CSE215
Foundations of Computer Science

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May 3, 2022

Agenda

- Attendance
- Midterm 2 passed !
- Homework09 solutions

To finish around 4h20

Descriptive Statistics:

Minimum	min =	53
Maximum	max =	100
Range	R =	47
Size	n =	28
Sum	sum =	2318
Mean	$\overline{x}$ =	82.7857143
Median	$\widetilde{x}$ =	84
Mode	mode =	70
Standard Deviation	s =	11.764218
Variance	s^2 =	138.396825
Mid Range	MR =	76.5
Quartiles		Quartiles: Q_1 --> 73.5 Q_2 --> 84 Q_3 --> 93

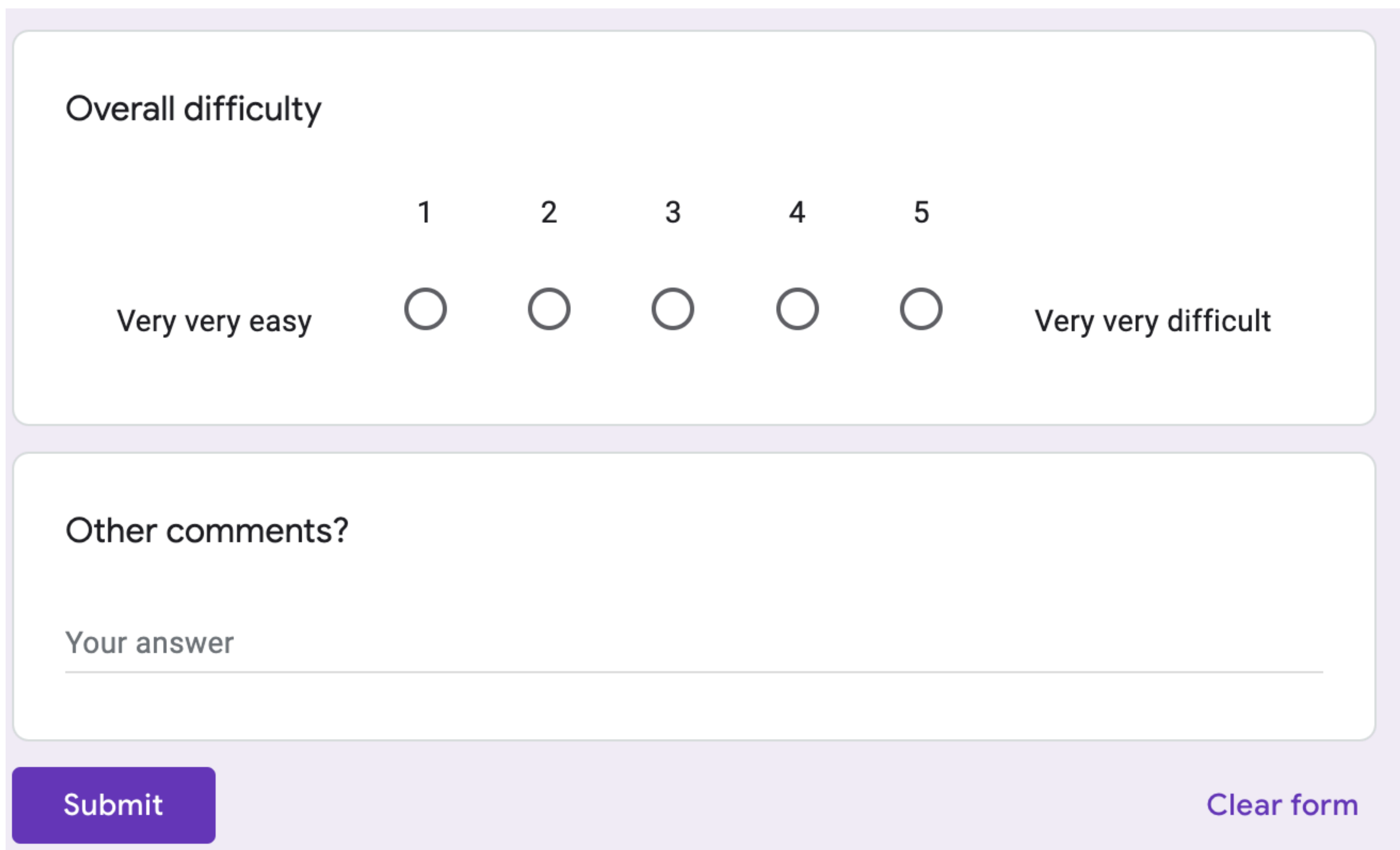
Frequency Table

Value	Frequency	Frequency %
53	1	3.57
70	4	14.29
72	1	3.57
73	1	3.57
74	2	7.14
76	2	7.14
78	1	3.57
82	1	3.57
84	2	7.14
86	2	7.14
88	1	3.57
90	2	7.14
92	1	3.57
94	1	3.57
96	3	10.71
98	1	3.57
100	2	7.14

https://www.calculatorsoup.com/calculators/statistics/descriptivestatistics.php

Anonymous Poll: How do you feel about midterm 2

https://forms.gle/Busfy6GUBTpgQeDM7


Overall difficulty

	1	2	3	4	5	
Very very easy	○	○	○	○	○	Very very difficult

Other comments?

Your answer

Submit

Clear form

Problem 1 (points = 10)

Determine if the following statements are true or false.
You do not need to explain the reasons.

1. $\{x \in \mathbf{Z} \mid x - 1 = 0\} = \{x \in \mathbf{Z} \mid x^2 - x = 0\}$ (where $\mathbf{Z}$ denotes the set of integers).
2. There are no integers a and b such that $12a + 4b = 1$.
3. Suppose x is a real number. If $x^2 + 5x < 0$ then $x < 0$.
4. Suppose a and b are two real numbers. If $a * b$ is rational and a is irrational, then b is irrational.
