CSE215
Foundations of Computer Science

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April 12, 2022

Agenda

- Exercises on mathematical induction

Finish around 4h20

True or False

- For any positive integer n, $n^2+1 \geq 2n$.
- There is no integer n such that $n^2 + n = 12$
- For any positive integer n, $3^n > n^3$
- For any positive integer n, $n! \leq n^2$
- for any pos. int. n, $4n^2 - 4n + 1$ is a perfect square
- The sum of the first n even positive integers is equal to $n(n+1)$
- For any integer n, n^2+n+2 is even

2021 Midterm-2

Problem 1. [20 points]

Use mathematical induction to prove the following identities.

(a) [5 points] For all integers $n \geq 1$,

$$1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \cdots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$

(b) [5 points] For all integers $n \geq 1$, $n(n^2+5)$ is a multiple of 6.

(c) [5 points] For all integers $n \geq 0$,

$$1 + \frac{2}{3} + \frac{4}{9} + \cdots + \left(\frac{2}{3}\right)^n = 3\left[1 - \left(\frac{2}{3}\right)^{n+1}\right]$$

(d) [5 points] Suppose that $c_1, c_2, c_3, \ldots$ is a sequence defined as follows:

$$\begin{aligned} &c_1 = 3, c_2 = -9 \\ &c_k = 7c_{k-1} - 10c_{k-2} \qquad \text{for all integers } k \geq 3 \end{aligned}$$

Prove that $c_n = 4 \cdot 2^n - 5^n$ for all integers $n \geq 1$.

2020 Midterm-2

Problem 1. [20 points]

Use mathematical induction to prove the following identities.

(a) [5 points] For integers $n \geq 1$**,**

$$\sum_{i=1}^{n} \frac{1}{(2i-1)(2i+1)} = \frac{n}{2n+1}$$

(b) [5 points] For integers $n \geq 1$**,**

$$1^2 - 2^2 + 3^2 - \cdots + (-1)^{n-1} n^2 = (-1)^{n-1} \frac{n(n+1)}{2}$$

(c) [5 points] $9^n + 3$ **is divisible by 4 for integers** $n \geq 1$.

(d) [5 points] Suppose that $g_1, g_2, g_3, \ldots$ **is a sequence defined as follows:**

$$g_1 = 3, g_2 = 5$$
$$g_k = 3g_{k-1} - 2g_{k-2} \qquad \text{for all integers } k \geq 3.$$

Prove that $g_n = 2^n + 1$ **for all integers** $n \geq 1$.

(a) **[5 points] For all integers $n \geq 1$,**

$$1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \cdots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$

- Proof.
  - Let P(n) be the predicate __________
  - Base step: We prove ___ holds: ____
  - Inductive step: We prove for any integer k>=1, P(k) -> P(k+1)
    - Let k be an arbitrary integer and k>=1.
    - Assume P(k) holds. That is _____
    - We need to prove P(k+1), namely, _____
      - ____
- QED.

(b) [5 points] For all integers $n \geq 1$, $n(n^2 + 5)$ is a multiple of 6.

- Proof.
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(c) [5 points] For all integers $n \geq 0$,

$$1 + \frac{2}{3} + \frac{4}{9} + \cdots + \left(\frac{2}{3}\right)^n = 3\left[1 - \left(\frac{2}{3}\right)^{n+1}\right]$$

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Prove that $c_n = 4 \cdot 2^n - 5^n$ for all integers $n \geq 1$.

- Proof.
  - Let P(n) be the predicate __________
  - Base step: We prove ____ holds: ____
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    - We need to prove P(k+1), namely, _____
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- QED.

(a) [5 points] For integers $n \geq 1$,

$$\sum_{i=1}^{n} \frac{1}{(2i-1)(2i+1)} = \frac{n}{2n+1}$$

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