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# Production Frontiers and Panel Data

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## 1. INTRODUCTION

This article considers estimation of a *stochastic frontier* production function—the type introduced by Aigner, Lovell, and Schmidt (1977) and Meeusen and van den Broeck (1977). Such a production frontier model consists of a production function of the usual regression type but with an error term equal to the sum of two parts. The first part is typically assumed to be normally distributed and represents the usual statistical noise, such as luck, weather, machine breakdown, and other events beyond the control of the firm. The second part is nonpositive and represents technical inefficiency—that is, failure to produce maximal output, given the set of inputs used. Realized output is bounded from above by a frontier that includes the deterministic part of the regression, plus the part of the error representing noise; so the frontier is stochastic. There also exist so-called *deterministic frontier* models, whose error term contains only the nonpositive component, but we will not consider them here (e.g., see Greene 1980). Frontier models arise naturally in the problem of efficiency measurement, since one needs a bound on output to measure efficiency. A good survey of such production functions and their relationship to the measurement of productive efficiency was given by Førsund, Lovell, and Schmidt (1980).

Previous work on production frontiers, with the exception of Pitt and Lee (1981), has assumed error terms that are independently distributed across observations; this assumption is reasonable only in a (single) cross section. Thus previous empirical implementations of frontier models have used cross-sectional data. There are great potential advantages to modifying existing frontier models to allow the use of panel data. In this article we exploit these advantages using a unique panel data set of U.S. domestic airlines and identify firm-specific productive efficiency.

Stochastic-frontier models currently suffer from three serious difficulties. First, the technical inefficiency of a particular firm (observation) can be estimated but not consistently. We can consistently estimate the (whole) error term for a given observation, but it contains statistical noise as well as technical inefficiency. The variance of the distribution of technical inefficiency, conditional on the whole error term, does not vanish when the sample size increases (see Jondrow et al. 1982 for a discussion of this point). Second, the estimation of the model and the separation of technical inefficiency from statistical noise require specific assumptions about the distribution of technical inefficiency (e.g., half-normal) and statistical noise (e.g., normal). It is not clear how robust one's results are to these assumptions. Another way to emphasize this point is to note that the evidence of technical inefficiency is skewness of the production-function error, and not everyone will agree that skewness should be regarded as evidence of inefficiency. Third, it may be incorrect to assume that inefficiency is independent of the regressors. If a firm knows its level of technical inefficiency, for example, this should affect its input choices.

All three of these problems are potentially avoidable if one has panel data, say $T$ observations on each of $N$ firms. The technical inefficiency of a particular firm can be estimated consistently at $T \to \infty$; adding more observations on the same firm yields information not attainable by adding more firms. Second, with a panel one need not make such strong distributional assumptions as are necessary with a single cross section. Essentially, evidence of inefficiency can be found in constancy over time as well as in skewness. Finally, estimates of the parameters and of the firms' inefficiency levels can be obtained without assuming that technical inefficiency is uncorrelated with the regressors. Therefore, we will consider a variety of different estimators, depending on what one is willing to assume about the distribution of technical inefficiency and its potential correlation with the regressors.

The model to be analyzed is presented in Section 2 of this article. Section 3 discusses estimation of the model by ordinary least squares. The *within* estimator is presented in Section 4. Section 5 presents the GLS estimator, whereas Section 6 discusses an estimator due to Hausman and Taylor (1981). Section 7 discusses MLE, given a distributional assumption on the effects. Section 8 discusses some tests of the assumptions that lead to the different estimators. Section 9 illustrates these methods using a new and fairly lengthy panel of U.S. domestic airlines, and Section 10 is the conclusion.

## 2. PRESENTATION OF THE MODEL

We begin with a single-equation production function. (Alternatively, with a change in the sign of the one-sided error, it could be a cost function.) The model to be analyzed is of the form

$$y_{it} = \alpha + X_{it}'\beta + v_{it} - u_i, \qquad i = 1, \ldots, N, t = 1, \ldots, T. \tag{1}$$

Here $i$ indexes firms and $t$ indexes time periods. The value $y_{it}$ is output (for firm $i$ in time $t$), whereas $X_{it}$ is a vector of $K$ inputs. The $v_{it}$ are uncorrelated with the regressors $X_{it}$—for example, by the Zellner, Kmenta, and Dreze (1966) argument. The $u_i$ represent technical inefficiency and, correspondingly, $u_i \geq 0$ for all $i$. We assume the $u_i$ to be iid with mean $\mu$ and variance $\sigma_u^2$ and independent of the $v_{it}$. A particular distribution (e.g., half-normal) may or may not be assumed for the $u_i$. Furthermore, the $u_i$ may or may not be assumed to be uncorrelated with the regressors ($X_{it}$); presumably this depends on whether $u_i$ is known to firm $i$ or not.

