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Cointegration, market integration, and market efficiency

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Abstract

This paper uses a standard dynamic general equilibrium asset-pricing model to study the relationship among the concepts of efficient markets, integrated markets, and cointegrated prices. Within this setting we show that these concepts are independent of one another and any zero, one, two, or three of these characteristics can emerge in equilibrium, depending upon taste, endowment, and technology parameters. In particular, the results of tests of cointegration among asset prices have no implications about market efficiency or market integration without additional restrictions on the economy or economies.

JEL classification: C32; E21; F15; G14; G15

Keywords: Efficient markets; Market integration; Cointegration

1. Introduction

Since Engle and Granger (1987) introduced the concept of cointegration into the time series literature, its implications for the time series behavior of economic data have been studied and tested in a wide variety of settings. This widespread interest in

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cointegration appears to have arisen from a combination of forces. First, many univariate economic time series appear to behave like difference-stationary or integrated of order one [$I(1)$] series, a requirement for cointegration. Second, cointegration often provides an appealing way to represent long-run equilibrium relationships implied by economic theory. Third, a voluminous specialized econometric literature has developed to facilitate estimation and inference when cointegration may be a feature of the data.

The efficiency and integration of asset markets are two basic issues in financial economics that have been examined using cointegration methods.[1] The empirical literature on cointegration and market efficiency is based on the premise that if asset prices in two efficient markets are $I(1)$ processes, then these prices cannot be cointegrated. For example, Granger (1986) states: "If x_t, y_t are a pair of prices from a jointly efficient, speculative market, they cannot be cointegrated." The empirical literature on cointegration and market integration is based on the premise that if two markets are (economically) integrated and their respective prices are $I(1)$, then these prices must be cointegrated. Of course, these two premises lead to the disconcerting conclusion that integrated financial markets cannot be efficient markets!

The conclusion that market efficiency and cointegrated prices are incompatible was first challenged by Dwyer and Wallace (1992). They used time series arguments to show that when the random walk condition is replaced with a no-arbitrage condition, any link between efficiency and cointegration disappears. Whether asset prices in efficient markets are cointegrated or not depends upon other features of the underlying model, a point subsequently reinforced by Engel (1996). To our knowledge, the premise that cointegrated prices are necessary for market integration has never been challenged or carefully studied.

The main objective of this paper is to study and clarify the relationship among market efficiency, market integration, and statistical cointegration from a theoretical point of view. We will work within the setting of a standard general equilibrium asset-pricing framework, specialized to assure that equilibrium asset prices emerge as $I(1)$ processes. However, any point in the product space formed by (markets efficient, markets not efficient) $\times$ (markets integrated, markets not integrated) $\times$ (prices cointegrated, prices not cointegrated) can be supported as an equilibrium outcome, depending upon the values of certain parameters. Consequently, we will argue that in the absence of a sufficiently well-specified model, cointegration test results are not informative with respect to either market efficiency or market integration.

[1] The following is a partial list of the approximately 40 relevant applications we found. The complete list of references is available upon request. Cointegration tests have been applied to test the efficiency of equity markets (Cerchi and Havenner, 1988) and foreign exchange markets (Baillie and Bollerslev, 1989; Hakkio and Rush, 1989; MacDonald and Taylor, 1989; Coleman, 1990; Booth and Mustafa, 1991; Copeland, 1991; Crowder, 1994). These tests have been applied to test market integration in the context of equity markets (Taylor and Tonks, 1989; Kasa, 1992; Corhay et al., 1993; Richards, 1995), security markets (Goodwin and Grennes, 1994; Alexakis et al., 1997; Bremnes et al., 1997), foreign exchange markets (Booth and Mustafa, 1991), commodity markets (Ardeni, 1989), and banking product markets (Jackson and Eisenbeis, 1997).

This paper extends the Dwyer and Wallace (1992) work in at least three important ways, the first being primarily a pedagogical contribution while the other two extensions are more substantive contributions. First, Dwyer and Wallace study the implications of market efficiency for the cointegration of asset prices from a time series point of view, whereas we study the issue from a general equilibrium asset-pricing model, making the economics of the relationship more transparent. Second, we use this model to examine the relationship between market efficiency and market integration. Finally, we use the model to study the relationship between market integration and statistical cointegration.

The remainder of the paper is organized as follows. Basic definitions are provided in Section 2, the model is described in Section 3, and implications are derived in Section 4. Section 5 gives an intuitive interpretation of the results, and Section 6 further discusses some relevant issues. A summary and conclusion are given in Section 7.

2. Basic definitions

To avoid ambiguities, before proceeding it will be useful to provide some standard definitions of market efficiency, market integration, and cointegrated time series. These definitions will be incorporated into the remainder of the paper.

2.1. Market efficiency

A capital market is said to be efficient if it fully and correctly reflects all relevant information in determining security prices. Formally, the market is said to be efficient with respect to some information set, Φ, if security prices would be unaffected by revealing that information to all participants (from the New Palgrave Dictionary of Money and Finance, p. 739).

2.2. (International capital) Market integration

Capital market integration can be defined as a situation where assets in different currencies or countries display the same risk-adjusted expected returns. Segmentation, in contrast, implies that the risk-return relationship in each national market is primarily determined by domestic factors (from the New Palgrave Dictionary of Money and Finance, p. 454).

2.3. Cointegration

If x_t, y_t are $I(1)$ but there exists a linear combination $z_t = m + ax_t + by_t$ which is both $I(0)$ and has a zero mean, then x_t, y_t are said to be cointegrated (Engle and Granger, 1991, p. 6).

