Multi-armed Contextual Bandits

Machine Learning and Causal Inference

Professor Susan Athey, Stanford

A/B Testing and Randomized Field Experiments

- Central to innovation in major tech companies, businesses, and (future) governments
- Used in economic evaluations, particularly development
  Future opportunities
- Many alternative treatments (phrasing of text message, variations of online training, etc.)
- Personalized treatment assignment

Schizophrenia

At the same time we use:

- Complex, sophisticated algorithms, econometric methods
- Fixed, preset experimentation among small number of alternatives

Cutting edge in tech companies today (Multi-world testing (MSFT), Google Optimize 360, Facebook):

- Adaptive, online experimentation
- For personalized policies

Bringing into economics

- Unlike most ML, this literature has explicit causal model from the start
- The setup is “good economics”: minimizing regret, balancing exploration and exploitation
- But almost no attention in econometrics or field experiments
- Sprawling literature is an impenetrable morass of mix and match heuristics and approaches

What do we need?

- Be able to understand the disparate literatures and jargon (contextual bandits, Gaussian processes, etc.)
- Justify the many choices in some sort of coherent way
- Efficiency in estimation, confidence intervals for evaluating final policy

1. Contextual Multi-armed Bandits

Treatments $w \in \mathbb{W} = \{1, 2, \ldots, M\}$,
potential outcomes $Y_i(1), \ldots, Y_i(M)$.
Expected outcome:

$$\mu(w, x) = \mathbb{E}[Y_i(w)|X_i = x]$$

Optimal rule:

$$\pi^*(x) = \arg\max_{w \in \mathbb{W}} \mu(w, x)$$

Unit i receives W_i, possibly different from optimal $W^*(X_i)$.
Expected average regret:

$$\mathbb{E}[\mathcal{R}_n] = \frac{1}{n} \sum_{i=1}^{n} \left(\mu(\pi^*(X_i), X_i) - \mu(W_i, X_i) \right)$$

We would like to choose a rule that assigns a new unit, say unit $n+1$, for $n = 0, 1, 2, \ldots, N$, optimally to a treatment, in order to minimize expected average regret, given the covariate/feature values, and given the outcomes, treatment, and covariate values for prior units:

$$\pi_n : \mathbf{W} \times \mathbf{X} \times \mathbf{W}^n \times \mathbf{Y}^n \times \mathbf{X}^n \mapsto [0,1]^{|\mathbf{W}|},$$

with $\sum_{w \in \mathbf{W}} \pi_n(w, x, W_1, \ldots, W_n, Y_1, \ldots, Y_n, X_1, \ldots, X_n) = 1$,
Challenge: how to balance **exploration** (information gained from assigning units to treatments that we are uncertain about) and **exploitation** (improvement in regret from assigning incoming units to the treatment that is currently viewed as the best).

Bandit problem choice:

- What heuristic to balance exploration and exploitation, when primitives of problem unknown? (UCB v. Thompson)

Contextual bandit choices

- Fixed set of policies, update weights on each using data (analog of non-contextual bandit where policy=arm) VS Estimate a more structural model, derive optimal policy
- How/whether to account for non-random assignment as data accumulates
- Parametric versus non-parametric models, Bayesian v. sort-of Bayesian v. Frequentist
- This is a problem where it is crucial to efficiently make use of available data. Efficiency theory may be insightful, and small sample properties are crucial.

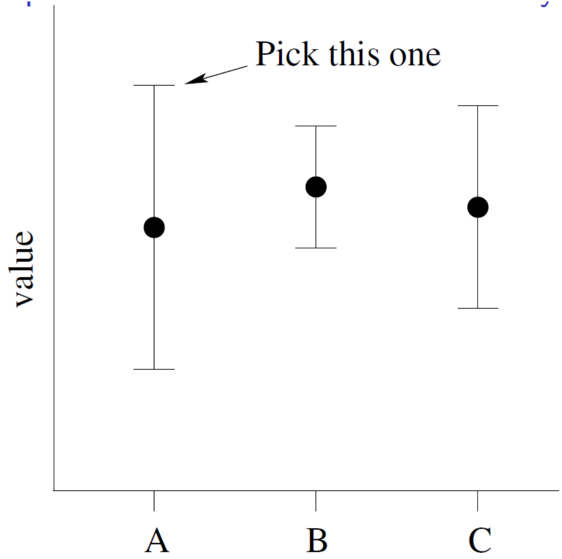
2. UCB Methods and Thompson Sampling without Covariates

Two general approaches to mult-armed bandit problems: UCB (Upper Confidence Bound) methods and Thompson sampling.
UCB methods: Develop estimator $\hat{\mu}_n(w)$ for $\mu(w)$, with measure of uncertainty, $\sigma_n(w)$, given first n units.
Then assign unit $n+1$ to treatment that solves

$$W_{n+1} = \arg\max_w \left\{ \hat{\mu}_n(w) + \sigma_n(w) \right\}.$$

$\sigma_n(w)$ goes to zero as more information about treatment level w accumulates.