For $T = 1$ (pure cross section of $N$ firms), the model in (1) is exactly the stochastic frontier of Aigner, Lovell, and Schmidt (1977). For $T > 1$, it is a straightforward generalization of that model, and it fits exactly the usual framework in the panel-data literature, with a *firm effect* but no *time effect*. The only difference from the standard panel-data literature is that our firm effects are one-sided, and we will in some cases assume a (nonnormal) distribution for them.

It may also be profitable to rewrite the model slightly in two ways. First, let $E(u_i) = \mu > 0$ (as before), and define

$$\alpha^* = \alpha - \mu, \; u_i^* = u_i - \mu \tag{2}$$

so that the $u_i^*$ are iid with mean 0. Then in the model

$$y_{it} = \alpha^* + X_{it}'\beta + v_{it} - u_i^*, \tag{3}$$

the error terms $v_{it}$ and $u_i^*$ have zero mean, and most of the results of the panel data literature apply directly, except of course those that hinge on normality. Second, define

$$\alpha_i = \alpha - u_i = \alpha^* - u_i^*, \tag{4}$$

and the model becomes

$$y_{it} = \alpha_i + X_{it}'\beta + v_{it}. \tag{5}$$

This is useful because we will have occasion to refer to the $\alpha_i$ shortly.

## 3. ESTIMATION BY ORDINARY LEAST SQUARES

Ordinary least squares may be applied to (3), treating $(v_{it} - u_i^*)$ as the disturbance. The resulting estimates of $\alpha^*$ and $\beta$ will be consistent as $N \to \infty$ (though not as $T \to \infty$ for fixed $N$) if the individual effects ($u_i$) are uncorrelated with the regressors ($X_{it}$). Under these circumstances, however, a better alternative exists (see Section 5), so ordinary least squares estimation is not recommended.

## 4. DUMMY VARIABLES (WITHIN ESTIMATOR)

The so-called *within* estimator treats the $u_i$ as fixed—that is, it estimates a separate intercept for every firm, as in (5). This can be done by suppressing the constant term and adding a dummy variable for each of the $N$ firms or, equivalently, by keeping the constant term and adding $(N - 1)$ dummies. Another equivalent procedure is to apply the within transformation—that is, to apply OLS after expressing all data in terms of deviations from the firm means (e.g., replace $y_{it}$ by $y_{it} - \bar{y}_i$, etc.). In the latter case, the $N$ intercepts are recovered as the means of the residuals by firm.

The chief advantage of the within estimator is that its consistency does not hinge on uncorrelatedness of the regressors and the individual effects. It also does not depend on the distribution of the effects, since in treating them as fixed it simply proceeds conditionally from whatever their realizations may be. The within estimate of $\beta$ is consistent as *either* $N$ or $T \to \infty$. Consistency of the individual estimated intercepts ($\alpha_i$), however, requires $T \to \infty$.

All of this is well known and requires basically no adaptation to the frontier case (e.g., see the usual panel data literature, Mundlak 1978 and Hausman and Taylor 1981). In the frontier case, however, we can use the fact that $u_i \geq 0$ to appropriately normalize the effects ($u_i$) and the overall constant ($\alpha$), at least if $N$ is large. If the $N$ estimated intercepts are $\hat{\alpha}_1, \ldots, \hat{\alpha}_N$, simply define

$$\hat{\alpha} = \max(\hat{\alpha}_i) \text{ and}$$
$$\hat{u}_i = \hat{\alpha} - \hat{\alpha}_i, \qquad i = 1, 2, \ldots, N. \tag{6}$$

This definition amounts to counting the most efficient firm in the sample as 100% efficient. Provided only that the density of $u$ is nonzero in some neighborhood $(0, \epsilon)$ for some $\epsilon > 0$, the efficiency of the most efficient firm in the sample will indeed approach 100% as $N \to \infty$. (This is essentially the argument of Greene 1980 in the single cross-section case.) Thus the estimates in (6) are consistent for $\alpha$ and the $u_i$ as $N$ and $T \to \infty$.

