

CSE215

Foundations of Computer Science

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Agenda

- Midterm 2 info
- Mock exam

Finish around 4h45

Midterm exam 2 info

To finish around 4h45

- Format — **Online**
- Content: Proof techniques, sequences, sets
- Difficulty ~ Homework

Summary on Sequences

- Sum
- Multiple of n
- Sequence explicit forms

Sequence 2021 SBU

Problem 1. [20 points]

Use mathematical induction to prove the following identities.

(a) [5 points] For all integers $n \geq 1$,

$$1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \cdots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$

(b) [5 points] For all integers $n \geq 1$, $n(n^2 + 5)$ is a multiple of 6.

(c) [5 points] For all integers $n \geq 0$,

$$1 + \frac{2}{3} + \frac{4}{9} + \cdots + \left(\frac{2}{3}\right)^n = 3 \left[1 - \left(\frac{2}{3}\right)^{n+1} \right]$$

(d) [5 points] Suppose that c_1, c_2, c_3, \dots is a sequence defined as follows:

$$c_1 = 3, c_2 = -9$$

$$c_k = 7c_{k-1} - 10c_{k-2} \quad \text{for all integers } k \geq 3$$

Prove that $c_n = 4 \cdot 2^n - 5^n$ for all integers $n \geq 1$.

Sequences 2020 SBU

Problem 1. [20 points]

Use mathematical induction to prove the following identities.

(a) [5 points] For integers $n \geq 1$,

$$\sum_{i=1}^n \frac{1}{(2i-1)(2i+1)} = \frac{n}{2n+1}$$

(b) [5 points] For integers $n \geq 1$,

$$1^2 - 2^2 + 3^2 - \cdots + (-1)^{n-1}n^2 = (-1)^{n-1} \frac{n(n+1)}{2}$$

(c) [5 points] $9^n + 3$ is divisible by 4 for integers $n \geq 1$.

(d) [5 points] Suppose that g_1, g_2, g_3, \dots is a sequence defined as follows:

$$g_1 = 3, g_2 = 5$$

$$g_k = 3g_{k-1} - 2g_{k-2} \quad \text{for all integers } k \geq 3.$$

Prove that $g_n = 2^n + 1$ for all integers $n \geq 1$.

Summary on Sets

- Venn diagram
- Prove set properties with set identities
- Prove set properties with element argument

Sets SBU 2021

Problem 2. [5 points]

Mention whether the following statements are true or false without giving any reasons. Assume all sets are subsets of a universal set U .

- (a) [1 point] $(A \cap B) \cap (A \cap C) = A \cap (B \cup C)$
- (b) [1 point] $A = A \cup (A \cap B)$
- (c) [1 point] $A \subseteq A \cup B$
- (d) [1 point] $A \cap (A \cup B) = A \cap B$
- (e) [1 point] $A \subseteq B$ if and only if $A \cup B = B$

Sets SBU 2020

Problem 2. [5 points]

Find if the statement is true or false. If the statement is true, prove it. If the statement is false, find a counterexample. Assume all sets are subsets of a universal set U . Let set A' be the complement of set A .

For all sets A and B , if $A' \subseteq B$, then $A \cup B = U$.

Summary of summary

- integer properties — even, odd, prime, composite
- sequence properties — sum, multiple of n , explicit form
- set properties — subsets, equality

Mock exam

To finish around 4h45

Problem 1 (points=20)

Let $B = \{2, \{2\}, \{3\}\}$. Which of the following are true

- $2 \in B$
- $\{2\} \in B$
- $\{2\} \subseteq B$
- $\{\{2\}\} \subseteq B$

How many subsets of B are there?

Problem 2 (points=10)

Determine all the elements of

$$\{1 + (-1)^n \mid n \in \mathbb{N}\}$$

$$\{n^2 + n^3 \mid n \in \{1, 2, 3\}\}$$

Problem 3 (points=10)

Let A, B, C be three sets. Prove that if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

Problem 4 (points=15)

Prove:

For any integers x and y , if x is even, then xy is even.

Problem 5 (points=15)

Prove:

$$\text{If } n \in \mathbb{N}, \text{ then } 1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + 4 \cdot 6 + \cdots + n(n+2) = \frac{n(n+1)(2n+7)}{6}.$$

Problem 6 (points=15)

True or False. Prove your conclusion.

If $n \in \mathbb{Z}$, then $n^2 + 3n + 4$ is even.

Problem 7 (points=15)

Prove that $\sqrt[3]{2}$ is irrational.