A Model of Spatial Arbitrage with Transport Capacity Constraints and Endogenous Transport Prices

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This article solves a high-frequency model of price arbitrage incorporating storage and trade when the amount of trade is limited by transport capacity constraints. In equilibrium there is considerable variation in transport prices because transport prices rise when the demand to ship goods exceeds the capacity limit. This variation is necessary to attract shipping capacity into the industry. In turn, prices in different locations differ by a time varying amount. Thus while the law of one price holds, it holds because of endogenous variation in transport prices.

Key words: capacity constraints, commodity prices, spatial arbitrage, transport costs.

"A boom in wheat exports, a larger-than-expected wheat crop in the Northern Plains, high wheat prices, and expectations of a huge corn crop have contributed to a sudden increase in secondary market graincar prices. Prices for shuttle cars for September delivery increased from $87.50 for the week ending July 21 to $925 for the week ending August 18. Prices for non-shuttle cars for September delivery increased from $153 to $504.50 for the same weeks." United States Department of Agriculture: Grain Transportation Report, August 23 2007 p. 1.

". . . lake rates from Chicago to Buffalo depend on the fluctuating demand for transportation. They will sometimes not only vary day by day but hourly through the day. If there happens to be a large influx of vessels brought in by a favorable wind the rates will go down, and the reverse will take place when there is a reverse condition of things, and this action takes place instantaneously, and ordinarily without any combination on the part of the vessel owners. Last week there was a sudden call for much transportation, I suppose caused by some sudden foreign grain demand. It was in excess of the capacity of the lake to furnish, and vessel owners rapidly advanced their prices from 6 to 15 cents a bushel." Mr. Hayes, General Manager of Blue Line Fast Freight, Detroit, in U.S. Congress (1874) Report of the Select Committee on Transportation Routes to the Seaboard, Part II p. 33–34.

For over a century, observers of grain transportation systems have noticed that shipping prices often fluctuate in line with the demand to ship goods. If there is only a limited supply of transport equipment available for immediate use, a large demand to ship goods can place considerable pressure on the transport system, and as a result prices increase. The supply limitations can be caused by a localized shortage of engines or rolling stock on a particular rail route, a shortage of barges or ships on inland waterways, congestion at ports, or sometimes a global shortage of transport equipment when the demand to ship all sorts of goods is high. Indeed, as Brennan, Williams, and Wright (1997) observed in their analysis of the Western Australian grain transportation system, capacity on a particular route may be deliberately limited to ensure that expensive transport equipment is utilized for as much of the year as possible.

This article develops a model of the way that transport prices and commodity prices in two locations are simultaneously determined when there is a fixed quantity of transport equipment and when agents use logistics management techniques to arbitrage prices across space and time. Formally, the model is a rational expectations model of trade between two centers that

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I would like to thank the economics department faculty at the University of Michigan for their helpful comments and support during the time that this article was written. I would also like to thank Shaun Vahey, seminar participants at the University of Indiana, and the European Science Foundation for financial support to attend the July 2004 conference "Market performance and the welfare gains of market integration in history."

DOI: 10.1111/j.1467-8276.2008.01183.x

incorporates storage, uncertainty, transport system capacity constraints, and transportation time. The two centers have separate demand functions and stochastic supply functions, and forward-looking, risk-neutral agents ship or store goods to take advantage of arbitrage possibilities.

Storage is an essential part of the model. Storage ensures the prices are dynamically consistent, and it means that arbitrageurs facing high transport prices recognize the opportunity cost of not shipping, namely storing and shipping at a cheaper time. However, incorporating storage into the model makes it difficult to solve. In particular, an analytic solution to the model cannot be found if arbitrageurs have rational expectations about future commodity and transport prices, because inventories cannot be negative. Rather, a numerical solution must be found, using techniques developed by Williams and Wright (1991) and Coleman (forthcoming). The solution comprises the optimal quantities of goods that are shipped, stored, and consumed, along with the equilibrium distributions of commodity and transport prices. These distributions depend on the primitives of the model, such the distribution of output shocks, the upper transport capacity limit, and marginal transport costs.

The key feature of the model is the way transport prices vary with the demand to ship goods. In the model, the marginal cost of shipping goods is constant, but there is a capacity limit T^*. When transport prices are determined competitively, transport prices have peak-pricing patterns similar to those exhibited by other capital intensive and capital constrained industries such as the electricity industry (Williamson 1966). The transport price is equal to the marginal transport cost when the amount shipped is less than full capacity. However, when the amount shipped is equal to full capacity, the price is equal to the difference between the simultaneously determined spot price in the exporting city and the discounted expected future price in the importing city. As such, demand or supply shocks in the commodity markets cause endogenous variation in transport prices. In parallel with other "peak-pricing" industries, the high prices earned when capacity is fully utilized provide shipping companies with a return on their capital.

The solution shows how commodity prices and transport prices are related. Ordinarily, inventories of goods in both centers are sufficiently large that small fluctuations in the local supply of the commodity can be smoothed. When a center has a small supply shock, arbitrageurs adjust their inventories to compensate for the shock and the difference between prices in the two cities is less than or equal to the marginal cost of shipping, adjusted for storage costs. If a city has a large negative supply shock, however, local inventories are run down, and new supplies are ordered from the other city. If inventories in the importing city are reduced to zero, the spot price there will temporarily exceed the spot price in the exporting center plus the transport price because the imported goods do not arrive immediately. The transport price depends on the amount shipped. If it is only necessary to ship small amounts, the transport price will be the marginal transport cost. If the shortage in the importing city is sufficiently severe or long lasting, however, shippers attempting to take advantage of the expected high prices in the importing city will drive up the transport price until it just equals the difference between the price in the export center and the discounted expected future price in the importing center. Transport prices and spatial price differentials will remain high so long as transport capacity is fully utilized and there is a shortage in the importing center.

The model is related to the Wright and Williams (1989) model that examines why storage occurs under price backwardation. Both models examine how prices are determined when arbitrageurs have limited transport facilities but can store goods. Their model offered an explanation of why prices can be in backwardation in one center while inventories are held elsewhere. The model in this article can be considered a formalization of their approach that allows equilibrium prices, storage quantities, and shipping volumes to be determined endogenously. It thus links the literature examining storage under backwardation with the literature examining the law of one price. Although the model is not designed to be directly estimated, it generates predictions about the behavior of transport prices and spot and future commodity prices that can be tested empirically. For example, the model predicts the conditions when local supply shocks will or will not induce a simultaneous change in local commodity prices and transport prices, conditions that could be tested with suitable data.

Although in many real world situations the supply of transport services is upward sloping rather than perfectly inelastic, the assumption that there is an upper capacity limit is maintained to focus attention on the way that the pattern of shipping prices is affected by the option of delaying shipping to a cheaper time. Nonetheless, it is possible to examine what happens to the pattern of shipping prices and commodity prices as the capacity limit is varied. As the shipping capacity increases, shipping constraints bind less frequently, so the transport price exceeds the marginal transport cost less frequently and the average earnings of the shipping industry decrease. In turn, the lower variability of transport prices means that the variance of the spatial price difference decreases.

