# Accelerated Variance Reduction Stochastic ADMM for Large-Scale Machine Learning

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**Abstract**—Recently, many stochastic variance reduced alternating direction methods of multipliers (ADMMs) (e.g., SAG-ADMM and SVRG-ADMM) have made exciting progress such as linear convergence rate for strongly convex (SC) problems. However, their best-known convergence rate for non-strongly convex (non-SC) problems is $\mathcal{O}(1/T)$ as opposed to $\mathcal{O}(1/T^2)$ of accelerated deterministic algorithms, where $T$ is the number of iterations. Thus, there remains a gap in the convergence rates of existing stochastic ADMM and deterministic algorithms. To bridge this gap, we introduce a new momentum acceleration trick into stochastic variance reduced ADMM, and propose a novel accelerated SVRG-ADMM method (called ASVRG-ADMM) for the machine learning problems with the constraint $Ax\!+\!By\!=\!c$. Then we design a linearized proximal update rule and a simple proximal one for the two classes of ADMM-style problems with $B\!=\!\tau I$ and $B\!\neq\!\tau I$, respectively, where $I$ is an identity matrix and $\tau$ is an arbitrary bounded constant. Note that our linearized proximal update rule can avoid solving sub-problems iteratively. Moreover, we prove that ASVRG-ADMM converges linearly for SC problems. In particular, ASVRG-ADMM improves the convergence rate from $\mathcal{O}(1/T)$ to $\mathcal{O}(1/T^2)$ for non-SC problems. Finally, we apply ASVRG-ADMM to various machine learning problems, e.g., graph-guided fused Lasso, graph-guided logistic regression, graph-guided SVM, generalized graph-guided fused Lasso and multi-task learning, and show that ASVRG-ADMM consistently converges faster than the state-of-the-art methods.

**Index Terms**—Stochastic optimization, ADMM, variance reduction, momentum acceleration, strongly convex and non-strongly convex, smooth and non-smooth

## 1 Introduction

THIS paper mainly considers the following composite finite-sum equality-constrained optimization problem,

$$\min_{x\in\mathbb{R}^{d_x},y\in\mathbb{R}^{d_y}} \Big\{f(x)+h(y),\ \text{s.t., } Ax+By=c\Big\} \tag{1}$$

where $c\!\in\!\mathbb{R}^{d_c}$, $A\!\in\!\mathbb{R}^{d_c\times d_x}$, $B\!\in\!\mathbb{R}^{d_c\times d_y}$, $f(x)\!:=\!\frac{1}{n}\sum_{i=1}^{n}f_i(x)$, each component function $f_i(\cdot)$ is convex, and $h(\cdot)$ is convex but possibly non-smooth. For instance, a popular choice of $f_i(\cdot)$ in binary classification problems is the logistic loss, i.e., $f_i(x)=\log(1+\exp(-b_i a_i^T x))$, where $(a_i,b_i)$ is the feature-label pair, and $b_i\!\in\!\{\pm1\}$. With regard to $h(\cdot)$, we are interested in a sparsity-inducing regularizer, e.g., $\ell_1$-norm [1, 2], group Lasso [3, 4] and nuclear norm [5–7].

Problem (1) arises in many places in machine learning, pattern recognition, computer vision, statistics, and operations research [8]. When the constraint in Eq. (1) is $Ax\!=\!y$, the formulation (1) becomes

$$\min_{x\in\mathbb{R}^{d_x},y\in\mathbb{R}^{d_y}} \Big\{f(x)+h(y),\ \text{s.t., } Ax=y\Big\} \tag{2}$$

where $A\!\in\!\mathbb{R}^{d_y\times d_x}$. Recall that this class of problems include the graph-guided fused Lasso [3], generalized Lasso [4] and graph-guided SVM [9] as notable examples. If the constraint degenerates $x=y$, this class of problems include the regularized empirical risk minimization (ERM) problem, e.g., logistic regression, Lasso and linear support vector machine.

For solving the large-scale optimization problem involving a large sum of $n$ component functions, stochastic gradient descent (SGD) [10] uses only one or a mini-batch of gradients in each iteration, and thus enjoys a significantly lower per-iteration complexity than deterministic methods including Nesterov's accelerated gradient descent (AGD) [11, 12] and accelerated proximal gradient (APG) [13, 14], i.e., $O(d_x)$ vs. $O(nd_x)$. Therefore, SGD has been successfully applied to many large-scale machine learning problems [9, 15, 16], especially training deep network models [17]. However, the variance of the stochastic gradient estimator may be large, and thus we need to gradually reduce its step-size, which leads to slow convergence [18], especially for equality-constrained composite convex problems [19].

This paper mainly focuses on the large sample regime. In this regime, even first-order deterministic methods such as FISTA [14] become computationally burdensome due to their per-iteration complexity of $O(nd_x)$. As a result, SGD with low per-iteration complexity $O(d_x)$ has witnessed tremendous progress in the recent years. Recently, a number of stochastic variance reduced methods such as SAG [23], SDCA [24], SVRG [18], Prox-SVRG [25] and VR-SGD [26] have been proposed to successfully address the problem of high variance of stochastic gradient estimators in ordinary SGD, resulting in linear convergence for strongly convex problems as opposed to sub-linear rates of SGD. More recently, the Nesterov's acceleration technique [27] was introduced in [28–31] to further speed up the stochastic

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*Manuscript received August 8, 2019.*

TABLE 1
Comparison of convergence rates and memory requirements of various stochastic ADMM algorithms, including stochastic ADMM (STOC-ADMM) [9], stochastic average gradient ADMM (SAG-ADMM) [19], stochastic dual coordinate ascent ADMM (SDCA-ADMM) [20], scalable stochastic ADMM (SCAS-ADMM) [21], stochastic variance reduced gradient ADMM (SVRG-ADMMM) [22], and our ASVRG-ADMM. It should be noted although all the methods except SDCA-ADMM apply the same update rule in (4), their algorithms do not actually work for solving the problem (1) with the constraint $Ax+By=c$, where $B\neq\tau I$, $\tau$ is an arbitrary bounded constant, and $I$ is an identity matrix.

| | Non-strongly convex | Strongly convex | Constraints | Space requirement |
|---|---|---|---|---|
| STOC-ADMM [9] | $\mathcal{O}(1/\sqrt{T})$ | $\mathcal{O}(\log T/T)$ | $Ax=y$ | $O(d_x d_y + d_x^2)$ |
| SAG-ADMM [19] | $\mathcal{O}(1/T)$ | unknown | $Ax=y$ | $O(d_x d_y + n d_x)$ |
| SDCA-ADMM [20] | unknown | linear rate | $Ax+By=c$ | $O(d_x d_y + n)$ |
| SCAS-ADMM [21] | $\mathcal{O}(1/T)$ | $\mathcal{O}(1/T)$ | $Ax=y$ | $O(d_x d_y)$ |
| SVRG-ADMM [22] | $\mathcal{O}(1/T)$ | linear rate | $Ax=y$ | $O(d_x d_y)$ |
| ASVRG-ADMM (ours) | $\mathcal{O}(1/T^2)$ | linear rate | $Ax+By=c$ | $O(d_x d_y)$ |

variance reduced algorithms, which results in the best-known convergence rates for both strongly convex (SC) and non-strongly convex (non-SC) problems, e.g., Katyusha [29]. This also motivates us to integrate the momentum acceleration trick into the stochastic alternating direction method of multipliers (ADMM) below.

### 1.1 Review of Stochastic ADMMs

It is well known that the ADMM is an effective optimization tool [32] to solve this class of composite optimization problems (1). The ADMM has shown attractive performance in a wide range of real-world problems, such as big data classification [33] and matrix and tensor recovery [5, 34, 35]. We refer the reader to [36–39] for some review papers on the ADMM. Recently, several faster deterministic ADMM algorithms have been proposed to solve some special cases of Problem (1). For instance, [40] proposed an accelerated ADMM, and proved that their algorithm has an $\mathcal{O}(1/T^2)$ convergence rate for SC problems, similar to [37, 41][1]. [42, 43] proposed a faster ADMM algorithm with a convergence rate $\mathcal{O}(1/T^2)$ for solving the special case of Problem (1) with the constraint $Ax=y$. However, the per-iteration complexity of all the full-batch ADMMs is $O(nd_x)$, and thus they become very slow and are not suitable for large-scale machine learning problems.

To tackle the issue of high per-iteration complexity of deterministic ADMM, [9, 44, 45] proposed some online or stochastic ADMM algorithms. However, all these variants only achieve the convergence rate of $\mathcal{O}(\log T/T)$ for SC problems and $\mathcal{O}(1/\sqrt{T})$ for non-SC problems, respectively, as compared with the linear convergence and $\mathcal{O}(1/T^2)$ rates of the accelerated deterministic ADMM algorithms mentioned above. Recently, several accelerated and faster converging versions of stochastic ADMMs such as SAG-ADMM [19], SDCA-ADMM [20] and SVRG-ADMM [22], which are all based on variance reduction techniques, have been proposed. With regard to strongly convex problems, [20, 22] proved that linear convergence can be obtained for the special ADMM form (i.e., Problem (2)) and the general ADMM form, respectively. [46] also proposed a fast stochastic variance reduced ADMM for stochastic composition optimization problems. More recently, [47, 48] proposed two accelerated stochastic ADMM algorithms for the problem (2) and four-composite optimization problems, respectively. For SAG-ADMM and SVRG-ADMM, an $\mathcal{O}(1/T)$ convergence rate can be guaranteed for non-strongly convex problems, which implies that there remains a gap in convergence rates between the stochastic ADMM and accelerated deterministic algorithms, i.e., $\mathcal{O}(1/T)$ vs. $\mathcal{O}(1/T^2)$.

### 1.2 Contributions

To fill in this gap, we design a new momentum acceleration trick similar to the ones in deterministic optimization and incorporate it into the stochastic variance reduction gradient (SVRG) based stochastic ADMM (SVRG-ADMM) [22]. Naturally, the proposed method has a low per-iteration cost as existing stochastic ADMM algorithms such as SVRG-ADMM, and does not require the storage of all gradients (or dual variables) as in SAG-ADMM [19] and SCAS-ADMM [21], as shown in Table 1.

The main differences between this paper and our previous conference paper [49] are listed as follows: 1) We briefly review recent work on stochastic ADMM for solving Problems (1) and (2). 2) When $B\neq\tau I$ in Eq. (1), where $\tau$ is an arbitrary bounded constant and $I$ is an identity matrix, the sub-problem with respect to $y$ (see Eq. (4) below) has no closed-form solution and has to be solved iteratively. To overcome this difficulty, we present a new linearized proximal update rule for both SC and non-SC problems (1) with the constraint $Ax+By=c$ when $B\neq\tau I$. In other words, the existing stochastic ADMM algorithms including the proposed ones in our previous work [49] do not work for this case. Although the theoretical guarantees of existing variance reduced stochastic ADMMs except SDCA-ADMM [20] are for Problem (1) with the general constraint $Ax+By=c$, they do not actually work for solving such problems. 3) For the case of $B=\tau I$, we use a simple proximal update rule as in our previous work [49] instead of the linearized proximal one. Then we propose two novel accelerated SVRG-ADMM algorithms (called ASVRG-ADMM) for both SC and non-SC problems. 4) We also theoretically analyze the convergence properties of the proposed ASVRG-ADMM algorithms for both SC and non-SC problems and the two cases of $B\neq\tau I$ and $B=\tau I$, respectively. 5) We further improve the theoretical results in our previous work [49] by removing the boundedness assumption. 6) Finally, we report more experimental results especially for the ADMM

[1] Note that, for simplicity, we do not differentiate the $\mathcal{O}(1/T^2)$ and $o(1/T^2)$ because they are of the same order in the worst-case nature and their difference is insignificant in general, where $T$ is the number of iterations.

problem (1) with the constraint $Ax+By=c$ to verify both the effectiveness and efficiency of ASVRG-ADMM.

