Heterogeneous Treatment Effects and Parameter Estimation with Generalized Random Forests

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See Wager and Athey (forthcoming, JASA)
and Athey, Tibshirani and Wager, forthcoming, AOS
https://arxiv.org/abs/1610.01271

Heterogeneous Parameter Estimates

- Heterogeneous treatment effects, elasticities, etc.
- Could estimate models within leaves of shallow trees; or regularize interaction effects in models
- Generalized random forests: look for parameter heterogeneity flexibly

Forests for GMM Parameter Heterogeneity

- Local GMM/ML uses kernel weighting to estimate personalized model for each individual, weighting nearby observations more.
  - Problem: curse of dimensionality
- We propose forest methods to determine what dimensions matter for "nearby" metric, reducing curse of dimensionality.
  - Estimate model for each point using "forest-based" weights: the fraction of trees in which an observation appears in the same leaf as the target
- We derive splitting rules optimized for objective
- Computational trick:
  - Use approximation to gradient to construct pseudo-outcomes
  - Then apply a splitting rule inspired by regression trees to these pseudo-outcomes

Related Work

(Semi-parametric) local maximum likelihood/GMM

- Local ML (Hastie and Tibshirani, 1987) weights nearby observations; e.g. local linear regression. See Loader, C. (1999); also Hastie and Tibshirani (1990) on GAM; see also Newey (1994)
- Lewbel (2006) asymptotic prop of kernel-based local GMM
- Other approaches include Sieve: Chen (2007) reviews

Score-based test statistics for parameter heterogeneity

- Andrews (1993), Hansen (1992), and many others, e.g. structural breaks, using scores of estimating equations
- Zeiles et al (2008) apply this literature to split points, when estimating models in the leaves of a single tree.

Splitting rules

- CART: MSE of predictions for regression, Gini impurity for classification, survival (see Bouhamad et al (2011))
- Statistical tests, multiple testing corrections: Su et al (2009)
- Causal trees/forests: adaptive v. honest est. (Athey and Imbens, 2016); propensity forests (Wager and Athey, 2015)

Solving estimating equations with random forests

We have $i = 1, \ldots, n$ i.i.d. samples, each of which has an **observable** quantity O_i, and a set of **auxiliary covariates** X_i.

Examples:

- Non-parametric regression: $O_i = \{Y_i\}$.
- Treatment effect estimation: $O_i = \{Y_i, W_i\}$.
- Instrumental variables regression: $O_i = \{Y_i, W_i, Z_i\}$.

Our **parameter of interest**, $\theta(x)$, is characterized by an estimating equation:

$$\mathbb{E}\left[\psi_{\theta(x),\,\nu(x)}(O_i) \,\middle|\, X_i = x\right] = 0 \quad \text{for all} \quad x \in \mathcal{X},$$

where $\nu(x)$ is an optional **nuisance parameter**.

The GMM Setup: Examples

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$$\mathbb{E}\left[\psi_{\theta(x),\,\nu(x)}(O_i)\,\middle|\,X_i = x\right] = 0 \ \text{ for all } \ x \in \mathcal{X},$$

where $\nu(x)$ is an optional **nuisance parameter**.

- **Quantile regression**, where $\theta(x) = F_x^{-1}(q)$ for $q \in (0,\, 1)$:

$$\psi_{\theta(x)}(Y_i) = q\,\mathbf{1}(\{Y_i > \theta(x)\}) - (1-q)\,\mathbf{1}(\{Y_i \leq \theta(x)\})$$

- **IV regression**, with treatment assignment W and instrument Z. We care about the treatment effect $\tau(x)$:

$$\psi_{\tau(x),\,\mu(x)} = \begin{pmatrix} Z_i(Y_i - W_i\,\tau(x) - \mu(x)) \\ Y_i - W_i\,\tau(x) - \mu(x) \end{pmatrix}.$$

Solving heterogeneous estimating equations

The classical approach is to rely on **local solutions** (Fan and Gijbels, 1996; Hastie and Tibshirani, 1990; Loader, 1999).

