

# **CSE215**

# **Foundations of Computer Science**

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# Agenda

- Attendance
- Midterm 2 passed !
- Homework09 solutions

**To finish around 4h20**

**Descriptive Statistics:**

Minimum	min =	53
Maximum	max =	100
Range	R =	47
Size	n =	28
Sum	sum =	2318
Mean	$\bar{x}$ =	82.7857143
Median	$\tilde{x}$ =	84
Mode	mode =	70
Standard Deviation	s =	11.764218
Variance	$s^2$ =	138.396825
Mid Range	MR =	76.5
Quartiles	Quartiles: Q <sub>1</sub> --> 73.5 Q <sub>2</sub> --> 84 Q <sub>3</sub> --> 93	

**Frequency Table**

Value	Frequency	Frequency %
53	1	3.57
70	4	14.29
72	1	3.57
73	1	3.57
74	2	7.14
76	2	7.14
78	1	3.57
82	1	3.57
84	2	7.14
86	2	7.14
88	1	3.57
90	2	7.14
92	1	3.57
94	1	3.57
96	3	10.71
98	1	3.57
100	2	7.14

# Anonymous Poll: How do you feed about midterm 2

<https://forms.gle/Busfy6GUBTpgQeDM7>

Overall difficulty

	1	2	3	4	5	
Very very easy	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	Very very difficult

Other comments?

Your answer

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Submit

Clear form

## Problem 1 (points = 10)

Determine if the following statements are true or false.  
You do not need to explain the reasons.

1.  $\{x \in \mathbf{Z} \mid x - 1 = 0\} = \{x \in \mathbf{Z} \mid x^2 - x = 0\}$   
(where  $\mathbf{Z}$  denotes the set of integers).
2. There are no integers  $a$  and  $b$  such that  $12a + 4b = 1$ .
3. Suppose  $x$  is a real number. If  $x^2 + 5x < 0$  then  $x < 0$ .
4. Suppose  $a$  and  $b$  are two real numbers. If  $a * b$  is rational and  $a$  is irrational, then  $b$  is irrational.
5. Suppose  $a$  and  $b$  are two real numbers. If  $a * b$  is irrational and  $a$  is rational, then  $b$  is irrational.

# Solution

1. False
2. True
3. True
4. False
5. True

## Problem 2 (points = 12)

Let  $A = \{0, 2, 4, 6, 8\}$ ,  $B = \{1, 3, 5, 7\}$  and the universal set  $U = \{0, 1, 2, \dots, 8\}$ . Find:

1.  $A'$
2.  $B'$
3.  $A \cap A'$
4.  $A \cup A'$
5.  $A - A'$
6.  $(A \cup B)'$

# Solution

1. B, namely  $\{1,3,5,7\}$
2. A, namely  $\{0,2,4,6,8\}$
3. empty set
4. U, namely  $\{0,1,2,\dots,8\}$
5. A, namely  $\{0,2,4,6,8\}$
6. empty set



### Problem 3 (points = 30)

Let  $A$  be the set  $\{\{1\}, \{1, 2\}, 2\}$ . First, determine if the following (1-6) are true or false. You do not need to explain the reasons.

1.  $1 \in A$
2.  $2 \in A$
3.  $\{2\} \subseteq A$
4.  $\{2\} \in A$
5.  $\{1, 2\} \in A$
6.  $\{1, 2\} \subseteq A$

And then calculate the following (7-10).

7.  $A - \{1, 2\}$
8.  $A - \{1\}$
9.  $A \cup \{1\}$
10.  $A \cup \{1, 2\}$

# Solution

1. false
2. true
3. true
4. false
5. true
6. false
7.  $\{\{1\},\{1,2\}\}$
8.  $\{\{1\},\{1,2\},2\}$
9.  $\{\{1\},\{1,2\},1,2\}$
10.  $\{1\},\{1,2\},1,2\}$

## **Problem 4 (points = 8)**

Suppose  $A$ ,  $B$  and  $C$  are three sets. Prove  
 $A - (B \cup C) = (A - B) \cap (A - C)$ .

# Solution

$$\begin{aligned} & A - (B \cup C) \\ &= A \cap (B \cup C)' \text{ Def. of complement} \\ &= A \cap (B' \cap C') \text{ De Morgan} \\ &= A \cap A \cap (B' \cap C') \text{ Idempotent} \\ &= (A \cap B') \cap (A \cap C') \text{ Communtative, associative} \\ &= (A - B) \cap (A - C) \text{ Def. of complement} \end{aligned}$$

## **Problem 5 (points = 8)**

Suppose  $A$ ,  $B$  and  $C$  are three sets. Prove that if  $A \subseteq B$ , then  $A - C \subseteq B - C$ .

