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EXPLORING THE INTERNATIONAL LINKAGES OF THE EURO AREA: A GLOBAL VAR ANALYSIS

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SUMMARY

This paper presents a quarterly global model combining individual country vector error-correcting models in which the domestic variables are related to the country-specific foreign variables. The global VAR (GVAR) model is estimated for 26 countries, the euro area being treated as a single economy, over the period 1979–2003. It advances research in this area in a number of directions. In particular, it provides a theoretical framework where the GVAR is derived as an approximation to a global unobserved common factor model. Using average pair-wise cross-section error correlations, the GVAR approach is shown to be quite effective in dealing with the common factor interdependencies and international co-movements of business cycles. It develops a sieve bootstrap procedure for simulation of the GVAR as a whole, which is then used in testing the structural stability of the parameters, and for establishing bootstrap confidence bounds for the impulse responses. Finally, in addition to generalized impulse responses, the current paper considers the use of the GVAR for 'structural' impulse response analysis with focus on external shocks for the euro area economy, particularly in response to shocks to the US.

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1. INTRODUCTION

Macro-economic policy analysis and risk management require taking account of the increasing interdependencies that exist across markets and countries, and national economic issues need to be considered from a global perspective. This invariably means that many different channels of transmission must be taken into account. A number of papers in the literature argue that a rapidly rising degree of financial market integration has induced a closer financial and real international interdependence. Kose *et al.* (2003), using a Bayesian latent factor model in output, consumption and investment for 63 countries, find evidence of a world business cycle. Monfort *et al.* (2003) show that G-7 countries share common dynamics in real economic activity, with clearly identifiable common swings across countries. Oil price changes also seem to play an important role in increasing business co-movements. Finally, strong and increasing unilateral spillover effects from the North American area to the European area are being found, often interpreted as being caused by the process of globalization.

In order to bridge the gap between the purely statistical analyses and the traditional modelling approaches, the present paper studies the transmission mechanisms of shocks at the world level

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using a global VAR (GVAR). Such a framework is able to account for various transmission channels, including not only trade relationships but also financial linkages, most notably through interest rates, stock prices and exchange rates. An overview is provided in Anderton *et al.* (2004).

Building on the work of Pesaran *et al.* (2004, PSW), this paper presents a global model covering 33 countries with 25 of these modelled separately and the remaining countries grouped into a single euro area economy comprising 8 of the 11 countries that joined the euro in 1999. The GVAR is constructed by combining separate models for each of the 26 economies linking core variables within each economy with corresponding trade-weighted foreign variables. In this way we are able to provide a general, yet practical, global modelling framework for a quantitative analysis of the relative importance of different shocks and channels of transmission mechanisms. To deal with the modelling issues that arise from the creation of the euro area (a single exchange rate and short-term interest rate post 1999), the GVAR model presented in this paper is estimated with the euro area being treated as a single economy. This allows us to consider the impact of external shocks on the euro area as a whole without the danger of being subject to possible inconsistencies that could arise if the different economies in the euro area were modelled separately. The effects of external shocks on the euro area will be examined based on different simulations using generalized as well as structural impulse response functions.

Compared to the earlier contribution of PSW, the current paper advances the work on GVAR modelling in the following directions:

(i) In addition to increasing the geographical coverage, the current version also extends the estimation period, and includes long-term as well as short-term interest rates, thus allowing more fully for the possible effects of bond markets on output, inflation and equity prices.
(ii) It provides a theoretical framework where the GVAR is derived as an approximation to a global unobserved common factor model. Also using average pair-wise cross-section error correlations, the GVAR approach is shown to be quite effective in dealing with the common factor interdependencies and international co-movements of business cycles.
(iii) It develops a sieve bootstrap procedure for simulation of the GVAR as a whole, which is then used in testing the structural stability of the parameters, and in establishing bootstrap confidence bounds for the impulse responses.
(iv) In addition to generalized impulse responses reported in PSW, in the current version we also show how the GVAR model can be used for 'structural' impulse response analysis. We focus on identification of shocks to the US economy, particularly the monetary policy shocks, and consider the time profiles of their effects on the euro area. Further to the US monetary policy shock, we also consider the effects of shocks to oil prices and US equity prices on the euro area.

The plan of the paper is as follows: Section 2 presents the GVAR approach. Section 3 gives details on the version of the GVAR used in this paper, presents tests of the weak exogeneity of the country-specific foreign variables and discusses the issue of structural breaks in the context of the GVAR model. Section 4 examines the ability of the model to account for interdependencies and international co-movements by computing pair-wise cross-section correlations of the endogenous variables and the associated residuals. Section 5 checks the robustness of the GVAR results to the choice of trade weights by estimating a model using time-varying weights. Section 6 derives generalized impulse response functions for the analysis of country-specific and global shocks.

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Section 7 considers the problem of structural identification of shocks to the US economy and their consequences for the euro area in particular. Section 9 offers some concluding remarks.

2. MODELLING INTERNATIONAL TRANSMISSIONS: A GVAR APPROACH

One of the most striking features of the business cycles across countries are the patterns of co-movement of output, inflation, interest rates and real equity prices. These co-movements have become more pronounced over the past two decades owing to increased economic and financial integration, with important implications for macro-economic policy spillovers across countries. The extent of co-movement of real GDP across countries has been empirically investigated, both by considering bivariate correlation of real GDP across countries and by decomposing the variations of real GDP into common and country-specific shocks. Multivariate and multi-country analyses have also been undertaken in the context of G-7 economies. For example, Gregory *et al.* (1997) using Kalman filtering and dynamic factor analysis provide a decomposition of aggregate output, consumption and investment for G-7 countries. Other similar decompositions have also been attempted by Canova and Marrinan (1998), Lumsdaine and Prasad (2003) and Kose *et al.* (2003).[1]

There are clearly many channels through which the international transmissions of business cycles can take place. In particular, they could be due to common observed global shocks (such as changes in oil prices), they could arise as a result of global unobserved factors (such as the diffusion of technological progress), or could be due to specific national or sectoral shocks.

Unobserved factor models with a large number of macro-economic variables have recently gained popularity with the work of Stock and Watson (2002a). A related literature on dynamic factor models has also been developed by Forni and Reichlin (1998) and Forni *et al.* (2000). The factor models, estimated using principal components, are generally used to summarize by a small set of factors the empirical content of a large number of variables. Although unobserved factor models have important applications in forecasting, the identification of factors is often problematic, especially when we wish to give them an economic interpretation.[2] It is also likely that even when all such 'common' factors are taken into account, there will be important residual interdependencies due to policy and trade spillover effects that remain to be explained. Therefore, a fairly detailed global framework would be needed if we are to investigate the relative importance of such diverse sources of co-movements in the world economy, and their impacts on the euro area. For this purpose we make use of the global vector autoregressive model (GVAR) recently developed by PSW.

[1] Other related references include Norrbin and Schlagenhauf (1996), Artis *et al.* (1997), Bergman *et al.* (1998), Clark and Shin (2000) and Kose (2002).

[2] For an attempt at structural identification of factor models see Forni *et al.* (2003). Recently Bernanke *et al.* (2005) have considered factor augmented vector autoregressions (FAVAR) in measuring the effects of monetary policy in the USA, where the factors are typically estimated by means of principal components analysis. While FAVAR could be viewed as an alternative approach to VARX* modelling of the individual countries, the number of estimated factors used in the former approach would be different for the different countries and it is not clear how these can be linked together in a global setting. Moreover, Monte Carlo experiments reported in Kapetanios and Pesaran (2007) show that the common correlated effects estimators that make use of cross-section averages (star variables in the context of VARX*) perform much better than the corresponding estimators based on principal components. Also, a recent application of the FAVAR approach to the UK economy by Laganá and Mountford (2005) shows that, while the additional variables embodied in the factors help in overcoming the price puzzle, the factor augmentation leads to new puzzles relating to the counter-intuitive effects of interest rate changes on house prices, equity prices and the exchange rate.

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To motivate the GVAR model for the analysis of the international transmission mechanisms and to relate it to the unobserved factor models, suppose there are $N+1$ countries (or regions) in the global economy, indexed by $i = 0, 1, \ldots, N$, where country 0 serves as the numeraire country (which we take as the USA, but could be any other country). The aim is to model a number of country-specific macro-economic variables such as real GDP, inflation, interest rates and exchange rates collected in the vector $\mathbf{x}_{it}$, over time, $t = 1, 2 \ldots, T$, and across the $N+1$ countries. Given the general nature of interdependencies that might exist in the world economy, it is clearly desirable that all the country-specific variables $\mathbf{x}_{it}$, $i = 0, 1, \ldots, N$, and observed global factors (such as oil prices) are treated endogenously. The following general factor model provides a good starting point and allows us also to relate the GVAR approach to the more familiar factor models used in the literature primarily for the analysis of G-7 economies.

Denote the observed global factors by the $m_d \times 1$ vector $\mathbf{d}_t$, and the unobserved global factors by the $m_f \times 1$ vector $\mathbf{f}_t$, and assume that[3]

$$\mathbf{x}_{it} = \delta_{i0} + \delta_{i1}t + \mathbf{\Gamma}_{id}\mathbf{d}_t + \mathbf{\Gamma}_{if}\mathbf{f}_t + \xi_{it}, \text{for } i = 0, 1, \ldots, N; t = 1, 2, \ldots, T \tag{1}$$

where $\mathbf{\Gamma}_i = (\mathbf{\Gamma}_{id}, \mathbf{\Gamma}_{if})$ is the $k_i \times m$, matrix of factor loadings, $m = m_d + m_f$, $\boldsymbol{\xi}_{it}$ is a $k_i \times 1$ vector representing the country-specific effects involving lagged values of $\mathbf{x}_{it}$ or country-specific dummy variables capturing major institutional and political upheavals, and δ_{i0} and δ_{i1} are the coefficients of the deterministics, here intercepts and linear trends. Other deterministics, such as seasonal dummies, can also be included in the model. The vector of observed global variables could include international variables such as oil or other commodity prices, world expenditure on R&D, or other indicators of global technology such as the number of international patents registered in the USA.

Unit root and cointegration properties of $\mathbf{x}_{it}$, $i = 0, 1, \ldots, N$, can be accommodated by allowing the global factors, $\mathbf{h}_t = (\mathbf{d}_t', \mathbf{f}_t')'$, and/or the country-specific factors, ξ_{it}, to have unit roots. More specifically, we assume that

$$\Delta\mathbf{h}_t = \Lambda(L)\eta_t, \eta_t \sim IID(\mathbf{0}, \mathbf{I}_m) \tag{2}$$

$$\Delta\xi_{it} = \mathbf{\Psi}_i(L)\mathbf{v}_{it}, \quad \mathbf{v}_{it} \sim IID(\mathbf{0}, \mathbf{I}_{k_i}) \tag{3}$$

where L is the lag operator and

$$\Lambda(L) = \sum_{\ell=0}^{\infty} \underset{m \times m}{\Lambda_\ell} L^\ell, \mathbf{\Psi}_i(L) = \sum_{\ell=0}^{\infty} \underset{k_i \times k_i}{\mathbf{\Psi}_{i\ell}} L^\ell \tag{4}$$

The coefficient matrices, Λ_ℓ and $\mathbf{\Psi}_{i\ell}$, $i = 0, 1, \ldots, N$, are absolute summable, so that $\text{var}(\Delta\mathbf{f}_t)$ and $\text{var}(\Delta\xi_{it})$ are bounded and positive definite, and $[\mathbf{\Psi}_i(L)]^{-1}$ exists. In particular we require that

$$\text{var}(\Delta\xi_{it}) = \sum_{\ell=0}^{\infty} \mathbf{\Psi}_{i\ell}\mathbf{\Psi}_{i\ell}' \leq \mathbf{K} < \infty \tag{5}$$

[3] Dynamic factor models of Forni and Lippi (1997) can also be accommodated by including lagged values of $\mathbf{d}_t$ and $\mathbf{f}_t$ as additional factors via suitable extensions of $\mathbf{d}_t$ and $\mathbf{f}_t$. For example, $\mathbf{f}_t$ in (1) can be replaced by $\mathbf{f}_t^* = (\mathbf{f}_t', \mathbf{f}_{t-1}', \ldots, \mathbf{f}_{t-p_f}')'$.

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where $\mathbf{K}$ is a fixed bounded matrix.

First differencing (1) and using (3) we have

$$[\boldsymbol{\Psi}_i(L)]^{-1}(1-L)(\mathbf{x}_{it}-\delta_{i0}-\delta_{i1}t-\boldsymbol{\Gamma}_{id}\mathbf{d}_t-\boldsymbol{\Gamma}_{if}\mathbf{f}_t)=\mathbf{v}_{it}$$

Using the approximation

$$(1-L)[\boldsymbol{\Psi}_i(L)]^{-1}\approx\sum_{\ell=0}^{p_i}\boldsymbol{\Phi}_{i\ell}L^{\ell}=\boldsymbol{\Phi}_i(L,p_i)$$

we obtain the following approximate VAR(p_i) model:

$$\boldsymbol{\Phi}_i(L,p_i)(\mathbf{x}_{it}-\delta_{i0}-\delta_{i1}t-\boldsymbol{\Gamma}_{id}\mathbf{d}_t-\boldsymbol{\Gamma}_{if}\mathbf{f}_t)\approx\mathbf{v}_{it} \tag{6}$$

Without the unobserved common factors, $\mathbf{f}_t$, the model for the ith country decouples from the rest of the country models and each country model can be estimated separately using the econometric techniques developed in Harbo *et al.* (1998) and Pesaran *et al.* (2000) with $\mathbf{d}_t$ treated as weakly exogenous. With the unobserved common factors included, the model is quite complex and its econometric analysis using Kalman filtering techniques would be quite involved unless N is very small. When N is relatively large a simple, yet effective, alternative would be to follow Pesaran (2006) and proxy $\mathbf{f}_t$ in terms of the cross-section averages of country-specific variables, $\mathbf{x}_{it}$, and the observed common effects, $\mathbf{d}_t$. To see how this procedure could be justified in the present more complicated context, initially assume $k_i=k$ and use the same set of weights, w_j, $j=0,1,\ldots,N$, to aggregate the country-specific relations defined by (1) to obtain

$$\sum_{j=0}^{N}w_j\mathbf{x}_{jt}=\sum_{j=0}^{N}w_j\delta_{j0}+\left(\sum_{j=0}^{N}w_j\delta_{j1}\right)t+\left(\sum_{j=0}^{N}w_j\boldsymbol{\Gamma}_{jd}\right)\mathbf{d}_t$$
$$+\left(\sum_{j=0}^{N}w_j\boldsymbol{\Gamma}_{jdf}\right)\mathbf{f}_t+\sum_{j=0}^{N}w_j\xi_{jt}$$

or

$$\mathbf{x}_t^*=\delta_0^*+\delta_1^*t+\boldsymbol{\Gamma}_d^*\mathbf{d}_t+\boldsymbol{\Gamma}_f^*\mathbf{f}_t+\xi_t^* \tag{7}$$

Also, note from (3) that

$$\xi_t^*-\xi_{t-1}^*=\sum_{j=0}^{N}w_j\boldsymbol{\Psi}_j(L)\mathbf{v}_{jt} \tag{8}$$

But using Lemma A.1 in Pesaran (2006), it is easily seen that for each t the left-hand side of (8) will converge to zero in quadratic mean as $N\to\infty$, if (5) holds, the country-specific shocks, $\mathbf{v}_{jt}$, are independently distributed across j, and if the weights, w_j, satisfy the atomistic conditions

$$\text{(i)}: w_j=O\left(\frac{1}{N}\right),\ \text{(ii)}:\sum_{j=0}^{N}|w_j|<K,\ \text{(iii)}:\sum_{j=0}^{N}w_j=1 \tag{9}$$

where K is a fixed constant. Under these conditions (for each t), $\xi_t^* - \xi_{t-1}^* \overset{q.m.}{\to} 0$, and hence $\xi_t^* \overset{q.m.}{\to} \xi^*$, where ξ^* is a time-invariant random variable. Using this result in (7) and assuming that the $k \times m_f$ average factor loading coefficient matrix, $\boldsymbol{\Gamma}_f^*$, has full column rank (with $k \geq m_f$) we obtain

$$\mathbf{f}_t \overset{q.m.}{\to} (\boldsymbol{\Gamma}_f^{*\prime}\boldsymbol{\Gamma}_f^*)^{-1}\boldsymbol{\Gamma}_f^*(\mathbf{x}_t^* - \delta_0^* - \delta_1^* t - \boldsymbol{\Gamma}_d^*\mathbf{d}_t - \xi^*)$$

which justifies using the observable vector $\{1, t, \mathbf{d}_t, \mathbf{x}_t^*\}$ as proxies for the unobserved common factors.[4] Substituting this result in (6), for N sufficiently large we have

$$\boldsymbol{\Phi}_i(L, p_i)(\mathbf{x}_{it} - \tilde{\delta}_{i0} - \tilde{\delta}_{i1}t - \tilde{\boldsymbol{\Gamma}}_{id}\mathbf{d}_t - \tilde{\boldsymbol{\Gamma}}_{if}\mathbf{x}_t^*) \approx \mathbf{v}_{it} \tag{10}$$

where $\tilde{\delta}_{i0}$, $\tilde{\delta}_{i1}$, $\tilde{\boldsymbol{\Gamma}}_{id}$ and $\tilde{\boldsymbol{\Gamma}}_{if}$ are given in terms of δ_{i0}, δ_{i1}, $\boldsymbol{\Gamma}_{id}$, $\boldsymbol{\Gamma}_{if}$, $\delta_0^* + \xi^*$, δ_1^*, $\boldsymbol{\Gamma}_d^*$, and $\boldsymbol{\Gamma}_f^*$.

