HAlg Note

1 Basic Knowledge

Useful Complex Number Properties: $|Re(z)|, |Im(z)| \leq |z| \quad Re(z) = \frac{z+\overline{z}}{2}, Im(z) = \frac{z-\overline{z}}{2i}, |z|^2 = z\overline{z}$

Triangle (Reverse) Inequality: $|z_1 + z_2| \leq |z_1| + |z_2| \quad |z_1| - |z_2| \leq |z_1 - z_2|$

Argument: $\arg(z) := \{\theta : z = |z|e^{i\theta}\} = \{Arg(z) + 2\pi k : k \in \mathbb{Z}\}$ **Principle Value of Argument**: $Arg(z) \in (-\pi, \pi]$

Open/Closed/Punctured ε-disc: $D_\varepsilon(z_0) := \{z \in \mathbb{C} : |z - z_0| < \varepsilon\} \quad \overline{D}_\varepsilon(z_0) := \{z \in \mathbb{C} : |z - z_0| \leq \varepsilon\} \quad D'_\varepsilon(z_0) := \{z \in \mathbb{C} : 0 < |z - z_0| < \varepsilon\}$

Open/Closed Set in $\mathbb{C}$: $U \subset \mathbb{C}$ is **open** if $\forall z_0 \in U, \exists \varepsilon > 0, D_\varepsilon(z_0) \subseteq U \quad U$ is **closed** if $\mathbb{C} \setminus U$ is open **Lemma**: $D_\varepsilon, D'_\varepsilon$ open, $\overline{D}_\varepsilon$ closed.

2 Holomorphic Functions

3 Basic Knowledge