WEB大数据挖掘(二)

Association Rules and Frequent Pattern Mining

Slides adapted from http://www.mmds.org

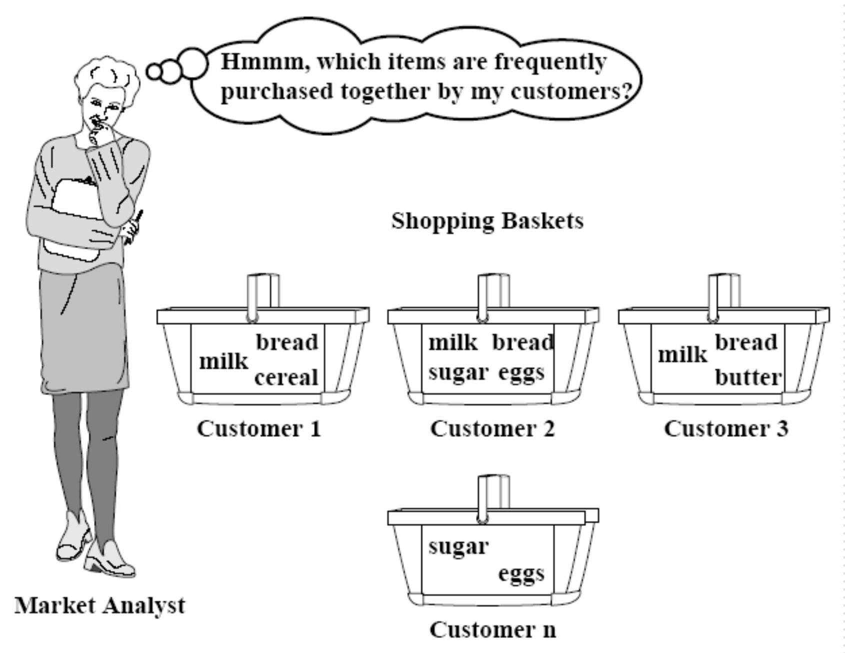
Agenda

High dim. data	Graph data	Infinite data	Machine learning	Apps
Locality sensitive hashing	**PageRank,** SimRank	Filtering data streams	SVM	Recommender systems
Clustering	Community Detection	Web advertising	Decision Trees	Association Rules
Dimensionality reduction	Spam Detection	Queries on streams	Perceptron, kNN	Duplicate document detection

Association Rule

- Items frequently purchased together:

beer → diaper



- Uses:
  - Placement
  - Advertising
  - Sales
  - Coupons
- Objective: increase sales and reduce costs

The Market-Basket Model

- A large set of *items*, e.g., things sold in a supermarket
- A large set of *baskets*, each of which is a small set of the items, e.g., the things one customer buys on one day

TID	*Items*
1	**Beer, Diaper, Milk**
2	**Coke, Diaper, Eggs**
3	**Beer, Coke, Diaper, Eggs**
4	**Coke, Eggs**

The Market-Basket Model

- A general many-many mapping (association) between two kinds of things
  - But we ask about connections among "items" not "baskets"
- The technology focuses on common events, not rare events ("long tail")

Applications – (1)

- Items = products; baskets = sets of products someone bought in one trip to the store
- Example application: given that many people buy beer and diapers together
  - Run a sale on diapers; raise price of beer
- Only useful if many buy diapers & beer

Applications – (2)

- Items = words; Baskets = Web pages;
- Unusual words appearing together in a large number of documents, e.g., "Brad" and "Angelina" may indicate an interesting relationship

Applications – (3)

- Items = sentences; baskets = documents containing those sentences
- Items that appear together too often could represent plagiarism

Association Rule Mining Applications

- Basket Data Analysis
- Genomic Data
- Telecommunication
- Credit Cards/ Banking Services
- Medical Treatments
- Web Personalization
- etc.

Scale of the Problem

- WalMart sells 100,000 items and can store billions of baskets
- The Web has billions of words and many billions of pages

Some Definition - Support

An itemset is supported by a basket (transaction) if it is included in the basket

Market-Basket transactions

TID	*Items*
1	Beer, Diaper, Milk
2	Coke, Diaper, Eggs
3	Beer, Coke, Diaper, Eggs
4	Coke, Eggs

<Beer, Diaper> is supported by basket 1, and 3, and its support is 2/4=50%.

Some Definition – Frequent Itemset

If the support of an itemset exceeds user specified *min_support* (threshold), this itemset is called a frequent itemset (pattern).

Market-Basket transactions

TID	*Items*
1	Beer, Diaper, Milk
2	Coke, Diaper, Eggs
3	Beer, Coke, Diaper, Eggs
4	Coke, Eggs

min_support=50%
<Beer, Diaper> is a frequent itemset
<Beer, Milk> is not a frequent itemset

Association Rules

- **Association Rules:**
  If-then rules about the contents of baskets
- $\{i_1, i_2, \ldots, i_k\} \rightarrow j$ means: "if a basket contains all of $i_1, \ldots, i_k$ then it is ***likely*** to contain j"
- **In practice there are many rules, want to find significant/interesting ones!**
- ***Confidence*** of this association rule is the probability of j given $I = \{i_1, \ldots, i_k\}$

$$\mathrm{conf}(I \rightarrow j) = \frac{\mathrm{support}(I \cup j)}{\mathrm{support}(I)}$$

Example: Confidence

$T_1 = \{m, c, b\}$ $T_2 = \{m, p, j\}$

$T_3 = \{m, b\}$ $T_4 = \{c, j\}$

$T_5 = \{m, p, b\}$ $T_6 = \{m, c, b, j\}$

$T_7 = \{c, b, j\}$ $T_8 = \{b, c\}$

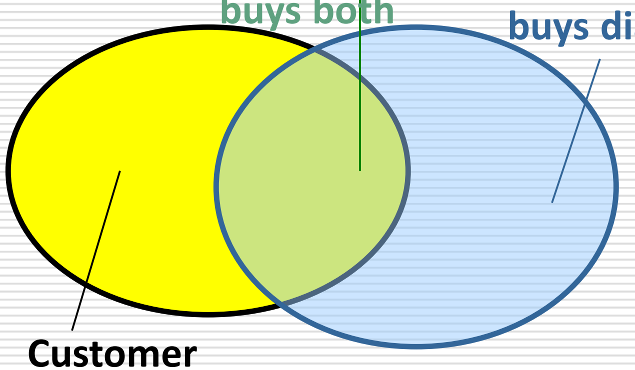
- Association rule: $\{m, b\} \rightarrow c$
  - Support(m,b)=4, Support(m,b,c)=2
  - Confidence = 2/4 = 0.5

Association Rules Mining

- Question: "find all association rules with support $\geq s$ and confidence $\geq c$"
- Hard part: finding the frequent itemsets

Frequent Patterns and Association Rules

Transaction-id	Items bought
1	A, B, D
2	A, C, D
3	A, D, E
4	B, E, F
5	B, C, D, E, F



- Itemset X = $\{x_1, \ldots, x_k\}$
- Find all the rules $X \rightarrow Y$ with minimum support and confidence
  - support, *s*, probability that a transaction contains $X \cup Y$
  - confidence, *c*, conditional probability that a transaction having X also contains *Y*

Let sup_{min} ***= 50%,*** $conf_{min}$ ***= 50%***

Freq. Pat.: {A:3, B:3, D:4, E:3, AD:3}

Association rules:

