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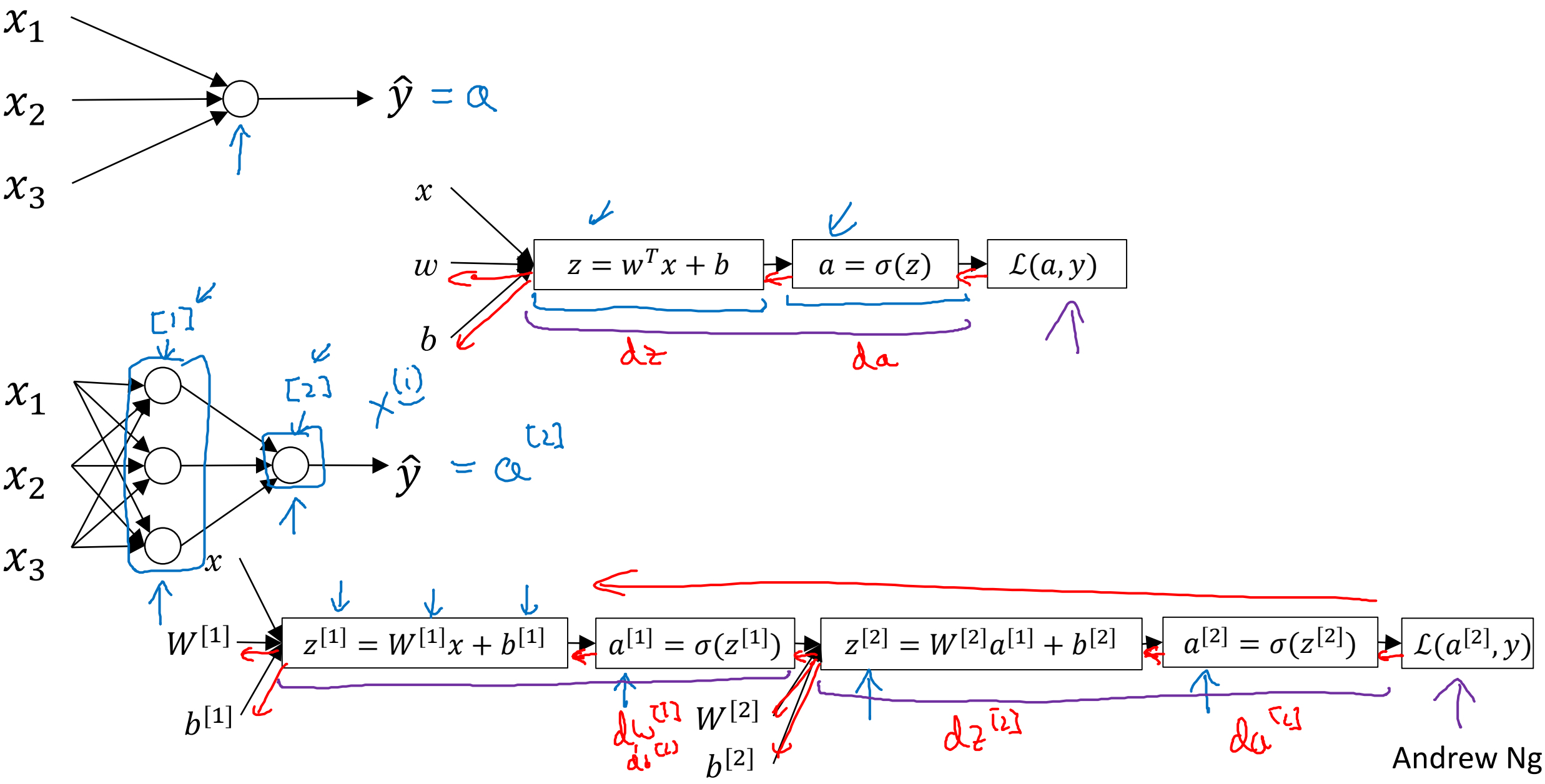
One hidden layer Neural Network

Neural Networks Overview

What is a Neural Network?



