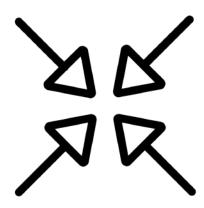
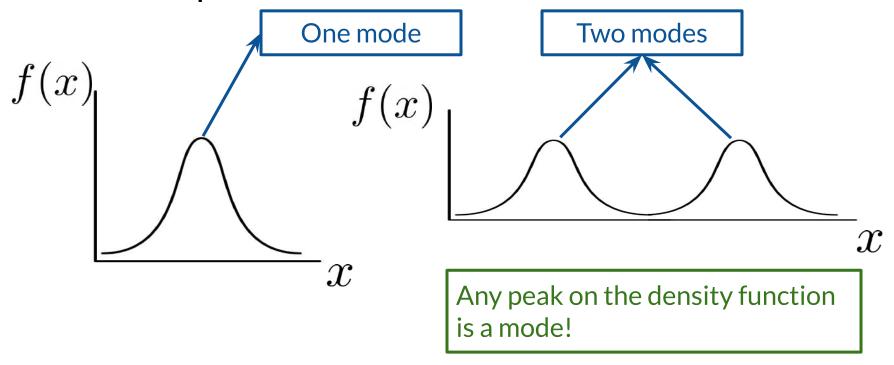
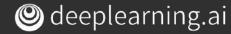


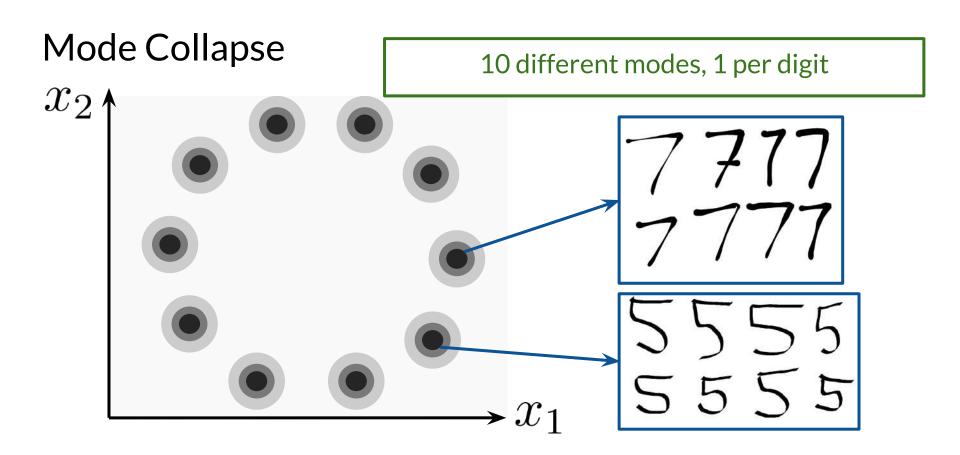
Outline

- Modes in distributions
- Mode collapse in GANs
- Intuition behind it during training

