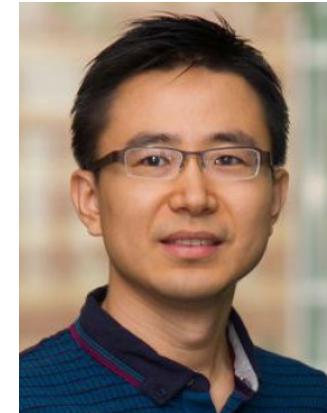




Low-Rank and Sparse Modeling for Visual Analytics Applications and Conclusions



Sheng Li & Yun Fu

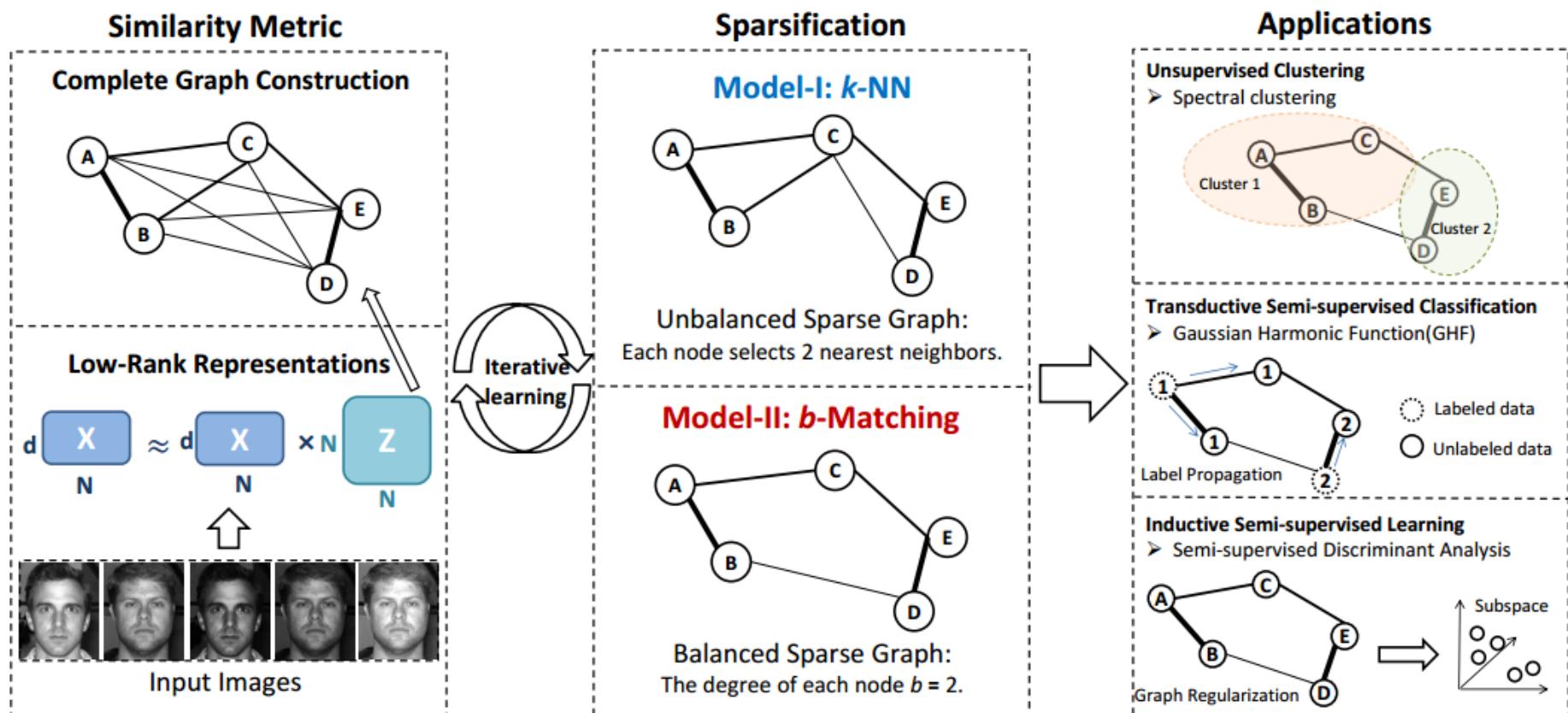
Northeastern University, Boston, MA
June 26, 2016

Outline

- ▶ **Applications**
 - Low-Rank and Sparse Models for Image Analysis
 - Low-Rank and Sparse Models for Video Analysis
- ▶ **Conclusions**

Applications: Image Analysis

- ▶ Low-Rank Coding based Graph Construction
 - ▶ Framework



Applications: Image Analysis

▶ Low-Rank Coding based Graph Construction

▶ Model I: Unbalanced Graph Construction

$$\begin{array}{ll} \min_{Z,E,S} & \text{rank}(Z) + \lambda_1 \|E\|_0 - \lambda_2 \sum_{i,j=1}^n S_{ij} (Z^T Z)_{ij} \\ \text{s.t. } & X = AZ + E, \sum_{j=1}^n S_{ij} = K, S_{ii} = 0, \end{array} \quad \xrightarrow{\hspace{1cm}} \quad \begin{array}{ll} \min_{Z,E,S} & \|Z\|_{\gamma_1} + \lambda_1 M_{\gamma_2}(E) - \lambda_2 \mathbf{1}_n^T (S \circ (Z^T Z)) \mathbf{1}_n \\ \text{s.t. } & X = AZ + E, \sum_{j=1}^n S_{ij} = K, S_{ii} = 0, \end{array}$$

- ▶ Minimax concave penalty (MCP) norm and Matrix-gamma norm are used.

▶ Model 2: Balanced Graph Construction

$$\begin{array}{ll} \min_{Z,E,S,J} & \|J\|_{\gamma_1} + \lambda_1 M_{\gamma_2}(E) - \lambda_2 \mathbf{1}_n^T (S \circ (Z^T J)) \mathbf{1}_n \\ \text{s.t. } & X = AZ + E, \sum_{j=1}^n S_{ij} = b, S_{ij} = S_{ji}, Z = J, \end{array}$$

b-Matching

- ▶ 95 Sheng Li and Yun Fu: *Learning Balanced and Unbalanced Graphs via Low-Rank Coding*, IEEE Trans. KDE, 2015.
Sheng Li and Yun Fu: *Low-Rank Coding with b-Matching for Semi-supervised Classification*, IJCAI 2013.

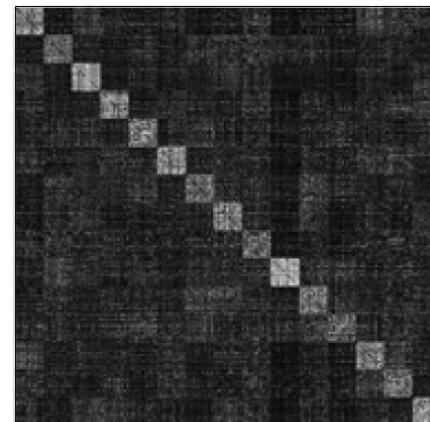
Applications: Image Analysis

▶ Low-Rank Coding based Graph Construction

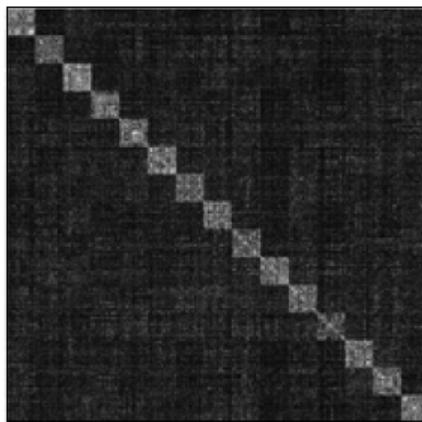
Visualization of Graphs
on CMU PIE Dataset



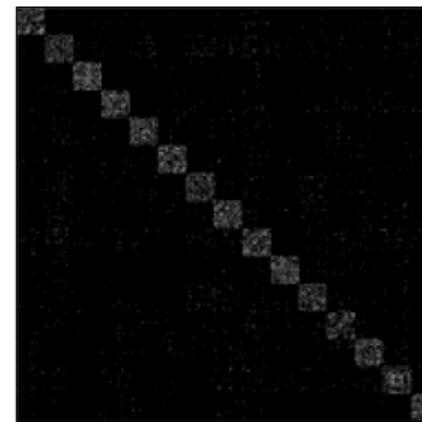
(a) KNN



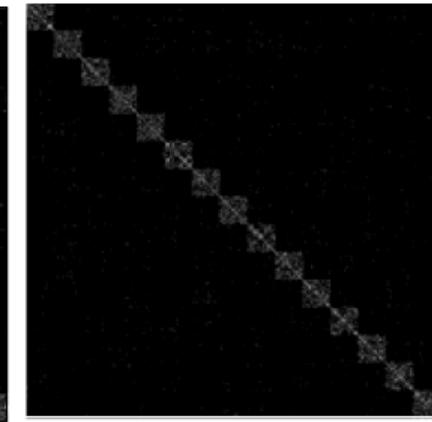
(b) L1



(c) LRR



(d) Ours-I



(e) Ours-II

- ▶ 96 Sheng Li and Yun Fu: *Learning Balanced and Unbalanced Graphs via Low-Rank Coding*, IEEE Trans. KDE, 2015.
Sheng Li and Yun Fu: *Low-Rank Coding with b-Matching for Semi-supervised Classification*, IJCAI 2013.

Applications: Image Analysis

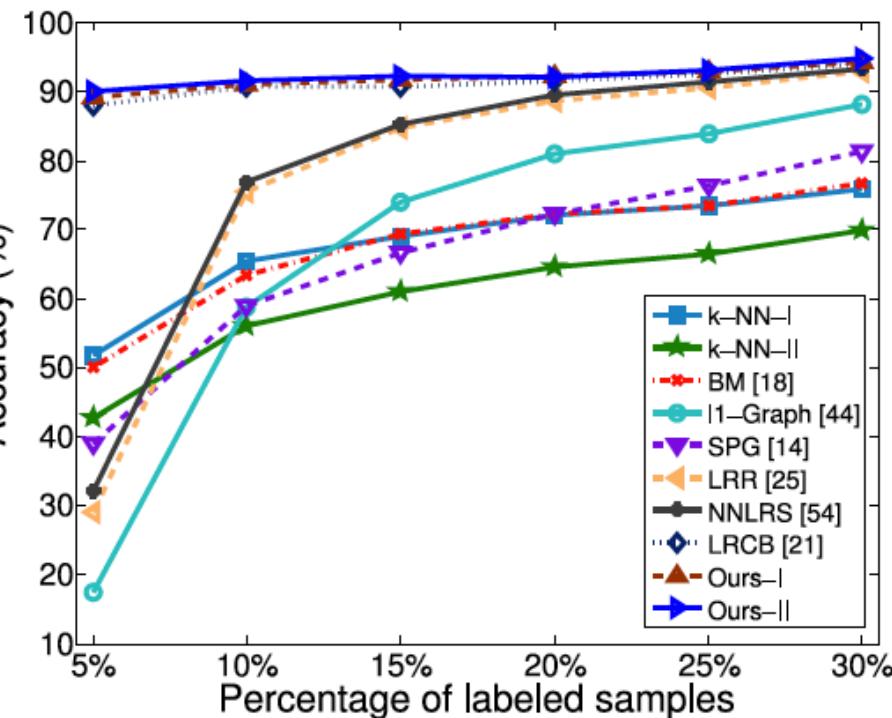
- ▶ Low-Rank Coding based Graph Construction
 - ▶ Results of [Image Clustering](#)

TABLE 2
Average Normalized Mutual Information of Different Graphs with Standard Deviations
for Clustering on Four Databases

