

CSE215

Foundations of Computer Science

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Agenda

- Attendance
- Midterm 2 passed !
- Homework09 solutions

To finish around 4h20

Descriptive Statistics:

Minimum	min = 53
Maximum	max = 100
Range	R = 47
Size	n = 28
Sum	sum = 2318
Mean	$\bar{x} = 82.7857143$
Median	$\tilde{x} = 84$
Mode	mode = 70
Standard Deviation	s = 11.764218
Variance	$s^2 = 138.396825$
Mid Range	MR = 76.5
Quartiles	Quartiles: Q ₁ --> 73.5 Q ₂ --> 84 Q ₃ --> 93

Frequency Table

Value	Frequency	Frequency %
53	1	3.57
70	4	14.29
72	1	3.57
73	1	3.57
74	2	7.14
76	2	7.14
78	1	3.57
82	1	3.57
84	2	7.14
86	2	7.14
88	1	3.57
90	2	7.14
92	1	3.57
94	1	3.57
96	3	10.71
98	1	3.57
100	2	7.14

Anonymous Poll: How do you feel about midterm 2

<https://forms.gle/Busfy6GUBTpgQeDM7>

Overall difficulty

	1	2	3	4	5	
Very very easy	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	Very very difficult

Other comments?

Your answer

Submit

Clear form

Problem 1 (points = 10)

Determine if the following statements are true or false.
You do not need to explain the reasons.

1. $\{x \in \mathbf{Z} \mid x - 1 = 0\} = \{x \in \mathbf{Z} \mid x^2 - x = 0\}$
(where \mathbf{Z} denotes the set of integers).
2. There are no integers a and b such that $12a + 4b = 1$.
3. Suppose x is a real number. If $x^2 + 5x < 0$ then $x < 0$.
4. Suppose a and b are two real numbers. If $a * b$ is rational and a is irrational, then b is irrational.
5. Suppose a and b are two real numbers. If $a * b$ is irrational and a is rational, then b is irrational.

Solution

1. False
2. True
3. True
4. False
5. True

Problem 2 (points = 12)

Let $A = \{0, 2, 4, 6, 8\}$, $B = \{1, 3, 5, 7\}$ and the universal set $U = \{0, 1, 2, \dots, 8\}$. Find:

1. A'
2. B'
3. $A \cap A'$
4. $A \cup A'$
5. $A - A'$
6. $(A \cup B)'$

Solution

1. B, namely $\{1,3,5,7\}$
2. A, namely $\{0,2,4,6,8\}$
3. empty set
4. U, namely $\{0,1,2,\dots,8\}$
5. A, namely $\{0,2,4,6,8\}$
6. empty set

Problem 3 (points = 30)

Let A be the set $\{\{1\}, \{1, 2\}, 2\}$. First, determine if the following (1-6) are true or false. You do not need to explain the reasons.

1. $1 \in A$
2. $2 \in A$
3. $\{2\} \subseteq A$
4. $\{2\} \in A$
5. $\{1, 2\} \in A$
6. $\{1, 2\} \subseteq A$

And then calculate the following (7-10).

7. $A - \{1, 2\}$
8. $A - \{1\}$
9. $A \cup \{1\}$
10. $A \cup \{1, 2\}$

Solution

1. false
2. true
3. true
4. false
5. true
6. false
7. $\{\{1\},\{1,2\}\}$
8. $\{\{1\},\{1,2\},2\}$
9. $\{\{1\},\{1,2\},1,2\}$
10. $\{1\},\{1,2\},1,2\}$

Problem 4 (points = 8)

Suppose A , B and C are three sets. Prove
 $A - (B \cup C) = (A - B) \cap (A - C)$.

Solution

$$\begin{aligned} & A - (B \cup C) \\ &= A \cap (B \cup C)' \text{ Def. of complement} \\ &= A \cap (B' \cap C') \text{ De Morgan} \\ &= A \cap A \cap (B' \cap C') \text{ Idempotent} \\ &= (A \cap B') \cap (A \cap C') \text{ Communtative, associative} \\ &= (A - B) \cap (A - C) \text{ Def. of complement} \end{aligned}$$

Problem 5 (points = 8)

Suppose A , B and C are three sets. Prove that if $A \subseteq B$, then $A - C \subseteq B - C$.