To summarize, we can estimate the individual intercepts (one for each firm) consistently as $T \to \infty$. Thus we can compare efficiency across firms. In addition, as $N \to \infty$ we can consistently separate the overall intercept from the one-sided individual effects, which allows us to measure efficiency relative to an absolute standard (100%).

The distributional properties of the estimators defined in (6) are not trivial; that is, if we wish to assign standard errors, compute confidence intervals, and so forth, some unsolved problems arise.

When $N$ is large relative to $T$, these problems are essentially avoided. In this case, the variability of the $\hat{\alpha}_i$ as estimates of the $\alpha_i$ is large relative to the variability of $\min(u_i)$ as an estimate of zero; that is, we can ignore the variability involved in the "max" operation. Then treating $\hat{\alpha}$ and the $\hat{u}_i$ as simple linear functions of the $\hat{\alpha}_i$, the distribution of $\hat{\alpha}$ and the $\hat{u}_i$ is easily calculated.

When $T$ is large relative to $N$, we can do the converse and ignore the variability of the $\hat{\alpha}_i$. Then essentially $\hat{\alpha} = \max(\alpha_i) = \alpha - \min(u_i)$, and we have the problem of the distribution of the smallest observation in a random sample of size $N$ from the distribution of $u$. For moderately large $N$, and for reasonable distributions of $u$, $\min(u_i)$ will follow an extreme-value (double-exponential) distribution. Hence standard results apply (e.g., see Galambos 1978). Alternatively, if we assume a particular distribution for the $u_i$, we can obtain more precise results. For example, if the $u_i$ are iid exponential with parameter $\Theta$, then $\min(u_i)$ is exponential with parameter $\Theta/N$ (e.g., see Johnson and Kotz 1970, pp. 211–212). Thus $\hat{\alpha} = \max(\alpha_i) = \alpha - \min(u_i)$ would have a mean equal to $\alpha - \Theta/N$ and a variance equal to $\Theta^2/N^2$. Given that the MLE of $\Theta$, say $\hat{\Theta}$, has a mean equal to $\Theta(N - 1)/N$, an unbiased estimator of $\alpha$ is $\hat{\alpha} + \hat{\Theta}/(N - 1)$, whereas an unbiased predictor of $u_i$ is $\hat{u}_i + \hat{\Theta}/(N - 1)$; furthermore, confidence intervals are easily constructed. (Of course, distributions other than exponential will not give such simple results.)

When we are not satisfied with ignoring either kind of variability, things are more complicated. The estimator $\hat{\alpha}$ in (6) is actually $\hat{\alpha} = \alpha + \max[(\hat{\alpha}_i - \alpha_i) - u_i]$. The terms $[(\hat{\alpha}_i - \alpha_i) - u_i]$ are mixtures of a normal and a one-sided distribution. Worse, they are not independent over $i$, so standard results do not apply. Just what can be done here remains to be seen.

A considerable disadvantage of the within estimator is that it is impossible to include in the specification regressors that are invariant over time, even though they vary across firms. In this case our estimated firm effects will include the effects of all variables that are fixed within the sample at the firm level, possibly including some (e.g., capital stock) that are not in any sense a representation of inefficiency. To avoid this problem one must make assumptions about uncorrelatedness of effects and regressors and/or about the distribution of the effects.

## 5. GENERALIZED LEAST SQUARES ESTIMATION

We now treat the effects $(u_i)$ as random, and we make the assumption that they are uncorrelated with the regressors. At this point, however, we still do not make any distributional assumption for the effects. This leads to the generalized least squares estimation of (3), exactly as in the panel-data literature, the covariance of the error $(v_{it} - u_i^*)$ being of the usual form. This covariance matrix depends on $\sigma_v^2$ and $\sigma_u^2$, and we distinguish the case in which these are known from the (realistic) case in which they are not known and must be estimated with $\alpha^*$ and $\beta$.