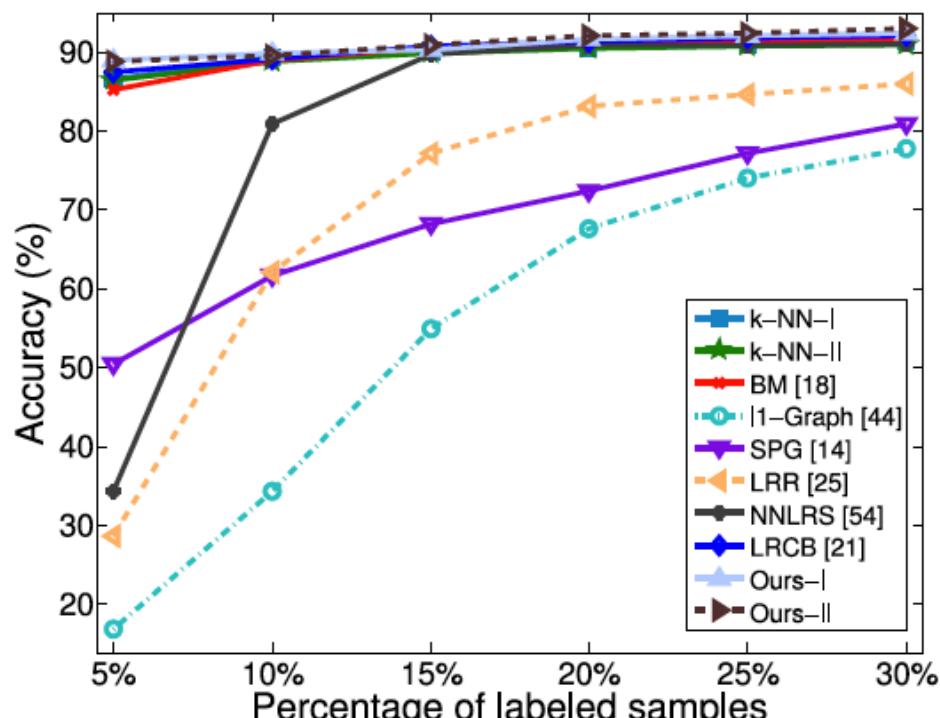
Methods	YaleB	PIE	ORL	USPS
k -NN-I	0.5988 ± 0.0463	0.3269 ± 0.0260	0.8127 ± 0.0002	0.6972 ± 0.0265
k -NN-II	0.4231 ± 0.0179	0.2636 ± 0.0152	0.7990 ± 0.0030	0.7100 ± 0.0191
BM [24]	0.4516 ± 0.0170	0.5127 ± 0.0185	0.8032 ± 0.0146	0.7020 ± 0.0169
l_1 -graph [35]	0.5216 ± 0.0167	0.4958 ± 0.0150	$0.7814 \pm .00294$	0.6272 ± 0.0249
LRR [28]	0.7122 ± 0.0078	0.6060 ± 0.0311	0.7799 ± 0.0259	0.6693 ± 0.0048
LRCB [29]	0.8541 ± 0.0104	0.6463 ± 0.0078	0.8126 ± 0.0125	0.7083 ± 0.0155
Ours-I	0.8716 ± 0.01387	0.6514 ± 0.0146	0.8424 ± 0.0216	0.7069 ± 0.0095
Ours-II	0.8673 ± 0.0166	0.6742 ± 0.0107	0.8751 ± 0.0094	0.7154 ± 0.0102

Applications: Image Analysis

- ▶ Low-Rank Coding based Graph Construction
 - ▶ Results of Transductive Semi-supervised Image Classification



(a) Extended YaleB



(d) USPS

Applications: Image Analysis

▶ Robust Subspace Discovery via Low-Rank Learning

▶ Motivation

Subspace Learning

- Find low-dimensional projection with specific properties.
- Unsupervised (e.g., PCA)/ Supervised (e.g., LDA)

Low-Rank Matrix Recovery

- Discover underlying subspaces in data set, and correct errors.
- Sparse subspace clustering (SSC), Low-rank representation (LRR).



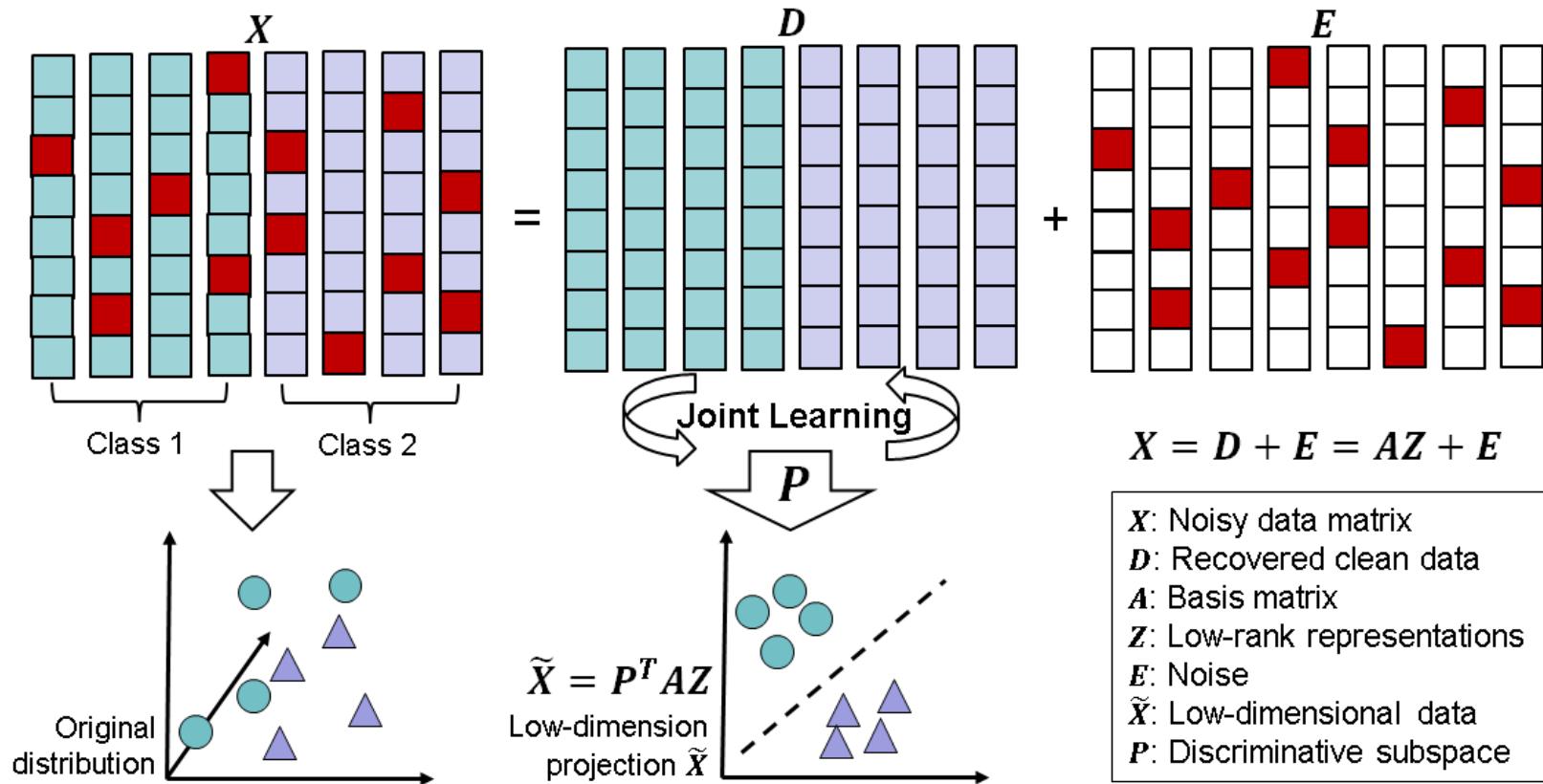
SRRS Approach

- Low-Rank** constraint is utilized to recover the clean data.
- Supervised** regularization is incorporated to improve the discriminability. Convexity is theoretically proven.

Applications: Image Analysis

▶ Robust Subspace Discovery via Low-Rank Learning

▶ Framework



- ▶ 100 Sheng Li, Yun Fu: *Learning Robust and Discriminative Subspace with Low-Rank Constraints*, IEEE Trans. NNLS.
Sheng Li, Yun Fu: *Robust Subspace Discovery through Supervised Low-Rank Constraints*, SDM 2014.

Applications: Image Analysis

- ▶ Robust Subspace Discovery via Low-Rank Learning
 - ▶ Model

$$\begin{aligned} & \min_{Z, P} \|Z\|_* + \lambda_1 \bar{f}(P, Z) \\ \text{s.t. } & X = AZ, \end{aligned}$$

Supervised Regularizer

↓

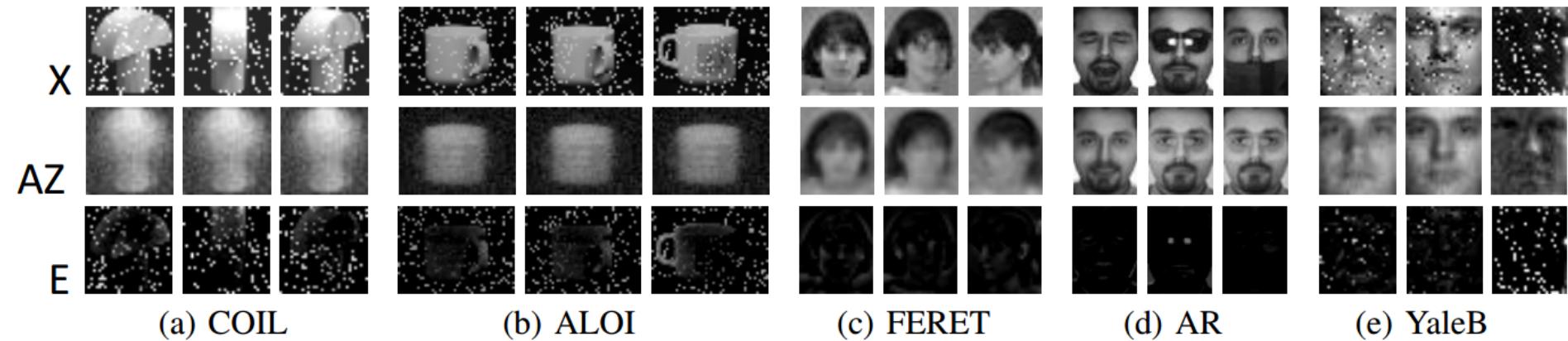
$$\begin{aligned} & \min_{Z, E, P} \|Z\|_* + \lambda_2 \|E\|_{2,1} + \lambda_1 (\|P^T AZ(\mathbf{I} - H_b)\|_F^2 \\ & - \|P^T AZ(H_b - H_t)\|_F^2 + \eta \|P^T AZ\|_F^2), \\ \text{s.t. } & X = AZ + E, P^T P = \mathbf{I}_p. \end{aligned}$$

- ▶ ADMM is used to solve the model

- ▶ 101 Sheng Li, Yun Fu: *Learning Robust and Discriminative Subspace with Low-Rank Constraints*, IEEE Trans. NNLS.
Sheng Li, Yun Fu: *Robust Subspace Discovery through Supervised Low-Rank Constraints*, SDM 2014.

Applications: Image Analysis

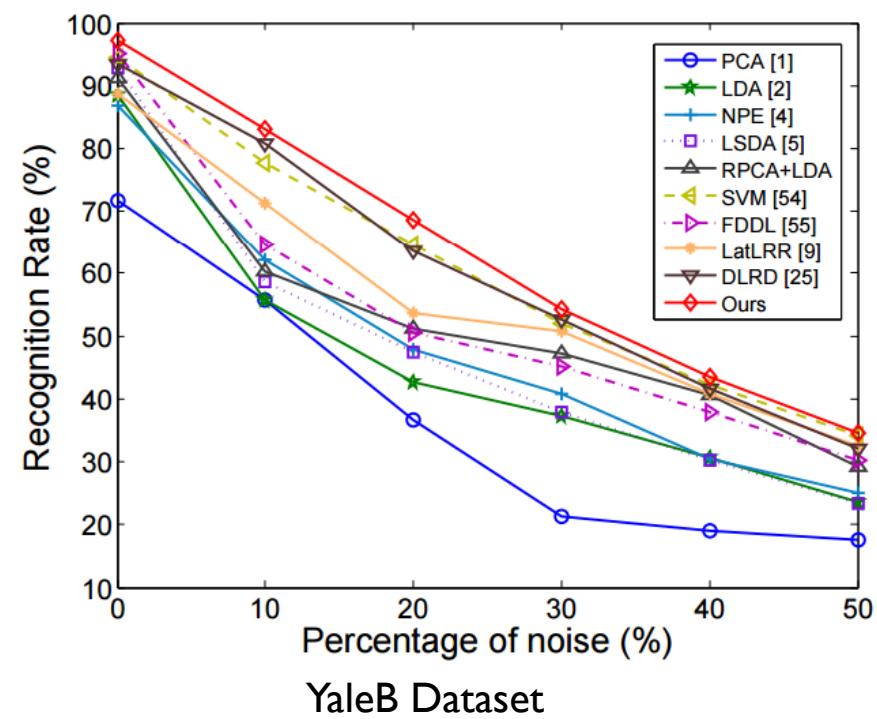
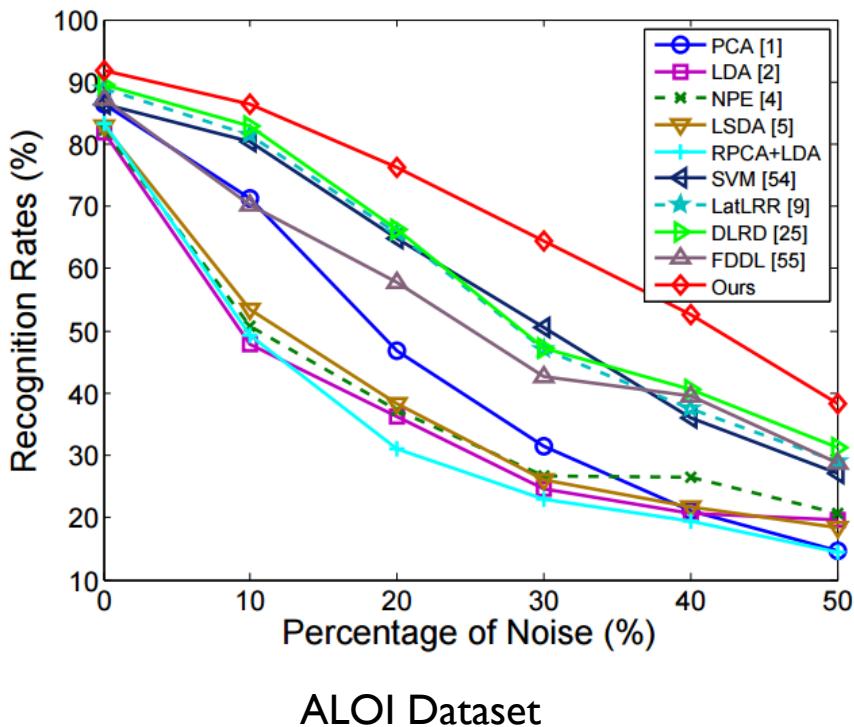
- ▶ Robust Subspace Discovery via Low-Rank Learning
 - ▶ Discriminative Image Recovery