2.4. Remarks

1. Note that the definitions do not imply that efficient markets are integrated markets or vice versa. Nor do they restrict whether or not prices are cointegrated in either efficient markets or integrated markets.
2. Pure random walks cannot be cointegrated, unless they are perfectly correlated. So, to the extent that market efficiency is interpreted to mean that asset prices are random walks, the argument that prices in efficient markets cannot be cointegrated is correct. However, once one has argued that asset prices are random walks, the restriction that they are not cointegrated is redundant.
3. Suppose that market efficiency is interpreted to mean that asset prices are random walks. Then rejection of the random walk null in favor of serially correlated price changes will imply rejection of market efficiency. However, serially correlated asset-price changes can occur regardless of whether asset prices are or are not cointegrated.
4. The definition of market integration is, in essence, a definition of no arbitrage opportunities across economies or "countries." If there are no trade frictions and agents across two economies make optimal decisions based on the same information set (which may be labeled as "efficient" or "inefficient", depending on the chosen definition of efficiency), under mild conditions on preferences markets will present no arbitrage opportunities (LeRoy and Warner, 2001). This implies not only that assets with identical payoffs must have the same price (i.e., the "Law of One Price" holds), but also that the prices of assets with different payoffs are related because they can all be priced by the same positive stochastic discount factor (LeRoy and Warner, 2001). As a result, the two economies may be treated as a single larger economy inhabited by a representative agent characterized by the aforementioned stochastic discount factor.

3. A representative-agent economy with multiple goods and multiple assets

Consider an economy inhabited by a representative agent who maximizes her expected utility of consumption. The agent's felicity function satisfies standard regularity conditions and is represented by $U(c_t)$, $c_t \equiv [c_{1t},\ldots,c_{Jt}]$. Variable c_{jt} ($j = 1,\ldots,J$) denotes the agent's consumption of nonstorable good j at time t, whose price and total supply are given by p_{jt} and y_{jt}, respectively. Vector $z_t \equiv [z_{1t},\ldots,z_{Jt}]$ consists of the agent's holding of J equity assets. One unit of equity asset j costs q_{jt} in period t, and yields a payoff consisting of dividends plus the ex-dividend value of the equity asset ($p_{jt+1}\, y_{jt+1} + q_{jt+1}$) in period $t + 1$. Without loss of generality, the total number of units of asset j in the economy is set equal to one.

In each period, the state of the economy is denoted by $s_t \in S$, and the agent's initial endowment of equity assets is given by z_{t-1}. Given such endowments and the state of the economy, the agent is assumed to choose the levels of consumption (c_t) and equity asset holdings (z_t) that maximize her expected utility over all possible states in the next period (s_{t+1}):

$$V(z_{t-1}, s_t, y_t) = \max_{c_t, z_t} \left[U(c_t) + \beta \int_S V(z_t, s_{t+1}, y_{t+1}) F(s_t, \mathrm{d}s_{t+1}) \right]. \tag{1}$$

In Eq. (1), $V(z_{t-1},s_t,y_t)$ is the agent's value function, $0 < \beta < 1$ is her subjective discount rate, and $F(s_t,\mathrm{d}s_{t+1})$ is the probability density function of next period's state conditional on this period's state of the economy. Optimization (1) is subject to the budget constraint (2) and to the feasibility constraints (3) and (4):

$$\sum_{j=1}^{J} p_{jt} c_{jt} + \sum_{j=1}^{J} q_{jt} z_{jt} \le \sum_{j=1}^{J} (p_{jt} y_{jt} + q_{jt}) z_{jt-1}, \tag{2}$$

$$0 \le c_{jt} \le y_{jt}, \quad j = 1, \ldots, J, \tag{3}$$

$$0 \le z_{jt} \le \bar{z}_{jt} > 1, \quad j = 1, \ldots, J. \tag{4}$$

Condition (4) states that, at the optimum, the agent holds a positive number of units of each asset, and that such holding has an upper bound strictly greater than the number of units of the respective asset in the economy $\left(\bar{z}_{jt} > 1\right)$.[2]

Assuming an interior solution, the first order conditions for optimization are:

$$U_j(c_t) - \eta_t p_{jt} = 0, \tag{5}$$

$$\beta \int_S \frac{\partial V(z_t, s_{t+1}, y_{t+1})}{\partial z_{jt}} F(s_t, \mathrm{d}s_{t+1}) - \eta_t q_{jt} = 0, \tag{6}$$

for all j. In Eq. (5), $U_j(c) \equiv \partial U(c)/\partial c_j$ and η_t denotes the Lagrangean multiplier corresponding to the budget constraint (2). But Eq. (5) implies that $\eta_t = U_j(c_t)/p_{jt}$, so that $\partial V(z_{t-1}, s_t, y_t)/\partial z_{jt-1} = \eta_t(p_{jt} y_{jt} + q_{jt}) = U_j(c_t)(y_{jt} + q_{jt}/p_{jt}) \,\forall j$. Hence:

$$\frac{\partial V(z_t, s_{t+1}, y_{t+1})}{\partial z_{jt}} = U_j(c_{t+1}) \left(y_{jt+1} + \frac{q_{jt+1}}{p_{jt+1}} \right), \tag{7}$$

[2] As pointed out later, market clearing requires that demand equal supply for each asset, so in equilibrium it must be the case that $z_{jt} = 1 \ \forall\, j$.

for all j. By using Eqs. (5) and (7), first order condition (6) can be rewritten as:

$$U_j(c_t)\frac{q_{jt}}{p_{jt}}=\beta\int_S U_j(c_{t+1})\left(y_{jt+1}+\frac{q_{jt+1}}{p_{jt+1}}\right)F(s_t,\mathrm{d}s_{t+1}), \tag{8}$$

for all j. The above expression may be manipulated further to yield:

$$\psi_{jt}=\beta\int_S\frac{U_j(c_{t+1})}{U_j(c_t)}\frac{y_{jt+1}}{y_{jt}}(1+\psi_{jt+1})F(s_t,\mathrm{d}s_{t+1}), \tag{9}$$

where $\psi_{jt}\equiv q_{jt}/(p_{jt}y_{jt})$. That is, ψ_{jt} is asset j's price/dividend ratio at time t.