We begin by summarizing some well-known results from the panel-data literature. With $\sigma_v^2$ and $\sigma_u^2$ known, the GLS estimator (of $\alpha^*$ and $\beta$) is consistent as either $N$ or $T \to \infty$. It is more efficient than the within estimator, but this difference in efficiencies disappears as $T \to \infty$. (It remains as $N \to \infty$ for fixed $T$, the usual panel case.) When $\sigma_v^2$ and $\sigma_u^2$ are not known (i.e., the realistic case), GLS is based on their consistent estimates, say $\hat{\sigma}_v^2$ requires $\hat{\sigma}_u^2$. Consistent estimation of $\sigma_u^2$ requires $N \to \infty$. Thus the strongest case for GLS is when $N$ is large and $T$ is small; the assumption of uncorrelatedness of effects and regressors buys extra efficiency. If $T$ is large and $N$ is small, GLS is useless (unless $\sigma_u^2$ were known a priori). If $N$ and $T$ are both large, GLS is feasible but not more efficient than within.

Given our estimate of $\beta$, say $\hat{\beta}$, we can recover estimates of the individual firm intercepts $(\alpha_i)$ from the residuals. If we define the residuals as $\hat{\epsilon}_{it} = y_{it} - X_{it}'\hat{\beta}$, we can estimate $\alpha_i$ by the mean (over time) of the residuals for firm $i$:

$$\hat{\alpha}_i = \frac{1}{T} \sum_t \hat{\epsilon}_{it}, \qquad i = 1, 2, \ldots, N. \tag{7}$$

These estimates are consistent as $T \to \infty$, provided that $\hat{\beta}$ is consistent (which requires $N \to \infty$ or $\sigma_u^2$ known). We can also decompose the $\hat{\alpha}_i$ into estimates of $\hat{\alpha}$ and the $\hat{u}_i$, as in (7), for which consistency requires $N \to \infty$ plus consistency of the $\hat{\alpha}_i$. Thus consistent estimation of technical inefficiency requires both $N \to \infty$ and $T \to \infty$, just as for within. (Another possibility is to use the best linear unbiased predictor (BLUP) of Taub 1979 and Lee and Griffiths 1979. After (3) is estimated, the BLUP of $u_i^*$ is $-\hat{\sigma}_u^2 \sum_t (y_{it} - \hat{\alpha}^* - X_{it}'\hat{\beta})/(T\hat{\sigma}_u^2 + \hat{\sigma}_v^2)$, and the resulting estimate of $\alpha_i$ is $\hat{\alpha}^* - \hat{u}_i^*$. For large $T$, this is equivalent to (7).)

The important advantage of the GLS estimator relative to the within estimator, in the present context, is not efficiency but rather the ability to include time-invariant regressors. In cases in which time-invariant regressors are relevant, this is important so that their effects do not contaminate measured efficiency.

## 6. THE HAUSMAN-TAYLOR ESTIMATOR

The GLS estimator hinges on the assumption that the effects and regressors are uncorrelated, whereas the within estimator does not. In a recent paper, Hausman and Taylor (1981) proposed an estimator that is a hybrid of the two, in the sense that one may assume the effects to be uncorrelated with some but not all of the regressors.

This estimator may be motivated in terms of the efficiency gains in imposing such (uncorrelatedness) restrictions. In the present context, however, a more compelling motivation is the potential to include time-invariant regressors. Hausman and Taylor gave an elegant and complete statement of the conditions for the coefficients of time-invariant regressors to be identified. Basically, the number of time-varying regressors that are uncorrelated with the effects must be at least as large as the number of time-invariant regressors that are correlated with the effects.

Individual effects can be estimated consistently from the residuals if $T$ is large and separated from the intercept if $N$ is large, exactly as for GLS.

## 7. MAXIMUM LIKELIHOOD, GIVEN INDEPENDENCE AND A DISTRIBUTION

In the previous frontier literature, the effects have been assumed independent of regressors, and specific distributional assumptions have been made for $v$ and $u$ (usually normal for $v$ and half-normal for $u$). As we have seen, these strong assumptions can be avoided when one has panel data. Nevertheless, it is still possible to make these assumptions, in which case a maximum-likelihood estimator is feasible.

We therefore assume that the $v_{it}$ are iid with density $f(v)$, known up to some parameters, that the $u_i$ are iid with density $g(u)$, also known up to some parameters, and that $u$ and $v$ are independent of each other and of the regressors. If we define $\epsilon_{it} = v_{it} - u_i$ and note that these are independent over $i$, then the likelihood function follows easily from the joint density of $(\epsilon_{i1}, \dots, \epsilon_{iT})$, which is given by

$$h(\epsilon_{i1}, \dots, \epsilon_{iT}) = \int_0^\infty g(u) \prod_{t=1}^{T} f(\epsilon_{it} + u)\, du. \quad (8)$$

Given this density, the likelihood function is

$$L = \prod_{i=1}^{N} h(y_{i1} - \alpha - X_{i1}'\beta, \dots, y_{iT} - \alpha - X_{iT}'\beta). \quad (9)$$

Its maximization yields the MLE's of the parameters ($\alpha$, $\beta$, and the parameters in the densities of $u$ and $v$).