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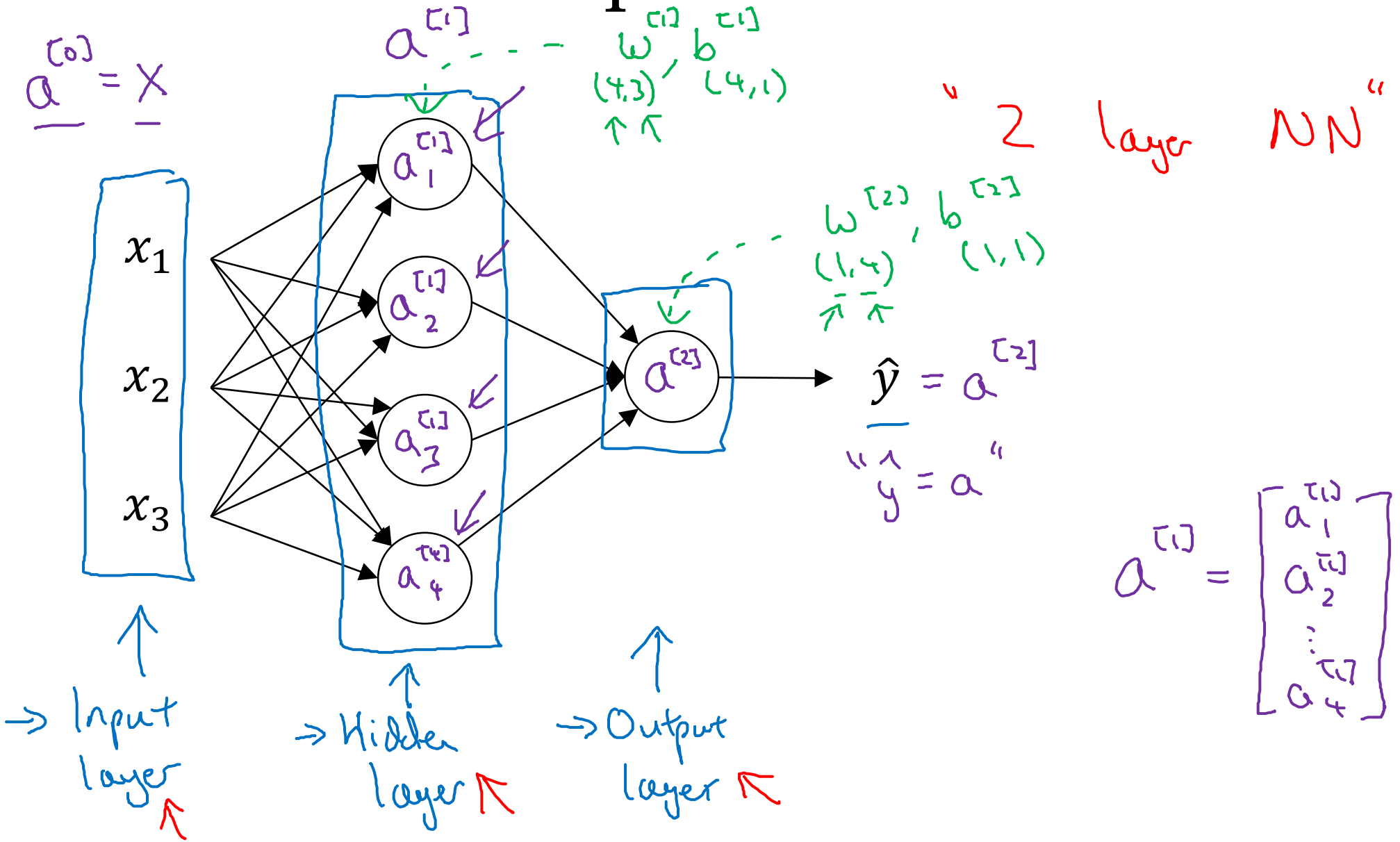
Neural Network Representation

Neural Network Representation



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Computing a Neural Network's Output

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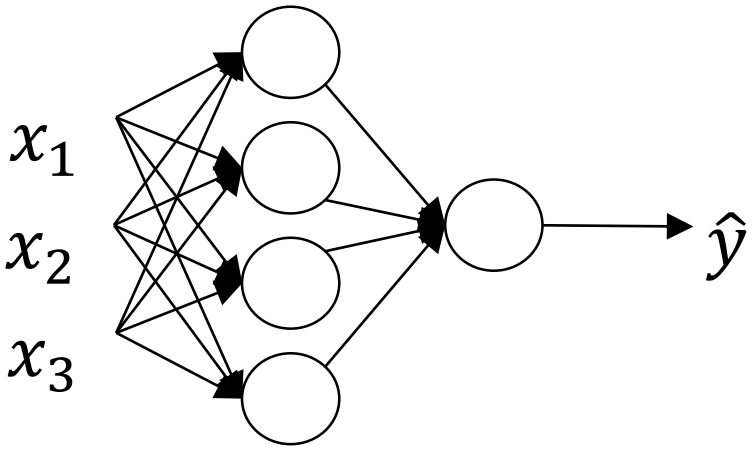
Vectorizing across multiple examples

Vectorizing across multiple examples



$$z^{[1]} = W^{[1]}x + b^{[1]}$$
$$a^{[1]} = \sigma(z^{[1]})$$
$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$
$$a^{[2]} = \sigma(z^{[2]})$$

$x \rightarrow a^{[2]} = \hat{y}$

$x^{(1)} \rightarrow a^{[2](1)} = \hat{y}^{(1)}$

$x^{(2)} \rightarrow a^{[2](2)} = \hat{y}^{(2)}$

$\vdots$

$x^{(m)} \rightarrow a^{[2](m)} = \hat{y}^{(m)}$

$a^{[2](i)}$ ← example i, layer 2

for $i = 1$ to m,

$z^{[1](i)} = W^{[1]}x^{(i)} + b^{[1]}$

$a^{[1](i)} = \sigma(z^{[1](i)})$

$z^{[2](i)} = W^{[2]}a^{[1](i)} + b^{[2]}$

$a^{[2](i)} = \sigma(z^{[2](i)})$

Vectorizing across multiple examples

```
for i = 1 to m:
```

$$z^{[1](i)} = W^{[1]}x^{(i)} + b^{[1]}$$

$$a^{[1](i)} = \sigma(z^{[1](i)})$$

$$z^{[2](i)} = W^{[2]}a^{[1](i)} + b^{[2]}$$

$$a^{[2](i)} = \sigma(z^{[2]}{}^{(i)})$$

$$Z^{[1]} = W^{[1]}X + b^{[1]}$$

$$A^{[1]} = g^{[1]}(Z^{[1]})$$

$$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$

$$A^{[2]} = g^{[2]}(Z^{[2]})$$

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

(n_x, m)

$$Z^{[1]} = \begin{bmatrix} | & | & & | \\ z^{[1](1)} & z^{[1](2)} & \cdots & z^{[1](m)} \\ | & | & & | \end{bmatrix}$$

$$A^{[1]} = \begin{bmatrix} | & | & & | \\ a^{[1](1)} & a^{[1](2)} & \cdots & a^{[1](m)} \\ | & | & & | \end{bmatrix}$$

training examples

hidden units.

