A STUDY OF ECONOMIC EFFICIENCY OF UTAH DAIRY FARMERS: A SYSTEM APPROACH

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Abstract—This study is primarily concerned with investigating the technical, allocative and scale inefficiency of owner-operators of dairy farms in Utah. A stochastic production frontier has been applied to analyse these inefficiencies. The results indicate that there is positive association between years of education and productivity of labor and capital. Productivity is also found to be negatively related to off-farm income. Regarding the effects of farm size on efficiency it is found that large farms are the most efficient of all sizes considered. Separate estimates of technical, allocative and scale inefficiencies indicate that large and medium-sized farms are technically more efficient than small farms. Large farms, on average, are found to be performing much better than medium-sized and small farms so far as allocative and scale inefficiency are concerned.

I. Introduction

THE study of efficiency in production is important. Whether one considers a free enterprise economy or a centrally planned economy, the agricultural sector or the manufacturing sector, a developed economy or a developing economy, the study of productive efficiency has important implications. This study is primarily concerned with investigating the productive efficiency of owner-operators of dairy farms in Utah. The main implication is that if significant inefficiencies exist, identification and elimination of these inefficiencies will result in more profit.

For a profit maximizing farm the observed output supply and input use will coincide with the profit maximizing output supply and input demand if and only if the farm is technically, allocatively, and scale efficient (see Førsund et al. (1980)). Thus one cannot infer anything about total or economic efficiency simply by estimating technical efficiency as in Aigner et al. (1977), Bagi and Huang (1983), Huang and Bagi (1984), Kumbhakar and Summa (1989), Kalirajan and Shand (1985), Pitt and Lee (1981). The objective of the present study is to investigate the three components of inefficiency—technical, allocative, and scale inefficiencies for Utah dairy farmers. To our knowledge, this is the first attempt to estimate the three types of inefficiency empirically. The motivation for this study is to explain the growth of dairy farms in Utah during the past twenty years. The market share of large farms (having more than 100 milking cows) has increased dramatically. This trend suggests that large farms gained ground relative to the small farms (having less than 50 milking cows). It is possible that large farms can cope with economic changes more efficiently than small farms. If large farms are more efficient than small farms, as a group, one could also ask whether each of the large farms is more efficient than the small farms. The present paper attempts to answer this question by estimating technical, allocative, and scale efficiency of Utah dairy farms individually.

We also examine the role of off-farm income in the analysis of farm efficiency. The larger the off-farm component of the operator's income the less time he will spend on the farm. One likely consequence is that the production decision, now being based on less economic information, will be relatively inefficient. Another source of inefficiency associated with part-time farming is rooted in the indivisibility of the farm operator. Given the kind of operating decisions that characterize dairy farms, the operator may be viewed as an indivisble factor. When the owner-operator takes a part-time job off the farm he will earn more total income but the farming sector may suffer from inefficiency.

The paper is organized as follows. Section II is devoted to modelling technical, allocative, and scale inefficiency in a simultaneous equation framework. This is followed by the method of estimation in section III. The description of data is in section IV and the empirical results are discussed in section V. Finally, section VI contains the conclusions of the present study.

II. Formulation of the Model

Let the production function for Utah dairy farmers be represented by

$$Y = A \prod_{i=1}^{n} X_i^{\alpha_i} \prod_{k=1}^{m} Z_k^{\beta_k} \exp(v) \tag{1}$$

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where Y is output, and X_i and Z_k are endogenous and exogenous inputs, respectively. A is the efficiency parameter and v is the random noise. The distinction between endogenous and exogenous inputs is relevant in any production decision since the choice of the endogenous inputs is based on some optimizing behavior like profit maximization or cost minimization whereas the exogenous inputs are not derived from such an optimization framework—at least in the short run. Thus the production relation in (1) can be treated as a short-run production function. It can be related to the stochastic production frontier, introduced by Aigner et al. (1977) and Meeusen and van den Broeck (1977), by specifying A as

$$A = a_0 \exp(\tau) \qquad \tau \leq 0 \tag{1a}$$

where a_0 is a parameter common to all farms and τ is the technical inefficiency that varies across farms. The production frontier can be viewed as composed of those parts of the farms' production functions that yield maximum output for a given set of inputs. Thus, some part of each farm's production function belongs to the frontier production function. However, it is quite possible that a farm with its scale of operation, observed at a single cross-section, may not be able to reach the frontier—the production function for the industry. On the other hand, there may be farms whose outputs are closer to the frontier, given their levels of inputs. The notion of how close the individual production plans are to the maximum levels as defined by the frontier, given input levels, is captured by $\exp(\tau)$ which essentially measures technical efficiency for each farm. Thus, farms with $\tau = 0$ operate on the production frontier. A farm is said to be technically inefficient for a given set of inputs if its output level lies inside the frontier. This is represented by a negative value of τ—which can be interpreted as the percentage reduction in potential output due to technical inefficiency. Alternatively, $\exp(\tau)$ can be interpreted as technical efficiency which is bounded between 1 and 0. This makes the range of technical efficiency to be 100% and 0%.

