NODEA Note

1 Basic Knowledge

Def of ODE & ODEs: (1st order) ODE: $\frac{dy}{dt} = f(t,y)$ & ODEs: $\frac{d\mathbf{y}}{dt} = \mathbf{f}(t,\mathbf{y})$, $\mathbf{y} = (y_1, \dots, y_d)^T, \mathbf{f}(t,\mathbf{y}) = (f_1(t,\mathbf{y}), \dots, f_d(t,\mathbf{y}))^T$

Autonomous: $\frac{d\mathbf{y}}{dt} = \mathbf{f}(\mathbf{y}) \Rightarrow$ autonomous ODE(s). || $\Downarrow$ New Autonomous ODEs: $\frac{d\mathbf{y}}{ds} = \mathbf{f}(y_{d+1}, \mathbf{y})$ and $\frac{dy_{d+1}}{ds} = 1$

· **Change to Autonomous**: For $\frac{d\mathbf{y}}{dt} = \mathbf{f}(t,\mathbf{y})$. Let $y_{d+1} = t$ and *new* independent variable s s.t. $\frac{dt}{ds} = 1$ $\Uparrow$

Linearity: ODE: $\frac{dy}{dt} = f(t,y)$ is linearity if $f(t,y) = a(t)y + b(t)$ || ODEs: If each ODE is linear, then the ODEs are linear.

Picard's Theorem: If $f(t,y)$ is continuous in $D := \{(t,y) : t_0 \le t \le T, |y - y_0| < K\}$ and $\exists L > 0$ (Lipschitz constant) s.t.

$\forall (t,u),(t,v) \in D \quad |f(t,u) - f(t,v)| \le L|u - v|$. And Assume that $M_f(T - t_0) \le K, M_f := \max\{|f(t,u)| : (t,u) \in D\}$

$\Rightarrow$ **Then**, $\exists$ a unique continuously differentiable solution $y(t)$ to the IVP $\frac{dy}{dt} = f(t,y), y(t_0) = y_0$ on $t \in [t_0, T]$.

Existence & Uniqueness Theorem: IVP $\frac{d\mathbf{y}}{dt} = \mathbf{f}(t,\mathbf{y}), \mathbf{y}(t_0) = \mathbf{y}_0$. If $f(t,y)$ and $\frac{\partial f}{\partial y_i}$ are continuous in a neighborhood of $(t_0, \mathbf{y}_0)$.

$\Rightarrow$ **Then**, $\exists I := (t_0 - \delta, t_0 + \delta)$ s.t. $\exists$ a unique continuously differentiable solution $\mathbf{y}(t)$ to the IVP on $t \in I$.

2 Euler's Method and Taylor Series Method