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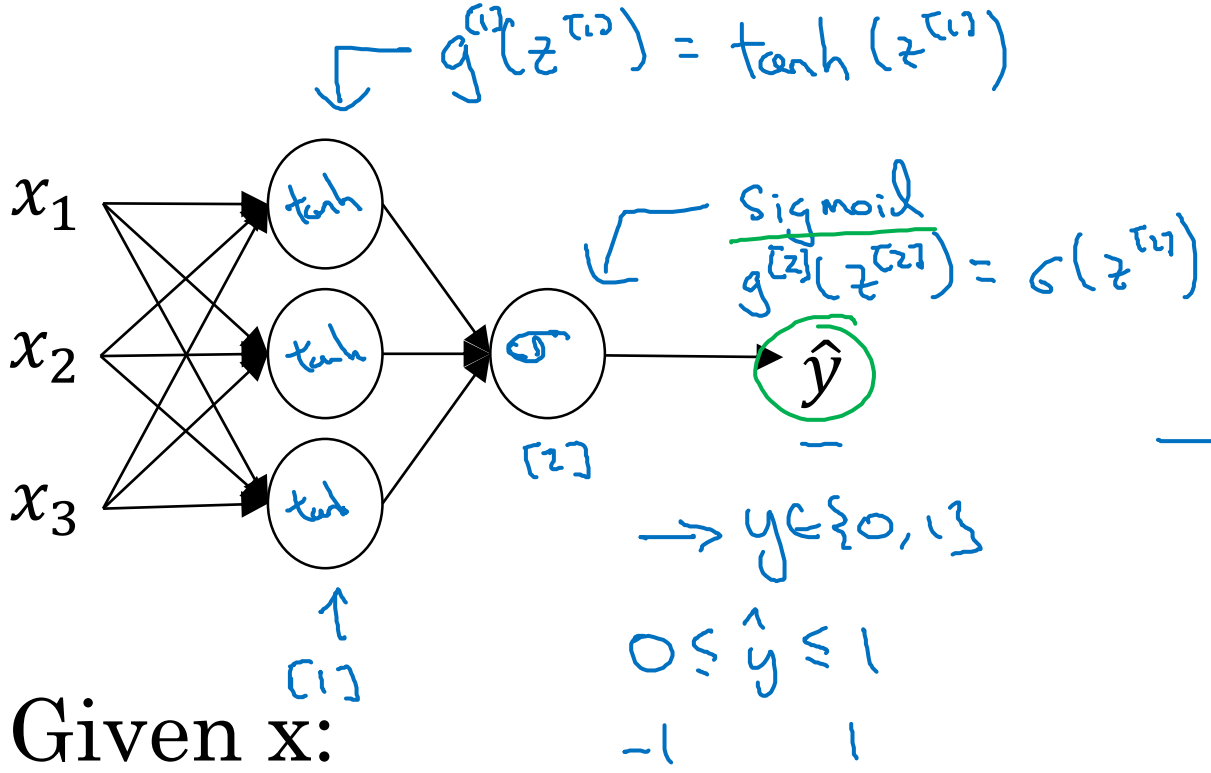
Activation functions

Activation functions



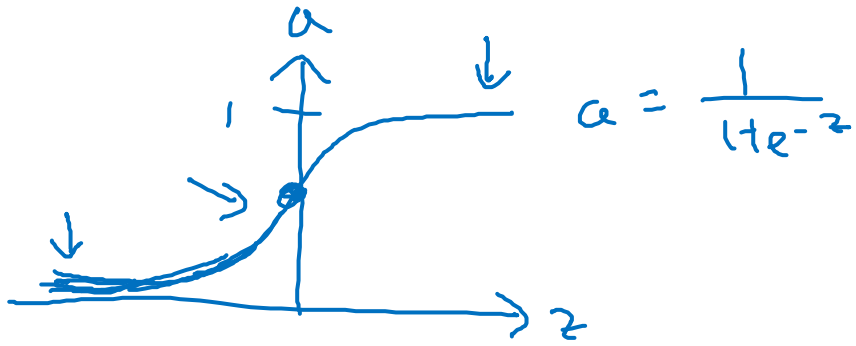
$g^{[1]}(z^{[1]}) = \tanh(z^{[1]})$

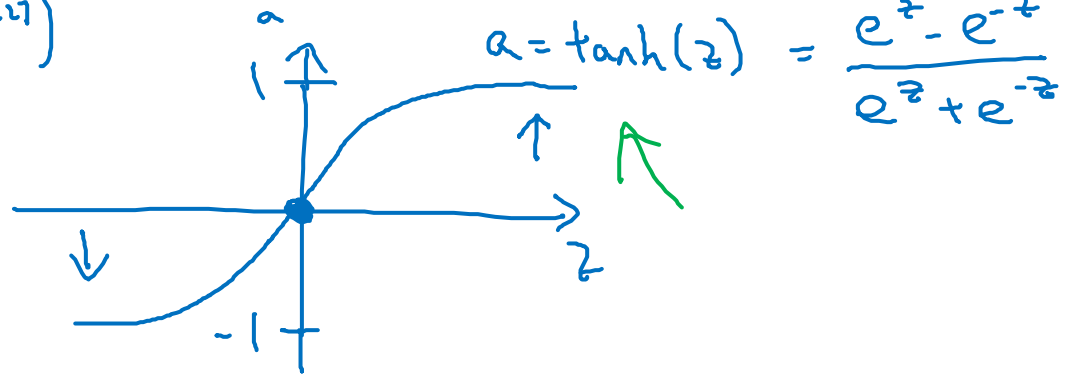
Sigmoid
$g^{[2]}(z^{[2]}) = \sigma(z^{[2]})$

