HAlg Note

1 Basic Knowledge

Def of Matrix: A mapping from $\{1,...,n\} \times \{1,...,m\}$ to a field F is called a $n \times m$ matrix over F.

· The set of all $n \times m$ matrices over F is denoted by $Mat(n \times m; F) := Maps(\{1,...,n\} \times \{1,...,m\}, F)$.

· If $n = m$, we sill speak of a **Square Matrix** and shorten the notation to $Mat(n; F)$.

Solution Sets of Inhomogeneous Systems of Linear Equations: Solution = 特解 (Particular Solution) + 通解 (Homogeneous solution)

Def of Group $(G, *)$: A set G with a operator $*$ is a group if: **Closure**: $\forall g, h \in G, g * h \in G$; **Associativity**: $\forall g, h, k \in G, (g * h) * k = g * (h * k)$;
Identity: $\exists e \in G, \forall g \in G, e * g = g * e = g$; **Inverse**: $\forall g \in G, \exists g^{-1} \in G, g * g^{-1} = g^{-1} * g = e$.

Field (F): A set F is a field with two operators: (addition)$+ : F \times F \to F; (\lambda, \mu) \to \lambda + \mu$ (multiplication)$\cdot : F \times F \to F; (\lambda, \mu) \to \lambda\mu$ if:
$(F, +)$ and $(F \setminus \{0_F\}, \cdot)$ are abelian groups with identity $0_F, 1_F$. and $\lambda(\mu + \nu) = \lambda\mu + \lambda\nu$

F**-Vector Space (V)**: A set V over a field F is a vector space if: V is an abelian group $V = (V, \dot{+})$ and $\forall\ \vec{v}, \vec{w} \in V \quad \lambda, \mu \in F$
a map $F \times V \to V : (\lambda, \vec{v}) \to \lambda\vec{v}$ satisfies: **I**: $\lambda(\vec{v} \dot{+} \vec{w}) = (\lambda\vec{v}) \dot{+} (\lambda\vec{w})$ **II**: $(\lambda + \mu)\vec{v} = (\lambda\vec{v}) \dot{+} (\mu\vec{v})$
III: $\lambda(\mu\vec{v}) = (\lambda\mu)\vec{v}$ **IV**: $1_F\vec{v} = \vec{v}$ ps:**I,II** are **Distributive Laws**; **III** is **Associative Law**. **Trivial Vector Space**: $V = \vec{0}$

· **Properties of** F**-Vector Space (V)**: **a.** $0_F\vec{v} = \vec{0}$ **b.** $(-1_F)\vec{v} = -\vec{v}$ **c.** $\lambda\vec{0} = \vec{0}$ **d.** If $\lambda\vec{v} = \vec{0}$, then $\lambda = 0$ or $\vec{v} = \vec{0}$ or both.