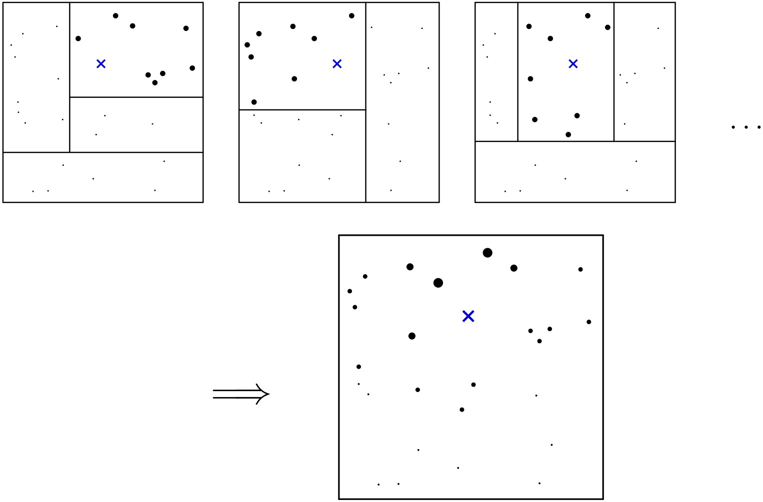
$$\sum_{i=1}^{n} \alpha(x;\, X_i)\; \psi_{\hat{\theta}(x),\, \hat{\nu}(x)}\left(O_i\right) = 0,$$

where the weights $\alpha(x;\, X_i)$ are obtained from, e.g., a **kernel**.

We use random forests to get good **data-adaptive** weights. Has potential to be help mitigate the **curse of dimensionality**.

- Building many trees with small leaves, then solving the estimating equation in each leaf, and finally **averaging the results** is a bad idea. Quantile and IV regression are badly **biased** in very small samples.
- Using RF as an "adaptive kernel" protects against this effect.

The random forest kernel



Forests induce a kernel via **averaging tree-based neighborhoods**.
This idea was used by Meinshausen (2006) for quantile regression.

Solving estimating equations with random forests

We want to use an estimator of the form

$$\sum_{i=1}^{n} \alpha(x; X_i)\, \psi_{\hat{\theta}(x),\, \hat{\nu}(x)}(O_i) = 0,$$

where the weights $\alpha(x; X_i)$ are from a random forest.

Key Challenges:

- How do we grow trees that yield an **expressive** yet **stable** neighborhood function $\alpha(\cdot; X_i)$?
- We do not have access to **"prediction error"** for $\theta(x)$, so how should we **optimize splitting**?
- How should we account for **nuisance parameters**?
- Split evaluation rules need to be **computationally efficient**, as they will be run many times for each split in each tree.

Step #1: Conceptual motivation

Following CART (Breiman et al., 1984), we use **greedy splits**. Each split directly seeks to improve the fit as much as possible.

- For regression trees, in large samples, the **best split** is that which **increases the heterogeneity** of the predictions the most.
- The same fact also holds **locally** for estimating equations.

We split a parent node P into two children C_1 and C_2. In **large samples** and with **no computational constraints**, we would like to maximize

$$\Delta\left(C_1,\, C_2\right) = n_{C_1} n_{C_2} \left(\hat{\theta}_{C_1} - \hat{\theta}_{C_2}\right)^2,$$

where $\hat{\theta}_{C_1}$, $\hat{\theta}_{C_2}$ **solve the estimating equation in the children**.

Step #2: Practical realization

Computationally, solving the estimating equation in each possible child to get $\hat{\theta}_{C_1}$ and $\hat{\theta}_{C_2}$ can be **prohibitively expensive**.

To avoid this problem, we use a **gradient-based approximation**. The same idea underlies gradient boosting (Friedman, 2001).

$$\hat{\theta}_C \approx \tilde{\theta}_C := \hat{\theta}_P - \frac{1}{|\{i : X_i \in C\}|} \sum_{\{i:X_i \in C\}} \xi^\top A_P^{-1} \, \psi_{\hat{\theta}_P, \hat{\nu}_p}(O_i),$$

$$A_P = \frac{1}{|\{i : X_i \in P\}|} \sum_{\{i:X_i \in P\}} \nabla \psi_{\hat{\theta}_P, \hat{\nu}_P}(O_i),$$

where $\hat{\theta}_P$ and $\hat{\nu}_P$ are obtained by solving the estimating equation once in the parent node, and ξ is a vector that picks out the θ-coordinate from the (θ, ν) vector.

