Stepsize anything: A unified learning rate schedule for budgeted-iteration training

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Abstract

The expanding computational costs and limited resources underscore the critical need for budgeted-iteration training, which aims to achieve optimal learning within predetermined iteration budgets. While learning rate schedules fundamentally govern the performance of different networks and tasks, particularly in budgeted-iteration scenarios, their design remains largely heuristic, lacking theoretical foundations. In addition, the optimal learning rate schedule requires extensive trial-and-error selection, making the training process inefficient. In this work, we propose the Unified Budget-Aware (UBA) schedule, a theoretically grounded learning rate schedule that consistently outperforms commonly-used schedules among diverse architectures and tasks under different constrained training budgets. First, we bridge the gap by constructing a novel training budget-aware optimization framework, which explicitly accounts for the robustness to landscape curvature variations. From this framework, we derive the UBA schedule, controlled by a single hyper-parameter φ that provides a trade-off between flexibility and simplicity, eliminating the need for per-network numerical optimization. Moreover, we establish a theoretical connection between φ and the condition number, adding interpretation and justification to our approach. Besides, we prove the convergence for different values of φ. We offer practical guidelines for its selection via theoretical analysis and empirical results. Extensive experimental results show that UBA *consistently surpasses* the commonly-used schedules across diverse vision and language tasks, spanning network architectures (e.g., ResNet, OLMo) and scales, under different training-iteration budgets.

1 Introduction

Deep learning has achieved remarkable success across various across a wide range of domains, including computer vision and natural language processing. However, despite continual advancements in hardware technologies [44, 52], the training cost of neural networks has increased dramatically due to the growing scale of models and datasets [4, 8, 46, 47]. As a result, resource constraints, including computational power, memory, energy consumption, and time budgets, are emerging as significant bottlenecks in the training process [40, 57]. These challenges highlight the pressing need for budgeted training, which aims to achieve optimal model performance under fixed hardware and limited time.

While existing budgeted training studies broadly address resource efficiency, a critical yet underexplored direction within budgeted training is achieving the best possible model performance under strictly fixed iteration constraints. This scenario is common and practically significant where practitioners work under limited computational or time budgets [29], and in extreme cases, models have to be completed within a few training iterations due to resource exhaustion. To formalize this specific

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research problem, we introduce the term 'budgeted-iteration training' , distinguishing it from the broader scope of budgeted training.

Budgeted-iteration training has received growing attention in research, given its significant real-world applicability. Several studies have developed relevant techniques that align with its goals. Smith et al. [42] propose the cyclical learning rate schedule (CLR), improving accuracy in fewer iterations without tuning [43]. Li et al. [29] introduce budget-aware adaptations for existing learning rate schedules. Chen et al. [5] propose a novel learning rate schedule called Reflected Exponential (REX). These approaches are primarily based on learning rate designing. Learning rate scheduling highlights key advantages: (i) It plays a critical role in general training of diverse neural architectures across tasks. (ii) It has demonstrated suitable and competitive in fixed training iteration budgets [19]. (iii) It is plug-and-play, requiring minimal adjustments to the underlying model architecture, which makes them easily adaptable to various deep learning frameworks. Leveraging these advantages, we adopt the learning rate design approach for budgeted-iteration training.

Despite their advantages, most learning rate schedules, whether tailored for budgeted-iteration training or standard training performance, are still heuristic and lack rigorous theoretical grounding. In addition, existing schedules typically rely on manually designed rules or empirical tuning. Consequently, selecting an optimal schedule often involves extensive trial-and-error, incurring substantial cost in terms of time and computation [33].

A natural question arises: Does there exist a theoretically grounded, unified schedule that eliminates heuristic selection while maintaining robust performance across tasks, networks, scales and training budgets?

In this paper, we provide an affirmative answer by proposing a theoretically grounded schedule. The proposed learning rate schedule should consistently outperforms existing schedules among diverse architectures and tasks under different constrained training budgets. By doing so, it avoids choosing suitable learning rate schedule after multiple trials for network training.

To achieve this, we first bridge the gap by constructing a unified budget-aware training optimization framework, which incorporates the robustness to landscape curvature variations induced by data distribution, sampling, network architectures and optimization. Then we obtain numerical solutions by gradient projection methods. To eliminate the need for repeated numerical optimization when applying our method to different networks, we propose a universal parametric function that approximates numerical solutions. We nominate the resulting schedule Unified Budget-Aware (UBA) schedule. It requires tuning only a single hyper-parameter φ, reducing the overhead of per-network numerical optimization. Moreover, we establish a theoretical connection between φ and the condition number, adding interpretation and optimization difficulty-aware theoretical grounding. Besides, we prove the convergence for different values of φ. These theoretical analysis along with empirical results offer practical guidelines for φ selection. We evaluate UBA through comprehensive experiments across vision and language tasks, spanning diverse architectures and iteration budgets. Specifically, for vision tasks, the UBA schedule demonstrates *consistent superiority* over baselines across all evaluated datasets and model scales under varying training iterations. For language tasks, we validate the effectiveness of UBA through extensive benchmarks with OLMo model(36M, 73M, 150M and 300M parameters). Results show that UBA achieves state-of-the-art performance across on approximately half of the benchmarks, and consistently outperforms baselines on the average scores.