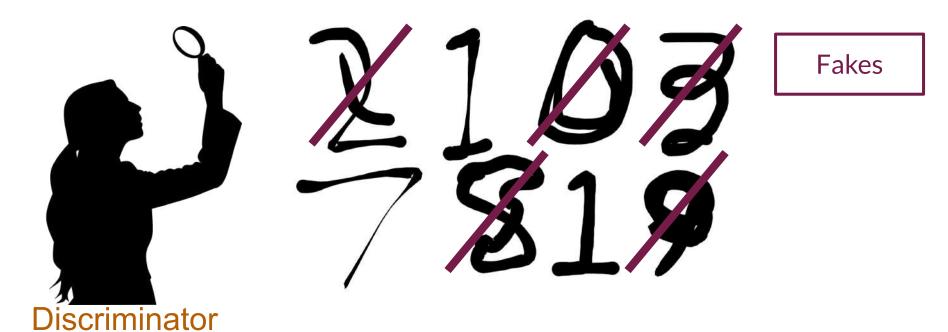


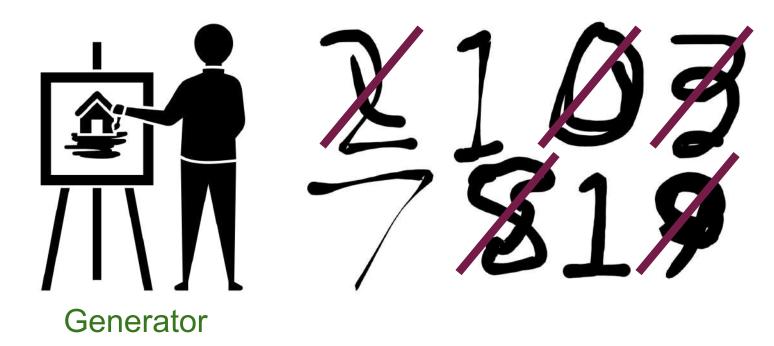


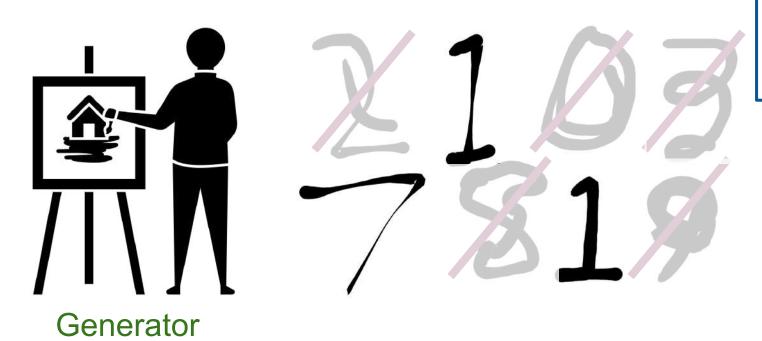




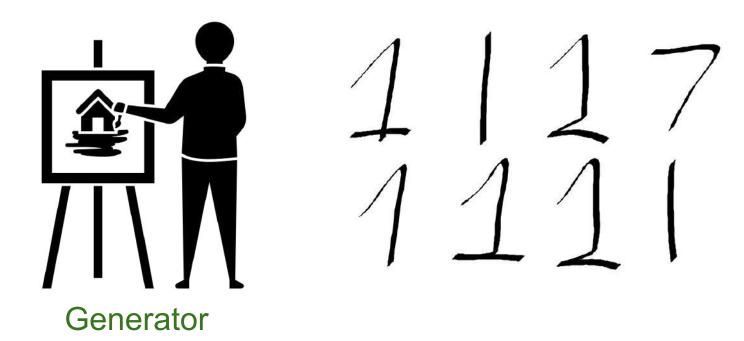




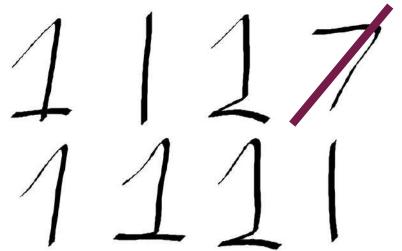




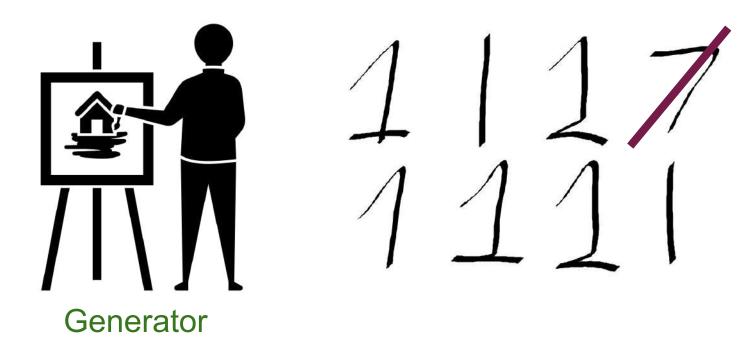
Fakes that fooled the discriminator



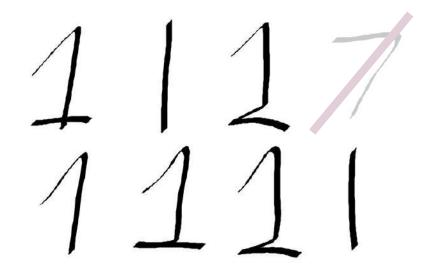




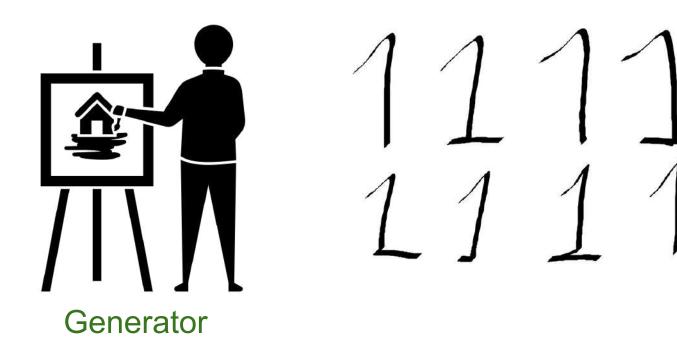
Fakes







Fakes that fooled the discriminator



Summary

- Modes are peaks in the distribution of features
- Typical with real-world datasets
- Mode collapse happens when the generator gets stuck in one mode

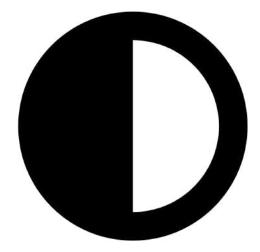




Problem with BCE Loss

Outline

- BCE Loss and the end objective in GANs
- Problem with BCE Loss



BCE Loss in GANs

$$J(\theta) = -\frac{1}{m} \sum_{i=1}^{m} \left[y^{(i)} \log h(x^{(i)}, \theta) + (1 - y^{(i)}) \log(1 - h(x^{(i)}, \theta)) \right]$$
Prediction