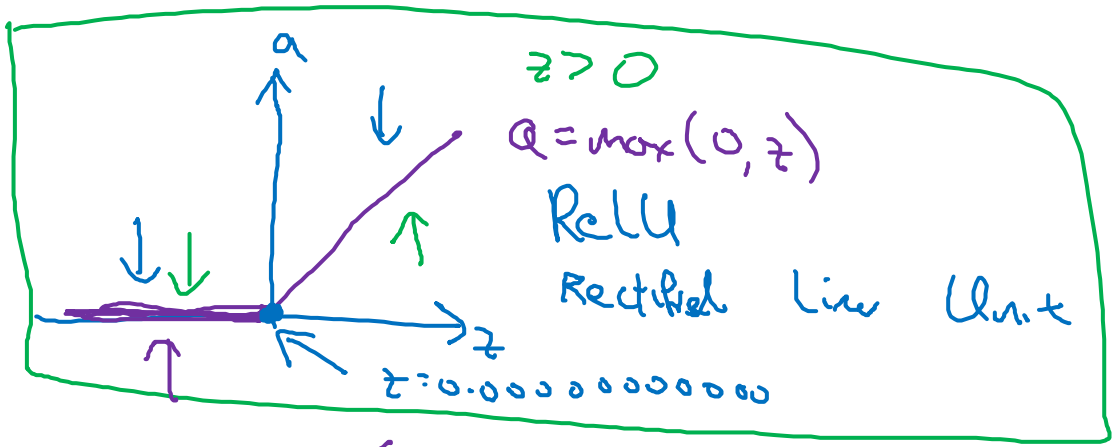
$\rightarrow y \in \{0, 1\}$

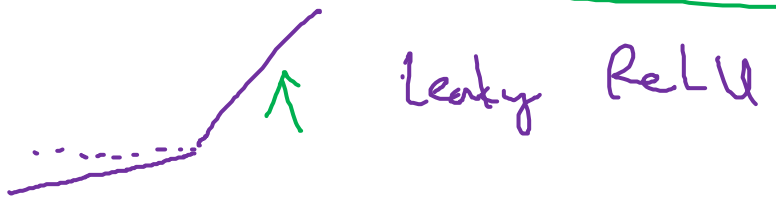
$0 \le \hat{y} \le 1$

$-1 \quad 1$









Given x:

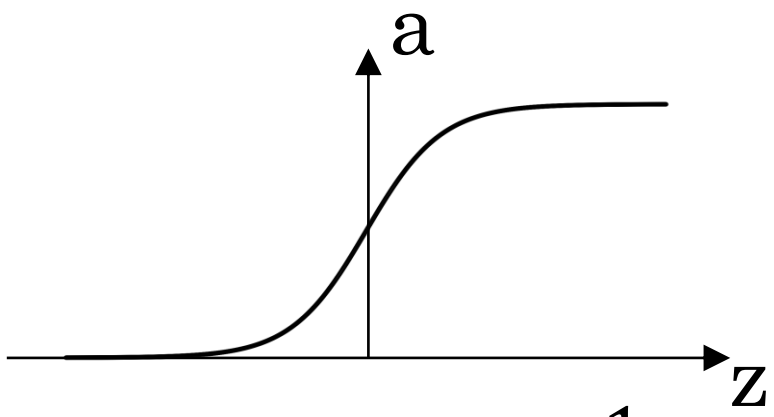
$$z^{[1]} = W^{[1]}x + b^{[1]}$$

$$a^{[1]} = \sigma(z^{[1]}) \quad g^{[1]}(z^{[1]})$$

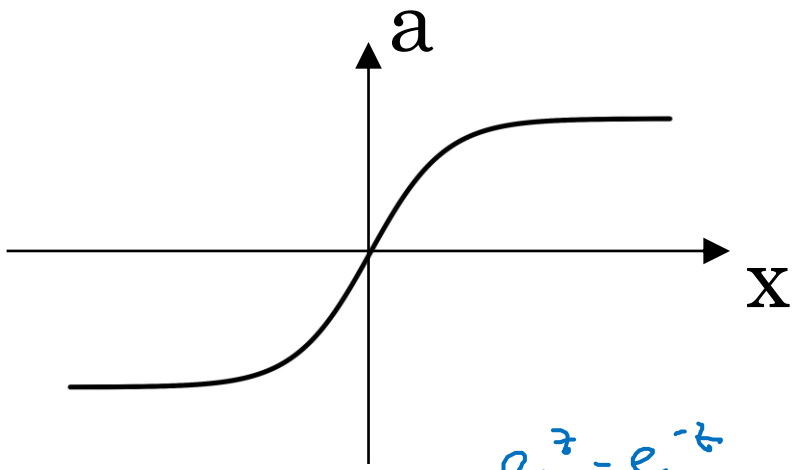
$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$a^{[2]} = \sigma(z^{[2]}) \quad g^{[2]}(z^{[2]})$$