3.1. A solution

To solve for an equilibrium in the present economy, the agent's preferences and the economy's technology need to be specified more explicitly. In particular, it is assumed that the agent's felicity is of the (standard) power separable form:

$$U(c)=\sum_{j=1}^{J}u(c_j,\gamma_j), \tag{10}$$

where $u(c_j,\gamma_j\neq 1)\equiv c_j^{1-\gamma_j}/(1-\gamma_j)$ and $u(c_j,\gamma_j=1)\equiv\ln(c_j)$. Hence, $U_j(c)=c_j^{-\gamma_j}$. In addition, it is assumed that next period's total supply of good j ($j=1,\ldots,J$) is given by:

$$y_{jt+1}=\lambda_j(s_{jt+1})y_{jt},\quad \lambda_j(s_{jt+1})=\exp(\mu_j+s_{jt+1}), \tag{11}$$

with $s_{jt+1}=\varepsilon_{jt+1}+\alpha_{1jt}+\alpha_{2jt}$, where ε_{jt+1}, α_{1jt}, and α_{2jt} are i.i.d. random technology shocks whose magnitudes become available as of times $t+1$, t, and t, respectively.

Random shocks α_{1t} and α_{2t} may be interpreted as time-t signals about s_{t+1}. The reason for distinguishing between α_{1t} and α_{2t} is to model efficient and inefficient markets separately from cointegrated and not cointegrated supply processes. More specifically, a financial market is efficient (inefficient) if its prices at any time t (do not) convey all of the information available at that time (see Section 2). Hence, when solving her optimization problem at time t, it is assumed that the representative agent always relies upon the realized values of s_t (i.e., ε_t, α_{1t-1}, and α_{2t-1}) and α_{1t}, but may or may not use the realized value of α_{2t}. The resulting equilibrium will be efficient if the representative agent does incorporate the realized value of α_{2t} in her time-t decisions, and inefficient otherwise.

Random shock α_{1jt} is the vehicle to incorporate statistical cointegration into the analysis. In the scenarios where supply processes are assumed to be not cointegrated, α_{1jt} is left unstructured. In contrast, scenarios assuming cointegration among supply processes restrict α_{1jt} to be a function of the concurrent differences in supplies across goods (see e.g., the discussion of Eq. (23) through Eq. (25) below).

3.2. Case 1: efficient markets

To model the efficient market scenario, it is assumed that the representative agent uses the realized values of both α_{1t} and α_{2t} when solving her optimization problem. This means that the distributions of α_{1t} and α_{2t} conditional on the information used by the agent at time t (and later) are degenerate, with means α_{1t} and α_{2t}, respectively, and zero variances. It is also assumed that, conditional on the information used by the agent at time $t-1$ (and earlier), α_{1jt} and α_{2jt} are identically and independently distributed as normal with zero means and variances $\sigma^2_{\alpha 1j}$ and $\sigma^2_{\alpha 2j}$, respectively:[3]

$$\alpha_{ijt} \sim \text{i.i.d. N}(0, \sigma^2_{\alpha ij}), \tag{12}$$

for $i = 1, 2$ and $j = 1,\ldots,J$.

The distribution of ε_{jt+1} conditional on the information used by the agent at time t (and earlier) is identically and independently normal with mean zero and variance $\sigma^2_{\varepsilon j}$:

$$\varepsilon_{jt+1} \sim \text{i.i.d. N}(0, \sigma^2_{\varepsilon j}), \tag{13}$$

for all j. Further, the distribution of ε_{t+1} conditional on the information used by the agent at time $t+1$ (and later) is degenerate, with mean ε_{t+1} and variance zero.

Given the above assumptions, the solution to Eq. (9) is as follows:

$$\psi_{jt} = \frac{E_j}{1 - A_{1j}A_{2j}E_j} \exp[(1-\gamma_j)(\alpha_{1jt} + \alpha_{2jt})], \tag{14}$$

where $E_j \equiv \beta \exp[(1-\gamma_j)\mu_j + 0.5(1-\gamma_j)^2\sigma^2_{\varepsilon j}] > 0$ and $A_{ij} \equiv \exp[0.5(1-\gamma_j)^2\sigma^2_{\alpha ij}] > 0$ for $i = 1, 2$. To prevent nonsensical negative dividend/price ratios, it must be the case that $A_{1j}A_{2j}E_j < 1$. It is straightforward to verify that Eq. (14) is indeed a solution to Eq. (9). To see this, note that nonsatiation implies that $c_{jt} = y_{jt}\ \forall\ j$ in equilibrium, so that $U_j(c_{t+1})/U_j(c_t) = (c_{jt+1}/c_{jt})^{-\gamma_1} = (y_{jt+1}/y_{jt})^{-\gamma_1}$.[4] Hence, Eq. (9) may be rewritten as:

$$\psi_{jt} = \beta \int_S \left[\frac{y_{jt+1}}{y_{jt}}\right]^{1-\gamma_j} (1 + \psi_{jt+1}) F(s_t, \mathrm{d}s_{t+1}), \tag{15}$$

$$= \beta \int_S [\lambda_j(s_{jt+1})]^{1-\gamma_j} (1 + \psi_{jt+1}) F(s_t, \mathrm{d}s_{t+1}),$$

$$= \beta \int_S [\exp(\mu_j + \varepsilon_{jt+1} + \alpha_{1jt} + \alpha_{2jt})]^{1-\gamma_j} (1 + \psi_{jt+1}) F(s_t, \mathrm{d}s_{t+1}),$$

[3] Lack of contemporaneous correlation among the α_{ijt}s is assumed only for simplicity. None of the main results require this assumption.