In practice, the number of countries, $N + 1$, may not be sufficiently large, and the individual countries not equally important in the global economy. The country-specific shocks might also be cross-sectionally correlated due to spatial or contagion effects that are not totally eliminated by the common factors, $\mathbf{d}_t$ and $\mathbf{f}_t$. Finally, k_i, the number of country-specific variables, need not be the same across i. For example, some markets may not exist or might not be sufficiently developed in some of the countries. Even if we focus on the same set of variables to model across countries, there will be one less exchange rate than there are countries in the global model. The GVAR framework developed in PSW addresses these considerations by using country-specific weights, w_{ij}. Specifically, instead of using the same $\mathbf{x}_t^*$ in all country models PSW use

$$\mathbf{x}_{it}^* = \sum_{j=0}^{N} w_{ij}\mathbf{x}_{jt}, \text{ with } w_{ii} = 0 \tag{11}$$

in the ith country model. The weights, w_{ij}, $j = 0, 1, \ldots, N$ could be used to capture the importance of country j for country ith economy. Geographical patterns of trade provide an obvious source of information for this purpose and could also be effective in mopping up some of the remaining spatial dependencies. The weights could also be allowed to be time-varying so long as they are predetermined. This could be particularly important in the case of rapidly expanding emerging economies with their fast-changing trade relations with the rest of the world. The use of the country-specific weights also allows a simple solution to the problem of k_i, the number of country-specific variables, being different across i. It would be sufficient to attach zero weights to the missing variable in country i, with the remaining weights being rebalanced to add up to unity.

With the above considerations in mind, the GVAR counterpart of (10) may now be written more generally as the individual country VARX$^*(p_i, q_i)$ models:

$$\boldsymbol{\Phi}_i(L, p_i)\mathbf{x}_{it} = \mathbf{a}_{i0} + \mathbf{a}_{i1}t + \boldsymbol{\Upsilon}_i(L, q_i)\mathbf{d}_t + \boldsymbol{\Lambda}_i(L, q_i)\mathbf{x}_{it}^* + \mathbf{u}_{it} \tag{12}$$

for $i = 0, 1, \ldots, N$, where for estimation purposes $\boldsymbol{\Phi}_i(L, p_i)$, $\boldsymbol{\Upsilon}_i(L, q_i)$ and $\boldsymbol{\Lambda}_i(L, q_i)$ can be treated as unrestricted. For the empirical implementation that will follow, for each country model we

[4] In a much simpler context Pesaran (2006) shows that it would still be valid to use $\{1, t, \mathbf{d}_t, \mathbf{x}_t^*\}$ as a proxy for $\mathbf{f}_t$ even if the rank condition is not satisfied. It seems reasonable to believe that the same would apply here.

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consider at most a VARX*(2, 2) specification which in its error correction form may be written as[5]

$$\Delta \mathbf{x}_{it} = \mathbf{c}_{i0} - \alpha_i \beta_i' [\zeta_{i,t-1} - \gamma_i (t-1)] + \mathbf{\Upsilon}_{i0} \Delta \mathbf{d}_t + \mathbf{\Lambda}_{i0} \Delta \mathbf{x}_{it}^* + \mathbf{\Upsilon}_{i1} \Delta \mathbf{d}_{t-1} + \Gamma_i \Delta \mathbf{z}_{i,t-1} + \mathbf{u}_{it} \quad (13)$$

where $\mathbf{z}_{it} = (\mathbf{x}_{it}', \mathbf{x}_{it}^{*\prime})'$, $\zeta_{i,t-1} = (\mathbf{z}_{i,t-1}', \mathbf{d}_{t-1}')'$, α_i is a $k_i \times r_i$ matrix of rank r_i and β_i is a $(k_i + k_i^* + m_d) \times r_i$ matrix of rank r_i. By partitioning β_i as $\beta_i = (\beta_{ix}', \beta_{ix*}', \beta_{id}')'$ conformable to $\zeta_{it} = (\mathbf{x}_{it}', \mathbf{x}_{it}^{*\prime}, \mathbf{d}_t')'$, the r_i error correction terms defined by (13) can now be written as

$$\beta_i'(\zeta_{it} - \gamma_i t) = \beta_{ix}' \mathbf{x}_{it} + \beta_{ix*}' \mathbf{x}_{it}^* + \beta_{id}' \mathbf{d}_t + (\beta_i' \gamma_i) t \quad (14)$$

that clearly allows for the possibility of cointegration both within $\mathbf{x}_{it}$ and between $\mathbf{x}_{it}$ and $\mathbf{x}_{it}^*$ and consequently across $\mathbf{x}_{it}$ and $\mathbf{x}_{jt}$ for $i \neq j$.

The number of cointegrating relations, r_i, and the vector of cointegrating relations for each country model can be consistently estimated separately, by treating $\mathbf{d}_t$ and $\mathbf{x}_{it}^*$ as weakly exogeniety $I(1)$ with respect to the long run parameters of the conditional model (13). Note that this assumption is compatible with a certain degree of weak dependence across $\mathbf{u}_{it}$, as discussed in PSW. Following Johansen (1992) and Granger and Lin (1995), the weak exogeneity assumption in the context of cointegrating models implies no long-run feedback from $\mathbf{x}_{it}$ to $\mathbf{x}_{it}^*$, without necessarily ruling out lagged short-run feedback between the two sets of variables. In this case $\mathbf{x}_{it}^*$ is said to be 'long-run forcing' for $\mathbf{x}_{it}$, and implies that the error correction terms of the individual country VECMs do not enter in the marginal model of $\mathbf{x}_{it}^*$. As discussed in more detail in Section (3.4), the weak exogeneity of these variables can then be tested in the context of each of the country-specific models. Furthermore, conditional on a given estimate of β_i, it is possible to show that the remaining parameters of the conditional model (namely $\mathbf{c}_{i0}$, α_i, $\mathbf{\Upsilon}_{i0}$, $\mathbf{\Lambda}_{i0}$, $\mathbf{\Upsilon}_{i1}$, Γ_i) can be consistently estimated either indirectly from a VAR in $\mathbf{z}_{it}$, or directly by OLS regressions of $\Delta \mathbf{x}_{it}$ on intercepts, the error correction terms, $\Delta \mathbf{x}_{it}^*$, $\Delta \mathbf{d}_t$, $\Delta \mathbf{d}_{t-1}$, and $\Delta \mathbf{z}_{i,t-1}$. The two approaches will yield identical estimates.[6]

Once the individual country models are estimated, all the $k = \sum_{i=0}^{N} k_i$ endogenous variables of the global economy, collected in the $k \times 1$ vector $\mathbf{x}_t = (\mathbf{x}_{0t}', \mathbf{x}_{1t}', \ldots, \mathbf{x}_{Nt}')'$, need to be solved simultaneously. PSW show how this can be done in the case where $p_i = q_i = 1$. In the present more general context we first rewrite (12) as

$$\mathbf{A}_i(L, p_i, q_i) \mathbf{z}_{it} = \varphi_{it}, \text{ for } i = 0, 1, 2, \ldots, N \quad (15)$$

where

$$\mathbf{A}_i(L, p_i, q_i) = [\mathbf{\Phi}_i(L, p_i), -\mathbf{\Lambda}_i(L, q_i)], \mathbf{z}_{it} = \begin{pmatrix} \mathbf{x}_{it} \\ \mathbf{x}_{it}^* \end{pmatrix},$$

$$\varphi_{it} = \mathbf{a}_{i0} + \mathbf{a}_{i1} t + \mathbf{\Upsilon}_i(L, q_i) \mathbf{d}_t + \mathbf{u}_{it}$$

[5] Here we consider the trend restricted version, case VI, discussed in Pesaran *et al.* (2000), which ensures that the deterministic trend property of the country-specific models remains invariant to the cointegrating rank assumptions.

[6] A proof of this statement is available in the Appendix. Further details concerning the estimation of the conditional model can also be found in Garratt *et al.* (2006, Ch. 6).

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Table I. Countries and regions in the GVAR model

USA	**Euro area**	**Latin America**
China	Germany	Brazil
Japan	France	Mexico
UK	Italy	Argentina
	Spain	Chile
Other developed economies	Netherlands	Peru
Canada	Belgium	
Australia	Austria	
New Zealand	Finland	
Rest of Asia	**Rest of W. Europe**	**Rest of the world**
Korea	Sweden	India
Indonesia	Switzerland	South Africa
Thailand	Norway	Turkey
Philippines		Saudi Arabia
Malaysia		
Singapore		

Let $p = \max(p_0, p_1, \ldots, p_N, q_0, q_1, \ldots, q_N)$ and construct $\mathbf{A}_i(L, p)$ from $\mathbf{A}_i(L, p_i, q_i)$ by augmenting the $p - p_i$ or $p - q_i$ additional terms in powers of L by zeros. Also note that

$$\mathbf{z}_{it} = \mathbf{W}_i \mathbf{x}_t, \quad i = 0, 1, 2, \ldots, N \tag{16}$$

where $\mathbf{W}_i$ is a $(k_i + k_i^*) \times k$ matrix, defined by the country specific weights, w_{ji}.

With the above notations (15) can be written equivalently as $\mathbf{A}_i(L, p)\mathbf{W}_i\mathbf{x}_t = \varphi_{it}$, $i = 0, 1, \ldots, N$, and then stacked to yield the VAR(p) model in $\mathbf{x}_t$:

$$\mathbf{G}(L, p)\mathbf{x}_t = \varphi_t \tag{17}$$

where

$$\mathbf{G}(L, p) = \begin{pmatrix} \mathbf{A}_0(L, p)\mathbf{W}_0 \\ \mathbf{A}_1(L, p)\mathbf{W}_1 \\ \vdots \\ \mathbf{A}_N(L, p)\mathbf{W}_N \end{pmatrix}, \varphi_t = \begin{pmatrix} \varphi_{0t} \\ \varphi_{1t} \\ \vdots \\ \varphi_{Nt} \end{pmatrix} \tag{18}$$

The GVAR(p) model (17) can now be solved recursively, and used for forecasting or generalized impulse response analysis in the usual manner. The issue of structural impulse response analysis poses special problems in the context of the GVAR model and will be dealt with in Section 7.

3. THE GVAR MODEL (1979–2003)

The version of the GVAR model developed in this paper covers 33 countries, where 8 of the 11 countries that originally joined the euro on 1 January 1999 are grouped together, and the remaining 25 countries are modelled individually (see Table I). The present GVAR model, therefore, contains 26 countries/regions. The original PSW model contained 11 countries/regions based on 25 countries. With increased country coverage, the countries in the present GVAR model account for 90% of world output as compared to 80% covered by the 11 countries/regions in PSW. Data sources are provided in Supplement A, available from the authors on request.

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The models are estimated over the period 1979(2)–2003(4). This considerably extends the 11 country/region models estimated in PSW over the shorter period 1979(2)–1999(4), most notably including the first years of EMU. The variables included in the current version of the GVAR differ also from those considered by PSW. In order to capture more fully the possible effects of bond markets on output and inflation, we now include, wherever possible, both a short rate (ρ_{it}^S), as well as a long rate of interest (ρ_{it}^L). However, given the data limitations and problems associated with compiling comparable money supply measures, we have decided against the inclusion of real money balances in the current version. Other variables included are real output (y_{it}), the rate of inflation, ($\pi_{it} = p_{it} - p_{i,t-1}$), the real exchange rate ($e_{it} - p_{it}$), and real equity prices (q_{it}), when available. More specifically

$$y_{it} = \ln(GDP_{it}/CPI_{it}), \quad p_{it} = \ln(CPI_{it}), q_{it} = \ln(EQ_{it}/CPI_{it}),$$
$$e_{it} = \ln(E_{it}), \rho_{it}^S = 0.25 \times \ln(1 + R_{it}^S/100), \rho_{it}^L = 0.25 * \ln(1 + R_{it}^L/100) \tag{19}$$

where GDP_{it} is the nominal Gross Domestic Product, CPI_{it} the consumer price index, EQ_{it} the nominal equity price index, E_{it} the exchange rate in terms of US dollars, R_{it}^S is the short rate, and R_{it}^L the long rate of interest, for country i during the period t.

The country-specific foreign variables, y_{it}^*, π_{it}^*, q_{it}^*, ρ_{it}^{*S}, ρ_{it}^{*L}, were constructed using trade weights. Baxter and Kouparitsas (2004) in studying the determinants of business cycle co-movements conclude that bilateral trade is the most important source of inter-country business cycle linkages.[7] Initially, we use fixed trade weights based on the average trade flows computed over the three years 1999–2001. Allowing for time-varying trade weights is straightforward and is considered in Section 5.

The time series data for the euro area was constructed by cross-section weighted averages of y_{it}, π_{it}, q_{it}, ρ_{it}^S, ρ_{it}^L, over Germany, France, Italy, Spain, Netherlands, Belgium, Austria and Finland, using the average Purchasing Power Parity GDP weights, also computed over the 1999–2001 period.[8]

With the exception of the US model, all models include the country-specific foreign variables, y_{it}^*, π_{it}^*, q_{it}^*, ρ_{it}^{*S}, ρ_{it}^{*L} and the log of oil prices (p_t^o), as weakly exogenous in the sense discussed above. In the case of the US model, oil prices are included as an endogenous variable, with $e_{US,t}^* - p_{US,t}^*$, $y_{US,t}^*$, and $\pi_{US,t}^*$ as weakly exogenous. Given the importance of the US financial variables in the global economy, the US-specific foreign financial variables, $q_{US,t}^*$, $R_{US,t}^{*S}$ and $R_{US,t}^{*L}$, were not included in the US model as they are unlikely to be long-run forcing with respect to the US domestic financial variables. The US-specific foreign output and inflation variables, $y_{US,t}^*$ and $\pi_{US,t}^*$, were, however, included in the US model (which were not included by PSW) in order to capture the possible second round effects of external shocks on the USA. Given the importance of the USA for the global economy, initially it was thought that the inclusion of $y_{US,t}^*$ and $\pi_{US,t}^*$ as weakly exogenous in the US model might result in the violation of the weak exogeneity assumption. However, as reported below this turns out not to be the case.