The article also examines how the pattern of shipping and commodity prices depends on fundamental economic factors when capacity levels are determined endogenously because firms enter or exit the shipping industry in response to long-term capital profitability. For example, if the variance of the underlying output shocks increases, the variance of shipping prices, the profitability of the shipping industry, and the variance of the spatial price difference increase at all shipping-capacity levels. The additional profitability of shipping attracts more shipping capacity into the market. However, if shipping capacity increases until the rate of return on capital is the same as its initial level, both the variance of shipping prices and the variance of the spatial price difference will have increased. Shipping firms increase their capacity in response to more volatile output, as their peak demands are higher; but they operate below capacity for a greater fraction of the time, so their earnings are more volatile.

The focus of this article is theoretical. Nonetheless, it is inspired by descriptions of spatial arbitrage in markets where transport capacity constraints have been important, such as the late nineteenth-century United States corn market (US Congress 1874; Coleman, forthcoming), the late twentieth-century Western Australian wheat market (Brennan, Williams, and Wright 1997), and contemporary midwestern grain markets. Transport capacity constraints will not be important in all markets, although in most markets one would expect that the owners of transportation and distribution systems restrict their purchases of capital intensive transport machinery and adopt logistics management practices to ensure their machinery is utilized as much as possible. Thus while the model is an abstraction, it provides some insight into the way commodity prices and transport prices are simultaneously determined in circumstances that transport capacity is not perfectly elastic, and transport markets are equilibrated through variable transport prices.

Previous Literature

In recent years, several authors have demonstrated that empirical examinations of spatial arbitrage must take into account transport price variation. In a widely copied approach, Spiller and Wood (1988) developed a switching regime model that estimated the distribution of transport prices from spatial price differences under the assumptions that arbitrage takes place instantaneously and that transport prices follow a parametric distribution such as a truncated normal or a gamma distribution. Their estimates of transport prices revealed considerable time-variation. This framework was significantly expanded by Baulch (1997) and Barrett and Li (2002), who developed parity bound models that directly incorporate transport price information into the estimation procedure. Time-varying transport prices were also shown to be an important factor in explaining international price differences in an empirical model of European, North American, and Japanese grain prices estimated by Goodwin, Grennes, and Wohlgenant (1990). In addition, they showed that these price differences were better explained if shipping times were incorporated into the model.

From a theoretical perspective, these approaches have two potential limitations. First, they are designed to examine commodity prices conditional on transport prices and trade flows, rather than to examine the potentially endogenous determinants of transport prices and trade flows. Thus while they show that spatially separated markets tend to be well-linked through arbitrage, conditional on transport prices, they do not explain why the variation in transport prices occurs. Second, these models ignore how storage affects the arbitrage process, not least by enabling shippers to substitute away from high-cost shipping times to low-cost shipping times. To this extent, the literature on spatial price arbitrage has evolved somewhat separately from the literature on logistics management, where the interaction of inventory management and transport systems is central, as well as much of the transport economics literature, where it is commonly assumed that transport prices vary endogenously because of capacity constraints (Stopford 1988; Tyworth 1991).[1]

The model in this article is designed to integrate these different approaches. In the taxonomy developed by Fackler and Goodwin (2001), the model is a dynamic point-location model of spatial arbitrage that incorporates uncertainty and rational expectations, transport delays, capacity constraints, storage, and endogenously determined transport prices. As Fackler and Goodwin note, models of spatial price determination with some of these features already exist, so some of the results of this article are not new. What is distinctive about the article, however, is the use of rational storage behavior to generate the intertemporal price dynamics while other features of the transport system such as time delays and capacity constraints are modeled.

Rational storage models have a long history, including Williams (1936), Gustafson (1958), Williams and Wright (1991), and Deaton and Laroque (1992, 1996). There are two small literatures that simultaneously analyze trade and storage behavior. The first, including Williams and Wright (1991) and Coleman (forthcoming), explicitly examines the implications for prices in different locations when agents use both storage and transport technologies to arbitrage prices. Both of these articles examine trade under uncertainty when the supply of transport services is infinitely elastic and the transport price is fixed. The second, including Wright and Williams (1989), Benirschka and Binkley (1995), and Frechette and Fackler (1999) examines why storage frequently takes place under price backwardation. Technically, the model in this article is most similar to that developed by Coleman. Conceptually, however, the model is more closely related to the Wright and Williams (1989) model that offers an explanation for storage under price backwardation.

Wright and Williams observed that most inventories are held in locations distant from the primary market for which spot and futures prices exist. They argued that when the spot price exceeds the future price, inventories in the primary market are run down and then attempts are made to obtain inventories held elsewhere. Only a small quantity of inventories are moved because transport facilities are limited; instead, transport prices are bid up to unusually high levels, making it unprofitable to ship most of the inventories to the primary market to take advantage of the unusually high spot prices. The model in this article has the same basic idea and can be considered a formalization of their approach. It extends the approaches of Benirschka and Binkley (1995), and Frechette and Fackler (1999) in three ways by allowing for uncertainty and by relaxing their assumptions that transport is instantaneous and infinitely elastic.

The Model

The model is an extension of the rational expectations models of storage and trade developed by Coleman (forthcoming) and Williams and Wright (1991). Following Samuelson (1952), the model links a set of equations representing supply and demand curves in different locations with a set of no-arbitrage conditions that preclude excess profits from either the storage of goods at a location or the shipment of goods between locations. The main difference from Coleman is that transport capacity constraints are imposed and the transport price is allowed to exceed the marginal transport cost.

There are two centers, A and B, each with a separate inverse demand function for a commodity:

$$(1) \qquad P_t^i = D_i^{-1}\left(Q_t^i\right): \quad D_i^{-1}(0) < \infty, \quad \lim_{Q \to \infty} D_i^{-1}(Q) = 0$$

where Q_t^i is the amount purchased for final use at time t and $i = A, B$. The output produced each period is stochastic but price inelastic, because it has a long gestation period. In the literature, output is usually modeled as a serially autocorrelated or seasonal stochastic process. Since the analytical structure of the model does not depend on this choice, in this article I follow Deaton and Laroque (1996) and assume that output in each center follows an independent first-order autoregressive process around a constant mean:

$$(2) \quad \left(X_t^i - \bar{X}^i\right) = \rho^i\left(X_{t-1}^i - \bar{X}^i\right) + e_t^i \quad i = A, B$$

where e_t^i is a white noise process and $|\rho^i| < 1$.[2]

[1] The logistics literature examines the optimal ways for a company with sales in several locations to minimize total procurement, transport, and storage costs. See Baumol and Vinod (1970) for an original statement, or Tyworth (1991) or de Jong (2000) for a review.

[2] The model can be solved for values of ρ in the range $-1 < \rho < 1$, although it is envisaged that ρ is high and positive, so that shocks are persistent. If ρ were negative, it would be optimal to ship to a center whenever output there was high, as it would be low the next period when the goods arrived.

