$$

The harvest can be consumed and/or stored until the next period. The amount stored is $x \geqslant 0$. The storage cost is given by a differentiable function $\phi : R_+ \to R_+$, with $\phi(0) = 0$, $\phi''(x) > 0$, and $0 \leqslant \phi'(x) < k$ for $x \geqslant 0$, $k \in R$. At time t price $p_t : K^{t+1} \to \overline{R}_+$ is defined as a (Borel) measurable function of the history ω^t. The representative competitive producer is risk-neutral and takes the price sequence, the sequence of measurable functions $\Pi = \{p_t\}_{t=0}^{\infty}$, as given. To simplify notation, the number of competitive producers is normalized at unity.

Given ω^t, the producer chooses storage x_t, effort λ_t and sales c_t, all functions of ω^t. Available supply at period t is z_t, where

$$z_t(\omega^t) = x_{t-1}(\omega^{t-1}) + \lambda_{t-1}(\omega^{t-1})\omega_t.$$

Consumption at time t equals sales by the representative producer at t. The consumer's demand is

$$f : R_+ \to \overline{R}_{++} \text{ with } f'(c) \equiv \frac{\mathrm{d}f}{\mathrm{d}c} < 0, \quad \text{and} \quad c \cdot f(c) < m < \infty, \quad \text{for all } c \geqslant 0.$$

Thus, following Scheinkman and Schechtman (1983), the model assumes an upper bound on the expenditure on consumption, but not on price as in for example Deaton and Laroque (1992).

Assume no market distortions, and that the representative consumer has infinite lifetime and is risk neutral with no income elasticity of demand for the stored good, and constant exogenous finite endowment $m_t = m$. U is a concave "surplus" function defined by

$$U(c) = \int_{\underline{c}}^{c} f(u)\,\mathrm{d}u,$$

where the arbitrary constant $\underline{c} > 0$ ensures that U is finite.

There is a constant interest rate $r > 0$, and discount factor $\delta \equiv 1/(1+r)$. Given z_0 and Π, and denoting $x = \{x_t\}_{t=0}^{\infty}$, and $\lambda = \{\lambda_t\}_{t=0}^{\infty}$, the producer chooses (x, λ) to solve the problem

$$\sup E_0 \left\{ \sum_{t=0}^{T} \delta^t \Big[p_t(\omega^t) c_t(\omega^t) - g\big(\lambda_t(\omega^t)\big) - \phi\big(x_t(\omega^t)\big) \Big] \right\},$$

subject to

$$\begin{aligned} &c_t(\omega^t) + x_t(\omega^t) = \lambda_{t-1}(\omega^{t-1})\omega_t + x_{t-1}(\omega^{t-1}), \quad && t \geqslant 1, \\ &c_0(\omega^0) + x_0(\omega^0) = z_0, && \\ &c_t(\omega^t) \geqslant 0, \quad x_t(\omega^t) \geqslant 0, \quad \lambda_t(\omega^t) \geqslant 0, && t \geqslant 0. \end{aligned}$$

The producer's choices in period t must satisfy the set of complementary inequalities that comprise the arbitrage conditions

$$\begin{aligned} &p_t(\omega^t) + \phi'\big(x_1(\omega^t)\big) \geqslant \delta E_t\big[p_{t+1}(\omega^t)\big], \quad && \text{with equality if } x_1(\omega^t) > 0, \\ &g'\big(\lambda_t(\omega^t)\big) \geqslant \delta E_t\big(\big(p_{t+1}(\omega^{t+1})\big) \cdot \omega^{t+1}\big), && \text{with equality if } \lambda_t(\omega^t) > 0, \end{aligned}$$

where E_1 denotes the expectation conditional on ω^t. The first of these conditions implies that the spread between next-period futures and the spot price can never exceed the cost of interest plus storage.

If there exist producer choices $(\hat{x}, \hat{\lambda})$ that solve the producer's problem such that, for each t, $f(\hat{z}_t(\omega^t) - \hat{x}_t(\omega^t)) = \hat{p}_t(\omega^t)$ almost surely, then $\widehat{\Pi} = \{\hat{p}_t\}_{t=0}^{\infty}$ is a rational expectations equilibrium.

Scheinkman and Schechtman (1983) use Benveniste and Scheinkman (1979) to prove the envelope condition that $\hat{p}_t(\omega^t)$ is the partial derivative of the value function W with respect to z_t. The value function is optimized social welfare in this simple general equilibrium competitive model, given initial availability z, and the history of disturbances ω^t:

$$W(z, t, \omega^t) = \sup E_t \left\{ \sum_{s=t}^{\infty} \delta^s \Big[u\big(c_s(\omega^w)\big) - \phi\big(x_s(\omega^s)\big) - g\big(\lambda_s(\omega^s)\big) \Big] \right\},$$

subject to

$$c_s(\omega^s) + x_s(\omega^s) = x_{s-1}(\omega^{s-1}) + \lambda_{s-1}(\omega^{s-1}) \cdot \omega_s \equiv z_s(\omega^s),$$
$$z_t(\omega^t) = z, \quad c_s(\omega^s) \geqslant 0, \quad x_s(\omega^s) \geqslant 0, \quad \lambda_s(\omega_s) \geqslant 0, \quad s \geqslant 0.$$

The strict concavity of W means that price is decreasing in available supply z.[4] If the disturbances are i.i.d. then z_t is the state variable. Below some threshold $z^* \geqslant 0$, storage is zero, and supply effort and expected price and consumption are all locally insensitive to z. Analytical propositions include [Scheinkman and Schechtman (1983, pp. 432–433)]

(1) consumption increases with z;
(2) storage increases with z, for $z \geqslant z^*$;
(3) supply of effort decreases with z, for $z \geqslant z^*$;
(4) the distribution of z converges to a stationary distribution [Scheinkman and Schechtman (1983, Theorem 4, p. 436)].

All of these propositions confirmed and generalized results originally obtained in Gustafson (1958) or later numerical models [Gardner (1979), Wright and Williams (1982a)]. The marginal propensity to store is always less than unity. Furthermore, storage is a non-increasing function of price, decreasing as price increases until stocks reach zero at $p^* = p(z^*)$. It follows that expected price is decreasing in stocks for positive levels of stocks, a fact that is important for the numerical solution of the model, discussed below.

A further analytical result for a similar model without effort response [Deaton and Laroque (1992, Theorem 3, p. 8)] is that if consumer demand is convex, so is the price function $p(z)$. Beyond this, the analytical results tell us little about the nature of the relations among available supply, price, consumption, storage, and effort. But by the time these analytical results were published, the numerical solutions developed by Gustafson (1958) and further elaborated by Gardner (1979) had revealed a great deal about the relationships for the zero supply case. Behavior of the model with supply response for many specifications had been solved numerically by Wright and Williams (1982a), who also show the numerically derived invariant distributions for the endogenous variables.

4 In a similar model with no supply (effort) response and proportional storage cost, Deaton and Laroque (1992) prove the existence of the price function $p(z) = \max \beta E[p(h + z - f^{-1}(p(z))), f(z)]$ and its uniqueness in the class of non-negative continuous non-increasing functions.