The production relation in (1) with (1a) can be rewritten as

$$Y = a_0 \prod_{i=1}^{n} X_i^{a_i} \prod_{k=1}^{m} Z_k^{\beta_k} \exp(\tau + v). \tag{1b}$$

Exogenous factors such as the education level of the owner-operator may enhance productivity of the endogenous inputs and hence increase output. Similarly, off-farm income of the owner-operator may have some adverse effect on production due to negligence, absenteeism, etc. Farm size may be another factor systematically affecting productivity and technical inefficiency. The random noise v includes factors not in the control of any farm, like weather variation, machine breakdown, etc. and can affect production both favorably and unfavorably. This makes the production frontier stochastic.

One can estimate the stochastic production function specified in (1b)—using data on inputs and output—by the maximum likelihood (ML) method provided some distributional assumptions are made about the error components τ and v. In doing so, one allows the production process to be only technically inefficient in the sense that actual output is less than the maximum possible output for a given input bundle. A production process can also be allocatively inefficient in the sense of not using inputs by equating ratios of marginal products with the input price ratios—when the objective is to minimize cost given output and input prices. Furthermore, in a profit maximizing framework a production process can be scale inefficient in the sense of not producing an output level by equating the product price with the marginal cost (see Førsund et al. (1980)). Thus in order to estimate total or economic efficiency, the three types of inefficiency are introduced in estimating the production function. For this we follow the framework in Kumbhakar (1987). However, we distinguish between scale and allocative inefficiency, which are combined in Kumbhakar (1987).

Allocative and scale inefficiency are introduced in the following form:

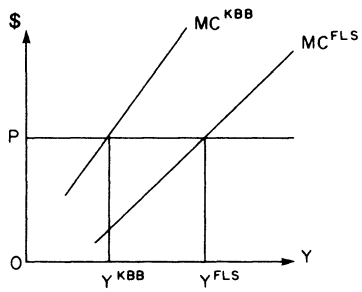
$$\frac{MP_j}{MP_1} = \frac{W_j}{W_1} \exp(u_j)$$

and

$$\frac{\partial C}{\partial Y} = P \exp(\xi), \tag{2}$$

where MP_j is the marginal product of input X_j, $C = \Sigma_{i=1}^{n} W_i X_i$ is the total variable cost, W_i is the price of X_i and P is the output price. $u_j (j = 2, \ldots, n)$ and ξ represent allocative, and

FIGURE 1.—MEASURES OF SCALE INEFFICIENCY



scale inefficiency, respectively. When $u_j = 0$ for all j—the production process is allocatively efficient. On the other hand, if $u_j > 0$ for some j, input X_j is relatively under-utilized given W_j and W_1. Similarly, input X_j is over-utilized if $u_j < 0$. The scale inefficiency "parameter" ξ is zero only when the production plan is optimal, given P. On the other hand, ξ is positive (negative) when actual output is less (greater) than the optimal level of output.

The definition of scale inefficiency, ξ, in (2) (hereafter KBB) is somewhat different from the Føsund et al. (1980) definition (hereafter FLS) which is based on efficient marginal cost (MC) (net of technical and allocative inefficiency). Thus according to the FLS definition, scale inefficiency, ξ_F, can be specified as

$$\frac{\partial C^*}{\partial Y} = P\exp(\xi_F), \tag{2a}$$

where C^* is the total variable cost net of technical and allocative inefficiency. The FLS measure is appealing in the sense that it makes the three sources of inefficiency complement one another. The measure based on actual cost—the KBB measure— is also interesting. It focuses on finding the optimal level of output for each farm conditional on its technical and allocative inefficiency. However, these two measures are not the same and therefore may not lead to the same conclusion so far as over-(under-)production of output is concerned. This is illustrated in figure 1.[1] Let MC^{FLS} be the MC curve based on total cost net of technical and allocative inefficiency. Since inefficiency always increases cost, MC^{KBB}—the MC curve based on observed total cost, is drawn above the MC^{FLS}. Thus, Y^{KBB} and Y^{FLS} are the scale efficient outputs according to the KBB and FLS measures, respectively. Consequently, the optimum level of output corresponding to the FLS measure is greater than that of the KBB measure. The magnitude of this difference will increase with the size of technical and allocative inefficiency. So far as the direction of scale inefficiency is concerned, a farm's output plan is said to be under-optimal (over-optimal) by both measures if $Y < Y^{KBB}(Y > Y^{FLS})$. If Y lies between Y^{KBB} and Y^{FLS}, the opposite conclusion will be reached regarding the direction of scale inefficiency. It can also be seen from the figure that these two measures give different magnitudes of scale inefficiency.