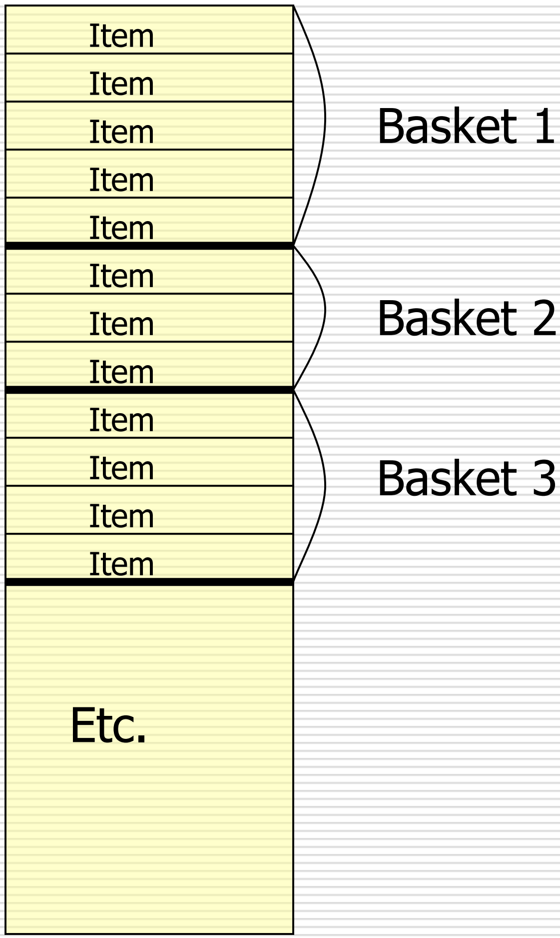
$A \rightarrow D$ **(60%, 100%)**

$D \rightarrow A$ **(60%, 75%)**

Computation Model

- Typically, data is kept in flat files rather than in a database system
  - Stored on disk
  - Stored basket-by-basket
  - Expand baskets into pairs, triples, etc. as you read baskets

File Organization



Example: items are positive integers, and boundaries between baskets are –1

Computation Model – (2)

- The true cost of mining disk-resident data is usually the number of disk I/O's
- In practice, association-rule algorithms read the data in *passes* – all baskets read in turn
- Thus, we measure the cost by the number of passes an algorithm takes

Main-Memory Bottleneck

- For many frequent-itemset algorithms, main memory is the critical resource
  - As we read baskets, we need to count something, e.g., occurrences of pairs
  - The number of different things we can count is limited by main memory
  - Swapping counts in/out is a disaster

Finding Frequent Pairs

- The hardest problem often turns out to be finding the frequent pairs
  - Why? Often frequent pairs are common, frequent triples are rare
    - Why? Probability of being frequent drops exponentially with size; number of sets grows more slowly with size
- We'll concentrate on pairs, then extend to larger sets

Naïve Algorithm

- Read file once, counting in main memory the occurrences of each pair
  - From each basket of *n* items, generate its *n*(*n* -1)/2 pairs by two nested loops
- Fails if (#items)^2 exceeds main memory
  - Remember: #items can be 100K (Wal-Mart) or 10B (Web pages)

Example: Counting Pairs

- Suppose 10^5 items
- Suppose counts are 4-byte integers
- Number of pairs of items: $10^5(10^5-1)/2 = 5*10^9$ (approximately)
- Therefore, $2*10^{10}$ (20 gigabytes) of main memory needed

Details of Main-Memory Counting

- **Two approaches**:

  (1) Count all pairs, using a triangular matrix

  - requires only 4 bytes/pair

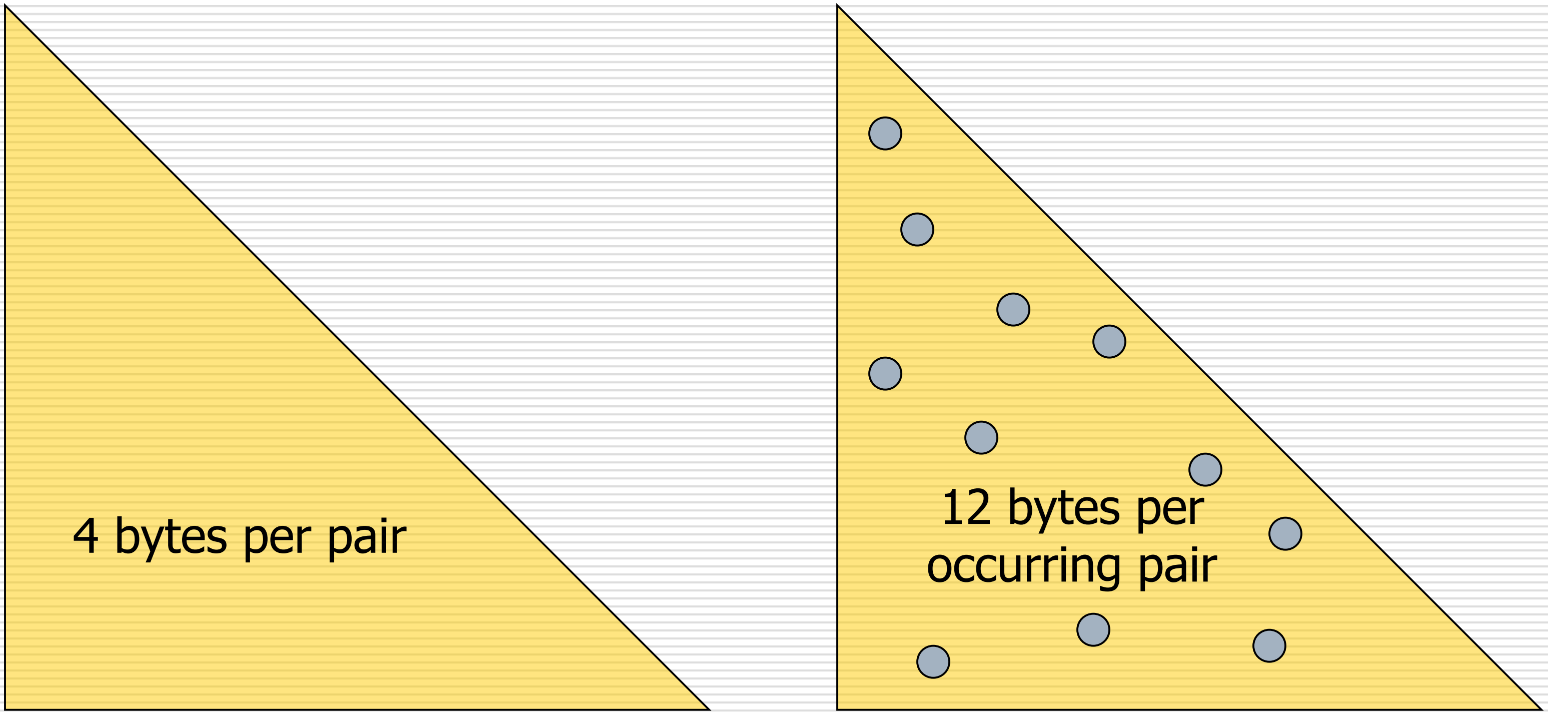
    always assume integers are 4 bytes

  (2) Keep a table of triples [*i*, *j*, *c*] = "the count of the pair of items {*i*, *j* } is *c*"