Pitt and Lee (1981) derived the likelihood function (9) for the case in which the $v_{it}$ are normal and the $u_i$ are half-normal, and they calculated maximum-likelihood estimates for a sample of Indonesian weaving firms.

The asymptotic properties of the MLE's in this model require further work, since they have not yet been worked out carefully. We conjecture that (given suitable regularity conditions) the MLE's are consistent and asymptotically efficient as $N \to \infty$, regardless of $T$. What happens as $T \to \infty$ for fixed $N$ is less clear. Certainly consistent estimation of the parameters of the distribution of $u$ must require $N \to \infty$, but results for the other parameters are less obvious. As far as efficiency is concerned, we conjecture that the MLE's are generally more efficient (asymptotically) than the estimators previously considered, since they exploit distributional information that the other estimators do not exploit. But it is conceivable that at least for some distributional choices, this information is useless asymptotically. For example, if both $v$ and $u$ are normal, then *within*, GLS, and MLE are all asymptotically equivalent (as both $N$ and $T \to \infty$). Whether *one-sided* distributions of $u$ exist, such that this equivalence occurs, is not yet clear.

The preceding estimator assumes both independence of effects and regressors and specific distributions for $v$ and $u$. If we relax the distributional assumptions but maintain independence, we are led to GLS, as discussed in Section 5. On the other hand, if we maintain the distributional assumptions but relax independence, things are less clear. As discussed in Section 4, we can estimate by within and then use the distributional assumption in normalizing the effects; but this is not entirely satisfactory, since the distributional assumption may be useful in estimating the parameters. A more promising possibility is to follow Mundlak (1978) and Chamberlain (1980) by modeling the correlation between $X$ and $u$. When this is done, GLS = within. For nonnormal $u$, however, GLS may not be the optimal estimator, and the optimal estimator may not equal within.

## 8. TESTS OF UNCORRELATEDNESS AND DISTRIBUTIONAL FORM

The estimators that have been presented differ in the extent to which they depend on the effects being uncorrelated with the regressors and/or on a distributional assumption for the effects. These assumptions can in turn be tested using Hausman-type (1978) tests, based on the differences between the various estimators.

Testing the null hypothesis that effects and regressors are uncorrelated was discussed in detail by Hausman and Taylor (1981, Sect. 2.2 and 3.3). The test they proposed is a Hausman-type test of the significance of the difference between the within estimator and the GLS estimator (to test the hypothesis that the effects are uncorrelated with *all* regressors) or of the significance of the difference between the within estimator and the Hausman–Taylor estimator (to test the hypothesis that the effects are uncorrelated with a specified subset of the regressors). This requires $N$ to be large, since the GLS and Hausman–Taylor estimators require large $N$ to estimate $\sigma_u^2$. Indeed, since only $N$ realizations of $u$ exist in the data, any asymptotic test about $u$ must, of necessity, require $N \to \infty$, so this is not surprising.

Given that the effects are uncorrelated with the regressors, a distributional assumption (e.g., normal $v$, half-normal $u$) can be tested by a Hausman test of the difference between the GLS estimator and the MLE. Similarly, the joint hypothesis that the effects are uncorrelated with the regressors and that the distributional assumptions are correct could be based on the difference between the MLE and the within estimator.

Since we have not provided an estimator that exploits a distributional assumption without assuming effects to be independent of regressors, no Hausman-type test is available of the distributional assumptions only. If both $N$ and $T$ are large, however, we could use standard goodness-of-fit tests to see whether the estimated effects from within follow the hypothesized distribution, and this test would not depend on the correlation between effects and regressors. A distributional assumption (e.g., normality) about the error terms $v$ could also be tested by standard methods using within residuals, and such asymptotic tests would require only that either $N$ or $T$ be large.