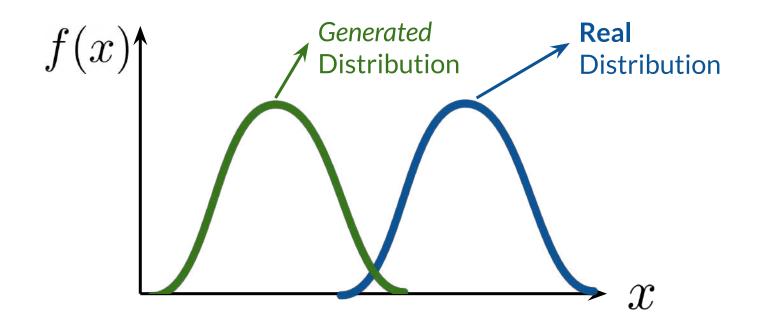
Parameters

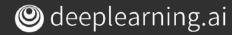


Maximize cost

Discriminator Cost

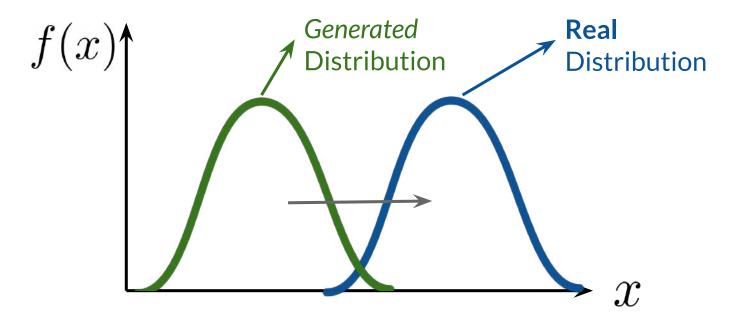
Objective in GANs





Objective in GANs

Make the generated and real distributions look similar



BCE Loss in GANs

Criticizing is more straightforward



Single output

Easier to train than the generator

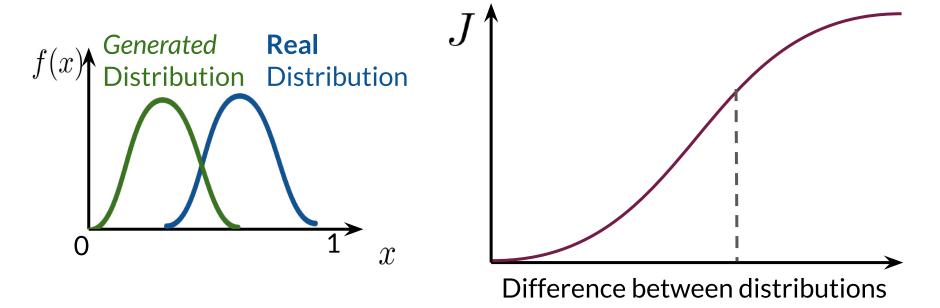


Complex output

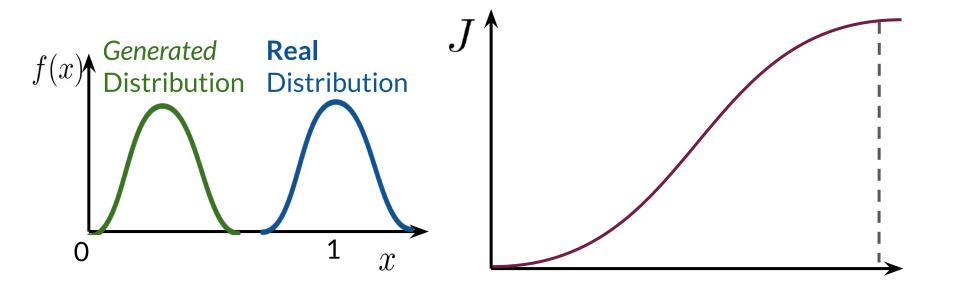
Difficult to train

Often, the discriminator gets better than the generator

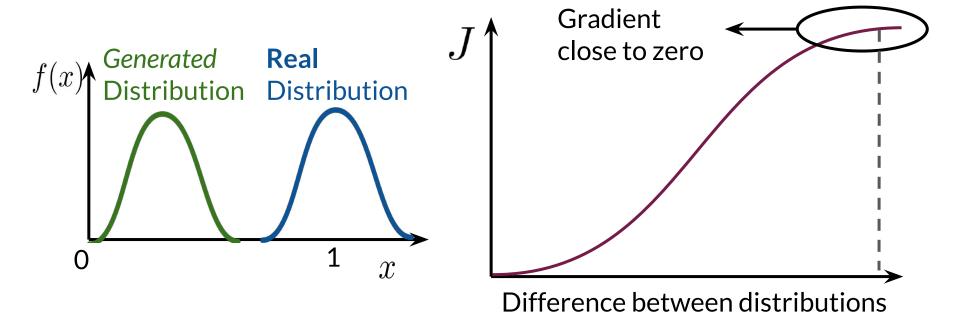