[4] In equilibrium, market clearing also requires that $z_{jt} = 1\ \forall\ j$.

$$= \exp[(1-\gamma_j)(\alpha_{1jt}+\alpha_{2jt})]\beta\int_S \exp[(1-\gamma_j)(\mu_j+\varepsilon_{jt+1})](1+\psi_{jt+1})F(s_t, \mathrm{d}s_{t+1}). \tag{16}$$

Expression (14) follows immediately from Eq. (16) by letting ψ_{jt} equal to a constant times $\exp[(1-\gamma_j)(\alpha_{1jt}+\alpha_{2jt})]$, and $F(s_t, \mathrm{d}s_{t+1})$ be the multivariate normal pdf corresponding to Eqs. (12) and (13).

3.3. Case 2: inefficient markets

Since an inefficient market means that its prices at time t do not convey all of the information available at that time, in the present framework this is modeled by the representative agent disregarding the realized value of α_{2jt} when solving for her optimization problem at time t. That is, conditional on the information used by the agent at time t (and earlier), α_{2jt} is identically and independently distributed as normal with mean zero and variance $\sigma^2_{\alpha 2j}$:

$$\alpha_{2jt} \sim \text{i.i.d. N}(0, \sigma^2_{\alpha 2j}), \tag{17}$$

for all j. And the distribution of α_{2t} conditional on the information used by the agent at time $t+1$ (and later) is degenerate, with mean α_{2t} and variance zero. The conditional distributions of ε_{jt+1} and α_{1jt} are the same as for the efficient market scenario.

In this instance, the solution to the dividend/price ratio in Eq. (9) is:

$$\psi_{jt} = \frac{A_{2j}E_j}{1-A_{1j}A_{2j}E_j}\exp\left[(1-\gamma_j)\alpha_{1jt}\right]. \tag{18}$$

Comparing Eqs. (14) and (18), it is clear that the dividend/price ratio at time t in the efficient market setting (Eq. (14)) reflects all of the information available at that time, and therefore fluctuates as the signals α_{1t} and α_{2t} do. In contrast, the inefficient-market price ratio (18) incorporates signal α_{1t} but not signal α_{2t} (although it reflects the (conditional) expectation of signal α_{2t}).

4. Cointegration, market integration, and market efficiency

There are four possible combinations concerning (market efficiency, market integration), namely, (yes, yes), (yes, no), (no, yes) and (no, no). To demonstrate that the concept of cointegration is unrelated either to market integration or to market efficiency, it will be shown that equity prices may be either cointegrated or not cointegrated in all four of the aforementioned (market efficiency, market integration) combinations. This is followed by an intuitive interpretation of the results in Section 5.

4.1. Market integration

The absence of arbitrage implies the existence of a positive stochastic discount factor, but the latter need not be unique (LeRoy and Warner, 2001). In the present study, this ambiguity is resolved by postulating a stylized representative-agent endowment economy and solving for its equilibrium. We first use this single economy to analyze the case of integrated markets, because in the postulated economy all of the J equity assets are priced by the same unique stochastic discount factor.[5] Hence, in such an economy the markets for all assets are integrated by construction. To make the cointegration analysis nontrivial, we focus on the implications of market integration for the cointegration between the prices of assets with different payoffs.[6]

4.1.1. Scenario 1.1: market integration and market efficiency with equity asset prices not cointegrated

Consider the price of any equity asset j under the joint assumptions of market integration and market efficiency. Given the price/dividend ratio (14), equity asset prices are:

$$q_{jt} = p_{jt} y_{jt} \frac{E_j}{1 - A_{1j} A_{2j} E_j} \exp\left[(1 - \gamma_j)(\alpha_{1jt} + \alpha_{2jt})\right]. \tag{19}$$

Without loss of generality, let good 1 be the numeraire (i.e., $p_{1t} = 1 \ \forall \ t$). Then, first order condition (5) implies that $p_{jt} = U_j(c_t)/U_1(c_t) = c_{jt}^{-\gamma_j}/c_{1t}^{-\gamma_1}$. Noting that equilibrium requires $c_{jt} = y_{jt} \ \forall \ j$, Eq. (19) may be rewritten as:

$$q_{jt} = y_{jt}^{1-\gamma_j} y_{1t}^{\gamma_1} \frac{E_j}{1 - A_{1j} A_{2j} E_j} \exp\left[\left(1 - \gamma_j\right)\left(\alpha_{1jt} + \alpha_{2jt}\right)\right]. \tag{20}$$

Finally, taking natural logarithms on both sides of Eq. (20) yields:

$$\ln(q_{jt}) = \ln\left(\frac{E_j}{1 - A_{1j} A_{2j} E_j}\right) + \gamma_1 \ln(y_{1t}) + (1 - \gamma_j) \ln(y_{jt}) + (1 - \gamma_j)(\alpha_{1jt} + \alpha_{2jt}). \tag{21}$$

[5] Recall from Remark 4 in Section 2 that two integrated economies may be treated as a single larger economy.

[6] The "Law of One Price" must hold in the postulated economy, so assets with identical payoffs have identical prices. Therefore, such prices are obviously cointegrated.

Expression (21) may also be used to obtain the following formula for the first difference in the logarithms of prices:

$$\Delta\ln(q_{jt})=\gamma_1\mu_1+(1-\gamma_j)\mu_j+\gamma_1(\varepsilon_{1t}+\alpha_{11t-1}+\alpha_{21t-1})+(1-\gamma_j)(\varepsilon_{jt}+\alpha_{1jt}+\alpha_{2jt}), \tag{22}$$

where $\Delta\ln(q_{jt})\equiv\ln(q_{jt})-\ln(q_{jt-1})$.

From Eqs. (21) and (22), it follows that the logarithm of any of the equity asset prices is integrated of order one. Further, it is also clear from Eqs. (21) and (22) that there cannot be any cointegration relationships among the logarithms of equity asset prices q_{jt} and q_{kt}, if there are no cointegration relationships among the supply processes $\ln(y_{jt})$ and $\ln(y_{kt})$.