  - requires 12 bytes

    but only for those pairs with count > 0

Details of Main-Memory Counting



Method (1)

Method (2)

Comparing the Two Approaches

- **Approach 1: Triangular Matrix**
  - **n** = total number of items
  - Count pair of items {*i*, *j*} only if *i*<*j*
  - Keep pair counts in lexicographic order:
    - {1,2}, {1,3},…, {1,*n*}, {2,3}, {2,4},…,{2,*n*}, {3,4},…{n-1,n}
  - Pair {*i*, *j*} is at position $(i-1)(n-i/2)+j-i$
  - Total number of pairs $\boldsymbol{n(n-1)/2}$; total bytes= $\boldsymbol{2n^2}$
  - **Triangular Matrix** requires 4 bytes per pair
- **Approach 2** uses **12 *bytes*** per occurring pair *(but only for pairs with count > 0)*
  - Beats Approach 1 if less than **1/3** of possible pairs actually occur

Comparing the Two Approaches

- **Approach 1: Triangular Matrix**
  - **n** = total number items
  - Count pair of items {i, j} only if $i<j$
  - s= $\mathbf{2n^2}$
- A pair
  (b
  - possible pairs actually occur

Problem is if we have too many items so the pairs do not fit into memory.

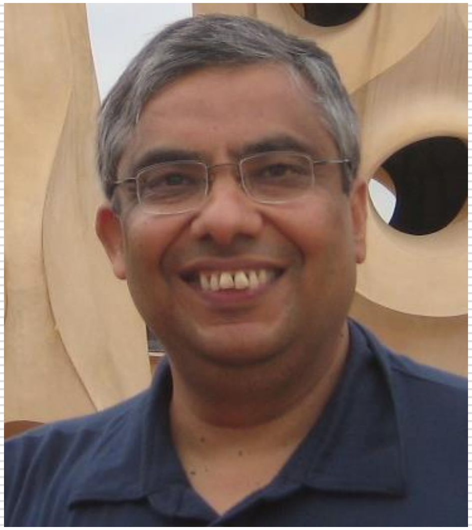
Can we do better?

Outline

- Association Rules
- Frequent Itemset Mining Algorithms
  - Apriori
  - FP-growth
- Sequential Pattern Mining Algorithms

Apriori Algorithm

- Proposed by Rakesh Agrawal [*VLDB'94*]
- Key idea:
  - Candidate generation-and-test
  - Anti-monotone property



[PDF] Fast Algorithms for Mining Association Rules - Rakesh Agrawal
rakesh.agrawal-family.com/papers/vldb94apriori.pdf ▾ 翻译此页
作者：R Agrawal - 被引用次数：20593 - 相关文章
Fast Algorithms for Mining Association Rules. Rakesh Agrawal. Ramakrishnan Srikant. IBM Almaden Research Center. 650 Harry Road, San Jose, CA 95120.

Apriori Algorithm – (1)

- A two-pass approach called *Apriori* limits the need for main memory
- *Monotonicity* : if a set of items appears at least s times, so does every subset
  - Contrapositive for pairs: if item i does not appear in s baskets, then no pair including i can appear in s baskets

Apriori Algorithm – (2)

- Pass 1: Read baskets and count in main memory the occurrences of each item
  - Requires only memory proportional to #items
- Items that appear at least *s* times are the *frequent items*

Apriori Algorithm – (3)

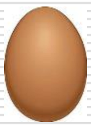
- Pass 2: Read baskets again and count in main memory only those pairs both of which were found in Pass 1 to be frequent
  - Requires memory proportional to square of *frequent* items only (for counts), plus a list of the frequent items (so you know what must be counted)

Apriori Algorithm

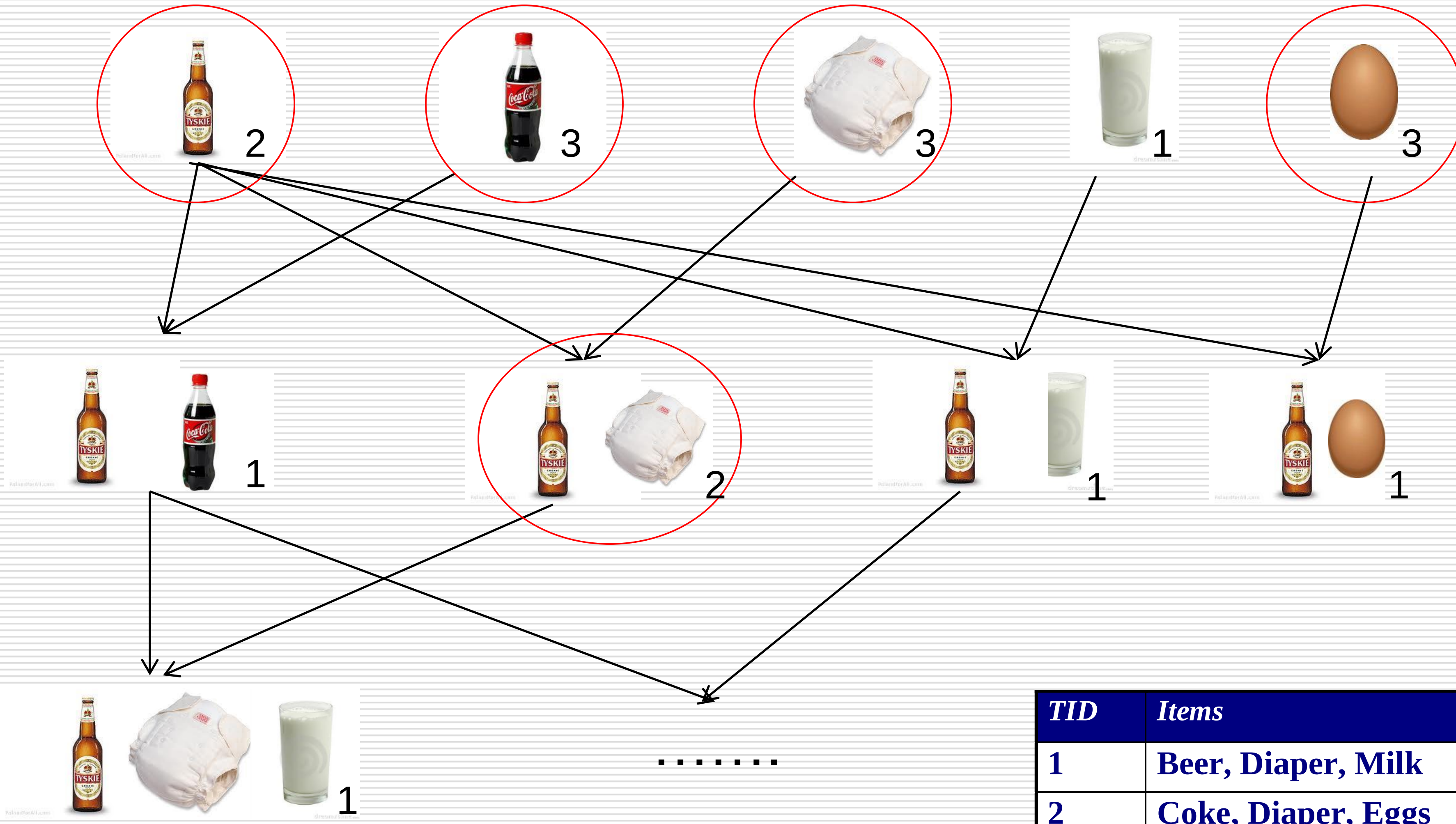
Market-Basket transactions

TID	*Items*
1	**Beer, Diaper, Milk**
2	**Coke, Diaper, Eggs**
3	**Beer, Coke, Diaper, Eggs**
4	**Coke, Eggs**





