# CSE215
# Foundations of Computer Science


**Instructor: Zhoulai Fu**

**State University of New York, Korea**


**May 17, 2022**

# Agenda

- Attendance
- Homework 11
- Exercises on functions

**To finish around 4h20**

**Zoom on today!**

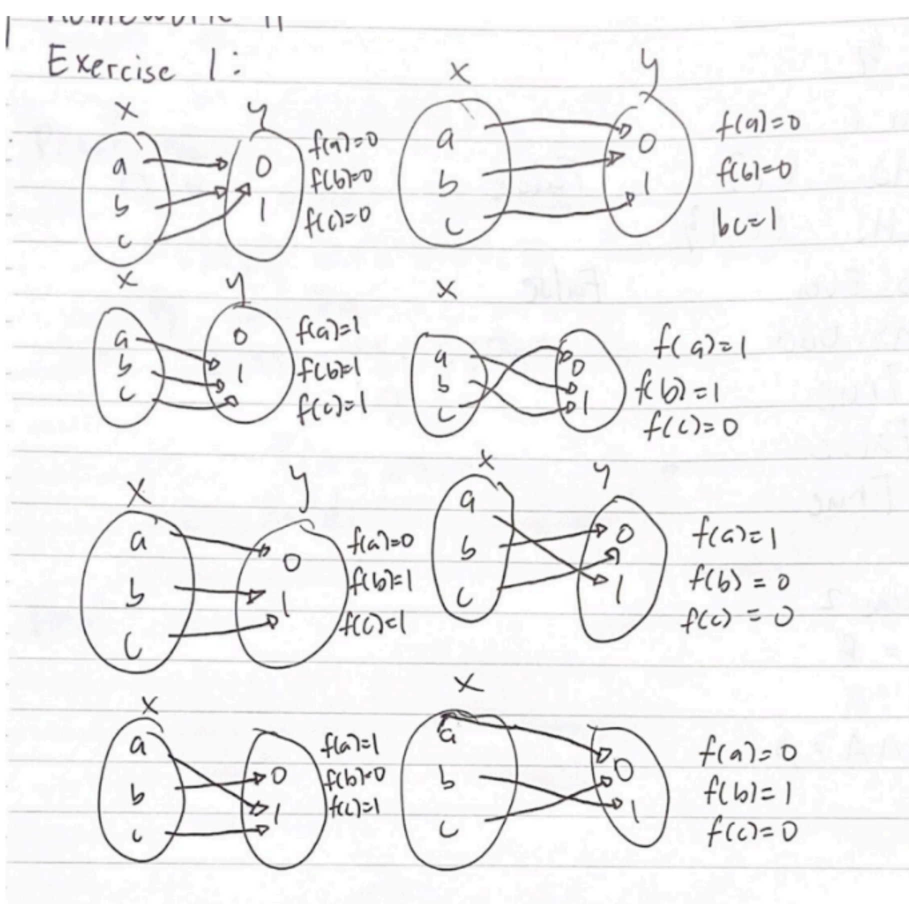
# Exercise 1. (points = 10)

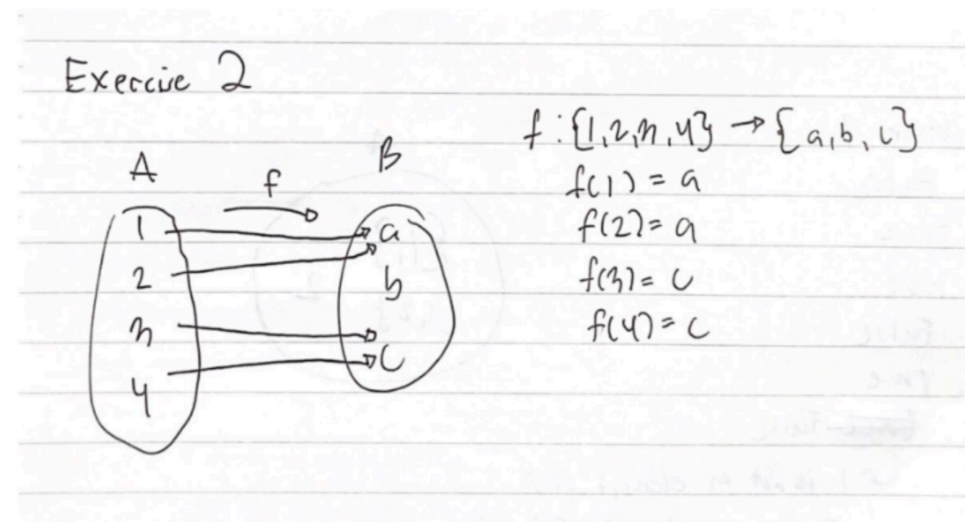
There are eight different functions f : {a, b, c} -> {0, 1} . List them all. Diagrams will suffice.