Step #2: Practical realization

In practice, this idea leads to a **split-relabel** algorithm:

1. **Relabel step:** Start by computing pseudo-outcomes

$$\tilde{\theta}_i = -\xi^\top A_P^{-1}\ \psi_{\hat{\theta}_P, \hat{\nu}_P}(O_i) \in \mathbb{R}.$$

2. **Split step:** Apply a CART-style regression split to the $\widetilde{Y}_i$.

This procedure has several advantages, including the following:

- **Computationally**, the most demanding part of growing a tree is in scanning over all possible splits. Here, we reduce to a regression split that can be efficiently implemented.
- **Statistically**, we only have to solve the estimating equation once. This reduces the risk of hitting a numerically unstable leaf—which can be a risk with methods like IV.
- From an **engineering** perspective, we can write a single, optimized split-step algorithm, and then use it everywhere.

Step #3: Variance correction

Conceptually, we saw that—in large samples—we want splits that maximize the heterogeneity of the $\hat{\theta}(X_i)$. In small samples, we need to account for **sampling variance**.

We need to penalize for the following two sources of variance.

- Our **plug-in estimates** for the heterogeneity of $\hat{\theta}(X_i)$ will be **overly optimistic** about the large-sample parameter heterogeneity. We need to correct for this kind of over-fitting.
- We **anticipate "honest" estimation**, and want to avoid leaves where the **estimating equation is unstable**. For example, with IV regression, we want to avoid leaves with an unusually weak 1st-stage coefficient.

This is a generalization of the analysis of Athey and Imbens (2016) for treatment effect estimation.

Generalized Random forests

Our label-and-regress splitting rules can be used to grow an ensemble of trees that yield a forest kernel. We call the resulting procedure a **generalized random forest**.

- Regression forests are a special case of GRF with a squared-error loss.

Available as an `R`-package, `GRF`, built on top of the `ranger` package for random forests (Wright and Ziegler, 2015).

Asymptotic normality of GRF

Theorem. (Athey, Tibshirani and Wager, 2016) Given regularity of both the estimating equation and the data-generating distribution, generalized random forests are **consistent** and **asymptotically normal**:

$$\frac{\hat{\theta}_n(x) - \theta(x)}{\sigma_n(x)} \Rightarrow \mathcal{N}\left(0,\, 1\right), \quad \sigma_n^2 \to 0.$$

Proof sketch.

- Influence functions: Hampel (1974); also parallels to use in Newey (1994).
- Influence function heuristic motivates approximating generalized random forests with a class of regression forests.
- Analyze the approximating regression forests using Wager and Athey (2015)
- Use coupling result to derive conclusions about GRF.

Asymptotic normality of GRF: Proof details

- Influence function heuristic motivates approximating GRFs with a class of regression forests. Start as if we knew true parameter value in calculating influence fn:
  - Let $\tilde{\theta}_i^*(x)$ denote the influence function of the i-th observation with respect to the true parameter value $\theta(x)$: $\tilde{\theta}_i^*(x) = -\xi^\top V(x)^{-1} \psi_{\theta(x),\, \nu(x)}(O_i)$
  - Pseudo-forest predictions: $\tilde{\theta}^*(x) = \theta(x) + \sum_{i=1}^n \alpha_i \tilde{\theta}_i^*(x)$.
- Apply Wager and Athey (2015) to this. Key points: $\tilde{\theta}^*(x)$ is linear function, so we can write it as an average of tree predictions, with trees built on subsamples. Thus it is U-statistic; can use the ANOVA decomposition.
- Coupling result: conclusions about GRFs.