The main contributions of this paper are summarized as follows.

- We propose an efficient accelerated variance reduced stochastic ADMM (ASVRG-ADMM) method, which integrates both our momentum acceleration trick and the variance reduction technique of SVRG-ADMM [22]. Moreover, ASVRG-ADMM has a linearized proximal rule and a simple proximal one for both cases of $B\neq\tau I$ and $B=\tau I$, respectively.
- We prove that ASVRG-ADMM achieves a linear convergence rate for SC problems, which is consistent with the best-known result in SDCA-ADMM [20] and SVRG-ADMM [22]. Besides, when ASVRG-ADMM uses its linearized proximal rule, it becomes more practical than existing algorithms, which have to solve the sub-problems iteratively.
- In particular, for the more general problem (1) with the constraint $Ax+By=c$ and $B\neq\tau I$, we also design a novel epoch initialization technique for the variable $y$ at each epoch of our linearized proximal acceleration algorithm for SC problems.
- We also prove that ASVRG-ADMM has a convergence rate $\mathcal{O}(1/T^2)$ for non-SC problems, which means that ASVRG-ADMM is a factor $T$ faster than SAG-ADMM and SVRG-ADMM, whose convergence rate is $\mathcal{O}(1/T)$. In particular, we design an adaptive increasing epoch length strategy and further improve the theoretical results by using this strategy and removing boundedness assumptions.
- Various experimental results on synthetic and real-world datasets further verify that our ASVRG-ADMM converges consistently much faster than the state-of-the-art stochastic ADMM methods.

The remainder of this paper is organized as follows. Section 2 discusses some recent advances in stochastic ADMM. Section 3 proposes a new accelerated stochastic variance reduction ADMM method (called ASVRG-ADMM) with the proposed momentum acceleration trick. Moreover, we analyze the convergence properties of ASVRG-ADMM in Section 4. Experimental results in Section 5 show the effectiveness of ASVRG-ADMM. In Section 6, we conclude this paper and discuss the future work.

## 2 Related Work

This section reveals recent progresses and efforts in stochastic optimization methods that are based on the stochastic alternating direction method of multipliers (ADMM).

### 2.1 Notation

Throughout this paper, the norm $\|\cdot\|$ denotes the standard Euclidean norm, and $\|\cdot\|_1$ is the $\ell_1$-norm, i.e., $\|x\|_1=\sum_i|x_i|$. We denote by $\nabla f(x)$ the gradient of $f(x)$ if it is differentiable, or $\partial f(x)$ any of the subgradients of $f(\cdot)$ at $x$ if $f(\cdot)$ is only Lipschitz continuous. To facilitate our discussion, we first make the following basic assumptions.

### 2.2 Basic Assumptions

**Assumption 1** (Smoothness). *Each convex component function $f_i(\cdot)$ is $L$-smooth if its gradients are $L$-Lipschitz continuous, that is*

$$\|\nabla f_i(x)-\nabla f_i(y)\|\leq L\|x-y\|,\ \text{for all } x,y\in\mathbb{R}^d.$$

**Assumption 2** (Strong Convexity). *A convex function $g(\cdot):\mathbb{R}^d\to\mathbb{R}$ is $\mu$-strongly convex, if there exists a constant $\mu>0$ such that*

$$g(y)\geq g(x)+\langle\nabla g(x),y-x\rangle+\frac{\mu}{2}\|y-x\|^2,\ \text{for all } x,y\in\mathbb{R}^d.$$

*If $g(\cdot)$ is non-smooth, we modify the above inequality by simply replacing $\nabla g(x)$ with an arbitrary sub-gradient $\partial g(x)$.*

### 2.3 Stochastic ADMM

It is easy to see that Problem (2) is only a special case of the general ADMM form (1) when $B=-I_{d_2}$ and $c=\mathbf{0}$. Thus, the purpose of this paper is to propose an accelerated stochastic variance reduced ADMM method for solving the more general problem (1). Although the stochastic (or online) ADMM algorithms and theoretical results in [9, 19, 22, 44] are all for the problem (1), they do not actually work.

The augmented Lagrangian function of Problem (1) is

$$\begin{aligned}\mathcal{L}(x,y,\lambda)\ &=\ f(x)+h(y)+\langle\lambda,Ax+By-c\rangle\\&\quad+\frac{\beta}{2}\|Ax+By-c\|^2\end{aligned}\tag{3}$$

where $\lambda$ is the vector of Lagrangian multipliers (also called the dual variable), and $\beta>0$ is a penalty parameter. To minimize Problem (1), together with the dual variable $\lambda$, the update steps of deterministic ADMM are

$$y_k=\arg\min\nolimits_y\left\{h(y)+\tfrac{\beta}{2}\|Ax_{k-1}+By-c+\lambda_{k-1}\|^2\right\},\tag{4}$$

$$x_k=\arg\min\nolimits_x\left\{f(x)+\tfrac{\beta}{2}\|Ax+By_k-c+\lambda_{k-1}\|^2\right\},\tag{5}$$

$$\lambda_k=\lambda_{k-1}+Ax_k+By_k-c.\tag{6}$$

To extend the deterministic ADMM to the online and stochastic settings, the update rules for $y_k$ and $\lambda_k$ remain unchanged, while in [9, 44], the update rule of $x_k$ is approximated as follows:

$$\begin{aligned}x_k=\arg\min_x\Big\{&\langle x,\nabla f_{i_k}(x_{k-1})\rangle+\frac{1}{2\eta_k}\|x-x_{k-1}\|_G^2\\&+\frac{\beta}{2}\|Ax+By_k-c+\lambda_{k-1}\|^2\Big\}\end{aligned}\tag{7}$$

where we draw $i_k$ uniformly at random from $[n]:=\{1,\ldots,n\}$, $\eta_k\propto 1/\sqrt{k}$ is the learning rate or step-size, and $\|z\|_G^2=z^TGz$ with a given positive semi-definite matrix $G$, e.g., $G\succeq I_{d_1}$ as in [22]. Analogous to SGD, the stochastic ADMM variants also use an unbiased estimate of the gradient at each iteration, i.e., $\mathbb{E}[\nabla f_{i_k}(x_{k-1})]=\nabla f(x_{k-1})$. However, all those algorithms have much slower convergence rates than their deterministic counterparts mentioned above. This barrier is mainly due to the large variance introduced by the stochasticity of the gradients [18]. Essentially, to guarantee convergence of SGD and its ADMM variants, we need to employ a decaying sequence of step-sizes $\{\eta_k\}$, which in turn leads to slower convergence rates.

Recently, a number of variance reduced stochastic ADMM methods (e.g., SAG-ADMM and SVRG-ADMM) have

**Algorithm 1** ASVRG-ADMM for strongly-convex problems

```
Input: $m, \eta, \beta > 0, 1 \leq b \leq n$.
Initialize: $\widetilde{x}^0 = \widetilde{z}^0, \widetilde{y}^0, \theta, \widetilde{\lambda}^0 = -\frac{1}{\beta}(A^T)^{\dagger}\nabla f(\widetilde{x}_0)$,
        $\nu = 1 + \frac{\eta\beta\|B^T B\|_2}{\theta}, \gamma = 1 + \frac{\eta\beta\|A^T A\|_2}{\theta}$;
for $s = 1, 2, \ldots, T$ do
    $\widetilde{p} = \nabla f(\widetilde{x}^{s-1}), x_0^s = z_0^s = \widetilde{x}^{s-1}, \lambda_0^s = \widetilde{\lambda}^{s-1}$;
    $y_0^s = \widetilde{y}^{s-1}$ for the case of $B = \tau I$, or $y_0^s = -\frac{1}{\beta}B^{\dagger}(Az_0^s - c)$ for the case of $B \neq \tau I$;
    for $k = 1, 2, \ldots, m$ do
        Choose $I_k \subseteq [n]$ of size b, uniformly at random;
        $\widetilde{\nabla} f_{I_k}(x_{k-1}^s) = \frac{1}{|I_k|}\sum_{i_k \in I_k}[\nabla f_{i_k}(x_{k-1}^s) - \nabla f_{i_k}(\widetilde{x}^{s-1})] + \widetilde{p}$;
        $y_k^s = \mathrm{Prox}_h^{\frac{1}{\beta\tau^2}}((-Az_{k-1}^s + c - \lambda_{k-1}^s)/\tau)$
                                        for the case of $B = \tau I$,
        $y_k^s = \mathrm{Prox}_h^{\frac{\eta\beta}{\theta\nu}}\left[y_{k-1}^s - \frac{\eta\beta}{\theta\nu}B^T(Az_{k-1}^s + By_{k-1}^s - c + \lambda_{k-1}^s)\right]$
                                        for the case of $B \neq \tau I$;
    $z_k^s = z_{k-1}^s - \frac{\eta}{\gamma\theta}\left[\widetilde{\nabla} f_{I_k}(x_{k-1}^s) + \beta A^T(Az_{k-1}^s + By_k^s - c + \lambda_{k-1}^s)\right]$;
        $x_k^s = (1-\theta)\widetilde{x}^{s-1} + \theta z_k^s$;
        $\lambda_k^s = \lambda_{k-1}^s + Az_k^s + By_k^s - c$;
    end for
    $\widetilde{x}^s = \frac{1}{m}\sum_{k=1}^m x_k^s,\ \widetilde{y}^s = (1-\theta)\widetilde{y}^{s-1} + \frac{\theta}{m}\sum_{k=1}^m y_k^s$;
    $\widetilde{\lambda}^s = -\frac{1}{\beta}(A^T)^{\dagger}\nabla f(\widetilde{x}^s)$;
end for
Output: $\widetilde{x}^T, \widetilde{y}^T$.
```

been proposed and made exciting progress such as linear convergence rates. SVRG-ADMM [22] is particularly attractive here because of its low storage requirement compared with the algorithms in [19, 20]. Within each epoch of mini-batch SVRG-ADMM, the full gradient $\widetilde{p} = \nabla f(\widetilde{x})$ is first computed, where $\widetilde{x}$ is the average point of the previous epoch. Then $\nabla f_{i_k}(x_{k-1})$ and $\eta_k$ in (7) are replaced by

$$\widetilde{\nabla} f_{I_k}(x_{k-1}) = \frac{1}{|I_k|}\sum_{i_k \in I_k}(\nabla f_{i_k}(x_{k-1}) - \nabla f_{i_k}(\widetilde{x})) + \widetilde{p} \quad (8)$$

and a constant step-size $\eta$, respectively, where $I_k \subset [n]$ is a mini-batch of size $b$. Note that mini-batching is a useful technique to reduce the variance of the stochastic gradients [26, 50]. In fact, $\widetilde{\nabla} f_{I_k}(x_{k-1})$ is also an unbiased estimator of the gradient $\nabla f(x_{k-1})$, i.e., $\mathbb{E}[\widetilde{\nabla} f_{I_k}(x_{k-1})] = \nabla f(x_{k-1})$.