# Solution

- Proof.
- Suppose  $x$  is an element of  $A - C$ .
- By definition of complement,  $x$  is an element of  $A$  and not an element of  $C$ .
- Since  $A$  is a subset of  $B$  and  $x$  is an element of  $A$ , we know  $x$  is an element of  $B$  by definition of subsets.
- Now  $x$  is an element of  $B$  and not element of  $C$ . Thus by definition of complement,  $x$  is an element of  $B - C$ .
- Thus  $A - C$  is a subset of  $B - C$  following definition of subsets.
- QED.

### **Problem 6 (points = 8)**

Prove that  $1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$  for every integer  $n \geq 1$ .

# Solution

- Proof.
- Let  $P(n)$  be the predicate  $1^3 + \dots + n^3 = n^2(n+1)^2/4$
- We will prove  $P(n)$  for all  $n \geq 1$  using mathematical induction.
- $P(1)$  is obviously true:  $LHS=RHS=1$
- Assume  $P(n)$  is true for an  $n \geq 1$ . We show  $P(n+1)$ .
- That is, we need to show  $1^3 + \dots + (n+1)^3 = (n+1)^2(n+2)^2/4$ 
  - $LHS = 1^3 + \dots + (n+1)^3 = n^2(n+1)^2/4 + (n+1)^3 = RHS$
- QED.



## Problem 7 (points = 8)

Prove that  $24 \mid 5^{2n} - 1$  for every integer  $n \geq 1$ .

# Solution

- Proof.
- Let  $P(n)$  be  $24 \mid 5^{\{2n\}} - 1$
- We will prove  $P(n)$  for all  $n \geq 1$  using mathematical induction.
- $P(1)$  is obviously true:  $24 \mid 24$
- Assume  $P(n)$  is true for some  $n \geq 1$ , we will prove  $P(n+1)$
- $5^{\{2(n+1)\}} - 1 = 25 ( 5^{\{2n\}} - 1 ) + 24$  which is a multiple of 24 following assumption  $P(n)$  and  $24 \mid 24$ . Thus  $P(n+1)$  holds.
- QED.

## **Problem 8 (points = 8)**

Prove that there are no integers  $a$  and  $b$  such that  $a^2 - 4b = 3$ .

# Proof

- Proof
- We use proof by contradiction. Assume there exists  $a$  and  $b$  such that  $a^2 - 4b = 3$
- Then  $a^2 = 4b + 3$ . This must be odd and can be written as  $2k + 1$  for some integer  $k$
- Thus  $4k^2 + 4k + 1 = 4b + 3$  which implies  $2k^2 + 2k = 2b + 1$
- LHS is an even number whereas RHS is odd. Contradiction.
- QED.

## Problem 9 (points = 8)

Suppose  $a$  is an integer. Prove that if  $7 \mid 4a$ , then  $7 \mid a$ .

# Solution

- Proof
- Suppose  $7 \mid 4a$ . We have  $4a = 7k$  for some integer  $k$ .
- Thus  $k$  is even and can be written  $2k'$  for some integer  $k'$ .
- Thus  $4a = 14k'$  which means  $2a = 7k'$ .
- Thus,  $k'$  is an even number and can be written as  $2k''$ .
- Thus  $2a = 14k''$  Thus  $a = 7k''$ . Thus  $7 \mid a$ .
- QED.

# **Solutions for Homework 09**

## Exercise 1.

$$1. \emptyset = \{0\} \rightarrow \text{False}$$

$$2. x \in \{x\} \rightarrow \text{True}$$

$$3. \emptyset = \{\emptyset\} \rightarrow \text{False}$$

$$4. \emptyset \in \{\emptyset\} \rightarrow \text{True}$$

$$5. 4 \notin B \rightarrow \text{True}$$

$$6. A \in N \rightarrow \text{False}$$

$$7. A \subseteq N \rightarrow \text{True}$$

$$8. B \subseteq A \rightarrow \text{False}$$



## Exercise 2

1.  $\{x \in \mathbb{N} \mid x = 2n - 5 \text{ for a natural number } n\} \Rightarrow \{1, 3, 5\}$
2.  $\{y \in \mathbb{N} \mid 2y^2 < 20, y \text{ is integer}\} \Rightarrow \{1, 2, 3\}$
3.  $\{z \in \mathbb{N} \mid \exists z = n^2, \text{ for a natural number } n\} \Rightarrow \{3, 12, 21\}$