[7] Imbs (2004) also provides further evidence on the effect of trade on business cycle synchronization. He concludes that while specialization patterns have a sizeable effect on business cycles, trade continues to play an important role in this process. He also notes that economic regions with strong financial links are significantly more synchronized. Focusing on global linkages in financial markets, Forbes and Chinn (2004) also show that direct trade appears to be one of the most important determinants of cross-country linkages.

[8] For the construction of the euro area exchange rate, each of the country members' exchange rate was converted to an index using 2000 as the base year and pre-multiplied by the euro/dollar rate for that year.

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In this paper, as the focus is mainly on the impact of external shocks on the euro area economy, from now on we shall concentrate the presentation of the results on countries/regions with special relevance to the euro area: USA, China, Japan, euro area, UK and the rest of Western Europe. A more detailed set of results are available in Supplement B, available from the authors on request.

3.1. Trade and Aggregation Weights

The trade shares used to construct the country-specific foreign variables are given in the 26×26 trade share matrix provided in Supplement A. Table II presents the trade shares for our eight focus economies (seven countries plus the euro area itself, composed of eight countries), with a 'Rest' category showing the trade shares with the remaining 10 countries in our sample. First considering the euro area, we can see that the USA, the UK and the rest of Western Europe have a similar share in euro area trade (around 1/5) accounting together for almost two-thirds of total euro area trade. Other important information that emerges from the trade matrix includes the very high share of the euro area in the trade of the UK and the rest of Western Europe (more than half of the trade relationships of these countries are with euro area countries). Hence, these countries are key in the transmission of shocks to the euro area via a third market, or through second-round effects.

Although we estimate models at a country level (the euro area being considered here as a single economy), we also wish to derive regional responses to shocks. Hence, for the rest of Western Europe (and also for rest of Asia, Latin America, other developed countries and rest of the world), we will aggregate impulse response functions by using weights based on the PPP valuation of country GDPs, which are thought to be more reliable than weights based on US dollar GDPs.

3.2. Unit Root Tests

Although the GVAR methodology can be applied to stationary and/or integrated variables, here we follow PSW and assume that the variables included in the country-specific models are integrated of order one (or $I(1)$). This allows us to distinguish between short-run and long-run relations and interpret the long-run relations as cointegrating. Therefore, we begin by examining the integration

Table II. Trade weights based on direction of trade statistics

Country/region	USA	Euro area	China	Japan	UK	Rest of W. Europe			Rest[a]
						Sweden	Switz.	Norway	
USA	0.000	0.155	0.073	0.124	0.052	0.008	0.012	0.004	0.570
Euro area	0.227	0.000	0.056	0.072	0.238	0.057	0.090	0.028	0.230
China	0.236	0.164	0.000	0.248	0.029	0.010	0.007	0.003	0.304
Japan	0.319	0.132	0.128	0.000	0.032	0.007	0.009	0.003	0.369
UK	0.180	0.537	0.020	0.042	0.000	0.027	0.028	0.023	0.146
Sweden	0.104	0.517	0.025	0.035	0.115	0.000	0.017	0.099	0.089
Switz.	0.113	0.670	0.015	0.039	0.066	0.015	0.000	0.004	0.079
Norway	0.090	0.449	0.020	0.030	0.181	0.132	0.008	0.000	0.091

Note: Trade weights are computed as shares of exports and imports displayed in rows by region such that a row, but not a column, sums to one. [a] 'Rest' gathers the remaining countries. The complete trade matrix used in the GVAR model is given in Supplement B, which can be obtained from the authors on request. Source: *Direction of Trade Statistics*, 1999–2001, IMF.

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properties of the individual series under consideration. In view of the widely accepted poor power performance of traditional Dickey–Fuller (DF) tests, we report unit root t-statistics based on weighted symmetric estimation of ADF type regressions introduced by Park and Fuller (1995). These tests, henceforth *WS*, exploit the time reversibility of stationary autoregressive processes in order to increase their power performance. Leybourne *et al.* (2004) and Pantula *et al.* (1994) provide evidence of superior performance of the *WS* test statistic compared to the standard ADF test or the GLS-ADF test proposed by Elliot *et al.* (1996). The lag length employed in the *WS* unit root tests is selected by the Akaike Information Criterion (AIC) based on standard ADF regressions. The results of the *WS* statistics for the level, first differences and the second differences of all the country-specific domestic and foreign variables in the GVAR model can be found in Supplement A.

Real output, interest rates (short and long), exchange rates and real equity prices (domestic and foreign) are $I(1)$ across the focus countries, with two notable exceptions. First, real output in the UK appears borderline $I(0)/I(1)$ according to the *WS* statistics, although ADF tests indicate that UK real output is $I(1)$. Second, e^* in the US model is an $I(2)$ variable. As in PSW, we deal with this problem by including $(e - p)$ instead of the nominal exchange rate variable, e, in the different country-specific models. Unit root tests applied to $(e - p)$ and $(e^* - p^*)$ indicate that these variables are $I(1)$ in all cases. Finally, consumer price indices turn out to be $I(2)$, so that inflation (Δp and Δp^*) appears to be $I(1)$ across all countries. The test results also generally support the unit root hypothesis in the case of the variables for the remaining countries except for $(e - p)$ for Canada.

3.3. Specification and Estimation of the Country-Specific Models

We begin the modelling exercise under the assumption that the country-specific foreign variables are weakly exogenous $I(1)$ variables, and that the parameters of the individual models are stable over time. The latter allows us to estimate and test the long-run properties of the different country-specific models separately and consistently. Both assumptions are needed for an initial implementation of the GVAR model, and their validity will be examined in what follows.

We do not impose the same specification across the country-specific models. For the euro area, Japan, the UK and countries belonging to the rest of Western Europe, we include real output (y), inflation rate (Δp), short-term interest rate (ρ^S), long-term interest rate (ρ^L), real equity prices (q) and real exchange rate $(e - p)$ as endogenous variables and foreign real output (y^*), foreign inflation (Δp^*), foreign real equity prices (q^*), foreign interest rates (short, ρ^{*S}; and long, ρ^{*L}) and oil prices (p^o) as weakly exogenous variables. In the case of China, owing to data constraints, real equity prices and long-term interest rates are excluded from the set of endogenous variables. The US model contains y, Δp, ρ^S, ρ^L, q and oil prices (p^o), as the endogenous variables. The US dollar exchange rate is determined outside the US model. As in PSW, the only exchange rate included in the US model is the foreign real exchange rate variable, $(e^*_{US} - p^*_{US})$, which is treated as weakly exogenous. The inclusion of oil prices in the US model as endogenous allows the evolution of the global macro-economic variables to influence oil prices, a feature that was absent from the PSW version, which treated oil prices as weakly exogenous in all country-specific models. Furthermore, unlike the PSW version, the present specification includes US-specific foreign real output (y^*_{US}) and foreign inflation (Δp^*_{US}) as weakly exogenous variables. This allows for the US model to be more fully integrated in the world economy and hence to take a more satisfactory account of second round effects in the global economic system as a whole. It is, of course, important that the weak exogeneity of these variables in the US model are tested, and this is done below.

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Once the variables to be included in the different country models are specified, the corresponding cointegrating VAR models are estimated and the rank of their cointegrating space determined. Initially we select the order of the individual country VARX*(p_i, q_i) models. It should be noted that p_i, the lag order of the domestic variables, and q_i, the lag order of the foreign ('star') variables in the VARX* models, need not be the same. In the empirical analysis that follows we entertain the case where the lag order of the domestic variables, p_i, is selected according to the Akaike information criterion. Owing to data limitations, the lag order of the foreign variables, q_i, is set equal to one in all countries with the exception of the USA and the euro area. For the same reason, we do not allow $p_{\max i}$ or $q_{\max i}$ to be greater than two. We then proceed with the cointegration analysis, where the country-specific models are estimated subject to reduced rank restrictions. To this end, the error correction forms of the individual country equations given by (12) are derived.[9]

The orders of the VARX* models, the number of cointegration relationships and diagnostic test results for all the models are provided in Supplement B. In Table III we give the lag orders and the number of cointegrating relations for the set of focus countries. For most countries a VARX*(2, 1) specification seemed to be satisfactory. For the USA and the euro area, however, a VARX*(2, 2) was favoured by the AIC. As regards the number of cointegrating relationships, we find 4 for Japan, 3 for the UK, Sweden and Switzerland, 2 for the euro area, Norway and the USA and 1 for China. The cointegration results are based on the trace statistic (at the 95% critical value level), which is known to yield better small sample power results compared to the maximal eigenvalue statistic.

3.4. Testing Weak Exogeneity

As noted earlier, the main assumption underlying our estimation strategy is the weak exogeneity of $\mathbf{x}_{it}^*$ with respect to the long-run parameters of the conditional model defined by (13). Here we provide a formal test of this assumption for the country-specific foreign variables (the 'star' variables) and the oil prices.

Weak exogeneity is tested along the lines described in Johansen (1992) and Harbo *et al.* (1998). This involves a test of the joint significance of the estimated error correction terms in auxiliary

Table III. VARX* order and number of cointegration relationships in the country-specific models

Country	VARX*(p_i, q_i) p_i	q_i	# Cointegrating relationships
USA	2	2	2
Euro area	2	2	2
China	2	1	1
Japan	1	1	4
UK	2	1	3
Sweden	2	1	3
Switzerland	1	1	3
Norway	2	1	2

[9] The rank of the cointegrating space for each country/region was computed using Johansen's trace and maximal eigenvalue statistics as set out in Pesaran *et al.* (2000) for models with weakly exogenous $I(1)$ regressors, in the case where unrestricted constants and restricted trend coefficients are included in the individual country error correction models.

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equations for the country-specific foreign variables, $\mathbf{x}_{it}^*$. In particular, for each lth element of $\mathbf{x}_{it}^*$ the following regression is carried out:

$$\Delta x_{it,l}^* = \mu_{il} + \sum_{j=1}^{r_i} \gamma_{ij,l} ECM_{i,t-1}^j + \sum_{k=1}^{s_i} \varphi_{ik,l} \Delta \mathbf{x}_{i,t-k} + \sum_{m=1}^{n_i} \vartheta_{im,l} \Delta \widetilde{\mathbf{x}}_{i,t-m}^* + \varepsilon_{it,l}$$

where $ECM_{i,t-1}^j$, $j = 1, 2, \ldots, r_i$ are the estimated error correction terms corresponding to the r_i cointegrating relations found for the ith country model and $\Delta \widetilde{\mathbf{x}}_{it}^* = (\Delta \mathbf{x}_{it}^{\prime *}, \Delta(e_{it}^* - p_{it}^*), \Delta p_t^o)'$. Note that in the case of the USA the term $\Delta(e_{it}^* - p_{it}^*)$ is implicitly included in $\Delta \mathbf{x}_{it}^*$. The test for weak exogeneity is an F-test of the joint hypothesis that $\gamma_{ij,l} = 0$, $j = 1, 2, \ldots, r_i$ in the above regression. The lag orders s_i and n_i, need not be the same as the orders p_i and q_i of the underlying country-specific VARX* models. We carried out two sets of experiments, one set using the lag orders of the underlying VARX* models given in Table III, and in another set of experiments we set $s_i = p_i$ and $n_i = 2$ for all countries. In both cases the exogeneity hypothesis could not be rejected for most of the variables being considered. Under the former specification of the lag orders 8 out of 153 cases were found to be significant at the 5% level, while under the latter only 5 out of 153 exogeneity tests turned out to be statistically significant.[10] The test results for this case are summarized in Table IV.

For the set of focus countries, as can be seen from this table, the weak exogeneity assumptions are rejected only for output in the UK model. We would have been concerned if the weak exogeneity assumptions were rejected in the case of the US or the euro area models, for example. But as can be seen from Table IV, the weak exogeneity of foreign variables and oil prices are not rejected in the euro area model. Aggregation of the euro area countries in a single model could have violated the weak exogeneity assumptions that underlie GVAR modelling. However, the tests suggest that the foreign euro area-specific variables can be considered as weakly exogenous. The same applies to the foreign variables (y_{US}^*, Δp_{US}^*, $e_{US}^* - p_{US}^*$) included in the US model. As expected, foreign real equity prices and foreign interest rates (both short and long term) cannot be considered as weakly exogenous and have thus not been included in the US model.

Table IV. F-statistics for testing the weak exogeneity of the country-specific foreign variables and oil prices

Country		Foreign variables						
		y^*	Δp^*	q^*	ρ^{*S}	ρ^{*L}	p^o	$e^* - p^*$
USA	F(2, 75)	0.30	1.89	—	—	—	—	1.83
Euro area	F(2, 67)	0.06	0.00	2.25	0.20	1.98	2.04	—
China	F(1, 72)	1.66	0.48	1.30	1.00	1.30	0.19	—
Japan	F(4, 71)	1.36	1.38	0.32	0.46	0.73	1.68	—
UK	F(3, 66)	$2.98^\dagger$	0.63	0.07	1.11	1.34	0.57	—
Sweden	F(3, 66)	2.52	0.81	0.16	0.40	0.40	0.90	—
Switzerland	F(3, 72)	0.40	0.27	0.42	0.90	0.04	0.36	—
Norway	F(2, 67)	0.95	0.57	0.41	0.14	0.87	0.28	—

Note: † denotes statistical significance at the 5% level.

[10] Increasing the lag order further resulted in no statistically significant outcomes.

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3.5. Testing for Structural Breaks

The possibility of structural breaks is one of the fundamental problems facing econometric modelling. The problem is likely to be particularly acute in the case of emerging economies that are subject to significant political and social changes. The GVAR model is clearly not immune to this problem. Unfortunately, despite the great deal of recent research in this area, there is little known about how best to model breaks. Even if in-sample breaks are identified using Bayesian or classical procedures, there are insurmountable difficulties in allowing for the possibility of future breaks in forecasting and policy analysis. See, for example, Stock and Watson (1996), Clements and Hendry (1998, 1999) and Pesaran *et al.* (2006).

However, the fact that country-specific models within the GVAR framework are specified conditional on foreign variables should help in alleviating the structural problem somewhat. For example, suppose that univariate equity return equations are subject to breaks roughly around the same time in different economies. This could arise, for example, due to a stock market crash in the USA with strong spillover effects to the rest of the world. However, since equity return equations in the country-specific models are specified conditional on the US equity returns, they need not be subject to similar breaks, and in this example the structural break problem could be confined to the US model. This phenomenon is related to the concept of 'co-breaking' introduced in macroeconometric modelling by Hendry (1996), and examined further by Hendry and Mizon (1998). The structure of the GVAR can readily accommodate co-breaking and suggests that the VARX* models that underlie the GVAR might be more robust to the possibility of structural breaks as compared to reduced-form single-equation models considered, for example, by Stock and Watson (1996).

In the context of cointegrated models, structural stability is relevant for both the long-run coefficients and the short-run coefficients, as well as the error variances.[11] As our interest is in exploring the transmission mechanisms of the US and the euro area, we will not consider the stability of the long-run coefficients and rather focus on the structural stability of the short-run coefficients. In fact, given the limited number of time series data available, a meaningful test of the stability of the long-run coefficients might not be feasible. Also to render the structural stability tests of the short-run coefficients invariant to exact identification of the long-run relations, we consider structural stability tests that are based on the residuals of the individual equations of the country-specific error correction models. It is well known that these residuals only depend on the rank of the cointegrating vectors and do not depend on the way the cointegrating relations are exactly identified. Fluctuation tests based on successive parameter estimates which reject the null of parameter constancy when the estimates fluctuate too much, such as those proposed by Ploberger *et al.* (1989), will not be invariant to the identification of the long-run parameters and will not be considered here.