4.1.2. Scenario 1.2: market integration and market efficiency with cointegrated equity asset prices

Although no cointegration has been assumed so far, the endowment process (11) is consistent with cointegration across endowments. To see this, note that taking logarithms on both sides of Eq. (11) and rearranging yields:

$$\ln(y_{jt+1})-\ln(y_{jt})=\mu_j+\alpha_{1jt}+\alpha_{2jt}+\varepsilon_{jt+1}, \tag{23}$$

and letting $\alpha_{1jt}\equiv\phi_{jk}[\ln(y_{jt})-\ln(y_{kt})]$ and $\alpha_{1kt}\equiv\phi_{kj}[\ln(y_{jt})-\ln(y_{kt})]$ for $k\neq$ j, where ϕ_{jk} and ϕ_{kj} are parameters, one obtains the following error correction model for endowments j and k:

$$\Delta\ln(y_{jt+1})=\mu_j+\phi_{jk}[\ln(y_{jt})-\ln(y_{kt})]+\alpha_{2jt}+\varepsilon_{jt+1}. \tag{24}$$

$$\Delta\ln(y_{kt+1})=\mu_k+\phi_{kj}[\ln(y_{jt})-\ln(y_{kt})]+\alpha_{2kt}+\varepsilon_{kt+1}. \tag{25}$$

Systems (24) and (25) imply that endowments j and k are cointegrated, with a long term relationship represented by $\ln(y_j)=\ln(y_k)$ (i.e., a cointegrating vector [1, −1]). Of course, in this particular example α_{1jt} and α_{1kt} are not independently distributed, as they are perfectly correlated. More complex cointegration relationships can be constructed in a similar fashion.

After plugging in the definition of α_{1jt} and α_{1kt} into Eq. (21), the following equations for the logarithms of asset prices j and k are obtained:

$$\begin{aligned}\ln(q_{jt})=&\ln\left(\frac{E_j}{1-A_{1j}A_{2j}E_j}\right)+\gamma_1\ln(y_{1t})+(1-\gamma_j)(1+\phi_{jk})\ln(y_{jt})\\&-(1-\gamma_j)\phi_{jk}\ln(y_{kt})+(1-\gamma_j)\alpha_{2jt},\end{aligned} \tag{26}$$

$$\begin{aligned}\ln(q_{kt})=&\ln\left(\frac{E_k}{1-A_{1k}A_{2k}E_k}\right)+\gamma_1\ln(y_{1t})+(1-\gamma_k)(1-\phi_{kj})\ln(y_{kt})\\&+(1-\gamma_k)\phi_{kj}\ln(y_{jt})+(1-\gamma_k)\alpha_{2kt}.\end{aligned} \tag{27}$$

Subtracting Eq. (27) from Eq. (26) and rearranging yields:

$$\ln(q_{jt}) - \ln(q_{kt}) = \ln\left(\frac{E_j}{1 - A_{1j}A_{2j}E_j}\right) - \ln\left(\frac{E_k}{1 - A_{1k}A_{2k}E_k}\right) - \gamma_j\left[\ln(y_{jt}) - \frac{\gamma_k}{\gamma_j}\ln(y_{kt})\right] + [1 + \phi_{jk}(1-\gamma_j) - \phi_{kj}(1-\gamma_k)][\ln(y_{jt}) - \ln(y_{kt})] + (1-\gamma_j)\alpha_{2jt} - (1-\gamma_k)\alpha_{2kt}. \tag{28}$$

It is clear from Eq. (28) that $\ln(q_{jt})$ and $\ln(q_{kt})$ are cointegrated (with cointegrating vector [1, −1]) as long as $\gamma_j = \gamma_k$. This is true because $\ln(y_{jt})$ and $\ln(y_{kt})$ are cointegrated by assumption. It is interesting to note that cointegration across endowments does not guarantee cointegration across asset prices, as evidenced by the non-stationarity of Eq. (28) when $\gamma_j \neq \gamma_k$.

4.1.3. Scenario 2.1: market integration and market inefficiency with equity asset prices not cointegrated

In the case of inefficient markets, the price/dividend ratio is Eq. (18). Following the same steps used to obtain Eq. (21), it is straightforward to derive the following expression for the logarithm of equity asset prices:

$$\ln(q_{jt}) = \ln\left(\frac{A_{2j}E_j}{1 - A_{1j}A_{2j}E_j}\right) + \gamma_1\ln(y_{1t}) + (1-\gamma_j)\ln(y_{jt}) + (1-\gamma_j)\alpha_{1jt}. \tag{29}$$

The same reasoning applied to Eq. (21) can be applied to Eq. (29) to conclude that the logarithm of any of the equity asset prices is integrated of order one, and that therecannot be any cointegration relationships among the logarithms of equity asset prices if there are no cointegration relationships among the supply processes.

4.1.4. Scenario 2.2: market integration and market inefficiency with cointegrated equity asset prices

Following the analysis performed under Scenario 1.2, one may obtain expressions (30) and (31) for the logarithms of asset prices j and k in the case of inefficient markets:

$$\ln(q_{jt}) = \ln\left(\frac{A_{2j}E_j}{1 - A_{1j}A_{2j}E_j}\right) + \gamma_1\ln(y_{1t}) + (1-\gamma_j)(1+\phi_{jk})\ln(y_{jt}) - (1-\gamma_j)\phi_{jk}\ln(y_{kt}), \tag{30}$$

$$\ln(q_{kt}) = \ln\left(\frac{A_{2k}E_k}{1 - A_{1k}A_{2k}E_k}\right) + \gamma_1\ln(y_{1t}) + (1-\gamma_k)(1-\phi_{kj})\ln(y_{kt}) + (1-\gamma_k)\phi_{kj}\ln(y_{jt}), \tag{31}$$

and the expression analogous to Eq. (28) is Eq. (32):

$$\ln(q_{jt}) - \ln(q_{kt}) = \ln\left(\frac{A_{2j}E_j}{1 - A_{1j}A_{2j}E_j}\right) - \ln\left(\frac{A_{2k}E_k}{1 - A_{1k}A_{2k}E_k}\right) - \gamma_j\left[\ln(y_{jt}) - \frac{\gamma_k}{\gamma_j}\ln(y_{kt})\right] + [1 + \phi_j(1 - \gamma_j) - \phi_k(1 - \gamma_k)][\ln(y_{jt}) - \ln(y_{kt})]. \quad (32)$$