All production, consumption, storage and trade activity takes place at the beginning of the period. It is assumed that unlimited quantities of the good can be stored in either center, and that at any time t goods that were produced in a different period but which are located in the same place are indistinguishable and trade at the same price. The length of a period is the time that it takes to ship goods from one center to another. Risk neutral arbitrageurs are assumed to predict future prices and purchase and hold inventories until the expected price increase just offsets the cost of storage; conversely, if the expected appreciation is less than the cost of storage, inventories will be zero. In keeping with the literature, there are three storage costs. First, there is an elevator charge K^S per unit to store goods each period; this charge is smaller than the marginal transport cost K^T as it is assumed that elevators are less expensive to build and operate than transport equipment. Second, the commodity depreciates at rate δ so if S_t is stored in period t, $(1-\delta)S_t$ will be available in period $t+1$. Third, there is an interest cost r foregone when storage is undertaken. The total quantity of stored and imported goods in a center at the beginning of the period is M_t^i,

$$(3) \qquad M_t^i = (1-\delta)\left(S_{t-1}^i + T_{t-1}^j\right)$$

where S_{t-1}^i is the non-negative quantity stored in center i and T_{t-1}^j is the nonnegative quantity exported from center j. The quantities consumed, stored and exported from a center at time t are such that $Q_t^i + S_t^i + T_t^i = X_t^i + M_t^i$.

The demand for shipping services depends on all the primitive factors of the model, and is solved endogenously. The amount arbitrageurs are willing to bid for transport services is determined by the opportunities to profit from trading. The demand curve to ship goods from one center to the other has an intercept at zero when exports would not be profitable even if shipping were free and shifts upward when the expected price in the destination center increases relative to the price in the source center. The slope of the curve depends on the slope of the commodity demand curve in each center and the availability of inventories. If the transport price were higher, arbitrageurs would reduce the volume they shipped until the expected price in the destination center was sufficiently high to offset the higher transport price. The rate at which commodity prices in the destination center increase as the volume of goods in transit declines depends on the rate at which inventories in the destination center are altered. Since this depends on the expected future transport price, the demand curve for shipping services at time t implicitly takes into account the opportunity to delay shipping to a later date.

When shipping takes time and there are transport capacity constraints, the amount of transport equipment in each center must be traced. To simplify the analysis, it is assumed all transport operators own two pieces of transport equipment, one in each center, and that whenever one is dispatched full the other one is shipped back in the opposite direction. In this way, there is a constant amount of transport equipment in each center at the beginning of each period. This is the transport capacity T^*. If less than T^* is shipped, shippers pay the marginal transport cost, K^T.[3] If T^* is shipped, shippers bid competitively for capacity and the price equals the difference between the spot price in the originating city and the appropriately discounted expected future price in the other city. (See equations (4g)–(4p) for a formal definition.) When goods are transported, they also depreciate at the rate δ.

It is assumed that risk-neutral, profit-maximizing, and rational speculators ship and store goods to take advantage of expected price differences. The speculators have expectations about future prices that incorporate all information about output, storage, and trade in both centers. The behavior of risk-neutral speculators can be represented by four inequalities. Let $y_t = [M_t^a, M_t^b, X_t^a, X_t^b]$ be the vector of state variables. Let K_t^a and K_t^b be the variable transport prices of shipping from A and B, respectively, and let the excess return to the carriers be $R_t^i = K_t^i - K^T$. Let $\Pi^{ij}(y_t) = (\frac{1-\delta}{1+r})E[P_{t+1}^i \mid y_t] - P^j(y_t)$ be the difference between the expected future price in center i and the price in center j at the point y_t. Then, at each point y_t:

$$\text{(4a)} \quad \Pi^{AA}(y_t) = \left(\frac{1-\delta}{1+r}\right) E\left[P_{t+1}^A \mid y_t\right] - P^A(y_t) \leq K^S$$

[3] Since there is only one good in the model, it is never profitable to simultaneously ship the good in both directions. The cost K^T is the cost of sending goods from one city to the other and simultaneously sending empty ships back in the other direction.

(4b,c)

$$\left[\Pi^{AA}(y_t) - K^S\right] \cdot S^A(y_t) = 0; \quad S^A(y_t) \geq 0$$

(4d)
$$\Pi^{BB}(y_t) = \left(\frac{1-\delta}{1+r}\right) E\left[P^B_{t+1} \mid y_t\right] - P^B(y_t) \leq K^S$$

(4e,f)

$$\left[\Pi^{BB}(y_t) - K^S\right].S^B(y_t) = 0; \quad S^B(y_t) \geq 0$$

(4g)
$$\Pi^{BA}(y_t) = \left(\frac{1-\delta}{1+r}\right) E\left[P^B_{t+1} \mid y_t\right] - P^A(y_t) \leq R^A_t(y_t) + K^T$$

(4h,i) $R^A_t(y_t) \geq 0; \quad 0 \leq T^A(y_t) \leq T^*;$

(4j,k)
$$\left[\Pi^{BA}(y_t) - \left(R^A_t(y_t) + K^T\right)\right] \cdot T^A(y_t) = 0; \quad \left[T^A(y_t) - T^*\right] \cdot R^A_t(y_t) = 0$$

(4l)
$$\Pi^{AB}(y_t) = \left(\frac{1-\delta}{1+r}\right) E\left[P^A_{t+1} \mid y_t\right] - P^B(y_t) \leq R^B_t(y_t) + K^T$$

(4m,n) $R^B_t(y_t) \geq 0; \quad 0 \leq T^B(y_t) \leq T^*$

(4o,p)
$$\left[\Pi^{AB}(y_t) - \left(R^B_t(y_t) + K^T\right)\right] \cdot T^B(y_t) = 0; \quad \left[T^B(y_t) - T^*\right] \cdot R^B_t(y_t) = 0$$

where

$$\left(X^i_{t+1} - \bar{X}^i\right) = \rho^i\left(X^i_t - \bar{X}^i\right) + e^i_{t+1}$$

$$P^i(y_t) = D^{-1}_i\left(X^i_t + M^i_t - S^i(y_t) - T^i(y_t)\right)$$

$$M^i_{t+1}(y_t) = (1-\delta)\left(S^i(y_t) + T^j(y_t)\right), \text{ and}$$

(5)
$$E\left[P^i_{t+1} \mid y_t\right] = \iint_x D^{-1}_i\left(X^i_{t+1} + M^i_{t+1}(y_t) - S^i(y_{t+1}) - T^i(y_{t+1})\right) f\left(X^i_{t+1}, X^j_{t+1}\right) dX^i_{t+1}\, dX^j_{t+1}.$$

The inequalities (4a)–(4c) and (4d)–(4f) are the conditions for profitable storage in the two centers. These are the same conditions describing optimal inventory holdings found in Coleman (forthcoming). Inventories in each center will be zero if the expected future price exceeds the current spot price by less than the costs of storage; otherwise, arbitrageurs hold a quantity that ensures the expected future price exactly equals the current price plus the storage costs.

Equations (4g)–(4k) are the conditions for trade from center A to center B. Equations (4g) and (4j) say that trade will be zero if the expected future price in center B exceeds the spot price in center A by less than the costs of trade; otherwise, arbitrageurs ship a quantity that ensures the expected future price exactly equals the current price plus the trade cost. Equations (4j) and (4k) say that the transport price will equal K^T if the traded amount is less than the shipping capacity; otherwise, the transport price will exceed K^T by the difference between the expected future price in center B (adjusted for the amount lost in transit and interest costs) and the spot price in center A. The conditions differ from those in Coleman, where the transport price always equaled the marginal cost K^T. The quintet (4l)–(4p) are similar, but describe the conditions for trade from center B to center A.