[1] We owe this point to an anonymous referee.

The system of simultaneous equations incorporating technical, allocative, and scale inefficiency (as defined in (2)) in a profit-maximizing framework can be written as

$$\begin{aligned}
\ln Y &= \ln a_0 + \sum_i a_i \ln X_i + \sum_k \beta_k \ln Z_k + \tau + v \\
\ln X_1 - \ln X_j &= \ln(a_1/a_j) - \ln W_1 \\
&\quad + \ln W_j + u_j, \\
&\qquad\qquad j = 2, \ldots, n \\
\ln X_1 - \ln Y &= \ln P - \ln W_1 + \ln a_1 + \ln r \\
&\quad - \ln\Big\{a_1 + \sum_j a_j \exp(-u_j)\Big\} + \xi,
\end{aligned} \tag{3}$$

where $r = \Sigma_i a_i$. Equations in (3) represent a system of $(n + 1)$ equations to be solved for $(n + 1)$ unknowns X_i and Y, which are the unconditional input demand and output supply functions. The conditional input demand functions can be obtained by solving the first n equations and the output supply function from the last equation. It is shown in Schmidt and Lovell (1979) that the conditional demand for each of the inputs will increase by the same proportion due to technical inefficiency. However, the presence of technical inefficiency reduces output supply which, in turn, requires less inputs. The net effect is to reduce demand for each input, as can be seen from the unconditional input demand and output supply functions (see Kumbhakar (1987) for details). For estimation it is easier to use the equations in (3) than the input demand and output supply functions.

III. Method of Estimation

To estimate the system of equations in (3) by the ML method, we need to derive the probability density function (pdf) of the error vector

$$\left(\tau + v, u', \xi - \ln\left(a_1 + \sum_i a_j \exp(-u_j)\right)\right)'$$

from the pdf of τ, v and ξ where $u = (u_2, \ldots, u_n)'$. The distributional assumptions on these random errors are

(i) τ is iid $N(0, \sigma_\tau^2)$ truncated at zero from above,
(ii) u is iid $N(\mu, \Sigma)$,
(iii) ξ is iid $N(0, \sigma_\xi^2)$, and
(iv) τ, u, and ξ are independent of each other and also independent of input prices W_i and exogenous inputs Z_k.

Let

$$z_1 = \tau + v$$

and

$$z_2 = \xi - \ln\left(a_1 + \sum_j a_j \exp(-u_j)\right).$$

Then the joint pdf of

$$(z_1, u, z_2),\ f(z_1, u, z_2) = f_1(z_1) f_2(u) f_3(z_2|u) = f_1(z_1) f_2(u) f_3(\xi),$$

where $f_1(.)$, $f_2(.)$ and $f_3(.)$ are the pdf of z_1, u and ξ, respectively. After some algebraic manipulations, the above joint pdf can be written as

$$f(z_1, u, z_2) = \frac{2\exp(-b/2)\sigma\Phi(-\mu_\tau/\sigma)}{(2\pi)^{n/2}\sigma_v\sigma_\tau\sigma_\xi|\Sigma|^{1/2}} \times \exp\left(-\frac{1}{2}(u-\mu)'\Sigma^{-1}(u-\mu)\right) \times \exp\left\{-\frac{1}{2\sigma_\xi^2}\left(z_2 + \ln\left(a_1 + \sum_j a_j \exp(-u_j)\right)\right)^2\right\}, \quad (4)$$

where

$$\sigma^2 = \frac{\sigma_v^2\sigma_\tau^2}{(\sigma_v^2 + \sigma_\tau^2)},$$

$$b = \frac{z_1^2}{(\sigma_v^2 + \sigma_\tau^2)},$$

$$\mu_\tau = \frac{z_1\sigma_v^2}{(\sigma_v^2 + \sigma_\tau^2)}$$

and $\Phi(.)$ is the cumulative pdf of a standard normal variable. The log likelihood function for a sample of F farms is then given by

$$L = \sum_{f=1}^{F} \ln f(z_1, u, z_2) + F \ln |J|, \quad (5)$$

where f indexes farm ($f = 1, 2, \ldots, F$). $f(z_1, u, z_2)$ is defined in (4) and z_1, u, ξ are to be replaced by their observable counterparts from (3).[2] $|J|$ is the determinant of the Jacobian of the transformation from z_1, u, z_2 to $\ln Y, \ln X_i$. In the present case it is $(1 - r)$. The ML estimates of the parameters can be obtained by maximizing L in (5). However, the burden of estimation can somewhat be reduced by concentrating the above log likelihood function with respect to μ and Σ.