Main contributions:

1. We construct a unified budget-aware training optimization framework that inherently adapts to landscape curvature variations, enhancing training robustness of leaning rates.

2. We propose the Unified Budget-Aware (UBA) schedule from our constructed optimization problem, controlled by a single hyper-parameter φ that provides a trade-off between flexibility and simplicity, i.e. adaptive curvature adjustment and minimal tuning cost.

3. We prove the convergence under φ and derive practical guidelines for its selection through analysis and experiments. Besides, theoretical analysis and empirical results show that φ is related to optimization difficulty such as condition number.

4. We perform experiments and demonstrate that UBA surpasses the commonly-used schedules across diverse vision and language tasks, spanning network architectures (e.g., ResNet, OLMo) and scales, under different training-iteration budgets.

2 Related work

Budgeted training Researchers face significant challenges in achieving optimal model performance under fixed hardware and limited time. To address these challenges, the concept of budgeted training has gained increasing attention, exploring techniques including computation efficiency, model compression, training stability and convergence improvement [40]. It focuses on: (i) emphasizing the allocation of resources, such as the balance between the model size and the amount of data [2, 6, 23, 27]. (ii) finding optimal configurations or improving performance within the given compute or time budget, such as memory efficiency and computation reduction [16, 25, 30, 37], optimization learning rate schedules [5, 29, 42, 43], batch size [16] and other weight averaging method [25, 26].

Within the context of budgeted training, a key area that remains under-explored is achieving the best possible model performance within a fixed number of iterations, i.e. 'budgeted-iteration training'. In this domain, learning rate scheduling based methods are particularly aligned with the objectives of budgeted-iteration training. Smith et al. [42] propose a new learning rate schedule, named cyclical learning rates (CLR). It improves accuracy in fewer iterations without tuning and is relevant to super-convergence phenomenon [43]. Li et al. [29] introduce an alternative setting of existing learning rate schedules for budgeted training. Chen et al. [5] propose the reflected exponential schedule (REX) via a profile and sampling fashion. Learning rate based approaches achieve robust and high-performing results under various lengths of training iterations, which corresponds to our purpose in this work, i.e. budgeted-iteration training [33]. Besides, this approach is plug-and-play, requiring no substantial alterations to the underlying model structure, making it readily adaptable to various deep network frameworks. Therefore, we explore the proper learning rate schedule to achieve budgeted-iteration training.

Learning rate schedule The learning rate plays a pivotal role in controlling the optimization process during network training. The common scheme is the step decay schedule. A typical instance decreases the learning rate by a decaying scalar 0.1 after 50% epochs and by a decaying scalar 0.01 after 75% epochs [20]. Then Loshchilov et al. [32] observe that sharp decreases may prevent models from escaping local minima and propose the cosine schedule function, which is the most popular schedule for language model pretraining. Although some schedules include the CLR [42], REX [5], Warmup-Stable-Decay (WSD) [24] and schedule from multi-power law [33], there is no consensus on the optimal choice. In addition, the detail can be found in Appendix F, including some works focus on adaptive learning rate methods.

Learning rate design is not only significant in general training, but also critical in budgeted-iteration training which still remains a topic of debate. Some analyses advocate for small, constant learning rates to ensure stability and convergence [11]. On the contrast, one prevailing hypothesis suggests that large learning rates may facilitate crossing over sharp local minima in the optimization landscape [55]. Despite the lack of comprehensive theoretical explanations, a range of learning rate schedules inspired by the above analyses as heuristic guidelines has been widely adopted in practice, using variable learning rates to budgeted-iteration training [5, 29]. In this work, we explore learning rate schedule from optimization problem tailored to budgeted-iteration training, aiming to balance iteration budget constraints and generalization.

3 Budgeted-iteration training

3.1 Finite optimization under limited training iterations

To design a one-size-fits-all learning rate schedule, we construct a robust optimization model of learning rates across varying training conditions (see more conceptual illustration in Appendix A). Specifically, we aim to guarantees minimal loss within constrained training iterations under the worst-case conditions, proposing a budget-iteration-aware framework for learning rate optimization.