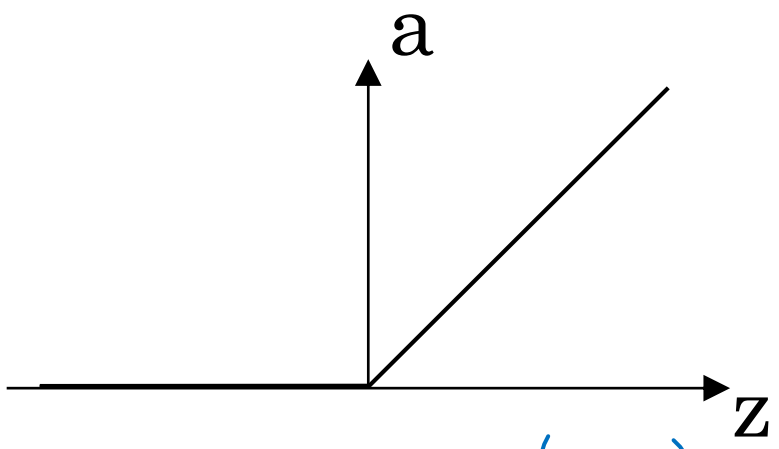
Pros and cons of activation functions



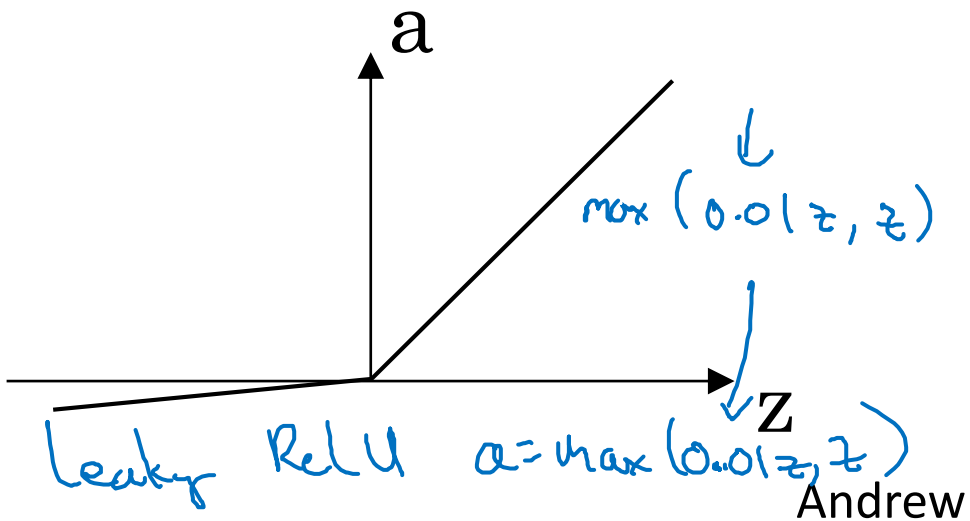
sigmoid: $a = \frac{1}{1+e^{-z}}$



tanh: $a = \frac{e^{z}-e^{-z}}{e^{z}+e^{-z}}$



ReLU $a = \max(0, z)$



Leaky ReLU $a = \max(0.01z, z)$

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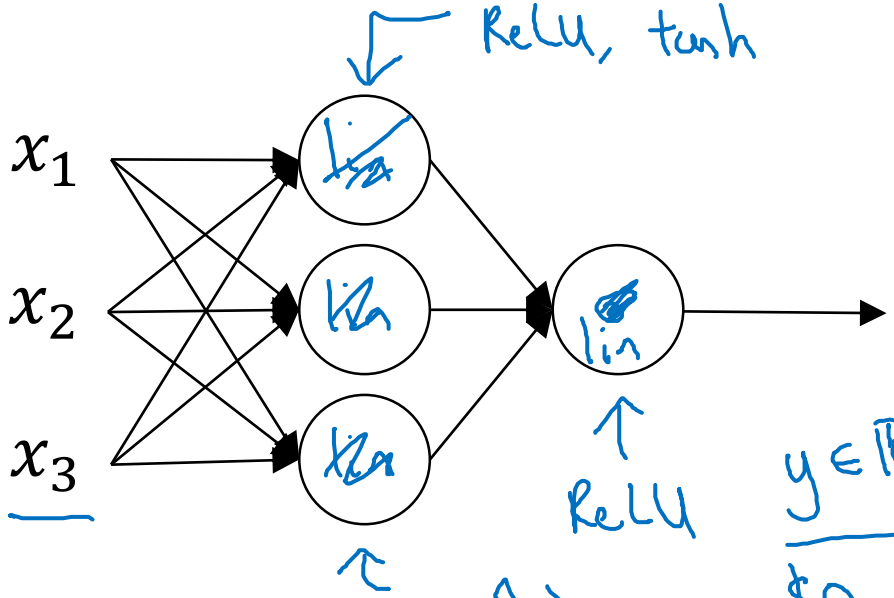
Why do you need non-linear activation functions?

Activation function



$y \in \mathbb{R}$

$\$0 \ldots \$1{,}000{,}000$s

$\hat{y} \geq 0$.

Given x:

$$z^{[1]} = W^{[1]}x + b^{[1]}$$

$$a^{[1]} = g^{[1]}(z^{[1]})$$

$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$a^{[2]} = g^{[2]}(z^{[2]})$$

$g(z) = z$

"linear activation function"

$$a^{[1]} = z^{[1]} = W^{[1]}x + b^{[1]}$$

$$a^{[2]} = z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$a^{[2]} = W^{[2]}\underbrace{\left(W^{[1]}x + b^{[1]}\right)}_{a^{[1]}} + b^{[2]}$$

$$= \underbrace{\left(W^{[2]}W^{[1]}\right)}_{W'}x + \underbrace{\left(W^{[2]}b^{[1]} + b^{[2]}\right)}_{b'}$$

$$= W'x + b'$$

$g(z) = z$

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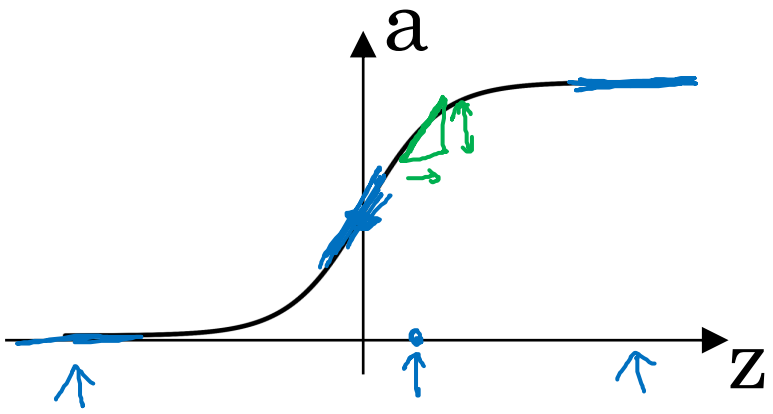
One hidden layer Neural Network