# Exercise 2. (points = 10)

Let A= {1,2,3,4} and B= {a,b,c}. Give an example of a function f:A->B that is neither one-to-one nor onto. Diagrams will suffice.

# Solution


Exercise 1:
f(a)=0
f(b)=0
f(c)=0
f(a)=0
f(b)=0
f(a)=1
f(b)=1
f(c)=1
f(a)=1
f(b)=1
f(c)=0
f(a)=0
f(b)=1
f(c)=1
f(a)=1
f(b)=0
f(c)=0
f(a)=1
f(b)=0
f(c)=1
f(a)=0
f(b)=1
f(c)=0


Exercise 2
f : {1,2,3,4} → {a,b,c}
f(1) = a
f(2) = a
f(3) = c
f(4) = c
A
B
f

# Exercise 3 (points = 20)

Consider the cosine function cos: R -> R. Determine

1. whether this function is one-to-one and
2. whether it is onto.

   Now consider the same cosine function, but defined as cos : R -> [−1, 1] (Here, [-1,1] is the closed interval from -1 to 1). Determine

3. whether this function is one-to-one and
4. whether it is onto.

# Solution

1) **The cosine function: R -> R**

   Let f(x) = cos x

   This function is not a one-to-one function because cos (1) = cos (-1): that is, f(1) = f(-1).

   By the definition, a function is one-to-one ⇔ $\forall x_1, x_2 \in X$, if $f(x_1) = f(x_2)$ then $x_1=x_2$.

   However, for the cosine function, cos (1) = cos (-1) but 1 ≠ -1

   So, the cosine function is not one-to-one.

   And this cosine function is not onto function because there does not exist a real number x for which cos x = 2.

   Hence, the cosine function: R -> R is neither a one-to-one nor onto function.

2) **The cosine function: R -> [-1, 1]**

   Let f(x) = cos x

   This function is not a one-to-one function because $\exists x_1, x_2 \in X$, if $f(x_1) = f(x_2)$ then $x_1 \neq x_2$, For example, f(1) = f(-1).

   And this cosine function is onto function because -1 ≤ cos x ≤ 1.

   Hence the cosine function: R -> [-1, 1] is not a one-to-one function, but a onto function.

# Exercise 4. (points = 15)

A function $f : \mathbb{Z} \to \mathbb{Z}$ is defined as $f(n)=2n+1$. Determine

1. whether this function is one-to-one and
2. whether it is onto.

# Solution

1)Suppose that $x_1$ and $x_2$ are integers such that $f(x_1) = f(x_2)$. By definition of f, then we have

$$2x_1 + 1 = 2x_2 + 1.$$

Adding -1 to both sides gives

$$2x_1 = 2x_2.$$

And dividing both sides by 2 gives

$$x_1 = x_2$$

Hence, f(n) is one-to-one.

2) The co-domain of f is Z and $0 \in Z$. For $f(n) = 0$, then $2n + 1 = 0$ which implies that $2n = -1$.

So, $n = -\frac{1}{2}$. However, $-\frac{1}{2}$ is not an integer. Hence there is no integer n for which $f(n) = 0$, and thus f(n) is not onto function.

# Exercise 5. (points = 15)

A function $f : Z \rightarrow Z \times Z$ is defined as $f(n) = (2n, n+3)$. Determine

1. whether this function is one-to-one and
2. whether it is onto.

[Hint: a pair $(p,q)$ is equal to another pair $(p',q')$ if and only if $p = p'$ and $q = q'$.]

# Solution

**Exercise 5**

**A function f :Z→Z×Z is defined as f(n)=(2n,n+3). Determine**

1. **whether this function is one-to-one and**
2. **whether it is onto.**

**[Hint: a pair (p,q) is equal to another pair (p',q') if and only if p = p' and q = q'.]**

1)Suppose that $n_1$ and $n_2$ are integers such that $f(n_1) = f(n_2)$. By definition of f,

$(2n_1, n_1+3) = (2n_2, n_2+3)$.