Suppose that the GRF estimator $\hat{\theta}(x)$ is consistent for $\theta(x)$. Then $\hat{\theta}(x)$ and $\tilde{\theta}^*(x)$ are coupled,

$$\tilde{\theta}^*(x) - \hat{\theta}(x) = o_P\left(\left\| \sum_{i=1}^n \alpha_i(x)\, \psi_{\theta(x),\, \nu(x)}\left(O_i\right) \right\|_2\right). \tag{1}$$

Simulation example: Quantile regression

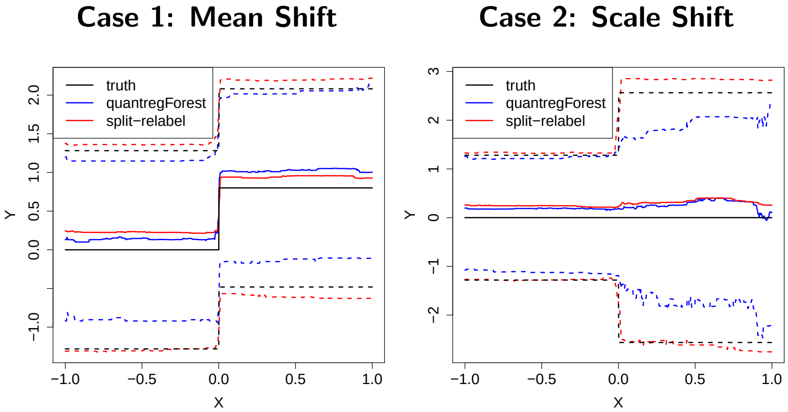
In quantile regression, we want to estimate the q-th quantile of the conditional distribution of Y given X, namely $\theta(x) = F_x^{-1}(q)$.

- Meinshausen (2006) used the random forest kernel for quantile regression. However, he used standard **CART regression splitting** instead of a tailored splitting rule.
- In our split-relabel paradigm, **quantile splits** reduce to **classification splits** ($\hat{\theta}_P$ is the q-th quantile of the parent):

$$\widetilde{Y}_i = \mathbf{1}(\{Y_i > \hat{\theta}_P\}).$$

- To estimate **many quantiles**, we do **multi-class** classification.

Simulation example: Quantile regression



The above examples show quantile estimates at $q = 0.1$, 0.5, 0.9, on Gaussian data with $n = 2,000$ and $p = 40$. The package `quantregForest` implements the method of Meinshausen (2006).

Simulation example: Instrumental variables

We want to estimate **heterogeneous treatment effects** with endogenous treatment assignment: Y_i is the treatment, W_i is the treatment assignment, and Z_i is an instrument satisfying:

$$\{Y_i(w)\}_{w \in \mathcal{W}} \perp\!\!\!\perp Z_i \,\big|\, X_i.$$

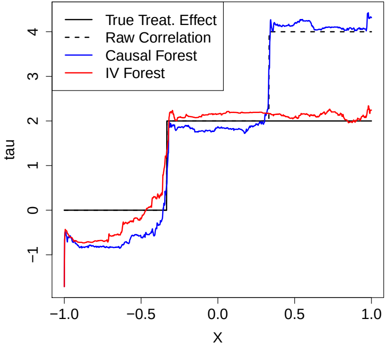
- Our **split-relabel** formalism tells us to use pseudo-outcomes

$$\tilde{\tau}_i = \left(Z_i - \bar{Z}_p\right)\left(\left(Y_i - \overline{Y}_p\right) - \hat{\tau}_P\left(W_i - \overline{W}_p\right)\right),$$

  where $\hat{\tau}_P$ is the IV solution in the parent, and $\overline{Y}_P$, $\overline{W}_p$, $\bar{Z}_p$ are averages over the parent.
- This is just IV regression residuals projected onto the instruments.

Simulation example: Instrumental variables

Using IV forests is important



We have **spurious correlations**:

- OLS for Y on W given X has two jumps, at $X_1 = -1/3$ and at $X_1 = 1/3$.
- The causal effect $\tau(X)$ only has a jump at $X_1 = -1/3$.
- $n = 10,000$, $p = 20$.