Among the tests included in our analysis are Ploberger and Krämer's (1992) maximal OLS cumulative sum (CUSUM) statistic, denoted by PK_{sup} and its mean square variant PK_{msq}. The PK_{sup} statistic is similar to the CUSUM test suggested by Brown *et al.* (1975), although the latter is based on recursive rather than OLS residuals. Also considered are tests for parameter constancy against non-stationary alternatives proposed by Nyblom (1989), denoted by $\mathfrak{N}$, as well as sequential Wald-type tests of a one-time structural change at an unknown change point. The latter include

[11] Tests of structural stability of the cointegrating vectors in VECM models have been considered by Quintos and Phillips (1993), Seo (1998) and Hansen and Johansen (1999), among others.

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the Wald form of Quandt's (1960) likelihood ratio statistic (*QLR*), the mean Wald statistic (*MW*) of Hansen (1992) and Andrews and Ploberger (1994) and the Andrews and Ploberger (1994) Wald statistic based on the exponential average (*APW*). The heteroskedasticity-robust version of the above tests is also presented.

Table V summarizes the results of the tests by variable at the 5% significance level. The critical values of the tests, computed under the null of parameter stability, are calculated using the sieve bootstrap samples obtained from the solution of the GVAR(p) model given by (17).[12] Note that the critical values employed in Stock and Watson (1996) are for the case of predetermined regressors and are therefore not applicable in the GVAR context.

The results vary across the tests and to a lesser extent across the variables. For example, using the *PK* tests (both versions) the null hypothesis of parameter stability is rejected at most 7 out of the possible maximum number of 134 cases, with the rejections spread quite evenly across the variables. Turning to the other three tests ($\mathfrak{R}$, *QLR* and *APW*) the outcomes very much depend on whether heteroskedasticity-robust versions of these tests are used. The results for the robust version are in line with those of the *PK* tests, although the rate of rejections are now in the range 9–10% rather than the 4–5% obtained in the case of the *PK* tests. Once possible changes in error variances are allowed for, the parameter coefficients seem to have been reasonably stable. At least based on the available tests, there is little statistical evidence with which to reject the hypothesis of coefficient stability in the case of 90% of the equations comprising the GVAR model. The non-robust versions of the $\mathfrak{R}$, *QLR* and *APW* tests, however, show a relatively large number of rejections, particularly the latter two tests that lead to rejection of the joint null hypothesis (coefficient and error variance stability) in the case of 60 (*QLR*) and 59 (*APW*) out of the 134 cases. In view of the test outcomes for the robust versions of these tests, the main reason for the rejection seem to be breaks in error variances and not the parameter coefficients. This conclusion

Table V. Number of rejections of the null of parameter constancy per variable across the country-specific models at the 5% level

Alternative test statistics	Domestic variables						Numbers(%)
	y	Δp	q	$e-p$	ρ^S	ρ^L	
PK_{sup}	0(0.0)	2(7.7)	3(15.8)	1(4.0)	0(0.0)	1(8.3)	7(5.2)
PK_{msq}	1(3.9)	1(3.9)	3(15.8)	0(0.0)	1(4.0)	1(8.3)	7(5.2)
$\mathfrak{R}$	0(0.0)	6(23.1)	4(21.1)	2(8.0)	7(28.0)	5(41.7)	24(17.91)
robust-N	1(3.9)	1(3.9)	3(15.8)	2(8.0)	3(12.0)	3(25.0)	13(9.7)
QLR	11(42.3)	9(34.6)	8(42.1)	9(36.0)	15(60.0)	7(58.3)	59(44.0)
robust-QLR	1(3.9)	2(7.7)	5(26.3)	2(8.0)	2(8.0)	1(8.3)	13(9.7)
MW	1(3.9)	8(30.8)	6(31.6)	6(24.0)	8(32.0)	6(50.0)	35(26.1)
robust-MW	2(7.7)	2(7.7)	2(10.5)	2(8.0)	2(8.0)	1(8.3)	11(8.2)
APW	11(42.3)	10(38.5)	6(31.6)	10(40.0)	15(60.0)	6(50.0)	58(43.3)
robust-APW	2(7.7)	1(3.9)	4(21.1)	2(8.0)	2(8.0)	1(8.3)	12(9.0)

Note: The test statistics PK_{sup} and PK_{msq} are based on the cumulative sums of *OLS* residuals, $\mathfrak{R}$ is the Nyblom test for time-varying parameters and *QLR, MW* and *APW* are the sequential Wald statistics for a single break at an unknown change point. Statistics with the prefix 'robust' denote the heteroskedasticity-robust version of the tests. All tests are implemented at the 5% significance level.

[12] Details of the bootstrap procedure and the mathematical expressions for the various test statistics are included in Supplement A, which is available upon request.

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is in line with many recent studies that find statistically significant evidence of changing volatility as documented, among others, by Stock and Watson (2002b), Artis *et al.* (2004) and Cecchetti *et al.* (2005).

Overall, not surprisingly there is evidence of structural instability but this seems to be mainly confined to error variances. We deal with the problem of possibly changing error variances by using robust standard errors when investigating the impact effects of the foreign variables, and base our analysis of impulse responses on the bootstrap means and confidence bounds rather than the point estimates.

3.6. Contemporaneous Effects of Foreign Variables on their Domestic Counterparts

Table VI presents the contemporaneous effects of foreign variables on their domestic counterparts together with robust t-ratios computed using White's heteroskedasticity-consistent variance estimator. These estimates can be interpreted as impact elasticities between domestic and foreign variables. Most of these elasticities are significant and have a positive sign, as expected. They are particularly informative as regards the international linkages between the domestic and foreign variables. Focusing on the euro area, we can see that a 1% change in foreign real output in a given quarter leads to an increase of 0.5% in euro area real output within the same quarter. Similar foreign output elasticities are obtained across the different regions.

We can also observe a high elasticity between long-term interest rates, ρ^L and ρ^{*L}, implying relatively strong co-movements between euro area and foreign bond markets. More importantly, the contemporaneous elasticity of real equity prices is significant and slightly above one. Hence, the euro area stock markets would seem to overreact to foreign stock price changes, although the extent of overreaction is not very large. Similar results are also obtained for Sweden and Norway.

Table VI. Contemporaneous effects of foreign variables on their domestic counterparts

Country	Domestic variables				
	y	Δp	q	ρ^S	ρ^L
USA	0.54	0.06	—	—	—
	[3.12]	[0.87]			
Euro area	0.53	0.25	1.15	0.09	0.63
	[4.03]	[3.31]	[8.90]	[3.84]	[7.86]
China	−0.10	0.61	—	0.12	—
	[−0.66]	[2.30]		[2.27]	
Japan	0.50	−0.04	0.67	−0.05	0.48
	[3.47]	[−0.38]	[5.53]	[−0.89]	[4.84]
UK	0.33	−0.15	0.84	0.27	0.67
	[2.33]	[−0.64]	[13.28]	[1.48]	[4.85]
Sweden	1.19	1.23	1.15	1.25	0.96
	[3.38]	[6.19]	[11.60]	[3.56]	[5.75]
Switzerland	0.47	0.52	0.70	0.16	0.41
	[3.81]	[3.68]	[2.17]	[3.10]	[5.88]
Norway	0.80	1.11	1.03	0.15	0.56
	[2.05]	[6.84]	[8.62]	[0.85]	[3.43]

Note: White's heteroskedastic-robust t-ratios are given in square brackets.

Contemporaneous financial linkages are likely to be very strong amongst the European economies through the equity and the bond market channels.

In contrast, we find rather low elasticities for inflation. For the euro area the foreign inflation elasticity is 0.25, suggesting that in the short run the euro area prices are not much affected by changes in foreign prices. The same is also true for the USA. For the remaining focus countries, with the exception of Japan and the UK, foreign inflation effects are much larger and are statistically significant.

Another interesting feature of the results are the very weak linkages that seem to exist across short-term interest rates (Sweden being an exception) and the high, significant relationships across long-term rates. This clearly shows a much stronger relation between bond markets than between monetary policy reactions.

4. PAIR-WISE CROSS-SECTION CORRELATIONS: VARIABLES AND RESIDUALS

One of the key assumptions of the GVAR modelling approach is that the 'idiosyncratic' shocks of the individual country models should be cross-sectionally 'weakly correlated', so that $\mathrm{cov}(\mathbf{x}_{it}^*, u_{it}) \to 0$, with $N \to \infty$, and as a result the weak exogeneity of the foreign variables is ensured. Direct tests of weak exogeneity assumptions discussed above indirectly support the view that the idiosyncratic shocks could only be weakly correlated. In this section we provide direct evidence on the extent to which this is likely to be true. The basic idea is similar to the cross-section dependence test proposed in Pesaran (2004). By conditioning the country-specific models on weakly exogenous foreign variables, viewed as proxies for the 'common' global factors, it is reasonable to expect that the degree of correlation of the remaining shocks across countries/regions will be modest. These residual interdependencies, as mentioned in Section 2, could reflect policy and trade spillover effects.

As a simple diagnostic of the extent to which the country-specific foreign variables have been effective in reducing the cross-section correlation of the variables in the GVAR model, we have computed average pair-wise cross-section correlations for the levels and first differences of the endogenous variables of the model, as well as those of the associated residuals over the estimation period, 1979–2003. We also computed average pair-wise cross-section correlations of the residuals obtained after re-estimating all of the individual country-specific models over the same period excluding the foreign (star) variables, including oil as endogenous in all the country models.[13] The results for all variables are summarized in Table VII.

The average cross-section correlations are generally high for the level of the endogenous variables and fall as first differences of these variables are considered. The results vary widely across variables and less so across countries, with inflation and real exchange rate for China being the exceptions. Output levels, sharing common trends, show the highest degree of cross-section correlations, of around 92–96%. This is followed by long-term interest rates (61–81%), real equity prices (35–61%), and short-term interest rates (32–52%). The effect of first differencing on cross-section correlations differ widely over variables as well as countries, and is most pronounced in the case of the output series. Average cross-section correlations of output changes, Δy_{it}, range between 1% for China to 16% for the USA, as compared to cross-section correlations of output

[13] For each country model we used the same VAR order as that specified in Table III and selected the number of cointegrating relationships based on Johansen's trace statistic computed for the individual VAR models (excluding the star variables).

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Table VII. Average pair-wise cross-section correlations of all variables and associated model's residuals

Country	Real output				Inflation			
	Levels	1st diff.	VAR Residuals	VARX* Residuals	Levels	1st diff.	VAR Residuals	VARX* Residuals
USA	0.96	0.16	0.05	−0.05	0.39	0.13	0.16	0.02
Euro area	0.96	0.14	0.12	−0.01	0.39	0.13	0.14	0.00
China	0.96	0.01	−0.01	−0.02	0.04	0.04	0.05	0.01
Japan	0.92	0.03	−0.03	−0.08	0.30	0.01	0.04	0.02
UK	0.95	0.11	0.07	0.01	0.34	0.03	0.10	0.02
Sweden	0.96	0.08	0.07	0.02	0.37	0.06	0.11	−0.01
Switz.	0.93	0.13	0.08	0.01	0.30	0.09	0.11	0.04
Norway	0.96	0.09	0.06	0.01	0.32	0.08	0.11	0.02

Country	Real equity prices				Real exchange rate			
	Levels	1st diff.	VAR Residuals	VARX* Residuals	Levels	1st diff.	VAR Residuals	VARX* Residuals
USA	0.58	0.40	0.34	−0.01	—	—	—	—
Euro area	0.58	0.43	0.39	−0.08	0.62	0.30	0.27	0.28
China	—	—	—	—	−0.19	0.07	0.05	0.03
Japan	0.35	0.31	0.21	−0.09	0.59	0.23	0.18	0.15
UK	0.61	0.41	0.37	−0.03	0.61	0.27	0.22	0.19
Sweden	0.57	0.39	0.36	−0.02	0.59	0.28	0.21	0.20
Switz.	0.54	0.26	0.19	−0.05	0.62	0.27	0.25	0.27
Norway	0.61	0.37	0.33	0.02	0.61	0.31	0.27	0.27

Country	Short-term interest rate				Long-term interest rate			
	Levels	1st diff.	VAR Residuals	VARX* Residuals	Levels	1st diff.	VAR Residuals	VARX* Residuals
USA	0.39	0.09	0.04	0.00	0.75	0.38	0.30	−0.04
Euro area	0.49	0.15	0.08	0.02	0.78	0.44	0.34	−0.05
China	0.32	0.03	0.01	−0.02	—	—	—	—
Japan	0.48	0.05	0.03	−0.01	0.78	0.27	0.26	−0.04
UK	0.52	0.12	0.09	0.00	0.79	0.37	0.29	−0.01
Sweden	0.46	0.03	0.04	−0.01	0.81	0.36	0.28	0.06
Switz.	0.33	0.09	0.08	0.00	0.61	0.36	0.31	0.02
Norway	0.40	0.02	0.02	0.00	0.73	0.26	0.19	0.03

Note: VAR residuals are based on cointegrating VAR models with domestic variables only and oil prices. VARX* residuals refer to the country models with country-specific foreign variables.

levels of 96% for both of these economies. Similar outcomes are also observed in the case of inflation and short-term interest rates. By comparison, first differencing of equity prices and long-term interest rates has only limited effect on cross-section correlations. For example, the average cross-section correlations of equity prices fall from 35–61% to 26–43% as one moves from levels of equity prices to their first differences. Overall, there is significant evidence of cross-country correlations for the variables in the GVAR model, although the extent of this correlation depends on the variable, whether it is transformed to stationarity by first differencing, and the country.

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Turning to the cross-section correlation of the residuals from the VARX* models (including domestic and foreign star variables), it is quite striking that except for real exchange rates these correlations are very small and do not depend on the choice of the variable or country. This is particularly apparent in the case of the equity and bond markets where the cross-section correlation of the residuals ranges between −8% and +6%, as compared to the values in the range 35% and 81% (or 26% and 44%) if cross-section correlations of the levels (or first differences) are considered. The model has clearly been successful in capturing the common effects driving bond and equity markets. The real exchange rate variable presents an important exception which requires further consideration.

With regard to the cross-section correlations of the residuals from the individual country models that include only the domestic variables, their value appears to lie between that of the first-differenced variables and the residuals from the VARX* models. Exceptions are noted in the case of inflation, where the correlations of the residuals from the individual country models excluding the star variables are slightly higher than those based on the first-differenced variables, and for the real exchange rates where the correlations of the residuals from the VARX* models and VAR models (excluding the star variables) are virtually identical.

Overall, the cross-section correlation results show the importance of country-specific variables in dealing with often significant dependencies that exist across macro-economic variables. Although these results do not constitute a formal statistical test of the importance of the foreign variables in the GVAR model, they do provide an important indication of their usefulness in modelling global interdependencies. The results also show that once country-specific models are formulated conditional on foreign variables, there remains only a modest degree of correlations across the shocks from different regions.

5. ROBUSTNESS OF THE GVAR RESULTS TO TIME-VARYING WEIGHTS

The preceding analysis was carried out using fixed trade weights, on the grounds that changes in trade weights tend to be rather gradual and secular changes in trade weights are often counteracted by the co-movements of the macro-economic variables, so that the foreign-specific variables computed using fixed and variable trade weights are often very close. To check the robustness of our results to the choice of trade weights we also estimated the GVAR model using rolling three-year moving averages of the annual trade weights.[14] But before discussing some of these results it would be instructive first to provide some evidence on the relationship of the two measures, $\mathbf{x}_{it}^{*}$ (based on fixed weights) and $\mathbf{x}_{it}^{**}$ (based on the time-varying weights). Since both measures are likely to be I(1), in Table VIII we provide the correlation coefficients between the levels as well as the first differences of the two measures. The two measures ($\mathbf{x}_{it}^{*}$ and $\mathbf{x}_{it}^{**}$) are very highly correlated, with the correlation coefficients of the levels being very close to unity. In terms of first differences, the correlations are not as high, particularly in the case of inflation rates and to some extent the real exchange rates. Given these results, it seems unlikely that the main conclusions of the paper would be much affected by whether fixed or time-varying trade weights are used.