Definition (Finite optimization): Let $f(W, D)$ denote the function parameterized by a given neural network with parameters $W \in \mathbb{R}^N$ on the dataset D, ξ denotes the data sampling on the dataset D, and let $\mathcal{F}$ be a function class.Let $\mathcal{L}$ be the loss function. Let η_t be the learning rate at the t-th iteration, T be a maximum number of training iterations, and t be the current learning step. A finite optimization is

$$\begin{aligned} \min_{\eta_1,\eta_2,...,\eta_{T-1}} \max_{f\in\mathcal{F}} \quad & \mathcal{L}(f(W_T,\xi)) \\ s.t. \quad & W_{t+1} = W_t - \eta_t \nabla \mathcal{L}(f(W_t,\xi)) \\ & t = 0,1,2,...,T-1 \end{aligned} \tag{1}$$

In the optimization model 1, the constraint represents the stochastic gradient descent process. The maximizing of $\mathcal{L}(f(W_T, \xi))$ represents the the worst-case among the training process on the network f. Then it minimizes the worst-case loss within given iterations, embodying its budget-aware property. By formulating the problem as a min-max optimization, we identify learning rates $\eta_t (t = 1, 2, \cdots, T-1)$ that are resilient to the uncertainties introduced by different training configuration, uniformly throughout the optimization trajectory.

The challenge lies in characterizing the f within the optimization process. f is primarily determined by variations in parameter configuration, datasets characteristics, batch ordering and network architecture. From an optimization standpoint, we assume that the characteristics of f shaped by these factors can be captured by the loss landscape of f. Therefore, we can analyze f by approximating its loss using a quadratic expansion around nearby strict local optima. Specifically, during the optimization process, the loss surface in the vicinity of the optimization trajectory can be approximated by sequence of strict local optima (or at least by one optimum), denoted as $\bar{W}^{(k)} \quad (k = 1, 2, \ldots K) \quad K \in \mathbb{Z}^+$, with the final optimum represented as $\bar{W}^K$. This approach enables us to capture the key features of the loss surface near these points. Therefore, the trajectory of optimization is impacted by the characteristics of the nearby strict local optima, since the optimization process is inherently shaped by the loss landscape and the key features of the surface are captured by these optima. Consequently, we can derive the learning rate within the optimization model by the information of these nearby strict local optima.

According to the second-order necessary condition for the strict minimum $\bar{W}^{(k)}$, this approximation is achieved through the positive semi-definite Hessian matrix $H_f^{(k)}(\xi) \in \mathbb{R}^{N\times N}$. In addition, the optimization problem (1) will be programmed sequentially. Then objective function of the optimization problem (1) can be reformulated as $\mathcal{L}(f(\bar{W}^{(k)}, \xi)) + \frac{1}{2}(W_{T_{k+1}} - \bar{W}^{(k)})^\top H_f^{(k)}(\xi)(W_{T_{k+1}} - \bar{W}^{(k)}) \quad (k = 1, 2, \ldots K)$. Given that the networks under consideration possess sufficient capacity to fit the data, the loss at the optimal point for different f can be made small enough. Thus the term $\mathcal{L}(f(\bar{W}^{(k)}, \xi))$ can be reasonably neglected in our analysis. We obtain the following sequential optimization problem.

$$\begin{aligned} \min_{\eta_{1+T_k},\eta_{2+T_k},\ldots,\eta_{T_{k+1}}} \max_{f(\xi)} \quad & \frac{1}{2}(W_{T_{k+1}} - \bar{W}^{(k)})^\top H_f^{(k)}(\xi)(W_{T_{k+1}} - \bar{W}^{(k)}) \\ s.t. \quad & W_{t+1} = W_t - \eta_t H_f^{(k)}(\xi)(W_t - \bar{W}^{(k)}) \\ & t = T_k + 1, T_k + 2, ..., T_{k+1} \quad (k = 1, 2, \ldots K-1) \end{aligned} \tag{2}$$

where $T_1 = 0$ and $T_K = T - 1$.

By the first constraint $W_{t+1} = W_t - \eta_t H_f^{(k)}(\xi)(W_t - \bar{W}^{(k)})$, we can obtain (see Appendix A for derivation)

$$\left\| W_{T_{k+1}} - \bar{W}^{(k)} \right\|_2^2 \leq \max_{\lambda_l^{(k)} \leq \lambda_i^{(k)} \leq \lambda_u^{(k)}} \left[\prod_{t=1+T_k}^{T_{k+1}} (1 - \eta_t \lambda_i^{(k)}) \right]^2 \left\| W_{T_k} - \bar{W}^{(k)} \right\|_2^2 \tag{3}$$