6.2. Solving the storage model

The general storage model has till now been solved only by what is now called dynamic programming, using an approach pioneered by Gustafson (1958).[5] The classic intuitive exposition of this method starts with the special case of a finite horizon, i.i.d. disturbances and no supply response. Imagine the world ends in period T. It is obvious if the commodity has only consumption value in period T that the optimal carryout stock is $x_T(\omega^T) = 0$. In period $T-1$, the planner's problem is to maximize with respect to carryout storage $x_{T-1} \geqslant 0$:

$$\begin{aligned} V(z_{T-1}, x_{T-1}, T-1) = {} & \int_0^{z_{T-1}-x_{T-1}} f(c)\,\mathrm{d}c - \phi(x_{T-1}) \\ & + \delta E_{T-1}\left[\int_0^{h_T(\omega^T)+x_{T-1}} f(c)\,\mathrm{d}c\right], \end{aligned}$$

where $h_t \equiv \lambda_{t-1}\omega_t$, $t \geqslant 0$.

This problem can be solved explicitly because future storage is zero. The solution satisfies

$$\begin{aligned} & \frac{\partial V(z_{T-1}, x_{T-1}, T-1)}{\partial x_{T-1}} \\ & \quad = -f\big(z_{T-1}\big(\omega^{T-1}\big) - x_{T-1}\big) - \phi'(x_{T-1}) \\ & \qquad + \delta E_{T-1}\big[f(h_T + x_{T-1})\big] \geqslant 0, \quad \text{with strict equality for } x_{T-1} > 0. \end{aligned}$$

From this expression the optimal choice $\hat{x}_{T-1}$ can be expressed as a function of z_{T-1}. By Bellman's principle of optimality, the decision in period $T-2$ assumes optimal choice of x in period $T-1$ and T, $\hat{x}_{T-1}(z_{T-1})$ and 0, respectively. By backward induction,

$$\begin{aligned} V(z_{T-j}, x_{T-j}, T-j) = {} & \int_0^{z_{T-j}-x_{T-j}} f(c)\,\mathrm{d}c - \phi(x_{T-j}) \\ & + \delta E_{T-j}\widehat{W}(h_{T-j+1} + x_{T-j}, T-j+1), \end{aligned}$$

where $\widehat{W}(\cdot,\cdot)$ is the value function, the discounted expected present value of social welfare in future periods, as of period $T-j+1$, given optimal contingent choices of

$$\hat{x}_t(z_t), \quad T-j < t \leqslant T.$$

[5] The pioneering work in solving the storage model draws on earlier work on inventory models by Arrow et al. (1951) as developed by Dvoretzky et al. (1952). Gustafson (1958) solved what would later have been called a rational expectations model, essentially similar to that outlined above, with no effort response. He does not use the term "dynamic programming", and indeed shows no signs of being aware of Bellman (1957), the classic book with that title.

Thus, knowing $V(z_{T-j+1}, x_{T-j+1}, T-j+1)$, one can solve for $\hat{x}_{T-j}$. It is not necessary to "remember" any information about decisions in later periods as the induction moves back in time from T. In the limit as j approaches infinity, the functions $\widehat{W}(z_{T-j+1}, T-j+1)$ and $\hat{x}_{T-j}(z_{T-j})$ converge on the functions $W(z)$ and $x(\hat{z})$, respectively. The latter is the "storage rule" relating stocks to current available supplies. Thus, in the limit, the fact that we assumed a "final period" is made irrelevant by discounting, given a transversibility condition that ensures that discounted future producer returns cannot grow indefinitely.[6] In the above model, this is ensured by the condition on consumer demand that limits consumer expenditures to less than m.

In practice, analytical solutions more than a few periods back from the terminal date T are unavailable, except in very special cases.[7] Fortunately, numerical methods have been available since Gustafson (1958), and these have become cheaper and easier to implement as computers have advanced in speed and sophistication. Gardner (1979) shows how the value function can be derived using a vector of couplets consisting of discrete values of carryout storage and matching values of the value function for the next period. (Equivalently, the marginal values can be used, as Gustafson showed.) This method becomes cumbersome and memory-intensive as more state variables are added.

If rational competitive supply (or effort) response is included with a single-period lag (much as described above for ideal stabilization), no state variable is added. But the Gardner method encounters computational problems when there is continuous supply response related to non-monotonicity of values in availability.

An alternative method approximates the stationary relation between expected price and carryout stocks with an nth-order polynomial in x, $\psi(x)$. The basic idea is to choose a function $\psi_0(x)$ as a "first guess", as shown in Figure 4. Then for each element of a predetermined vector of M, positive values of x solve the arbitrage equation for each of the N possible harvest outcomes h^i:

$$p\left(h^i + x^j - x^{*ij}\right) + \phi\left(x^{*ij}\right) - \delta\psi_0\left(x^{*ij}\right) = 0, \quad i \in [1, N],\ j \in [1, M],$$

for x^{*ij}, the "next-period" value of x given h^i, x^j and $\psi_0(\cdot)$. If the solution $x^{*ij} \leqslant 0$, set $x^{*ij} = 0$. Then for each x^j, calculate the expected price $p^{*j} = \sum_{i=1}^{M}[p(h^i + x^j - x^{*ij})\text{prob}(h^i)]$ where $\text{prob}(h^i)$ is the probability of h^i.

Solving for the equilibrium price function as a function of the carryout is a simple problem in applied numerical projection methods. The approach adopted in Wright and Williams (1982a, 1984a) is to regress the values p^{*j} on an nth-order polynomial in x^{*ij}, where the elements of that vector increase in constant increments, to yield an update $\psi_1(x)$ of the initial estimate $\psi_0(x)$. The process is repeated till the first period,

[6] See Gardner (1979) for a nice numerical exposition, and the Appendix in Deaton and Laroque (1992) for an existence proof using contraction mapping for their zero supply response model.

[7] See Newbery and Stiglitz (1981, pp. 437–438) or Aiyagari et al. (1989) for analytically tractable cases, and Williams and Wright (1991, Ch. 3, Appendix) for analytic solutions to a model like that above for $T-1$ and $T-2$.

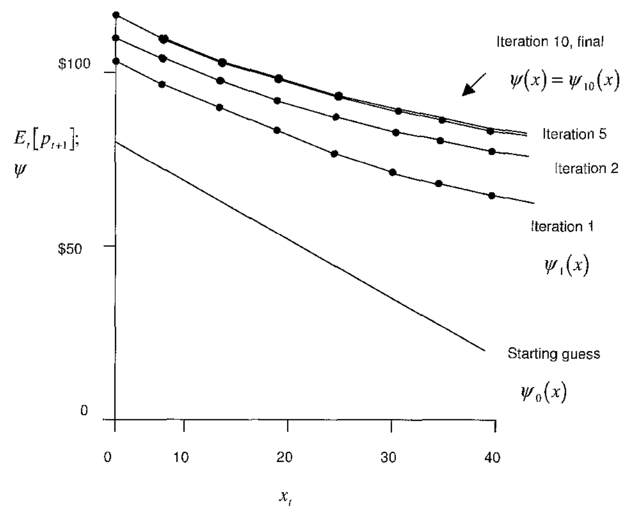

Figure 4. Iteration to find the polynomial approximation ψ.

k, in which the estimated polynomial $\psi_k(x)$ is, by a convergence criterion, sufficiently close to $\psi_{k-1}(x)$. Finally, set $\psi(x) = \psi_k(x)$.

