CSE215
Foundations of Computer Science

Instructor: Zhoulai Fu

State University of New York, Korea

April 21, 2022

Agenda

- Properties on sets
- Exercises on set properties

Finish around 4h45

Properties on Sets

Procedural versions of set definitions

Definition

Let X and Y be subsets of a universal set U and suppose x and y are elements of U.

- $x \in X \cup Y \Leftrightarrow x \in X$ or $x \in Y$
- $x \in X \cap Y \Leftrightarrow x \in X$ and $x \in Y$
- $x \in X - Y \Leftrightarrow x \in X$ and $x \notin Y$
- $x \in X' \Leftrightarrow x \notin X$
- $(x, y) \in X \times Y \Leftrightarrow x \in X$ and $y \in Y$

Some basic set properties

- Inclusion of intersection: For all sets A and B, $A \cap B \subseteq A$ and $A \cap B \subseteq B$.
- Inclusion in union: For all sets A and B, $A \subseteq A \cup B$ and $B \subseteq A \cup B$.
- Transitive property of subsets: For all sets A, B, and C, if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

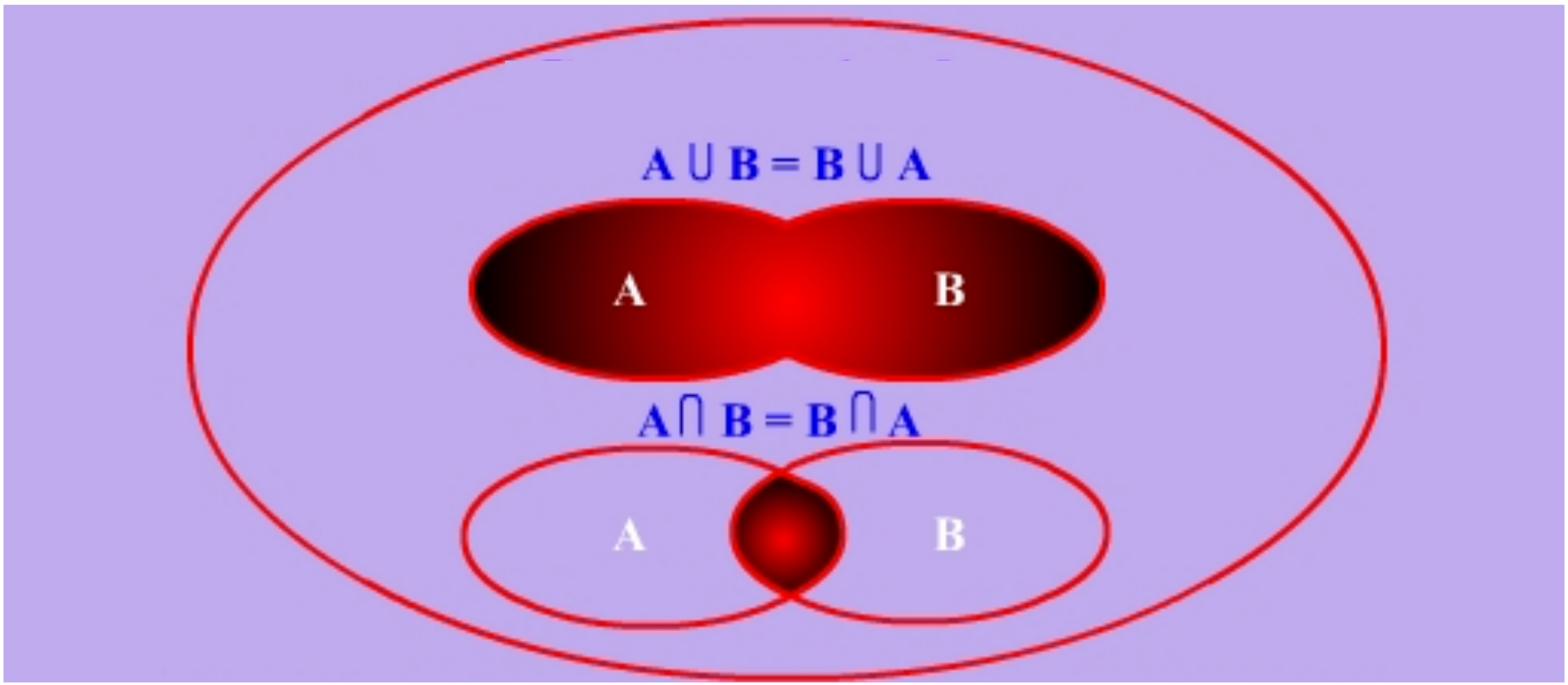
Set identities

Laws	Formula	Formula
Commutative laws	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Associative laws	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
Distributive laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identity laws	$A \cup \phi = A$	$A \cap U = A$
Complement laws	$A \cup A' = U$	$A \cap A' = \phi$
Double comp. law	$(A')' = A$	
Idempotent laws	$A \cup A = A$	$A \cap A = A$