Derivatives of activation functions

Sigmoid activation function



$$g(z) = \frac{1}{1+e^{-z}}$$

$a = g(z) = \frac{1}{1+e^{-z}}$

$g'(z) = \frac{d}{dz} g(z)$ = slope of $g(x)$ at z

$= \frac{1}{1+e^{-z}} \left(1 - \frac{1}{1+e^{-z}}\right)$

$= g(z)\left(1 - g(z)\right)$ ←

$= a(1-a)$

$g'(z) = a(1-a)$ ↑

$z = 10$. $g(z) \approx 1$

$\frac{d}{dz} g(z) \approx 1\,(1-1) \approx 0$

$z = -10$ $g(z) \approx 0$

$\frac{d}{dz} g(z) \approx 0 \cdot (1-0) \approx 0$

$z = 0$ $g(z) = \frac{1}{2}$

$\frac{d}{dz} g(z) = \frac{1}{2}\left(1 - \frac{1}{2}\right) = \frac{1}{4}$

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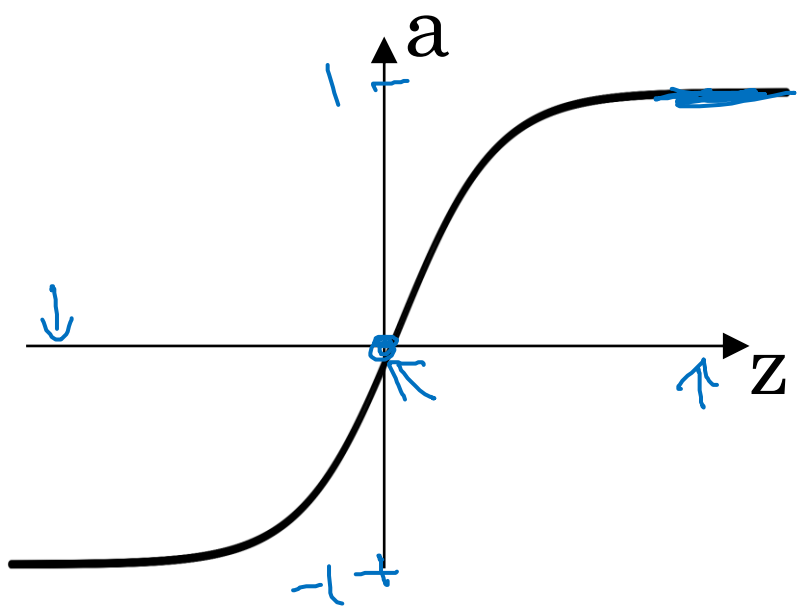
Tanh activation function



$$g(z) = \tanh(z)$$

$$= \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

$$g'(z) = \frac{d}{dz} g(z) = \text{slope of } g(z) \text{ at } z$$

$$= 1 - (\tanh(z))^2$$

$z = 10 \quad \tanh(z) \approx 1$

$g'(z) \approx 0$

$z = -10 \quad \tanh(z) \approx -1$

$g'(z) \approx 0$

$z = 0 \quad \tanh(z) = 0$

$g'(z) = 1$

$a = g(z)$, $\quad g'(z) = 1 - a^2$

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Gradient descent for neural networks

Gradient descent for neural networks

Parameters: $w^{[1]}$ $(n^{[1]}, n^{[0]})$, $b^{[1]}$ $(n^{[1]}, 1)$, $w^{[2]}$ $(n^{[2]}, n^{[1]})$, $b^{[2]}$ $(n^{[2]}, 1)$

$n_x = n^{[0]}$, $n^{[1]}$, $n^{[2]} = 1$

Cost function: $J(w^{[1]}, b^{[1]}, w^{[2]}, b^{[2]}) = \frac{1}{m} \sum_{i=1}^{m} \mathcal{L}(\hat{y}, y)$

($w^{[2]}$, $b^{[2]}$; $\hat{y} = a^{[2]}$)

Gradient descent:

→ Repeat {

→ Compute predicts $(\hat{y}^{(i)}, i = 1, \dots, m)$

$dw^{[1]} = \frac{\partial J}{\partial w^{[1]}}$, $db^{[1]} = \frac{\partial J}{\partial b^{[1]}}$, ...

$w^{[1]} := w^{[1]} - \alpha\, dw^{[1]}$

$b^{[1]} := b^{[1]} - \alpha\, db^{[1]}$

$w^{[2]} := \dots$ $b^{[2]} := \dots$

}

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Formulas for computing derivatives

Forward propagation:

$$Z^{[1]} = W^{[1]} X + b^{[1]}$$

$$A^{[1]} = g^{[1]}(Z^{[1]})$$

$$Z^{[2]} = W^{[2]} A^{[1]} + b^{[2]}$$

$$A^{[2]} = g^{[2]}(Z^{[2]}) = \sigma(Z^{[2]})$$

Back propagation:

$$dZ^{[2]} = A^{[2]} - Y$$

$Y = [y^{(1)}\ y^{(2)} \cdots y^{(m)}]$

$$dW^{[2]} = \frac{1}{m} dZ^{[2]} A^{[1]T}$$

$(n^{[2]}, m)$

$$db^{[2]} = \frac{1}{m} \text{np.sum}(dZ^{[2]}, \text{axis}=1, \text{keepdims}=\text{True})$$

$(n^{[2]}, 1)$

$$dZ^{[1]} = W^{[2]T} dZ^{[2]} * g^{[1]'}(Z^{[1]})$$

$(n^{[1]}, m)$ $\quad$ $(n^{[1]}, m)$ $\quad$ element-wise product

$$dW^{[1]} = \frac{1}{m} dZ^{[1]} X^T$$

$$db^{[1]} = \frac{1}{m} \text{np.sum}(dZ^{[1]}, \text{axis}=1, \text{keepdims}=\text{True})$$