Solution

- Proof.
- Suppose x is an element of $A - C$.
- By definition of complement, x is an element of A and not an element of C .
- Since A is a subset of B and x is an element of A , we know x is an element of B by definition of subsets.
- Now x is an element of B and not element of C . Thus by definition of complement, x is an element of $B - C$.
- Thus $A - C$ is a subset of $B - C$ following definition of subsets.
- QED.

Problem 6 (points = 8)

Prove that $1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$ for every integer $n \geq 1$.

Solution

- Proof.
- Let $P(n)$ be the predicate $1^3 + \dots + n^3 = n^2(n+1)^2/4$
- We will prove $P(n)$ for all $n \geq 1$ using mathematical induction.
- $P(1)$ is obviously true: $LHS=RHS=1$
- Assume $P(n)$ is true for an $n \geq 1$. We show $P(n+1)$.
- That is, we need to show $1^3 + \dots + (n+1)^3 = (n+1)^2(n+2)^2/4$
 - $LHS = 1^3 + \dots + (n+1)^3 = n^2(n+1)^2/4 + (n+1)^3 = RHS$
- QED.

Problem 7 (points = 8)

Prove that $24 \mid 5^{2n} - 1$ for every integer $n \geq 1$.

Solution

- Proof.
- Let $P(n)$ be $24 \mid 5^{\{2n\}} - 1$
- We will prove $P(n)$ for all $n \geq 1$ using mathematical induction.
- $P(1)$ is obviously true: $24 \mid 24$
- Assume $P(n)$ is true for some $n \geq 1$, we will prove $P(n+1)$
- $5^{\{2(n+1)\}} - 1 = 25 (5^{\{2n\}} - 1) + 24$ which is a multiple of 24 following assumption $P(n)$ and $24 \mid 24$. Thus $P(n+1)$ holds.
- QED.

Problem 8 (points = 8)

Prove that there are no integers a and b such that $a^2 - 4b = 3$.

Proof

- Proof
- We use proof by contradiction. Assume there exists a and b such that $a^2 - 4b = 3$
- Then $a^2 = 4b + 3$. This must be odd and can be written as $2k + 1$ for some integer k
- Thus $4k^2 + 4k + 1 = 4b + 3$ which implies $2k^2 + 2k = 2b + 1$
- LHS is an even number whereas RHS is odd. Contradiction.
- QED.

Problem 9 (points = 8)

Suppose a is an integer. Prove that if $7 \mid 4a$, then $7 \mid a$.

Solution

- Proof
- Suppose $7 \mid 4a$. We have $4a = 7k$ for some integer k .
- Thus k is even and can be written $2k'$ for some integer k' .
- Thus $4a = 14k'$ which means $2a = 7k'$.
- Thus, k' is an even number and can be written as $2k''$.
- Thus $2a = 14k''$ Thus $a = 7k''$. Thus $7 \mid a$.
- QED.

Solutions for Homework 09

Exercise 1.

$$1. \emptyset = \{0\} \rightarrow \text{False}$$

$$2. x \in \{x\} \rightarrow \text{True}$$

$$3. \emptyset = \{\emptyset\} \rightarrow \text{False}$$

$$4. \emptyset \in \{\emptyset\} \rightarrow \text{True}$$

$$5. 4 \notin B \rightarrow \text{True}$$

$$6. A \in N \rightarrow \text{False}$$

$$7. A \subseteq N \rightarrow \text{True}$$

$$8. B \subseteq A \rightarrow \text{False}$$

Exercise 2

1. $\{x \in \mathbb{N} \mid x = 2n - 5 \text{ for a natural number } n\} \Rightarrow \{1, 3, 5\}$
2. $\{y \in \mathbb{N} \mid 2y^2 < 20, y \text{ is integer}\} \Rightarrow \{1, 2, 3\}$
3. $\{z \in \mathbb{N} \mid \exists z = n^2, \text{ for a natural number } n\} \Rightarrow \{3, 12, 27\}$