5. Suppose a and b are two real numbers. If $a * b$ is irrational and a is rational, then b is irrational.

Solution

1. False
2. True
3. True
4. False
5. True

Problem 2 (points = 12)

Let $A = \{0, 2, 4, 6, 8\}$, $B = \{1, 3, 5, 7\}$ and the universal set $U = \{0, 1, 2, \ldots . 8\}$. Find:

1. A'
2. B'
3. $A \cap A'$
4. $A \cup A'$
5. $A - A'$
6. $(A \cup B)'$

Solution

1. B, namely {1,3,5,7}
2. A, namely {0,2,4,6,8}
3. empty set
4. U, namely {0,1,2,…8}
5. A, namely {0,2,4,6,8}
6. empty set

Problem 3 (points = 30)

Let A be the set $\{\{1\},\{1,2\},2\}$. First, determine if the following (1-6) are true or false. You do not need to explain the reasons.

1. $1 \in A$
2. $2 \in A$
3. $\{2\} \subseteq A$
4. $\{2\} \in A$
5. $\{1,2\} \in A$
6. $\{1,2\} \subseteq A$

And then calculate the following (7-10).

7. $A - \{1,2\}$
8. $A - \{1\}$
9. $A \cup \{1\}$
10. $A \cup \{1,2\}$

Solution

1. false
2. true
3. true
4. false
5. true
6. false
7. {{1},{1,2}}
8. {{1},{1,2},2}
9. {{1},{1,2},1,2}
10. {1},{1,2},1,2}

Problem 4 (points = 8)

Suppose A, B and C are three sets. Prove $A - (B \cup C) = (A - B) \cap (A - C)$.

Solution

$$
\begin{aligned}
& A - (B \cup C) \\
=& A \cap (B \cup C)' \text{ Def. of complement} \\
=& A \cap (B' \cap C') \text{ De Morgen} \\
=& A \cap A \cap (B' \cap C') \text{ Idempotent} \\
=& (A \cap B') \cap (A \cap C') \text{ Communtative, associative} \\
=& (A - B) \cap (A - C) \text{ Def. of complement}
\end{aligned}
$$

Problem 5 (points = 8)

Suppose A, B and C are three sets. Prove that if $A \subseteq B$, then $A - C \subseteq B - C$.