Naive Algorithm

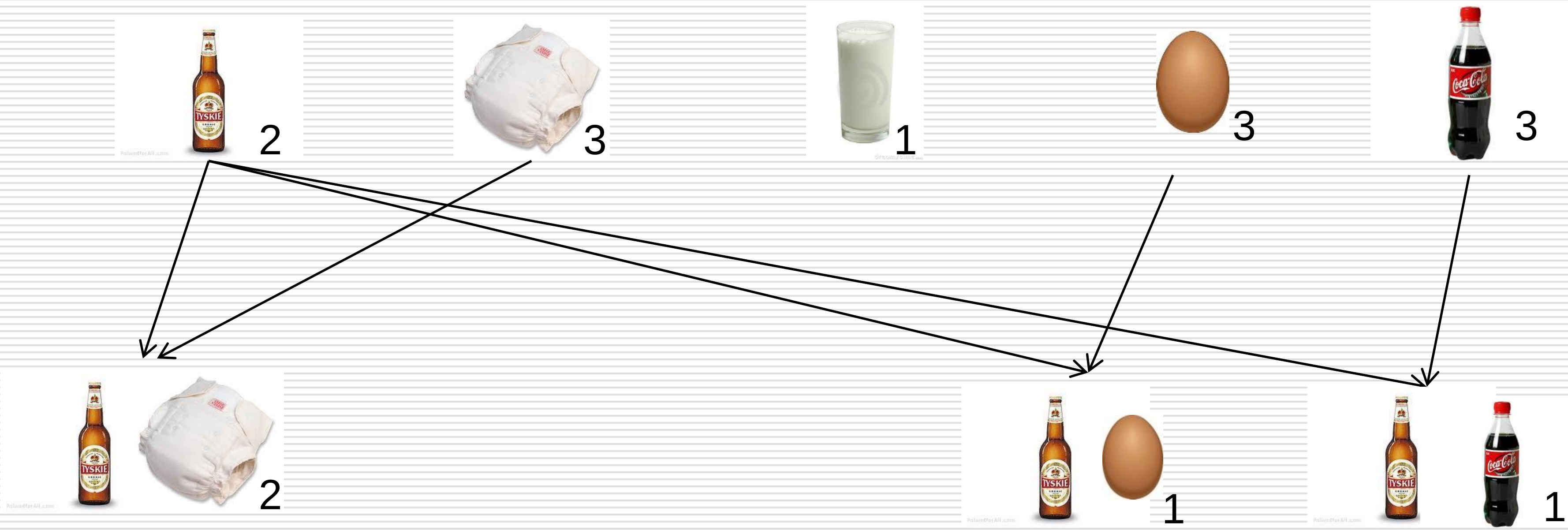


TID	Items
1	Beer, Diaper, Milk
2	Coke, Diaper, Eggs
3	Beer, Coke, Diaper, Eggs
4	Coke, Eggs

$Sup_{min}=2$

Apriori Algorithm

- Anti-monotone property: If an itemset is not frequent, then any of its superset is not frequent



TID	Items
1	Beer, Diaper, Milk
2	Coke, Diaper, Eggs
3	Beer, Coke, Diaper, Eggs
4	Coke, Eggs

$Sup_{min}=2$

Apriori Algorithm

$Sup_{min} = 2$

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1^{st} scan → C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

→ L_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

2^{nd} scan → C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

→ L_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

C_3

Itemset
{B, C, E}

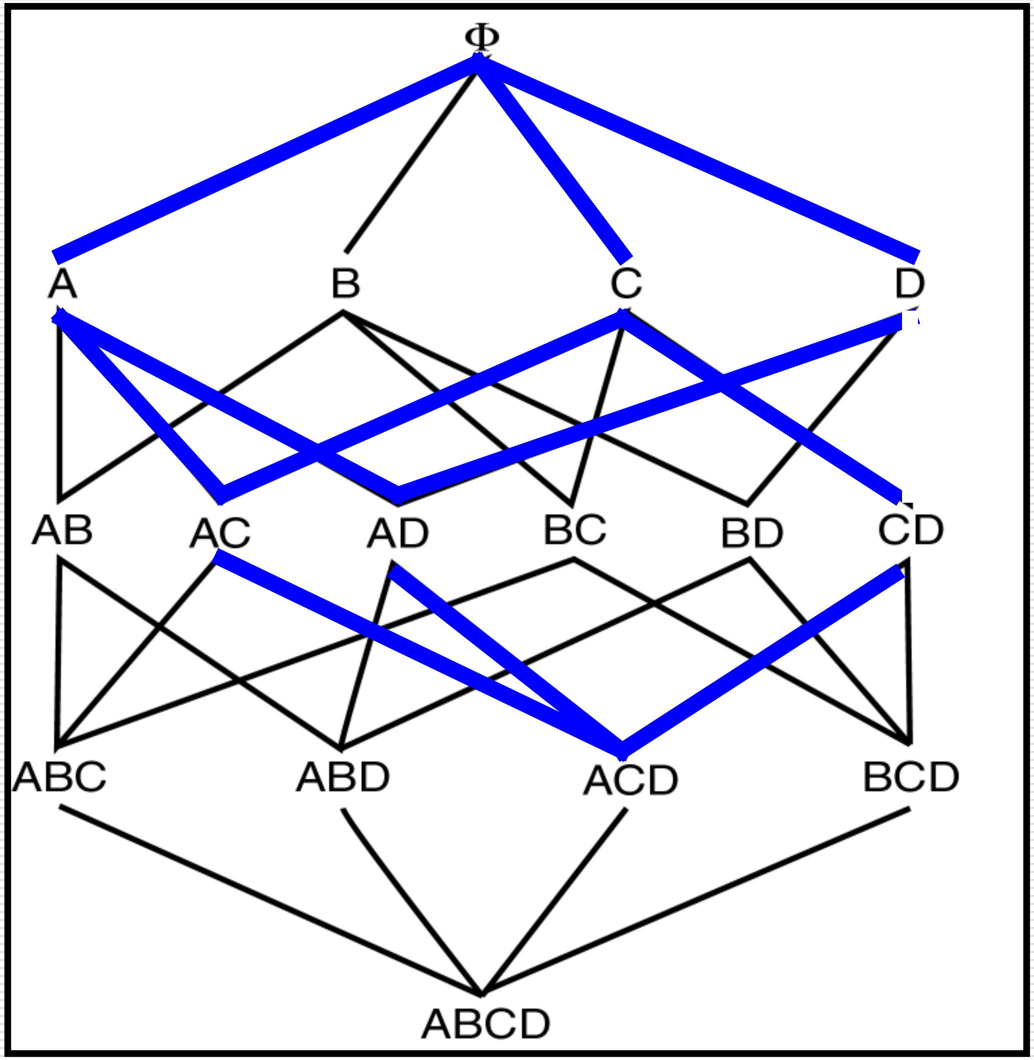
3^{rd} scan → L_3

Itemset	sup
{B, C, E}	2

Apriori Algorithm

1. C_1 = Itemsets of size one in I;
2. Determine all large itemsets of size 1, $L_{1;}$
3. i = 1;
4. Repeat
5.   i = i + 1;
6. C_i = Apriori-Gen(L_{i-1});
7.   Count C_i to determine $L_{i;}$
8. until no more large itemsets found;