To check this conjecture we re-estimated the GVAR model, allowing for p_i in the individual country VARX* models to be unrestricted and q_i to be the same as in the fixed-weights case, and

[14] The process of computing time-varying trade weights was initialized by using the same set of weights for the first three years of the sample period.

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Table VIII. Correlation coefficients of country-specific foreign variables using fixed and time-varying trade weights

Country	Output		Inflation		Real equity prices	
	Levels	1st diff.	Levels	1st diff.	Levels	1st diff.
USA	1.00	0.92	0.89	0.70	1.00	1.00
Euro area	1.00	0.91	0.87	0.52	1.00	1.00
China	1.00	0.92	0.42	0.03	0.99	0.98
Japan	1.00	0.88	0.78	0.53	0.99	0.99
UK	1.00	0.99	0.95	0.75	1.00	1.00
Sweden	1.00	0.98	0.89	0.62	1.00	1.00
Switz.	1.00	0.99	0.90	0.60	1.00	1.00
Norway	1.00	0.99	0.91	0.64	1.00	1.00

Country	Short-term interest rates		Long-term interest rates		Real exchange rates	
	Levels	1st diff.	Levels	1st diff.	Levels	1st diff.
USA	0.98	0.96	1.00	0.99	0.97	0.85
Euro area	0.99	0.99	1.00	1.00	0.80	0.85
China	0.96	0.93	0.99	0.94	0.79	0.64
Japan	0.99	0.99	1.00	0.99	0.87	0.62
UK	1.00	0.99	1.00	1.00	0.98	0.97
Sweden	0.99	0.98	1.00	1.00	0.99	0.99
Switz.	0.99	0.98	1.00	1.00	1.00	0.99
Norway	1.00	0.98	1.00	0.99	0.99	0.95

obtained very similar numbers of cointegrating relations. The differences between the two sets of results were Japan (3 cointegrating relations as compared to 4 obtained when fixed weights were used) and Sweden (2 instead of 3). We obtained the same number of cointegrating relations for the remaining countries, except for Argentina, Mexico, Australia, Indonesia, Philippines and Singapore, where the number of cointegrating relations was decreased by one, and for Chile where the number of cointegrating relations was increased by one.

Turning to the impact effects of the foreign variables, once again we obtain very similar results, particularly in the case of real equity prices and long-term interest rates. The results for output effects are also very close, with the exception of the estimates obtained for Japan and Norway. Not surprisingly, the results are affected most in the case of China, where none of the estimates based on the time-varying weights are now statistically significant, as compared to the statistically significant estimates obtained when using the fixed weights. Similar conclusions are also reached if one considers average pair-wise cross-section correlations of the residuals or the impulse responses (to be reported below) under the two weighting schemes.[15]

6. GENERALIZED IMPULSE RESPONSE FUNCTIONS

To study the dynamic properties of the global model and to assess the time profile of the effects of shocks to foreign variables on the euro area economy, we investigate the implications of three different external shocks: (a) a one standard error negative shock to US real equity prices; (b) a

[15] Estimation details for the GVAR model using the time-varying weights are available on request.

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one standard error positive shock to US interest rates; and (c) a one standard error positive shock to oil prices. Here we make use of the Generalized Impulse Response Function (GIRF), proposed in Koop *et al.* (1996), developed further in Pesaran and Shin (1998) for vector error-correcting models.[16] The GIRF is an alternative to the Orthogonalized Impulse Responses (OIR) of Sims (1980). The OIR approach requires the impulse responses to be computed with respect to a set of orthogonalized shocks, while the GIR approach considers shocks to individual errors and integrates out the effects of the other shocks using the observed distribution of all the shocks without any orthogonalization. Unlike the OIR, the GIRF is invariant to the ordering of the variables and the countries in the GVAR model, which is clearly an important consideration. Even if a suitable ordering of the variables in a given country model can be arrived at from economic theory or general *a priori* reasoning, it is not clear how to order countries in the application of the OIR to the GVAR model.

In the absence of strong *a priori* beliefs on ordering of the variables and/or countries in the GVAR model, the GIRFs provide useful information with respect to changes in oil prices, equity prices and even interest rates. Although the approach is silent as to the reasons behind the changes, the GIRFs can be quite informative about the dynamics of the transmission of shocks from the rest of the world to the euro area.

In the discussion of the results, we focus only on the first two years following the shock. This seems a reasonable time horizon over which the model presents credible results. However, in Figures 1–5 we provide results over a longer period, partly as visual aids for the analysis of the model's convergence properties. The figures display the bootstrap estimates of the GIRFs and their associated 90% confidence bounds. The computations are carried out using the same sieve bootstrap procedure discussed above in the case of the structural stability tests.

The figures show that the GIRFs settle down reasonably quickly, suggesting that the model is stable. This is supported by the eigenvalues of the GVAR model, which are 268 in total.[17] From the individual country models and the theorem in PSW we do not expect the rank of the cointegrating matrix in the global model to exceed 63 (namely the number of cointegrating relations in all the individual country models). Hence, the global system should have at least 71 eigenvalues (i.e., 134–63) that fall on the unit circle. The GVAR satisfies these properties and indeed has 71 eigenvalues equal to unity, with the remaining 198 eigenvalues having moduli all less than unity.[18]

6.1. Shock to US Equity Prices

Consider first the GIRFs for a one standard error negative shock to US equity prices. This shock is equivalent to a fall of around 4–5% in US real equity prices per quarter. The equity price shock is accompanied by a decline in US real GDP of around 0.1% on impact, by 0.4% on average over the first year and by 0.5% on average over the second year (see Figure 1). The transmission of

[16] For an account of the GIRF applied to VARX and cointegrating VAR models see Garratt *et al.* (2006, Chs 6 and 10).

[17] The GVAR contains 134 endogenous variables with a maximum lag order of 2, which give rise to a companion VAR(1) model in 268 variables.

[18] Of these 197 eigenvalues, 128 (64 pairs) are complex, introducing cyclical features in the impulse responses. The eigenvalues with the largest complex part are $0.030098 \pm 0.724360i$, $0.141048 \pm 0.615778i$ and $-0.403860 \pm 0.593683i$, where $i = \sqrt{-1}$. After the unit roots, the three largest eigenvalues (in moduli) are 0.907389, 0.884077 and 0.879361, implying a reasonable rate of convergence of the model after a shock to its long-run equilibrium. Given the unit eigenvalues of the system, some shocks will have permanent effects on the levels of the endogenous variables.

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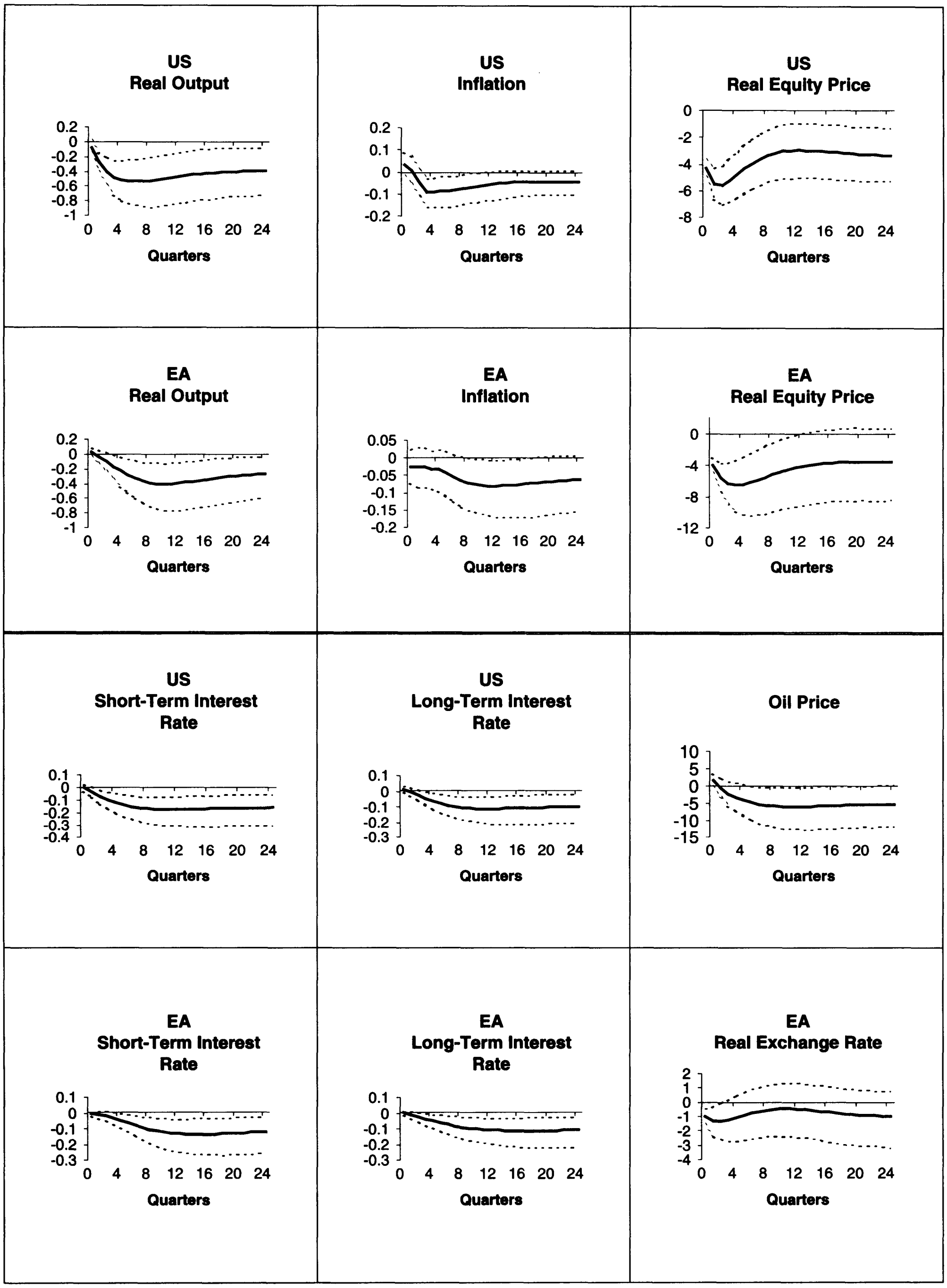

Figure 1. Generalized impulse responses of a negative unit (1 s.e.) shock to US real equity prices (bootstrap mean estimates with 90% bootstrap error bounds)

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the shock to the euro area equity markets takes place rather quickly and the effects of the shock are generally statistically significant. On impact, equity prices fall by similar amounts (around 4.0%) in both the USA and the euro area, but the effects of the US shock on the euro area equity markets become more pronounced over the first two years, suggesting a mild overreaction of equity prices in the European markets to the US shock. This partly reflects the higher volatility of the European equity markets as compared to the volatility of the S&P 500 used as the market index for the USA.

Like in the USA, real output in the euro area is negatively affected by the adverse equity shock, although to a lesser extent. Inflation tends to decrease although the magnitude of the reaction remains limited. As to be expected, short-term and long-term interest rates are also negatively affected by the shock. The impact of the shock on the short-term rate is stronger in the USA than in the euro area, which may be related to the different reaction functions of monetary authorities to asset price movements in these economies. Finally, real exchange rates in the euro area appreciate, although the effects cease to be statistically significant after the first two or three quarters.

6.2. Shock to Oil Prices

Figure 2 presents the GIRFs of a positive one standard error shock to oil prices on the USA and the euro area. A one standard error positive shock results in a 10–11% increase per quarter in the price of oil.

On impact the oil price shock has a negative effect on real output in the USA, and for the first couple of quarters on real output in the euro area. However, these effects are not statistically significant. In contrast, the effects of the oil price shock on inflation are unambiguously positive and statistically significant in both the USA and the euro area. The effects are stronger on the US inflation as compared to the euro area inflation, which is consistent with what we observe on the real side, and is in line with a rise in short-term interest rates, triggered in turn by increased inflationary pressures.

As regards the financial variables, not surprisingly the increase in oil prices adversely affects equity prices and places an upward pressure on the long-term interest rates. The increase in long-term interest rates shows that the bond markets tend to react more to inflation expectations rather than to the growth prospects. Bond and equity market reactions are consistent with each other and are common to both regions. The euro area real exchange rate, however, does not react to the oil price shock, and the associated GIRFs are not statistically significant.

6.3. Shock to US Short-Term Interest Rate

The GIRFs results of a positive one standard error shock to US short-term interest rates are displayed in Figure 3. In the USA, the one standard error positive shock to the interest rate equation amounts to a 0.2% increase in the short-term rate (i.e., around 80 basis points), measured on a quarterly basis.

The effects of the shock on real output and inflation are generally uncertain, particularly in the case of the euro area. Initially, the shock raises output and inflation in the USA, which are counter-intuitive, but these responses become statistically insignificant after one or two quarters. The positive impact effects of the interest rate shock on the US inflation is reminiscent of the price puzzle observed by a number of researchers working with VAR models of the USA (Sims, 1992;

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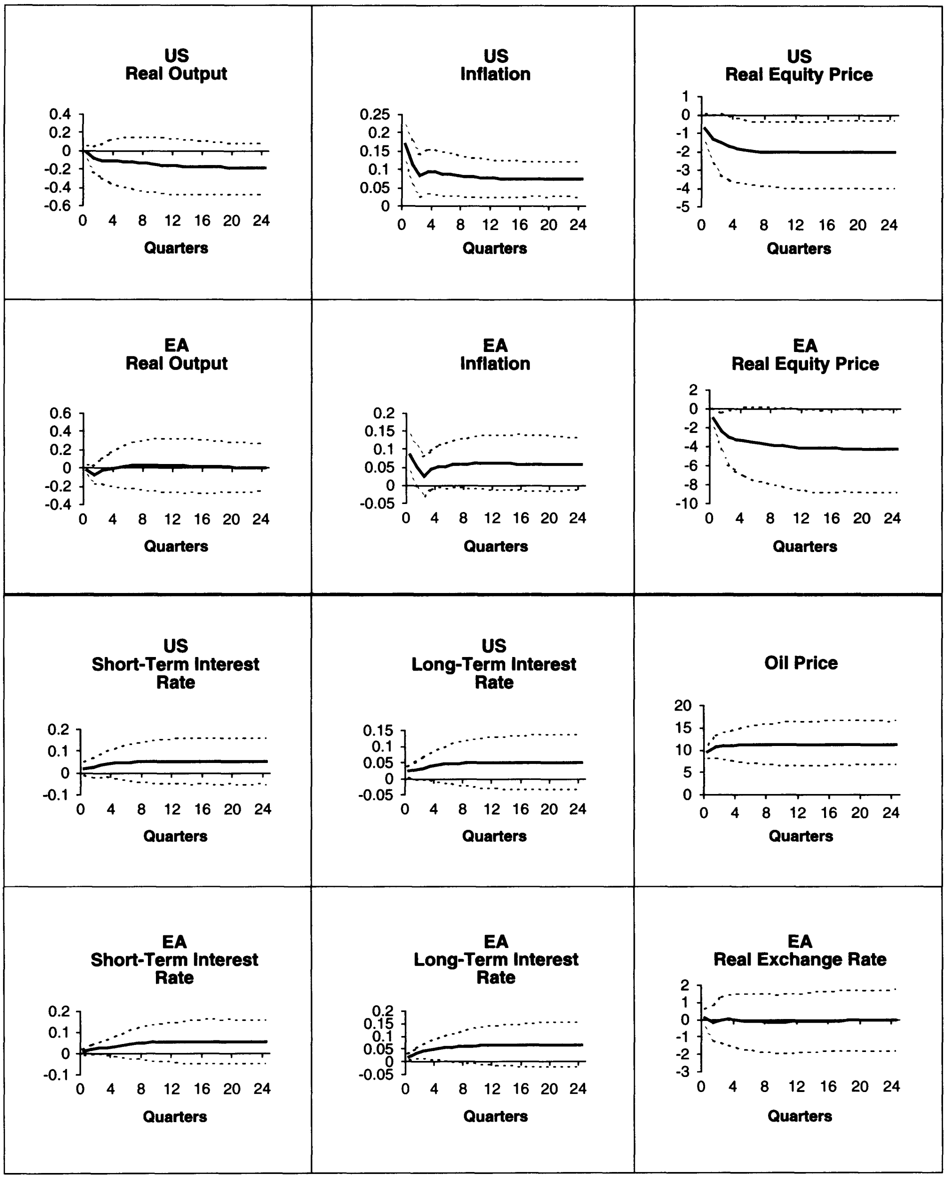

Figure 2. Generalized impulse responses of a positive unit (1 s.e.) shock to oil prices in the US model (bootstrap mean estimates with 90% bootstrap error bounds)

Eichenbaum, 1992). But the reappearance of the puzzle in the GVAR context is somewhat more surprising considering the many other transmission channels that are included. We shall return to this issue when we consider the impulse responses of structural US monetary policy shocks below. However, note that the effects of the US interest shock on the euro area output and inflation are very small and statistically insignificant at all horizons.