Uni. bound laws	$A \cup U = U$	$A \cap \phi = \phi$
De Morgan's laws	$(A \cup B)' = A' \cap B'$	$(A \cap B)' = A' \cup B'$
Absorption laws	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$
Complements	$U' = \phi$	$\phi' = U$
Set diff. laws	$A - B = A \cap B'$	

Comparison: Logical laws

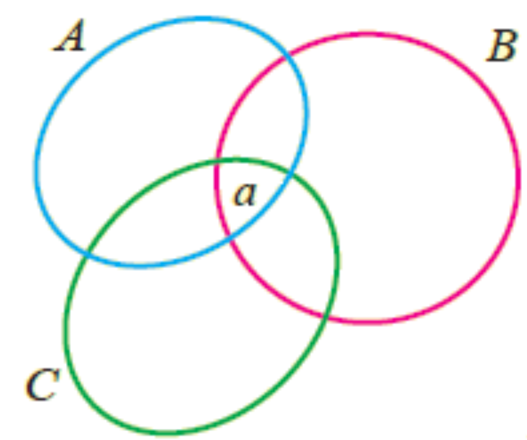
Laws	Formula	Formula
Commutative laws	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
Associative laws	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
Distributive laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
Identity laws	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
Negation laws	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
Double neg. law	$\sim (\sim p) \equiv p$	
Idempotent laws	$p \wedge p \equiv p$	$p \vee p \equiv p$
Uni. bound laws	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$
De Morgan's laws	$\sim (p \wedge q) \equiv \sim p \vee \sim q$	$\sim (p \vee q) \equiv \sim p \wedge \sim q$
Absorption laws	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Negations	$\sim \mathbf{t} \equiv \mathbf{c}$	$\sim \mathbf{c} \equiv \mathbf{t}$

Commutative laws



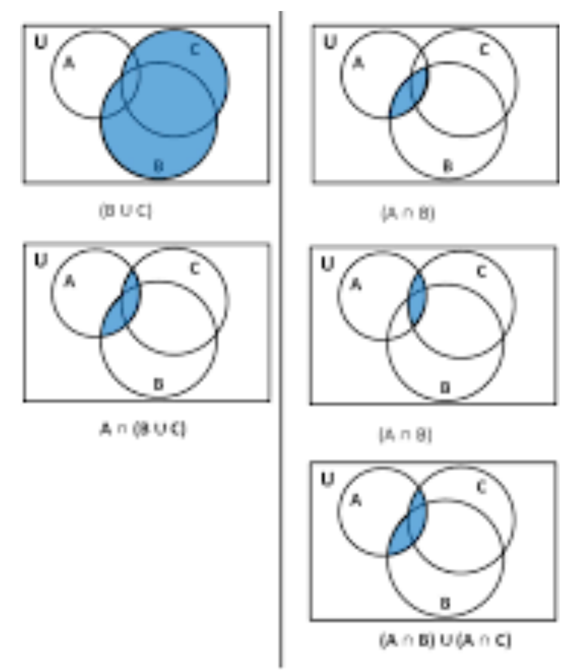
Associative laws

$$(A \cup B) \cup C = A \cup (B \cup C) \quad (A \cap B) \cap C = A \cap (B \cap C)$$



Distributive laws

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $\quad$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$