The model solution comprises two parts. The first part is the set of optimal storage and trade functions $[S^A(.), S^B(.), T^A(.), T^B(.)]$ that satisfy the inequalities (4a)–(4p). The second part is the equilibrium distribution of the state variables, which depends on the assumed stochastic process determining output and the optimal storage and trade functions. The solution fulfils two conditions: first, that storage and trading decisions are profit-maximizing conditional on expectations of future prices; and, second, that price expectations are consistent with the storage and trading decisions and expectations of future output quantities.

A numerical solution is found using the methodology used by Coleman (forthcoming). The solution is calculated over a discrete four-dimensional grid corresponding to the four state variables. There are three steps. First, a discrete joint probability distribution over the grid values for the stochastic variables X^A and X^B is chosen, and the double-integral formula in equation (5) is replaced by the equivalent summation formula. The joint probability density for X is chosen to mimic an autocorrelated process with normal innovations, and is represented by an $m^2 \times m^2$ Markov transition matrix Π specifying the probability of going from one point (X^A_{i1}, X^B_{j1}) to a second point (X^A_{i2}, X^B_{j2}). Second, an algorithm is used to calculate the optimal amounts of storage and trade in the two centers. The algorithm constructs a series of successive approximations to the optimal storage and trade functions, and is repeated until the difference between successive values of the control values is small. Piece-wise linear demand functions for each center are specified

$$(6)\quad D_i^{-1}(Q_t^i) = \begin{cases} \alpha^i & \text{if} \quad Q_t^i = 0 \\ \alpha^i - \beta^i Q_t^i & \text{if} \quad 0 < Q_t^i \le \alpha^i/\beta^i \\ 0 & \text{if} \quad Q_t^i > \alpha^i/\beta^i \end{cases}.$$

Third, the equilibrium probability distribution—the unconditional probability of being at every grid point—is found, and moments of interest are calculated.

Properties of the Model

The model solution has two key properties. The first concerns price relationships when inventories in both centers are positive. In particular, if both centers have positive inventories, prices in the two centers will not differ by more than the difference between the transport price and the storage cost. To derive this result, note that if $S_t^A > 0$ and $S_t^B > 0$ equations (4a) and (4d) hold with equality. Equations (4a) and (4l) together imply

$$(7a)\quad P_t^A - P_t^B \le R_t^B + K^T - K^S.$$

Similarly, equations (4d) and (4g) together imply

$$(7b)\quad P_t^A - P_t^B \ge -(R_t^A + K^T - K^S).$$

Consequently,

$$(8)\quad -(R_t^A + K^T - K^S) \le P_t^A - P_t^B \le R_t^B + K^T - K^S.$$

When the transport capacity constraint is so large that it never binds, $R_t^A = R_t^B = 0$ and prices in the two centers do not differ by more than $K^T - K^S$. If transport constraints bind and either R_t^A or R_t^B is positive, prices in the two centers can differ by more than $K^T - K^S$, but they cannot differ by more than actual transport price.

The second property concerns the conditions when exporting occurs and is best analyzed by considering what happens when the transport capacity constraint is so large that it never binds. When the capacity constraint never binds, goods are only exported if there is some probability that inventories will be zero in the importing center when they arrive in the subsequent period. To see this, consider a point y_t at which it is profitable to export from B to A, so equation (4l) holds with equality. In combination with equation (4d) describing storage in center B,

$$(9)\quad E\left[P_{t+1}^A - P_{t+1}^B \mid y_t\right] \ge \frac{1+r}{1-\delta}(K^T - K^S)$$

with the inequality holding exactly if $S_t^B > 0$: that is, exports take place only if the expected future price differential is greater than $K^T - K^S$. If there are positive inventories in center A when the goods arrive at time $t+1$, however, equation (4a) applied at time $t+1$ holds exactly. In combination with equation (4l) applied at $t+1$, it follows that

$$(10)\quad P_{t+1}^A - P_{t+1}^B \le K^T - K^S.$$

Consequently, exports will only take place if there is some probability that inventories in center A are zero at time $t+1$—that is, if the goods arrive when supplies are so short that inventories are reduced to nothing and prices spike up. Exporters will not cover the full cost of exporting on every shipment, but will make sufficiently large profits when their goods arrive when prices are temporarily high in the importing center to offset the losses they make at other times. Indeed, a sufficiently small quantity of goods is shipped to ensure that temporary price spikes sometimes occur.

When the transport capacity constraint sometimes binds, the result is more complex. Arbitrageurs will still cover their costs only if their goods sometimes arrive when they are unusually scarce in the destination market. Now, however, there are two reasons why prices can be high in the importing center at $t+1$. Again consider the case that center B exports to A at time t, so equation (4l) holds with equality; then

$$(11)\quad E\left[P_{t+1}^A - P_{t+1}^B \mid y_t\right] \ge \frac{1+r}{1-\delta}(R_t^B + K^T - K^S)$$

with $R_t^B = 0$ if $T_t^B < T^*$. As previously, the price differential $P_{t+1}^A - P_{t+1}^B$ can exceed $R_t^B + K^T - K^S$ if $S_{t+1}^A = 0$. But it can also exceed $R_t^B + K^T - K^S$ if $S_{t+1}^A > 0$, $T_{t+1}^B = T^*$, and transport prices at $t+1$ are sufficiently high, for in this case

$$(12)\quad P_{t+1}^A - P_{t+1}^B \ge R_{t+1}^B + K^T - K^S.$$

When these circumstances occur, the importing center will import even though it has positive inventories and is certain to have positive inventories in the next period. This is not possible in most models in the literature, where inventories must be reduced to zero before imports occur.

This case is interesting as it extends the Wright and Williams (1989) explanation of why storage may occur when prices are in backwardation. They demonstrated that it is possible for inventories to be positive in one center while prices in the other center are in backwardation when there are capacity constraints. In this model, it is also possible for inventories to be positive while local prices are in long-term backwardation, so long as prices are expected to rise in the short term before they subsequently fall.

Suppose center A has a small quantity of inventories at time t when an adverse output shock occurs. Further suppose that the transport capacity is small, and that the shock is sufficiently large and persistent that output is not expected to return to near-normal levels for several periods, say until $t + k$. Arbitrageurs will start to run down their inventories and simultaneously import. Because the amount they can import in this and subsequent periods is limited, they will not want to deplete their inventories too fast as they know they will need to use them to supplement the limited amounts they will be able to import in the future. Consequently, even though prices will be expected to rise in the short term to justify storage, it is also possible that prices are in long-term backwardation because they are higher than they will be when output returns to normal: that is, $\frac{1-\delta}{1+r} E_t P^A_{t+1} = P^A_t + K^S$ and $P^A_t > E_t P^A_{t+k}$.[4] Note that if center B also has inventories, the spatial price differential will be expected to increase at rate $\frac{1+r}{1-\delta}$ and transport prices will be expected to rise until the transport capacity constraint is no longer binding.