Given the final function $\psi(\cdot)$, it is possible to solve for all the other endogenous variables, given the state z_t. For this application, the polynomial approximation illustrated performs well because the function ψ is smooth and monotonic when restricted to its support, as shown in Figure 4. (A demanding test of performance used by Wright and Williams is to check the deviation of average expected profits of storage from zero in large stochastic simulations of the equilibrium model.) More generally, numerical methods need not be applied to the problem as it is most easily conceived (determination of the kinked "storage model" in this case) if a smooth relationship exists that embodies the same information. Here that smooth relationship is the marginal value function as a function of stocks.

Results using this approach, extended to include responsive supply, were first published in Wright and Williams (1982a). The algorithm was first described in the Appendix of Wright and Williams (1984a), written contemporaneously with Wright and Williams (1982a). It is outlined in greater detail in Williams and Wright (1991, Chapter 3, Appendix). The algorithm is easily extended to several state variables [see for example the study of public and private storage in Wright and Williams (1982b), in which the private storage rule has multiple kinks in some cases], though at the cost of a substantial increase in the (otherwise modest) demand on computer time. A somewhat similar approach, termed "parameterized expectations", was introduced to the macroeconomic literature by Marcet (1990); the survey by Taylor and Uhlig (1990) indicates that the macroeconomic literature on this topic lagged behind the agricultural economics literature in the 1980s. The recent paper by Christiano and Fisher (2000) indicates that much of the lost ground has been regained.

Although no significant problems with the accuracy of the polynomial regression method have been reported in the storage literature, when applied to an appropriate smooth relationship, the main methodological modifications to the implementation of this solution strategy over the past decade have concentrated on substitution of alternative means of approximation of the price function. Given the smoothness of the equilibrium price function (in contrast to the function, $x(z)$), a more generally numerically accurate and efficient method is to apply Chebyshev collocation, introduced into the economic literature by Judd (1992). This method uses as a basis the Chebyshev polynomials, defined over $[-1, 1]$ by $T_i(x) \equiv \cos(i \arccos x)$. Using the identity $\cos n\phi = T_n(\cos\phi)$, and the initial values, $T_0(x) = 1$, $T_1(x) = x$, one can recursively calculate [Boehm and Prautzsch (1993, p. 108)], $T_{i+1}(x) = 2xT_i(x) - T_{i-1}(x)$, $i = 1, \ldots, \mu$. Adapted to the interval, $[0, x^{\max}]$, for stock values, the polynomial $\sum_{i=1}^{M} a_i T_i(x)$ is interpolated at the Chebyshev nodes

$$x^j = 0.5x^{\max}\bigl(1 + \cos\bigl((2j + 1/M + 1)\pi/2\bigr)\bigr), \quad j = 0, \ldots, M.$$

As the number of nodes, M, increases, the above interpolant converges rapidly on the price function. [See Judd (1992, p. 421), for the appropriate theorem; its proof is in Rivlin (1990, p. 14)].

The first published application of this methodology in the commodity storage literature appears to be by Miranda and Glauber (1995), and it subsequently has been used by others [for example, Park (1996), Makki et al. (1996), McNew and Gardner (1999)]. If the price function is as smooth as typically observed for this type of problem, the gain over simple polynomial approximation should indeed be modest. If cases should arise in which curvature changes very rapidly, methods other than Chebyshev collocation may be superior. Extensive reviews of numerical methods of use for economists are found in Judd (1998) and Miranda and Fackler (1999, forthcoming 2002).

7. Storage behavior and its effects on consumption, price and production

7.1. Storage behavior in the simple model

The responsiveness of storage affects the behavior of other endogenous variables, making some less variable, others more so. An example of the infinite-horizon "storage rule" is shown in the lower panel of Figure 5.[8] For simplicity, we assume i.i.d. production dis-

[8] The rule is in general nonlinear beyond the kink, though this is difficult to see in the figure. The storage rule tends to exhibit a significantly increasing marginal propensity to store when there is no supply response. The qualitative results are generally robust to differences in numerical specification. In the example, consumer demand is linear with price elasticity of -0.2, supply elasticity is 0.5, marginal storage costs are constant at 2 percent of mean price, and the interest rate is 5 percent. The distribution of yield disturbance is a discrete approximation to the normal distribution with a coefficient of variation of 10 percent.

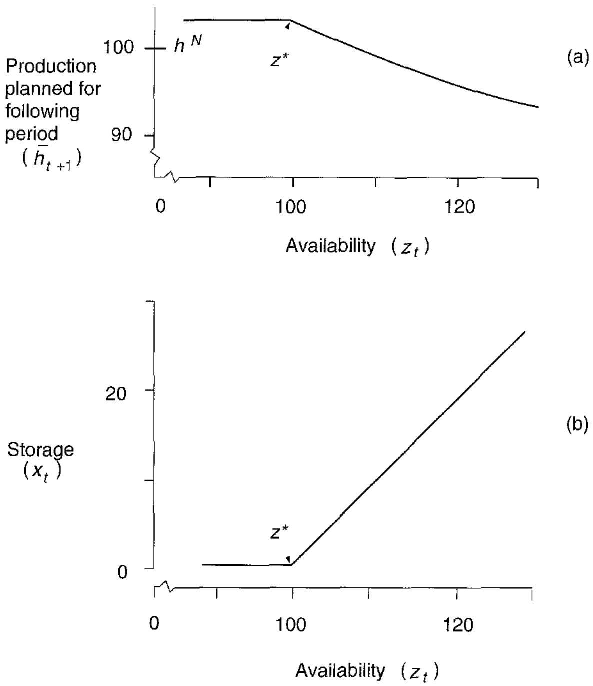

Figure 5. Joint rules for equilibrium planned production and storage.

turbances, except where noted below; availability is the sole state variable. Storage in this model is zero for $z < z^*$, then increases monotonically. The storage density has a mass point at zero, a mode at a low amount of stocks and a long upper tail. Consistent with intuition, the storage rule shifts to the right if output variance falls, interest costs or storage costs rise, or the length of horizon falls [Gustafson (1958)]. The storage rule implies that storage demand is positive and increasing in availability z for $z > z^*$, and this means that market demand has a kink at z^* and becomes more elastic beyond z^*, as shown in Figure 6. Storage reduces the dispersion of price, and truncates the density at the low-price end, leaving a long tail to the right. Given constant cost of capital, storage can eliminate very low prices but in general can only reduce, not eliminate, occurrence of famines.

Responsiveness of planned production tends to make the storage rule steeper and more nearly linear. Planned output decreases with availability for $z > z^*$, as shown in the upper panel of Figure 5. Its density has a mass-point at its maximum value and is skewed to the left, as shown in Figure 7. For $z < z^*$ and $x = 0$, planned production is highest but marginally unresponsive to current or future price. Importantly, "supply elasticity" to current price cannot be constant and positive in this model. The model with responsive production has higher expected availability $E_t(z_{t+1})$ when z_t is low,

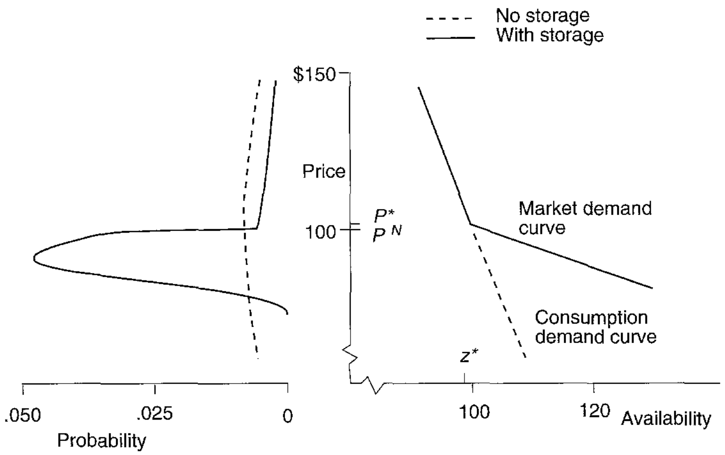

Figure 6. Probability distribution of availability and price.