At the maximum of L, the estimate of μ is

$$\mu_j = \frac{1}{F}\sum_{f=1}^{F} u_{jf} = \overline{\ln X_1} + \overline{\ln W_1} - \overline{\ln X_j} - \overline{\ln W_j} + \ln(a_j/a_1), \quad (6)$$

where a bar over a variable designates an arithmetic mean. Similarly, σ_{jk}, the $(j,k)^{\text{th}}$ element of Σ can be estimated from

$$\sigma_{jk} = \frac{1}{F}\sum_f (u_{jf} - \overline{u_j})(u_{kf} - \overline{u_k}) \qquad j, k = 2, \ldots, n, \quad (7)$$

which is independent of the parameters. Thus Σ becomes a constant. The concentrated log likelihood function can be written as (for a single

[2] The farm subscript f on z_1, ξ and u are omitted for notational convenience. Similar are the cases with inputs, input prices, output and output price—X_i, W_i, Y and P, respectively.

observation)

$$\begin{aligned} L_c = \text{const.} &- \frac{b}{2} + \ln\sigma - \ln\sigma_v \\ &- \ln\sigma_\tau - \ln\sigma_\xi + \ln\Phi(-\mu_\tau/\sigma) \\ &- \frac{1}{2\sigma_\xi^2}\left(z_2 - \ln\left(a_1 + \sum_j a_j \exp(-u_j)\right)\right)^2 \\ &+ \ln(1-r). \end{aligned} \tag{8}$$

The ML estimates of $\ln a_0, a_1, \ldots, a_n, \sigma_v, \sigma_\tau, \sigma_\xi$ can be obtained by maximizing (8) after adding it over all the farms. The ML estimates of μ and elements of Σ can be obtained from (6) and (7).

Following Kumbhakar (1987), it can be shown that τ given z_1 (the residual of the production function) is normally distributed with mean μ_τ and variance σ^2 truncated at zero. Thus, point estimate of technical inefficiency for each farm can be obtained from the mean of τ, i.e.,

$$\hat{\tau} = \hat{\mu}_\tau - \hat{\sigma}\frac{\phi(\hat{\mu}_\tau/\hat{\sigma})}{\Phi(-\hat{\mu}_\tau/\hat{\sigma})}, \tag{9}$$

where $\hat{\mu}_\tau, \hat{\sigma}$ are the estimates of μ_τ and σ. Given X and Z, the percentage loss in output due to technical inefficiency, τ, can be obtained from

$$PT = (Y - Y^*)/Y^* = 1 - \exp(\tau), \tag{9a}$$

where Y^* is the frontier output (conditional on X and Z) obtained by setting $\tau = 0$.

Allocative inefficiency for each farm and for every endogenous input (X) can be obtained from

$$\begin{aligned} \hat{u}_j = \ln X_1 - \ln X_j - \ln(\hat{a}_1/\hat{a}_j) \\ + \ln W_1 - \ln W_j, \qquad j=2,\ldots,n. \end{aligned} \tag{10}$$

Since both positive and negative u_j increase cost, it might be of some interest to estimate the percentage increase in cost due to allocative inefficiency, CA, for each farm, Once u_j is estimated, CA can be estimated from (see Schmidt and Lovell (1979) for details)

$$CA = \exp(E - \ln\hat{r}) - 1, \tag{11}$$

where

$$E = \sum_{j=2}^{n}\hat{a}_j\hat{u}_j/\hat{r} + \ln\left\{\hat{a}_1 + \sum_{j=2}^{n}\hat{a}_j\exp(-\hat{u}_j)\right\}.$$

The effect of scale inefficiency on output can be obtained from the output supply function—the solution of $\ln Y$ from (3)—which is

$$\begin{aligned} \ln Y = m &- \frac{1}{1-r}\sum_i a_i \ln W_i \\ &+ \frac{1}{1-r}\sum_k \beta_k \ln Z_k + \frac{r}{1-r}\ln P \\ &+ \frac{\tau + v}{1-r} \\ &- \frac{r}{1-r}\ln\left\{a_1 + \sum_j a_j\exp(-u_j)\right\} \\ &- \frac{1}{1-r}\sum_j a_j u_j + \frac{r}{1-r}\xi, \end{aligned} \tag{12}$$