- ▶ X: corrupted images
- ▶ AZ: recovered images
- ▶ E: noise

Applications: Image Analysis

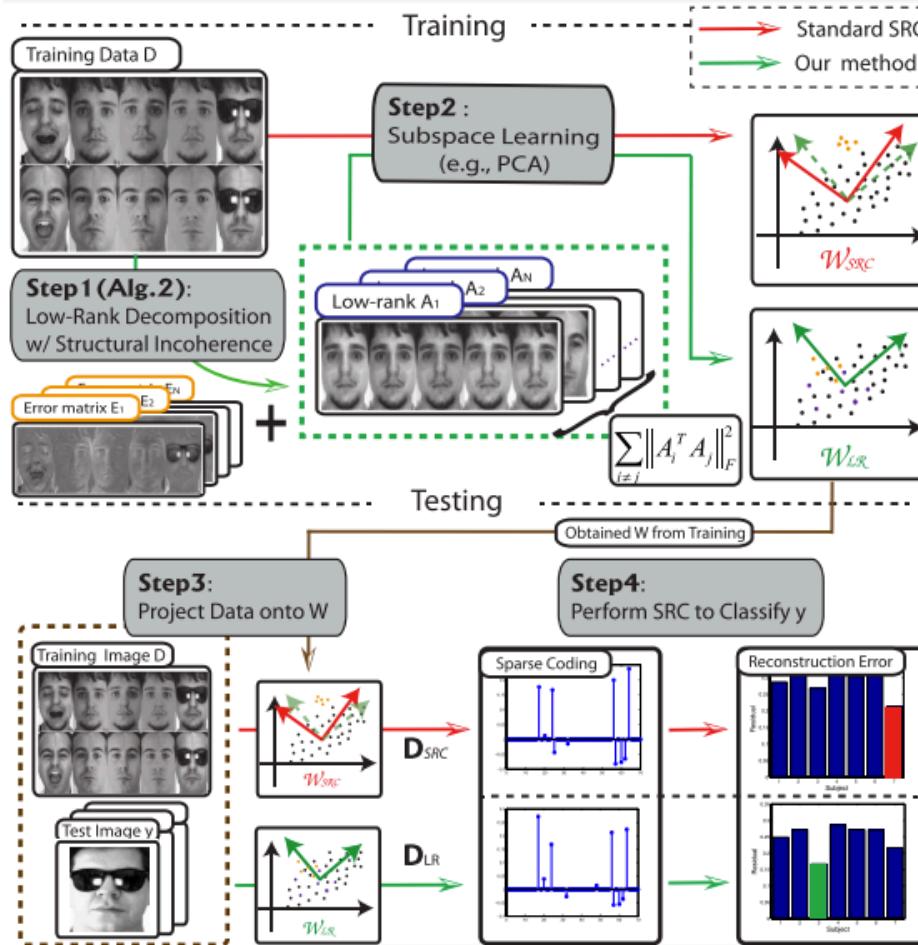
- ▶ Robust Subspace Discovery via Low-Rank Learning
 - ▶ Noisy Image Classification



- ▶ 103 Sheng Li, Yun Fu: *Learning Robust and Discriminative Subspace with Low-Rank Constraints*, IEEE Trans. NNLS.
Sheng Li, Yun Fu: *Robust Subspace Discovery through Supervised Low-Rank Constraints*, SDM 2014.

Applications: Image Analysis

▶ Structurally Incoherent Low-Rank Matrix Decomposition



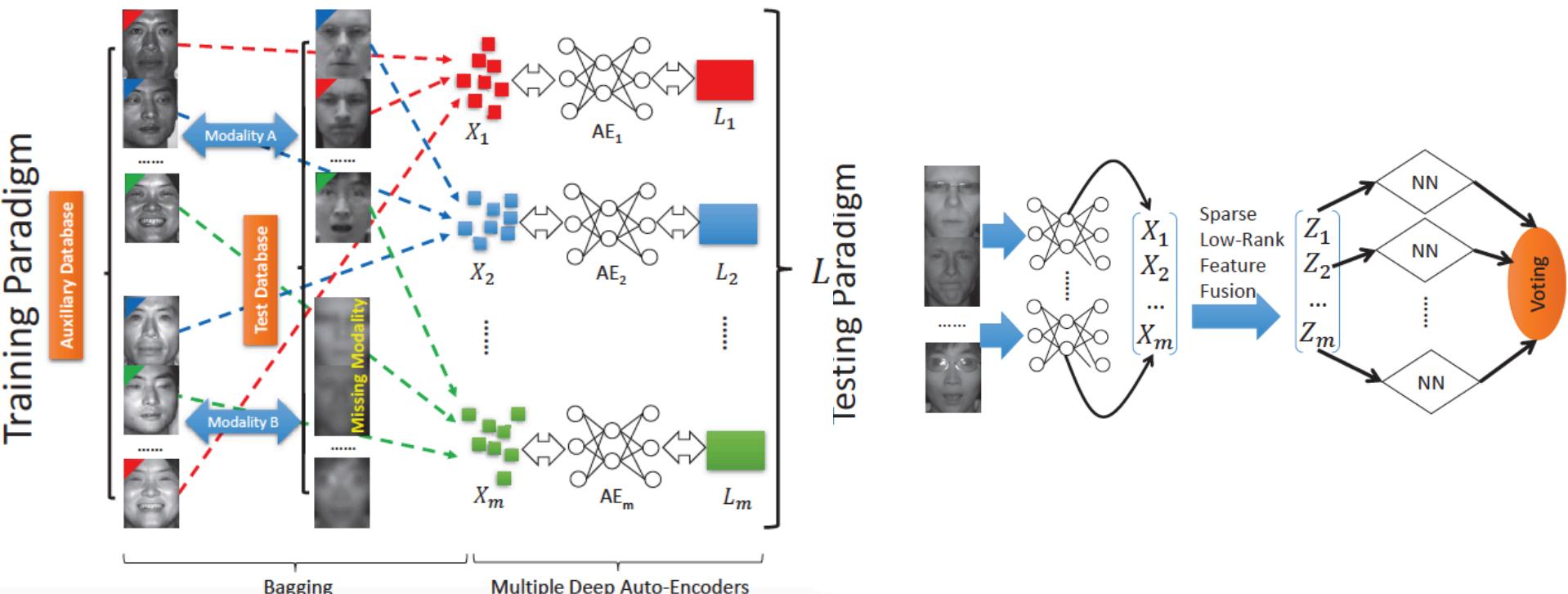
Face Recognition

$$\min_{\mathbf{A}_i, \mathbf{E}_i} \|\mathbf{A}_i\|_* + \lambda \|\mathbf{E}_i\|_1 + \eta \sum_{j \neq i} \|\mathbf{A}_j^T \mathbf{A}_i\|_F^2$$

s.t. $\mathbf{D}_i = \mathbf{A}_i + \mathbf{E}_i$.

Applications: Image Analysis

▶ Low-Rank Feature Fusion for Missing Modality Face Recognition



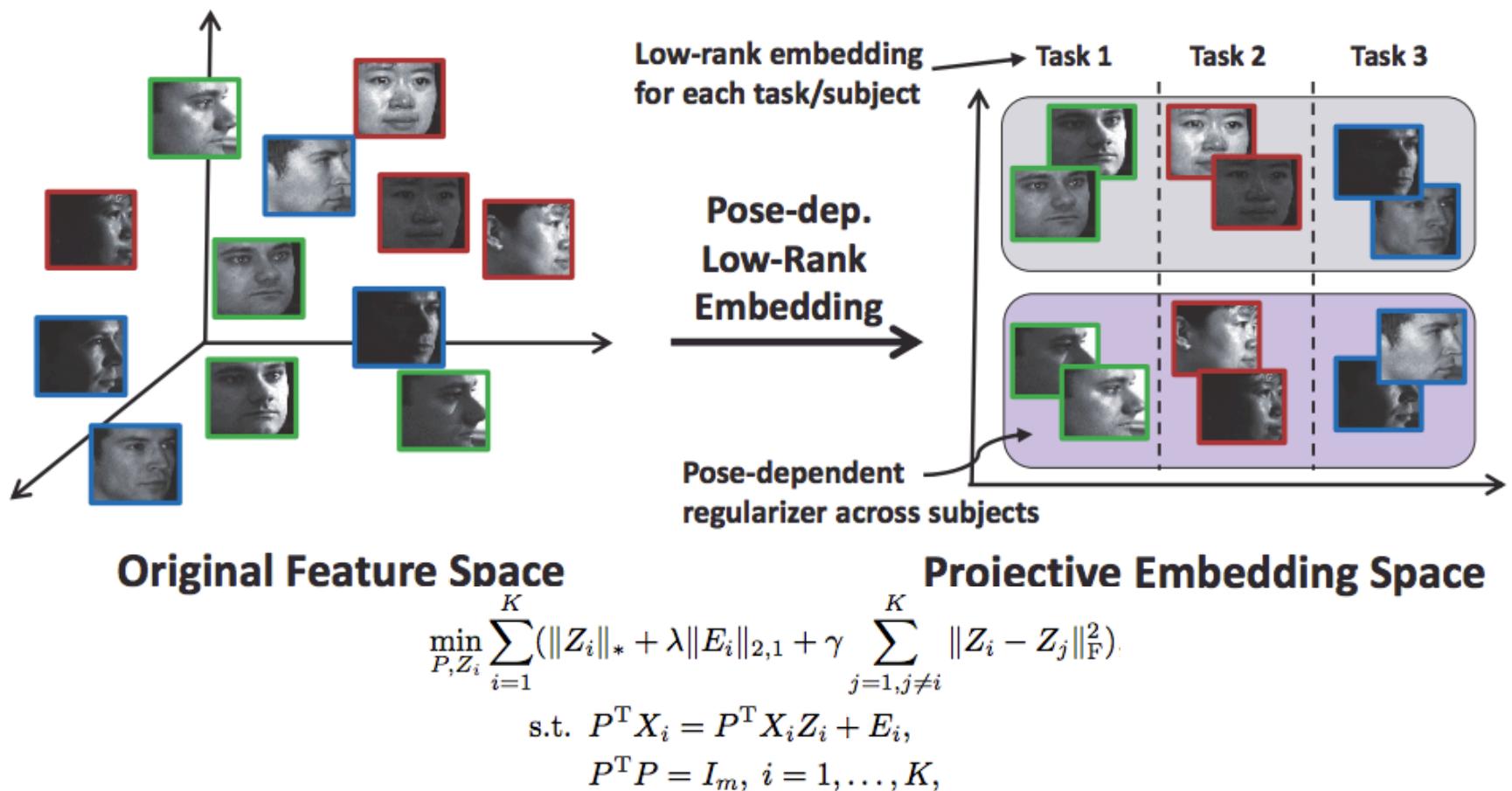
$$\min_{Z, E} \|Z\|_* + \lambda_1 \|E\|_{2,1} + \lambda_2 \|\Psi(Z)\|_{2,1},$$

$$\text{s.t. } X = XZ + E.$$

- ▶ 105 Ming Shao, Zhengming Dong, Yun Fu: *Sparse Low-Rank Fusion based Deep Features for Missing Modality Face Recognition*, FG 2015.

