A quick note on implied volatility
Applied Stochastic Processes (FIN 514)

Instructor: Jaehyuk Choi

Peking University HSBC Business School, Shenzhen, China

2019-20 Module 1 (Fall 2019)

Overview

- The Black-Scholes-Merton (BSM), Bachelier (normal) models are seldom used to *predict* the option price. The option prices are determined from the supply and demand of market.
- However, the pricing models are still important because they provide a consistent measure to intuitively understand the prices of the options with different strikes, time to maturity, etc.
- *Implied volatility* (IV) is the value of the volatility in pricing model which returns (or solve) the price of an option given (i.e., from market):

$$C(K, S_0, \sigma, T_e) = \text{Price}$$

- How can you compare the two prices?

$$C(K = 105, S_0 = 100, T_e = 1) = 5.9 \quad P(K = 98, S_0 = 100, T_e = 1) = 8.7$$

The implied volatility is same as 20%.

General rule

- Option prices (both call and put) increase as the volatility increases: $C(K, S_0, \sigma, T_e)$ is monotonically increasing function.
- Option value = Time value + Intrinsic Value
  - Intrinsic value: the value you get by exercising option now. (> 0)
  - Time value: the extra value from the change of the underlying price until the expiry. (> 0)
- The intrinsic value can be understood as the option value wih $\sigma = 0$, $C(K, S_0, \sigma = 0, T_e)$, hence the minimum value.
- The call option value as $\sigma \to \infty$:
  - BSM model ($S_T \geq 0$): S_0. The underlying stock is always worth more than any call option with $K > 0$.
  - Normal model (S_T can be negative): ∞. Call option can protect the (infinite) loss.

IV computation

- The computation of IV depends on numerical root-solving method. [Demo]
  - Newton method: using vega,

$$\sigma^{(k+1)} = \sigma^{(k)} - \frac{C(\sigma^{(k)}, \cdots) - \text{Price}}{V(\sigma^{(k)})}, \quad V(\sigma) = \frac{\partial C(\sigma, \cdots)}{\partial \sigma}$$

  - Brent's method: [Demo]
- BSM model:
  - Jackel (2015): Let's Be Rational
    Machine epsilon error within two step iterations. (partial final project)
- Normal model:
  - Choi et al (2007). Numerical Approximation of the Implied Volatility Under Arithmetic Brownian Motion:
    Polynomial approximation with error ($< 10^{-9}$)