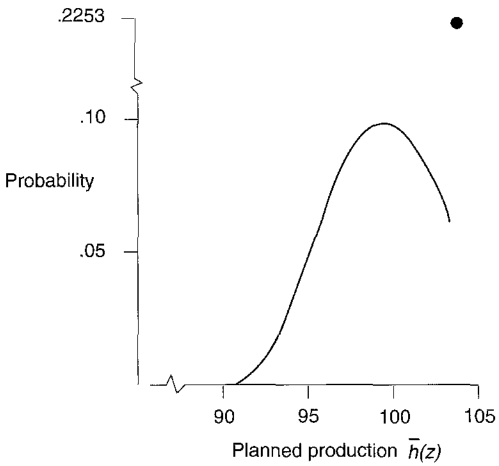

Figure 7. Probability distribution of planned production.

relative to the case with planned production fixed at its mean, and vice versa for high z_t. This lagged supply response, liberated in the i.i.d. case by the presence of storage, is highly stabilizing for consumption and price.

7.2. *Market dynamics of a storable commodity*

In a market with i.i.d. disturbances and no storage, realizations in one period have no implications for future realizations. Output incentives are constant, so supply response is irrelevant for time series behavior. In such a model, the real effects of dynamic interventions such as ideal production stabilization, discussed above, can be analyzed by comparative statics. Given a one-period production response, distributions of outcomes of price, consumption, and production for period t, expected in period $t - 1$ after a regime change for period t has been announced, are the steady-state distributions in the new regime. But we have seen that even in this case, the incidence effects are dynamic.

In a model with storage, expectations are conditional on the state(s). Consider for example Figure 8 which shows the expectations of prices in future periods conditional on the state (the initial available supply z_0) in period 0. For given z_0, these expectations can be estimated by generating thousands of simulations of the model, each for periods $1, 2, 3, \ldots$, taking random draws from the harvest distribution for each future period in each simulation, and taking the mean of the prices realized for each period by each of those simulations. The mean in period 1 is an estimate of the expected value in period 1 price, conditional on available supply in period 0, that is, $E_0(p_1)$. Note that $E_0(p_1)$ is not the price expected in period 1, and indeed $E_0(p_1)$ may not equal the realized price in any state; consider the case of a discrete two-point harvest density.

The evolution of future expected prices is shown for two values of supply elasticity and of initial availability in Figure 8. When initial availability is high and there is no supply response, price is low and storage is high, dampening expectations of next period's price, which exceeds current price by the cost of storage including interest cost. This

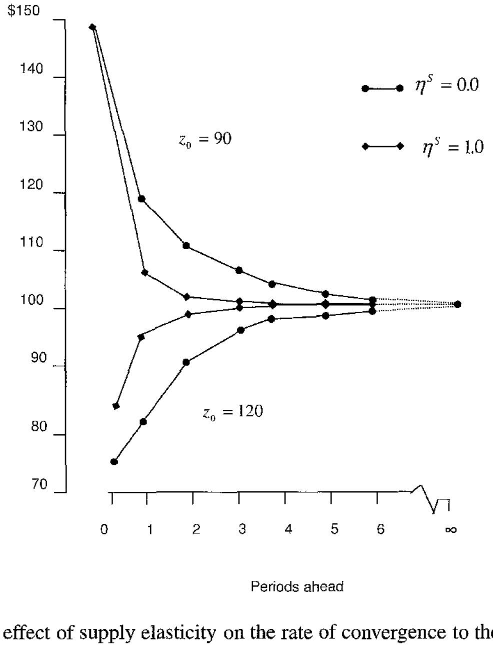

Figure 8. The effect of supply elasticity on the rate of convergence to the steady state.

overhang is expected to decline over time, allowing expected prices, conditional on z_0, to rise to converge on their limit, the mean of the steady-state distribution. As expected prices rise (and expected stocks fall), expected consumption falls. Expected production rises, but not enough to counteract the expected decline in available supplies. Note that in general $E_0(p_1) \neq E_0(f(E_0(c_1)))$.

If one assumes futures markets to be efficient and risk-neutral, the expected prices in Figure 8 can be called futures prices for contracts with maturities $1, 2, 3, \ldots$ periods from period 0. In period 1, after (unanticipated) realization of the harvest h_1, there is a jump in the path of futures prices starting in period 1 opposite in direction to the change in available supply $(z_1 - z_0)$. The new path of expectations (not shown) is conditional on z_1.

Figure 8 also shows the expected prices if available supplies z_0 are low and stocks are zero. The chain of expected prices declines toward the steady state as expected stocks are rebuilt towards their steady-state expectation. Paths of current futures prices for different maturities like those in Figure 8 can be observed in the array of actual futures prices for storable commodities reported in the financial press for any given trading day. They are not necessarily monotonic. If storage is positive at the steady-state mean price, then expected price the next period is sufficiently higher to satisfy the arbitrage condition, but the expected price conditional on z_0 for periods further in the future converges to the steady-state mean. [See Bresnahan and Spiller (1986), and Williams and Wright (1991, Figure 3.5, p. 136).]

The convergence of the futures price profiles to the same steady state values, independent of the current states, helps show why markets for long-term futures do not exist, despite the frequent lamentations of economists. The reason is that they are not needed. Futures prices are not reflections in a crystal ball. They embody relevant current information, and several periods out, the effect of current information on futures prices and on the spreads between them is greatly diminished. There are no markets for far-out futures because there is too little current information flow about them (not reflected in prices of existing futures contracts) to motivate trade [see Williams (1986, Chapter 6)].

The dynamic response of the land price associated with a change in current product price depends on the discount rate and (speaking loosely) the expected rate of convergence to the steady state. Discounting makes the near-term rent realizations more important than later realizations, but high interest rates also discourage storage and hasten convergence. As Figure 8 shows, supply response also speeds up convergence, thus increasing the importance of the steady state (comparative statics) effects, relative to the case with no supply response, while also changing those effects, in ways quantitatively similar to those noted for ideal stabilization above.

8. Storage and market stabilization

A major motive for studying storage is to analyze policy interventions that have long been controversial features of many commodity markets. These include controls on private "hoarding" or "speculation", buffer stocks, strategic reserves, rationing of low-price supplies, marketing boards, price floors, variable tariffs and other trade barriers, and production controls. In all of these interventions, efficiency effects tend to be dwarfed by distributional effects. Hence the underlying motivation, generally viewed by the public as stabilization and by economists as substitution for a missing risk market, is more often redistribution to those who most enthusiastically and effectively support such measures. To comprehend these distributional effects, it is necessary to recognize the dynamic nature of the problem and the importance of private responses to public actions.