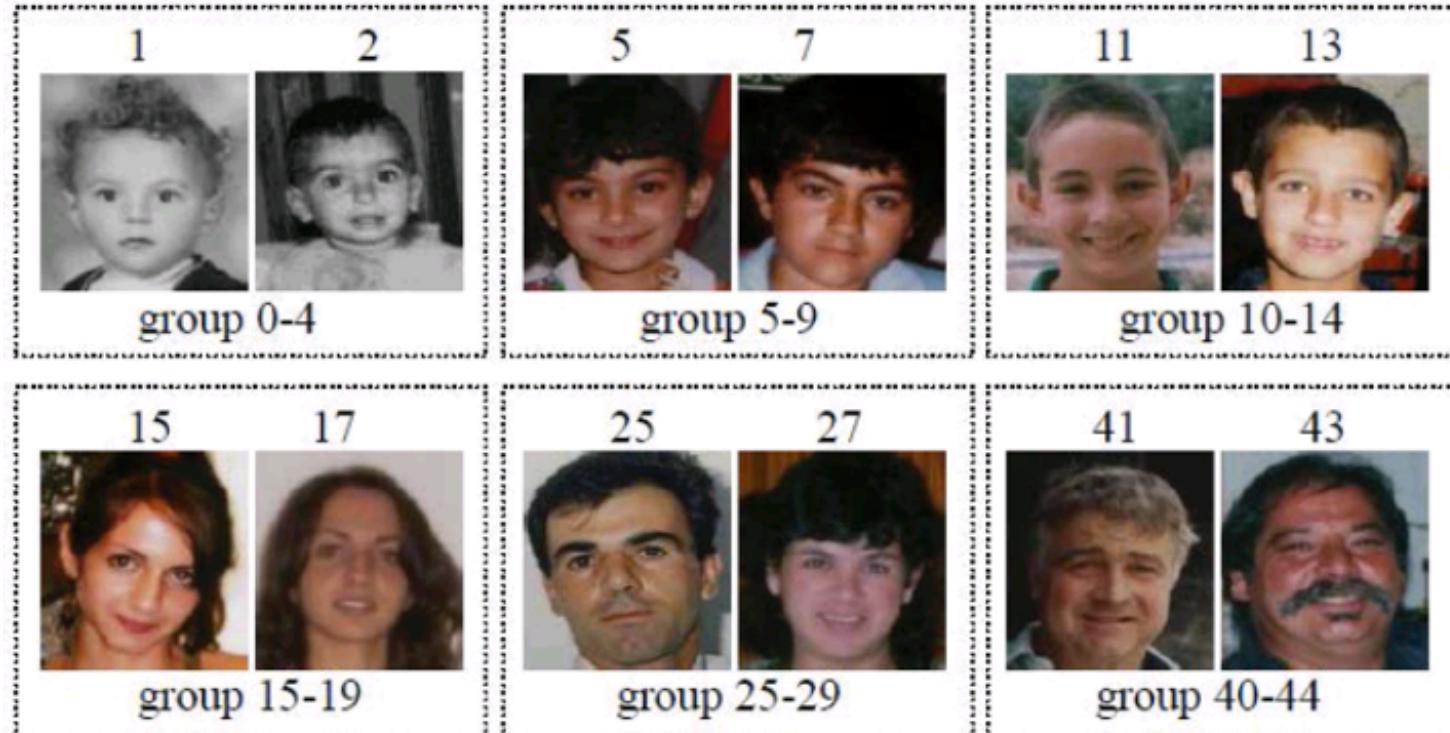
Applications: Image Analysis

► Head Pose Estimation



Applications: Image Analysis

► Facial Age Estimation

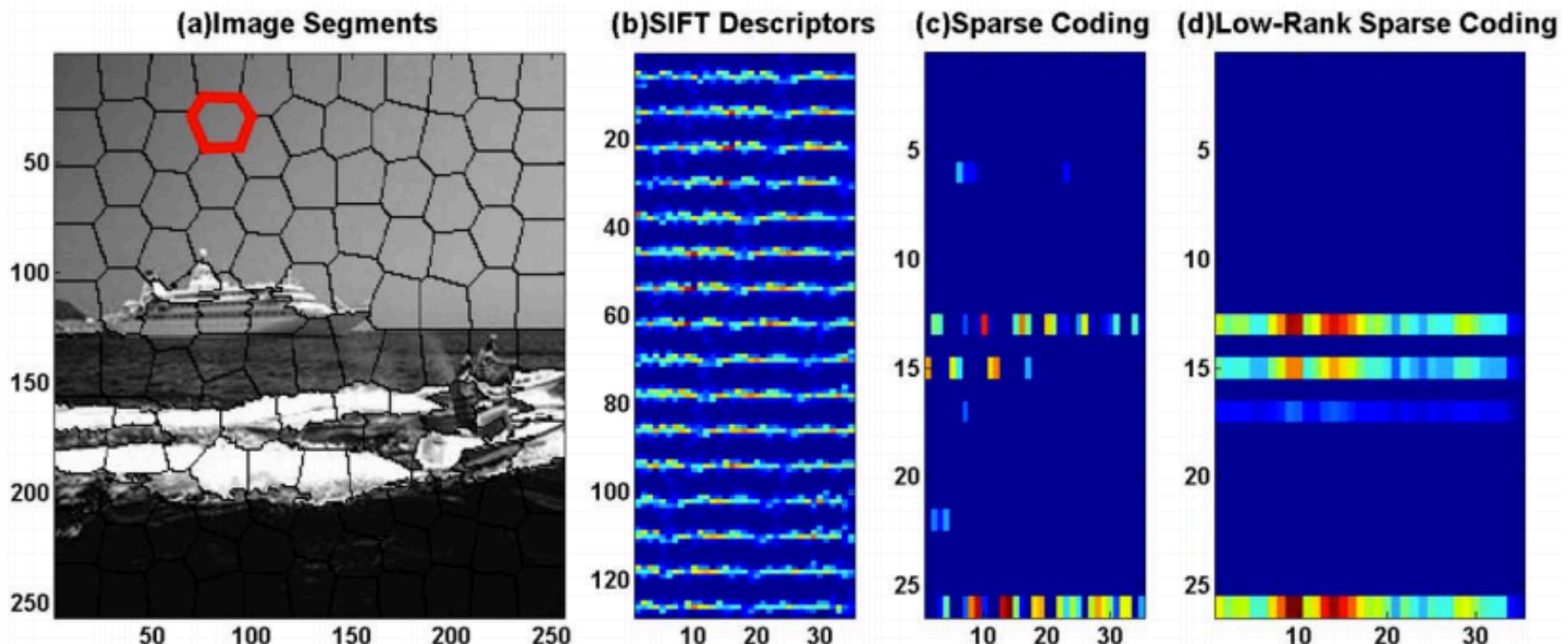


$$\min_{Z,E} \|Z\|_* + \lambda \|E\|_1 + \beta \|Z\|_1 + \mu \left\| \tilde{Z} - Q \right\|_F^2$$

$$s.t X = DZ + E, D = \tilde{X}$$

Applications: Image Analysis

▶ Low-Rank Sparse Coding for Image Classification

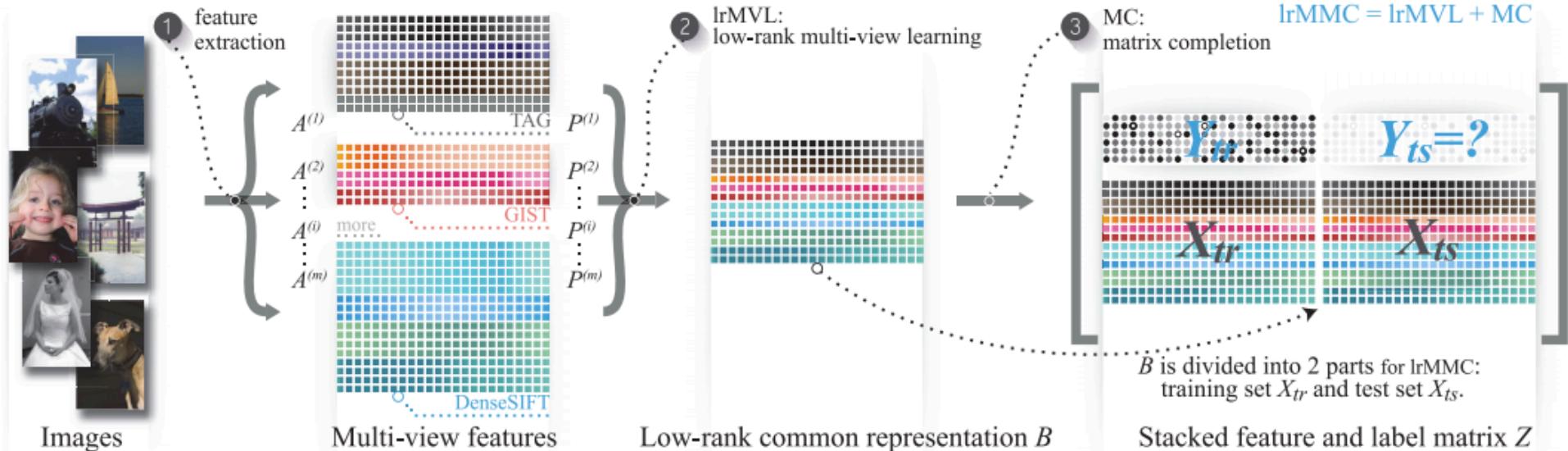


$$\min_{\mathbf{Z}} \frac{1}{2} \|\mathbf{X} - \mathbf{D}\mathbf{Z}\|_F^2 + \lambda_1 \|\mathbf{Z}\|_* + \lambda_2 \|\mathbf{Z}\|_{1,1}$$

- ▶ 108 Tianzhu Zhang, Bernard Ghanem, Si Liu, Changsheng Xu, Narendra Ahuja: *Low-Rank Sparse Coding for Image Classification*. ICCV 2013.

Applications: Image Analysis

► Multi-Label Image Classification



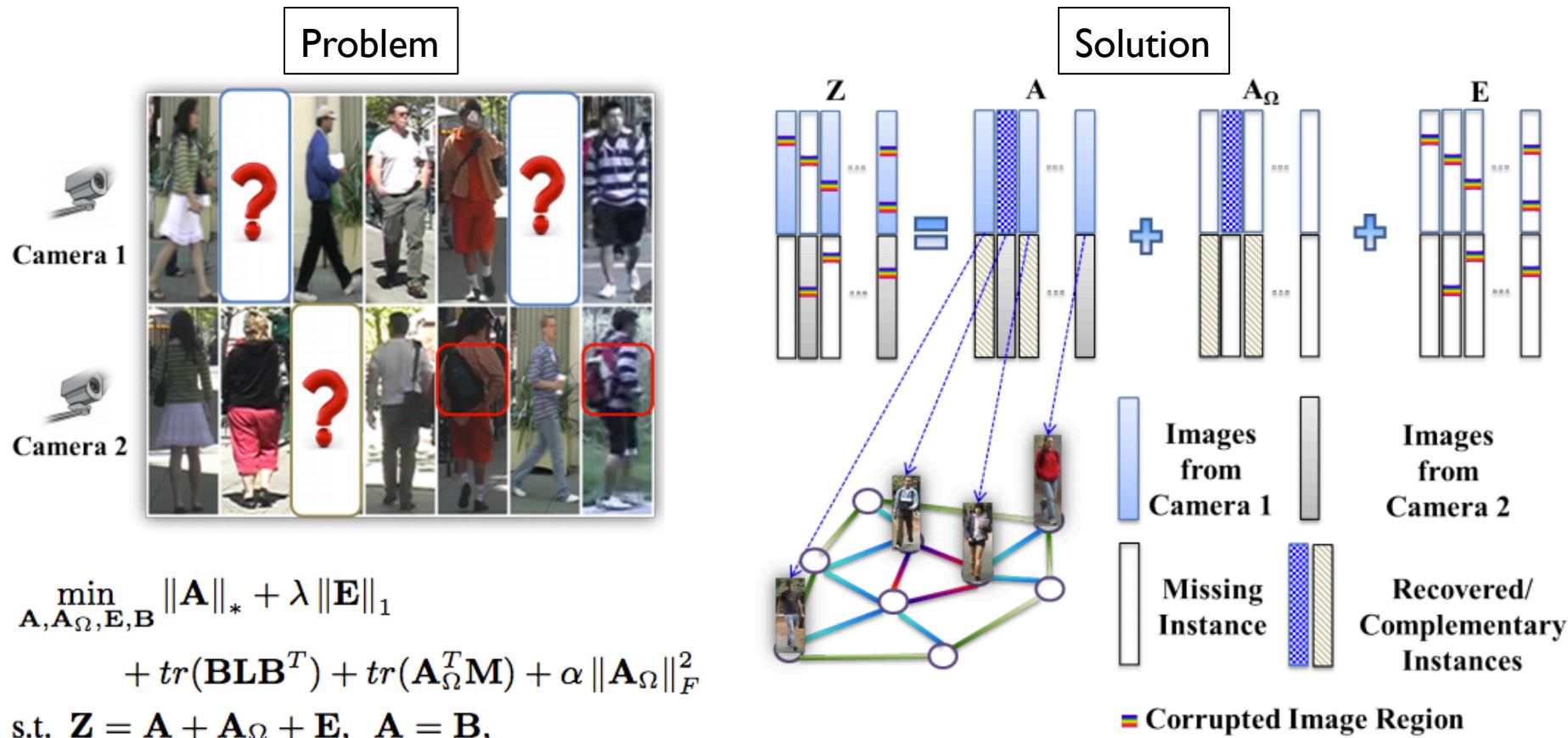
$$\min_{B, P, \theta} \mu \|B\|_* + \|PB - A\|_F^2 + \frac{\gamma}{2} \|\theta\|_2^2$$

$$\text{s.t. } \theta_i \geq 0, \sum \theta_i = 1, i = 1, \dots, m.$$

- 109 Meng Liu, Yong Luo, Dacheng Tao, Chao Xu, Yonggang Wen: *Low-Rank Multi-View Learning in Matrix Completion for Multi-Label Image Classification*. AAAI 2015.