Problems with BCE Loss



Problems with BCE Loss

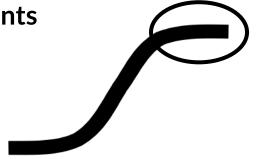


Problems with BCE Loss



Summary

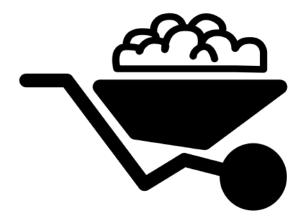
- GANs try to make the real and generated distributions look similar
- When the discriminator improves too much, the function approximated by BCE Loss will contain flat regions
- Flat regions on the cost function = vanishing gradients

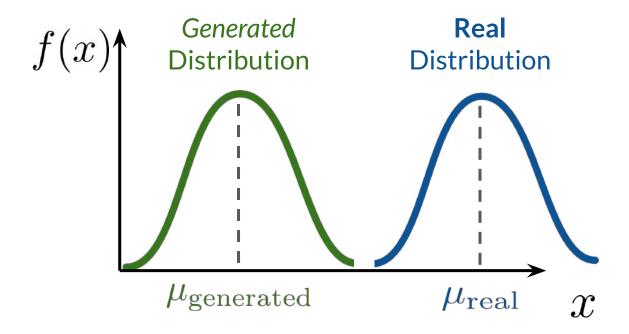


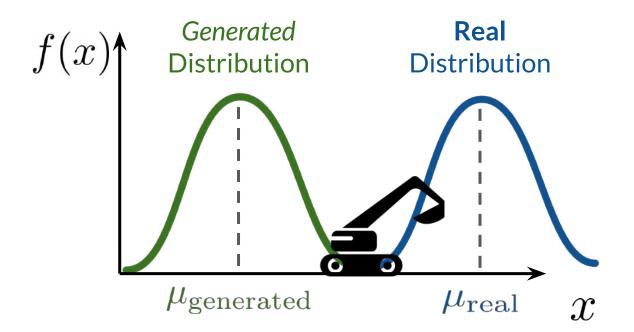


Outline

- Earth Mover's Distance (EMD)
- Why it solves the vanishing gradient problem of BCE Loss