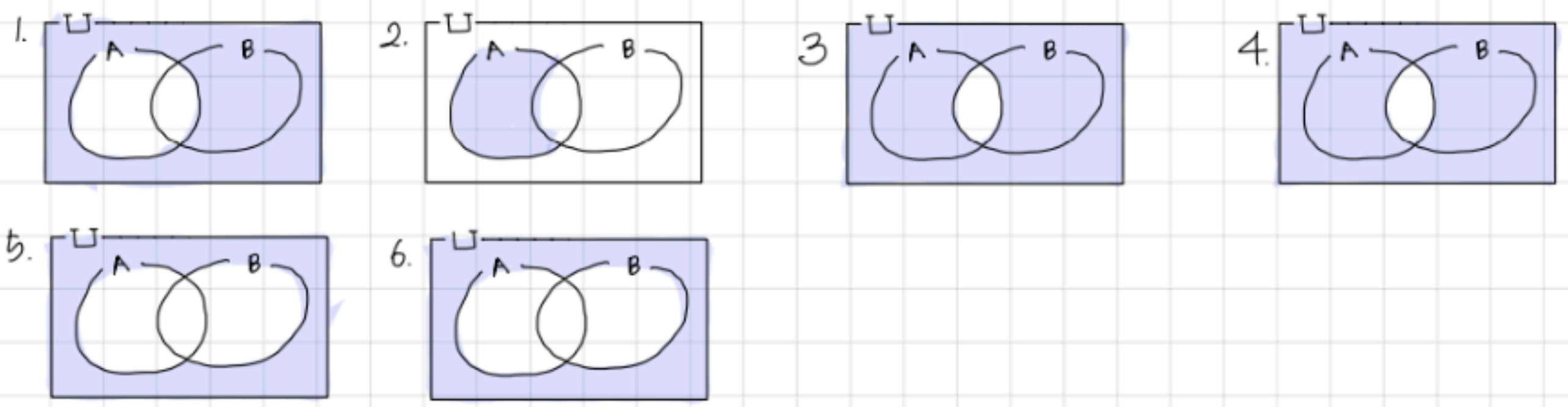
## Exercise 3

1.  $X \times Y = \{a, a\}, \{a, b\}, \{a, e\}, \{a, f\}, \{c, a\}, \{c, b\}, \{c, e\}, \{c, f\}$
2.  $Y \times X = \{a, a\}, \{a, c\}, \{b, a\}, \{b, c\}, \{e, a\}, \{e, c\}, \{f, a\}, \{f, c\}$
3.  $X \times X = \{a, a\}, \{a, c\}, \{c, a\}, \{c, c\}$

### Exercise 4



### Exercise 5



## Exercise 6.

$$1. B \cup (\emptyset \cap A) = B$$

$$\begin{aligned} & \text{Distributive} \\ & (B \cup \emptyset) \cap (B \cup A) \\ & \text{Identity} \downarrow \\ & B \cap (B \cup A) \\ & \text{Absorption} \end{aligned}$$

$$2. (A' \cap U)' = A$$

$$\begin{aligned} & \text{Identity} \downarrow \\ & (A')' \\ & \text{Double Comp.} \end{aligned}$$

$$3. (C \cup A) \cap (B \cup A) = A \cup (B \cap C)$$

$$\begin{aligned} & \text{Distributive} \\ & ((C \cup A) \cap B) \cup ((C \cup A) \cap A) \\ & \text{Distributive} \quad \text{Absorption} \downarrow \\ & ((B \cap C) \cup (B \cap A)) \cup A \\ & \text{Associative} \\ & (A \cup (B \cap A)) \cup (B \cap C) \\ & \text{Absorption} \downarrow \quad \downarrow \\ & A \cup (B \cap C) \end{aligned}$$

$$5. (A \cap B) \cup (A \cup B)' = B$$

$$\begin{aligned} & \text{De Morgan} \\ & (A \cap B) \cup (A' \cap B) \\ & \text{distributive} \\ & ((A \cap B) \cup A') \cap ((A \cap B) \cup B) \\ & \text{distributive} \quad \text{Absorption} \downarrow \\ & (A' \cup A) \cap (A \cup B) \\ & \text{Complement} \downarrow \\ & U \cap (A \cup B) \\ & \text{Identity} \downarrow \\ & (A \cup B) \cap B \\ & \text{Absorption} \end{aligned}$$

$$6. A \cap (A \cup B) = A$$

$$\text{Absorption}$$