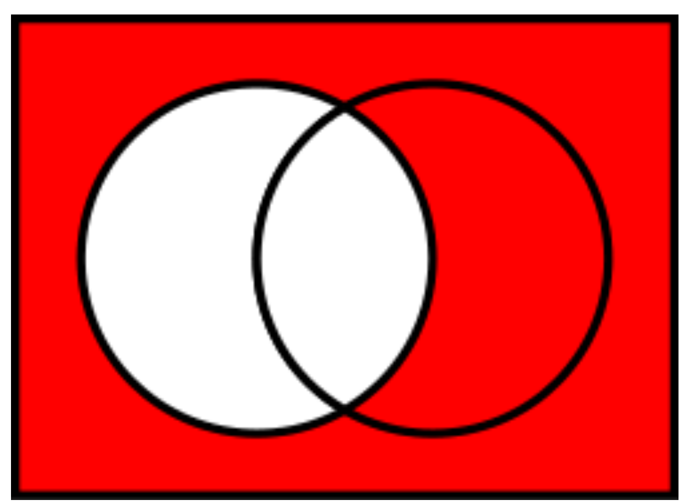
Identity, Idempotent & Universal Bound laws

Identity laws	$A \cup \phi = A$	$A \cap U = A$
Idempotent laws	$A \cup A = A$	$A \cap A = A$
Uni. bound laws	$A \cup U = U$	$A \cap \phi = \phi$