Applications: Image Analysis

► Robust Person Re-identification



Applications: Image Analysis

► Conformal and Low-Rank Model for [Image Restoration](#)

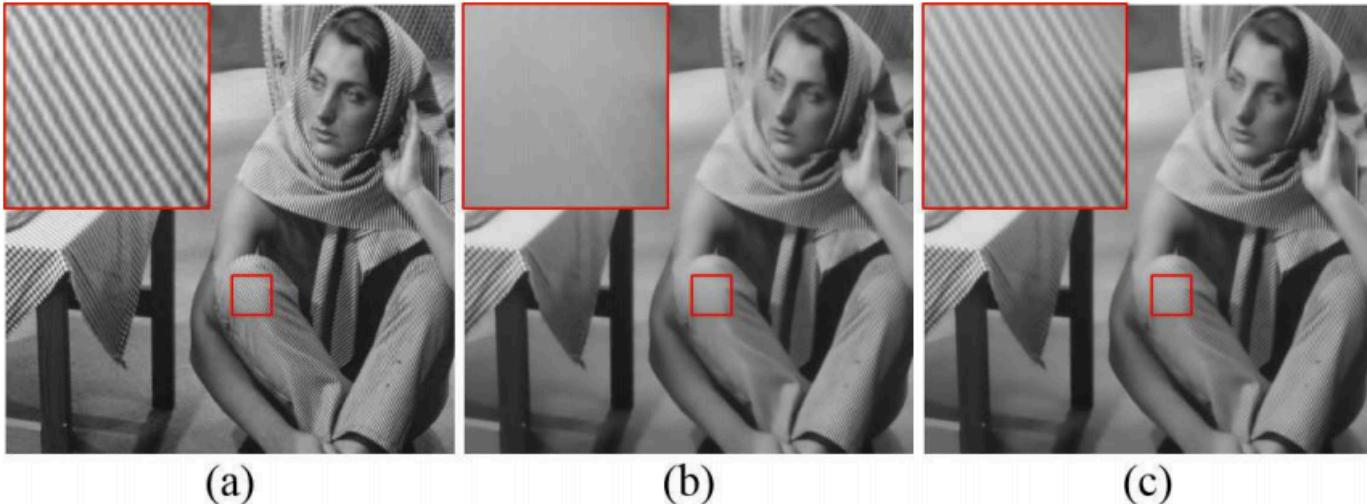


Figure 2. The reconstruction results of image “*Barbara*”. (a) is the source image, (b) and (c) are the reconstruction results without and with conformal and low-rank constraints, respectively.

$$\begin{aligned} & \min_{D, A, S} \sum_i \|x_i - D\alpha_i\|_2^2 + \lambda_1 \sum_i \|\alpha_i\|_1 + \lambda_3 \|A\|_* \\ & + \lambda_2 \sum_i \sum_{j, k \in N_i} (\|x_j - x_k\|^2 - s_i \|\alpha_j - \alpha_k\|^2)^2. \end{aligned}$$

Applications: Image Analysis

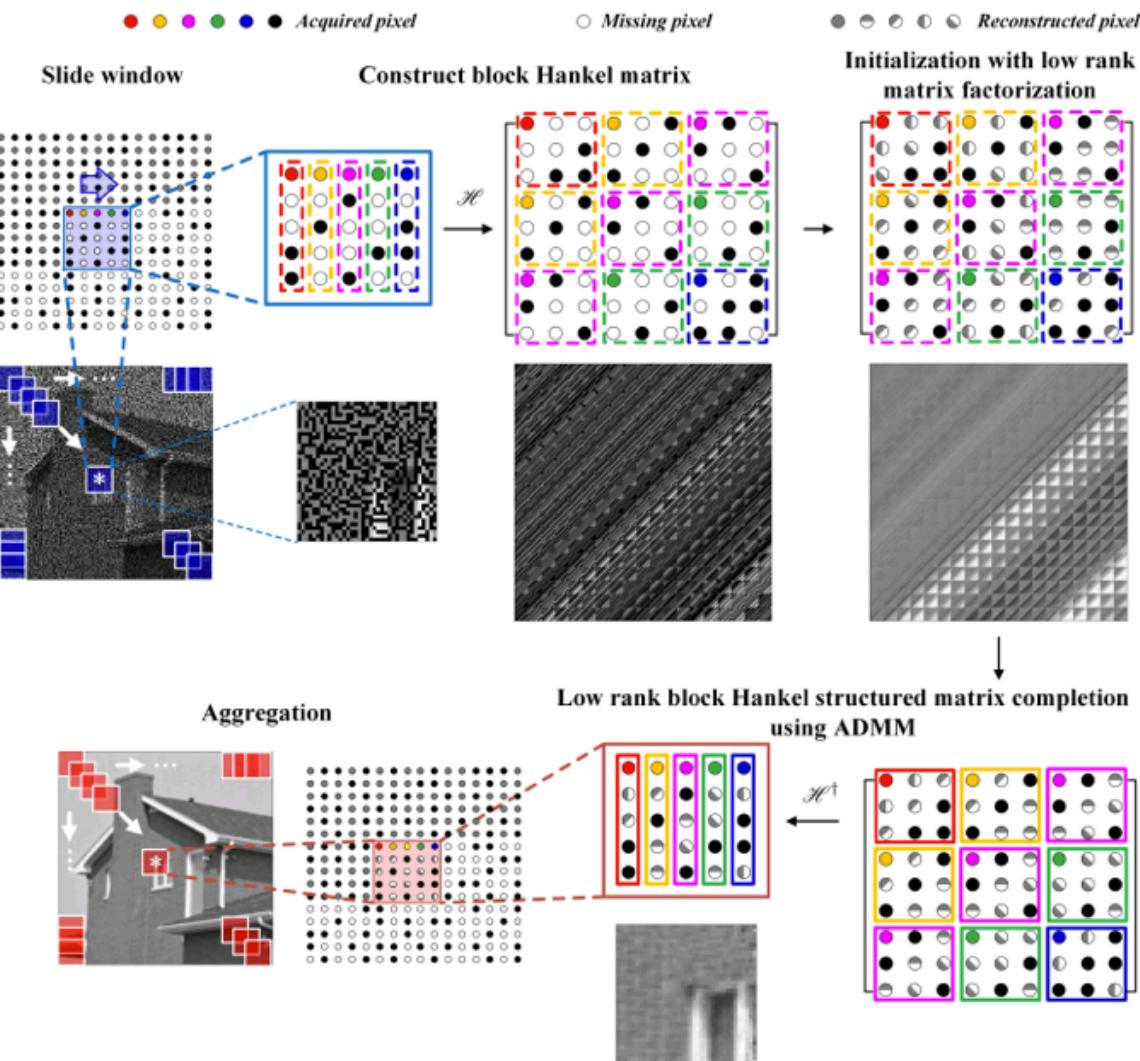
▶ Image Inpainting

$$\|\mathbf{A}\|_* = \min_{\mathbf{U}, \mathbf{V}: \mathbf{A} = \mathbf{U}\mathbf{V}^H} \|\mathbf{U}\|_F^2 + \|\mathbf{V}\|_F^2$$

$$\min_{\mathbf{X}} \quad \|\mathbf{U}\|_F^2 + \|\mathbf{V}\|_F^2$$

subject to $\mathcal{H}\{\mathbf{X}\} = \mathbf{U}\mathbf{V}^H$

$$\mathbf{X}(i, j) = m_{ij}, \quad \forall (i, j) \in \Omega$$



Applications: Image Analysis

► Single Image Super-resolution

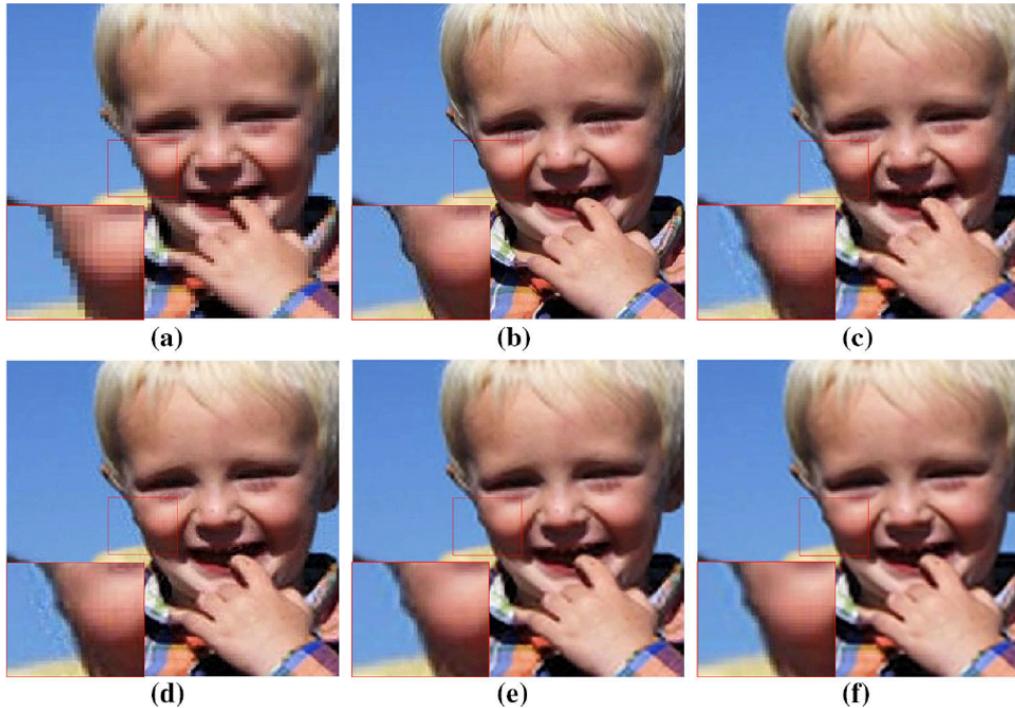
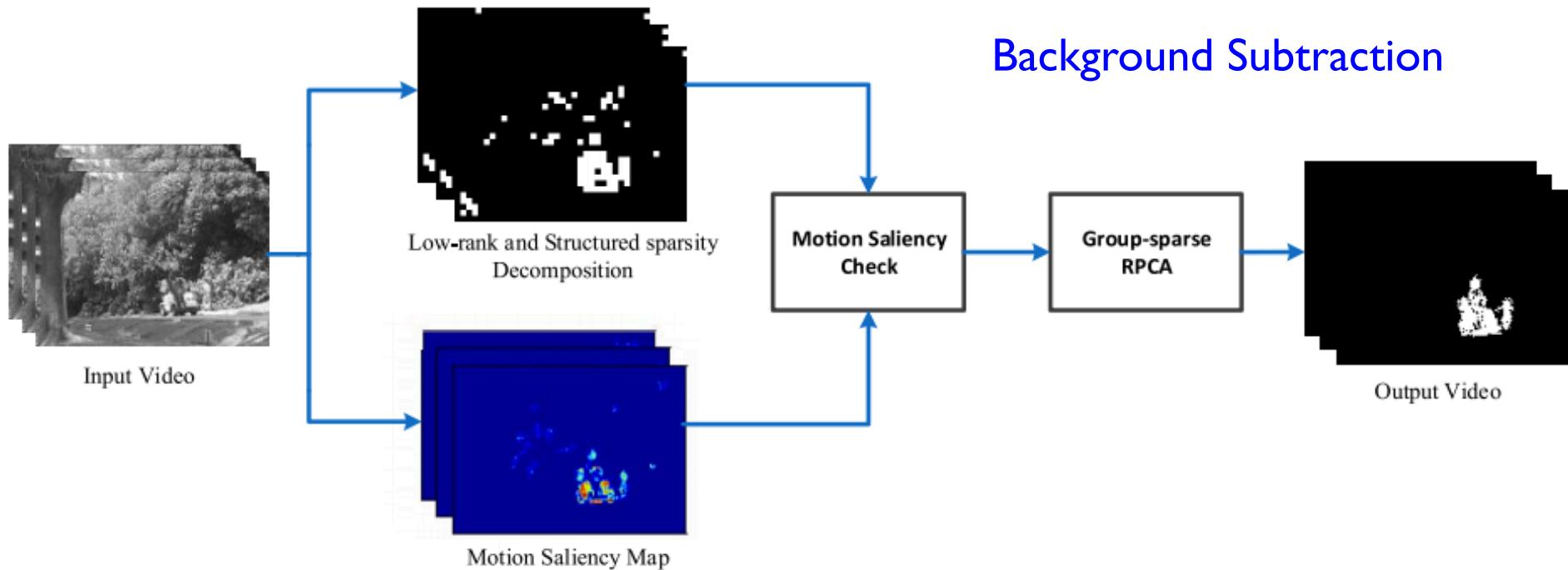


Fig. 5 Comparison of SR results with magnification factor 3 on “Boy”. The areas in the large red rectangles are the $2 \times$ local magnification (bicubic) of the small red rectangles in each example. **a** LR image; **b** original image; **c** results by SC [42]; **d** results by SC-BP; **e** results by CSC-BP-2 [44]; **f** results by the proposed SLR (color figure online)

$$\begin{aligned} \min_{Z, E} \quad & \frac{1}{2} \|\bar{Y} - SHZ\|_F^2 + \frac{\alpha}{2} \|Z - WZ\|_F^2 + \beta \|E\|_1 \\ \text{s.t. } \quad & \hat{X} = Z + E, \quad \text{Rank}(Z) \leq r, \end{aligned}$$