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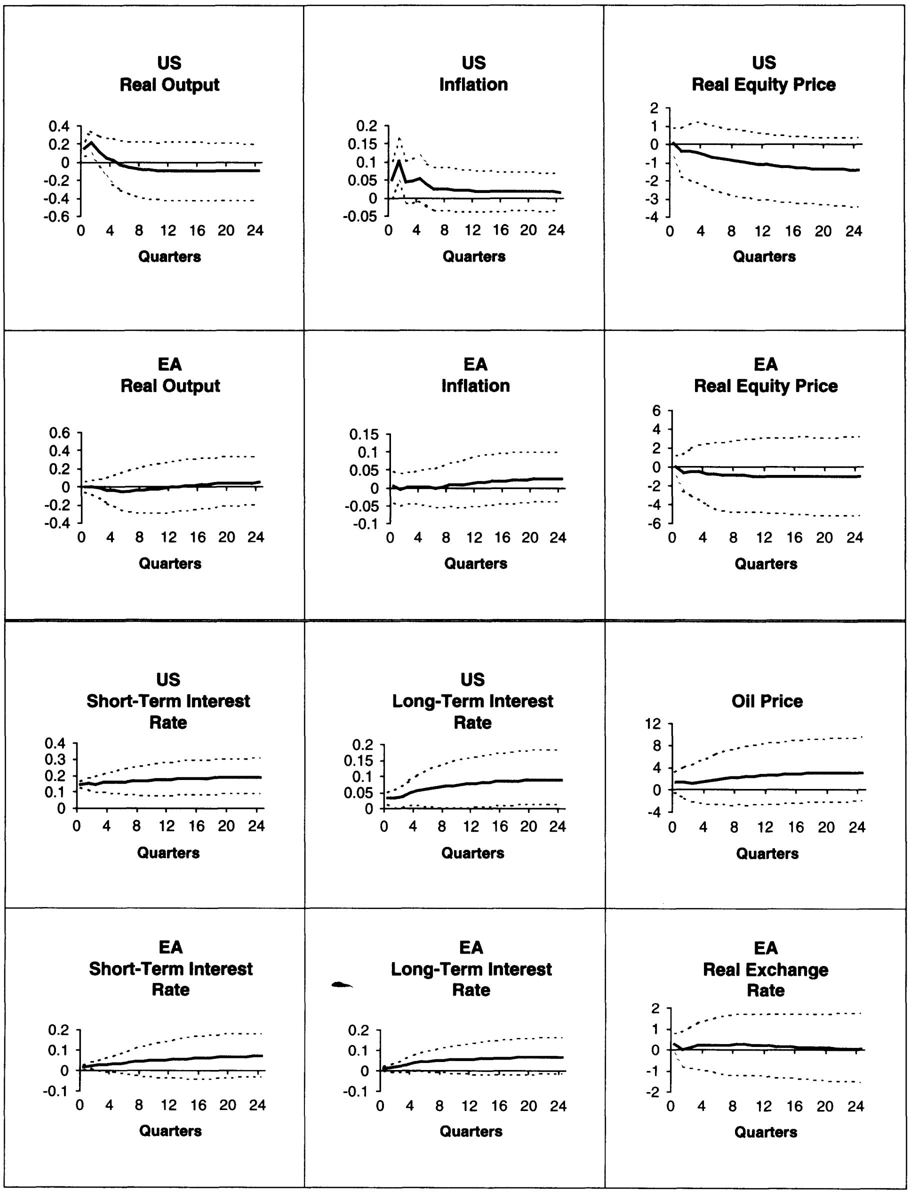

Figure 3. Generalized impulse response of a positive unit (1 s.e.) shock to US short-term interest rates (bootstrap mean estimates with 90% bootstrap error bounds)

The effects of the shock on long-term interest rates are, however, positive and statistically significant for most horizons in the case of the US rate, and for the initial few periods in the case of the long-term rate in the euro area. By contrast, the shock has sustained significant effects on the US short-term rate, but not on the short-term rate in the euro area, reflecting the weak interdependence of the short-term rates across the two regions and the stronger interdependence

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of the long-term rates globally. The interest rate shock has the expected negative effects on the real equity prices, but these effects are not statistically significant. The same also applies to the effects of the interest rate shock on oil prices and the euro area real exchange rates.

6.4. Global Shocks

So far we have considered the effects of variable/country-specific shocks, with particular emphasis on the shocks originating from the USA viewed possibly as global shocks, considering the dominant role of the USA in the world economy. While such a strategy might be appropriate in the case of shocks to the US equity market, it might be less defensible for other types of shocks. Therefore, it might be desirable to consider the effects of 'global' shocks which might not necessarily originate from a particular country, but could be common to the world economy as a whole. Examples of such shocks include major developments in technology or global shocks to commodity or equity markets. Apart from explicitly including global effects, such as oil prices, in the GVAR model, it is also possible to consider the effects of global shocks defined as a weighted average of variable-specific shocks across all the countries in the model. To see how this can be done consider the GVAR model (17), and abstracting from deterministic terms and higher order lags write it as

$$\mathbf{G}\mathbf{x}_t = \mathbf{H}\mathbf{x}_{t-1} + \cdots + \mathbf{u}_t, \mathbf{u}_t \sim IID(\mathbf{0}, \boldsymbol{\Sigma}_u) \tag{20}$$

A global shock can be defined as a weighted average of shocks to the same variable in all $N+1$ countries, using a set of weights reflecting the relative importance of the individual countries in the world economy. For example, using *PPP* GDP weights a global shock to the lth variable can be defined as $u^g_{\ell t} = \mathbf{a}'_\ell \mathbf{u}_t$, where a_ℓ is a $k \times 1$ selection vector, $a_\ell = (a'_{0\ell}, a'_{1\ell}, \ldots, a'_{N\ell})'$ and $a_{i\ell}$ is the $k_i \times 1$ vector with zero elements, except for its element that corresponds to the lth variable, which is set equal to w_i, the weight of the ith country in the world economy. By construction $\sum_{i=0}^{N} w_i = 1$.

The GIRF in the case of a one standard error global shock is given by

$$\psi(h, \mathbf{x} : u^g_\ell) = E(\mathbf{x}_{t+h} | \Omega_{t-1}, u^g_{\ell t} = \sqrt{\mathbf{a}'_\ell \boldsymbol{\Sigma}_u \mathbf{a}_\ell}) - E(\mathbf{x}_{t+h} | \Omega_{t-1})$$

and in the case of the above GVAR model is easily seen to be

$$\psi(0, \mathbf{x} : u^g_\ell) = \frac{\mathbf{G}^{-1} \boldsymbol{\Sigma}_u \mathbf{a}_\ell}{\sqrt{\mathbf{a}'_\ell \boldsymbol{\Sigma}_u \mathbf{a}_\ell}} \tag{21}$$

The effect of a one standard error global shock on expected values of x at time $t+h$, for $h = 1, 2, \ldots$ can then be obtained recursively by using (21) and solving forward in the light of the difference equation (20).

GIRFs of the impacts of real equity price and output shocks on the main variables are provided in Supplement B. In the case of the global equity shock, the results are very similar to those of a shock to the US equity prices discussed above. This result confirms the predominant role of the US stock market in the equity price developments across countries. In the case of the global output shock, beyond the fact that the USA is relatively less affected (since the shock hits all countries at the same time), the results are broadly similar when compared with those of the shock to US

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real output. The main difference concerns the impacts of the global output shock on real exchange rates, which tend to depreciate *vis-à-vis* the US dollar, while they appreciate in most cases when the shock originates in the USA.

7. IDENTIFICATION OF SHOCKS USING THE GVAR MODEL

Identification of all the 134 different shocks (the total number of endogenous variables) in the GVAR model will be a formidable undertaking, and might not be necessary since in practice monetary policy, demand and supply shocks are likely to be highly correlated across countries. In what follows we focus on identification of shocks to the US economy, particularly the monetary policy shocks, and consider the time profiles of their effects on the euro area.

7.1. Methodology

We include the US model as the first country model and, following Sims (1980), consider alternative orderings of the variables within the US model. The outcome of this exercise will be invariant to the ordering of the rest of the variables in the GVAR model, so long as the contemporaneous correlations of these shocks are left unrestricted (both in relation to themselves and with respect to the US shocks). Ordering of the rest of the variables in the GVAR model will be important for the analysis of the US monetary policy shocks, only if short-run over-identifying restrictions are imposed on the parameters of the models.

In the light of the arguments advanced in Sims and Zha (1998), one possible identification scheme for the US pursued below is to adopt the ordering of the variables in the US model as follows: $\mathbf{x}_{0t}$ = (oil, short-term interest rate, long-term interest rate, equity prices, inflation, output). We denote this ordering by $\mathbf{x}_{0t}^{A}$. It is also assumed that the variance matrix of the structural errors (ε_{0t}) associated with these variables are orthogonal. In terms of ε_{0t} a $VARX^*(1,1)$ model for the USA can be written as

$$\mathbf{P}_0\mathbf{x}_{0t} = \mathbf{P}_0\mathbf{\Phi}_0\mathbf{x}_{0,t-1} + \mathbf{P}_0\mathbf{\Psi}_{01}\mathbf{x}_{0t}^{*} + \mathbf{P}_0\mathbf{\Psi}_{02}\mathbf{x}_{0,t-1}^{*} + \mathbf{P}_0\mathbf{u}_{0t}$$

where $\varepsilon_{0t} = \mathbf{P}_0\mathbf{u}_{0t}$ are the structural shocks, and $\mathbf{P}_0$ is a $k_0 \times k_0$ matrix of coefficients to be identified. The identification conditions à la Sims (1980) require $\mathbf{\Sigma}_{\varepsilon 0} = \text{cov}(\varepsilon_{0t})$ to be diagonal and $\mathbf{P}_0$ to be lower triangular. Let $\mathbf{Q}_0$ to be the upper Cholesky factor of $\text{cov}(\mathbf{u}_{0t}) = \mathbf{\Sigma}_{u0} = \mathbf{Q}_0'\mathbf{Q}_0$ so that $\mathbf{\Sigma}_{\varepsilon 0} = \mathbf{P}_0\mathbf{\Sigma}_{u0}\mathbf{P}_0'$, and hence

$$\mathbf{P}_0\mathbf{Q}_0' = \mathbf{\Sigma}_{\varepsilon 0}^{1/2}, \text{ a diagonal matrix} \tag{22}$$

Consider now the GVAR model (20) and pre-multiply it by

$$\mathbf{P}_G^0 = \begin{pmatrix} \mathbf{P}_0 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} \end{pmatrix} \tag{23}$$

to obtain

$$\mathbf{P}_G^0\mathbf{G}\mathbf{x}_t = \mathbf{P}_G^0\mathbf{H}\mathbf{x}_{t-1} + \cdots + \varepsilon_t$$

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where $\varepsilon_t = (\varepsilon'_{0t}, \mathbf{u}'_{1t}, \ldots, \mathbf{u}'_{Nt})'$ and

$$\boldsymbol{\Sigma}_\varepsilon = \operatorname{cov}(\varepsilon_t) = \begin{pmatrix} V(\varepsilon_{0t}) & \operatorname{cov}(\varepsilon_{0t}, \mathbf{u}_{1t}) & \cdots & \operatorname{cov}(\varepsilon_{0t}, \mathbf{u}_{Nt}) \\ \operatorname{cov}(\mathbf{u}_{1t}, \varepsilon_{0t}) & V(\mathbf{u}_{1t}) & \cdots & \operatorname{cov}(\mathbf{u}_{1t}, \mathbf{u}_{Nt}) \\ \vdots & \vdots & & \vdots \\ \operatorname{cov}(\mathbf{u}_{Nt}, \varepsilon_{0t}) & \operatorname{cov}(\mathbf{u}_{Nt}, \mathbf{u}_{1t}) & \cdots & V(\mathbf{u}_{Nt}) \end{pmatrix} \tag{24}$$

with

$$V(\varepsilon_{0t}) = \boldsymbol{\Sigma}_{\varepsilon,00} = \mathbf{P}_0 \boldsymbol{\Sigma}_{u,00} \mathbf{P}'_0,$$
$$\operatorname{cov}(\varepsilon_{0t}, \mathbf{u}_{jt}) = \operatorname{cov}(\mathbf{P}_0 \mathbf{u}_{0t}, \mathbf{u}_{jt}) = \mathbf{P}_0 \boldsymbol{\Sigma}_{u,0j}$$

Generalized impulse responses with respect to the structural shocks are now defined as

$$\psi(h, \mathbf{x} : \varepsilon) = E(\mathbf{x}_{t+h} | \Omega_{t-1}, \mathbf{e}'_i \varepsilon_t = \sqrt{\mathbf{e}'_i \boldsymbol{\Sigma}_\varepsilon \mathbf{e}_i}) - E(\mathbf{x}_{t+h} | \Omega_{t-1})$$

But the contemporaneous effects are

$$\mathbf{P}^0_G \mathbf{G} E(\mathbf{x}_t | \Omega_{t-1}, \mathbf{e}'_i \varepsilon_t = \sqrt{\mathbf{e}'_i \boldsymbol{\Sigma}_\varepsilon \mathbf{e}_i}) = \mathbf{P}^0 \mathbf{H} \mathbf{x}_{t-1} + \frac{\boldsymbol{\Sigma}_\varepsilon \mathbf{e}_i}{\sqrt{\mathbf{e}'_i \boldsymbol{\Sigma}_\varepsilon \mathbf{e}_i}}$$

where $\mathbf{e}_i$ is a selection vector applied to all the elements of $\mathbf{x}_t$. Thus, the contemporaneous effects are given by

$$\psi(0, \mathbf{x} : \varepsilon_0) = \frac{(\mathbf{P}^0_G \mathbf{G})^{-1} \boldsymbol{\Sigma}_\varepsilon \mathbf{e}_i}{\sqrt{\mathbf{e}'_i \boldsymbol{\Sigma}_\varepsilon \mathbf{e}_i}} = \frac{\mathbf{G}^{-1} (\mathbf{P}^0_G)^{-1} \boldsymbol{\Sigma}_\varepsilon \mathbf{e}_i}{\sqrt{\mathbf{e}'_i \boldsymbol{\Sigma}_\varepsilon \mathbf{e}_i}}$$

The impulse responses for other horizons can be derived using the same recursive relations used for the computation of the generalized impulse responses.