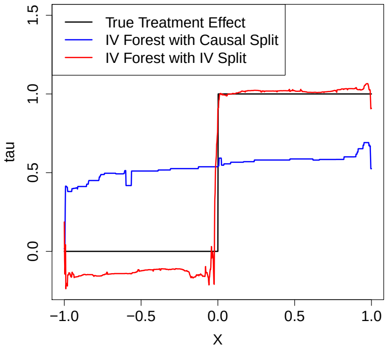
The response function is

$$Y_i = (2W_i - 1)\,\mathbf{1}(\{X_{1,i} > -1/3\}) + (3A - 1.5)\,\mathbf{1}(\{X_{1,i} > 1/3\}) + 2\varepsilon_i.$$

A_i is correlated with W_i.

Simulation example: Instrumental variables

Using IV splits is important



We have **useless correlations**:

- The joint distribution of (W_i, Y_i) is independent of the covariates X_i.
- But: the causal effect $\tau(X)$ has a jump at $X_1 = 0$.
- $n = 5,000$, $p = 20$.

The response function is

$$Y_i = 2 \cdot \mathbf{1}(\{X_{1,i} \leq 0\})\, A_i + \mathbf{1}(\{X_{1,i} > 0\})\, W_i + \mathbf{1}(1 + 0.73 \cdot \mathbf{1}(\{X_{1,i} > 0\}))\varepsilon_i.$$

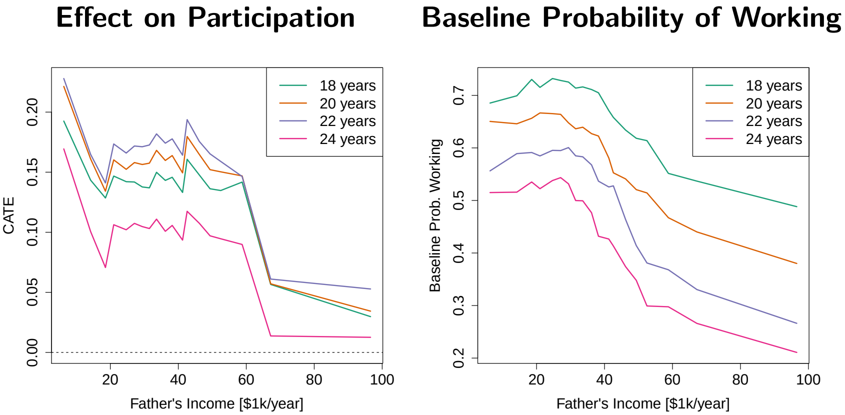
A_i is correlated with W_i.

Empirical Application: Family Size

Angrist and Evans (1998) study the effect of family size on women's labor market outcomes. Understanding heterogeneity can guide policy.

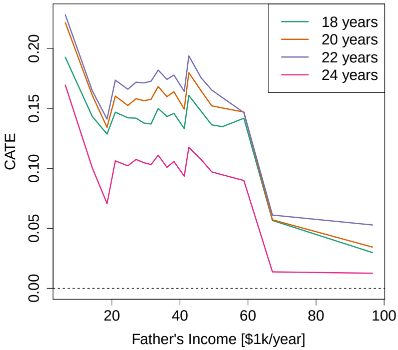
- Outcomes: participation, female income, hours worked, etc.
- Treatment: more than two kids
- Instrument: first two kids same sex
- First stage effect of same sex on more than two kids: .06
- Reduced form effect of same sex on probability of work, income: .008, $132
- LATE estimates of effect of kids on probability of work, income: .133, $2200

Treatment Effects: Magnitude of Decline

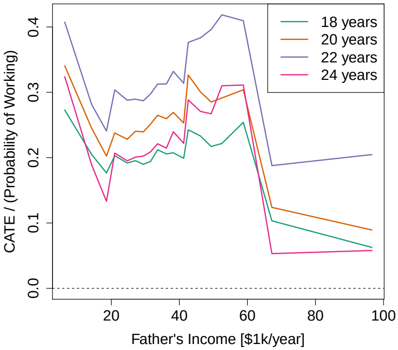


Treatment Effects: Magnitude of Decline

Effect on Participation



Effect relative to Baseline



Treatment Effects: Magnitude of Decline