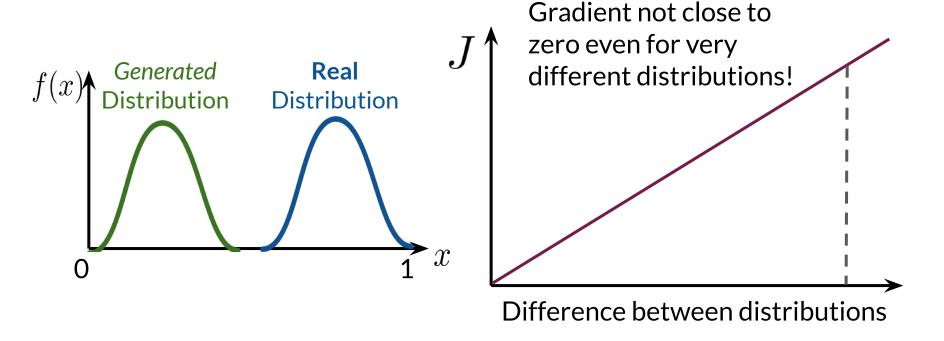






Effort to make the generated distribution equal to the real distribution

Depends on the distance and amount moved



Summary

- Earth mover's distance (EMD) is a function of amount and distance
- Doesn't have flat regions when the distributions are very different
- Approximating EMD solves the problems associated with BCE





Wasserstein Loss

Outline

- BCE Loss Simplified
- W-Loss and its comparison with BCE Loss



BCE Loss Simplified

$$J(\theta) = -\frac{1}{m} \sum_{i=1}^{m} \left[y^{(i)} \log h(x^{(i)}, \theta) + (1 - y^{(i)}) \log(1 - h(x^{(i)}, \theta)) \right]$$



Minimize cost



Maximize cost



$$J(\theta) = \begin{bmatrix} \frac{1}{m} \sum_{i=1}^{m} y^{(i)} \log h(x^{(i)}, \theta) + (1 - y^{(i)}) \log(1 - h(x^{(i)}, \theta)) \end{bmatrix}$$

$$\min_{d} \max_{q} - \left[\mathbb{E}(\log (d(x))) + \mathbb{E}(y^{(i)} - h(x^{(i)}, \theta)) \right]$$



Minimize cost



Maximize cost



$$J(\theta) = \frac{1}{m} \sum_{i=1}^{m} y^{(i)} \log h(x^{(i)}, \theta) + (1 - y^{(i)}) \log(1 - h(x^{(i)}, \theta))$$

$$\min_{d} \max_{g} - \left[\mathbb{E}(\log (d(x))) + \mathbb{E}(1 - \log (d(g(z)))) \right]$$



Minimize cost



Maximize cost

W-Loss

W-Loss approximates the Earth Mover's Distance

W-Loss

W-Loss approximates the Earth Mover's Distance

$$\min_{g} \max_{c} \ \mathbb{E}(c(x)) - \mathbb{E}(c(g(z)))$$

W-Loss

W-Loss approximates the Earth Mover's Distance

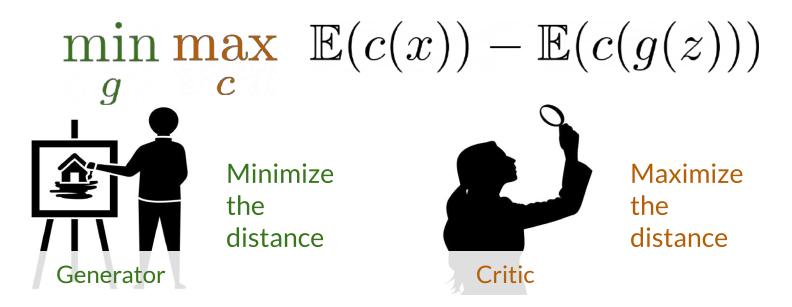
$$\min_{q} \max_{c} \mathbb{E}(c(x)) - \mathbb{E}(c(g(z)))$$