## 9. EMPIRICAL ILLUSTRATION

In this section we illustrate the methods outlined before by estimating a production function for the U.S. domestic airline industry. The source of the data was the Civil Aeronautics Board Form 41 data base provided by the Air Transport Association of America and maintained by the Boeing Computer Services, Inc. A detailed description of the accounts in the Form 41 from which expense and quantity indexes were compiled is available on request. The data are by airline by quarter from 1970 I to 1978 III. The airlines used in the study are American, Allegheny, Braniff, Continental, Delta, Eastern, Frontier, North Central, Ozark, Piedmont, United, and Western. Each observation on the calculated Divisia indexes of price and quantity required information on 230 separate accounts. Appendix A contains a discussion of the broad categories of inputs and output and the main contents of these categories. For a lengthier discussion, see Sickles (1983).

The input categories are capital, labor, energy, and materials, and the output is capacity ton miles (CTM). We implicitly assume that any unfilled space is wastage. On the other hand, it is obviously cheaper to fly an airplane from one point to another if it is empty and if it does not make intermediate stops. We therefore controlled for differences in the airlines' networks by including, as arguments in the production function, load factor, average stage length (miles between each takeoff and landing), and their interaction. We also included quarterly seasonal dummy variables. The Divisia indexes were constructed using industry price weights so that the major portion of estimated inefficiencies should be due to inefficient use of inputs instead of being purchased at suboptimal prices. We assume Cobb–Douglas technology and Hicks-neutral technological change. We also abstract from the complications that would be introduced by a more plausible treatment of the dynamics of production (e.g., by allowing for quasi-fixed factors of production or sluggish adjustment to desired production levels).

Appendix B reports the within, GLS, and MLE estimates of the production function. The estimated factor-productivity growth of between 1.5% and 2% per year for each of the models is close to the 2.5% growth rate calculated directly from the data as the difference in the Divisia indexes of output and of the inputs. All three sets of results are close in terms of $\bar{R}^2$, estimated output elasticities, and significance of coefficients.

Given the considerable differences in the sizes of the firms, one might suspect heteroscedasticity to be a problem. We, however, did not find it to be so. Running separate regressions for each airline, estimated error variances were not very different. For example, we have $s^2 = .00015$ for the largest airline in the sample and $s^2 = .00013$ for the smallest; this difference is insignificant.

Table 1 displays the estimated technical efficiency of the 12 airlines and their average output. The efficiencies are close for all three models, and the rankings are almost identical. Two comments about these efficiencies are in order. First, because of the small number of firms in the sample ($N = 12$), the normalization of the most efficient firm as 100% technically efficient is questionable. We have a reasonably long sample ($T = 35$), however, so we can have some faith in the *relative* efficiency rankings. (In other words, we should have more faith in the statement that Delta is 12% more efficient than Eastern than we should in the specific technical efficiencies of 95.2% and 83.2%, respectively.) Second, our efficiency rankings for the period 1970–1978 do not seem to do a good job of predicting post-1978 financial success; some of our most efficient firms are now bankrupt or nearly so. An obvious explanation (which we believe) is that this is simply due to the difference between regulated and unregulated environments. An airline may have been very good at flying

*Table 1. Technical Efficiencies*

| Firm | Average Capacity Output (thousands) | Firm Efficiency (%) | | |
|---|---|---|---|---|
| | | Within | GLS | MLE |
| American | 1,138,244 | 89.2 | 83.3 | 85.6 |
| Allegheny | 146,727 | 75.1 | 78.0 | 77.8 |
| Braniff | 341,447 | 89.2 | 87.7 | 89.6 |
| Continental | 332,006 | 100.0 | 98.0 | 99.4 |
| Delta | 763,683 | 95.2 | 94.0 | 96.2 |
| Eastern | 841,230 | 83.2 | 80.4 | 80.8 |
| Frontier | 68,620 | 95.7 | 100.0 | 100.0 |
| North Central | 58,267 | 71.9 | 77.0 | 76.8 |
| Ozark | 48,116 | 70.7 | 75.7 | 75.6 |
| Piedmont | 49,656 | 70.2 | 75.1 | 74.7 |
| United | 1,430,228 | 93.1 | 87.8 | 91.0 |
| Western | 310,974 | 98.0 | 95.9 | 98.6 |

from point $A$ to point $B$ but poor at choosing $A$ and $B$ or the fare to change.