For two ordered pairs to be equal, both the first and second components must be equal. Thus,

$2n_1 = 2n_2$

$n_1+3 = n_2+3$

Adding these two equations gives that $n_1 = n_2$.

Thus, this function is one-to-one.

2) The co-domain of f is Z x Z and (1, 1) ∈ Z x Z. By definition of f, we have

$$2n = 1,\ n + 3 = 1$$

$2n = 1 \Rightarrow n = \frac{1}{2}$. However, $\frac{1}{2}$ is not an integer.

Hence, this function is not onto.

# Exercise 6. (points = 15)

A function $f : Z \times Z \rightarrow Z$ is defined as $f(m,n) = 2n - 4m$. Determine

1. whether this function is one-to-one, and
2. whether it is onto.

# Solution

1)Let $m_1 = 0$ & $n_1 = 0$ and $m_2 = 1$ & $n_2 = 2$, then by definition of f,

$f(m_1, n_1) = 0$ and also $f(m_2, n_2) = 0$. That is, $f(m_1, n_1) = f(m_2, n_2)$ but $m_1 \neq m_2$ and $n_1 \neq n_2$.

Hence, f is not one-to-one.

2) $f(m,n) = 2n-4m = 2(n-2m)$. It means that $f(m,n)$ is always even. If $k \in Z$ and k is odd, $f(m,n) \neq k$. Hence, f is not onto.

# Exercise 7. (points = 15)

A function $f : Z \times Z \rightarrow Z$ is defined as $f(m,n) = 3n - 4m$. Determine

1. whether this function is one-to-one and

2, whether it is onto.

# Solution

Answer:

1)Let $m_1 = 0$ & $n_1 = 0$ and $m_2 = 3$ & $n_2 = 4$, then by definition of f,

$f(m_1, n_1) = 0$ and also $f(m_2, n_2) = 0$. That is, $f(m_1, n_1) = f(m_2, n_2)$ but $m_1 \neq m_2$ and $n_1 \neq n_2$.

Hence, f is not one-to-one.

2) For any $k \in Z$, we have $f(m, n) = k$; that is, $3n - 4m = k$. The function f is $Z \times Z \to Z$, so we can have $f(-k, -k) = 3(-k) - 4(-k) = k$. Since $(-k, -k) \in Z \times Z$ and $k \in Z$, then the function f is onto.

Break?

Exercises on functions

# SBU 2021

**Problem 4. [5 points]**

The rotate-by-90-degree function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ is defined as follows.

$$f(x, y) = (y, -x)$$

(This function is used in image processing for rotating a face or an image by 90 degree in clockwise direction.) Is this function a one-to-one correspondence? Prove or disprove.

# SBU 2021

**Problem 5. [10 points]**

Mention whether the following statements are true or false without giving any reasons. Assume that the functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are arbitrary functions.

(a) [1 point] $f \circ f = f$.

(b) [1 point] $f \circ g = g \circ f$.

(c) [1 point] $f$ and $g$ are both one-to-one correspondences implies that $f \circ g$ and $g \circ f$ are both one-to-one correspondences.

(d) [1 point] $f$ and $g$ are both onto does not imply that $f \circ g$ and $g \circ f$ are both onto.

(e) [1 point] $f$ and $g$ are both one-to-one implies that $f \circ g$ and $g \circ f$ are both one-to-one.

(f) [1 point] If $f \circ g$ is the identity function, then $f$ and $g$ are one-to-one correspondences.

(g) [1 point] Suppose $f^{-1}$ exists. Then $f^{-1}$ need not be an onto function.

# More from Textbook

20. Let $f: W \to X$, $g: X \to Y$, and $h: Y \to Z$ be functions. Must $h \circ (g \circ f) = (h \circ g) \circ f$? Prove or give a counterexample.

21. True or False? Given any set $X$ and given any functions $f: X \to X$, $g: X \to X$, and $h: X \to X$, if $h$ is one-to-one and $h \circ f = h \circ g$, then $f = g$. Justify your answer.

22. True or False? Given any set $X$ and given any functions $f: X \to X$, $g: X \to X$, and $h: X \to X$, if $h$ is one-to-one and $f \circ h = g \circ h$, then $f = g$. Justify your answer.