where $m = (\ln a_0 + r\ln r + \Sigma_i a_i \ln a_i)/(1-r)$. It can be seen from (12) that once ξ is estimated, an estimate of the degree of under-(over-)production of output (in logarithms) can be obtained from

$$OU = \frac{\hat{r}\hat{\xi}}{1-\hat{r}}, \tag{13}$$

where r and ξ are to be replaced by their estimates. Similarly, OU^F can be calculated replacing ξ by ξ_F in (13), namely,

$$OU^F = \frac{\hat{r}\hat{\xi}_F}{1-\hat{r}}. \tag{13a}$$

As pointed out before, the difference in the magnitudes of OU and OU^F will depend on the sizes of technical and allocative inefficiency. If no such inefficiency exists both OU and OU^F will be the same. On the other hand, if over-production is indicated by both the measures, the magnitude of over-production will be higher under the KBB measure. The opposite is true when under-production is evidenced by both the measures.

To make the formulas in (13) and (13a) operational, we need to find the estimates of ξ and ξ_F. Once allocative inefficiency is estimated, ξ for each farm can be estimated from the last equation in (3). This is given by

$$\begin{aligned} \hat{\xi} = \ln X_1 - \ln Y - \ln P + \ln W_1 - \ln\hat{a}_1 \\ - \ln\hat{r} + \ln\left\{\hat{a}_1 + \sum_j \hat{a}_j\exp(-\hat{u}_j)\right\}, \end{aligned} \tag{14}$$

where $\hat{u}_j$ is obtained from (10). On the other hand, ξ_F can be obtained from the relation (2a) which can be rewritten as

$$\xi_F = \ln C^* - \ln r - \ln Y - \ln P$$
$$= \ln X_1 - \ln Y - \sum_j a_j u_j / r$$
$$+ z_1/r - \ln P + \ln W_1 - \ln a_1. \quad (14a)$$

Given that firms can make a mistake by producing suboptimum levels of output and that these mistakes are costly, it might be worth estimating loss in profit due to such mistakes. The percentage loss in profit due to scale inefficiency can be estimated from

$$\Pi_S = 1 - \frac{\Pi(P, W, Z, \xi, v)}{\Pi^*}$$
$$= 1 - \frac{\exp\{\hat{r}\hat{\xi}/(1-\hat{r})\}\{1 - \hat{r}\exp(\hat{\xi})\}}{(1-\hat{r})}, \quad (15)$$

where Π^* is the profit frontier defined by $\Pi^* = PY - \Sigma_i W_i X_i$ when $\xi = \tau = u_j = 0$ and $\Pi(P, (W, Z, v)$ is profit with only scale inefficiency (i.e., $\tau = u_j = 0$). Similarly, Π_s^F can be calculated by using the FLS measure of scale inefficiency in (15). It has been shown in section II that if ξ and ξ_F are both positive (negative), the magnitude of over-(under-)production measured by the KBB definition is greater (less) than that of the FLS definition. This in turn indicates that the percentage loss of profit using the former measure is greater (less) than that using the latter measure. No such conclusion can, however, be reached if ξ and ξ_F are of different signs. If the interest is in the measure of dollar cost of scale inefficiency (in terms of forgone profit), C_s, for each farm, it can be calculated from $C_s = \Pi^* \cdot \Pi_S$ using both measures of scale inefficiency.

IV. Description of Data

Data for this study were obtained from a random sample of dairy farmers in Utah surveyed by the Department of Economics, Department of Family and Human Development and the Department of Home Economics and Consumer Education at Utah State University, Logan. The purpose of the survey was to determine major factors leading to either financial success or failure and economic efficiency of Utah dairy farmers. The survey was based on the five counties that were the major dairy production centers in Utah. A sample of 116 farm families, from a population of 510 in these counties, was interviewed. The sample was stratified by county dairy farm population and farm size. Eighty nine of the surveys were complete enough to be included in this analysis. Questions regarding a wide range of farm and family characteristics were obtained including debt situation, management style, numbers of acres and cows, input costs, capital structure, operator's education level, off-farm income, etc. The response rate,[3] defined as the percentage of families actually interviewed out of the total contacted, was 66.67%.

TABLE 1.—MARKET SHARES OF UTAH DAIRY FARMS (MILK PRODUCTION)

Size of Farm[a] (percentage of Utah total)	1969	1974	1978	1982
Small	43.0%	26.0%	17.3%	10.6%
Medium	35.7%	32.5%	33.3%	29.3%
Large	21.3%	41.5%	49.4%	60.1%

[a] Small = less than 50 milk cows. Medium = between 50 and 100 milk cows. Large = over 100 milk cows.