A detailed analysis of all types of public interventions in commodity markets is well beyond the scope of this chapter. But many of the important considerations in "stabilization" schemes can be discussed in the context of a very simple public program, a "price floor" p^F that is implemented by an open public offer to buy any quantity at this price and to sell any available stocks at the same price, upon demand.

8.1. *A simple floor price program*

Consider, for example, the announcement and introduction of such a floor price in a market with positive harvest with an i.i.d. distribution (no supply response), and consumer demand of constant price elasticity in the inelastic range, with relative curvature parameter $C > 2$ and constant marginal storage cost.

If the initial available supply z_0 is low enough, there is no storage. As shown in the top panel of Figure 9, introduction of the floor price p^F may have no immediate effect on producer surplus, given storage remains at zero for available supply z. In the steady state, expected producer surplus is decreased for this set of parameters by the price stabilization induced by p^F, although the difference is too small to clearly show on the right-hand side of the top panel of Figure 9. This effect is anticipated from the analysis of ideal output stabilization for $C > 2$, discussed above. So initially there is no change in producer surplus, and in the steady state, expected producer surplus is lower. Yet the lower panel shows that stabilization favors those who own the fixed production assets ("land") at period 0, at the expense of near-term consumers.

The earliest nonzero effects of the price floor scheme on commodity price must be positive, since purchases precede sales. Thus in the medium term it is expected that storage and price will be higher, and consumption lower, because of the effects of the price floor on market price. In the illustrated example, the influence of these medium-term responses on land price dominates that of the steady state (comparative statics) adjustment because of the "front-end loading" caused by discounting. As shown in the bottom panel of Figure 9 the land price jumps up upon announcement of the price floor, benefiting the current land owners even though current commodity price is unaffected. After the announcement, expected land price eventually becomes lower than its initial steady state mean, consistent with the result for producer surplus, but this has no direct distributional significance in a partial equilibrium setting.

If initial available supplies are sufficiently larger, there will be positive initial stocks. Imposition of a price floor then causes an immediate positive jump in the value of

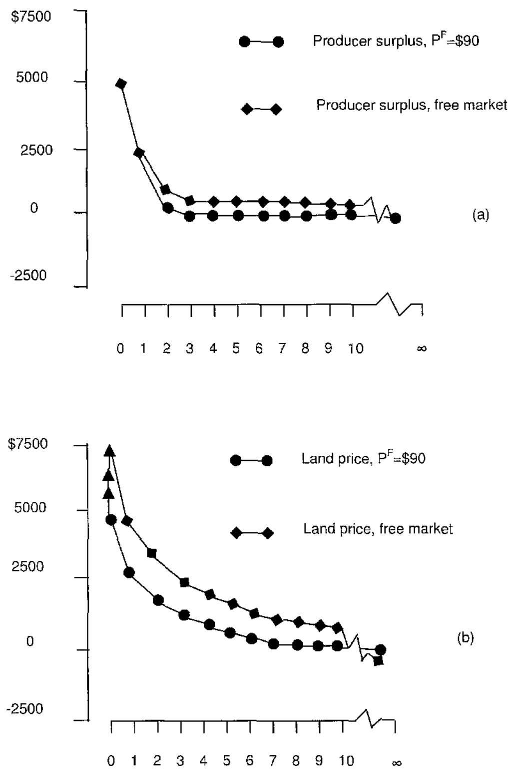

Figure 9. The effects of a price floor on producer surplus and its capitalization.

these stocks. Their owners unambiguously gain from the announcement of the price floor, whereas the price of land may jump up or down, depending upon the parameter values [see Miranda and Helmberger (1988); Wright and Williams (1988b)]. In general, the former conventional wisdom based on comparative statics (see the Kanbur quote above) underestimates the aggregate gains of landowners and stockholders.

Indeed analysis of this type of scheme can be used to illustrate several pitfalls of incidence and efficiency analysis of storage interventions in a dynamic model. As shown in Wright and Williams (1988b), comparative statics might not only give the wrong signs to the incidence effects on stockholders-landholders versus present and future consumers, it might also show the intervention to have negative deadweight loss, because it ignores the front-loaded cost of buffer stock accumulation. Analyses that ignore private storage may similarly conclude that a public buffer stock increases aggregate welfare; the "invention" of storage is incorrectly attributed to the government's use of (pre-existing) storage technology.

8.2. *Time-series behavior of "self-liquidating" floor price debt*

Perhaps the most important and certainly the least understood aspect of a public price floor program and other buffer stock schemes is the time series behavior of their money balances. When price p^F is set at the mean (or the mean of a floor and ceiling price equals the free-market mean), the program is often assumed to be "self-liquidating". This quality is frequently used to justify ignoring the evolution of the program funds in analyzing its implications, apparently based on the intuition that we expect funds from purchases and sales after several years of operation to be close to their initial value. But this intuition is wide of the mark, a point that seems to have been understood by Waugh at least as far back as 1967 [Waugh (1967, p. 31)].

To see this, consider the simple case in which demand is linear and planned production is constant, so the mean price is exogenous. Assume further that the harvest has a symmetric two-point distribution, there is no private storage, and p^F is set at mean price. Imagine a "buffer fund" scheme whereby the government pays $(p^F - p_t)$ for each unit sold at each time t. Negative payments are receipts by the government. The fund's monetary balance, B_t, with initial value B_0, follows a random walk. Given an infinite horizon, the balance passes any finite negative bound in finite time, and the probability that it is zero at any future date is the same as the probability that it is never zero before that date, and quickly becomes negligible [see Feller (1967, Lemma 1, p. 76)].

Similarly, a price floor scheme backed by a buffer stock generates a fund balance that behaves as a martingale with absorbing barrier at zero. The balance hits zero with probability one in finite time (that is, "infinitely often"). Though the theoretical inevitability of failure of similar public programs with finite resources was long ago noted by Townsend (1977), when they do fail there is generally a public consensus that the intervention price was wrongly set. There is scant recognition that failure is inevitable at any relevant intervention price; higher floor prices merely hasten its occurrence.

8.3. *Private storage and speculative attack*

Assume now that the floor price program has infinite financial resources, and consider the behavior of private stocks. The public floor price is set below the mean at $p^F = f(q^F)$ where $f(\cdot)$ is the consumption demand. A public storage rule for this type of model is shown in Figure 10 as the piecewise linear curve that follows the horizontal axis till z^{**} rises vertically, then turns to follow a straight line at a 45° angle.

Private storage, if allowed, tends to respond to the price floor by increasing stocks for a range of prices above p^F. In the example shown in Figure 10, the private storage rule, the dashed line, starts to the left of the free-market rule, and also to the left of q^F. Public acquisitions do not start at $z = q^F$, as they would in the absence of private storage, but at z^{**}. At levels of z below A^*, there is no storage. For levels between A^* and z^{**}, private stocks and consumption both increase in z as price decreases towards p^F.