Numerical Results

In the rest of this section, numerical simulations are used to illustrate how transport capacity constraints affect trade flows, storage quantities, and prices. The solution depends on the basic parameters of the model including the demand and supply functions, the interest rate, and the marginal transport cost. An important parameter is the extent to which output is regionally specialized. When the centers are identical, trade occurs infrequently in either direction, and average trade flows are small; but when one center produces a large fraction of output, it exports most of the time and average trade flows are larger. The solutions are sufficiently distinctive that numerical results are presented for the case that output is the same in each center and for the case that output is regionally specialized. The parameters are chosen so that the centers have identical demand functions, the price elasticity in each center at average consumption is one, the storage cost K^S is zero, the marginal transport price K^T is 5% of average prices, the period is one week, and the annual interest and depreciation rates are 5%.[5] The simulations show the probability of each set of complementary conditions (table 1); how inventories, trade flows, and prices are distributed for different values of the capacity constraint (tables 2–3); and how they depend on the size of the underlying shocks to the economy (table 4). The capacity constraint is then endogenized to show how much capacity is needed to reach a target rate of return as the size of the shocks to the economy is varied (table 5).

Table 1 shows the spatial price differentials and expected future price differentials corresponding to the twenty different combinations of complementary conditions. It gives the probability of each set of conditions occurring when center A produces 50% or 35% of total output, and when the capacity constraint is 20 units (10% of total output). Two aspects of the probability distributions are characteristic of the distributions found for a wide range of different parameterizations: both centers have positive inventories most of the time; and the spatial price difference exceeds the transport price (adjusted for the storage cost) less than 5% of the time. For example, when A produces 50% of total output, both centers have positive inventories 94% of the time and the spatial price difference exceeds the transport price (adjusted for the storage cost) 4% of the time.

[4] This possibility should not be considered a theoretical aberration. In the late nineteenth century, New York corn prices were frequently in backwardation over winter while New York inventories were positive. In particular, it was often the case that the December spot price (or the price for delivery that month) was higher than the price for delivery the following May even though millions of bushels of corn were in local storage. The price for January delivery was higher than the price for December delivery, however. This is because inexpensive lake transport of grain was unavailable over winter, and shippers in Chicago delayed shipment until May. Corn inventories in New York that were amassed in late fall were run down to near zero levels by early spring in anticipation of a resumption of the imports from Chicago.

[5] In the baseline case, the following model parameters are used: the demand function $D_i^{-1}(Q) = \alpha - \beta Q = 200 - Q$; mean production is 100 in each center; the production conditional variance $\sigma^2 = 100$; the production autocorrelation $\rho = 0.9$; the weekly interest rate $r = 0.001$; the weekly depreciation rate $\delta = 0.001$; $K^S = 0$; and $K^T = 5$. The mean price is 100.

Table 1. Analytical Results Corresponding to Different Complementary Conditions

T_t^A	T_t^B	S_t^A	S_t^B	Prob(.) $X^A = 50$	Prob(.) $X^A = 35$	$P_t^A - P_t^B$	$E[P_{t+1}^A - P_{t+1}^B \mid y_t]$
$=0$	$=0$	>0	>0	63.7%	2.4%	$\leq (K^T - K^S)$	$\frac{1+r}{1-\delta}(P_t^A - P_t^B)$
$=0$	$=0$	>0	$=0$	0.04%	1.5%	$\leq (K^T - K^S)$	Uncertain
$=0$	$=0$	$=0$	>0	0.04%	0%	$\geq -(K^T - K^S)$	Uncertain
$=0$	$=0$	$=0$	$=0$	0.11%	0%	Uncertain	Uncertain
$=0$	$0 < T_t^B < T^*$	>0	>0	5.9%	1.7%	$= (K^T - K^S)$	$= \frac{1+r}{1-\delta}(K^T - K^S)$
$=0$	$0 < T_t^B < T^*$	>0	$=0$	0.27%	3.9%	$= (K^T - K^S)$	$\geq \frac{1+r}{1-\delta}(K^T - K^S)$
$=0$	$0 < T_t^B < T^*$	$=0$	>0	0.36%	0.01%	$\geq (K^T - K^S)$	$= \frac{1+r}{1-\delta}(K^T - K^S)$
$=0$	$0 < T_t^B < T^*$	$=0$	$=0$	0.30%	0.04%	$\geq (K^T - K^S)$	$\geq \frac{1+r}{1-\delta}(K^T - K^S)$
$=0$	$=T^*$	>0	>0	9.5%	85.6%	$= (K_t^B - K^S)$	$= \frac{1+r}{1-\delta}(K_t^B - K^S)$
$=0$	$=T^*$	>0	$=0$	0.00%	0.69%	$= (K_t^B - K^S)$	$\geq \frac{1+r}{1-\delta}(K_t^B - K^S)$
$=0$	$=T^*$	$=0$	>0	1.8%	2.9%	$\geq (K_t^B - K^S)$	$= \frac{1+r}{1-\delta}(K_t^B - K^S)$
$=0$	$=T^*$	$=0$	$=0$	0.00%	0.01%	$\geq (K_t^B - K^S)$	$\geq \frac{1+r}{1-\delta}(K_t^B - K^S)$
$0 < T_t^A < T^*$	$=0$	>0	>0	5.9%	0.01%	$= -(K^T - K^S)$	$= -\frac{1+r}{1-\delta}(K^T - K^S)$
$0 < T_t^A < T^*$	$=0$	>0	$=0$	0.36%	0.21%	$\leq -(K^T - K^S)$	$= -\frac{1+r}{1-\delta}(K^T - K^S)$
$0 < T_t^A < T^*$	$=0$	$=0$	>0	0.27%	0%	$= -(K^T - K^S)$	$\leq -\frac{1+r}{1-\delta}(K^T - K^S)$
$0 < T_t^A < T^*$	$=0$	$=0$	$=0$	0.30%	0%	$\leq -(K^T - K^S)$	$\leq -\frac{1+r}{1-\delta}(K^T - K^S)$
$=T^*$	$=0$	>0	>0	9.5%	0.01%	$= -(K_t^A - K^S)$	$= -\frac{1+r}{1-\delta}(K_t^A - K^S)$
$=T^*$	$=0$	>0	$=0$	1.8%	0.17%	$\leq -(K_t^A - K^S)$	$= -\frac{1+r}{1-\delta}(K_t^A - K^S)$
$=T^*$	$=0$	$=0$	>0	0.00%	0%	$= -(K_t^A - K^S)$	$\leq -\frac{1+r}{1-\delta}(K_t^A - K^S)$
$=T^*$	$=0$	$=0$	$=0$	0.00%	0%	$\leq -(K_t^A - K^S)$	$\leq -\frac{1+r}{1-\delta}(K_t^A - K^S)$

Notes: The table gives the values of the price differential and the expected future price differential $E[P_{t+1}^A - P_{t+1}^B \mid y_t]$ for different combinations of the complementary conditions 4c, f, i, n. The probabilities of each set of conditions occurring pertain to the cases that marginal transport costs equal 5, the capacity constraint equals 20, and A either produces 50% of output or 35% of output.