where $\lambda_i^{(k)} \quad (i = 1, 2, \cdots, N)$ are the eigenvalues of $H_f^{(k)}(\xi)$ around the k-th optimum, which satisfy $0 < \lambda_l^{(k)} \leq \lambda_i^{(k)}(f(\xi)) \leq \lambda_u^{(k)}$ for all i, $u_i^{(k)} \quad (i = 1, 2, \cdots, N)$ denote N linearly independent eigenvectors and $s_i^{(k)}$ are the coefficients corresponding to the eigenvector components. Since the term $\left\| W_{T_k} - \bar{W}^{(k)} \right\|_2^2$ is a certain constant, the optimization process of weights $W_{T_{k+1}}$ is

equivalent to the least upper bound of the $\prod_{t=1+T_k}^{T_{k+1}}(1-\eta_t\lambda_i^{(k)})$. Thus, the optimization of learning rate schedule can be formulated as follow,

$$\begin{aligned}\min_{\eta_{1+T_k},\eta_{2+T_k},\ldots,\eta_{T_{k+1}}} \max_{\lambda_l^{(k)}\le\lambda_i^{(k)}\le\lambda_u^{(k)}} & \prod_{t=1+T_k}^{T_{k+1}}\left[(1-\eta_t\lambda_i^{(k)})\right]^2 \\ s.t. \quad & \eta_{1+T_k},\eta_{2+T_k},\ldots,\eta_{T_{k+1}}\in[\eta_{\min},\eta_{\max}] \\ & k=1,2,\ldots K\end{aligned} \tag{4}$$

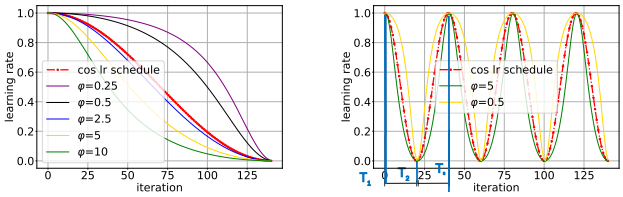
To solve the constrained min-max problem (4), we adopt an iterative projected gradient method that alternates between minimizing over the variables η_t and maximizing over the parameters $\lambda_i^{(k)}$. To avoid repeated numerical optimization, we fit the solutions with a parametric function. Details regarding the numerical solution and curve fitting process are provided in Appendix B. Then, we obtain the η_t within the interval $t\in[1+T_k,T_{k+1}]$ as follows

$$\eta_t=(\eta_{\max}-\eta_{\min})\frac{2(1+\cos(\frac{(2(t-T_k)-1)\pi}{2(T_{k+1}-T_k)}+(k-1)\pi))}{2\varphi+(2-\varphi)(1+\cos(\frac{(2(t-T_k)-1)\pi}{2(T_{k+1}-T_k)}+(k-1)\pi)}+\eta_{\min}, \tag{5}$$

where φ is the hyper-parameter controlling the variation speed of the learning rate η_t.

When setting $(K>2)$, UBA extends to a multi-phase formulation, which offers three potential advantages. (i) Hierarchical optimization: by partitioning the budget-aware optimization into multiple phases, each near a different local minima, it improves approximation accuracy and captures the dynamic features of the loss surface. (ii) It helps escape saddle points, (iii) It generalizes the single-phase method, allowing for future extensions. Notably, a single-phase approach $(K=2)$ remains effective, as the robust budget-aware model inherently derives the optimal schedule function, as shown in Figure 1(a). For consistency with common practice and to better isolate scheduling effects, this paper focuses primarily on the single-phase implementation. The multi-phase approach is also compared in Appendix, with its schedule shown in Figure 1(b).

We name the proposed schedule Unified Budget-Aware (UBA) schedule for reasons. The robust budget-aware model minimizes the loss function **uniformly** across the optimization trajectory, resulting in stable performance. UBA provides a reliable, **unified** choice for practitioners, eliminating the need for case-by-case baseline comparisons and delivering consistent superiority across datasets, architectures, and training budgets. Lastly, UBA can **uniformly** approximate the behavior of existing schedules through simple parameter adjustments.



(a) (b)

Figure 1: Evolution of the learning rate in UBA schedule across training iterations.