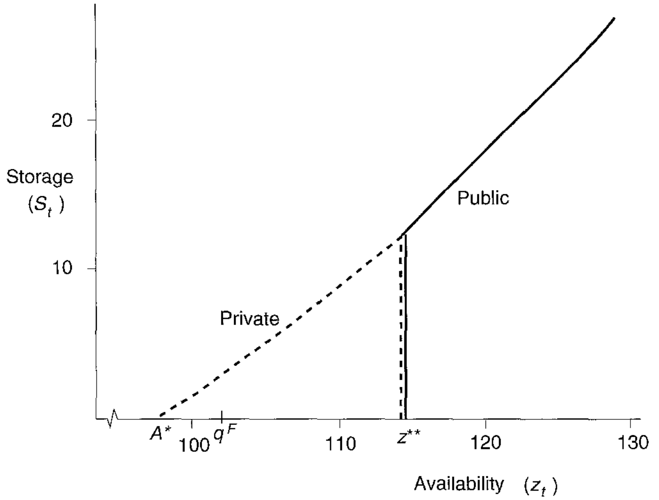

Figure 10. Public and private storage rules with a price floor.

At z equal to z^{**}, a marginal increase in z depresses future price below the level needed to sustain the private storage arbitrage. All private stocks are sold to ("dumped on") the public floor price program in any period in which the harvest is large enough to increase available supply above z^{**}. Similarly, the next harvest that is low enough to reduce available supply below z^{**} coincides with a "run" or "speculative attack" on government stocks that reduces those stocks to zero.

The rationality of this type of speculative behavior was proved by Salant (1983). The social welfare implications of speculative attack in general are more controversial. It is often viewed as undesirable speculative disruption by public stabilization authorities, including defenders of national currencies and operators of emergency food reserves. But Williams and Wright show numerical results indicating that, given the public program, private speculation can stabilize consumption, in the sense of inducing a mean-preserving contraction of consumption in a model with fixed planned supply. Though it increases the frequency of price changes, it reduces the occurrence of large price swings.

If the public sector has a finite budget, private speculation may reappear when public stock accumulation has exhausted that budget, and it will tend to reduce the otherwise abrupt changes that occur around that stock level. In this case, the public policy is a price "peg" of the kind discussed in Wright and Williams (1991, Chapter 14).

Indeed if these numerical results can be generalized, limits on private speculation must be justified on second-best grounds; perhaps speculation is too stabilizing, given the public intervention, or prevents risky behavior (gambling on "leaning against the wind") by public authorities that is for some reason desirable. This is an interesting and promising area for further theoretical and numerical investigation. The storage model is a useful "test bed" for ideas that drive government policies regarding defense of exchange rates against speculative attack, and more generally fixed versus variable exchange rates.

A phenomenon related to speculative attack might be observed in poaching that leads to extinction of animals valued for their storable or durable products. Pursuing a model suggested by Martin Weitzman, Kremer and Morcom (2000) note that if extinction is anticipated, future price of the product (e.g., ivory from elephants) will rise, increasing current price and hence the incentive for competitive poaching. But there may also be a non-extinction equilibrium, in which future supplies are anticipated to be larger, reducing future price and hence current price and eliminating the incentive to poach. Government policy might ensure the second (survival) equilibrium by credibly committing to prevent extinction, as in the United States Endangered Species Act. Alternatively, it might be possible that public (or private) storage of confiscated contraband or harvested materials, with the threat of sale to the market if price rises above a threshold or the species becomes endangered, could eliminate the extinction equilibrium.

8.4. Buffer stock schemes: theory and practice

The floor price scheme described above is pedagogically useful for its simplicity. The commodity programs that have been tried over the past seventy years have often combined the floor price with a higher "ceiling" or "release" price. These are not so simple. Numerical models of this type of arrangement show important interactions between band width, private storage within the band, the expected rate of accumulation of losses, and the maximum level of stocks [see Williams and Wright (1991, Chapter 14)]. For many, a strong intuition is that such a program keeps price around the middle of the "price band" most of the time, if the band is judiciously chosen. But numerical examples show the price density has mass points at both ends of the band, and little mass between the mid-point of the band and the top. Most of the time, the market may appear to be "challenging" either the floor or the release price.[9]

In practice, postwar experience has affirmed that the "finite time" within which we expect such programs to fail is disconcertingly short, often less than a decade. Recent failures in programs for tin and wool [Bardsley (1994), Gilbert (1996), Haszler (1998)], among others, have shown that the largest and most catastrophic price effect of these interventions is the severe price collapse that accompanies their inevitable failure.

[9] In an interesting empirical paper, Zant (1997) finds that in the Indian rubber market, operation of a buffer stock did not significantly reduce relative real price variation, compared to periods before and after the scheme. The market was subject to trade interventions throughout.

9. Storage with market power

Discussions of the effects of market power on storage behavior cover a range of quite distinct cases that are easily confused. If output demand is price elastic, a monopoly over risky production may extend to storage if demand is not too convex (that is, if marginal revenue is steeper than demand), as Newbery (1984) showed. Otherwise competitive storage will dominate monopolistic storage in this model unless there are entry barriers to storage as well as production.

When storage and production are monopolized, the nature of storage behavior depends upon the flexibility of decisions over pricing and production. [See Williams and Wright (1991, Chapter 11, Section 11), for a review.] For example, storage can help reduce the cost of a constraint on price adjustment (as in Keynesian models), or on production planning. The storage model most familiar to operations researchers is the (S, s) model. In this model, ordering cost of a firm is linear with a fixed cost component, and demand is random. Scarf (1959) proved the optimality of a discrete ordering policy in which stocks are raised to S whenever they fall to s. Caplin (1985) extended the previously firm-oriented focus of this model to take an aggregate perspective on the behavior of stocks.

But this literature has not taken the market-wide perspective of commodity storage models. The firm in the (S, s) model for some reason does not adjust price as demand fluctuates. It is implicitly assumed that the firm cannot sell excess inventory at an equilibrium "market price". The lack of such a market price implies market power, transaction costs, or other market imperfections not explicitly specified.

If storage alone is monopolized, the storage service is reduced in supply relative to competition. As Adam Smith recognized, charges of excessive hoarding by monopolists are misdirected. But the precise behavior of the profit-maximizing monopolist depends upon the ability of the monopolist to commit to announced policy, in the presence of competitive producers and speculators who respond to the monopolist's actions. This "time consistency" problem is like that faced by government in its storage interventions.

10. Optimizing (second-best) government interventions

When a government, instead of choosing the usual rather simple behavioral rule, attempts an optimizing intervention in a commodity market, justified by either a market distortion or an attempt to exploit market power vis-à-vis a trade partner, the optimization via dynamic programming breaks down. [See Wright and Williams (1982b) for an example of operating a strategic petroleum reserve in the presence of private storage.] Public storage interventions announced for next period, based on current available supply, may not be credible; other market participants might anticipate that the government will have an incentive to re-optimize next period, rather than follow its pre-announced strategy. Government then finds itself in a strategic game in which pre-commitment is valuable but difficult. This can make flexibility, normally very valuable in handling risk, actually disadvantageous.

Commitment problems are, paradoxically, at the heart of the most cogent argument for public intervention in the market for consumer commodities with stochastic supply or price. In an extreme shortage, government may be unable to withstand pressure to put a ceiling on price (for example, by rationing the good). Given this inability is common knowledge, private speculators will adjust their price expectations and store less of any given available supply. The inadequacy of private storage then justifies the public intervention on efficiency grounds.

The lack of a capacity to store can be advantageous as a source of commitment in a two-period game between an oil importer and an oil exporter [Eaton and Eckstein (1984)]. Another context in which storage capacity may be disadvantageous is in a context of self-motivated, stabilizing risk-sharing such as the "sovereign borrowing" studied by Eaton and Gersovitz (1981) and, in a renegotiation-proof context, by Kletzer and Wright (2000), and the related literature on risk-sharing in village economies [Ligon (1998)]. In these models, storage can increase the value of autarchy, weakening the force of punishments that reduce a defector's utility to the autarchy level, thus diminishing the potential for consumption-smoothing achieved by a sequence of unilateral equilibrium transfers [Ligon et al. (2000)].