Tables 2 and 3 show how trade flows, transport prices, storage quantities, and commodity prices vary as the transport capacity is increased. Table 2 describes the case when center A produces 50% of total output on average; table 3 describes what happens when center A produces 35% of output and frequently imports. The effects of increasing capacity on shipping prices and commodity prices are similar in each case. First, as transport capacity is increased the mean volume of trade increases and the fraction of occasions when the capacity constraint binds decreases. This lowers the mean and variance of transport prices. There is a small reduction in the size of inventories in the importing and exporting centers. Second, price dispersion decreases, measured either by the variance of $P_t^A - P_t^B$ or the fraction of times that $P_t^A - P_t^B$ exceeds $K^T - K^S$. The effect of relaxing capacity constraints is greater when one center is the dominant producer, for inventory management is less useful for smoothing prices than when both centers are the same.

Table 4 shows how the effects of capacity constraints depend on the size of the underlying shocks to the economy. In the simulations, center A produces 35% of output and the standard deviation of the production shock is either 5 or 10% of average output. As output volatility increases, inventory management becomes more important, and average inventory levels increase markedly. Trade occurs less frequently, but the transport capacity is more likely to be fully utilized when trade does occur so the fraction of times that the transport price is bid above the marginal cost increases.[6] This increases the mean and variance of transport prices, which in turn increases price dispersion, as measured by the variance of $P_t^A - P_t^B$. For a given level of transport capacity, therefore, high output volatility generates large spatial price dispersion and large returns to transport operators.

Table 5 shows how transport capacity responds to an increase in output volatility when firms are assumed to enter or exit the transport market until a target rate of return is obtained. It shows the transport capacity needed

[6] Note that under this parameterization goods would always be sent from B to A if there were no uncertainty. Uncertainty increases the fraction of time that no trade occurs, and the fraction of time that A exports to B because it has a temporary surplus and B a temporary shortfall.

Table 2. Prices, Storage, and Transport Price Statistics Corresponding to the Model (Center *A* Produces 50% of Output, σ = 10, Changing Transport Capacity)

Statistic	$T^*=5$	$T^*=10$	$T^*=20$	$T^*=30$	$T^*=40$	$T^*=50$	$T^*=\infty$
Prices							
Mean(P^A)	100.4	100.3	100.3	100.3	100.3	100.3	100.3
Std. Dev−(P^A)	10.3	9.6	8.5	8.6	8.6	8.5	8.5
Mean($P^A - P^B$)	0	0	0	0	0	0	0
Std. Dev.($P^A - P^B$)	13.0	10.5	7.3	5.8	5.1	4.8	4.6
% ($\mid P^A - P^B \mid > K^T - K^S$)	54.2%	40.2%	23.8%	13.8%	7.8%	4.4%	3.1%
Storage							
Mean(S^A)	361	325	288	273	265	269	261
Std. Dev.(S^B)	305	275	252	253	253	257	251
% ($S^A > 0$)	95%	96%	97%	97%	97%	97%	97%
Trade							
Mean(T^A)	1.4	2.2	2.9	3.1	3.1	3.1	3.2
Std. Dev.(T^A)	2.2	4.1	6.8	8.2	9.0	9.1	10.5
Mean(T^A)$\mid T^A > 0$	4.9	9.4	16.7	19.4	19.7	20.0	20.0
% ($T^A > 0$)	28.4%	23.5%	17.5%	15.9%	15.9%	15.7%	16.0%
% ($T^A = T^*$)	27.1%	20.1%	11.4%	6.1%	2.8%	0.8%	0%
Transport Price							
Mean (K_t^A)	7.27	6.40	5.54	5.20	5.08	5.04	5
Std. Dev (K_t^A)	5.9	4.5	2.5	1.5	0.9	0.6	0
Excess return	4.54	2.80	1.08	0.32	0.16	0.08	0

P^A: the price in center A. S^A: storage in center A. T^A: trade from center A to center B.
K_t^A: the transport price incurred sending a ship from A to B.
% $|P^A - P^B| > K^T - K^S$: the fraction of time the price difference exceeds the difference between the marginal trade cost and the storage cost. $K^S = 0$ in these simulations.
%($S^A[T^A] > 0$): the fraction of time storage [exports] > 0.
% ($T^A = T^*$): the fraction of times the quantity traded is equal to the transport capacity and the fraction of times $K_t^A > K^T$.

to obtain an average transport price of 7 or 40% higher than marginal cost.[7] Output in center A equals 50, 45, or 35% of total output. The optimal capacity is calculated by simple iterative guesswork: two initial guesses of the capacity needed to achieve a particular target return were made, the return for these capacity levels were calculated, and then the estimates were updated using the bisection method. After the optimal capacity was found, various statistics about the equilibrium patterns of commodity prices, transport prices, storage quantities, and trade volumes were calculated.

Two results are of interest. First, for any level of specialization the equilibrium amount of transport capacity increases as the variance of output shocks increases. The increase in transport capacity reduces the fraction of time that capacity constraints bind. Yet the variance of transport prices increases overall, for even though capacity limits bind less often, when they do bind the transport price is very high. In turn, spatial price dispersion increases. Thus even when one takes into account the expansion of transport capacity, an economy with a high variance of output shocks has greater price dispersion and greater transport price volatility than an economy with a low variance of output shocks.

Second, for any level of output variance, the equilibrium amount of transport capacity increases as the degree of specialization increases. This result should be of no surprise. However, the table indicates that capacity increases by less than one for one with the output asymmetry. For example, with σ = 10, optimal capacity increases by 18.7 units and mean trade from B to A increases by 18.3 units when A reduces its average share of output by 20 units from 45% of the total to 35% of the total. Since there is a reduction in capacity compared to the output asymmetry, there is less ability for trade to offset the effects of large output shocks in the importing center. As a result, transport prices are more likely to exceed K^T as the output asymmetry increases, and the spatial price difference is more likely to exceed $K^T - K^S$.

[7] The target return can be used to calculate the return to capital if the cost of a unit of capital is specified.

Table 3. Prices, Storage, and Transport Price Statistics Corresponding to the Model (Center A Produces 35% of Output, $\sigma = 10$, Changing Transport Capacity)