The observations were separated by size based on dollar sales during 1985 (U.S. Department of Commerce). Dollar sales represented approximate gross farm income based on average herd production, annual average milk price for the type of milk sold (grade AA and/or manufacturing grade), income from the sale of dairy cattle and other livestock, and crop income.

Table 1 depicts the change in the size distribution of Utah dairy farms. The number of dairy cows is a convenient proxy for farm size. In 1969–1982, the production share of small farms (having less than 50 milking cows) declined from 43% to 10.6% while large farms (having more than 100 milking cows) gained ground. The market share of large farms increased from 21.3% in 1969 to 60.1% in 1982. This was accomplished through several avenues including new entry, expanding herd size on existing farms, and increases in average milk production per cow caused by improved feeding programs and improved genetics.

Table 2 presents some socio-economic characteristics of the farmers surveyed. Larger farms tended to be operated by the slightly younger and

[3] Since the non-respondents and the twenty-seven excluded families were not in a "special category" in terms of the farm and family characteristics, the possibility of selectivity bias could be ignored.

TABLE 2.—AVERAGE FARM AND FARM FAMILY CHARACTERISTICS FROM SURVEY OF UTAH DAIRY FARMERS, 1985

Category (Average Values)	Small (under $100,000 sales)	Medium ($100,000–$250,000 sales)	Large (over $250,000 sales)
Operator's age	50.4	52.5	49.7
Operator's education level (years)	14.0	13.0	13.0
Milk cows	34.0	71.7	159.3
Rolling herd average (lbs.)	14,613	16,519	17,685
Annual off-farm income/year	$11,145	$5,553	$4,512
Farm assets	$237,078	$433,689	$759,706
Farm debts	$59,500	$108,704	$302,428

slightly less educated farmers. Large farmers were also carrying the largest debt loads but had 21% higher average production per cow than small producers. Small producers had the largest off-farm incomes, tended to be slightly more educated and worked more hours off the farm (both husband and wife).

We considered two endogenous inputs—capital and labor. Labor represented the time, in hours, spent in activities on the farm by the husband, wife and hired labor. Additional labor was considered to be available at a cost of $5 per hour (Utah Agricultural Statistics). The opportunity cost of capital consisted of depreciation and interest expenses on the farm (Jorgenson and Griliches (1970), Jorgenson (1969)). All capital was depreciated using the straight line method. An interest rate of 11% was used to calculate the opportunity cost of capital. We considered land an exogenous input. Consequently a short-run production function is estimated.

Other exogenous inputs are years of formal education of the farm operator, off-farm income, and two dummy variables for three farm sizes. Off-farm income affects not only the farm's cash flow situation, but is also a measure of effort spent in non-farm activities. Size may influence efficiency if economies of size are present. The farm's output is measured in pounds of milk produced (adjusted for waste and disease).

V. Empirical Results

Considering the diverse farm sizes we divided the sample into three subgroups, viz, small, medium, and large, and estimated a separate model for each of these groups. The results are listed under models B–D in the order of small, medium and large farms.[4] This was done to determine whether the production structure differs across farms of different sizes. If so, as evidenced by the likelihood ratio (LR) test, estimates for the production parameters as well as inefficiency based on model A are inappropriate. However, for purposes of comparison, we present the results for all the models (models A–D).

The parameter estimates are reported in table 3. The labor coefficient (elasticity) is about twice as large as that of capital in each of these models. For labor the highest value is 0.382 and is associated with small farms (model B). The large farm has the smallest labor elasticity. This indicates that increasing management and/or labor time on the small farms will have a greater impact on a percentage basis than on the large farms, suggesting that small farms would benefit if the owner-operator spent more time on the farm or hired additional labor. The order of the magnitude for the capital coefficients is similar to that of labor—highest for small farms and lowest for large farms. The coefficient of land is quite small but statistically significant in all the groups as well as in the full model (model A).

For exogenous factors affecting productivity, the crucial variables are education, off-farm income,

[4] The appropriateness of dividing the sample into groups is tested by the likelihood ratio (LR) test—where the restricted version is model A (with pooled data from all the groups) and the unrestricted version is models B–D. The hypothesis of a single production function (model A) for all the farms of different sizes is rejected at the 1% level of significance.