Storage itself may be destabilizing in other special if not bizarre cases, such as the highly stylized model of Hart and Kreps (1986) in which demand alternates between radically different forms from period to period. Leach (1997) shows that storage can arise as a method of increasing the firm's strategic strength in the context of wage bargaining, and strikes evolve as a means of limiting that strength, in a model with no exogenous uncertainty. A related point is that, by decreasing the cost of strikes, storage may increase the frequency of labor market instability [Paarsch (1990)].

McLaren (1996) makes the interesting claim that the International Coffee Agreement, in which consumer countries have participated, is an example of a Pareto-improving commitment mechanism. He argues that it reassures suppliers, who have a long production lag between planting and first harvest, against *ex post* exploitation by monopsonistic consumers.

11. Spatial and trade models

Commodity models are made richer, and more complex, when storage and trade are both explicitly included as costly activities in a stochastic context with rational behavior. The unidirectionality (non-negativity) of storage combines with the unidirectionality of transport costs (returning an import doubles, rather than eliminates, transport costs) to make for interesting behavioral relations. [See Knapp (1982), Miranda and Glauber (1995), Williams and Wright (1991, Chapter 9).] Among the interesting results are the following:

- If disturbances in each country are not negatively correlated, storage tends to be more stabilizing than trade, but the interaction of the two is highly complementary.

- Two countries actively trading a commodity between them should not both be storing it unless transport takes significant time.
- The f.o.b.-c.i.f. bounds are not good guides to the range of competitive prices in a small country that can store the commodity [Anderson (1985)].

Intertemporal considerations also make traditional spatial models in the tradition of Von Thunen much more interesting. A major problem in constructing such a model is in general the lack of sufficient spatially disaggregated data. Taking advantage of data made available through the extraordinary powers of a Royal Commission, Brennan et al. (1997) modeled the major region of the Western Australian wheat market, using data including storage technologies, capacities, and costs for 104 delivery locations, capacities and costs of two different rail systems, and costs of road hauling. They modeled efficient transportation of a wheat harvest to the export terminal over the harvest (direct delivery) and off-peak (store then deliver) periods, for different combinations of peak and off-peak export prices. As price pressure for immediate delivery increased, storage decreased overall but increased in locations "inconvenient" to the port in terms of intensity of use of scarce transport resources. Rail trips became shorter and concentrated on locations with high-throughput loading facilities, to save scarce locomotive time.

When the spread between the post-harvest price and the harvest price is plotted against aggregate stocks after harvest, a typical "supply of storage" curve with "convenience yield" [Kaldor (1939)] at low levels of stocks is obtained in this model. As first documented by Working [for example, Working (1934)], low but positive stocks are held at negative spreads, and high stocks are held at prices reflecting positive storage returns. This showed, as conjectured in Wright and Williams (2000), that convenience yield could occur as an aggregation phenomenon in which stocks are held at full carry according to local prices but appear to be held at a loss according to spreads in a related market separated by transport costs that vary with delivery pressure.

The idea that marketing costs can explain the "convenience yield" phenomenon is embodied in the extension of the Scheinkman and Schechtman (1983) model by Bobenrieth and Wright (1998). In this model, marketing costs are modeled as a concave increasing function of available supplies and a convex decreasing function of carryout stocks. Thus the marketing cost function is not convex. This formulation is actually consistent with some intuitive discussions of the notion of "convenience yield" (as distinct from efforts to formalize that notion) in the literature. In contrast to the notion that at low levels stocks offer a positive dividend-like "convenience yield", as in modern empirical models of commodity price behavior [for example, Fama and French (1987), Pindyck (1993, 1994)], stocks on hand have a shadow price below the market price, yet the typical "supply of storage" behavior is exhibited by the model. This model shows promise of helping explain the related phenomenon of "liquidity preference", in which the transaction cost function has been assumed to be convex in consumption and money [e.g., Brock (1974, p. 769), Bougheas (1994)].

12. Testing the storage model

The lack of a closed-form solution for the storage model long hindered testing of the storage model. Recently, however, substantial progress has been made on two fronts. First, in an important and innovative line of research, Deaton and Laroque (1992) have tested simple versions of the model with annual commodity price data. They estimated the equation implied by the model with i.i.d. disturbances, storage decay rate δ, and discount factor β,

$$E_t\left(\frac{p_{t+1}}{p_t}\right) = \min \frac{(p_t, p^*)}{[\beta(1-\delta)]},$$

using the Generalized Method of Moments with lagged prices as instruments.

The results show mean values of $1/[\beta(1-\delta)]$ in excess of unity, for twelve of thirteen commodity price series, where r is the interest rate, and p^* below the maximum sample observation (so that the frequency of stockouts is positive though in some cases very low). There is also heteroscedasticity as implied by the storage model, though the stockout frequency was too low to test the prediction of theory that the variance of the disturbance is constant conditional on $p_t > p^*$, and there is little evidence of residual autocorrelation. There is substantial evidence in favor of the storage model and against the random walk hypothesis.[10]

However, the coefficient $1/[\beta(1-\delta)]$ is not significantly different from unity for each commodity, and the i.i.d. model does not appear to reproduce the degree of autocorrelation seen in the data at high prices. Chambers and Bailey (1996), using a similar approach on monthly data, found some informal support for a model with "periodic disturbances", that is, different disturbance distributions and associated threshold prices for different groups of months in the year. In further work, Deaton and Laroque (1996) assumed linear demand; their pseudo maximum-likelihood estimation results imply that their parameterization of the storage model does not track prices when they are high, and that adding autocorrelation in harvest does not adequately solve this problem.

In a very interesting paper, Miranda and Rui (1996) reassessed the model specification of Deaton and Laroque and Chambers and Bailey. They point out that the restriction of storage cost to a "constant decay" specification implied a "supply of storage" function which has a slope that is roughly the *negative* of the *ad hoc* empirical "supply of storage" curve pioneered by Working [see Working (1934)]: storage becomes more expensive at high prices in the Deaton and Laroque model, discouraging storage.

Instead, Miranda and Rui use a semilog storage cost ("supply of storage") function, $c(x) = a + \beta \ln(x)$, that has the qualitative characteristics of the numerous empirical estimates for various storable commodities dating back to the studies of Holbrook Working. Like Deaton and Laroque (1996), they assume linear consumer demand and a fixed

[10] Ardeni and Wright (1992) also reject the martingale hypothesis for the aggregate barter terms of trade between primary commodities and manufacturers using the state-space approach of Harvey (1989). However, the long price swings that are a major feature of these data are not explained by the storage model.

interest rate of 5 percent. Using maximum likelihood methods and the Chebyshev orthogonal collocation method with Gaussian quadrature [Miranda and Rui (1996), Judd (1992, 1998)], they estimate the model for the same commodities studied by Deaton and Laroque. They find that their storage model explains the autocorrelation of commodity price data very well, in sharp contrast to earlier results. What remains is to make the empirical connection between their storage cost specification and the progress on its microfoundations that has recently been made in terms of spatial aggregation [Wright and Williams (2000), Brennan et al. (1997)] or in terms of marketing costs more generally [Bobenrieth and Wright (1998)], as discussed above.