Maximize the distance

W-Loss

W-Loss approximates the Earth Mover's Distance



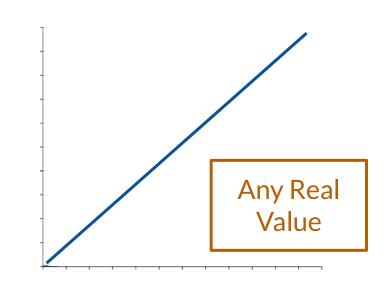
Discriminator Output

Discriminator output

 $z^{[l]} \ge 0$ 0.5

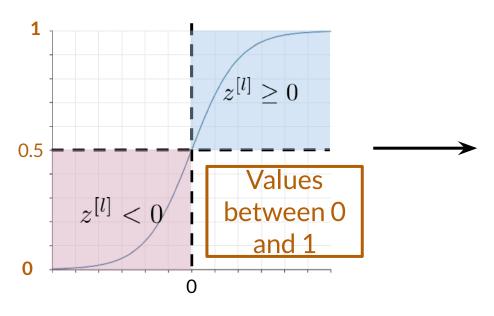
Values
between 0
and 1

Discriminator output

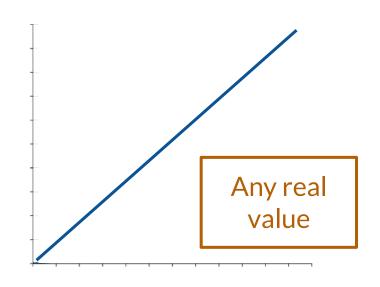


Discriminator Output

Discriminator output



Discriminator output Critic



W-Loss vs BCE Loss

BCE Loss

W-Loss

Discriminator outputs between 0 and 1

$$-\left[\mathbb{E}(\log (d(x))) + \mathbb{E}(1 - \log (d(g(z))))\right]$$

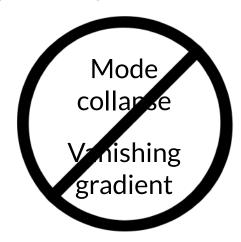
Critic outputs any number

$$\mathbb{E}(c(x)) - \mathbb{E}(c(g(z)))$$

W-Loss helps with mode collapse and vanishing gradient problems

Summary

- W-Loss looks very similar to BCE Loss
- W-Loss prevents mode collapse and vanishing gradient problems





Condition on Wasserstein Critic

Outline

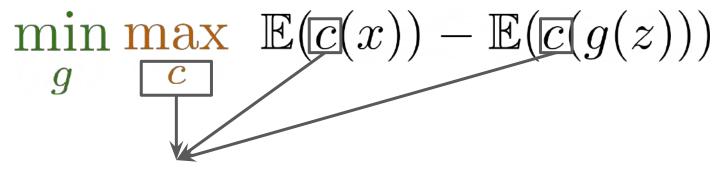
- Continuity condition on the critic's neural network
- Why this condition matters



$$\min_{g} \max_{c} \ \mathbb{E}(c(x)) - \mathbb{E}(c(g(z)))$$

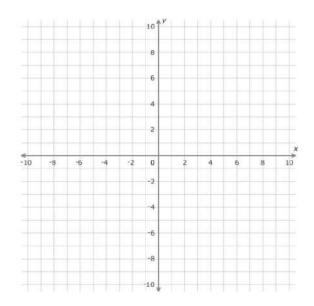
$$\min_{q} \max_{c} \mathbb{E}(c(x)) - \mathbb{E}(c(g(z)))$$