Frequent Itemset Property



Drawbacks of Apriori

- Multiple scans of transaction database
  - Multiple database scans are costly
- Huge number of candidates
  - To find frequent itemset $i_1 i_2 \ldots i_{100}$
    - # of scans: 100
    - # of Candidates: $2^{100}-1 = 1.27*10^{30}$

Improving Apriori: General Ideas

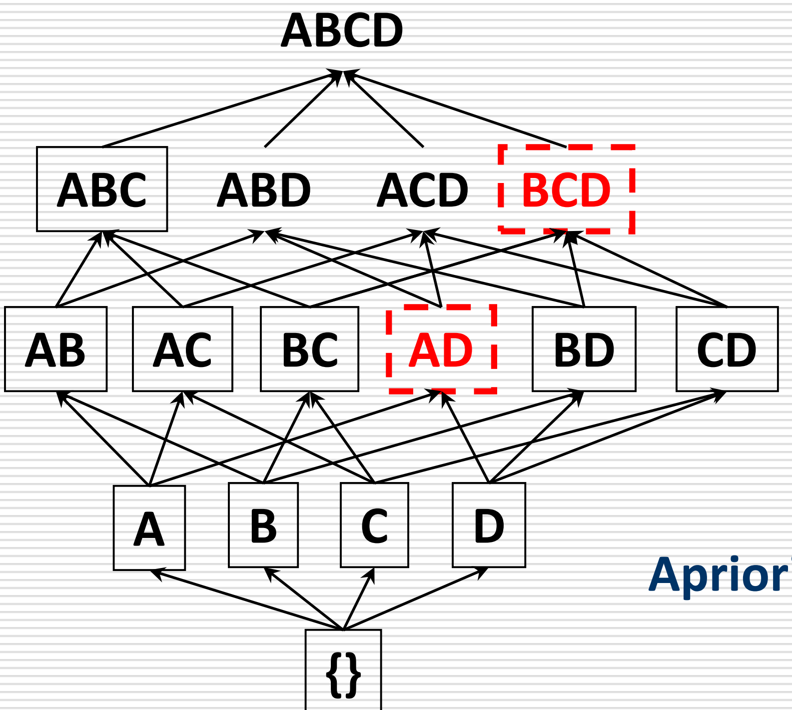
- Reduce passes of transaction database scans
- Shrink number of candidates
- Facilitate support counting of candidates

Improving Apriori's Efficiency

- Hash-based itemset counting: A *k*-itemset whose corresponding hashing bucket count is below the threshold cannot be frequent
- Transaction reduction: A transaction that does not contain any frequent k-itemset is useless in subsequent scans
- Partitioning: Any itemset that is potentially frequent in DB must be frequent in at least one of the partitions of DB
- Sampling: mining on a subset of given data, need a lower support threshold + a method to determine the completeness
- Dynamic itemset counting: add new candidate itemsets immediately (unlike Apriori) when all of their subsets are estimated to be frequent

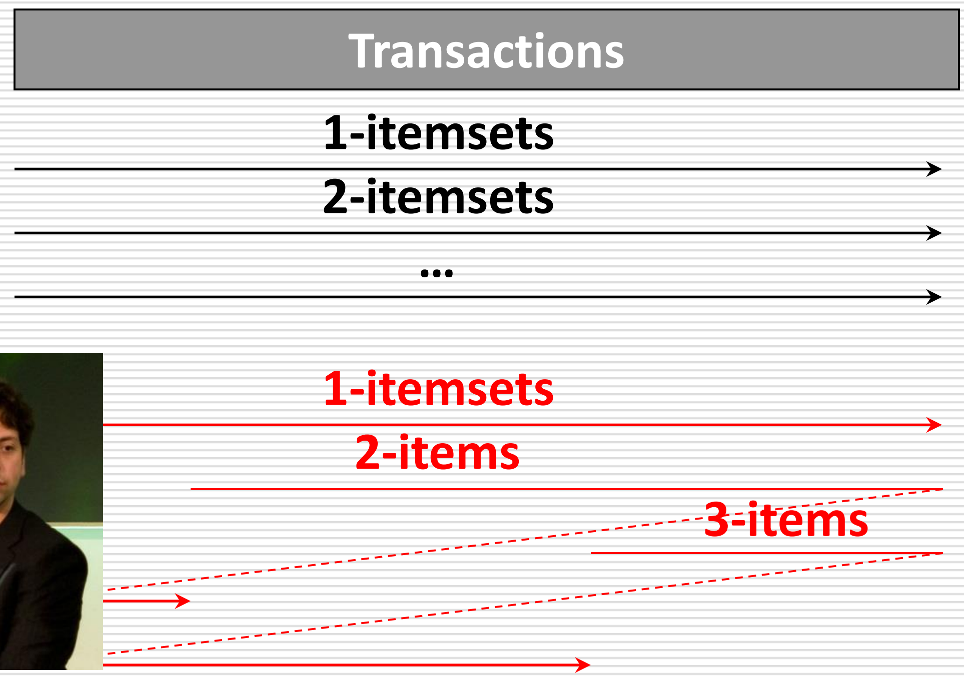
DIC: Reduce Number of Scans

- Once both A and D are determined frequent, the counting of AD begins
- Once all length-2 subsets of BCD are determined frequent, the counting of BCD begins



Itemset lattice

Apriori



S. Brin , R. Motwani, J. Ullman, and S. Tsur. Dynamic itemset counting and implication rules for market basket data. In *SIGMOD'97*

FP-growth Algorithm

FP-Growth

- Proposed by Jiawei Han [*SIGMOD'00*]
- Uses the Apriori pruning principle
- Scan DB only twice
  - Once to find frequent 1-itemset (single item pattern)
  - Once to construct FP-tree (prefix tree, Trie), the data structure of FP-growth

Mining Frequent Patterns without Candidate Generation

- Compress a large database into a compact, Frequent-Pattern tree (FP-tree) structure
  - highly condensed, but complete for frequent pattern mining
  - avoid costly database scans
- Develop an efficient, FP-tree-based frequent pattern mining method
  - A divide-and-conquer methodology: decompose mining tasks into smaller ones
  - Avoid candidate generation: sub-database test only!