For the equality-constrained composite convex problem (2), Xu *et al.* [47] proposed a faster variant of SVRG-ADMM with an adaptive penalty parameter scheme. Fang *et al.* [48] proposed an accelerated stochastic ADMM with Nesterov's extrapolation and variance reduction techniques for solving four-composite optimization problems. Moreover, Huang *et al.* [51], and Huang and Chen [52] proposed several variants of SVRG-ADMM for solving non-smooth and non-convex optimization problems.

## 3 Momentum Accelerated Variance Reduction Stochastic ADMM

In this section, we propose an efficient accelerated variance reduced stochastic ADMM (ASVRG-ADMM) method for solving both SC and non-SC problems (1). In particular, we design two new linearized proximal accelerated algorithms for both SC and non-SC problems with the constraint $Ax + By = c$ and $B \neq \tau I$, respectively.

### 3.1 ASVRG-ADMM for Strongly Convex Problems

In this part, we first consider the case of Problem (1) when each $f_i(\cdot)$ is convex, $L$-smooth, and $f(\cdot)$ is $\mu$-strongly convex. Recall that this class of problems include graph-guided logistic regression and support vector machines (SVM) as notable examples. To efficiently solve this class of problems, we incorporate both the momentum acceleration trick proposed in our previous work [49] and the variance reduced stochastic ADMM [22], as shown in Algorithm 1. All our algorithms including Algorithm 1 are divided into $T$ epochs, and each epoch consists of $m$ stochastic updates, where $m$ is usually chosen to be $m = \Theta(n)$ as in [18, 49].

#### *3.1.1 Update Rule of $y$*

As in both SVRG-ADMM [22] and ASVRG-ADMM [49], the variable $y$ is updated by solving the following problem for both strongly convex and non-strongly convex cases:

$$y_k^s = \arg\min_y \left\{ h(y) + \frac{\beta}{2}\|Az_{k-1}^s + By - c + \lambda_{k-1}^s\|^2 \right\} \quad (9)$$

where the superscript $s$ indicates the $s$-th epoch, the subscript $k$ denotes the $k$-th inner-iteration, $z_{k-1}^s$ is an auxiliary variable and its update rule is given in Section 3.1.2.

When $B = \tau I$ (e.g., $B$ is an identity matrix), the solution to the problem in Eq. (9) can be relatively easily obtained. In other words, we still apply the simple proximal rule proposed in our previous work [49] to solve such problems. For this case, we give the following proximal update rule:

$$y_k^s = \mathrm{Prox}_h^{\frac{1}{\beta\tau^2}}((-Az_{k-1}^s + c - \lambda_{k-1}^s)/\tau)$$

where the proximal operator $\mathrm{Prox}_h^{\delta}(\cdot)$ is defined as

$$\mathrm{Prox}_h^{\delta}(w) = \arg\min_x \left\{ \frac{1}{2\delta}\|x - w\|^2 + h(x) \right\}.$$

However, when $B \neq \tau I$ (e.g., $B$ is not a diagonal matrix), it is often hard to solve the problem (9) in practice [32]. To address this issue, in this paper we design the following linearized proximal rule to update $y$,

$$y_k^s = \arg\min_y \left\{ h(y) + \frac{\beta}{2}\left\|Az_{k-1}^s + By - c + \lambda_{k-1}^s\right\|^2 + \frac{\theta_{s-1}}{2\eta}\|y - y_{k-1}^s\|_{Q_s}^2 \right\}$$

where $Q_s = \nu I_{d_2} - \frac{\eta\beta}{\theta_{s-1}}B^T B$ with $\nu \geq 1 + \frac{\eta\beta\|B^T B\|_2}{\theta_{s-1}}$ to ensure that $Q_s \succeq I$, where $\|\cdot\|_2$ is the spectral norm, i.e., the largest singular value of the matrix. The above problem is equivalent to the following problem,

$$y_k^s = \arg\min_y \left\{ h(y) + \frac{\nu\theta_{s-1}}{2\eta}\left\|y - y_{k-1}^s + \frac{\eta\beta}{\theta_{s-1}\nu}p_k^s\right\|^2 \right\} \quad (10)$$

where $p_k^s = B^T(Az_{k-1}^s + By_{k-1}^s - c + \lambda_{k-1}^s)$. We can easily obtain the following proximal update rule for Problem (10):

$$y_k^s = \mathrm{Prox}_h^{\frac{\eta\beta}{\theta_{s-1}\nu}}\left[y_{k-1}^s - \frac{\eta\beta}{\theta_{s-1}\nu}B^T(Az_{k-1}^s + By_{k-1}^s - c + \lambda_{k-1}^s)\right].$$

From the above analysis, it is clear that we introduce the linearized proximal operation into the proposed algorithms (including Algorithm 1 and Algorithm 2 below) and make our algorithms much more practical than existing stochastic ADMM algorithms including SVRG-ADMM [22] and the algorithms proposed in [49]. Besides, the proposed linearized proximal rule can also avoid the calculation of the pseudo-inverse matrix at each inner-iteration. Then the new algorithms proposed in this paper as well as their convergence analysis are different from those in our previous work [49]. To ensure linear convergence of the proposed linearized proximal algorithm for strongly convex problems as SVRG-ADMM, we also design the following new epoch initialization strategy for $y_0^s$ at each epoch instead of $y_0^s\!=\!\widetilde{y}^{s-1}$ in [49], where the snapshot point $\widetilde{y}^{s-1}$ is defined in Algorithm 1.

$$y_0^s = -B^{\dagger}(Az_0^s - c) \tag{11}$$

where $B$ is required to be a matrix of full column rank, and $(\cdot)^{\dagger}$ denotes the pseudo-inverse of a matrix. Note that the epoch initialization strategy in Eq. (11) plays a key role in our linear convergence guarantees of our linearized proximal acceleration algorithm for the general case of $B\neq\tau I$. For the case of $B=\tau I$, we still use the proximal rule and the initialization strategy (i.e., $\widetilde{\lambda}^s=-\frac{1}{\beta}(A^T)^{\dagger}\nabla f(\widetilde{x}^s)$) in our previous work [49] to guarantee linear convergence, while only this strategy cannot guarantee the convergence of our algorithm for the general case of $B\neq\tau I$. Therefore, we require both the initialization strategies of $y_0^s$ and $\widetilde{\lambda}^s$ to guarantee linear convergence of our algorithm. Note that the initialization techniques involve the pseudo-inverses of $A^T$ and $B$. As $A$ and $B$ are often sparse, these can be efficiently computed by the Lanczos algorithm [53].

### 3.1.2 Update Rule of $z$

$z$ is an auxiliary variable, and its update rule is given as follows. Similar to [19, 22], we also use the inexact Uzawa method [54] to approximate (7), which can avoid computing the inverse of the matrix $(\frac{1}{\eta}I_{d_1}+\beta A^TA)$. Moreover, the momentum parameter $\theta_s$ ($0\leq\theta_s\leq 1$ and its update rule is provided in Section 3.1.4) is introduced into the proximal term $\frac{1}{2\eta}\|z-z_{k-1}^s\|^2_{G_s}$ similar to that of (7), and then the problem with respect to $z$ is formulated as follows:

$$\begin{aligned}\min_{z}\Big\{\Big\langle z-z_{k-1}^s,\,\widetilde{\nabla}f_{I_k}(x_{k-1}^s)\Big\rangle+\frac{\theta_{s-1}}{2\eta}\|z-z_{k-1}^s\|^2_{G_s}\\ +\frac{\beta}{2}\|Az+By_k^s-c+\lambda_{k-1}^s\|^2\Big\}\end{aligned} \tag{12}$$

where $\widetilde{\nabla}f_{I_k}(x_{k-1}^s)$ is the stochastic variance reduced gradient estimator independently introduced in [18, 55], and $G_s=\gamma I_{d_1}-\frac{\eta\beta}{\theta_{s-1}}A^TA$ with $\gamma>1+\frac{\eta\beta\|A^TA\|_2}{\theta_{s-1}}$ to ensure that $G_s\succeq I$ similar to [22]. In fact, there is also an alternative to set $G_s$ as an identity matrix, and then the problem (12) can be solved through matrix inversion [9, 19].

### 3.1.3 Our Momentum Accelerated Update Rule for $x$

In particular, our momentum accelerated update rule for $x$ is defined as follows:

$$x_k^s=\widetilde{x}^{s-1}+\theta_{s-1}(z_k^s-\widetilde{x}^{s-1})=(1-\theta_{s-1})\widetilde{x}^{s-1}+\theta_{s-1}z_k^s \tag{13}$$

where $\theta_{s-1}(z_k^s-\widetilde{x}^{s-1})$ is a new momentum term similar to those as in accelerated deterministic methods [27], which helps accelerate the convergence speed of our algorithms by using the iterate of the previous epoch, i.e., $\widetilde{x}^{s-1}$. Note that $\theta_{s-1}$ is a momentum parameter, and its update rule is given below. The momentum term, $\theta_{s-1}(z_k^s-\widetilde{x}^{s-1})$, plays a key role as the Katyusha momentum in [29]. Different from Katyusha [29], which uses both the Nesterov's momentum and Katyusha momentum, our ASVRG-ADMM algorithms (including Algorithm 1 and Algorithm 2 below) have only one momentum term.

### 3.1.4 Update Rule of $\theta_s$

In all epochs of Algorithm 1, the momentum parameter $\theta_s$ can be set to a constant $\theta$, which must satisfy the condition $0\leq\theta\leq 1-\delta(b)/(\alpha-1)$, where $\alpha=\frac{1}{L\eta}$ and $\delta(b)=\frac{n-b}{b(n-1)}$. In particular, we also provide the selecting schemes for the momentum parameter $\theta$ and corresponding theoretical analysis for the two cases of $B\!=\!\tau I$ and $B\neq\tau I$, which all are presented in the Supplementary Material.

The detailed procedure for solving the strongly convex problem (1) is shown in Algorithm 1, where we use the same epoch initialization technique for $\widetilde{\lambda}^s$ as in [22]. Similar to $x_k^s$, $\widetilde{y}^s\!=\!(1\!-\!\theta_{s-1})\widetilde{y}^{s-1}\!+\!\frac{\theta_{s-1}}{m}\sum_{k=1}^{m}y_k^s$. When $\theta\!=\!1$, ASVRG-ADMM degenerates to the linearized proximal variant of SVRG-ADMM in [22], as shown in the Supplementary Material.