Applications: Video Analysis

▶ Low-Rank and Structure Sparsity Decomposition (LSD)

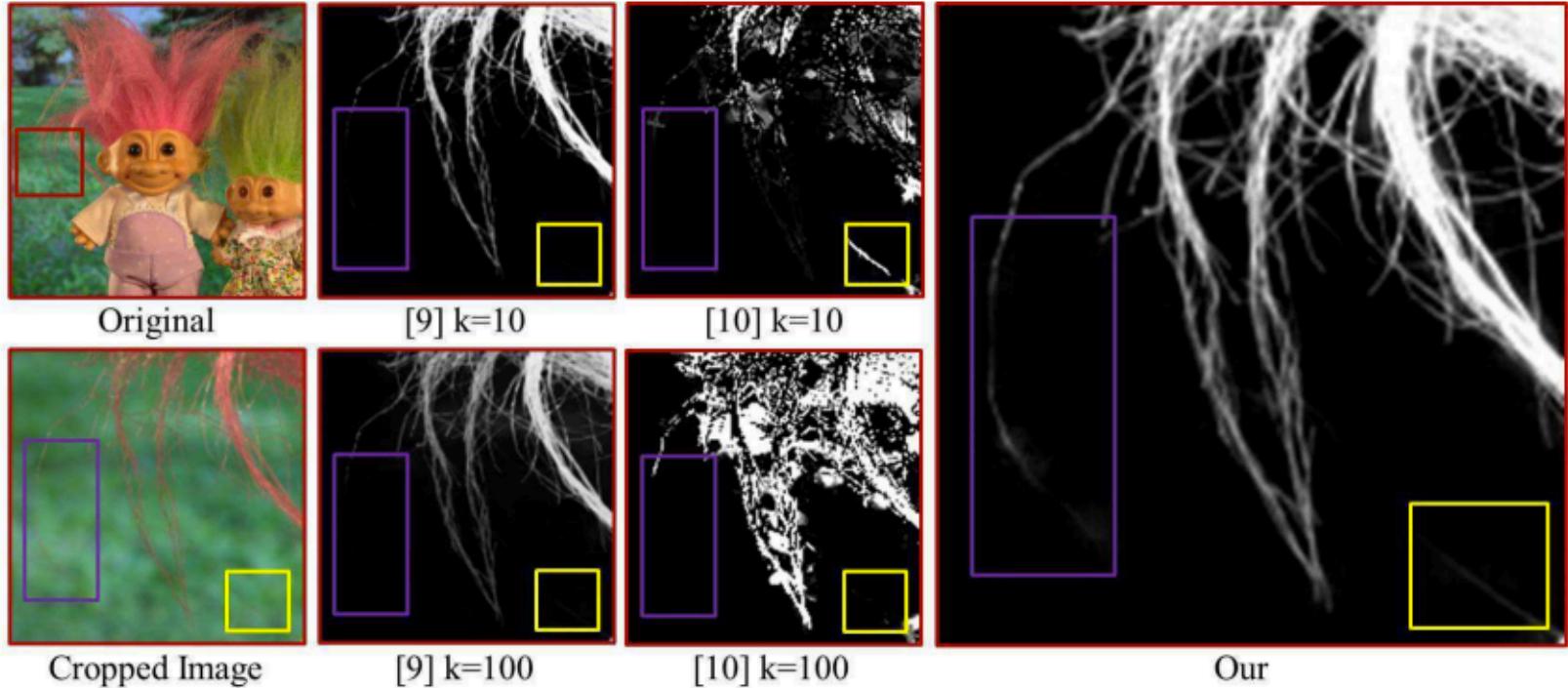


$$\min_{L,S} \|L\|_* + \lambda \Omega(S) \quad s.t. \quad D = L + S \quad \Omega(S) = \sum_{j=1}^n \sum_{g \in \mathcal{G}} \|s_g^j\|_\infty$$

- ▶ 114 Xin Liu, Guoying Zhao, Jiawen Yao, Chun Qi: *Background Subtraction Based on Low-Rank and Structured Sparse Decomposition*, IEEE Trans. IP, 2015.

Applications: Video Analysis

▶ Video Matting



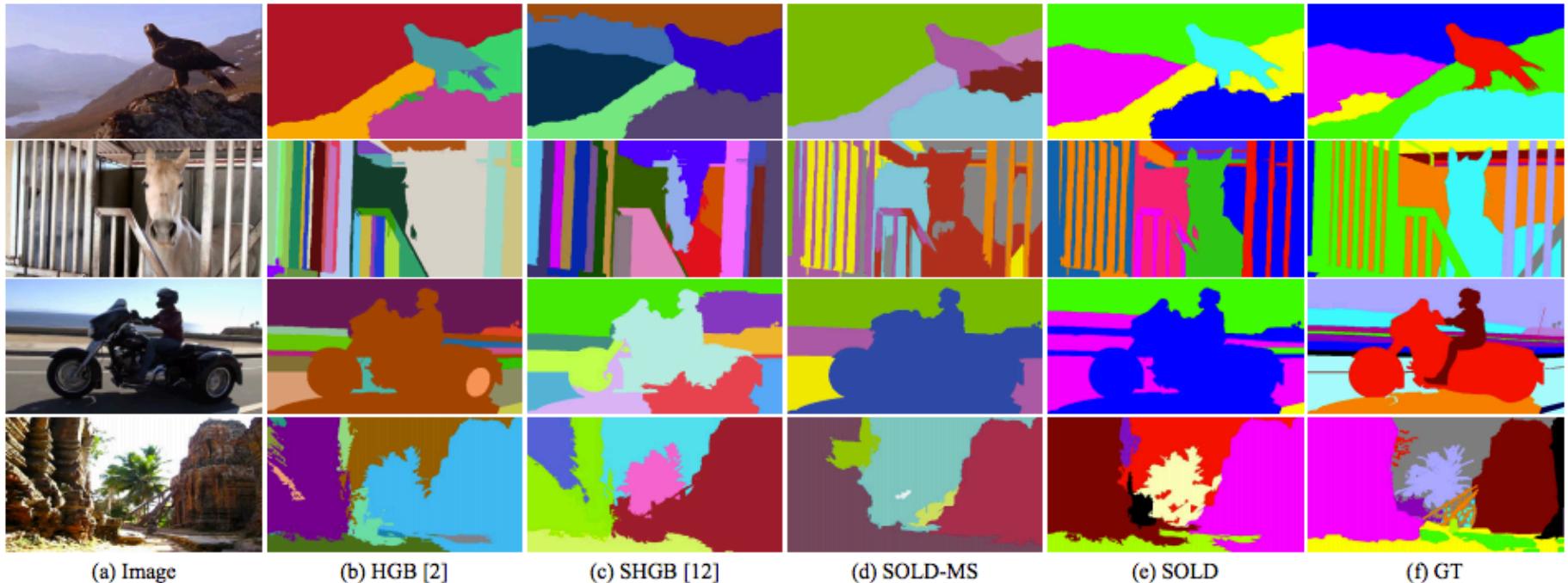
$$\min \sum_i^n (\|\mathbf{X}_i - \mathbf{DW}_i\|_0 + \|\mathbf{W}_i\|_0) + \|\mathbf{W}\|_*,$$

$$\forall p, q, (w_i)_{p,q} \in \mathbf{W}_i, \text{ s.t. } (w_i)_{p,q} \geq 0.$$

- ▶ 115 Xin Liu, Guoying Zhao, Jiawen Yao, Chun Qi: *Background Subtraction Based on Low-Rank and Structured Sparse Decomposition*, IEEE Trans. IP, 2015.

Applications: Video Analysis

▶ Video Segmentation

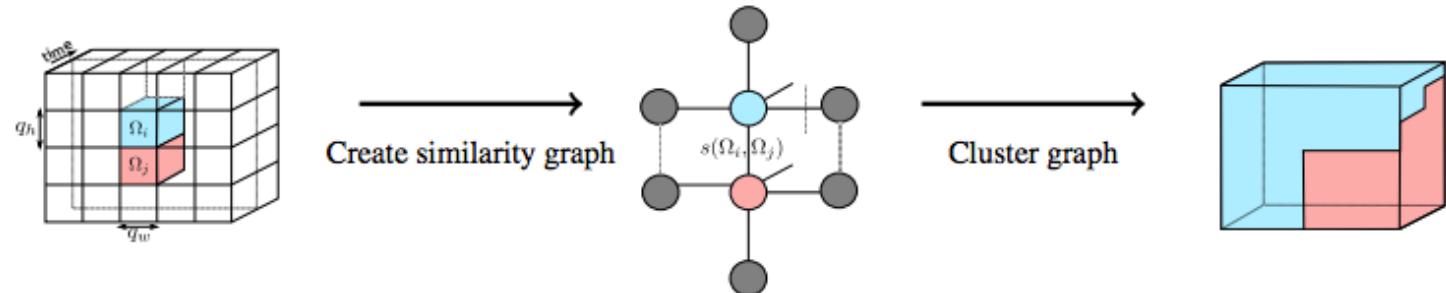


$$\min_{\mathbf{Z}, \mathbf{E}} \frac{1}{2} \|\mathbf{X} - \mathbf{X}\mathbf{Z} - \mathbf{E}\|_F^2 + \alpha \|\mathbf{Z}\|_* + \lambda \|\mathbf{E}\|_1 + \gamma \text{tr}(\mathbf{Z}^T \mathbf{Q})$$

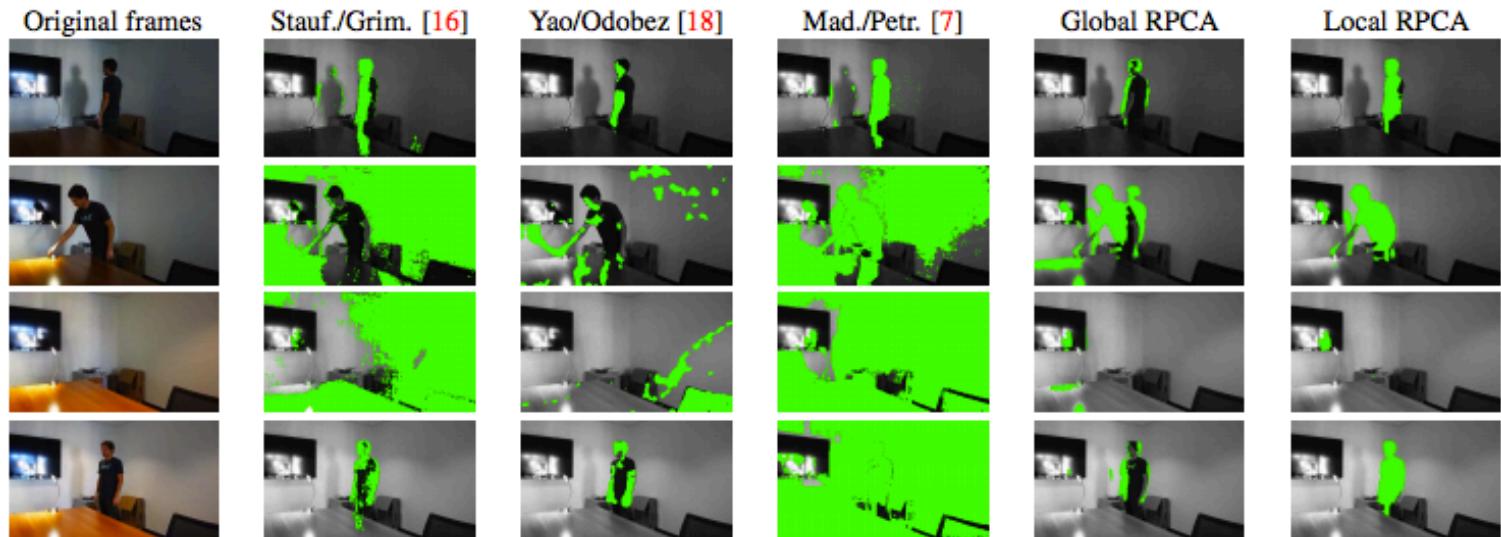
- ▶ 116 Chenglong Li, Liang Lin, Wangmeng Zuo, Shuicheng Yan, Jin Tang: *SOLD: Sub-optimal low-rank decomposition for efficient video segmentation*, CVPR 2015

Applications: Video Analysis

► Spatial-Temporal Video Segmentation



Qualitative (visual) evaluation



Applications: Video Analysis

▶ Video Denoising



Original Frame

Noisy Frame

VBM3D

PCA

Low-Rank

$$\begin{aligned} & \min_Q \|Q\|_* \\ \text{s.t. } & |Q|_\Omega - P|_\Omega \|_F^2 \leq \#(\Omega) \hat{\sigma}^2 \end{aligned}$$