TABLE 3.—MAXIMUM LIKELIHOOD ESTIMATES OF MODELS A–D
(ASYMPTOTIC STANDARD ERRORS IN PARENTHESES)

Parameter	Model A	Model B	Model C	Model D
Constant	9.9890	7.6250	8.9983	10.0921
	(0.423)	(.2061)	(0.733)	(.0555)
Labor	.2681	.3820	.2558	.1902
	(.0028)	(.0199)	(.0036)	(.0035)
Capital	.1197	.1651	.1151	.1021
	(.0034)	(.0074)	(.0038)	(.0063)
Land	.0324	.0102	.0031	.0647
	(.0005)	(.0013)	(.0005)	(.0005)
Education	.1372	.1054	.3367	.3071
	(.0106)	(.0159)	(.0143)	(.0119)
Off-farm income	−.0156	−.0105	−.0090	−.0046
	(.0003)	(.0018)	(.0005)	(.0004)
Size dummy (medium)	−.3965	—	—	—
	(.0035)			
Size dummy (small)	−.7399	—	—	—
	(.0063)			
σ_v	.0393	.0371	.0402	.0326
	(.0002)	(.0005)	(.0003)	(.0003)
σ_τ	.5522	.6166	.6653	.1491
	(.0787)	(.0105)	(.2011)	(.0052)
σ_ξ	.6131	.7425	.5217	.4204
	(.1216)	(.1210)	(.0729)	(.1234)
Incidental parameters:				
μ_k	−.0213	−.2797	−.0248	.0083
σ_{kk}	.3808	.5366	.3415	.3106

and size-related dummy variables.[5] Education has a very high positive and statistically significant coefficient. This suggests that education increases output by increasing productivity of labor and capital. The effect of education is strongest for the medium-sized farms (model C). Off-farm income on the other hand has a negative effect on output. This negative effect is strongest for the small farms which have the largest off-farm incomes. As noted earlier, the larger the off-farm income the less time the operator will spend on the farm. Consequently, the production decision will be based on less economic information, which tends to reduce efficiency.

Effects of education and off-farm income on output can be examined more closely from the estimates of elasticities. It can be seen from (12) that the elasticity of Z_k on Y is $\beta_k/(1 - r)$. Moreover, the elasticity of Z_k on X_i is also $\beta_k/(1 - r)$ for all i. Concentrating on models B–D, we can see that education has the highest elasticity (0.5352) on medium-sized farms and the lowest elasticity (0.2327) on small farms. Thus gains from increasing education are highest for the medium-sized farms. For off-farm income, these elasticities are −0.0232, −0.0143 and −0.0065 on small, medium, and large farms, respectively. This indicates that the loss in productivity due to off-farm income is highest on the small farms.

The size-related dummy variables in model A show that, on average, outputs of small farms are 52.28% $(1 - \exp(-.7399))$ lower and those of medium-sized farms are 32.73% $(1 - \exp(-.3965))$ lower compared with large farms, ceteris paribus.[6] Some economies of size are likely to exist in the dairy industry which would account for large farms being more efficient than smaller farms. However, since the data reject model A in favor of separate models by farm size, it would be interesting to see whether similar conclusions could be reached from the latter models.

We now return to the estimates of technical inefficiency, τ, allocative inefficiency, u_j and scale inefficiency, ξ. These inefficiencies can be estimated for each farm by using (9), (10) and (13). Instead of reporting τ, u_j, and ξ, we report PT, percentage loss of output due to technical inefficiency; CA, percentage increase in cost due to allocative inefficiency; and Π_s, percentage loss of profit due to scale inefficiency. These are given in equations (9a), (11) and (15). To conserve space we report only the mean values of PT, CA, and Π_s by farm size. In table 4a, we report these results based on the pooled data (model A). The estimates of PT show that the large farms are technically more efficient than small and medium-sized farms. Output of small farms, on the average, could have been 52.12% higher had these farms been operating on the production frontier. The corresponding figures for the medium and large farms are 31.56% and 20.02%, respectively. Thus, groups of small and medium-sized farms are $(.5212 - .2002)100 = 32.10\%$ and $(.3156 - .2002)100 = 11.54\%$ less efficient (technically) relative to the group of large farms. These numbers indicate how much potential exists for raising

[5] It is to be noted that these dummy variables are used in model A with the implicit assumption that farm sizes affect the intercepts only in the logarithmic form of the production function.

[6] If the objective is only to see whether large farms are, on the average, technically more efficient than medium and small farms frontier models may not be necessary (see Lau and Yotopoulos (1971), Sidhu (1974)).

TABLE 4A.—MEASURES OF INEFFICIENCY BY SIZE OF DAIRY FARM (BASED ON MODEL A)

Measure	Small	Medium	Large
PT^a	52.12%	31.56%	20.02%
CA^b	5.89%	3.62%	3.87%
Π_s^c	18.56%	11.18%	7.96%
Π_s^F	13.16%	8.12%	6.06%

[a] Percentage loss in output due to technical inefficiency.
[b] Percentage increase in cost due to allocative inefficiency.
[c] Percentage loss in profit due to scale inefficiency. The superscript F in Π_s indicates the Forsund et al. measure.