Given the similarities in the results, it is not surprising that the null hypothesis of no correlation between effects and regressors is accepted. The Hausman test $\chi^2_{11}$ is 3.74. Given the uncorrelatedness of the effects and regressors, the distributional test can be carried out by comparing the GLS estimator with the MLE estimator. For this test the $\chi^2_{11}$ is 13.73. The joint hypothesis of uncorrelated regressors *and* correct distribution is tested by comparing the within with the MLE estimates. In this case the $\chi^2_{11}$ was 13.64. All of these are well within the acceptance region at the .05 level.

It is also interesting to examine the within inefficiencies directly. They do not look too different from drawings from a half-normal distribution, though of course, with only 12 observations this is hard to tell. If we split their possible range into the three cells $u \leq .10$, $.10 < u \leq .20$, and $.20 < u$, the observed counts are 5, 3, and 4. Treating the inefficiencies as half-normal data, the MLE of $\sigma^2_u$ is .03208, which leads to expected cell counts of 5.08, 3.17, and 3.75; these are surprisingly close to those observed. Presumably other tests of fit would also fail to reject half-normality, given the small sample size.

## 10. CONCLUSIONS AND FURTHER DIRECTIONS

In this article we have considered estimation of a stochastic frontier production function model, given panel data. We have provided a variety of estimators, depending on whether or not one is willing to assume that technical inefficiency (the individual effect, in panel-data jargon) is uncorrelated with the regressors and on whether or not one is willing to make specific distributional assumptions for the errors (e.g., normal for the general error term and half-normal for technical inefficiency). We have also indicated how to test these assumptions.

Since we rely here on asymptotics, it is important that either $N$ or $T$ (or both) be large. The most favorable case is naturally when both $N$ and $T$ are large, since we can then estimate the parameters of the model and the technical efficiency of each firm consistently, regardless of which of the preceding sets of assumptions we choose; all of the methods discussed in this article are potentially applicable.

If $T$ is large but $N$ is small, we are restricted to using the within estimator, which exploits neither a distributional assumption nor uncorrelatedness of effects and regressors and which does not allow time-invariant regressors. We can consistently estimate the intercept for each firm, but there is no consistent way of separating the overall intercept from the one-sided effects. Thus we can compare efficiencies across firms but not relative to an absolute standard.

If $N$ is large but $T$ is small, we are closest in spirit to both the usual panel-data literature and the usual frontier literature. We can choose any of the estimators described before, depending on what we are willing to assume, and we can test our assumptions. Although we can estimate the intercept of each firm (or the technical efficiency of each firm, if we use MLE), we cannot do so consistently; consistency of estimated-firm effects inherently requires $T \rightarrow \infty$.

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## APPENDIX A: DATA

The source of data was the Civil Aeronautics Board Form 41 data base provided to Sickles by the Air Transport Association of America and maintained by the Boeing Computer Services, Inc., from 1970 I to 1978 III. Mergers during this time were handled by combining accounts for the relevant parties at the time of merger. The airlines in the study are American, Allegheny, Braniff, Continental, Delta, Eastern, Frontier, North Central, Ozark, Piedmont, United, and Western. Each observation on the calculated indexes of price and quantity required information on 230 separate accounts.

The labor input was composed of 55 separate labor accounts aggregated into five major employment classes. They are pilots, flight attendants, machinists, passenger/cargo and aircraft handlers, and other personnel. Labor-related expenses such as insurance, pensions, and payroll taxes were allocated to each class on the basis of the expense share of the class. Expense/person quarters in 1972 III were normalized to 1.0 before the Divisia indexes for price and quantity were calculated.

The capital input was developed by constructing four categories of expenses that were directly or indirectly identified with capital. These expense categories were flight equipment purchased and rented, ground equipment purchased, ground equipment rented, and landing fees. Quantity indexes for flight equipment purchased and rented were calculated by imputing to purchased aircraft the rental price of a comparably configured aircraft. This assumes of course that depreciation of the aircraft is negligible. Because of the strict Federal Aviation Administration maintenance requirements, this assumption seems quite reasonable. It is one that was adopted in a previous study by Caves, Christensen, and Tretheway (1981). We adjusted for differing aircraft utilization rates by scaling the capital quantity on the basis of average hours ramp to ramp during the day relative to the maximum average quarterly usage during the period. Ground-equipment rental expenses and the implicit deflator for nonresidential fixed investment were used for the second category. Ground-equipment rental expenses were calculated using the perpetual inventory approach, a 1955 benchmark, and the Jorgensen–Hall user price for capital formula. A 15-year replacement rate and accelerated or straight-line (depending on the firm's current profitability) depreciation schedules were assumed. The fourth capital category is landing fees or the rental cost of the airport facilities. A price deflator for landing fee expenses was cost/capacity ton landed.