FP-Growth

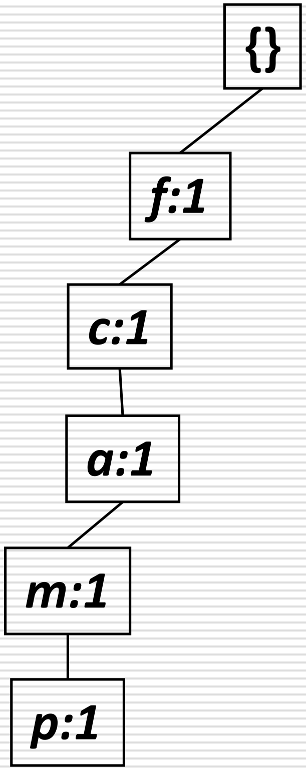
TID	Items bought
10	{f, a, c, d, g, i, m, p}
20	{a, b, c, f, l, m, o}
30	{b, f, h, j, o, w}
40	{b, c, k, s, p}
50	{a, f, c, e, l, p, m, n}

$Sup_{min} = 2$

Header Table

Item	frequency
f	4
c	4
a	3
b	3
m	3
p	3

TID	(ordered) frequent items
10	{f, c, a, m, p}
20	{f, c, a, b, m}
30	{f, b}
40	{c, b, p}
50	{f, c, a, m, p}



FP-Growth

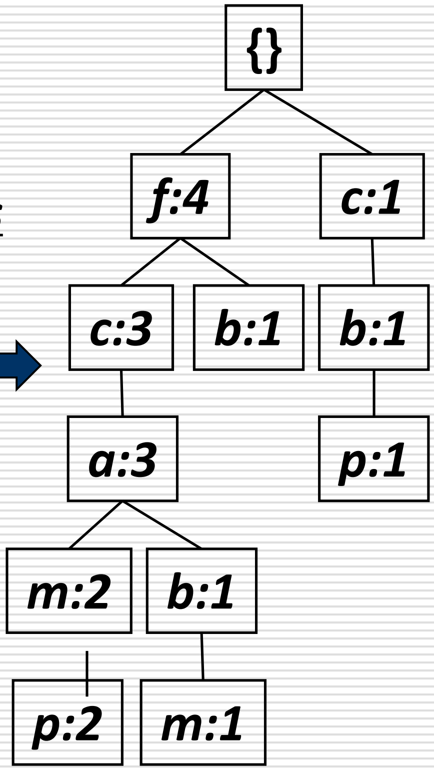
TID	*Items bought*
10	*{f, a, c, d, g, i, m, p}*
20	*{a, b, c, f, l, m, o}*
30	*{b, f, h, j, o, w}*
40	*{b, c, k, s, p}*
50	*{a, f, c, e, l, p, m, n}*

$Sup_{min} = 2$

Header Table

Item	*frequency*
f	*4*
c	*4*
a	*3*
b	*3*
m	*3*
p	*3*

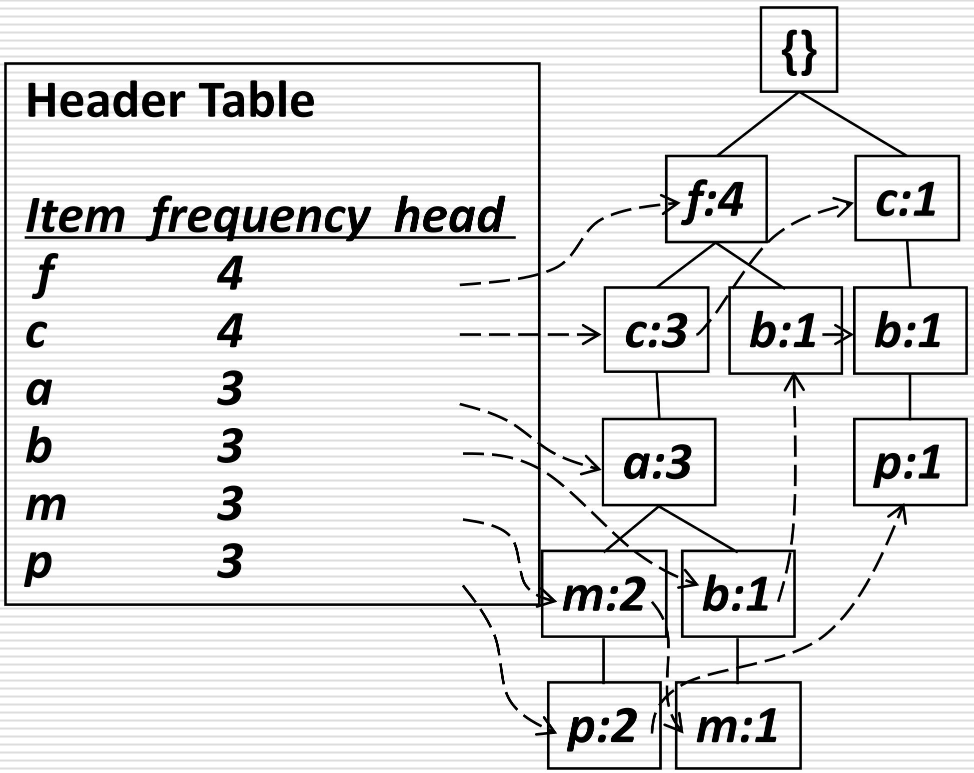
TID	*(ordered) frequent items*
10	*{f, c, a, m, p}*
20	*{f, c, a, b, m}*
30	*{f, b}*
40	*{c, b, p}*
50	*{f, c, a, m, p}*



FP-Growth

$Sup_{min} = 2$

TID	*(ordered) frequent items*
10	{*f, c, a, m, p*}
20	{*f, c, a, b, m*}
30	{*f, b*}
40	{*c, b, p*}
50	{*f, c, a, m, p*}

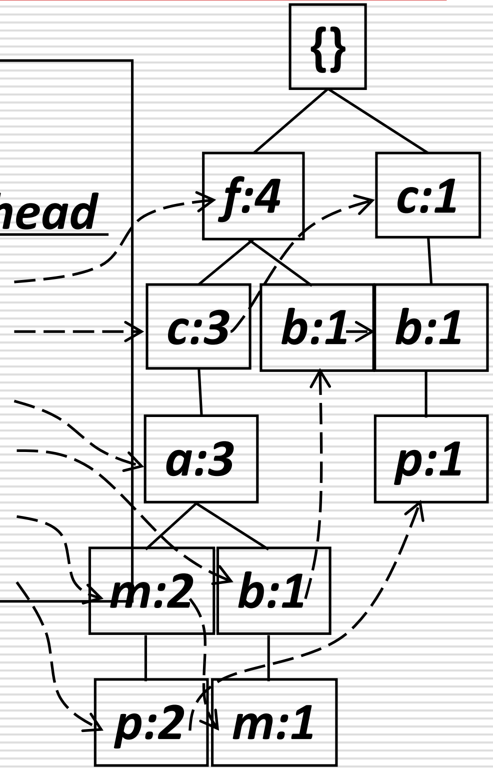


FP-Growth

$Sup_{min} = 2$

Header Table

Item	*frequency*	*head*
f	*4*	
c	*4*	
a	*3*	
b	*3*	
m	*3*	
p	*3*	



***Conditional* pattern bases**

Item	*cond. pattern base*	*freq. itemset*
p	*fcam:2, cb:1*	*fp, cp, ap, mp, fcp, fap, fmp, cap, cmp, amp, facp, fcmp, famp, fcamp*
m	*fca:2, fcab:1*	*fm, cm, am, fcm, fam, cam, fcam*
b	*fca:1, f:1, c:1*	*…*
a	*fc:3*	*…*
c	*f:3*	*…*

Principles of Frequent Pattern Growth

- Pattern growth property
  - Let α be a frequent itemset in DB, B be α's conditional pattern base, and β be an itemset in B. Then $\alpha \cup \beta$ is a frequent itemset in DB iff β is frequent in B.
- "*abcdef* " is a frequent pattern, if and only if
  - "*abcde* " is a frequent pattern, and
  - "*f* " is frequent in the set of transactions containing "*abcde* "

Why Is Frequent Pattern Growth Fast?