As for Scenario 1.2, when $\ln(y_{jt})$ and $\ln(y_{kt})$ are cointegrated with cointegrating vector [1, −1], $\ln(q_{jt})$ and $\ln(q_{kt})$ are also cointegrated with cointegrating vector [1, −1] provided $\gamma_j = \gamma_k$.

4.2. No market integration

A simple way to model a lack of market integration in the present framework consists of postulating M autarkic economies equal in all respects, except for being characterized by different representative agents and different random processes.[7] In equilibrium, equity assets in the various economies are priced by different representative agents; therefore, such markets (economies) are not integrated by construction. To be able to compare prices across economies, all prices are expressed in terms of the first consumption good ($j = 1$).

4.2.1. Scenario 3.1: no market integration and market efficiency with equity asset prices not cointegrated

For each individual economy m, the situation is the same as discussed earlier in Section 4.1. In particular, asset prices in economy m obey Eq. (21), reproduced below as Eq. (33) with the addition of superscripts to identify the economy they correspond to:

$$\ln(q_{jt}^m) = \ln\left(\frac{E_j^m}{1 - A_{1j}^m A_{2j}^m E_j^m}\right) + \gamma_1^m \ln(y_{1t}^m) + (1 - \gamma_j^m)\ln(y_{jt}^m) + (1 - \gamma_j^m)(\alpha_{1jt}^m + \alpha_{2jt}^m). \quad (33)$$

As argued earlier in connection with Eq. (21), the logarithm of equity asset prices is integrated of order one if there are no cointegrating relationships among the supply processes within economy m. Further, equity asset prices are not cointegrated across economies if there are no cointegrating relationships among supply processes across economies.

[7] In particular, subscript j refers to exactly the same type of consumption good in all economies.

4.2.2. Scenario 3.2: no market integration and market efficiency with cointegrated equity asset prices

Expression (34) is the generalization of Eq. (28) to allow for different economies:

$$\begin{aligned}\ln(q_{jt}^{m}) - \ln(q_{kt}^{n}) = {} & \ln\left(\frac{E_j^m}{1 - A_{1j}^m A_{2j}^m E_j^m}\right) - \ln\left(\frac{E_k^n}{1 - A_{1k}^n A_{2k}^n E_k^n}\right) \\ & + \gamma_1^m\left[\ln(y_{1t}^m) - \frac{\gamma_1^n}{\gamma_1^m}\ln(y_{1t}^n)\right] - \gamma_j^m\left[\ln(y_{jt}^m) - \frac{\gamma_k^n}{\gamma_j^m}\ln(y_{kt}^n)\right] \\ & + [1 + \phi_{jk}^{mn}(1-\gamma_j^m) - \phi_{kj}^{nm}(1-\gamma_k^n)][\ln(y_{jt}^m) - \ln(y_{kt}^n)] \\ & + (1-\gamma_j^m)\alpha_{2jt}^m - (1-\gamma_k^n)\alpha_{2kt}^n, \end{aligned} \quad (34)$$

where ϕ_{jk}^{mn} and ϕ_{kj}^{nm} are defined by expressions $\alpha_{1jt}^m \equiv \phi_{jk}^{mn}[\ln(y_{jt}^m) - \ln(y_{kt}^n)]$ and $\alpha_{1kt}^n \equiv \phi_{kj}^{nm}[\ln(y_{jt}^m) - \ln(y_{kt}^n)]$, respectively. It follows from Eq. (34) that if supply processes $\ln(y_{jt}^m)$ and $\ln(y_{kt}^n)$ are cointegrated with cointegrating vector [1, −1], the corresponding asset prices $\ln(q_{jt}^m)$ and $\ln(q_{kt}^n)$ will be cointegrated with cointegrating vector [1, −1] provided that $\gamma_j^m = \gamma_k^n$ and the supply processes for the numeraire good 1 are cointegrated with cointegrating vector $[1, -\gamma_1^m/\gamma_1^n]$.

A nontrivial implication from the latter condition on the numeraire good's supply processes is that the choice of numeraire is crucial for whether prices are cointegrated across markets or not.

4.2.3. Scenario 4.1: no market integration and market inefficiency with equity asset prices not cointegrated

The expression analogous to Eq. (29) is Eq. (35):

$$\ln(q_{jt}^m) = \ln\left(\frac{A_{2j}^m E_j^m}{1 - A_{1j}^m A_{2j}^m E_j^m}\right) + \gamma_1^m \ln(y_{1t}^m) + (1-\gamma_j^m)\ln(y_{jt}^m) + (1-\gamma_j^m)\alpha_{1jt}^m. \quad (35)$$

From the discussions regarding Eqs. (21), (33), and (29), it is clear that prices are integrated of order one but are not cointegrated across economies if the supply processes are integrated of order one but not cointegrated.