3.2 Theoretical analysis

Proposition 1. *The fit function* (5) *is the exact closed-form solution to the min-max optimization problem:*

$$\min_{\eta_{1+T_k},\eta_{2+T_k},\ldots,\eta_{T_{k+1}}} \max_{\lambda_l^{(k)}\le\lambda_i^{(k)}\le\lambda_u^{(k)}} \prod_{t=1+T_k}^{T_{k+1}}\left[\left(1-\left(\left(\frac{1}{\lambda_l^{(k)}}-\frac{1}{\lambda_u^{(k)}}\right)\eta_t+\frac{1}{\lambda_u^{(k)}}\right)\lambda_i^{(k)}\right)\right]^2 \tag{6}$$

when the hyper-parameter φ are determined by $\lambda_l^{(k)}$ and $\lambda_u^{(k)}$ through the relation $\varphi=2\frac{\lambda_u^{(k)}}{\lambda_l^{(k)}}$ and $\eta_{\max}=1, \quad \eta_{\min}=0.$

The min-max model in Proposition 1 represents a special case of our generalized optimization framework. We show that UBA is the exact solution to this special case optimization problem when the learning rate is scaled by $\left(\left(\frac{1}{\lambda_l^{(k)}} - \frac{1}{\lambda_u^{(k)}}\right)\eta_t + \frac{1}{\lambda_u^{(k)}}\right)$. It provides a theoretical foundation for our choice of learning rate instead of choosing it heuristically or empirically, adding rigor and justification to our approach. Moreover, in this case, φ is linked condition number. It indicates how the learning rate is shaped by the local curvature of the model, which could guide the optimization process more effectively. The relationship between φ and condition number suggests an adaptive nature for the learning rate. In regions with sharp curvatures (large condition number), φ reduces the learning rate more rapidly to avoid overshooting. Conversely, in flatter regions, φ allows the learning rate to remain large for several iterations, facilitating faster convergence. The transformation of the learning rate trend is shown in Figure 1. *This is a step toward establishing a principled connection between learning rate and local loss landscape geometry along with optimization difficulty.* By this special case, we generalize that φ is related to optimization difficulty. The empirical results support these conclusions, with further details in Section 4.3 and Appendix E.5.

Proposition 2. *Consider the training process within the interval $t \in [1+T_k, T_{k+1}]$. When the hyper-parameter φ is set sufficiently close to 2, the proposed learning rate scheduling formula reduces to the cosine learning rate schedule.*

By Proposition 1, the hyper-parameter φ is related to the optimization difficulty. In that case, the optimization difficulty is explicitly quantified as the condition number and $\varphi = 2\kappa$, where $\kappa = \frac{\lambda_u^{(k)}}{\lambda_l^{(k)}}$. Furthermore, Proposition 2 shows that when φ approaches 2, the proposed learning rate schedule converges to the standard cosine schedule. It means that, in regions where the optimization difficulty is not large (i.e., the curvature is relatively flat), setting $\varphi \approx 2$ allows the learning rate schedule to naturally reduce to the cosine form. Besides, our schedule can approximate the behavior of existing schedules (e.g.step decay, cosine annealing, cyclic schedule or Rex schedule) through simple parameter adjustments, shown in Table 1. The detail can be found in Appendix C. More importantly, it outperforms these schedules by training convergence and final accuracy, supporting results can be found in experiment sections 4.1,4.2 and Appendix E.4.

Table 1: The adaptive simulation of existing schedules.

Schedule	Parameter adjustments	Schedule	Parameter adjustments
Cosine	$\varphi = 2$	Step	$\varphi = 0, \eta_{\max} = 0.5^k, T_{k+1} - T_k =$decaying step,$k = 1, 2, ...$
Exponential	$\varphi = 30$	Cyclic	$\varphi = 2, k \leftarrow k+1, T_{k+1} - T_k =$cyclic step ,$k = 1, 2, ...$
Rex	$\varphi = 0.8$	OneCycle	$\varphi = 2, k \leftarrow k+1, T_2 - T_1 =$ pct_start step

Theorem 1. *Let $n_t = H_f^{(k)}(W_t - \bar{W}^{(k)}) - H_f^{(k)}(\xi)(W_t - \bar{W}^{(k)})$ be the stochastic curvature noise introduced by sampling at iteration t. Assume that the sampling Hessian satisfies: $\mathbb{E}_\xi\left[n_t n_t^\top\right] \preceq \sigma^2 H_f^{(k)}$ for some constant σ. Denote $\tau := \frac{4\lambda_l^{(k)}(\eta_{\max}-\eta_{\min})\left(1+\cos\left(\frac{(2(t-T_k)-1)\pi}{2(T_{k+1}-T_k)}\right)\right)(T_{k+1}-T_k)}{(\varphi-2)\pi}$. If we set the learning rate as the proposed form* (5)*, the loss uncertainty introduced by stochastic gradient method within the interval $t \in [1+T_k, T_{k+1}]$ can be bounded by two terms,*