- The performance study shows
  - FP-growth is faster than Apriori (in most cases), and is also faster than tree-projection (an order of magnitude on some datasets)
- Reasoning
  - No candidate generation (claimed by the authors)
  - Use compact data structure
  - Eliminate repeated database scan
  - Basic operation is counting and FP-tree building

Extension of Association Rule Mining

- Association rule mining has been extensively studied in the data mining community.
- There are many efficient algorithms and model variations.
- Other related work includes
  - Multi-level or generalized rule mining
  - Sequential pattern mining
  - Constrained rule mining
  - Incremental rule mining
  - Maximal and closed frequent itemset mining
  - Numeric association rule mining
  - Rule interestingness and visualization
  - Parallel algorithms
  - …

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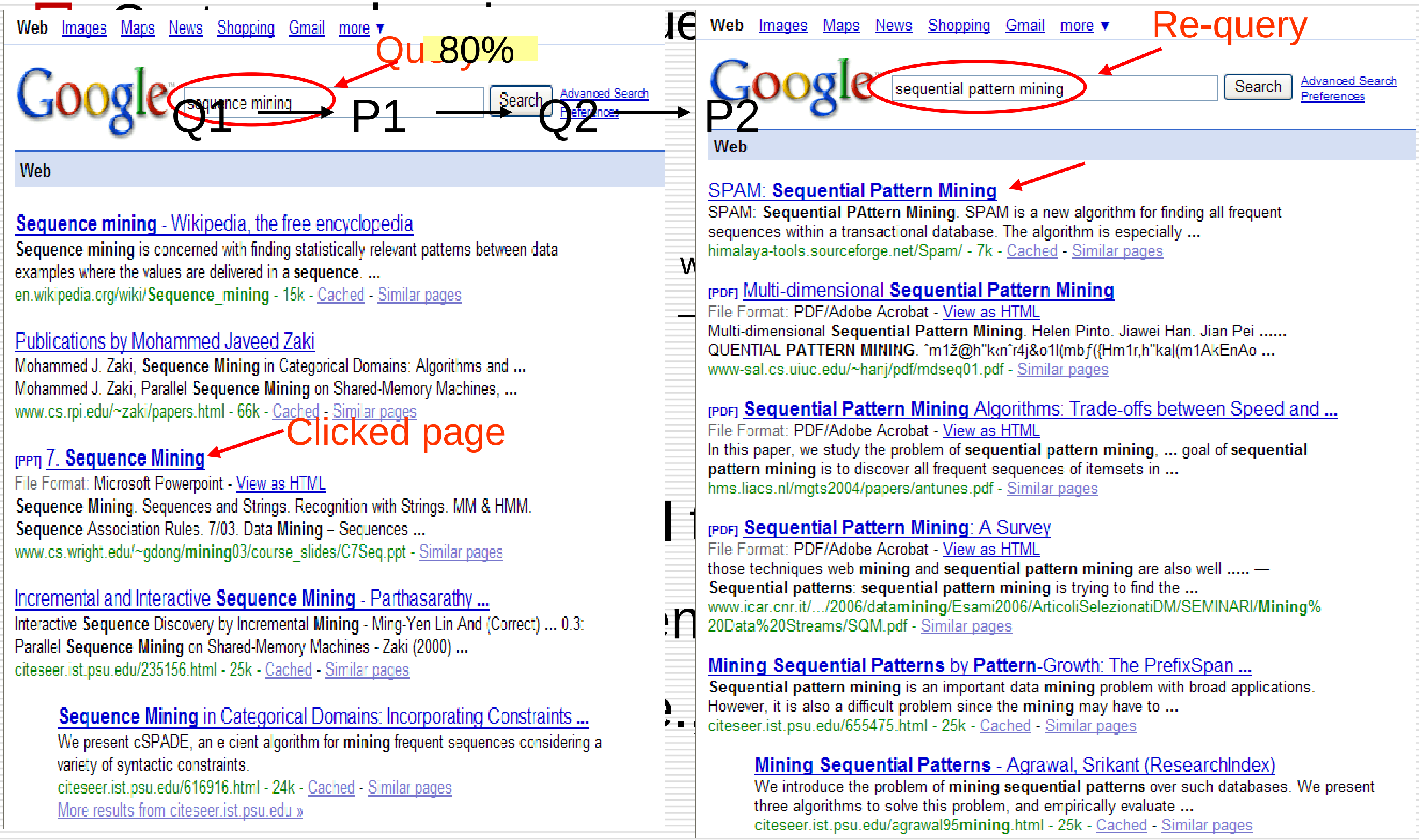
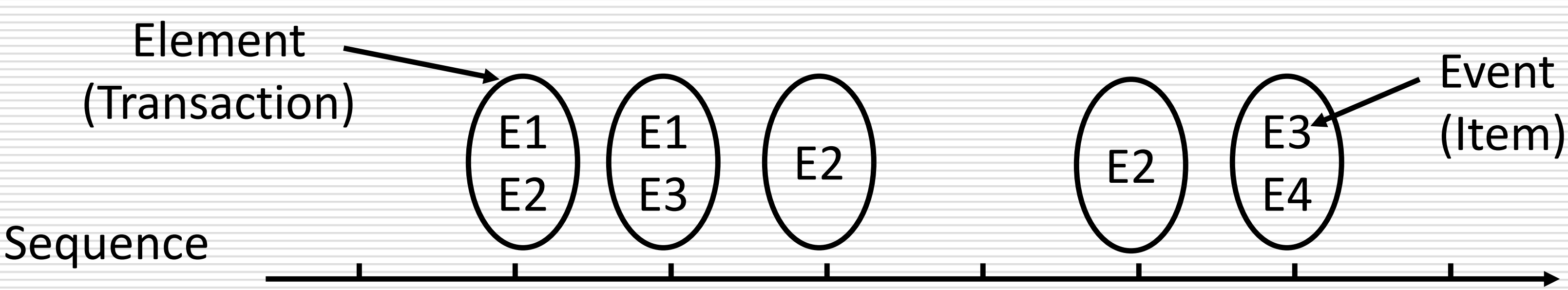
Applications



Examples of Sequence Data

Sequence Database	Sequence	Element (Transaction)	Event (Item)
Customer	Purchase history of a given customer	A set of items bought by a customer at time t	Books, diary products, CDs, etc
Web Data	Browsing activity of a particular Web visitor	A collection of files viewed by a Web visitor after a single mouse click	Home page, index page, contact info, etc
Event data	History of events generated by a given sensor	Events triggered by a sensor at time t	Types of alarms generated by sensors
Genome sequences	DNA sequence of a particular species	An element of the DNA sequence	Bases A,T,G,C



Formal Definition of a Sequence

- A sequence is an ordered list of elements (transactions)

$$s = < e_1 \, e_2 \, e_3 \ldots >$$

  - Each element contains a collection of events (items)

$$e_i = \{i_1, i_2, \ldots, i_k\}$$

  - Each element is attributed to a specific time or location

- Length of a sequence, |s|, is given by the number of elements of the sequence

- A k-sequence is a sequence that contains k events (items)

Formal Definition of a Subsequence

- A sequence $<a_1\ a_2 \ldots a_n>$ is contained in another sequence $<b_1\ b_2 \ldots b_m>$ ($m \geq n$) if there exist integers $i_1 < i_2 < \ldots < i_n$ such that $a_1 \subseteq b_{i1}$, $a_2 \subseteq b_{i1}$, …, $a_n \subseteq b_{in}$

Data sequence	Subsequence	Contain?
< {2,4} {3,5,6} {8} >	< {2} {3,5} >	Yes
< {1,2} {3,4} >	< {1} {2} >	No
< {2,4} {2,4} {2,5} >	< {2} {4} >	Yes

- The support of a subsequence w is defined as the fraction of data sequences that contain w
- A *sequential pattern* is a frequent subsequence (i.e., a subsequence whose support is ≥ *minsup*)

Sequential Pattern Mining: Definition

- Given:
  - a database of sequences
  - a user-specified minimum support threshold, *minsup*
- Task:
  - Find all subsequences with support ≥ *minsup*

Some Definitions

If the support of a sequence exceeds user specified *min_support*, this sequence is called a sequential pattern.