Under the orthogonalization scheme, $\boldsymbol{\Sigma}_\varepsilon$, defined by (24), is specified as $V(\varepsilon_{0t}) = \mathbf{I}_{k_0}$, and

$$\operatorname{cov}(\varepsilon_{0t}, \mathbf{u}_{jt}) = \mathbf{P}^0_G \boldsymbol{\Sigma}_{u,0j}, \text{for} j = 1, 2, \ldots, N \tag{25}$$

Under this specification, using (22) we have $\mathbf{P}_0 = (\mathbf{Q}'_0)^{-1}$ and hence $(\mathbf{P}^0_G)^{-1}$ is a block diagonal matrix with $\mathbf{Q}'_0$ on its first block and identity matrices on the remaining blocks. This covariance specification given ensures that the impulse responses of structural shocks to the US economy will be invariant to any reordering of the variables in the rest of the GVAR model.[19] Also the structural impulse responses of the shocks to the oil prices (the first variable in the *VARX** model of the USA) will be the same as the corresponding generalized impulse responses (see Pesaran and Shin, 1998).

[19] Setting $\operatorname{cov}(\varepsilon_{0t}, \mathbf{u}_{jt}) = 0$ as an alternative option can also be entertained. This covariance specification imposes further restrictions and should be used with care.

7.2. US Monetary Policy Shocks

We consider identification of a US monetary policy shock under two different orderings of the variables in the US model, namely Sims and Zha type ordering $\mathbf{x}_{0t}^{A} =$ (oil, short-term interest rate, long-term interest rate, equity prices, inflation, output) discussed above, and the alternative ordering B, $\mathbf{x}_{0t}^{B} =$ (oil, long-term interest rate, equity prices, inflation, output, short-term interest rate), where the monetary policy variable is placed last, after inflation and output.[20] The impulse responses associated with these two identification schemes are displayed in Figures 4 and 5, respectively.

The results are similar across the two orderings, and are not that different from the GIRF outcomes in Figure 3. The main differences are in the effects on US long-term rate, output and inflation, particularly over the first three or four quarters after the shock. Out of the two orderings, A and B, the effects of the latter are more pronounced and differ more markedly from the 'non-structural' GIRFs presented in Figure 3. This is particularly so in the case of the effects of the shock on US output which are now negative and statistically significant after one or two quarters under $\mathbf{x}_{0t}^{B}$. Also, under this ordering the effects of the monetary policy shock on the US long-term rate is no longer statistically significant, which contrasts to the results obtained under $\mathbf{x}_{0t}^{A}$.

The price puzzle continues to be present under both orderings, although it is now confined to the first one or two quarters immediately after the shock where the effects remain statistically significant. These short-term positive responses are more difficult to justify in the case of identified monetary policy shocks as compared to the GIRFs of an interest rate shock. However, Christiano *et al.* (1999) show that such a response can be expected when output comes after the monetary policy variables in the ordering of variables, which is actually the case under ordering A. This feature is less pronounced and more short-lived when considering ordering B. Overall, as far as the effects of the monetary policy shocks on output and inflation are concerned, ordering B yields results that are more in line with *a priori* expectations.

Finally, as regards the transmission of the US monetary policy shock to the euro area, there are very few differences across the two orderings, and the effects of the shock on the euro area variables are not that large and tend to be statistically insignificant.

8. CONCLUDING REMARKS

This paper updates and extends the GVAR model of Pesaran *et al.* (2004) in a number of directions, provides an unobserved common factor interpretation of the country-specific foreign variables included in the GVAR, addresses the issue of structural stability and shows how the model can be used for structural impulse response analysis. It also extends the geographical coverage from 11 country/regions to 26 countries with the euro area being treated as a single economy, updates the estimation period to the end of 2003 (from end of 1999 previously), adds the long-term interest rate in country-specific models, and includes oil prices as an endogenous variable in the US model rather than treating it as a global exogenous variable. Also, the US model now allows for feedback effects from changes in output and inflation outside the US variables. The current version, therefore, captures more fully the interactions in the world economy and includes new channels of transmissions via bond markets, the feedback effects on oil prices from the

[20] This alternative ordering was suggested to us by one of the referees.

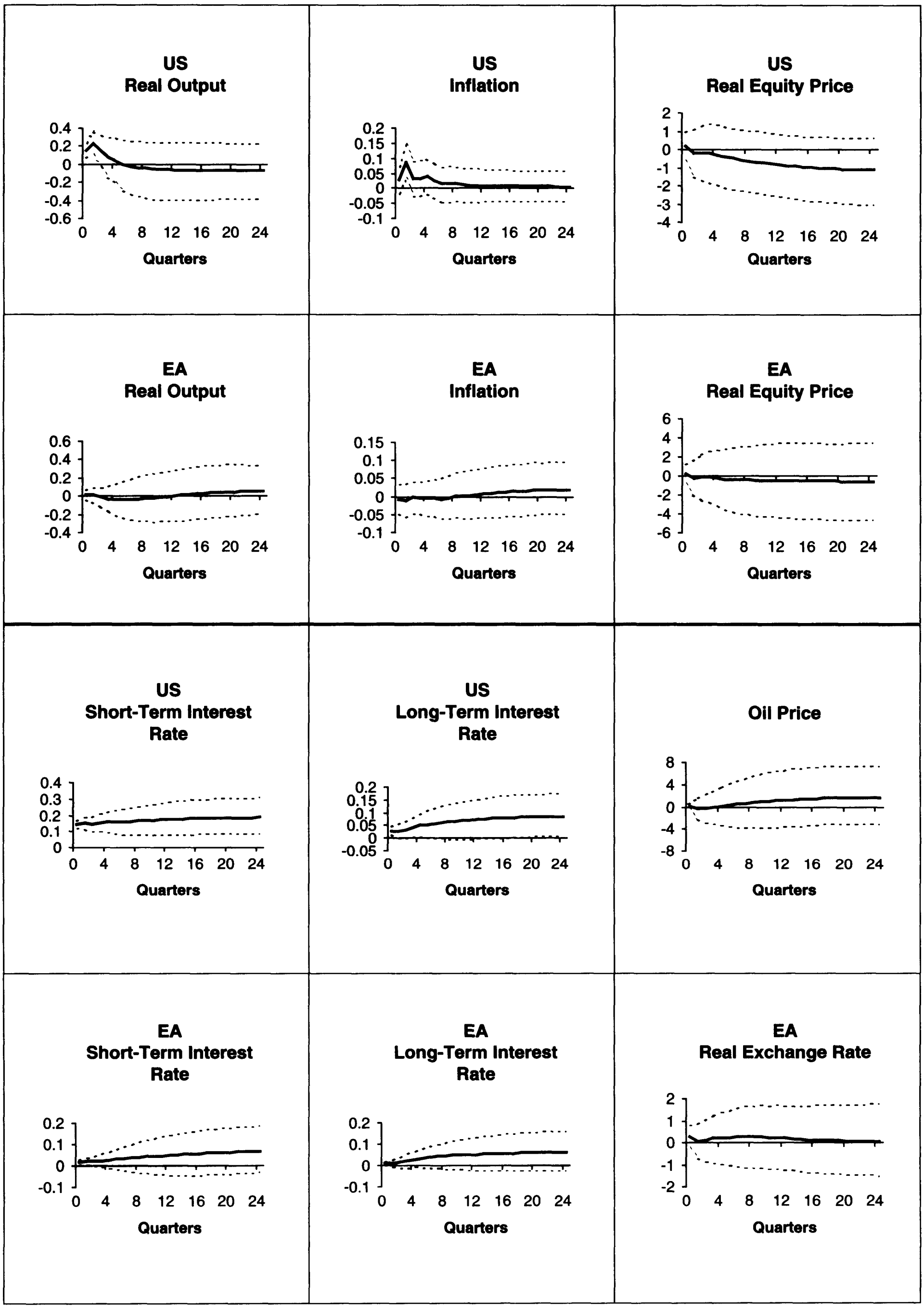

Figure 4. Impulse responses of a positive unit (1 s.e.) shock to US monetary policy under ordering A: {OIL, IR, LIR, EQ, INFL, GDP} (bootstrap mean estimates with 90% bootstrap error bounds)

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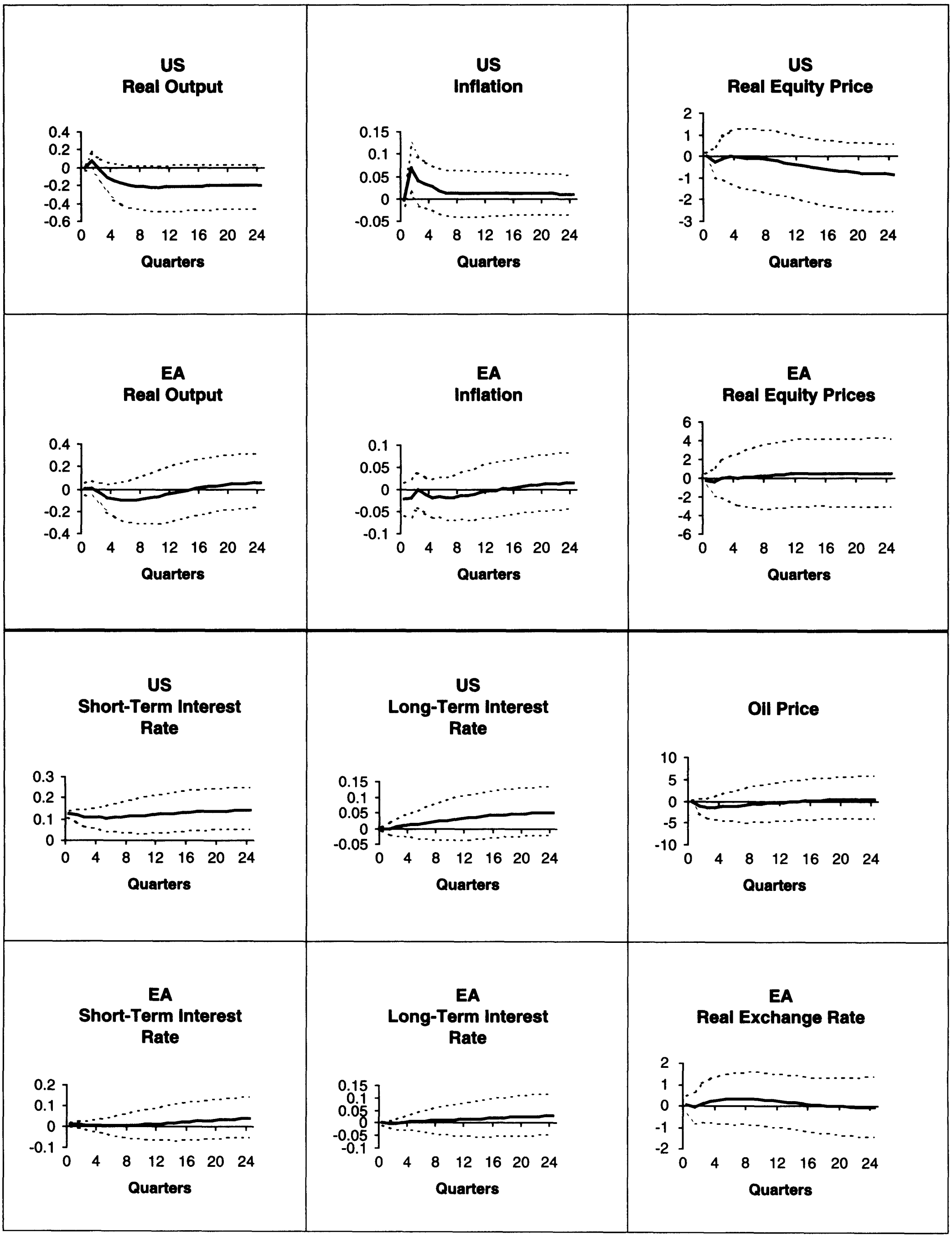

Figure 5. Impulse responses of a positive unit (1 s.e.) shock to US monetary policy under ordering B: {OIL, LIR, EQ, INFL, GDP, IR} (bootstrap mean estimates with 90% bootstrap error bounds)

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global economy, and the changes in output and inflation from the rest of the world to the US economy.

Although the new GVAR model can be used for many different purposes, in this paper we have focused on its short-term and long-term implications of external shocks for the euro area economy. We provide impact effects of external changes in interest rates (short-term and long-term rates), inflation, output, real equity prices, real exchange rates and oil prices on the euro area and present the time profiles of these shocks using both generalized and structural impulse response functions.

The key to the GVAR modelling is the systematic inclusion of the country-specific foreign variables in the individual country models in order to deal with the common factor dependencies that exist in the world economy. The average pair-wise cross-section correlations computed for the endogenous variables, their first differences and the residuals from the GVAR model show that very little cross-section correlations remain once the effects of foreign variables have been taken into account. This is in line with the results of the tests of weak exogeneity of the foreign variables also reported in the paper. Considering the problem of structural breaks, we have found that structural instability is mainly confined to error variances and does not seem to adversely affect the coefficient estimates. To this end, we use robust standard errors when investigating the impact of the foreign variables and we base the analysis of the impulse responses on the bootstrap means and confidence bounds rather than point estimates.

In addition to generalized impulse response functions, we also consider structural identification of shocks in the global economy, and emphasize that, unlike the GIRFs, the results of structural impulse responses in general depend on the order in which different countries are included in the GVAR model. It is partly for this reason that in our structural impulse response analysis we focus on identification of shocks to the US economy, which we order as the first economy in the GVAR model. In particular, we consider the short-term and long-term effects of a US monetary policy shock on the euro area.

From a policy analysis perspective, a number of interesting results emerge. The simulations clearly show that financial shocks are transmitted relatively rapidly, and often get amplified as they travel from the USA to the euro area. Equity and bond markets seem to be far more synchronous as compared to real output, inflation and short-term interest rates. While the impact of an oil price shock on inflation is statistically significant, the impact on output remains limited despite some deterioration in the financing conditions through a tightening of monetary policy, an increase in long-term interest rates and a decrease in real equity prices.

Our analysis of monetary policy shocks has shown that the transmission of a change in US monetary policy to the euro area is limited and statistically insignificant. This result has been confirmed both from the GIRFs of a shock to US short-term interest rates and from the IRFs of a monetary policy shock irrespective of the chosen ordering.

The model also highlights the importance of second-round effects of the shocks. A shock in the USA can be amplified because the USA will also be affected over time through the return impacts of output and inflation shocks in the rest of the world. The euro area in turn reacts to the US shocks directly as well as indirectly through the impact of the US shocks on euro area trade partners and so on. The transmission of shocks does not take place only through trade, but also as importantly through the impacts on financial variables with subsequent spillover effects on real variables. The GVAR presents a complicated, yet simple-to-follow, spatio-temporal structure for the analysis of the world economy. To be sure, it can be modified and extended further. But it is hoped that the present version makes a further step towards the development of a transparent and coherent framework for the analysis of global interdependencies.

APPENDIX

Consistent Estimation of the Cointegrating VARX* Model

Using a simple version of the VARX* model we consider the consistent estimation of the parameters of the conditional model, and provide a formal proof of the statement that the weak exogeneity of $\mathbf{x}_t^*$ with respect to the long-run coefficients, β, is sufficient for consistent estimation of the remaining parameters of interest that enter the conditional model.