Statistic	$T^* = 5$	$T^* = 10$	$T^* = 20$	$T^* = 30$	$T^* = 40$	$T^* = 50$	$T^* = \infty$
Prices							
Mean(P^A)	125	120	112	106	104	103	103
Std. Dev.(P^A)	13	13	12	11	10	9	9
Mean (P^B)	75	80	89	95	97	98	98
Std. Dev(P^B)	10	10	9	8	8	9	10
Mean($P^A - P^B$)	50	40.3	22.6	10.8	6.6	5.5	5.1
Std. Dev.($P^A - P^B$)	16.6	16.4	14.4	9.8	5.7	3.2	1.9
% $\|P^A - P^B\| > K^T - K^S$	99%	98%	92%	76%	52%	31%	2%
Storage							
Mean(S^A)	384	405	417	357	297	213	91
Std. Dev.(S^A)	329	332	330	286	242	173	81
Mean (S^B)	498	458	367	233	178	213	300
Std.Dev(S^B)	421	399	350	252	184	227	304
Trade							
Mean(T^A)	0	0.0	0.1	0.1	0.1	0.0	0.0
Std. Dev.(T^A)	0.0	0.2	1.0	1.4	1.2	0.8	0.4
Mean(T^A)$\|T^A$>0	5	10	15	20	16.7	14.3	12.8
% ($T^A > 0$)	0.01%	0.04%	0.4%	0.4%	0.3%	0.1%	0.1%
% ($T^A = T^*$)	0.01%	0.04%	0.2%	0.1%	0.0%	0.0%	0%
Mean(T^B)	5.0	9.8	18.8	24.7	26.8	27.3	27.4
S. Dev.(T^B)	0.4	1.3	4.4	11	17	21	21
Mean(T^B)$\|T^B$>0	5.0	9.94	19.7	28.3	34.8	37.0	32.7
% ($T^B > 0$)	99%	99%	96%	87%	77%	74%	84%
% ($T^B = T^*$)	98%	98%	91%	74%	54%	35%	0%
Transport Price							
Mean (K_t^A)	5.00	5.00	5.00	5.00	5.00	5.00	5
Std. Dev (K_t^A)	0.0	0.1	0.1	0.1	0.0	0.0	0
Mean (K_t^B)	49.7	40.18	22.58	10.96	6.60	5.42	5
Std. Dev (K_t^B)	16.0	15.6	13.2	8.5	4.3	1.9	0
Excess return	44.72	35.18	17.58	5.96	1.60	0.42	0

P^A: the price in center A. S^A: storage in center A. T^A: trade from center A to center B.
K_t^A: the transport price incurred sending a ship from A to B.
% | $P^A - P^B$ | $> K^T - K^S$: the fraction of time the price difference exceeds the difference between the marginal trade cost and the storage cost. $K^S = 0$ in these simulations.
% ($S^A[T^A] > 0$): the fraction of time storage [exports] > 0.
% ($T^A = T^*$): the fraction of times the quantity traded is equal to the transport capacity and the fraction of times $K_t^A > K^T$.

Empirical Relevance of the Model

How relevant is the model? The model describes arbitrage behavior for commodities that are traded in competitive markets, and which are transported by competitive but capacity constrained shipping firms. These conditions may exist for commodities that are transported by pipelines. It is plausible that they are important in developing countries, where a scarcity of capital is an everyday occurrence. More generally, as Williamson (1966) suggested, it is possible that many transport markets have bottlenecks and capacity constraints and are equilibrated using a mixture of peak-load pricing and logistics management techniques because transport equipment is capital intensive. According to Stopford (1988) the maritime shipping firms that transport most bulk commodities are competitive, and shipping prices vary substantially in the short term in accordance with supply and demand.[8] Contemporary data on grain shipments between the United States and Japan, and the United States and Europe show that trade volumes and shipping prices are positively correlated, which is consistent with this notion.[9]

Whether or not goods prices and transport prices in a particular industry can be

[8] According to Stopford (p. 15, p. 229) 74% of total seaborne cargoes in 1985 were bulk goods, of which 36% was oil and 23% was iron ore, coal or grain.

[9] For example, for 1995–2000, the correlation coefficient between monthly Gulf of Mexico-Antwerp grain shipping prices and the total volume of US grain shipments is 0.54. Data are from International Grains Council (1995–2000).

Table 4. Prices, Storage, and Transport Price Statistics Corresponding to the Model (Center *A* Produces 35% of Output, Changing Transport Capacity, $\sigma = 5$ or $\sigma = 10$)

Statistics	$T^* = 10$ $\sigma = 5$	$T^* = 20$ $\sigma = 5$	$T^* = 40$ $\sigma = 5$	$T^* = \infty$ $\sigma = 5$	$T^* = 10$ $\sigma = 10$	$T^* = 20$ $\sigma = 10$	$T^* = 40$ $\sigma = 10$	$T^* = \infty$ $\sigma = 10$
Prices								
Mean$(P^A - P^B)$	39.5	7.1	4.7	4.5	40.3	22.6	6.6	5.1
Std. Dev.$(P^A - P^B)$	10.8	9.6	1.7	1.2	16.4	14.4	5.7	1.9
Storage								
Mean(S^A)	102	102	55	27	405	417	357	91
Mean (S^B)	137	104	50	73	458	367	178	300
Trade								
Mean(T^A)	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.0
Mean(T^B)	9.8	19.7	27.3	27.4	9.8	18.8	26.8	27.4
% $(T^A + T^B > 0)$	99%	99%	93%	96%	99%	96%	77%	84%
% $(T^A$ or $T^B = T^*)$	99%	92%	27%	0%	98%	91%	54%	0%
Transport price								
Mean (K_t^A)	5.00	5.00	5.00	5	5.00	5.00	5.00	5
Std. Dev (K_t^A)	0.1	0.0	0.0	0	0.1	0.1	0.0	0
Mean (K_t^B)	39.24	20.41	5.17	5	40.18	22.58	6.60	5
Std. Dev (K_t^B)	10.4	9.1	0.9	0	15.6	13.2	4.3	0
Excess return	34.24	15.41	0.17	0	35.18	17.58	1.60	0

σ: the standard deviation of the shock hitting output in each center.
P^A: the price in center *A*. S^A: storage in center *A*. T^A: trade from center *A* to center *B*.
K_t^A: the transport price incurred sending a ship from *A* to *B*.
$\%|P^A - P^B| > K^T - K^S$: the fraction of time the price difference exceeds the difference between the marginal trade cost and the storage cost. $K^S = 0$ in these simulations.
$\%(S^A[T^A] > 0)$: the fraction of time storage [exports] > 0.
$\%(T^A = T^*)$: the fraction of times the quantity traded is equal to the transport capacity and the fraction of times $K_t^A > K^T$.

appropriately described by this model needs to be determined on a case by case basis. It may be the case, for instance, that a better description for some industries is that the supply of transport services is upward sloping but not perfectly inelastic. However, there is some direct evidence that the model has direct applicability, at least in grain markets.

The first evidence concerns the grain market in Western Australia. Wright and Williams (1989) suggested that storage under price backwardation might occur because of localized capacity constraints in the transport sector. Brennan, Williams, and Wright (1997) investigated this idea directly by examining the way that rail and road networks were used in conjunction with storage elevators to transport wheat from rural hinterlands to a port. They argued that empirical evidence strongly supported the thesis that the rail and elevator network was used to minimize total transport and storage costs. In essence, elevators in distant locations were used to store grain during the peak harvest period while the rolling stock was used to move grain to the port from closer locations. Grain was kept in the distant locations even when prices in the port were in backwardation because rolling stock was limited and the implicit transport price was too high.

The second evidence concerns the operation of the grain transport market between Chicago and New York in the late nineteenth century. This market was described in detail by Coleman (forthcoming). He demonstrated that localized stock-outs frequently occurred in New York when inventories fell to very low levels. He also showed the spot price in New York was close to the spot price in Chicago plus the transport price when inventories were normal or large in New York, but that it exceeded the Chicago spot price plus the transport price when inventories were very small. This confirms a major prediction of the current model. He further showed that transport prices between Chicago and New York varied substantially from week to week, but did not analyze why they varied. However, there is historical evidence that transport prices varied as a result of fluctuating demand for transport services and a fixed supply of transport equipment.