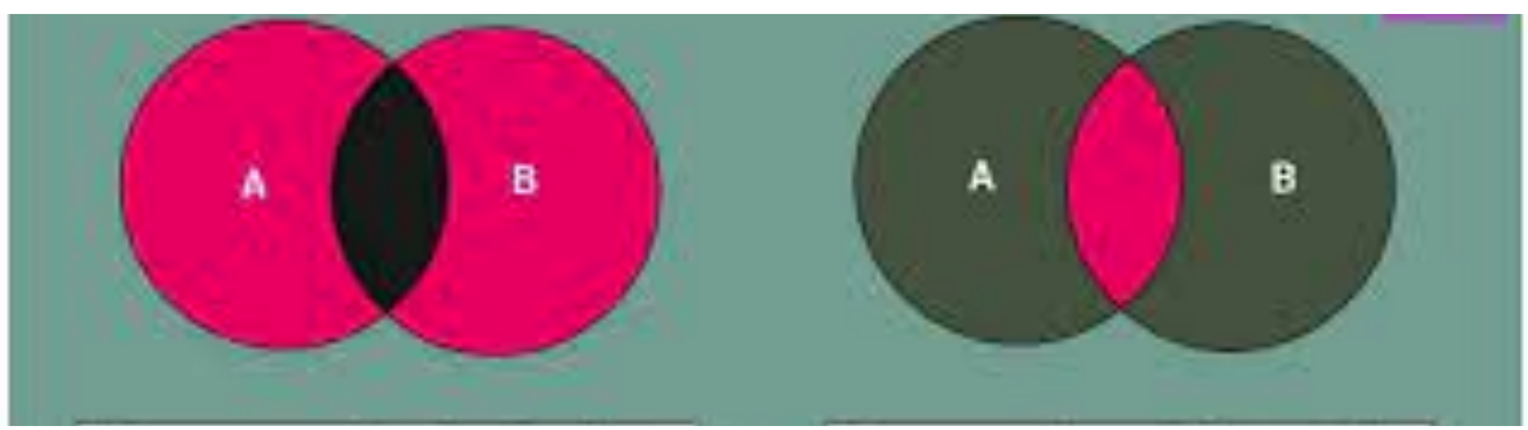
Complement & Double complement laws

$A \cup A' = U$	$A \cap A' = \phi$

$$(A')' = A$$



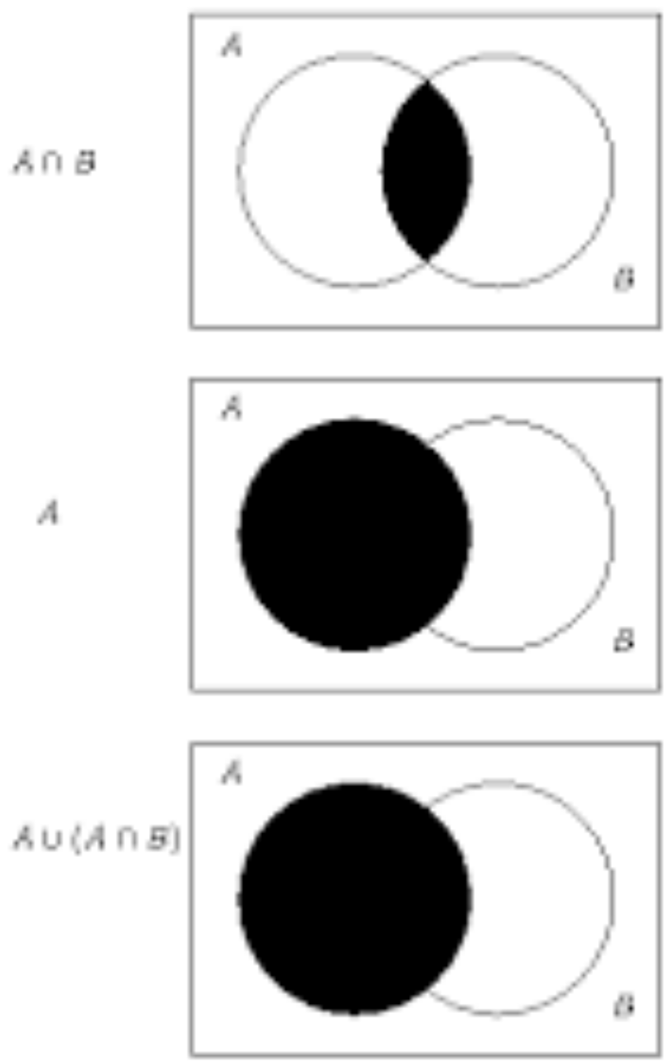
De Morgen Laws



$(A \cup B)' = A' \cap B'$ $\qquad$ $(A \cap B)' = A' \cup B'$

Absorption laws

Absorption laws $A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$



Break

Exercises on Sets

Finish around 4h45

SBU midterm 2021

Problem 2. [5 points]

Mention whether the following statements are true or false without giving any reasons. Assume all sets are subsets of a universal set U.

(a) [1 point] $(A \cap B) \cap (A \cap C) = A \cap (B \cup C)$

(b) [1 point] $A = A \cup (A \cap B)$

(c) [1 point] $A \subseteq A \cup B$

(d) [1 point] $A \cap (A \cup B) = A \cap B$

(e) [1 point] $A \subseteq B$ if and only if $A \cup B = B$

SBU midterm 2020

Problem 2. [5 points]

Find if the statement is true or false. If the statement is true, prove it. If the statement is false, find a counterexample. Assume all sets are subsets of a universal set U. Let set A' be the complement of set A.

For all sets A and B, if $A' \subseteq B$, then $A \cup B = U$.

Solution

Proof.

We want to prove, for any sets A, B, C, if $A' \subset B$ then $A \cup B = U$

Suppose A, B, C are three arbitrarily chosen sets, and $A' \subset B$

Since $A' \subset B$, we know $A \cup A' \subset A \cup B$

As any set unions its complement is the set of the universe, we have $U \subset A \cup B$.

Since any set is a subset of U, we obtain $A \cup B \subset U$.

Thus $A \cup B = U$.

QED.

Alternative solution

Proof.

We want to prove, for any sets A, B, C, if $A' \subset B$ then $A \cup B = U$.

Let a be any element in U.

If a is an element of A, then a is an element of $A \cup B$.

I a is not an element of A', then a is an element of B since $A' \subset B$.

Therefore, $U \subset A \cup B$. Since $A \cup B \subset U$, we obtain $A \cup B = U$.

QED.

SBU midterm 2020

Problem 2. [5 points]

For each of the following statements, find a counterexample to show that the statement is false. Assume all sets are subsets of a universal set U.

1. For all sets A, B, and C, $(A \cap B) \cup C = A \cap (B \cup C)$.
2. For all sets A, B, and C, if $A \not\subseteq B$ and $B \not\subseteq C$, then $A \not\subseteq C$.