## 3.2 ASVRG-ADMM for Non-Strongly Convex Problems

In this part, we consider the non-strongly convex (non-SC) problems of the form (1) when each $f_i(\cdot)$ is convex, $L$-smooth, and $h(\cdot)$ is not necessarily strongly convex (possibly non-smooth), e.g., graph-guided fused Lasso. The detailed procedure for solving the non-SC problem (1) is shown in Algorithm 2, which has slight differences in the initialization and output of each epoch from Algorithm 1. In addition, the key difference between them is the update rule for the momentum parameter $\theta_s$. Different from the strongly convex case, the momentum parameter $\theta_s$ for the non-SC case is required to satisfy the following inequalities:

$$\frac{1-\theta_s}{\theta_s^2}=\frac{1}{\theta_{s-1}^2}\;\;\text{and}\;\;0\leq\theta_s\leq 1-\frac{\delta(b)}{\alpha-1} \tag{14}$$

where $\delta(b):=\frac{n-b}{b(n-1)}$ is a decreasing function with respect to the mini-batch size $b$. The condition (14) allows the momentum parameter to decease, but not too fast, similar to the requirement on the step-size $\eta_k$ in classical SGD and stochastic ADMM [56]. Unlike deterministic acceleration methods, $\theta_s$ must satisfy both inequalities in (14).

Motivated by the momentum acceleration techniques in [27, 57] for deterministic optimization, we give the update rule of the momentum parameter $\theta_s$ for the mini-batch case:

$$\theta_s=\frac{\sqrt{\theta_{s-1}^4+4\theta_{s-1}^2}-\theta_{s-1}^2}{2}\;\;\text{and}\;\;\theta_0=1-\frac{\delta(b)}{\alpha-1}. \tag{15}$$

For the special case of $b=1$, we have $\delta(1)=1$ and $\theta_0=1-\frac{1}{\alpha-1}$, while $b\!=\!n$ (i.e., the deterministic version), $\delta(n)=0$ and $\theta_0=1$. Since the sequence $\{\theta_s\}$ is decreasing, $\theta_s\leq 1-\frac{\delta(b)}{\alpha-1}$ is satisfied. That is, $\theta_s$ in Algorithm 2 is adaptively adjusted as in (15).

**Algorithm 2** ASVRG-ADMM for non-SC problems

```
Input: $m, \eta, \beta > 0, 1 \le b \le n$.
Initialize: $\widetilde{x}^0 = \widetilde{z}^0, \widetilde{y}^0 = y_m^0, \widetilde{\lambda}^0, \theta_0 = 1 - \frac{L\eta\delta(b)}{1-L\eta}$.
for $s = 1, 2, \ldots, T$ do
    $x_0^s = (1-\theta_{s-1})\widetilde{x}^{s-1} + \theta_{s-1}\widetilde{z}^{s-1}, y_0^s = y_m^{s-1}, \lambda_0^s = \widetilde{\lambda}^{s-1}$;
    $\widetilde{p} = \nabla f(\widetilde{x}^{s-1}), z_0^s = \widetilde{z}^{s-1}$;
    $\nu = 1 + \frac{\eta\beta\|B^T B\|_2}{\theta_{s-1}}, \gamma = 1 + \frac{\eta\beta\|A^T A\|_2}{\theta_{s-1}}$;
    for $k = 1, 2, \ldots, m$ do
        Choose $I_k \subseteq [n]$ of size b, uniformly at random;
        $\widetilde{\nabla} f_{I_k}(x_{k-1}^s) = \frac{1}{|I_k|}\sum_{i_k \in I_k}[\nabla f_{i_k}(x_{k-1}^s) - \nabla f_{i_k}(\widetilde{x}^{s-1})] + \widetilde{p}$;
        $y_k^s = \mathrm{Prox}_h^{\frac{1}{\beta\tau^2}}\left((-Az_{k-1}^s + c - \lambda_{k-1}^s)/\tau\right)$
                                                for the case of $B = \tau I$,
        $y_k^s = \mathrm{Prox}_h^{\frac{\eta\beta_{s-1}}{\nu\theta_{s-1}}}\left[y_{k-1}^s - \frac{\eta\beta B^T(Az_{k-1}^s + By_{k-1}^s - c + \lambda_{k-1}^s)}{\nu\theta_{s-1}}\right]$
                                                for the case of $B \neq \tau I$;
        $z_k^s = z_{k-1}^s - \frac{\eta}{\gamma\theta_{s-1}}\left[\widetilde{\nabla} f_{I_k}(x_{k-1}^s) + \beta A^T(Az_{k-1}^s + By_k^s - c + \lambda_{k-1}^s)\right]$;
        $x_k^s = (1-\theta_{s-1})\widetilde{x}^{s-1} + \theta_{s-1} z_k^s$;
        $\lambda_k^s = \lambda_{k-1}^s + Az_k^s + By_k^s - c$;
    end for
    $\widetilde{x}^s = \frac{1}{m}\sum_{k=1}^m x_k^s, \widetilde{y}^s = (1-\theta_{s-1})\widetilde{y}^{s-1} + \frac{\theta_{s-1}}{m}\sum_{k=1}^m y_k^s$;
    $\widetilde{\lambda}^s = \lambda_m^s, \widetilde{z}^s = z_m^s, \theta_s = \frac{\sqrt{\theta_{s-1}^4 + 4\theta_{s-1}^2} - \theta_{s-1}^2}{2}$;
end for
Output: $\widetilde{x}^T, \widetilde{y}^T$.
```

# 4 Convergence Analysis

In this section, we theoretically analyze the convergence properties of our ASVRG-ADMM algorithms (i.e., Algorithms 1 and 2) for SC and non-SC problems with the cases of $B \neq \tau I$ and $B = \tau I$, respectively. We first make the following assumption for the case of SC problems.

**Assumption 3.** *The matrices $A$ and $B^T$ both have full row rank.*

The first two assumptions (i.e., Assumptions 1 and 2) are common in the convergence analysis of first-order optimization methods, while the last one (i.e., Assumption 3) has been used in the convergence analysis of deterministic ADMM [7, 58, 59] and stochastic ADMM [22] for only the strongly convex case. Following [22], we first introduce the following function as a convergence criterion, where $h'(y)$[2] is the (sub)gradient of $h(\cdot)$ at $y$.

$$P(x,y) := f(x) - f(x^*) - \langle \nabla f(x^*), x - x^* \rangle + h(y) - h(y^*) - \langle h'(y^*), y - y^* \rangle$$

where $(x^*, y^*)$ denotes an optimal solution of Problem (1). By the convexity of $f(\cdot)$ and $h(\cdot)$, $P(x,y) \ge 0$ for all $x, y \in \mathbb{R}^d$.

Note that we present a new linearized proximal technique in (10) to update $y_k^s$, and thus we need to provide new convergence guarantees for our algorithms (i.e., Algorithms 1 and 2), which are different from those in our previous work [49]. Next, we present five main theoretical results for the convergence properties of Algorithms 1 and 2. And the detailed proofs of all the theoretical results are provided in this paper or the Supplementary Material.

We first sketch the proofs of our main theoretical results as follows: The proofs of our main results rely on the one-epoch inequalities in Lemma 4 ($B \neq \tau I$) below and Lemma 7 ($B = \tau I$) in the Supplementary Material. That is, the proofs of Theorems 1-5 below rely on the one-epoch inequalities in Lemmas 4 and 7, but require telescoping such inequalities in different manners. Furthermore, $P(x,y)$ in Lemma 4 consists of two terms, and thus we give the upper bounds of the two terms in Lemmas 2 and 3 to obtain Lemma 4, as well as applying Lemmas 2 and 6 to get Lemma 7 in the Supplementary Material. In addition, to remove the strong assumption used in Theorems 3 and 4, we also design an adaptive strategy of increasing epoch length for Algorithm 2, and the corresponding theoretical result is given in Theorem 5, which shows that Algorithm 2 with an adaptive increasing epoch length attains the same convergence rate without the boundedness assumption.

## 4.1 Key Lemmas

In this part, we give and prove some intermediate key results for our convergence analysis.

**Lemma 1.**

$$\mathbb{E}[\|\widetilde{\nabla} f_{I_k}(x_{k-1}^s) - \nabla f(x_{k-1}^s)\|^2] \le 2L\delta(b)\left[f(\widetilde{x}^{s-1}) - f(x_{k-1}^s) + \langle \nabla f(x_{k-1}^s, x_{k-1}^s - \widetilde{x}^{s-1}\rangle\right]$$

*where $\delta(b) = \frac{n-b}{b(n-1)} \le 1$ and $1 \le b \le n$.*

The proofs of Lemmas 1, 2 and all the theorems below are provided in the Supplementary Material. Lemma 1 provides an upper bound on the expected variance of the mini-batch SVRG estimator $\widetilde{\nabla} f_{I_k}(x_{k-1}^s)$.

**Lemma 2.** *Let $(x^*, y^*)$ be an optimal solution of Problem (1), and $\lambda^*$ be the corresponding Lagrange multiplier that maximizes the dual. Let $\varphi_k^s = \beta(\lambda_k^s - \lambda^*)$, and suppose that each $f_i(\cdot)$ is L-smooth. If the inequality $1 - \theta_{s-1} \ge \frac{\delta(b)}{\alpha-1}$ is satisfied, then*

$$\begin{aligned}
&\mathbb{E}[f(\widetilde{x}^s) - f(x^*) - \langle \nabla f(x^*), \widetilde{x}^s - x^* \rangle] \\
&\quad - \mathbb{E}\left[\frac{\theta_{s-1}}{m}\sum_{k=1}^{m}\left\langle A^T\varphi_k^s, x^* - z_k^s \right\rangle\right] \\
&\le (1-\theta_{s-1})\mathbb{E}[f(\widetilde{x}^{s-1}) - f(x^*) - \langle \nabla f(x^*), \widetilde{x}^{s-1} - x^* \rangle] \\
&\quad + \frac{\theta_{s-1}^2}{2m\eta}\mathbb{E}[\|x^* - z_0^s\|_{G_s}^2 - \|x^* - z_m^s\|_{G_s}^2].
\end{aligned}$$

For the case of $B \neq \tau I$, we have the following result, which corresponds to Lemma 7 in the Supplementary Material for the case of $B = \tau I$.

**Lemma 3.** *Let $\{(\widetilde{y}^s, y_k^s)\}$ be the sequence generated by Algorithm 1 (or Algorithm 2), we have*

$$\begin{aligned}
&\mathbb{E}[h(\widetilde{y}^s) - h(y^*) - \langle h'(y^*), \widetilde{y}^s - y^* \rangle] \\
&\quad - \frac{\theta_{s-1}}{m}\sum_{k=1}^{m}\mathbb{E}\left[\langle B^T\varphi_k^s, y^* - y_k^s \rangle\right] \\
&\le (1-\theta_{s-1})\mathbb{E}[h(\widetilde{y}^{s-1}) - h(y^*) - \langle h'(y^*), \widetilde{y}^{s-1} - y^* \rangle] \\
&\quad + \frac{\beta\theta_{s-1}}{2m}\mathbb{E}\left[\|Az_0^s - Ax^*\|^2 - \|Az_m^s - Ax^*\|^2 + \sum_{k=1}^{m}\|\lambda_k^s - \lambda_{k-1}^s\|^2\right] \\
&\quad + \frac{\theta_{s-1}^2}{2m\eta}\mathbb{E}[\|y^* - y_0^s\|_{Q_s}^2 - \|y^* - y_m^s\|_{Q_s}^2].
\end{aligned}$$

[2] Note that $\nabla f(x)$ is the gradient of a smooth function $f(\cdot)$ at $x$, while $h'(y)$ denotes a subgradient (or the gradient) of a non-smooth (or smooth) function $h(\cdot)$ at $y$.