The spatial model of storage of Wright and Williams (2000) has itself been indirectly tested by Benirschka and Binkley (1995) on United States corn data. They find that the loan rate, a proxy for distance from market, in which it is decreasing, is significantly negatively related to the amount of grain storage capacity, in line with a theory that carryover stocks will be held where they have the least opportunity cost and are least "convenient" to the market. They also show that the rate of drawdown of stocks within the year is slower for states like Iowa relative to states like Illinois, Indiana, and Ohio, that are closer to the market. Further, locations with the highest prices at harvest tend to exhibit lower rates of price increase during the remainder of the crop year. Clearly, further tests of the storage model should follow the example of this work in paying closer attention to the disaggregation of price and quantity data. This is no easy task. For example, data limitations render the results of Frechette and Fackler (1999) inconclusive.

A less formal but no less important test of the storage model is its usefulness in interpreting market behavior. A particularly instructive application of the model to make sense of volatile commodity prices is Verleger (1994).

13. Tests using the storage model

The storage model can be used to generate sample data series for use in evaluating econometric tests of market behavior [Williams and Wright (1991, Chapter 7)]. Monte Carlo evaluations of published tests of forecasting ability raise serious questions about statistical inference using standard methodologies. For example, common tests of forecasting ability [e.g., Stein (1981)] that rely on R^2 as an index of predictive value are shown to be misleading, especially when the response to predicted changes is very effective. Similarly, the interpretation of R^2 in tests of the ability of the spot-futures spread to forecast movements in the spot [Fama and French (1987), Peck (1989)] is clarified by explicit reference to the commodity model [Williams and Wright (1991, pp. 180–181)]. Furthermore, common tests of relative bias in futures markets, and tests of excessive price variability can be quite unreliable when applied to markets for storable commodities. In addition, Monte Carlo tests of adaptive and "rational" expectations estimates to agricultural supply response reveal serious downward bias when the source of variability is yield variation.

This section would be longer if the potential of the model for quality control on tests of market behavior had been adequately exploited. The results so far merely scratch the surface, but they raise serious questions generally neglected in the relevant literature.

14. Challenges for the future

Storage behavior in response to market risk is only one element of commodity market dynamics. The possibility of persistent productivity shifts is another issue that bears on the adequacy of storage as a protection against shortfalls in output. Progress is being made on the effects of El Niño and La Niña on medium-term agricultural output. The evidence on more persistent disturbances is entirely inadequate. Studies of long time series of commodity prices are difficult to interpret, partly because a century or so is not really "long" enough, and the world is not static enough for inference about the longer-frequency movements that appear to be important features of the data. One way to begin investigating this question is to examine the long-run behavior of the underlying production disturbance. However, sample data on yields and weather rarely cover much more than a century. The best long-run evidence regarding annual growing conditions may be indirect, in such forms as tree rings and ice cores. One attempt that points to the possibility of persistent changes in the long-run local growth environment is the study of California tree-ring histories presented in Yoo and Wright (2000).

Macroeconomic effects on storage may well be very important. Using the many commodity price series available, we should be able to make more progress on the dynamic effects of cyclical changes in the cost of capital, exchange rates, and aggregate demand.

For some developing economies, the macroeconomic benefits of price stability deserve another look in the light of advances in macroeconomic theory. On the supply side, a challenge is to address the poor performance of current econometric approaches in Monte Carlo studies. It should be possible to make better inferences about aggregate supply responses in commodity markets by incorporating insights garnered from storage models.

A continuing challenge is to distinguish persistent structural changes including technical change in production and utilization and the effects of global warming from short-term fluctuations and positive or negative price bubbles. Adaptive learning, pioneered in an agricultural context by Rausser (1978) and Rausser and Hochman (1979), will be an important topic on the research agenda. Given the amount of historical data we have, it would be a mistake to expect great precision from inferences in this area, but it would be a greater mistake to neglect it.

In the context of very high frequency (daily) price data, the work of Bobenrieth (1996), applying the approach of Hamilton (1989), shows how statistical inferences about short-run, spot-price regime changes can reconcile questions implicit in earlier studies [e.g., Yang and Brorsen (1992)] about the consistency of economic and statistical theory and evidence with respect to the distribution of average daily price changes in some storable agricultural commodities.

Another recent research initiative [Bobenrieth et al. (forthcoming 2001)] derives the behavior of price in a model like that of Scheinkman and Schechtman (1983), but assuming that probability of zero output is positive, and that price (but not market revenue) approaches infinity as consumption goes to zero. (Think of the case of consumption demand with constant elasticity of unity.) In this model, if storage is strictly positive, it remains strictly positive. The path of expected price conditional on current information approaches infinity monotonically, but price falls with probability one in finite time, as in common conceptions of a commodity price bubble. Yet the price has a unique invariant distribution ("long-run price distribution") with infinite mean.

These results are easiest to understand by noting that price is a mapping from consumption, which in this model has an invariant distribution with finite positive mean. When consumption is low, it is expected to rise, but equilibrium stocks ensure that consumption remains positive when harvest is zero, and that expected price always exceeds the spot price by the current cost of storage. Interestingly, the sample mean of future prices, conditional on current information, underpredicts the path of "full carry" price expectations with arbitrarily high probability at sufficiently long horizon, in a manner similar to the behavior of "mean reversion" models in finance. Successive price realizations are always positively correlated, as in models with "convenience yield". The econometric implications of this type of model are currently under investigation.

15. Conclusion

Large fluctuations in output and prices are prominent features of markets for agricultural commodities. Analyses of the implications of such fluctuations are highly sensitive to assumptions about consumption demand, risk aversion, and the nature of "stabilization". Too often, storage is neglected, or the effects of storage interventions on welfare are overstated by ignoring private storage or other means of stabilizing consumption.

In a market with storage, interventions induce dynamic responses that tend to dominate any comparative static effects. Because stocks must be bought before they can be sold, initiation of storage interventions tends to favor producers more strongly than indicated in current analyses of price stabilization.

Developed within agricultural economics beginning with Gustafson (1958), the storage model is an essential tool for learning about the dynamics of commodity market behavior under rational expectations, and the potential and limitations of market manipulation and government interventions. It is also a useful "test bed" for Monte Carlo studies evaluating econometric results regarding the performance of futures markets and agricultural supply response. Finally it has potential, till recently unexploited, for elucidating bubble-like price behavior, and "mean reversion" in commodity prices.

A current challenge is to extend the storage model to include learning about changes in the agricultural environment, such as possible global weather changes on the one hand, or biological innovations on the other. Uncertainty about the occurrence of such changes and their implications may be a factor in apparent bubble-like commodity price behavior, a subject that certainly merits further research.

Many commodity policies involving storage had their genesis in the Great Depression of the 1930s. It is not surprising that they have seemed less appropriate in the more robust economic environment that has existed since then, especially since Keynes' (1942, p. 309) apprehension that market-stabilizing intervention might degenerate into supply restriction schemes has been validated. But should a Depression-like global market collapse recur, it is not clear that economists are equipped with theories and policy prescriptions for early detection of a commodity market crisis or for crisis intervention much superior to those of the 1930s. Given recent instability in global markets, this topic merits attention from commodity economists, macroeconomists, and historians.

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