TABLE 4B.—MEASURES OF INEFFICIENCY BY SIZE OF DAIRY FARM (BASED ON MODELS B–D)

Measure	Small (Model B)	Medium (Model C)	Large (Model D)
PT^a	31.69%	11.46%	20.16%
CA^b	5.91%	3.74%	3.58%
Π_s^c	19.52%	11.22%	5.59%
Π_s^F	13.73%	9.21%	3.45%

Note: See notes to table 4a.

output without increasing input use.[7] The estimates of CA show that cost of the small farms, on the average, is increased by 5.89% due to allocative inefficiency. For medium-sized and large farms such costs are 3.62% and 3.87%, respectively. These results suggest that technical inefficiency is more serious for these dairy farms than allocative inefficiency.

The estimates of OU (based on model A) show that actual outputs exceed optimum levels for 31.58% of the small farms. Actual output exceeds the optimum, on the average, by 68.89% and 81.48%, respectively, for medium- and large-sized farms. If the FLS definition of scale inefficiency is used, the percentage of farms producing excess output reduces to 26.31, 53.33 and 70.37, respectively.[8] These results suggest that the Utah industry is characterized by over-production, especially by the large farms. Milk prices decreased during the three years prior to this study period (Utah Agricultural Statistics). Consequently, these farms may not have fully adjusted their factors of production (cows, barns, corrals, etc.) to lower milk prices. However, the estimates of Π_s, suggest that small farms as a group could increase their profit by 13.16% (18.56% if the FLS definition is used) by producing milk at optimum levels. For medium and large farms these numbers are 8.17% (11.18%) and 6.06% (7.96%), respectively.

The estimates of PT, CA, and Π_s reported in table 4a are based on model A, the specification of which is rejected in favor of separate models by farm size. Results based on such separate models by farm size are reported in table 4b. Examining the results only for small farms (model B) it can be seen that the estimates of loss of output due to technical inefficiency, PT, are much lower compared with model A. The estimates of percentage increase in cost due to allocative inefficiency and percentage loss of profit due to scale inefficiency are more or less the same. Similar results are found for the medium-sized farms (model C). For large farms, the results in the two models are quite similar except for Π_s, which are somewhat smaller in model D for both measures of scale inefficiency.

Results relating to technical, allocative, and scale inefficiencies from the single as well as the separate models by farm size explain why over time the percentage of large farms increased relative to small and medium-sized dairy farms in Utah. Large farms, on the average, are 13.52% more efficient (technically) relative to the small farms (table 4b). These are, on the average, more efficient than small farms in allocating inputs as well as choosing optimal level of output. These are reflected in lower values of CA and Π_s for the large farms.

VI. Conclusions

This paper has been primarily concerned with investigating technical, allocative, and scale inefficiency of owner-operators of dairy farms in Utah. The distinguishing feature of the model is estimating these inefficiencies in a simultaneous equation framework. A stochastic production frontier technique, which gives estimates of each type of inefficiency for every individual farm, has been applied for this purpose.

The results obtained indicate that there is a positive association between farmer education and productive efficiency. Education is associated with

[7] One can, however, question whether all the inputs have been captured, or whether inputs are correctly captured. Similarly, if soil quality varies, the inefficient farms may be those with poor quality of soil (Schmidt (1985)).

[8] We also find that numbers of farms for which the direction of scale inefficiency changes (from positive to negative) are 1, 7 and 3 in the groups of small, medium and large farms, respectively, when the FLS definition is used.

greater productivity because it improves managerial ability and enhances the productivity of capital and labor. The empirical findings also indicate that productivity is negatively related to off-farm income. The larger the off-farm income the less time the farm-operator spends managing farm operations. Consequently, production decisions based on insufficient information are less efficient. On farm size and efficiency the large farms were found to be the most efficient.

Separate estimates of technical, allocative, and scale inefficiencies have been made for all farms. Results indicate that large farms are technically more efficient than small farms. Output for these farms, on the average, is 11.53% higher than the small farms, ceteris paribus. However, the output of large farms, on the average, would have increased by 20.16% had these farms been operating on the production frontier. The corresponding figure for medium-sized farms is 11.46%. Due to allocative inefficiency, costs of small farms, on the average, are increased by 5.91% whereas the figures are 3.74% for medium-sized farms and 3.58% for large farms. Most of the farmers in all size categories are found to be scale inefficient. Loss of profit due to scale inefficiency ranges from 5.59% (for large farms in model D) to 13.73% (for small farms in model B). One probable explanation is that milk prices decreased during the three years prior to this study period and the farms may not have fully adjusted their outputs to the change in prices.

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