The evidence is mainly anecdotal, and comes from a Senate enquiry into the operation of the transport market between the midwest and the eastern seaboard (U.S. Congress 1874). As part of this enquiry, Congress interviewed shipping

Table 5. Equilibrium Transport Capacity as a Function of Output Shock Variance

Statistics Center A Output σ	50% σ = 2.5	50% σ = 5	50% σ = 10	45% σ = 2.5	45% σ = 5	45% σ = 10	35% σ = 2.5	35% σ = 5	35% σ = 10
Capacity limit	0.01[a]	3.0	13.6	8.2	11.3	19.6	28.1	30.8	38.3
Prices									
Mean($P^A - P^B$)	0	0	0	7.1	6.4	4.9	7.2	7.2	6.9
Std. Dev.($P^A - P^B$)	6.0	8.1	9.2	3.4	5.3	7.8	3.4	4.2	6.1
% $\|P^A - P^B\| > K^T - K^S$	36%	41%	33%	61%	55%	43%	61%	58%	57%
Storage									
Mean(S^A)	25	86	308	21	73	286	25	86	323
Mean (S^B)	25	86	308	13	59	274	13	34	179
Trade									
Mean(T^A)	0.0	0.6	2.5	0.0	0.1	0.8	0.0	0.0	0.1
Mean(T^B)	0.0	0.6	2.5	6.5	6.9	8.4	26.4	26.4	26.7
% ($T^A + T^B > 0$)	36%	44%	42%	90%	68%	53%	100%	99%	80%
% (T^A *or* $T^B = T^*$)	36%	41%	34%	61%	52%	41%	56%	55%	57%
Transport Prices									
Mean (K_t^A)	5.54	5.99	6.01	5.00	5.01	5.14	5.00	5.00	5.00
Std. Dev (K_t^A)	1.6	2.8	3.7	0.0	0.1	1.3	0.0	0.0	0.1
Mean (K_t^B)	5.54	5.99	6.01	7.00	6.98	6.86	7.00	7.00	7.00
Std. Dev (K_t^B)	1.6	2.8	3.7	2.9	3.8	4.9	2.9	3.7	4.8
Excess return	1.04[a]	1.98	2.01	2.00	1.99	2.00	2.00	2.00	2.00

Note: The table shows equilibrium values of prices, transport prices, and trade when transport capacity is determined endogenously to generate an excess return of 2.00. Center A output is the fraction of output made in center A on average. Variable definitions are as follows:
σ is the standard deviation of the shock hitting output in each center.
P^A is the price in center A. S^A: storage in center A. T^A: trade from center A to center B.
K_t^A is the transport price incurred sending a ship from A to B.
%$|P^A - P^B| > K^T - K^S$ is the fraction of time the price difference exceeds the difference between the marginal trade cost and the storage cost. $K^S = 0$ in these simulations.
% ($S^B[T^A] > 0$) is the fraction of time storage [exports] > 0.
% ($T^A = T^*$) is the fraction of times the quantity traded is equal to the transport capacity and the fraction of times $K_t^A > K^T$.
[a]The minimum return to capital is not earned in this case. Output has so little volatility that consumption is almost completely smoothed through inventory adjustment.

agents and commodity merchants involved in the trade. Several of these agents argued that transport prices fluctuated daily in response to the supply and demand of shipping: the quote from Mr. Hayes in the introduction is an example. Many of these agents also stated that the high prices obtained when shipping capacity was fully utilized were necessary to generate a reasonable average return to shipping companies.[10] For example, Mr. Hayes said, this time in the context of railways:

> *"...at no time in any year in my knowledge, whether the crop was large or small, was there a regular demand for transportation up to the amount that the various lines could supply. Even when the crop was large cars laid idle at certain seasons, and because many laid idle the service was performed at a loss. If there is to be a fair average annual result to the transporter, then, when the demand again picks up, there must be a sufficient increase of charges to make a good average price."* p. 37.

This testimonial is clearly in concordance with the way transport is modeled in the article, and supports the statistical findings on the interrelationships among commodity prices, transport prices, and inventories that are reported by Coleman.

Discussion and Conclusions

Most papers analyzing the law of one price have assumed transport prices are exogenously determined and transport is supplied elastically to move goods. This article suggests these assumptions may not be innocuous. Rather, if transport operators plan to use their equipment as much as possible, transport systems will have capacity constraints, transport prices will have a peak-pricing pattern, and commodity prices will vary substantially

[10] A very similar set of explanations for why cattle prices in New York and Liverpool varied so much were given to the 1890 U.S. Congress Select Committee on the Transportation and Sale of Meat Products. (U. S. Congress 1890.) See the testimony by T.C. Eastman pp. 513–27, and Mr. F. W. J. Hurst pp. 555–57.

between locations. This variation is not evidence against the law of one price but rather the mechanism by which transport operators earn their cost of capital.

The model suggests that prices in different locations adjust to shocks in a way quite different to that often postulated. In most models, a region experiencing a negative output shock will smooth consumption by importing sufficient quantities of goods to ensure the local price does not exceed the world price by more than an exogenously determined transport price. In this model, the same region would not be able to fully smooth consumption because not enough goods can be imported; rather, the output shock causes spatial price dispersion and temporarily large profit margins for the shipping companies.

The model assumes transport services are sold in competitive markets. Yet if the service was not sold—the shipping was done on own account, for example, or the transport service was broadly interpreted to include distribution services such as retailing—the same logic would remain. Capital-intensive distribution services would have limited capacity, and the profit in a particular period would vary with the size of supply-and-demand shocks. There would be imperfect price pass-through from one location to another. Evidence that price differences between locations varied significantly over time would not be an evidence that the arbitrage was not working, but a necessary condition to attract arbitrageurs into the industry.

The article raises several issues for further enquiry. The first concerns the number of sectors and time periods for which this model is applicable. Whether or not goods prices and transport prices can be described by this model needs to be determined on a case-by-case basis. However, it is possible that the model may have many applications. If, as Wright and Williams (1989) suggest, storage under price backwardation occurs because of localized capacity constraints in the transport sector, the model may be relevant wherever storage under backwardation occurs.

Second, the model could be extended to allow for the possibility that capacity constraints vary systematically through time, possibly because transport equipment is moved from market to market in response to peak demand periods. Such adjustments occur in the passenger airline industry, for instance, although this does not eradicate seasonal peak-pricing patterns. A more complex extension would be to allow the transport supply curve to have a more general upward sloping form.

Third, the model uses an autoregressive rather than a seasonal process to model output uncertainty. While the set of arbitrage conditions (equations 4) is the same in both cases, the relative importance of each of the complementary slackness conditions will change. While I expect the overall flavor of the results to be similar—one would expect localized stock-outs and variable transport prices under both assumptions—it may be of interest to explicitly model the seasonal case for commodities where seasonal patterns in either transport availability or production are important.

Fourth, the model suggests that the way in which the transport sector is incorporated into trade models is important. Seemingly small changes to a model such as the incorporation of transport capacity constraints have large effects on the properties of the model. It may be the case that more realistic modeling of the transport systems used in other trade models would also have large effects on the properties of these models. If so, it may be necessary to better understand the structure and pricing of the transport and logistics sector to better understand price determination in spatially separate markets.

*[Received May 2006;
accepted May 2008.]*

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