$(n^{[1]}, 1)$ $\quad$ $(n^{[1]},)$ $\quad$ reshape

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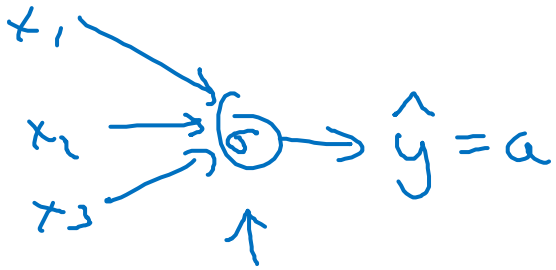
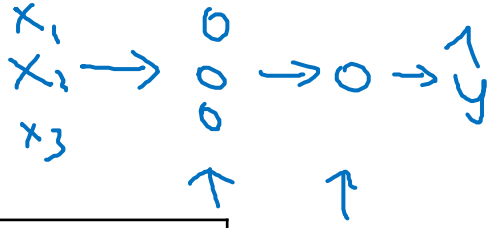
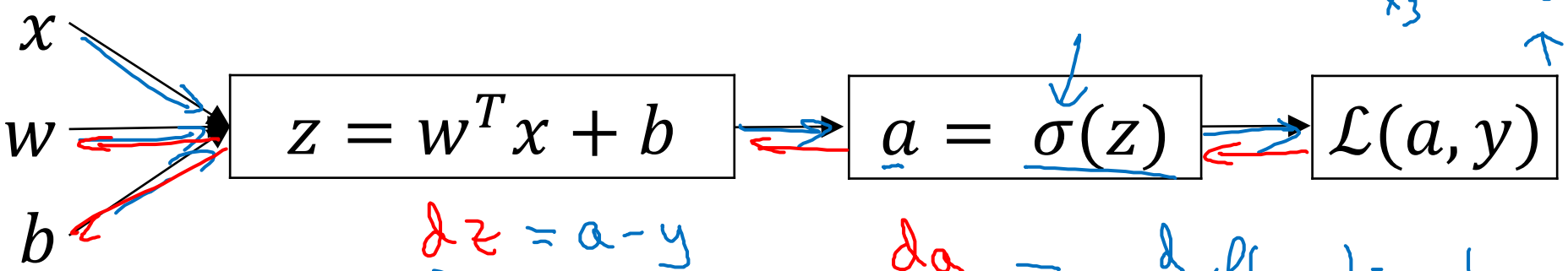
Backpropagation intuition (Optional)

Computing gradients

Logistic regression







$dz = a - y$

$dw = dz \cdot x$

$db = dz$

$dz = da \cdot g'(z)$

$da = \frac{d}{da}\mathcal{L}(a,y) = -y\log a - (1-y)\log(1-a)$

$= -\frac{y}{a} + \frac{1-y}{1-a}$

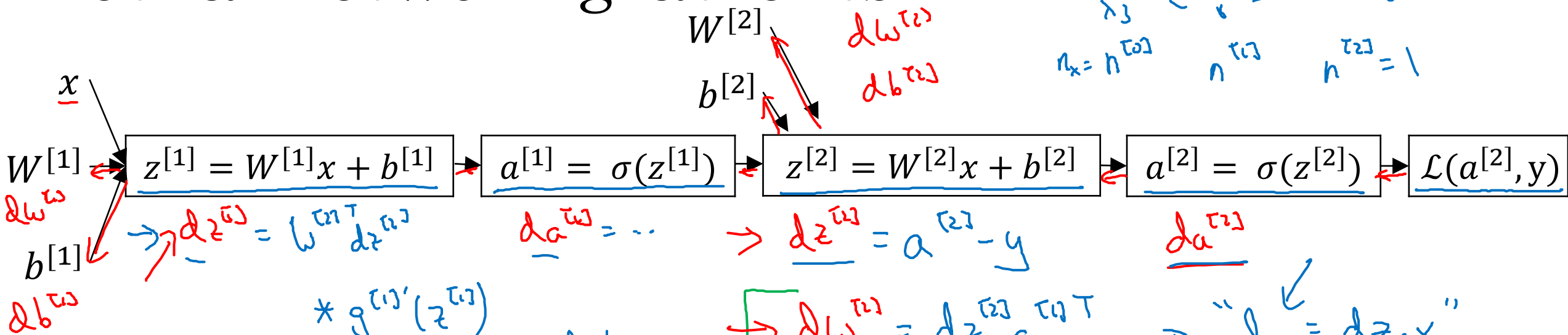
$g(z) = \sigma(z)$

$\frac{d\mathcal{L}}{dz} = \frac{d\mathcal{L}}{da} \cdot \frac{da}{dz}$

"dz" = "da"

$\frac{d}{dz} g(z) = g'(z)$

Neural network gradients



$dz^{[1]} = W^{[2]T} dz^{[2]}$

$da^{[1]} = \ldots$

$dz^{[2]} = a^{[2]} - y$

$da^{[2]}$

$* g^{[1]'}(z^{[1]})$

↑ element-wise product

$dW^{[2]} = dz^{[2]} a^{[1]T}$

$db^{[2]} = dz^{[2]}$

"$dw = dz \cdot x$"