$$\min_{q} \max_{c} \mathbb{E}(c(x)) - \mathbb{E}(c(g(z)))$$

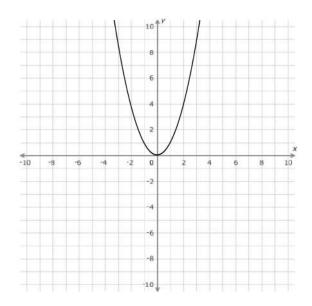


Needs to be 1-Lipschitz Continuous

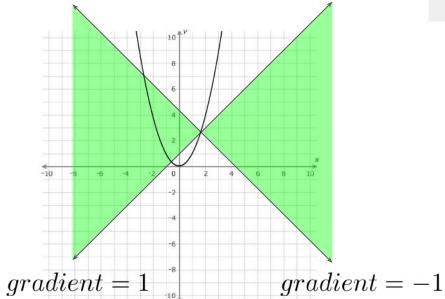
Critic needs to be 1-L Continuous

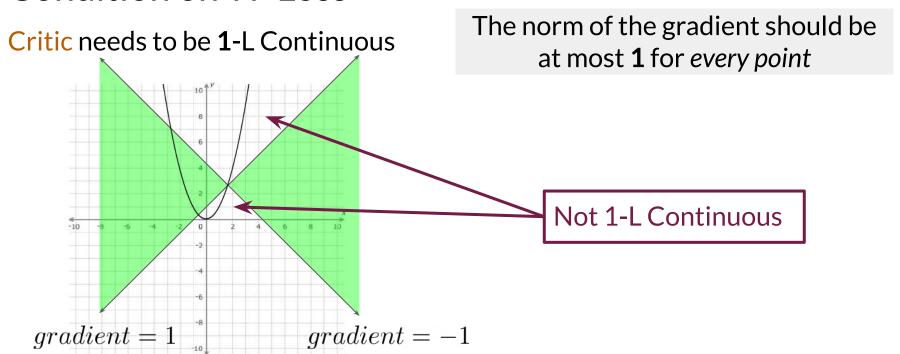


Critic needs to be 1-L Continuous

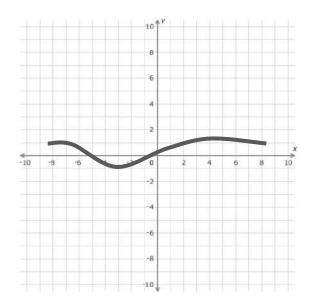


Critic needs to be 1-L Continuous

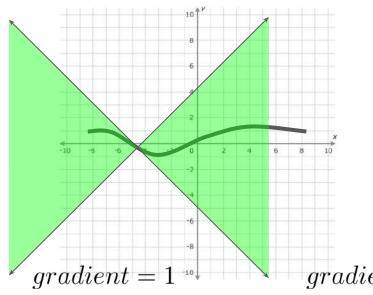




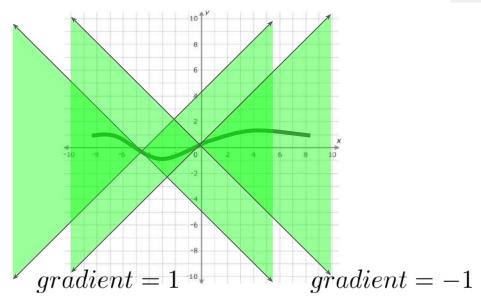
Critic needs to be 1-L Continuous



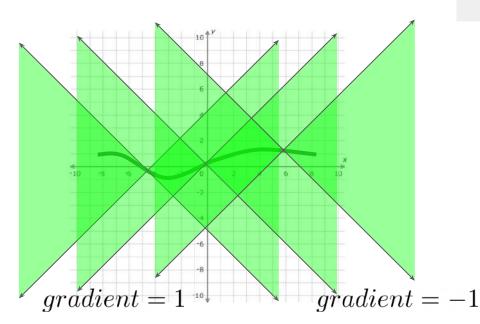
Critic needs to be 1-L Continuous



Critic needs to be 1-L Continuous

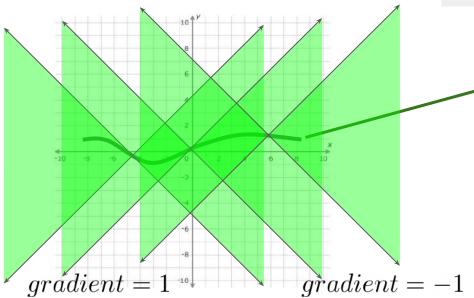


Critic needs to be 1-L Continuous



Critic needs to be 1-L Continuous

The norm of the gradient should be at most **1** for every point



W-Loss is valid

Needed for training stable neural networks with W-Loss

Summary

- Critic's neural network needs to be 1-L Continuous when using W-Loss
- This condition ensures that W-Loss is validly approximating Earth
 Mover's Distance