4.2.4. Scenario 4.2: no market integration and market inefficiency with cointegrated equity asset prices

The generalization of Eq. (32) to potentially different economies is Eq. (36):

$$\begin{aligned}\ln(q_{jt}^{m}) - \ln(q_{kt}^{n}) = {} & \ln\left(\frac{A_{2j}^m E_j^m}{1 - A_{1j}^m A_{2j}^m E_j^m}\right) - \ln\left(\frac{A_{2k}^n E_k^n}{1 - A_{1k}^n A_{2k}^n E_k^n}\right) \\ & + \gamma_1^m\left[\ln(y_{1t}^m) - \frac{\gamma_1^n}{\gamma_1^m}\ln(y_{1t}^n)\right] - \gamma_j^m\left[\ln(y_{jt}^m) - \frac{\gamma_k^n}{\gamma_j^m}\ln(y_{kt}^n)\right] \\ & + [1 + \phi_{jk}^{mn}(1-\gamma_j^m) - \phi_{kj}^{nm}(1-\gamma_k^n)][\ln(y_{jt}^m) - \ln(y_{kt}^n)]. \end{aligned} \quad (36)$$

The key conclusions drawn in connection with Eq. (34) apply to Eq. (36) as well, because of the structural similarities between the two expressions. That is, if supply processes $\ln(y_{jt}^m)$ and $\ln(y_{kt}^n)$ are cointegrated with cointegrating vector [1, −1], the corresponding asset prices $\ln(q_{jt}^m)$ and $\ln(q_{kt}^n)$ will be cointegrated with cointegrating vector [1, −1] if $\gamma_j^m = \gamma_k^n$ and the supply processes for the numeraire good 1 are cointegrated with cointegrating vector $[1, -\gamma_1^m/\gamma_1^n]$. Again, this implies that whether prices are cointegrated across markets or not depends crucially on the choice of numeraire.

5. Intuition

The distinction between market integration and cointegration may be best understood with the help of some simple examples. First, consider why it is possible to have cointegrated asset prices across markets even if such markets are not integrated in an economic sense. To this end, assume two countries C_A and C_B, both characterized by endowment economies with a good such as wheat. Assume also that C_A and C_B are barred from trading with each other, so that markets C_A and C_B are not integrated by construction. Then, if wheat endowments in C_A and C_B are cointegrated (e.g., because wheat supply in both countries experience the same process of technological progress), the prices of assets paying dividends in the form of wheat in markets C_A and C_B will be cointegrated.[8] Note that prices may be different across markets in the short run, if short-run shocks on wheat endowments (e.g., due to weather) are uncorrelated across the two countries.

The case of market integration without cointegration can be illustrated by one country with a representative-agent endowment economy characterized by two goods, e.g., wheat and rice. The same representative agent prices all assets; therefore, by construction, the market for an asset paying wheat dividends is integrated with the market for an asset paying rice dividends. Wheat- and rice-based asset prices may be highly correlated in the short run.[9] This may happen, for example, if weather shocks have similar effects on wheat and rice production. However, asset prices will not be cointegrated if wheat and rice endowments are not cointegrated.[10] This would happen if the nature of technological progress driving the increase in supply over time was different across crops.

Regarding the distinction between cointegration and market efficiency, it must be pointed out that market inefficiency arises when asset prices do not reflect all information available at the time of decision making. In the economies used in our

[8] The prices of wheat will also be cointegrated across markets C_A and C_B.

[9] More precisely, non-zero correlation in the short run means that $\mathrm{Cov}_t(q_{\mathrm{wheat},t+1}, q_{\mathrm{rice},t+1})/[\mathrm{StDev}_t(q_{\mathrm{wheat},t+1})\ \mathrm{StDev}_t(q_{\mathrm{rice},t+1})] \neq 0$, where $\mathrm{Cov}_t(\cdot)$ and $\mathrm{StDev}_t(\cdot)$ denote, respectively, the covariance and the standard deviation conditional on information at time t.

[10] Further, as discussed in connection with Eq. (28), cointegrated endowments are necessary but not sufficient. In addition to cointegrated endowments, preferences for wheat and rice need to be the same for asset prices to be cointegrated.

analysis, the fundamental difference between asset price behavior under efficient markets as opposed to inefficient markets is that the former displays higher short-run volatility. This is to be expected, as information shocks are reflected sooner in the prices of assets. However, the behavior of asset prices in the long run is fundamentally driven by the long-run behavior of their dividends (i.e., endowments), regardless of whether markets are efficient or not. Therefore, two inefficient asset prices may be cointegrated if their underlying endowment processes are cointegrated. Conversely, if the respective endowments are not cointegrated, asset prices in an efficient market will not be cointegrated.

6. Further discussion

Richards (1995, p. 637) claims that "cointegration of [stock market] return indices is directly at odds with market efficiency." According to Richards, the discrepancy between this claim and Dwyer and Wallace's (1992) claim that equity prices in efficient markets can be cointegrated is based on how equity prices are measured. In particular, Richards argues that although ex-dividend equity prices can be cointegrated in efficient markets, equity prices cum dividends cannot be cointegrated.[11]

The preceding analysis was performed in terms of ex-dividend equity asset prices. However, it is straightforward to demonstrate that similar results can be derived from the analysis of equity asset prices cum dividends. For example, the expression analogous to Eq. (21) for equity asset prices cum dividends ($r_{jt} \equiv q_{jt} + p_{jt} y_{jt}$) is:

$$\begin{aligned} \ln(r_{jt}) = {} & \gamma_1 \ln(y_{1t}) + (1-\gamma_j)\ln(y_{jt}) \\ & + \ln\left\{1 + \frac{E_j}{1 - A_{1j}A_{2j}E_j} \exp[(1-\gamma_j)(\alpha_{1jt} + \alpha_{2jt})]\right\}. \end{aligned} \tag{37}$$

Given the structure of Eq. (37), it is clear that this expression leads to the same conclusions drawn from Eq. (21).

Another important issue is whether the possibility of cointegration in the presence of efficient markets can only arise if the error in the cointegrating relationship is i.i.d. This is relevant because there exists a literature that implies that if markets are efficient, prices may be cointegrated but the error in the cointegrating relationship must be white noise. However, as shown next, it is possible to have efficient markets with cointegrated prices, and the errors in the cointegrating relationship not be white noise.