The energy input was developed by combining information on aircraft gallons used with expense data per period. Furthermore, we transformed the gallons used into the BTU equivalent using the conversion rate for turbo fuel, the predominant fuel used by the carriers since the mid-1960's.

The fourth input, *materials*, is comprised of many broad classes of materials, which were themselves aggregates of 56 different accounts. These categories included advertising, communications, insurance, outside services, supplies, passenger food, commissions, and other operating and nonoperating expenses.

The capacity ton mile quantity index was generated from data on total capacity ton miles for first class and coach. Price deflators for the three categories were derived from the revenue output accounts. Thus our measure of output is transferred space. We are implicitly assuming that unused space is wastage and is a demand consideration that is outside the scope of this study.

## APPENDIX B: ESTIMATION RESULTS

*Within*: $\bar{R}^2 = .992$ and $\sigma_v^2 = .00142$

$$
\begin{aligned}
\ln CTM = {} & \underset{(1.25)}{.675\text{AA}} + \underset{(1.05)}{.533\text{AL}} + \underset{(1.30)}{.675\text{BN}} \\
& + \underset{(1.51)}{.783\text{CO}} + \underset{(1.39)}{.734\text{DL}} + \underset{(1.14)}{.615\text{EA}} \\
& + \underset{(1.51)}{.739\text{FR}} + \underset{(1.02)}{.502\text{NC}} + \underset{(1.00)}{.490\text{OZ}} \\
& + \underset{(0.99)}{.485\text{PI}} + \underset{(1.32)}{.714\text{UA}} + \underset{(1.48)}{.763\text{WA}} \\
& + \underset{(8.86)}{.00383t} + \underset{(5.67)}{.147K} + \underset{(7.06)}{.218L} \\
& + \underset{(16.12)}{.605E} + \underset{(5.30)}{.148M} - \underset{(-1.05)}{.00566\text{ Winter}} \\
& + \underset{(3.33)}{.0179\text{ Spring}} + \underset{(6.07)}{.0346\text{ Summer}} \\
& - \underset{(-12.6)}{1.33\text{ Load F}} - \underset{(-2.64)}{.000418\text{ STGL}} \\
& + \underset{(7.34)}{.00149\text{ Load F} * \text{STGL}}.
\end{aligned}
$$

*GLS*: $\bar{R}^2 = .986$, $\sigma_v^2 = .00142$, and $\sigma_u^2 = .0259$

$$
\begin{aligned}
\ln CTM = {} & \underset{(.077)}{.345} + \underset{(9.26)}{.00352t} + \underset{(6.11)}{.154K} \\
& + \underset{(7.45)}{.226L} + \underset{(16.1)}{.597E} + \underset{(5.78)}{.157M} \\
& - \underset{(-1.11)}{.00597\text{ Winter}} + \underset{(3.35)}{.0180\text{ Spring}} \\
& + \underset{(6.02)}{.0344\text{ Summer}} - \underset{(-12.96)}{1.356\text{ Load F}} \\
& - \underset{(-2.05)}{.000303\text{ STGL}} \\
& + \underset{(7.50)}{.00151\text{ Load F} * \text{STGL}}.
\end{aligned}
$$

*MLE*: $\bar{R}^2 = .985$, $\sigma_v^2 = .00129$, and $\sigma_u^2 = .1990$

$$
\begin{aligned}
\ln CTM = {} & \underset{(-2.22)}{-.0957} + \underset{(12.26)}{.00294t} + \underset{(31.8)}{.201K} \\
& + \underset{(13.6)}{.238L} + \underset{(70.4)}{.527E} + \underset{(15.9)}{.215M} \\
& - \underset{(-1.24)}{.00548\text{ Winter}} + \underset{(3.99)}{.0184\text{ Spring}} \\
& + \underset{(8.64)}{.0386\text{ Summer}} \\
& - \underset{(-18.8)}{1.412\text{ Load F}} - \underset{(-22.3)}{.00345\text{ STGL}} \\
& + \underset{(11.4)}{.00147\text{ Load F} * \text{STGL}}.
\end{aligned}
$$

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