1-Lipschitz Continuity Enforcement

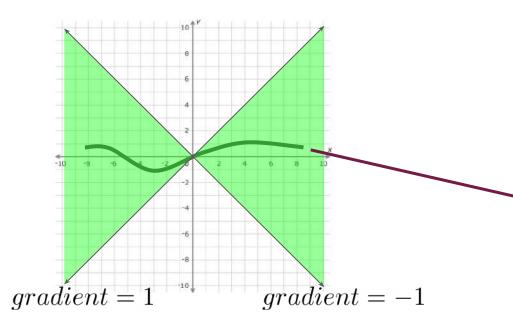
Outline

- Weight clipping and gradient penalty
- Advantages of gradient penalty



1-L Enforcement

Critic needs to be 1-L Continuous



Norm of the gradient at most 1

$$||\nabla f(x)||_2 \le 1$$

Slope of the function at most 1

1-L Enforcement: Weight Clipping

Weight clipping forces the weights of the critic to a fixed interval

Gradient descent to update weights

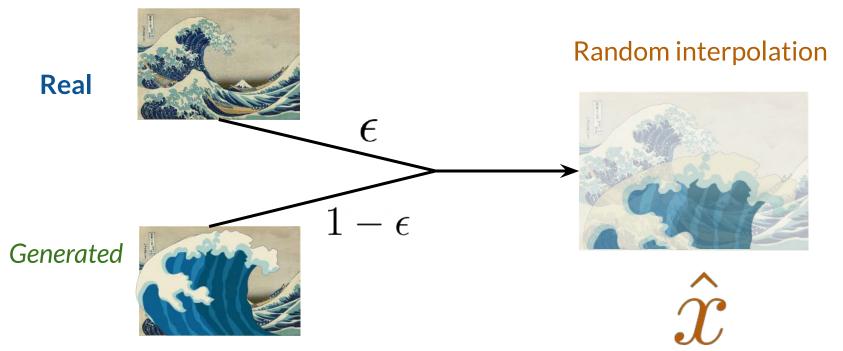
Clip the critic's weights

Limits the learning ability of the critic

$$\min_{g} \max_{c} \ \mathbb{E}(c(x)) - \mathbb{E}(c(g(z))) + \lambda \operatorname{reg}$$

Regularization of the critic's gradient

 $\textbf{Real} \qquad \qquad \boldsymbol{\epsilon}$



$$\mathbb{E}(||\nabla c(\hat{x})||_2-1)^2$$
 Regularization term

$$\mathbb{E}(||\nabla c(\hat{x})||_2-1)^2$$
 Regularization term

$$\mathbb{E}(||\nabla c(\hat{x})||_2-1)^2$$
 Regularization term $\epsilon x + (1-\epsilon)g(z)$ Interpolation

$$\mathbb{E}(||\nabla c(\hat{x})||_2-1)^2$$
 Regularization term $ex+(1-\epsilon)g(z)$ Interpolation

$$\mathbb{E}(||\nabla c(\hat{x})||_2 - 1)^2$$

Regularization term

$$\epsilon x + (1 - \epsilon)g(z)$$
Real Generated

Interpolation

Putting It All Together

$$\min_{\boldsymbol{q}} \max_{\boldsymbol{c}} \mathbb{E}(c(x)) - \mathbb{E}(c(g(z))) + \lambda \mathbb{E}(||\nabla c(\hat{x})||_2 - 1)^2$$

Putting It All Together

$$\min_{\boldsymbol{g}} \max_{\boldsymbol{c}} \mathbb{E}(c(x)) - \mathbb{E}(c(g(z))) + \lambda \mathbb{E}(||\nabla c(\hat{x})||_2 - 1)^2$$

Makes the GAN less prone to mode collapse and vanishing gradient

Putting It All Together

$$\min_{\boldsymbol{g}} \max_{\boldsymbol{c}} \mathbb{E}(c(x)) - \mathbb{E}(c(g(z))) + \lambda \mathbb{E}(||\nabla c(\hat{x})||_2 - 1)^2$$

Makes the GAN less prone to mode collapse and vanishing gradient

Tries to make the critic be 1-L Continuous, for the loss function to be continuous and differentiable

Summary

- Weight clipping and gradient penalty are ways to enforce 1-L continuity
- Gradient penalty tends to work better