The strategy to prove the claim above consists of building an economy where market efficiency is imposed by assumption, and such that equity asset prices display cointegration without white noise errors. More specifically, suppose α_{2jt} follows the

[11] Richards also argues that $I(1)$ return indices in integrated financial markets need not be cointegrated. His argument is based on an example in which the CAPM model is assumed to hold for all assets, and rests on whether the sum of the two terms in his Eq. (6) is $I(0)$. According to Richards, each term must be $I(0)$ for their sum to be $I(0)$, but he argues that it is implausible for each term to be $I(0)$ and that it is more plausible that each term is $I(1)$. However, a possibility not considered by Richards is that the sum of the $I(1)$ terms will be $I(0)$ if they are cointegrated, in which case his argument breaks down.

moving average process of order 1 [MA(1)] shown in Eq. (38), instead of being distributed as postulated in Eq. (12):

$$\alpha_{2jt} = \xi_{jt} + \varphi_j \xi_{jt-1}, \quad \xi_{jt} \sim \text{i.i.d. N}(0, \sigma^2_{\xi j}), \left|\varphi_j\right| < 1, \tag{38}$$

for all j. Then, the expression analogous to Eq. (28) is:

$$\begin{aligned}\ln(q_{jt}) - \ln(q_{kt}) = {} & \ln\left(\frac{E_j}{1 - A_{1j}\Xi_{\varphi j}E_j}\right) - \ln\left(\frac{E_k}{1 - A_{1k}\Xi_{\varphi k}E_k}\right) \\ & - \gamma_j\left[\ln(y_{jt}) - \frac{\gamma_k}{\gamma_j}\ln(y_{kt})\right] + [1 + \phi_{jk}(1-\gamma_j) - \phi_{kj}(1-\gamma_k)] \\ & \times[\ln(y_{jt}) - \ln(y_{kt})] + (1-\gamma_j)\alpha_{2jt} - (1-\gamma_k)\alpha_{2kt} \\ & + \ln\{1 - A_{1j}E_j[\Xi_{\varphi j} + \Xi_j \exp((1-\gamma_j)\varphi_j\xi_{jt})]\} \\ & - \ln\{1 - A_{1k}E_k[\Xi_{\varphi k} + \Xi_k \exp((1-\gamma_k)\varphi_k\xi_{kt})]\},\end{aligned} \tag{39}$$

where $\Xi_j \equiv \exp[0.5(1-\gamma_j)^2\sigma^2_{\xi j}]$ and $\Xi_{\varphi j} \equiv \exp[0.5(1-\gamma_j)^2(1+\varphi_j)^2\sigma^2_{\xi j}]$.[12]

Since $\ln(y_{jt})$ and $\ln(y_{kt})$ were assumed to be cointegrated with cointegrating vector [1, −1] in deriving Eq. (39), it follows that $\ln(q_{jt})$ and $\ln(q_{kt})$ are also cointegrated with cointegrating vector [1, −1] if $\gamma_j = \gamma_k$. However, the errors from the resulting cointegrating relationship are not white noise, because α_{2jt} and α_{2kt} are the MA(1) processes defined in Eq. (38). This proves our earlier assertion.

7. Conclusion

As our long list of references illustrates, there is an extensive body of empirical literature that relies on the results of cointegration tests to determine the economic efficiency and/or integration of asset markets. We have argued within the context of a standard, general equilibrium asset-pricing framework, specialized to assure that equilibrium prices are unit root processes, that market efficiency, market integration, and cointegrated prices are independent restrictions. That is, depending upon the parameters characterizing technology, endowments, and preferences, any point in the product space formed by (markets efficient, markets not efficient) × (markets integrated, markets not integrated) × (prices cointegrated, prices not cointegrated)

[12] More specifically, the equilibrium price-dividend ratio ($\psi_{jt} \equiv q_{jt}/(p_{jt}\, y_{jt})$) corresponding to Eq. (38) is:

$$\psi_{jt} \equiv \frac{E_j}{1 - A_{1j}\Xi_{\varphi j}E_j}\exp[(1-\gamma_j)(\alpha_{1jt} + \alpha_{2jt})]\{1 - A_{1j}E_j[\Xi_{\varphi j} + \Xi_j\exp((1-\gamma_j)\varphi_j\xi_{jt})]\}.$$

It can be verified that $\Xi_j = \Xi_{\varphi j} = A_{2j}$ when $\varphi_j = 0$, in which case the above expression for ψ_{jt} simplifies to Eq. (14). This should be expected, because although Eq. (14) and the above expression for ψ_{jt} assume that α_{2jt} is distributed as indicated in Eqs. (12) and (38), respectively, Eq. (12) can be obtained by setting $\varphi_j = 0$ in Eq. (38).

can emerge as an equilibrium outcome. Consequently, in the absence of a sufficiently well-specified model, cointegration tests are not informative with respect to either market efficiency or market integration. Similarly, tests of market efficiency are not informative with respect to market integration and vice versa.

The fundamental insight obtained from using an equilibrium model is that asset prices are ultimately determined by the relevant preferences and endowment processes, irrespective of whether markets are integrated and/or efficient. In the present model, asset-pricing expressions (21), (26), (29), (30), and their no-market-integration counterparts all depend on the preference parameters (γs) and the endowments (y_{jt}s). Market integration and/or efficiency affects the functional form of the asset-pricing equations, as the aforementioned expressions do differ from each other, but they all depend on the relevant preferences and endowments. As a result, the issue of whether asset prices are cointegrated or not rests on the characteristics of preferences and endowment processes, not on market integration and/or market efficiency. More specifically, asset prices are cointegrated if endowments are cointegrated and preferences satisfy certain conditions, and asset prices are not cointegrated otherwise. This holds regardless of whether markets are efficient and/or integrated. In short, cointegration tests of asset prices may be used to draw inferences about preferences and endowment processes, but not for assessing market integration and/or efficiency.

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