Applications: Video Analysis

▶ Video Deraining and Desnowing

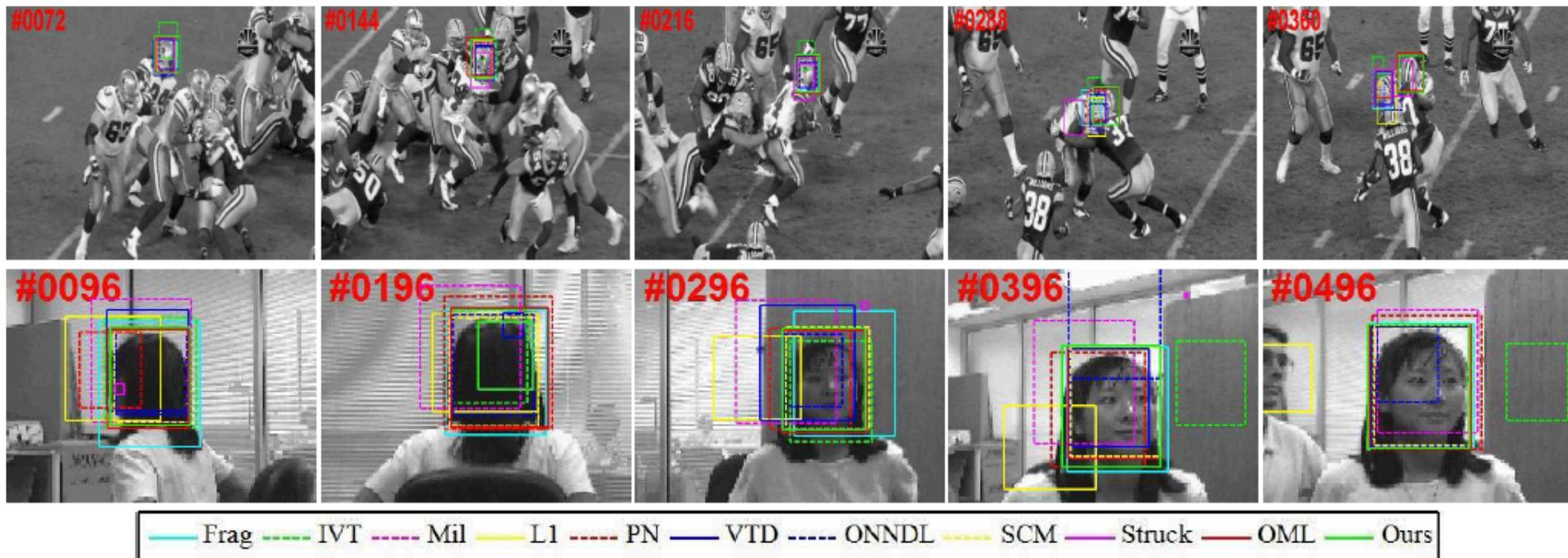


$$\begin{aligned} & \text{minimize} \quad \|\mathbf{X}\|_* \\ & \text{subject to } (\mathbf{1} - \mathbf{M}) \circ \mathbf{X} = (\mathbf{1} - \mathbf{M}) \circ \mathbf{B}, \\ & \quad \mathbf{M} \circ \mathbf{X} \leq \mathbf{M} \circ \mathbf{B}, \end{aligned}$$

- ▶ 119 Jin-Hwan Kim, Jae-Young Sim, Chang-Su Kim: *Video Deraining and Desnowing Using Temporal Correlation and Low-Rank Matrix Completion*, IEEE Trans. Image Processing, 2015.

Applications: Video Analysis

► Object Tracking



$$\min_W: f(W) := \mathbb{E}_t[l(W, t)] + \gamma \|W\|_*$$

$$\text{s.t.: } W \succeq 0.$$

- 120 Yang Cong, Baojie Fan, Ji Liu, Jiebo Luo, Haibin Yu: Speeded Up Low-Rank Online Metric Learning for Object Tracking, IEEE Trans. CSVT, 2015.

Applications: Video Analysis

► Face Clustering in Videos

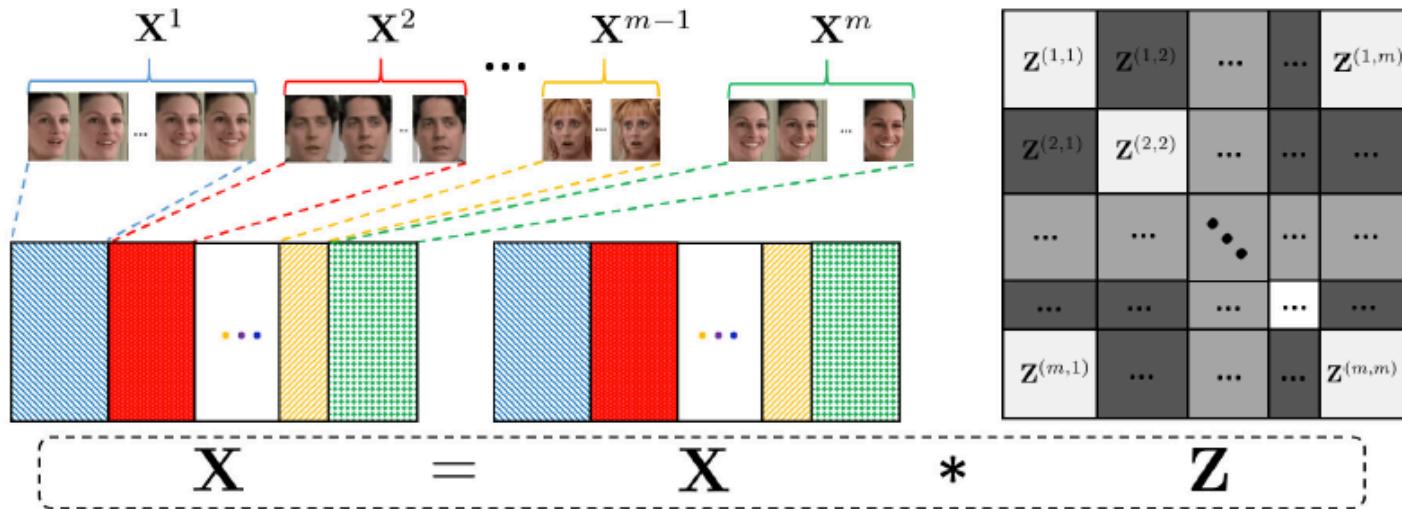
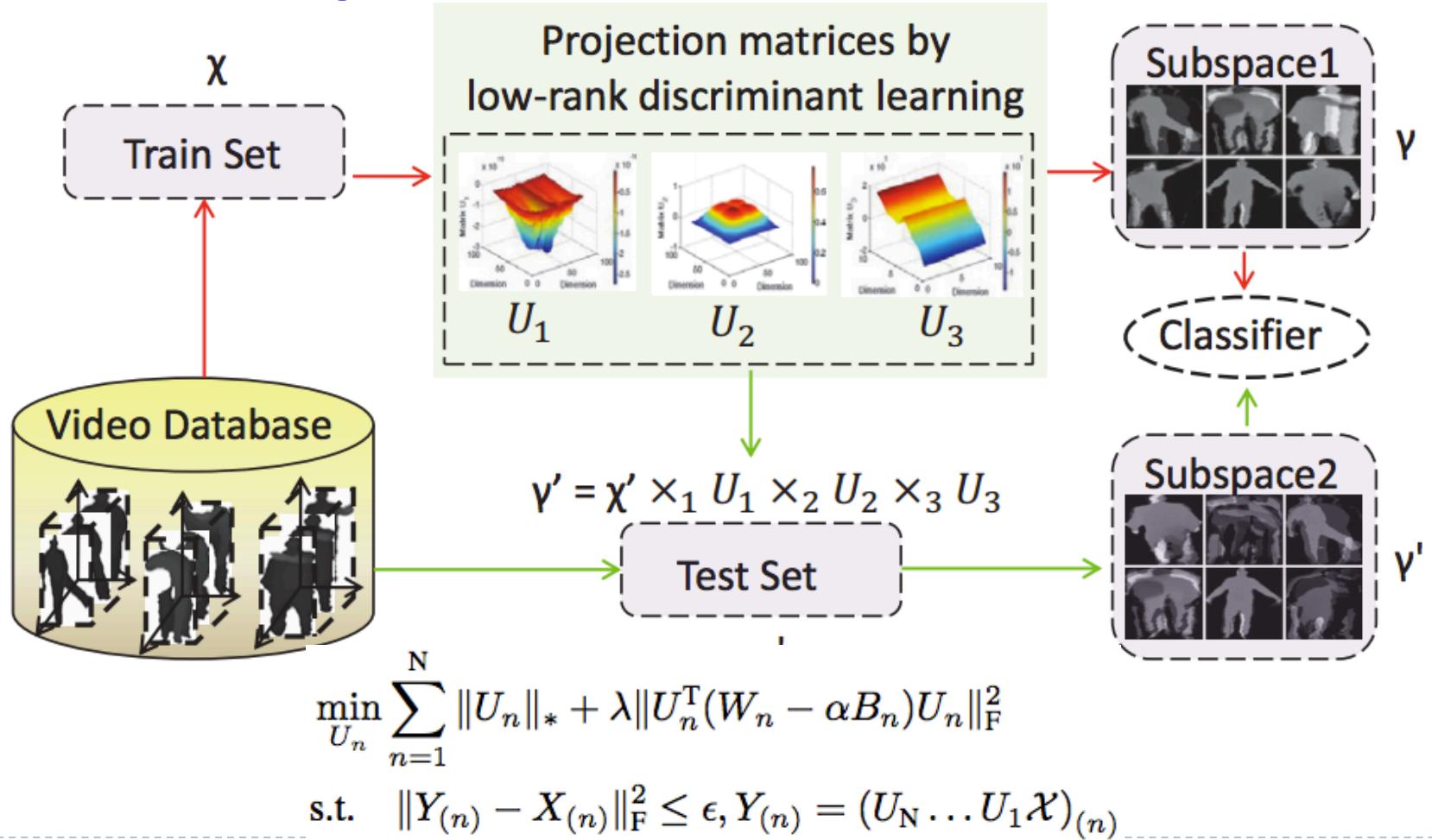


Fig. 1. Illustration of the block-sparse property of \mathbf{Z} , as well as the relationship between \mathbf{X} and \mathbf{Z} in the *noise-free* case. The different colors in \mathbf{X} denote different face tracks.

$$\min_{\mathbf{Z}} \|\mathbf{Z}\|_* + \gamma \sum_{i=1}^m \sum_{j=1}^m Q_{i,j} \|\mathbf{Z}^{(i,j)}\|_F + \frac{\lambda}{2} \|\mathbf{X} - \mathbf{XZ}\|_F^2$$

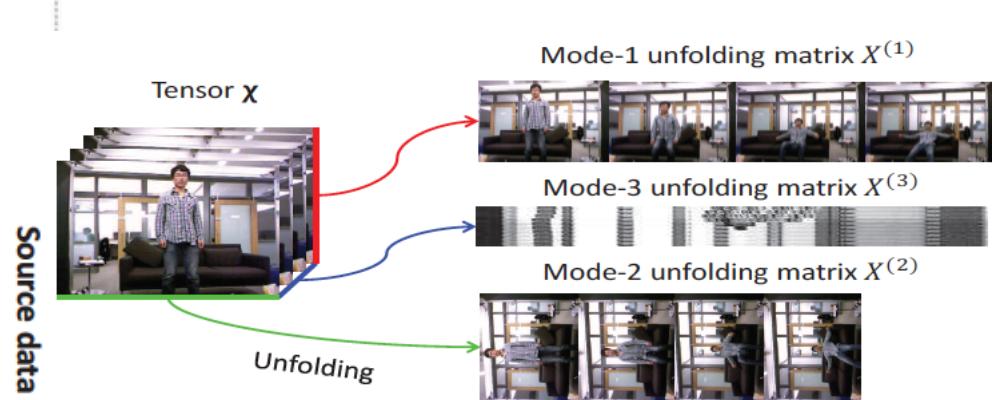
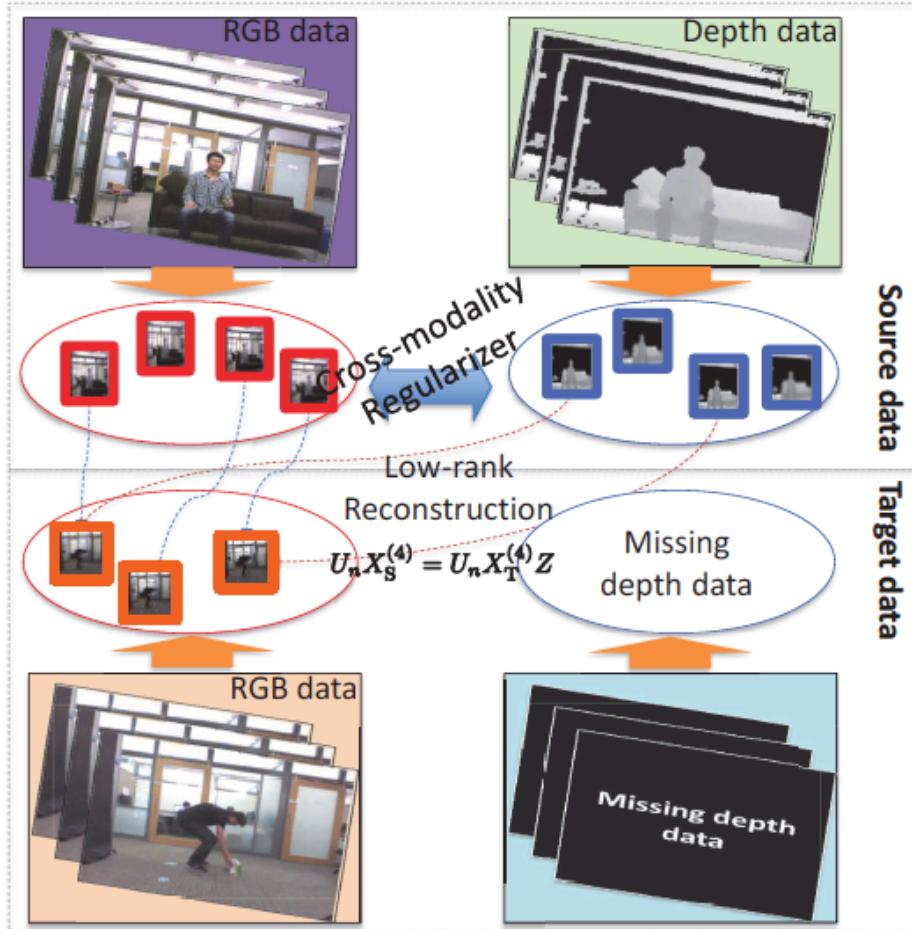
Applications: Video Analysis