For** $\varphi > 2$**:

$$
\begin{aligned}
&\mathbb{E}\left[\mathcal{L}(f(W_t,\xi)) - \mathcal{L}(f(\bar{W}^{(k)},\xi))\right] \\
\leq&\Big(\frac{4(T_{k+1}-T_k)+(\varphi-2)\pi}{4(T_{k+1}-T_k)+(\varphi-2)\pi(t-T_k)}\Big)^\tau \cdot \exp\Big(-2\lambda_i^{(k)}\eta_{\min}(t-T_k)\Big)\lambda_u^{(k)}\|W_{1+T_k} - \bar{W}^{(k)}\|_2^2 \\
+&\sigma^2 \sum_{i=1+T_k}^{t} \eta_i^2 \sum_{j=1}^{N} (\lambda_j^{(k)})^2 \exp\Big(-2\lambda_j^{(k)}\eta_{\min}(t-i)\Big) \cdot \left(\frac{4(T_{k+1}-T_k)+(\varphi-2)\pi(i-T_k)}{4(T_{k+1}-T_k)+(\varphi-2)\pi(t-T_k)}\right)^\tau
\end{aligned}
\tag{7}
$$

For $\varphi < 2$***:***

$$\begin{aligned}
&\mathbb{E}\left[\mathcal{L}(f(W_t,\xi)) - \mathcal{L}(f(\bar{W}^{(k)},\xi))\right] \\
\leq&\Big(\frac{(2\varphi+2\pi-\varphi\pi) - \frac{(2-\varphi)\pi}{(T_{k+1}-T_k)}(t-T_k-0.5)}{(2\varphi+2\pi-\varphi\pi)+\frac{(2-\varphi)\pi}{2(T_{k+1}-T_k)}}\Big)^{-\tau} \cdot \exp\Big(-2\lambda_i^{(k)}\eta_{\min}(t-T_k)\Big)\lambda_u^{(k)}\|W_{1+T_k} - \bar{W}^{(k)}\|_2^2 \\
&+\sigma^2 \sum_{i=_{1+T_k}}^{t} \eta_i^2 \sum_{j=1}^{N} (\lambda_j^{(k)})^2 \exp\Big(-2\lambda_j^{(k)}\eta_{\min}(t-i)\Big) \cdot \left(\frac{(2\varphi+2\pi-\varphi\pi) - \frac{(2-\varphi)\pi}{(T_{k+1}-T_k)}(t-T_k-0.5)}{(2\varphi+2\pi-\varphi\pi) - \frac{(2-\varphi)\pi}{(T_{k+1}-T_k)}(i-T_k-0.5)}\right)^{-\tau}
\end{aligned} \tag{8}$$

4 Experiment results

We conduct comprehensive evaluations along three dimensions: (i) Modality diversity, including vision and language tasks; (ii) Training budget, including model scales (small to large) and training iterations (short to long); (iii) Ablation studies, such as parameter sensitivity and cross-optimizer performance. This evaluation strategy ensures the robustness and generalization of our findings.

A key advantage of UBA lies in its critical parameter φ. While an optimal φ can further improve model performance, we intentionally fix its value across all tasks and architectures to ensure fair evaluation. We fix $\varphi = 5$ for SGD and $\varphi = 0.5$ for AdamW (see learning rate variations in Figure 1(a)). The rationale for these choices is systematically analyzed in our ablation study 4.3.

Baselines Research on budgeted-iteration training remains limited, with most existing approaches focusing on learning rate scheduling strategies [5, 29, 43]. To provide a fair comparison, we adopt several widely used and empirically effective learning rate schedules as baselines: **Step(SS)** [15], **Cosine(CS)** [32], **Cyclical (CLR) and OneCycle (1C)** [42, 43], **Budgeted training(BT)** [29] and **Reflected Exponential (REX)** [5]. Details of these baselines are provided in Appendix E.1.

4.1 Experiments for Vision Classification Tasks

We evaluate our proposed UBA schedule on vision benchmarks (e.g., CIFAR10/100 and ImageNet) using different architectures (VGG16, ResNet18, ResNet34, ResNet50). In addition, we independently train models using fixed epoch budgets of 25%, 50%, and 100% of the maximum training epochs, without reusing or interpolating results from longer training runs. This setup ensures that each budget setting is evaluated in isolation, thereby preserving the integrity of comparisons across low and high training budgets. Table 2 presents the validation accuracy across different training budgets, comparing UBA against six baseline schedules (see detailed results in Appendix E.3).

We find that: (i) UBA demonstrates the strongest performance across all training budgets, achieving superior results on both small-scale (CIFAR10/100 with ResNet18/34) and large-scale (ImageNet with ResNet50) benchmarks. This consistent improvement highlights the *generalizability across model and dataset scales*. (ii) UBA outperforms baselines not only at 100% training budget but also at 25% and 50% iteration budgets, demonstrating its *budget efficiency*, i.e. effectiveness in computation-constrained scenarios. (iii)Notably, UBA shows robust performance even while the second-best schedule varies depending on both the datasets, architectures and training budgets. For practitioners seeking reliable schedules without extensive method selection, UBA provides a default-strong choice. UBA eliminates the need for case-by-case baseline comparison and delivers *stable superiority and reliability* regardless of datasets, architectures, training budgets and scales.