Since a new linearized proximal rule is proposed to update the variable $y$ for Algorithms 1 and 2 in the case of $B\neq\tau I$, we need to give the following proof for Lemma 3, which is different from Lemma 6 in the Supplementary Material for the case of $B=\tau I$.

*Proof:* Since $\lambda^s_k=\lambda^s_{k-1}+Az^s_k+By^s_k-c$, and using the optimality condition of Problem (10) (i.e., $h'(y^s_k)+\beta B^T(Az^s_{k-1}+By_k-c+\lambda_{k-1})+\frac{\theta_{s-1}}{\eta}Q_s(y^s_k-y^s_{k-1})=0$), we have

$$\begin{aligned}
&h(y^s_k)-h(y^*)\\
\leq&\langle h'(y^s_k),\ y^s_k-y^*\rangle\\
=&\left\langle\beta B^T(Az^s_{k-1}+By_k-c+\lambda_{k-1})+\frac{\theta_{s-1}Q_s(y^s_k-y^s_{k-1})}{\eta},y^*-y^s_k\right\rangle\\
=&\left\langle\beta B^T\lambda^s_k+\frac{\theta_{s-1}Q_s}{\eta}(y^s_k-y^s_{k-1}),\ y^*-y^s_k\right\rangle\\
&+\left\langle\beta B^T(Az^s_{k-1}-Az^s_k),\ y^*-y^s_k\right\rangle\\
=&\frac{\beta}{2}\left\langle B^T\lambda^s_k,\ y^*-y^s_k\right\rangle+\frac{\theta_{s-1}}{2\eta}(\|y^*-y^s_{k-1}\|^2_{Q_s}-\|y^*-y^s_k\|^2_{Q_s})\\
&+\frac{\beta}{2}(\|Az^s_{k-1}-Ax^*\|^2-\|Az^s_k-Ax^*\|^2+\|\lambda^s_k-\lambda^s_{k-1}\|^2)
\end{aligned}$$

where the last equality follows from $Ax^*+By^*=c$ and Property 1 in the Supplementary Material. Taking expectation over the random choice of $i_k$, we have

$$\begin{aligned}
&\mathbb{E}\Big[h(y^s_k)-h(y^*)-\langle h'(y^*),y^s_k-y^*\rangle-\langle B^T\varphi^s_k,\ y^*-y^s_k\rangle\Big]\\
\leq&\frac{\beta}{2}\mathbb{E}[\|Az^s_{k-1}-Ax^*\|^2-\|Az^s_k-Ax^*\|^2]\\
&+\frac{1}{2}\mathbb{E}\left[\beta\|\lambda^s_k-\lambda^s_{k-1}\|^2+\frac{\theta_{s-1}}{\eta}(\|y^*-y^s_{k-1}\|^2_{Q_s}-\|y^*-y^s_k\|^2_{Q_s})\right].
\end{aligned}$$

Using the update rule of $\widetilde{y}^s=(1-\theta_{s-1})\widetilde{y}^{s-1}+\frac{\theta_{s-1}}{m}\sum_{k=1}^m y^s_k$, $h(\widetilde{y}^s)\leq(1-\theta_{s-1})h(\widetilde{y}^{s-1})+\frac{\theta_{s-1}}{m}\sum_{k=1}^m h(y^s_k)$, and taking expectation over whole history and summing up the above inequality for all $k=1,\ldots,m$, we have

$$\begin{aligned}
&\mathbb{E}\left[h(\widetilde{y}^s)-h(y^*)-\langle h'(y^*),\widetilde{y}^s-y^*\rangle-\frac{\theta_{s-1}}{m}\sum_{k=1}^m\langle B^T\varphi^s_k,y^*-y^s_k\rangle\right]\\
\leq&\frac{\theta_{s-1}}{m}\mathbb{E}\left[\sum_{k=1}^m\Big(h(y^s_k)-h(y^*)+\langle h'(y^*)-\theta_{s-1}B^T\varphi^s_k,y^*-y^s_k\rangle\Big)\right]\\
&+\mathbb{E}\left[\frac{\theta^2_{s-1}}{2\eta}(\|y^*-y^s_{k-1}\|^2_{Q_s}-\|y^*-y^s_k\|^2_{Q_s})\right]\\
&+(1-\theta_{s-1})\mathbb{E}[h(\widetilde{y}^{s-1})-h(y^*)-\langle h'(y^*),\widetilde{y}^{s-1}-y^*\rangle]\\
\leq&\frac{\beta\theta_{s-1}}{2m}\mathbb{E}\left[\|Az^s_0-Ax^*\|^2-\|Az^s_m-Ax^*\|^2+\sum_{k=1}^m\|\lambda^s_k-\lambda^s_{k-1}\|^2\right]\\
&+(1-\theta_{s-1})\mathbb{E}[h(\widetilde{y}^{s-1})-h(y^*)-\langle h'(y^*),\widetilde{y}^{s-1}-y^*\rangle]\\
&+\frac{\theta^2_{s-1}}{2\eta}[\|y^*-y^s_{k-1}\|^2_{Q_s}-\|y^*-y^s_k\|^2_{Q_s}].
\end{aligned}$$

This completes the proof. □

For the case of $B\neq\tau I$, we also have the following one-epoch inequality, which is a key lemma to prove Theorems 2, 4 and 5 below and is corresponding to Lemma 7 in the Supplementary Material for the case of $B=\tau I$, and Lemma 7 is also a main result to prove Theorems 1, 3 and 6 below.

**Lemma 4.** *(One-epoch Upper Bound) Using the same notation as in Lemma 2, let $\{(z^s_k,x^s_k,y^s_k,\lambda^s_k,\widetilde{x}^s,\widetilde{y}^s)\}$ be the sequence generated by Algorithm 1 (or Algorithm 2) with $\theta_s\leq 1-\frac{\delta(b)}{\alpha-1}$. Then the following inequality holds for all $k$,*

$$\begin{aligned}
&\mathbb{E}\Big[P(\widetilde{x}^s,\widetilde{y}^s)-\frac{\theta_{s-1}}{m}\sum_{k=1}^m\Big((x^*-z^s_k)^TA^T\varphi^s_k+(y^*-y^s_k)^TB^T\varphi^s_k\Big)\Big]\\
\leq&\mathbb{E}\Big[\frac{P(\widetilde{x}^{s-1},\widetilde{y}^{s-1})}{1/(1-\theta_{s-1})}+\frac{\theta^2_{s-1}}{2m\eta}\Big(\|x^*-z^s_0\|^2_{G_s}-\|x^*-z^s_m\|^2_{G_s}\Big)\Big]\\
&+\frac{\beta\theta_{s-1}}{2m}\mathbb{E}\Big[\|Az^s_0-Ax^*\|^2-\|Az^s_m-Ax^*\|^2+\sum_{k=1}^m\|\lambda^s_k-\lambda^s_{k-1}\|^2\Big]\\
&+\frac{\theta^2_{s-1}}{2m\eta}\mathbb{E}[\mathcal{R}^s-\|y^*-y^s_m\|^2_{Q_s}]
\end{aligned}$$

*where $\mathcal{R}^s$ is defined as follows:*

$$\mathcal{R}^s=\begin{cases}\sigma\|Ax^*-Az^s_0\|^2, & \textit{if } f(x) \textit{ is SC},\\ \|y^*-y^s_0\|^2_{Q_s}, & \textit{if } f(x) \textit{ is non-SC}\end{cases}\tag{16}$$

*and $\sigma=\|B^\dagger\|^2_2(\frac{2\eta\beta\|B^TB\|_2}{\theta_{s-1}}+1)$.*

*Proof:* Using Lemmas 2 and 3 and the definition of $P(x,y)$, we have

$$\begin{aligned}
&\mathbb{E}\left[P(\widetilde{x}^s,\widetilde{y}^s)-\frac{\theta_{s-1}}{m}\sum_{k=1}^m\Big((x^*-z^s_k)^TA^T\varphi^s_k+(y^*-y^s_k)^TB^T\varphi^s_k\Big)\right]\\
\leq&\mathbb{E}\left[\frac{P(\widetilde{x}^{s-1},\widetilde{y}^{s-1})}{1/(1-\theta_{s-1})}+\frac{\theta^2_{s-1}(\|x^*-z^s_0\|^2_{G_s}-\|x^*-z^s_m\|^2_{G_s})}{2m\eta}\right]\\
&+\frac{\beta\theta_{s-1}}{2m}\mathbb{E}\left[\|Az^s_0-Ax^*\|^2-\|Az^s_m-Ax^*\|^2+\sum_{k=1}^m\|\lambda^s_k-\lambda^s_{k-1}\|^2\right]\\
&+\frac{\theta^2_{s-1}}{2m\eta}\mathbb{E}[\|y^*-y^s_0\|^2_{Q_s}-\|y^*-y^s_m\|^2_{Q_s}].
\end{aligned}$$

When $f(\cdot)$ is $\mu$-strongly convex and $Ax^*+By^*=c$, we have $y^*=B^\dagger(c-Ax^*)$. Using the update rule of $y^s_0=B^\dagger(c-Az^s_0)$ and $\nu=1+\frac{\eta\beta\|B^TB\|_2}{\theta_{s-1}}$, we have

$$\begin{aligned}
\|y^*-y^s_0\|^2_{Q_s}&=\|B^\dagger(Az^s_0-Ax^*)\|^2_{Q_s}\\
&\leq\|B^\dagger\|^2_2\|Az^s_0-Ax^*\|^2_{Q_s}\\
&\leq\|B^\dagger\|^2_2\left\|\nu I-\frac{\eta}{\theta_{s-1}}B^TB\right\|_2\|Az^s_0-Ax^*\|^2\\
&\leq\|B^\dagger\|^2_2\left(\frac{2\eta\beta\|B^TB\|_2}{\theta_{s-1}}+1\right)\|Az^s_0-Ax^*\|^2.
\end{aligned}$$

Therefore, the result of Lemma 4 holds. □

## 4.2 Linear Convergence

For Algorithm 1, we first give the following results for the two cases of $B=\tau I$ and $B\neq\tau I$, respectively.

**Theorem 1.** *(Case of $B=\tau I$) Using the same notation as in Lemma 2 with $\theta\leq 1-\frac{\delta(b)}{\alpha-1}$, suppose that $f(\cdot)$ is $\mu$-strongly convex, each $f_i(\cdot)$ is $L$-smooth and Assumption 3 holds, and $m$ is sufficiently large so that*

$$\rho_1=\underbrace{\frac{\theta\|\theta G+\eta\beta A^TA\|_2}{\eta m\mu}}_{1}+\underbrace{(1-\theta)}_{2}+\underbrace{\frac{L\theta}{\beta m\sigma_{\min}(AA^T)}}_{3}<1\tag{17}$$

*where $\sigma_{\min}(AA^T)$ is the smallest eigenvalue of the positive semi-definite matrix $AA^T$, and $G_s\equiv G$ as in Eq. (12). Then*

$$\mathbb{E}\Big[P(\widetilde{x}^T,\widetilde{y}^T)\Big]\leq\rho_1^T P(\widetilde{x}^0,\widetilde{y}^0).$$

The theoretical result in our previous work [49] can be viewed as the special case of Theorem 1 when $B=I$. From Theorem 1, we can see that ASVRG-ADMM achieves linear convergence, which is consistent with that of SVRG-ADMM, while SCAS-ADMM has only an $\mathcal{O}(1/T)$ convergence rate.