► Action Recognition



Applications: Video Analysis

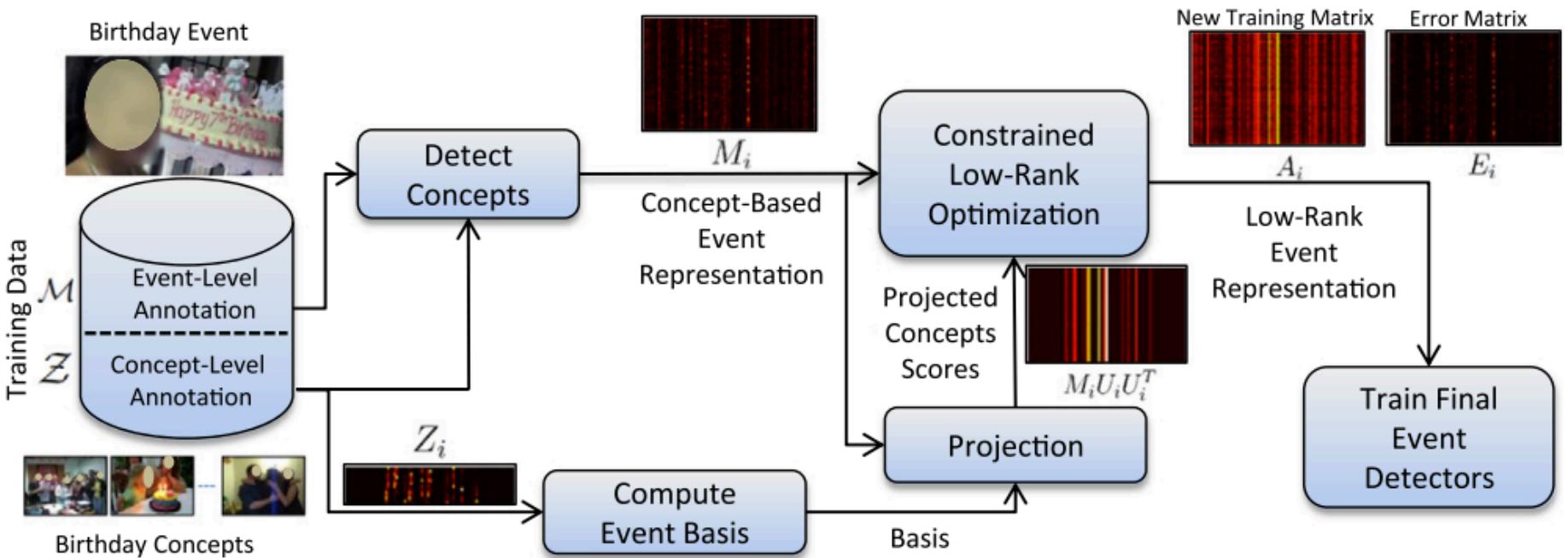
► RGB-D Action Recognition



$$\begin{aligned} & \min_{Z, L, E} \|Z\|_* + \|L\|_* + \lambda \|E\|_{2,1} + \frac{\beta}{2} \text{tr}(Z^T \mathcal{L} Z), \\ \text{s.t. } & U_n X_S = U_n X_T Z + L U_n X_S + E, \\ & U_n^T U_n = I, \quad n = 1, 2, 3, \end{aligned}$$

Applications: Video Analysis

► Complex Event Recognition



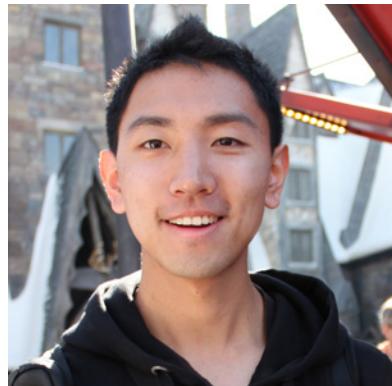
$$\min_{A_i, E_i} \|A_i\|_* + \lambda \|E_i\|_1 + \frac{\tau}{2} \left\| A_i - M_i U_i U_i^T \right\|_F^2 \text{ s.t. } M_i = A_i + E_i$$

- 124 Afshin Dehghan, Omar Oreifej, Mubarak Shah: *Complex event recognition using constrained low-rank representation*. IVC, 2015.

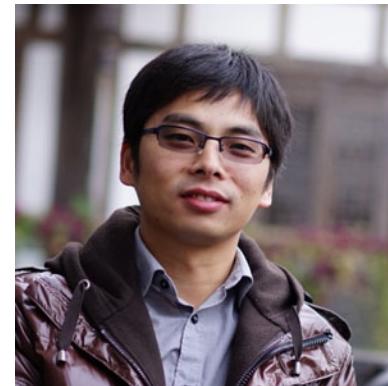
Conclusions

- ▶ Convex/non-convex optimization algorithms
- ▶ Randomized and distributed algorithms
- ▶ Low-rank models for domain adaptation and multi-view learning
- ▶ Applications on image and video analysis

Collaborators



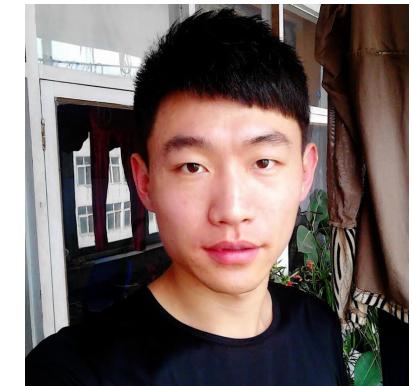
Ming Shao



Allan Ding



Chengcheng Jia



Handong Zhao

Thank you!

SMILE Lab: www.northeastern.edu/smilelab/

References

1. Yun Fu, Low-Rank and Sparse Modeling for Visual Analysis, Springer, 2014.
 2. Sheng Li and Yun Fu: Low-Rank Coding with b-Matching for Semi-supervised Classification, IJCAI 2013.
 3. Sheng Li and Yun Fu: Learning Balanced and Unbalanced Graphs via Low-Rank Coding, IEEE TKDE, 2015.
 4. Sheng Li, Kang Li, Yun Fu: Temporal Subspace Clustering for Human Motion Segmentation. ICCV 2015.
 5. Sheng Li, Yun Fu: Robust Subspace Discovery through Supervised Low-Rank Constraints, SDM 2014.
 6. Sheng Li, Yun Fu: Learning Robust and Discriminative Subspace with Low-Rank Constraints, IEEE TNNLS.
 7. Sheng Li, Ming Shao, Yun Fu: Multi-View Low-Rank Analysis for Outlier Detection. SDM 2015.
 8. Ming Shao, Dmitry Kit, Yun Fu: Generalized Transfer Subspace Learning Through Low-Rank Constraint. IJCV, 2014.
 9. Zhengming Ding and Yun Fu: Low-Rank Common Subspace for Multi-View Learning. ICDM 2014.
 10. Zhengming Ding, Ming Shao, Yun Fu: Deep Low-Rank Coding for Transfer Learning. IJCAI 2015.
 11. Zhengming Ding and Yun Fu: Robust Multi-View Subspace Learning through Dual Low-Rank Decompositions. AAAI 2016.
 12. Chengcheng Jia, Guoqiang Zhong, Yun Fu: Low-Rank Tensor Learning with Discriminant Analysis for Action Classification and Image Recovery., AAAI 2014.
 13. Handong Zhao, Zhengming Ding, Yun Fu: Pose-Dependent Low-Rank Embedding for Head Pose Estimation, AAAI 2016.
-

References

14. Zhouchen Lin, Minming Chen, Yi Ma. The augmented lagrange multiplier method for exact recovery of corrupted low-rank matrices. arXiv, 2010.
15. Zhouchen Lin, Risheng Liu, Zhixun Su: Linearized Alternating Direction Method with Adaptive Penalty for Low-Rank Representation. NIPS 2011: 612-620
16. Guangcan Liu, Zhouchen Lin, Shuicheng Yan, Ju Sun, Yong Yu, Yi Ma: Robust Recovery of Subspace Structures by Low-Rank Representation. *IEEE Trans. PAMI*, 2013
17. Hu et al. Fast and accurate matrix completion via truncated nuclear norm regularization, IEEE TPAMI, 2013.
18. Zhong et al. A Nonconvex Relaxation Approach for Rank Minimization Problems. AAAI 2015.
19. Lu et al. Generalized Nonconvex Nonsmooth Low-Rank Minimization. CVPR 2014.
20. Peng et al. Subspace clustering using log-determinant rank approximation. KDD 2015.
21. Kang et al. Robust PCA via Nonconvex Rank Approximation. ICDM 2015.
22. Oh et al. Fast Randomized Singular Value Thresholding for Nuclear Norm Minimization. CVPR 2015.
23. Ameet Talwalkar, Lester W. Mackey, Yadong Mu, Shih-Fu Chang, Michael I. Jordan: Distributed Low-Rank Subspace Segmentation. ICCV 2013.
24. I-Hong Jhuo, Dong Liu, D.T. Lee, Shih-Fu Chang. Robust Visual Domain Adaptation with Low-Rank Reconstruction, CVPR 2012.
25. Chang Xu, Dacheng Tao, Chao Xu: A Survey on Multi-view Learning. arXiv 2013.
26. Tianzhu Zhang, Bernard Ghanem, Si Liu, Changsheng Xu, Narendra Ahuja: Low-Rank Sparse Coding for Image Classification. ICCV 2013.

References

27. Meng Liu, Yong Luo, Dacheng Tao, Chao Xu, Yonggang Wen: Low-Rank Multi-View Learning in Matrix Completion for Multi-Label Image Classification. AAAI 2015.
28. Chia-Po Wei, Chih-Fan Chen, Yu-Chiang Frank Wang: Robust Face Recognition With Structurally Incoherent Low-Rank Matrix Decomposition, IEEE TIP, 2014.
29. Chenjing Yan, Congyan Lang, Songhe Feng: Facial Age Estimation Based on Structured Low-rank Representation, ACM MM, 2015
30. Ming-Chia Tsai, Chia-Po Wei, Yu-Chiang Frank Wang: Graph regularized low-rank matrix recovery for robust person re-identification, ICIP 2015
31. Jianwei Li, Xiaowu Chen, Dongqing Zou, Bo Gao, Wei Teng: Conformal and Low-Rank Sparse Representation for Image Restoration. ICCV 2015.
32. Kyong Hwan Jin, Jong Chul Ye: Annihilating Filter-Based Low-Rank Hankel Matrix Approach for Image Inpainting, IEEE TIP, 2015.
33. Jialin Peng, Benny Y. C. Hon, Dexing Kong: A structural low rank regularization method for single image super-resolution, Machine Vision and Applications, 2015.
34. Xin Liu, Guoying Zhao, Jiawen Yao, Chun Qi: Background Subtraction Based on Low-Rank and Structured Sparse Decomposition, IEEE TIP, 2015.

References

35. Chenglong Li, Liang Lin, Wangmeng Zuo, Shuicheng Yan, Jin Tang: SOLD: Sub-optimal low-rank decomposition for efficient video segmentation, CVPR 2015
36. Alasdair Newson, Mariano Tepper, Guillermo Sapiro: Low-Rank Spatio-Temporal Video Segmentation. BMVC 2015.
37. Hui Ji, Chaoqiang Liu, Zuowei Shen, Yuhong Xu: Robust video denoising using low rank matrix completion, CVPR 2010.
38. Jin-Hwan Kim, Jae-Young Sim, Chang-Su Kim: Video Deraining and Desnowing Using Temporal Correlation and Low-Rank Matrix Completion, IEEE TIP, 2015.
39. Yang Cong, Baojie Fan, Ji Liu, Jiebo Luo, Haibin Yu: Speeded Up Low-Rank Online Metric Learning for Object Tracking, IEEE TCSVT, 2015.
40. Shijie Xiao, Mingkui Tan, Dong Xu, Weighted Block-Sparse Low Rank Representation for Face Clustering in Videos. ECCV 2014.