4.2 Experiments for language models

We evaluate UBA schedule on the OLMo[18], a truly open language model based on decoder-only transformer architecture, across diverse benchmarks in language model evaluation. Since large language models commonly provide multiple scales to accommodate different compute constraints. We adjust both model size and training steps to explore budgets. We evaluate UBA on OLMo networks spanning four parameter scales: 36M, 73M, 151M, and 300M, covering normal to large-scale mod-

Table 2: Validation accuracy for vision classification tasks. We present validation accuracy on vision benchmarks (e.g., CIFAR10/100 and ImageNet) using different architectures (ResNet18, ResNet34, ResNet50) under fixed epoch budgets of 25%, 50%, and 100% of the maximum training epochs.

Schedule	CIFAR10-ResNet18 training budget (epoch(%))			CIFAR100-ResNet34 training budget (epoch(%))			ImageNet–ResNet50 training budget (epoch(%))		
	75 (25%)	150 (50%)	300 (100%)	75 (25%)	150 (50%)	300 (100%)	75 (25%)	150 (50%)	300 (100%)
SS	92.41	93.90	93.94	70.85	*75.58*	77.28	74.84	76.96	78.10
CS	93.66	94.15	95.40	63.88	68.84	70.43	*75.99*	*77.79*	*79.10*
CLR	91.89	92.23	95.32	72.63	73.81	74.80	72.87	74.88	76.91
1C	94.36	95.02	95.48	*73.72*	75.53	77.90	75.28	77.37	78.79
BT	93.24	94.28	95.55	72.53	75.40	78.49	75.71	77.48	78.66
REX	**94.79**	*94.96*	*95.59*	73.12	75.48	*77.99*	75.28	77.03	78.46
UBA(ours)	*94.54*	**95.26**	**95.74**	**74.57**	**76.68**	**78.97**	**76.00**	**77.99**	**79.32**

els. Due to space limitation, we defer details such as benchmarks introduction, experimental setting and overall results in Appendix E.4).

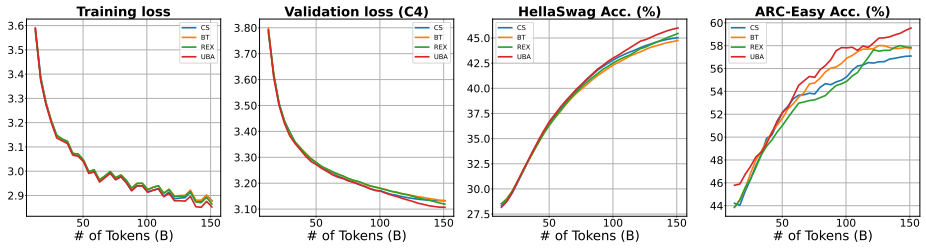


Figure 2: Training dynamics and performance for language tasks under 150B tokens on 300M OLMo. We present the training loss, validation loss, and downstream performance on HSWAG and ARC-E, demonstrating that UBA schedule achieves superior performance.

From Table 3, UBA achieves state-of-the-art performance on approximately 50% of the benchmarks across all scales while the second-best schedule varies. Furthermore, it achieves *consistent superior average performance* among all baselines. It highlights the stable superiority and reliability of UBA, providing a *default-strong choice*. Moreover, it demonstrates significant improvements in SciQ-73M($+1.7$) and ARC-E-300M($+2.63$), highlighting its ability to enhance generalization across diverse benchmarks. Besides, in Figure 2, UBA consistently achieves lower training loss and validation loss throughout the training, indicating the *efficient training ability* and downstream performance enhancement. Notably, while task-specific tuning of φ can further improve model performance, we intentionally fix its value across all tasks and architectures to ensure fair evaluation. Remarkably, even with this universal φ setting, UBA consistently outperforms baselines on diverse benchmarks, demonstrating *inherent robustness* to varying benchmarks and model scales.

4.3 Ablation Study

Performance across different optimizers UBA originates from the optimization problem under gradient descent dynamics. While modern optimizers (e.g., AdamW [31]) introduce momentum and adaptive mechanisms, they maintain the fundamental property of gradient-based updates. To verify the cross-optimizer performance, we conduct ablation studies between SGD and AdamW optimizers(see detailed results in Appendix E.5). The results show that UBA achieves SOTAs on both SGD and AdamW optimizers, demonstrating the *cross-optimizer robustness* of UBA. This highlights its *broad applicability* beyond standard gradient descent.