**Remark 1.** *Theorem 1 shows that our result improves slightly upon the rate $\rho_1$ in SVRG-ADMM [22] with the same $\eta$ and $\beta$. Specifically, $\rho_1$ in Eq. (17) consists of three components, which are corresponding to those of Theorem 1 in [22]. In Algorithm 1, recall that $\theta \leq 1$ and $G$ is defined in Eq. (12). Thus, the upper bound of Eq. (17) is slightly smaller than that of Theorem 1 in [22]. In particular, we can set $\eta = 1/8L$ (i.e., $\alpha = 8$) and $\theta = 1-\delta(b)/(\alpha-1) = 1-\delta(b)/7$. Therefore, the second term in Eq. (17) equals to $\delta(b)/7$, while that of SVRG-ADMM is approximately equal to $4L\eta\delta(b)/(1-4L\eta\delta(b)) \geq \delta(b)/2$. In summary, the convergence speed of SVRG-ADMM can be slightly improved by ASVRG-ADMM.*

**Theorem 2.** *(Case of $B \neq \tau I$) Using the same notation as in Lemma 2 with $\theta \leq 1-\frac{\delta(b)}{\alpha-1}$, suppose that $f(\cdot)$ is $\mu$-strongly convex, each $f_i(\cdot)$ is $L$-smooth and Assumption 3 holds, and $m$ is sufficiently large so that*

$$\rho_2 = \frac{\theta\varrho}{\eta m\mu} + (1-\theta) + \frac{L\theta}{\beta m \sigma_{\min}(AA^T)} < 1 \tag{18}$$

*where $\varrho = \|\theta G+\eta(\beta+\theta\sigma)A^T A)\|_2$. Then*

$$\mathbb{E}\Big[P(\widetilde{x}^T,\widetilde{y}^T)\Big] \leq \rho_2^T P(\widetilde{x}^0,\widetilde{y}^0).$$

From Theorem 2, ASVRG-ADMM can also achieve linear convergence for the more complex ADMM-style problem (1) with $B \neq \tau I$. It is not hard to see that the convergence rate $\rho_2$ in Theorem 2 is slightly larger than that (i.e., $\rho_1$) of Theorem 1, meaning slow convergence for more complex optimization problems, as verified by our experiments.

### 4.3 Convergence Rate of $\mathcal{O}(1/T^2)$

We first assume that $y \in \mathcal{Y}$ and $z \in \mathcal{Z}$, where $\mathcal{Y}$ and $\mathcal{Z}$ are the convex compact sets with diameters $D_{\mathcal{Y}} = \sup_{y_1,y_2\in\mathcal{Y}}\|y_1-y_2\|$ and $D_{\mathcal{Z}} = \sup_{z_1,z_2\in\mathcal{Z}}\|z_1-z_2\|$, respectively, and $D_{\Lambda} = \sup_{\lambda_1,\lambda_2\in\Lambda}\|\lambda_1-\lambda_2\|$. The above assumption is called the boundedness assumption. We also denote $D_{x^*} = \|\widetilde{x}^0-x^*\|$, $D_{y^*} = \|\widetilde{y}^0-y^*\|$ and $D_{\lambda^*} = \|\widetilde{\lambda}^0-\lambda^*\|$, where $(\widetilde{x}^0,\widetilde{y}^0,\widetilde{\lambda}^0)$ are initial points, $(x^*,y^*)$ is an optimal solution of Problem (1) and $\lambda^*$ is the corresponding dual variable. The boundedness of $D_{x^*}$, $D_{y^*}$ and $D_{\lambda^*}$ are easily satisfied, which is called the basic conditions in this paper.

For Algorithm 2, we give the following results for the cases of $B=\tau I$ and $B\neq\tau I$, respectively, whose proofs are provided in the Supplementary Material.

**Theorem 3.** *(Case of $B = \tau I$) Let $\varsigma$ be a positive constant, suppose that each $f_i(\cdot)$ is $L$-smooth, $\mathcal{Z}$ and $\Lambda$ are the convex compact sets with diameters $D_{\mathcal{Z}}$ and $D_{\Lambda}$, then*

$$\begin{aligned}&\mathbb{E}\Big[P(\widetilde{x}^T,\widetilde{y}^T)+\varsigma\|A\widetilde{x}^T+\tau\widetilde{y}^T-c\|\Big]\\ \leq\; &\frac{4(\alpha-1)\delta(b)\left(P(\widetilde{x}^0,\widetilde{y}^0)+\varsigma\|A\widetilde{x}^0+\tau\widetilde{y}^0-c\|\right)}{(\alpha-1-\delta(b))^2(T+1)^2}\\ &+\frac{2L\alpha D_{x^*}^2}{m(T+1)^2}+\frac{4\alpha\beta(\|A^TA\|_2D_{\mathcal{Z}}^2+D_{\Lambda}^2)}{m(\alpha-1)(T+1)}.\end{aligned} \tag{19}$$

**Remark 2.** *With $m=\Theta(n)$, Theorem 3 shows that the convergence bound consists of the three components, which converge as $\mathcal{O}(1/T^2)$, $\mathcal{O}(1/nT^2)$ and $\mathcal{O}(1/nT)$, respectively, while the three components of SVRG-ADMM converge as $\mathcal{O}(1/T)$, $\mathcal{O}(1/nT)$ and $\mathcal{O}(1/nT)$. Clearly, ASVRG-ADMM achieves the convergence rate of $\mathcal{O}(1/T^2)$ as opposed to $\mathcal{O}(1/T)$ of SVRG-ADMM and SAG-ADMM ($m \gg T$ in general). All the components in the bound of SCAS-ADMM converge as $\mathcal{O}(1/T)$. Thus, it is clear that ASVRG-ADMM is at least a factor $T$ faster than existing stochastic ADMM algorithms including SAG-ADMM, SVRG-ADMM and SCAS-ADMM. Theorem 3 shows that the convergence result in our previous work [49] can be viewed as the special case of Theorem 3. In addition, Theorems 3 and 4 below require the boundedness assumption and the basic conditions (i.e., $D_{x^*}$, $D_{y^*}$ and $D_{\lambda^*}$ are bounded by some constants).*

**Theorem 4.** *(Case of $B \neq \tau I$) Using the same notation as in Lemma 2, and suppose that each $f_i(\cdot)$ is $L$-smooth, and $\mathcal{Y}$, $\mathcal{Z}$ and $\Lambda$ are the convex compact sets with diameters $D_{\mathcal{Y}}$, $D_{\mathcal{Z}}$ and $D_{\Lambda}$, then we have*

$$\begin{aligned}&\mathbb{E}\Big[P(\widetilde{x}^T,\widetilde{y}^T)+\varsigma\|A\widetilde{x}^T+B\widetilde{y}^T-c\|\Big]\\ \leq\; &\frac{4(\alpha-1)\delta(b)\left(P(\widetilde{x}^0,\widetilde{y}^0)+\varsigma\|A\widetilde{x}^0+B\widetilde{y}^0-c\|\right)}{(\alpha-1-\delta(b))^2(T+1)^2}\\ &+\frac{2\alpha\beta\left(2\|A^TA\|_2D_{\mathcal{Z}}^2+\|B^TB\|_2D_{\mathcal{Y}}^2+2D_{\Lambda}^2\right)}{m(\alpha-1)(T+1)}\\ &+\frac{2L\alpha(D_{x^*}^2+D_{y^*}^2)}{m(T+1)^2}.\end{aligned} \tag{20}$$

### 4.4 $\mathcal{O}(1/T^2)$ without Boundedness Assumption

The result in Theorem 4 shows that ASVRG-ADMM attains the optimal convergence rate $O(1/T^2)$ for the non-SC problem (1) with $B \neq \tau I$. Compared with SVRG-ADMM and SAG-ADMM, ASVRG-ADMM attains a better convergence rate for non-SC problems, but with the price on the boundedness of the feasible primal sets $\mathcal{Z}$, $\mathcal{Y}$, and the feasible dual set $\Lambda$. Note that many previous works such as [60, 61] also require such assumptions of boundedness when proving the convergence of ADMMs. In order to remove the strong assumption and further improve our theoretical results, we design an adaptive strategy of increasing epoch length, i.e., $m_{s+1} = \lceil\frac{\theta_{s-1}}{\theta_s}m_s\rceil$, while a constant epoch length $m$ is used in original Algorithm 2. The increasing epoch length strategy is similar to that in [62], that is, $m_{s+1} = \lceil\frac{\theta_{s-1}}{\theta_s}m_s\rceil$ instead of $m_{s+1}=2m_s$ in [62]. By replacing the epoch length $m$ in Algorithm 2 with $m_s$, we can obtain the following improved theoretical result. It should be noted that the increasing factor $\frac{\theta_{s-1}}{\theta_s}$ approaches 1 as the number of epochs increases, which means that the epoch length increases very slowly. Below we only present the convergence result for the general case of $B\neq\tau I$, the theoretical result for the case of $B = \tau I$ and the detailed proofs for all the results are provided in the Supplementary Material.

**Theorem 5.** *(Without boundedness assumption) Using the same notation as in Lemma 2, suppose that each $f_i(\cdot)$ is $L$-smooth. Let $\{(\widetilde{x}^s,\widetilde{y}^s,\widetilde{\lambda}^s)\}$ be the sequence generated by Algorithm 2 with our* *adaptive increasing epoch length strategy for the case of $B \neq \tau I$, then*

$$
\begin{aligned}
&\mathbb{E}\Big[P(\widetilde{x}^T, \widetilde{y}^T) + \varsigma\|A\widetilde{x}^T + B\widetilde{y}^T - c\|\Big] \\
\le\; &\frac{4(\alpha-1)\delta(b)\left(P(\widetilde{x}^0, \widetilde{y}^0) + \varsigma\|A\widetilde{x}^0 + B\widetilde{y}^0 - c\|\right)}{(\alpha-1-\delta(b))^2(T+1)^2} \\
+\, &\frac{2(\alpha-1)\beta\left(2\|A^TA\|_2 D_{x^*}^2 + \|B^TB\|_2 D_{y^*}^2 + 2D_{\lambda^*}^2\right)}{(\alpha-1-\delta(b))m_1(T+1)^2} \\
+\, &\frac{2L\alpha(D_{x^*}^2 + D_{y^*}^2)}{m_1(T+1)^2}.
\end{aligned} \tag{21}
$$

**Remark 3.** *With the setting $m_1 = \Theta(n)$, Theorem 5 shows that ASVRG-ADMM with our adaptive epoch length strategy obtains the rate of $\mathcal{O}(1/T^2)$. The upper bound only relies on the constants $D_{x^*}$, $D_{y^*}$ and $D_{\lambda^*}$, while the theoretical result in Theorem 4 requires that $\mathcal{Y}$, $\mathcal{Z}$ and $\Lambda$ are all bounded with the diameters $D_{\mathcal{Y}}$, $D_{\mathcal{Z}}$ and $D_{\Lambda}$. That is, ASVRG-ADMM with our adaptive epoch length strategy achieves the convergence rate $\mathcal{O}(1/T^2)$ without the boundedness assumption.*