Upper Confidence Bounds



[9]

Thompson Sampling

- Specify parametric joint distribution of $(Y_i(1), \ldots, Y_i(M))$, given parameter θ, e.g., $Y_i(w) \sim \mathcal{N}(\beta(w), \sigma^2(w))$, with $\theta = (\beta(1), \sigma^2(1), \ldots, \beta(M), \sigma^2(M))$.
- Specify prior distribution for θ.
- Calculate posterior distribution for θ given information for units 1 through n, and implied posterior for $\mu(1), \ldots, \mu(M)$.
- Assign unit $n+1$ to treatment w with probability equal to the posterior probability that treatment w is the best one given current information, $\mathrm{pr}(\mu(w) = \max_{w' \in \mathbf{W}} \mu(w'))$.

Bayesian way of balancing exploration and exploitation: if $\hat{\mu}(1)$ is less than $\hat{\mu}(2)$, it may still be choosen with substantial probability if we are uncertain about $\mu(2) - \mu(1)$.

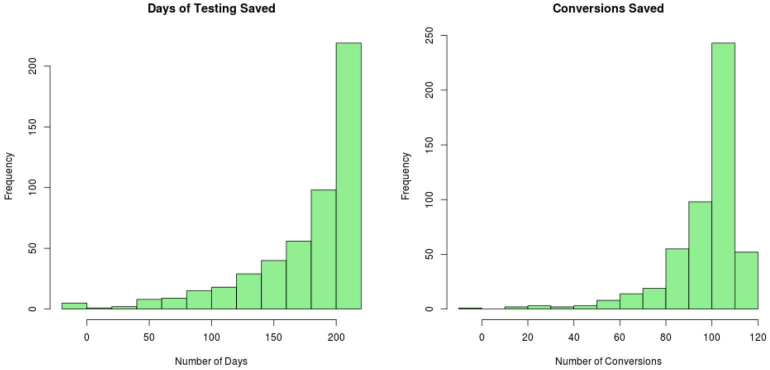
Epsilon-Greedy

- Experiment randomly across arms with low probability that decreases to zero as more observations come in, otherwise choose the best arm.
- Theory says this eventually finds the optimal policy, and further, it is hard to show that something else does much better, if at all.
  - Theory type one: a bandit eventually discovers the best policy
  - Theory type two: an upper bound on the overall regret of the bandit
- These are popular for theory because they are easy to analyze
- Is this a problem with the theory? One might conclude that the theory does not put meaningful bounds on performance if epsilon-greedy is fine.

Bandits use data more efficiently than A/B test

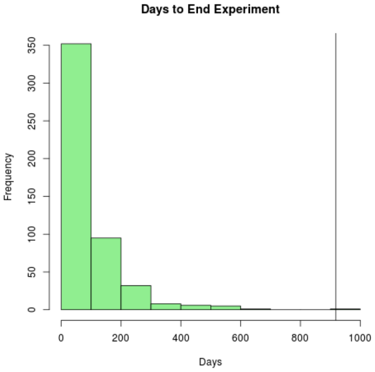
- A/B test: can do power calculation to design experiment in advance, compare to bandit with stopping rule
- Stop when "value remaining in experiment" (optimal choice versus best draw by draw choice when drawing from posterior) small enough, 95th percentile
- Example: Experiment to find ad that maximizes conversions. 100 people exposed per day. Arm 1 has conversion rate .04, arm 2 has .05.
- A/B test takes 220 days to reach 22,000 exposures

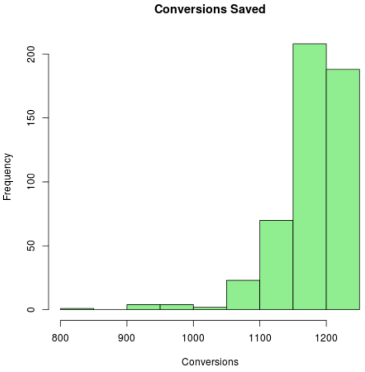
Comparison against pre-planned A/B test with correct power calculation (2 arms):



Source: https://support.google.com/analytics/answer/2844870?hl=en

Comparison against pre-planned A/B test with correct power calculation (6 arms requires more than 2 years with 100 exposures per day):





What to do with covariates?

- Run separate bandits for covariate values.
- Build parametric model for potential outcomes given covariates.

What to do with many covariates?

- Specify set of policy/assignment rules and run bandits to choose between them (Beygelzimer et al, 2011, Agarwal et al 2016)
- Use Ridge regression to model outcomes, UCB/Thompson sampling for each x (lin-UCB)
- Langford et al (2016): update policies/add to mix after batches, using weighted classifier to estimate new policies
- Gaussian process approaches: Eytan Bakshy et al (Facebook)