Parameter analysis of φ We perform a sensitivity analysis of $\varphi \in \{0.25, 0.5, 1.0, 2.5, 5, 10\}$ in equation (5) , which controls the variation speed of learning rate (see detailed setting, results and analyses in Appendix E.6). Experiments 11 show AdamW prefers smaller φ while SGD requires larger φ. We attribute this phenomenon to the preconditioning effect of AdamW. As the relationship

Table 3: Performance comparison between UBA and the best-performing baseline schedules on OLMo. We report the results of top-performing baseline schedule for the corresponding benchmark and model scale. The names of the top-performing baseline schedules are listed below the results. Bold values indicate UBA's superiority (See overall results of each schedule in Appendix E.4).

		Benchmark(accuracy %)										
Size	Sched.	PIQA	HSWAG	OBQA	SciQ	ARC-E	ARC-C	COPA	SIQA	SOC	OTH	Avg.
36M	Best baseline	61.15 (CLR)	27.91 (1C)	27.80 (REX)	68.10 (REX)	45.26 (SS)	23.41 (1C)	63.00 (SS)	41.45 (CLR)	25.18 (REX)	29.86 (1C)	40.92 (REX)
	UBA	60.39	**27.98**	27.40	**68.20**	**45.79**	21.74	**63.00**	40.69	24.33	**30.14**	**40.97**
73M	Best baseline	62.46 (REX)	30.22 (REX)	28.80 (CS)	72.60 (CS)	47.54 (1C)	26.42 (1C)	66.00 (BT)	41.97 (BT)	26.32 (SS)	29.34 (REX)	42.53 (CS)
	UBA	**63.17**	30.09	**28.80**	**74.30**	45.79	22.74	65.00	41.45	**27.13**	28.85	**42.73**
150M	Best baseline	66.00 (CS)	35.72 (REX)	32.80 (1C)	78.20 (BT)	53.16 (1C)	26.42 (BT)	69.00 (REX)	43.71 (CLR)	27.68 (REX)	32.64 (REX)	45.91 (REX)
	UBA	65.23	35.50	29.80	**78.30**	50.35	**27.42**	67.00	43.50	**29.29**	**32.72**	**45.91**
300M	Best baseline	70.18 (REX)	46.30 (REX)	33.40 (CS)	84.40 (REX)	57.72 (REX)	28.43 (REX)	72.00 (BT)	44.58 (CS)	29.50 (REX)	36.74 (REX)	49.70 (REX)
	UBA	69.48	**46.44**	**34.60**	83.90	**60.35**	**29.10**	**72.00**	44.42	28.53	35.41	**50.42**

formalized in Proposition 1), smaller φ is favored for low optimization difficult, while larger φ is beneficial for difficult optimization. AdamW adapts the learning rate by scaling gradients with the second moment estimate $\sqrt{v_t}$, implicitly reducing the condition number of the optimization landscape. Thus the schedule with smaller φ is preferred. In contrast, SGD lacks such adaptive mechanisms, thus a larger φ is more suitable for this situation. This alignment between theory and experiment underscores the importance of tailoring φ to the optimizer's characteristics. *Overall, φ is related to a generalized optimization difficulty, where optimization precondition effect, datasets distribution and network architecture are all related to optimization difficulty.*

Performance across different periods Our schedule has a periodic phase-based learning rate adjustment setting, where the learning rate at the k-th phase is dynamically determined by the k-th local minimum of the loss landscape. To validate our scheduling strategy, we conduct experiments by varying K (see detailed results in Appendix E.7). The results suggests that multi-phase scheduling captures the dynamic features of the loss surface more finely, but it needs careful selection for φ, where φ reflects optimization difficulty. However, selecting optimal φ values per phase for multi-phase remains non-trivial, which motivates future work on automated landscape-aware φ tuning.

5 Conclusion

In this paper, we construct a unified budget-aware training optimization framework that inherently adapts to landscape curvature variations, enhancing training robustness. From this optimization framework, we propose the Unified Budget-Aware (UBA) schedule. Extensive experiments demonstrate UBA *consistently surpasses* the commonly-used schedules across diverse vision and language tasks, spanning network architectures (e.g., ResNet, OLMo) and scales, under different training-iteration budgets.

Theoretically and empirically, we observe that the parameter φ correlates with the optimization difficulty of the training process, influences the optimal choice of φ and UBA's performance. However, the explicit relationship between φ and optimization difficulty remains unexplored and no established evaluation metric exists to quantify the optimization difficulty. These limitations motivate future work on optimization difficulty-aware φ tuning.

Despite these open questions, the effectiveness, implementation simplicity and ease of tuning make the UBA a practical, must-try schedule for deep learning practitioners. We hope this work could motivate more studies on learning rate scheduling.

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