### 4.5 Discussion

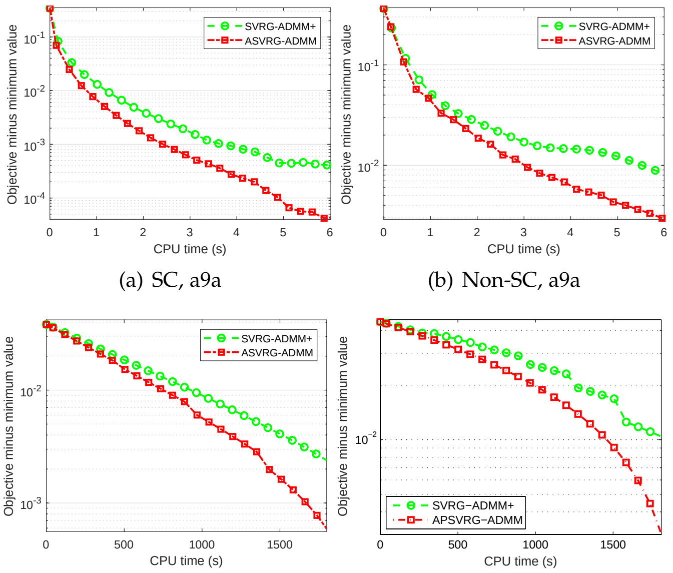
All our algorithms and convergence results can be extended to the following settings: When the mini-batch size $b = n$ and $m = 1$, then $\delta(n) = 0$, that is, the first term of both (19) and (20) vanishes, and ASVRG-ADMM degenerates to the deterministic two-block[3] ADMM version [63]. The convergence rate of (20) becomes $\mathcal{O}\Big(\frac{D_{x^*}^2+D_{y^*}^2}{(T+1)^2} + \frac{D_{\mathcal{Z}}^2+D_{\mathcal{Y}}^2+D_{\Lambda}^2}{T+1}\Big)$, which is consistent with the result for accelerated deterministic ADMM [34, 37]. Many empirical risk minimization problems can be viewed as the special case of Problem (2) when $A = I$. Thus, our method can be extended to solve them, and has an $\mathcal{O}(1/T^2 + 1/(nT^2))$ rate, which is consistent with the best-known result as in [29, 30].

## 5 Experimental Results

In this section, we apply ASVRG-ADMM to solve various machine learning problems, e.g., non-SC graph-guided fused Lasso, SC and non-SC graph-guided logistic regression, and SC graph-guided SVM problems. We compare ASVRG-ADMM with the state-of-the-art methods: STOC-ADMM [9], OPG-ADMM [45], SAG-ADMM [19], SCAS-ADMM [21] and SVRG-ADMM [22]. All methods were implemented in MATLAB, and the experiments were performed on a PC with an Intel i5-2400 CPU and 16GB RAM.

### 5.1 Synthetic Data

We first evaluate the empirical performance of the proposed algorithms for solving both SC and non-SC problems (1) on some synthetic data. Here, each $f_i(x)$ is the logistic loss function on the feature-label pair $(a_i, b_i)$, i.e., $f_i(x) = \log(1+\exp(-b_i a_i^T x))$ $(i = 1, 2, \ldots, n)$ for the non-SC case and $f_i(x) = \log(1+\exp(-b_i a_i^T x)) + \frac{\lambda_2}{2}\|x\|_2^2$ for the SC case, where $\lambda_2 \ge 0$ is the regularization parameter. We used a relatively small data set, a9a (about 733K), and a relatively large data set, epsilon (about 11G), as listed in Table 2. Note that the constraint matrix $A$ is set to $A = [G; I]$ as in [9, 19, 22, 61], where $G$ is the sparsity pattern of the graph obtained by sparse inverse covariance selection [64], while both $B$ and $c$ are randomly generated. In particular, the generated matrix $B$ has full column rank, but is not an identity matrix. Since the original SVRG-ADMM [22] cannot be used to solve the general problem (1) with $B \neq \tau I$, we also present its linearized proximal variant (called SVRG-ADMM+), as shown in the Supplementary Material.



(a) SC, a9a

(b) Non-SC, a9a

(c) SC, epsilon

(d) Non-SC, epsilon

Fig. 1. Comparison of the linearized proximal SVRG-ADMM and our ASVRG-ADMM algorithms for both SC and non-SC problems on the two data sets: a9a (top) and epsilon (bottom).

Fig. 1 shows the training loss (i.e., the training objective value minus the minimum) of SVRG-ADMM+ and ASVRG-ADMM for both SC and non-SC problems on a9a and epsilon. All the experimental results show that our ASVRG-ADMM method (i.e., Algorithms 1 and 2) converges consistently much faster than SVRG-ADMM+, which empirically verifies our theoretical results of ASVRG-ADMM.

### 5.2 Real-world Applications

We also apply our ASVRG-ADMM method to solve a number of real-world machine learning problems such as graph-guided fused Lasso, graph-guided logistic regression, graph-guided SVM, generalized graph-guided logistic regression and multi-task learning.

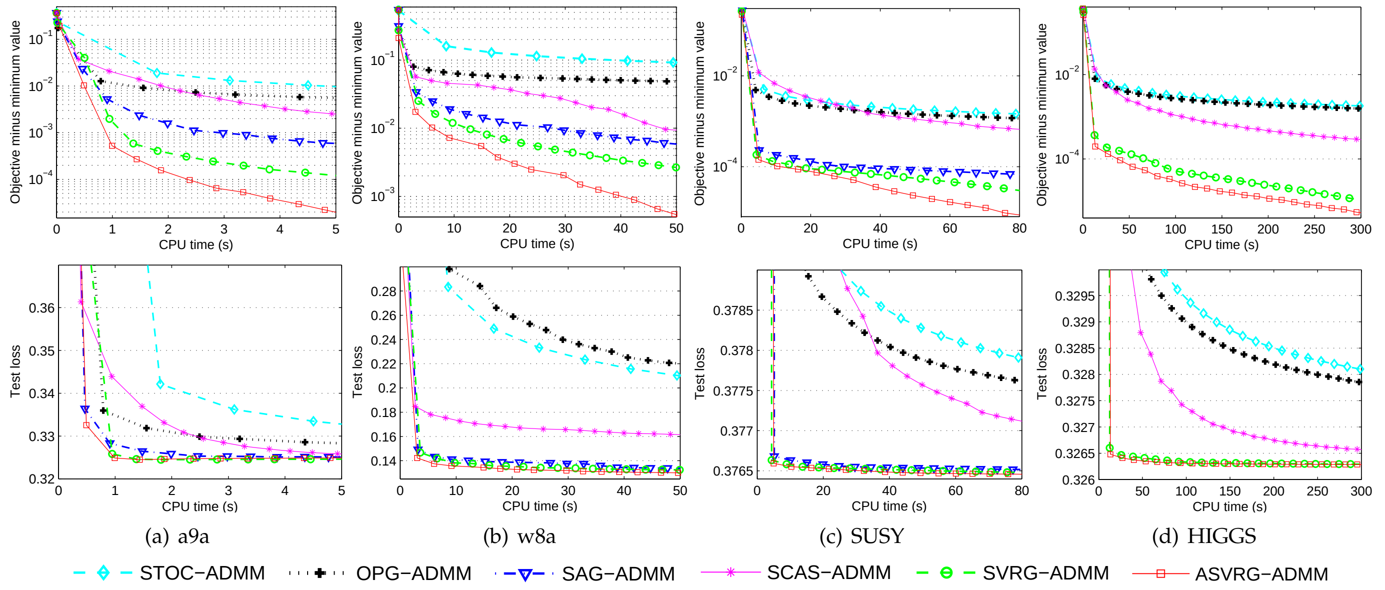
#### *5.2.1 Graph-Guided Fused Lasso*

We evaluate the empirical performance of ASVRG-ADMM for solving the non-SC graph-guided fused Lasso problem:

$$
\min_{x,y} \left\{ \frac{1}{n}\sum_{i=1}^{n} f_i(x) + \lambda_1\|y\|_1, \text{ s.t., } Ax = y \right\} \tag{22}
$$

where $f_i(\cdot)$ is the logistic loss function on the feature-label pair $(a_i, b_i)$, i.e., $f_i(x) = \log(1+\exp(-b_i a_i^T x))$, and $\lambda_1 \ge 0$ is the regularization parameter. As in [9, 22, 61], we set $A = [G; I]$, where $G$ is the sparsity pattern of the graph obtained by sparse inverse covariance selection [64]. We used four

[3] Note that the formulation (1) is called two-block because of the two sets of variables $(x, y)$, which are updated alternately.



(a) a9a
(b) w8a
(c) SUSY
(d) HIGGS

Fig. 2. Comparison of different stochastic ADMM methods for non-SC graph-guided fused Lasso problems on the four data sets. The $y$-axis represents the objective value minus the minimum value (top) or test loss (bottom), and the $x$-axis corresponds to the running time (seconds).

publicly available data sets[4] in our experiments, as listed in Table 2. The parameter $m$ of ASVRG-ADMM is set to $m = \lceil 2n/b \rceil$ as in [19, 22], as well as $\eta$ and $\beta$. All the other algorithms except STOC-ADMM adopted the linearization of the penalty term $\frac{\beta}{2}\|Ax - y + z\|^2$ to avoid the inversion of $\frac{1}{\eta_k} I_{d_1} + \beta A^T A$ at each iteration, which can be computationally expensive for large matrices.

TABLE 2
Summary of data sets and regularization parameters $\lambda_1$ and $\lambda_2$ used in our experiments.

| Data sets | ♯ training | ♯ test | ♯ mini-batch | $\lambda_1$ | $\lambda_2$ |
|---|---|---|---|---|---|
| a9a | 16,281 | 16,280 | 20 | 1e-5 | 1e-2 |
| epsilon | 400,000 | 100,000 | 30 | 1e-4 | 1e-4 |
| w8a | 32,350 | 32,350 | 20 | 1e-5 | 1e-2 |
| SUSY | 3,500,000 | 1,500,000 | 100 | 1e-5 | 1e-2 |
| HIGGS | 7,700,000 | 3,300,000 | 150 | 1e-5 | 1e-2 |

Fig. 2 shows the training loss (i.e., the training objective value minus the minimum) and test error of all the algorithms for non-SC problems on the four data sets. SAG-ADMM could not generate experimental results on the HIGGS data set because it ran out of memory. These figures clearly indicate that the variance reduced stochastic ADMM algorithms (i.e., SAG-ADMM, SCAS-ADMM, SVRG-ADMM and ASVRG-ADMM) converge much faster than those without variance reduction techniques, e.g., STOC-ADMM and OPG-ADMM. In particular, ASVRG-ADMM consistently outperforms the other algorithms in terms of convergence speed in all the settings, which empirically verifies our theoretical result that ASVRG-ADMM has a faster convergence rate of $\mathcal{O}(1/T^2)$, as opposed to the best-known rate of $\mathcal{O}(1/T)$. Moreover, the test error of ASVRG-ADMM is consistently better than those of the other methods.