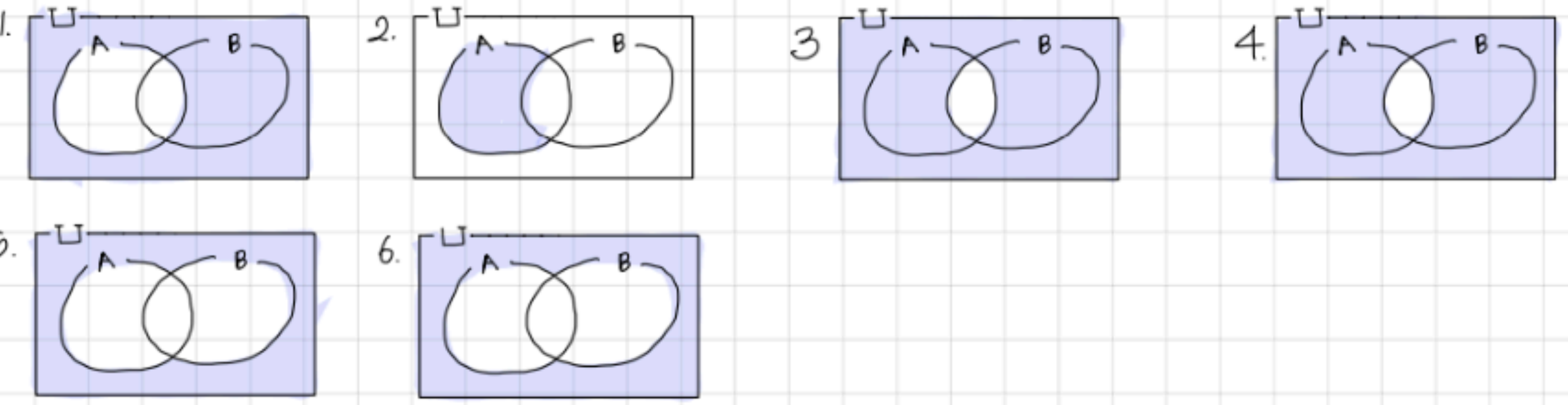
Exercise 3

1. $X \times Y = \{a, a\}, \{a, b\}, \{a, e\}, \{a, f\}, \{c, a\}, \{c, b\}, \{c, e\}, \{c, f\}$
2. $Y \times X = \{a, a\}, \{a, c\}, \{b, a\}, \{b, c\}, \{e, a\}, \{e, c\}, \{f, a\}, \{f, c\}$
3. $X \times X = \{a, a\}, \{a, c\}, \{c, a\}, \{c, c\}$

Exercise 4



Exercise 5



Exercise 6.

$$1. B \cup (\emptyset \cap A) = B$$

$$\begin{aligned} & \text{Distributive} \\ & (B \cup \emptyset) \cap (B \cup A) \\ & \text{Identity} \downarrow \\ & B \cap (B \cup A) \\ & \text{Absorption} \end{aligned}$$

$$2. (A' \cap U)' = A$$

$$\begin{aligned} & \text{Identity} \downarrow \\ & (A')' \\ & \text{Double Comp.} \end{aligned}$$

$$3. (C \cup A) \cap (B \cup A) = A \cup (B \cap C)$$

$$\begin{aligned} & \text{Distributive} \\ & ((C \cup A) \cap B) \cup ((C \cup A) \cap A) \\ & \text{Distributive} \quad \text{Absorption} \downarrow \\ & ((B \cap C) \cup (B \cap A)) \cup A \\ & \text{Associative} \\ & (A \cup (B \cap A)) \cup (B \cap C) \\ & \text{Absorption} \downarrow \quad \downarrow \\ & A \cup (B \cap C) \end{aligned}$$

$$5. (A \cap B) \cup (A \cup B)' = B$$

$$\begin{aligned} & \text{De Morgan} \\ & (A \cap B) \cup (A' \cap B) \\ & \text{distributive} \\ & ((A \cap B) \cup A') \cap ((A \cap B) \cup B) \\ & \text{distributive} \quad \text{Absorption} \downarrow \\ & (A' \cup A) \cap (A \cup B) \\ & \text{Complement} \downarrow \\ & U \cap (A \cup B) \\ & \text{Identity} \downarrow \\ & (A \cup B) \cap B \\ & \text{Absorption} \end{aligned}$$

$$6. A \cap (A \cup B) = A$$

$$\text{Absorption}$$