$W^{[1]} = [\longrightarrow]$

$W^{[2]}$ $(n^{[2]}, n^{[1]})$

$z^{[2]}, dz^{[2]}$ $(n^{[2]}, 1)$ $-(1,1)$

$z^{[1]}, dz^{[1]}$ $(n^{[1]}, 1)$

foo

dfoo

$W^{[1]}$

$dW^{[2]}$

$dW^{[1]} = dz^{[1]} \cdot x^T$

$db^{[1]} = dz^{[1]}$

$a^{[0]T}$

$dz^{[1]} = W^{[2]T} dz^{[2]} * g^{[1]'}(z^{[1]})$

$(n^{[1]}, 1) = (n^{[1]}, n^{[2]})\ (n^{[2]}, 1) * (n^{[1]}, 1)$

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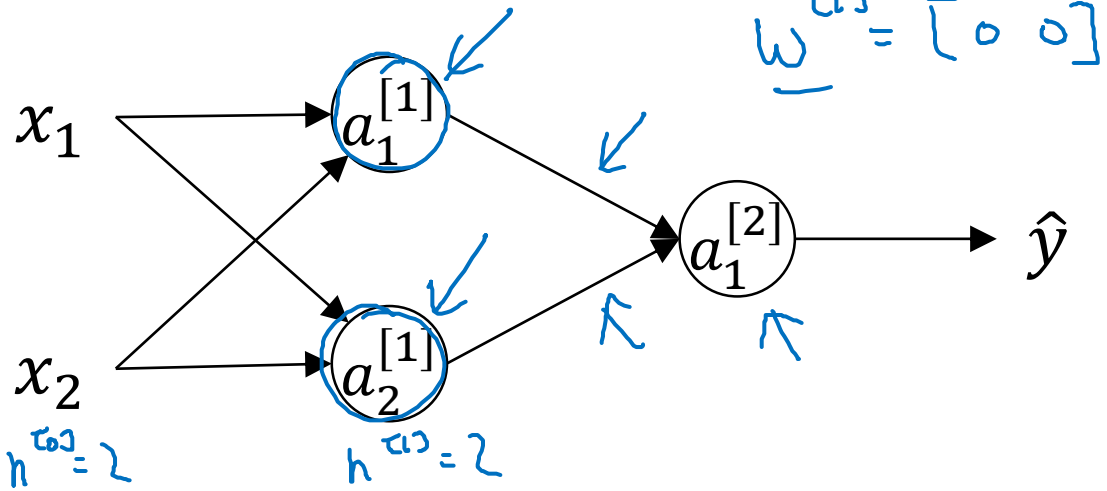
Random Initialization

What happens if you initialize weights to zero?



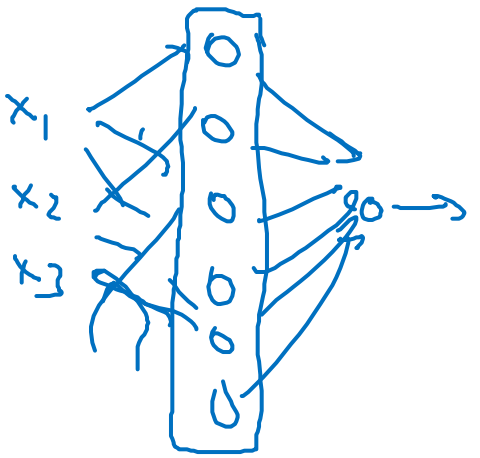
$$w^{[1]} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad b^{[1]} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$a_1^{[1]} = a_2^{[1]} \qquad dz_1^{[1]} = dz_2^{[1]}$$

$$dw = \begin{bmatrix} u & v \\ u & v \end{bmatrix}$$

$$w^{[1]} = w^{[1]} - \alpha\, dw$$

Symmetric



$$w^{[1]} = \begin{bmatrix} \cdots & \cdots \\ \cdots & \cdots \end{bmatrix}$$

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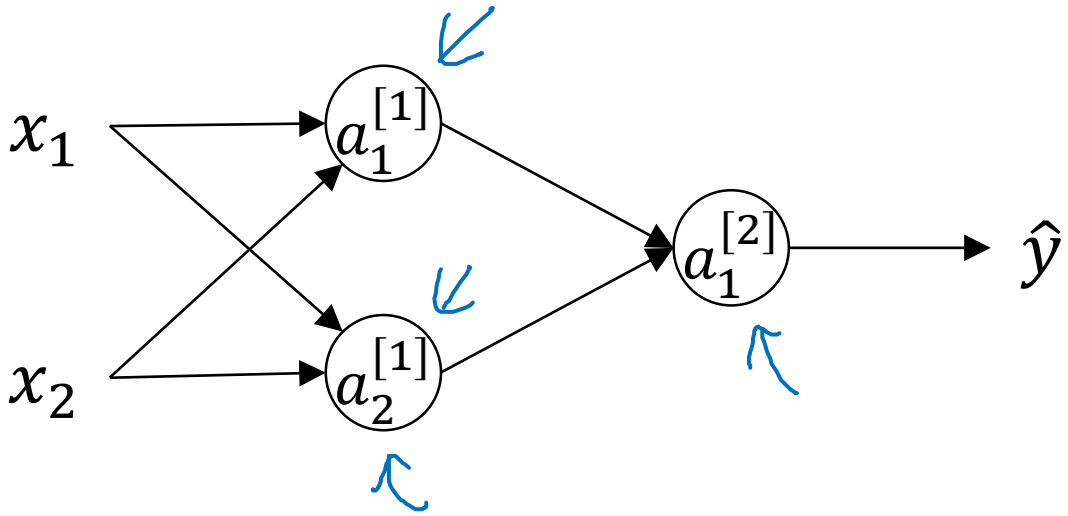
Random initialization



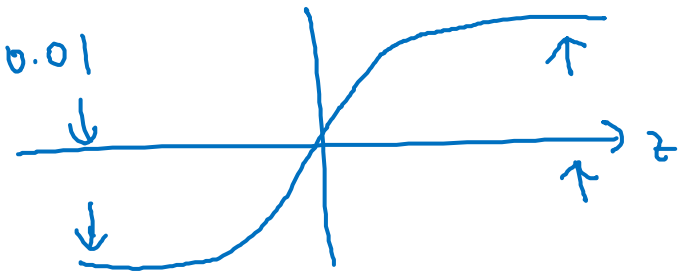
$W^{[1]} = np.random.randn((2,2)) * 0.01$

$b^{[1]} = np.zeros((2,1))$

$W^{[2]} = np.random.randn((1,2)) * 0.01$

$b^{[2]} = 0$

0.01 / 100 ?



$z^{[1]} = W^{[1]}x + b^{[1]}$

$a^{[1]} = g^{[1]}(z^{[1]})$