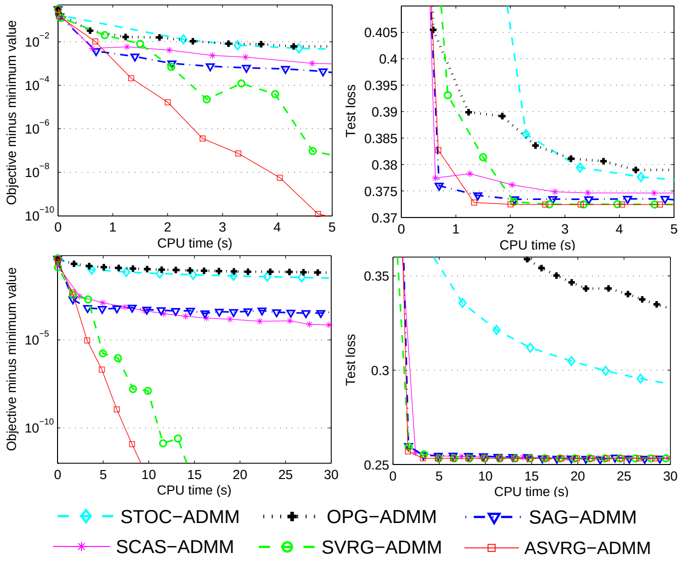


Fig. 3. Comparison of the stochastic ADMM methods for SC graph-guided logistic regression problems on a9a (top) and w8a (bottom).

### 5.2.2 Graph-Guided Logistic Regression

We also discuss the performance of ASVRG-ADMM for the SC graph-guided logistic regression problem:

$$\min_{x,y} \left\{ \frac{1}{n}\sum_{i=1}^{n}\left( f_i(x) + \frac{\lambda_2}{2}\|x\|^2 \right) + \lambda_1 \|y\|_1, \ \text{s.t.}, \ Ax = y \right\}. \quad (23)$$

Due to limited space and similar experimental phenomena on the four data sets, we only report the experimental results on the a9a and w8a data sets in Fig. 3, from which we can see that SVRG-ADMM and ASVRG-ADMM achieve comparable performance, and they significantly outperform the other methods in terms of convergence speed, which is consistent with their linear (geometric) convergence guarantees. Moreover, ASVRG-ADMM converges slightly faster than SVRG-ADMM, which shows the effectiveness of the

[4] http://www.csie.ntu.edu.tw/~cjlin/libsvmtools/datasets/

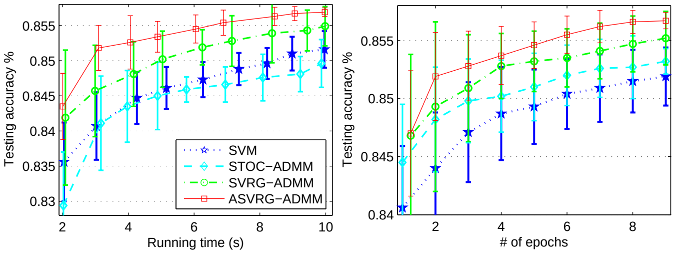


Fig. 4. Accuracy comparison of multi-class classification on 20newsgroups: accuracy vs running time (left) or number of epochs (right).

proposed momentum trick to accelerate variance reduced stochastic ADMM, as we expected.

### 5.2.3 Graph-Guided SVM

We also evaluate the performance of ASVRG-ADMM for solving the SC graph-guided SVM problem,

$$\min_{x,y}\left\{\frac{1}{n}\sum_{i=1}^{n}\left([1-b_i a_i^T x]_+ + \frac{\lambda_2}{2}\|x\|_2^2\right)+\lambda_1\|y\|_1\right\}, \qquad (24)$$
$$\text{s.t., } Ax=y$$

where $[x]_+ = \max(0, x)$ is the non-smooth hinge loss. To effectively solve (24), we use the smooth Huberized hinge loss in [65] to approximate the hinge loss. For the 20newsgroups data set[5], we randomly divide it into 80% training set and 20% test set. Following [9], we set $\lambda_1 = \lambda_2 = 10^{-5}$, and use the one-vs-rest scheme for the multi-class classification.

Fig. 4 shows the average prediction accuracies and standard deviations of testing accuracies over 10 different runs. Since STOC-ADMM, OPG-ADMM, SAG-ADMM and SCAS-ADMM consistently perform worse than SVRG-ADMM and ASVRG-ADMM in all settings, we only report the results of STOC-ADMM. We can see that SVRG-ADMM and ASVRG-ADMM consistently outperform the classical SVM and STOC-ADMM. Moreover, ASVRG-ADMM performs much better than the other methods in all settings, which further verifies the effectiveness of ASVRG-ADMM.

### 5.2.4 Generalized Graph-Guided Logistic Regression

Moreover, we apply ASVRG-ADMM to solve the non-SC graph-guided logistic regression problem as in [66]:

$$\min_{x,y}\left\{\frac{1}{n}\sum_{i=1}^{n} f_i(x)+\lambda_1\|x\|_1+\lambda_2\|y\|_1, \text{ s.t., } Ax=y\right\}. \qquad (25)$$

All the problems in (22), (23) and (24) can be cast as the form (2), while Problem (25) can be cast as the form (1), i.e., $\min_{x,v}\{f(x)+\|v\|_1, \text{s.t.}\, Cx+Bv=0\}$, where $v=[\lambda_1 z^T, \lambda_2 y^T]^T$ and $z$ are slack variables, $C=[I_{d_x}, A^T]^T, B=-\begin{bmatrix}\frac{1}{\lambda_1}I_{d_x} & 0\\ 0 & \frac{1}{\lambda_2}I_{d_y}\end{bmatrix}$.

The experimental results on the a9a data set are shown in Fig. 5, from which we can see that SVRG-ADMM+ and ASVRG-ADMM converge significantly faster than STOC-ADMM+. Note that SVRG-ADMM+ and STOC-ADMM+ are the linearized proximal variants of SVRG-ADMM and STOC-ADMM. Moreover, ASVRG-ADMM outperforms them in terms of both convergence speed and test error, which shows the effectiveness of our momentum trick to accelerate variance reduced stochastic ADMM.

[5] http://www.cs.nyu.edu/~roweis/data.html

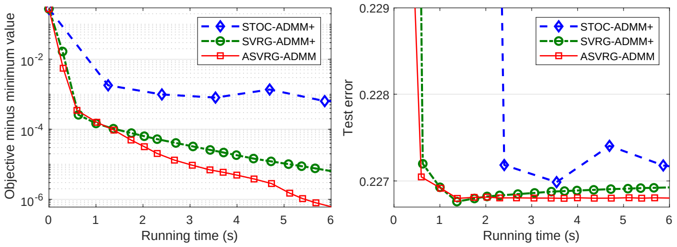


Fig. 5. Comparison of all the methods for generalized graph-guided fused Lasso on a9a, where regularization parameters $\lambda_1 = \lambda_2 = 10^{-5}$.

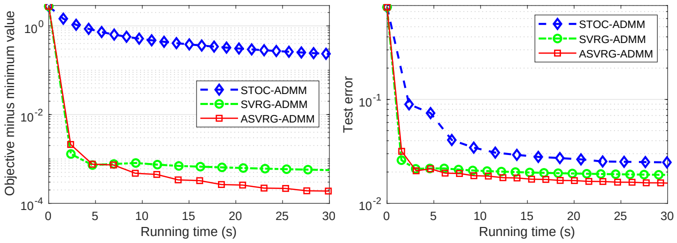


Fig. 6. Comparison of all the methods for multi-task learning problems on 20newsgroups, where the regularization parameter $\lambda_1 = 10^{-4}$.

### 5.2.5 Multi-Task Learning

Finally, we consider the multi-task learning problem and can cast it as the non-SC constrained problem: $\min_{X,Y}\{\sum_{i=1}^{N} f_i(X)+\lambda_1\|Y\|_*, \text{s.t.}, X=Y\}$, where $X, Y \in \mathbb{R}^{d\times N}$, $N$ is the number of tasks, $f_i(X)$ is the multinomial logistic loss on the $i$-th task, and $\|Y\|_*$ is the nuclear norm. The experimental results in Fig. 6 show that ASVRG-ADMM outperforms the other methods including SVRG-ADMM in terms of both convergence speed and test error.

## 6 Conclusions and Further Work

In this paper, we proposed an efficient accelerated stochastic variance reduced ADMM (ASVRG-ADMM) method, in which we combined both our proposed momentum acceleration trick and the variance reduction stochastic ADMM [22]. We also designed two different update rules for the general ADMM (i.e., $B \neq \tau I$) and special ADMM (i.e., $B = \tau I$) problems, respectively. That is, we presented a new linearized proximal scheme for the case of $B \neq \tau I$, and adopted a simple proximal scheme in our previous work [49] for the case of $B = \tau I$. Moreover, we theoretically analyzed the convergence properties of the proposed linearized proximal accelerated SVRG-ADMM algorithms, which show that ASVRG-ADMM achieves linear convergence and $\mathcal{O}(1/T^2)$ rates for strongly convex and non-strongly convex cases, respectively. In particular, ASVRG-ADMM is at least a factor $T$ faster than existing stochastic ADMM methods for non-strongly convex problems.

Our empirical study showed that the convergence speed of ASVRG-ADMM is much faster than those of the state-of-the-art stochastic ADMM methods such as SVRG-ADMM. We can apply our proposed momentum acceleration trick to accelerate existing incremental gradient descent algorithms such as [67, 68] for solving regularized empirical risk minimization problems. An interesting direction of future work is the research of our proposed momentum acceleration trick for accelerating incremental gradient descent ADMM algorithms such as SAG-ADMM [19] and SAGA-ADMM [52]. In addition, it is also interesting to extend our algorithms and theoretical results from the two-block version to the multi-block ADMM case [69].

## Acknowledgments

We thank all the reviewers for their valuable comments. This work was supported by the National Natural Science Foundation of China (Nos. 61876221, 61876220, 61976164, 61836009, U1701267, 61871310, and 61801353), the Project supported the Foundation for Innovative Research Groups of the National Natural Science Foundation of China (No. 61621005), the Major Research Plan of the National Natural Science Foundation of China (Nos. 91438201 and 91438103), the Program for Cheung Kong Scholars and Innovative Research Team in University (No. IRT_15R53), the Fund for Foreign Scholars in University Research and Teaching Programs (the 111 Project) (No. B07048), and the Science Foundation of Xidian University (Nos. 10251180018 and 10251180019). Z. Lin is supported by NSF China (grant no.s 61625301 and 61731018), Major Scientific Research Project of Zhejiang Lab (grant no.s 2019KB0AC01 and 2019KB0AB02), Beijing Academy of Artificial Intelligence, and Qualcomm.

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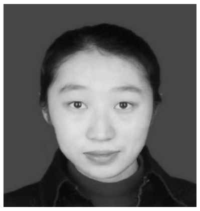

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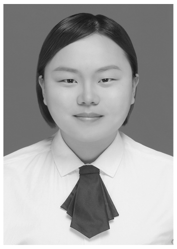

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