ID	Customer Sequence
10	<bdbcbdabad>
20	<dcaabcbdab>
30	<cadadcadca>

min_support=50%
<bdb> is a sequential pattern
<adc> is not a sequential pattern

Sequential Pattern Mining: Challenge

- Given a sequence: <{a b} {c d e} {f} {g h i}>
  - Examples of subsequences:

    <{a} {c d} {f} {g} >, < {c d e} >, < {b} {g} >, etc.

- How many k-subsequences can be extracted from a given n-sequence?

<{a b} {c d e} {f} {g h i}> n = 9

k=4: Y _ _ Y Y _ _ _ Y

<{a} {d e} {i}>

Answer :

$$\binom{n}{k} = \binom{9}{4} = 126$$

Outline

- Association Rules
- Frequent Itemset Mining Algorithms
- Sequential Pattern Mining Algorithms
  - GSP

GSP (Generalized Sequential Pattern Mining)

- Proposed by Srikant and Agrawal [*EDBT'96*]
- Uses the Apriori pruning principle

Finding Length-1 Sequential Patterns

- Initial candidates:
  - <a>, <b>, <c>, <d>, <e>, <f>, <g>, <h>
- Scan database once, count support for candidates

min_sup =2

Seq. ID	Sequence
10	<(bd)cb(ac)>
20	<(bf)(ce)b(fg)>
30	<(ah)(bf)abf>
40	<(be)(ce)d>
50	<a(bd)bcb(ade)>

Cand	Sup
<a>	**3**
<b>	**5**
<c>	**4**
<d>	**3**
<e>	**3**
<f>	**2**
~~<g>~~	1
~~<h>~~	1

Generating Length-2 Candidates

51 length-2 Candidates

	<a>	<b>	<c>	<d>	<e>	<f>
<a>	<aa>	<ab>	<ac>	<ad>	<ae>	<af>
<b>	<ba>	<bb>	<bc>	<bd>	<be>	<bf>
<c>	<ca>	<cb>	<cc>	<cd>	<ce>	<cf>
<d>	<da>	<db>	<dc>	<dd>	<de>	<df>
<e>	<ea>	<eb>	<ec>	<ed>	<ee>	<ef>
<f>	<fa>	<fb>	<fc>	<fd>	<fe>	<ff>

	<a>	<b>	<c>	<d>	<e>	<f>
<a>		<(ab)>	<(ac)>	<(ad)>	<(ae)>	<(af)>
<b>			<(bc)>	<(bd)>	<(be)>	<(bf)>
<c>				<(cd)>	<(ce)>	<(cf)>
<d>					<(de)>	<(df)>
<e>						<(ef)>
<f>						

Without Apriori property, 8*8+8*7/2=92 candidates

Apriori prunes 44.57% candidates

GSP Mining Process

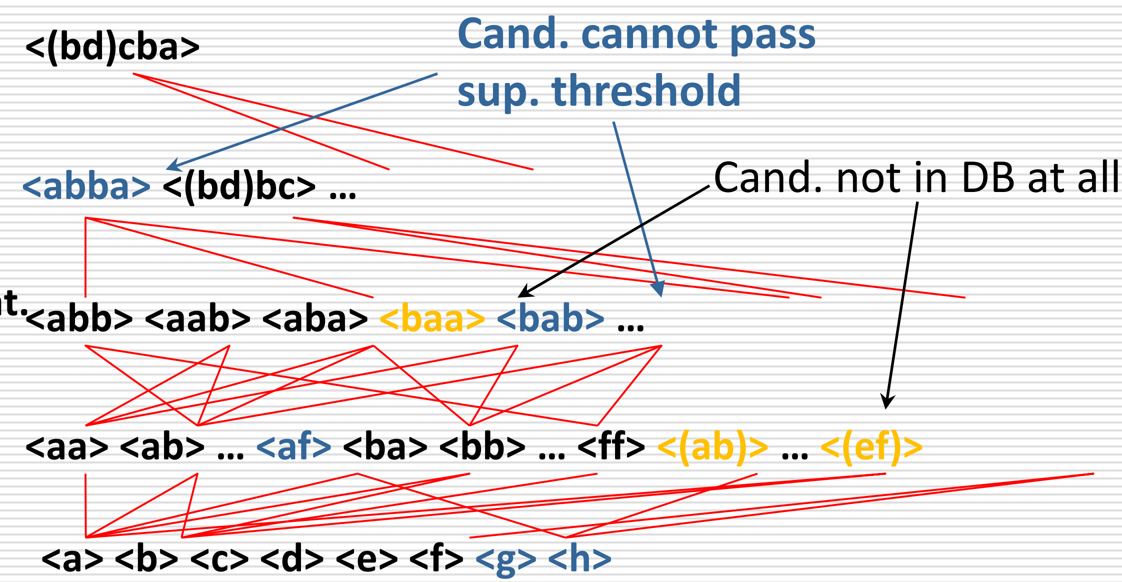
5th scan: 1 cand. 1 length-5 seq. pat.

4th scan: 8 cand. 6 length-4 seq. pat.

3rd scan: 46 cand. 19 length-3 seq. pat.
20 cand. not in DB at all

2nd scan: 51 cand. 19 length-2 seq. pat. 10 cand. not in DB at all

1st scan: 8 cand. 6 length-1 seq. pat.



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GSP Algorithm

- Take sequences in form of <x> as length-1 candidates
- Scan database once, find F_1, the set of length-1 sequential patterns
- Let k=1; while F_k is not empty do
  - Form C_{k+1}, the set of length-(k+1) candidates from F_k;
  - If C_{k+1} is not empty, scan database once, find F_{k+1}, the set of length-(k+1) sequential patterns
  - Let k=k+1;

GSP Algorithm

- Benefits from the Apriori pruning
  - Reduces search space
- Bottlenecks
  - Scans the database multiple times
  - Generates a huge set of candidate sequences

Extension of Association Rule Mining

- Association rule mining has been extensively studied in the data mining community.
- There are many efficient algorithms and model variations.
- Other related work includes
  - Multi-level or generalized rule mining
  - Sequential pattern mining
  - Constrained rule mining
  - Incremental rule mining
  - Maximal and closed frequent itemset mining
  - Numeric association rule mining
  - Rule interestingness and visualization
  - Parallel algorithms
  - …

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- Slides are adapted from:
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  - Dr. Jure Leskovec
  - Dr. Wujun Li

Quiz

TID	Items
10	{a, d, e}
20	{a, b, c, e}
30	{a, b, d, e}
40	{a, c, d, e}
50	{b, c, e}
60	{b, d, e}
70	{c, d}
80	{a, b, c}
90	{a, d, e}
100	{a, b, e}

Confidence({bd} → {e}) = ?

Confidence({e} → {bd}) = ?