Consistent Estimation of the Conditional Model

To simplify the exposition and without loss of generality we abstract from the deterministics and set the order of the underlying VAR in $\mathbf{z}_t = (\mathbf{x}_t', \mathbf{x}_t^{*\prime})'$ to 2, and consider the VAR(2) specification

$$\Delta \mathbf{z}_t = \alpha_z \beta' \mathbf{z}_{t-1} + \mathbf{\Gamma}_z \Delta \mathbf{z}_{t-1} + \varepsilon_{zt} \tag{26}$$

where $\mathbf{x}_t$ and $\mathbf{x}_t^*$ are $k \times 1$ and $k^* \times 1$ vectors, β is $(k + k^*) \times r$ matrix $(r < k + k^*)$, and $\varepsilon_{zt} \sim IID(\mathbf{0}, \mathbf{\Sigma}_{zz})$. Use the following partitioned matrices to derive the conditional model:

$$\mathbf{\Sigma}_{zz} = \begin{pmatrix} \mathbf{\Sigma}_{xx} & \mathbf{\Sigma}_{xx*} \\ \mathbf{\Sigma}_{x*x} & \mathbf{\Sigma}_{x*x*} \end{pmatrix},$$

$$\alpha_z = \begin{pmatrix} \alpha \\ \alpha^* \end{pmatrix}, \mathbf{\Gamma}_z = \begin{pmatrix} \mathbf{\Gamma} \\ \mathbf{\Gamma}^* \end{pmatrix}, \text{ and } \varepsilon_{zt} = \begin{pmatrix} \varepsilon_t \\ \varepsilon_t^* \end{pmatrix}$$

Under the assumption that $\mathbf{x}_t^{*\prime}s$ are weakly exogenous with respect to β we have $\alpha^* = \mathbf{0}$, and the remaining parameters, α, β, $\mathbf{\Gamma}_z$, and $\mathbf{\Sigma}_{zz}$, can be estimated consistently subject to r^2 exact identifying restrictions on β. In fact, given the super-consistency property of the estimator of β (say $\hat{\beta}$) all the remaining parameters can be consistently estimated by the OLS regressions:

$$\Delta \mathbf{x}_t \text{ on } \hat{\xi}_{t-1} \text{ and } \Delta \mathbf{z}_{t-1},$$
$$\Delta \mathbf{x}_t^* \text{ on } \Delta \mathbf{z}_{t-1}$$

where $\hat{\xi}_{t-1} = \hat{\beta}' \mathbf{z}_{t-1}$ is taken as data. Denoting these estimators by $\hat{}$, $\mathbf{\Sigma}_{zz}$ can also be consistently estimated by

$$\hat{\mathbf{\Sigma}}_{zz} = T^{-1} \sum_{t=1}^{T} \hat{\varepsilon}_{zt} \hat{\varepsilon}_{zt}',$$

where

$$\hat{\varepsilon}_t = \Delta \mathbf{x}_t - \hat{\alpha} \hat{\xi}_{t-1} - \hat{\mathbf{\Gamma}} \Delta \mathbf{z}_{t-1},$$
$$\hat{\varepsilon}_t^* = \Delta \mathbf{x}_t^* - \hat{\mathbf{\Gamma}}^* \Delta \mathbf{z}_{t-1}$$

For the construction of the GVAR from the country-specific models a consistent estimator of the conditional VARX* model of $\mathbf{x}_t$ given $\mathbf{x}_t^*$ is needed. To this end we first note that

$$E(\Delta \mathbf{x}_t | \Delta \mathbf{x}_t^*, \mathcal{I}_{t-1}) = \alpha \beta' \mathbf{z}_{t-1} + \mathbf{\Gamma} \Delta \mathbf{z}_{t-1} + E(\varepsilon_t | \Delta \mathbf{x}_t^*, \mathcal{I}_{t-1})$$

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where $\mathcal{I}_{t-1}$ is a non-decreasing information set that contains at least $\mathbf{z}_{t-1}, \mathbf{z}_{t-2}, \ldots$. But under our assumptions

$$E(\varepsilon_t|\Delta\mathbf{x}_t^*, \mathcal{I}_{t-1}) = \mathbf{\Sigma}_{xx*}\mathbf{\Sigma}_{x*x*}^{-1}\varepsilon_t^*$$
$$= \mathbf{\Sigma}_{xx*}\mathbf{\Sigma}_{x*x*}^{-1}(\Delta\mathbf{x}_t^* - \mathbf{\Gamma}^*\Delta\mathbf{z}_{t-1})$$

and

$$\Delta\mathbf{x}_t = \alpha\beta'\mathbf{z}_{t-1} + \mathbf{\Gamma}\Delta\mathbf{z}_{t-1} + \mathbf{\Sigma}_{xx*}\mathbf{\Sigma}_{x*x*}^{-1}(\Delta\mathbf{x}_t^* - \mathbf{\Gamma}^*\Delta\mathbf{z}_{t-1}) + \mathbf{v}_t$$

where

$$\mathbf{v}_t = \Delta\mathbf{x}_t - E(\Delta\mathbf{x}_t|\Delta\mathbf{x}_t^*, \mathcal{I}_{t-1})$$

and by construction $E(\mathbf{v}_t|\Delta\mathbf{x}_t^*, \beta'\mathbf{z}_{t-1}, \Delta\mathbf{z}_{t-1}) = \mathbf{0}$.

Hence the VARX* model is given by

$$\Delta\mathbf{x}_t = \alpha\beta'\mathbf{z}_{t-1} + \mathbf{\Gamma}\Delta\mathbf{z}_{t-1} + \mathbf{\Sigma}_{xx*}\mathbf{\Sigma}_{x*x*}^{-1}(\Delta\mathbf{x}_t^* - \mathbf{\Gamma}^*\Delta\mathbf{z}_{t-1}) + \mathbf{v}_t$$

or

$$\Delta\mathbf{x}_t = \alpha\beta'\mathbf{z}_{t-1} + \mathbf{\Lambda}_0\Delta\mathbf{x}_t^* + \mathbf{\Lambda}_1\Delta\mathbf{z}_{t-1} + \mathbf{v}_t \tag{27}$$

where

$$\mathbf{\Lambda}_0 = \mathbf{\Sigma}_{xx*}\mathbf{\Sigma}_{x*x*}^{-1},\ \mathbf{\Lambda}_1 = \mathbf{\Gamma} - \mathbf{\Sigma}_{xx*}\mathbf{\Sigma}_{x*x*}^{-1}\mathbf{\Gamma}^*$$

The parameters of interest that enter the GVAR model are α, β, $\mathbf{\Lambda}_0$, and $\mathbf{\Lambda}_1$, and these can be estimated consistently and efficiently using the SURE estimators of α, β, $\mathbf{\Gamma}$, $\mathbf{\Gamma}^*$, $\mathbf{\Sigma}_{xx*}$ and $\mathbf{\Sigma}_{x*x*}$ computed in the case of the error correction model (26) subject to the restrictions $\alpha^* = \mathbf{0}$, namely by applying SURE to the system:

$$\Delta\mathbf{x}_t = \alpha\hat{\xi}_{t-1} + \mathbf{\Gamma}\Delta\mathbf{z}_{t-1} + \mathbf{e}_t,$$
$$\Delta\mathbf{x}_t^* = \mathbf{\Gamma}^*\Delta\mathbf{z}_{t-1} + \mathbf{e}_t^*$$

taking $\hat{\xi}_{t-1}$ as given. In the present set-up the OLS estimators of $\mathbf{\Gamma}^*$ obtained from the OLS regressions of $\Delta\mathbf{x}_t^*$ on $\Delta\mathbf{z}_{t-1}$ will be identical to the SURE estimator of $\mathbf{\Gamma}^*$. However, the OLS estimators of α and $\mathbf{\Gamma}$ obtained from the OLS regressions of $\Delta\mathbf{x}_t$ on $\hat{\xi}_{t-1}$ and $\Delta\mathbf{z}_{t-1}$ will not be the same as the corresponding SURE estimators. Denoting these SURE estimators by $\tilde{\alpha}$, $\tilde{\mathbf{\Gamma}}$ etc. we have

$$\tilde{\mathbf{\Sigma}}_{xx*} = T^{-1}\sum_{t=1}^{T}\tilde{\mathbf{e}}_t^*\tilde{\mathbf{e}}_t', \text{ and } \tilde{\mathbf{\Sigma}}_{x*x*} = T^{-1}\sum_{t=1}^{T}\tilde{\mathbf{e}}_t^*\tilde{\mathbf{e}}_t^{*\prime},$$
$$\tilde{\mathbf{e}}_t = \Delta\mathbf{x}_t - \tilde{\alpha}\hat{\xi}_{t-1} - \tilde{\mathbf{\Gamma}}\Delta\mathbf{z}_{t-1},\ \tilde{\mathbf{e}}_t^* = \Delta\mathbf{x}_t^* - \tilde{\mathbf{\Gamma}}^*\Delta\mathbf{z}_{t-1}$$

Using these estimates the parameters of the VARX* model, (27) are then given by $\tilde{\alpha}$, $\hat{\beta}$,

$$\tilde{\mathbf{\Lambda}}_0 = \tilde{\mathbf{\Sigma}}_{xx*}\tilde{\mathbf{\Sigma}}_{x*x*}^{-1}, \text{ and } \tilde{\mathbf{\Lambda}}_1 = \tilde{\mathbf{\Gamma}} - \tilde{\mathbf{\Sigma}}_{xx*}\tilde{\mathbf{\Sigma}}_{x*x*}^{-1}\tilde{\mathbf{\Gamma}}^*$$

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It is also worth noting that OLS regressions of $\Delta\mathbf{x}_t$ on $\hat{\boldsymbol{\xi}}_{t-1}$, $\Delta\mathbf{x}_t^*$ and $\Delta\mathbf{z}_{t-1}$ would also yield the same results, namely estimates of α, $\boldsymbol{\Lambda}_0$ and $\boldsymbol{\Lambda}_1$ that are identical to the estimates $\tilde{\alpha}$, $\tilde{\boldsymbol{\Lambda}}_0$ and $\tilde{\boldsymbol{\Lambda}}_1$ obtained using the above indirect method.

Proof of Sufficiency

To prove that the long-run forcing assumption is sufficient for the consistent and efficient estimation of the parameters of the conditional model we shall assume that $\varepsilon_{zt} \sim IIDN(\mathbf{0}, \boldsymbol{\Sigma}_{zz})$, and make use of the Engle *et al.* (1983) likelihood framework.

Let $\theta = (\text{vec}(\alpha)', vec(\beta)', vec(\boldsymbol{\Lambda}_0)', vec(\boldsymbol{\Lambda}_1)')'$ be the parameters of interest and note that the log-likelihood function of (26) for the sample of observations over $t = 1, 2, \ldots, T$ is given by

$$\ell(\alpha_z, \beta, \Gamma_z, \boldsymbol{\Sigma}_{zz}) = -\frac{T}{2}\ln|\boldsymbol{\Sigma}_{zz}| - \frac{1}{2}\sum_{t=1}^{T}\varepsilon_{zt}'\boldsymbol{\Sigma}_{zz}^{-1}\varepsilon_{zt}$$

Also it is easily seen that

$$\varepsilon_{zt}'\boldsymbol{\Sigma}_{zz}^{-1}\varepsilon_{zt} = \varepsilon_t'\boldsymbol{\Sigma}^{xx}\varepsilon_t + 2\varepsilon_t'\boldsymbol{\Sigma}^{xx*}\varepsilon_t^* + \varepsilon_t^{*\prime}\boldsymbol{\Sigma}^{x*x*}\varepsilon_t^*$$

where

$$\boldsymbol{\Sigma}_{zz}^{-1} = \begin{pmatrix} \boldsymbol{\Sigma}^{xx} & \boldsymbol{\Sigma}^{xx*} \\ \boldsymbol{\Sigma}^{x*x} & \boldsymbol{\Sigma}^{x*x*} \end{pmatrix}$$

and

$$|\boldsymbol{\Sigma}_{zz}| = |\boldsymbol{\Sigma}_{x*x*}||\boldsymbol{\Sigma}_{xx} - \boldsymbol{\Sigma}_{xx*}\boldsymbol{\Sigma}_{x*x*}^{-1}\boldsymbol{\Sigma}_{x*x}|$$

Let $\mathbf{v}_t = \varepsilon_t - \boldsymbol{\Sigma}_{xx*}\boldsymbol{\Sigma}_{x*x*}^{-1}\varepsilon_t^*$ and note that

$$\varepsilon_{zt}'\boldsymbol{\Sigma}_{zz}^{-1}\varepsilon_{zt} = \mathbf{v}_t'\boldsymbol{\Sigma}_{vv}^{-1}\mathbf{v}_t + \varepsilon_t^{*\prime}\boldsymbol{\Sigma}_{x*x*}^{-1}\varepsilon_t^*$$

where

$$\boldsymbol{\Sigma}_{vv} = \boldsymbol{\Sigma}_{xx} - \boldsymbol{\Sigma}_{xx*}\boldsymbol{\Sigma}_{x*x*}^{-1}\boldsymbol{\Sigma}_{x*x}$$

Also

$$\mathbf{v}_t = \varepsilon_t - \boldsymbol{\Lambda}_0\varepsilon_t^* = \Delta\mathbf{x}_t - \alpha\beta'\mathbf{z}_{t-1} - \boldsymbol{\Lambda}_0\Delta\mathbf{x}_t^* - \boldsymbol{\Lambda}_1\Delta\mathbf{z}_{t-1}$$

which is the same as $\mathbf{v}_t$ defined by (27). Hence, under $\alpha^* = \mathbf{0}$, the log-likelihood function can be decomposed as

$$\ell(\alpha, \beta, \boldsymbol{\Gamma}_z, \boldsymbol{\Sigma}_{zz}) = \ell_v(\theta, \boldsymbol{\Sigma}_{vv}) + \ell_*(\boldsymbol{\Sigma}_{x*x*}, \boldsymbol{\Gamma}^*)$$

where

$$\ell_v(\theta, \beta, \boldsymbol{\Sigma}_{vv}) = -\frac{T}{2}\ln|\boldsymbol{\Sigma}_{vv}| - \frac{1}{2}\sum_{t=1}^{T}$$
$$(\Delta\mathbf{x}_t - \alpha\beta'\mathbf{z}_{t-1} - \boldsymbol{\Lambda}_0\Delta\mathbf{x}_t^* - \boldsymbol{\Lambda}_1\Delta\mathbf{z}_{t-1})'\boldsymbol{\Sigma}_{vv}^{-1}(\Delta\mathbf{x}_t - \alpha\beta'\mathbf{z}_{t-1} - \boldsymbol{\Lambda}_0\Delta\mathbf{x}_t^* - \boldsymbol{\Lambda}_1\Delta\mathbf{z}_{t-1})$$

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and

$$\ell_*(\mathbf{\Sigma}_{x*x*}, \mathbf{\Gamma}^*) = -\frac{T}{2}\ln|\mathbf{\Sigma}_{x*x*}| - \frac{1}{2}\sum_{t=1}^{T}(\Delta\mathbf{x}_t^* - \mathbf{\Gamma}^*\Delta\mathbf{z}_{t-1})'\mathbf{\Sigma}_{x*x*}^{-1}(\Delta\mathbf{x}_t^* - \mathbf{\Gamma}^*\Delta\mathbf{z}_{t-1})$$

Hence, under $\alpha^* = \mathbf{0}$, the parameters of interest, θ, that enter the conditional model, $\ell_v(\theta, \mathbf{\Sigma}_{vv})$, are variation free with respect to the parameters of the marginal model, $\ell_*(\mathbf{\Sigma}_{x*x*}, \mathbf{\Gamma}^*)$, and the ML estimators of $\boldsymbol{\theta}$ based on the conditional model will be identical to the ML estimators computed indirectly (as set out above) using the full model, $\ell(\alpha, \beta, \mathbf{\Gamma}_z, \mathbf{\Sigma}_{zz})$.

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