Solution

- Proof.
- Suppose x is an element of A - C.
- By definition of complement, x is an element of A and not an element of C.
- Since A is a subset of B and x is an element of A, we know x is an element of B by definition of subsets.
- Now is an element of B and not element of C. Thus by definition of complement, x is an element of B-C
- Thus A-C is a subset of B-C following definition of subsets.
- QED.

Problem 6 (points = 8)

Prove that $1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$ for every integer $n \geq 1$.

Solution

- Proof.
- Let P(n) be the predicate 1^3 + … + n^3 = n^2(n+1)^2/4
- We will prove P(n) for all n>=1 using mathematical induction.
- P(!) is obviously true: LHS=RHS=1
- Assume P(n) is true for an n>=1 We show P(n+1).
- That is, we need to show 1^3 + … + (n+1)^3 = (n+1)^2(n+2)^2/4
  - LHS = 1^3 + … + (n+1)^3 = n^2(n+1)^2/4 + (n+1)^3 = RHS
- QED.

Problem 7 (points = 8)

Prove that $24 \mid 5^{2n} - 1$ for every integer $n \geq 1$.

Solution

- Proof.
- Let P(n) be 24 | 5^{2n}-1
- We will prove P(n) for all n>=1 using mathematical induction.
- P(1) is obviously true: 24 }24
- Assume P(n) is true for some n >=1, we will prove P(n+1)
- 5^{2(n+1)} - 1 =25 (5^{2n} - 1} + 24 which is a multiple of 24 following assumption P(n) and 24}24. Thus P(n+1) holds.
- QED.

Problem 8 (points = 8)

Prove that there are no integers a and b such that $a^2 - 4b = 3$.

Proof

- Proof
- We use proof by contradiction. Assume there exists a and b such that $a^2-4b=3$
- Then $a^2 = 4b+3$. Thus must be odd and can be written as $2k+1$ for some integer k
- Thus $4k^2+4k + 1 = 4b+3$ which implies $2k^2+2k=2b+1$
- LHS is an even number whereas RHS is odd. Contradiction.
- QED.

Problem 9 (points = 8)

Suppose a is an integer. Prove that if $7 \mid 4a$, then $7 \mid a$.

Solution

- Proof
- Suppose $7 \mid 4a$. We have $4a = 7k$ for some integer k.
- Thus k is even and can be written $2k'$ for some integer k'.
- Thus $4a = 14k'$ which means $2a = 7k'$.
- Thus, k' is an even number and can be written as $2k''$.
- Thus $2a = 14k''$ Thus $a = 7k''$. Thus $7 \mid a$.
- QED.

Solutions for Homework 09

Exercise 1.

1. $\emptyset = \{0\} \rightarrow$ False
2. $x \in \{x\} \rightarrow$ True
3. $\emptyset = \{\emptyset\} \rightarrow$ False
4. $\emptyset \in \{\emptyset\} \rightarrow$ True
5. $4 \notin B \rightarrow$ True
6. $A \in N \rightarrow$ False
7. $A \subseteq N \rightarrow$ True
8. $B \subseteq A \rightarrow$ False

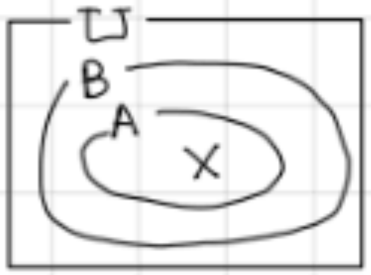
Exercise 2

1. $\{x \in N \mid x = 2n-5 \text{ for a natural number } n\} \Rightarrow \{1, 3, 5\}$
2. $\{y \in N \mid 2y^2 < 20, y \text{ is integer}\} \Rightarrow \{1, 2, 3\}$
3. $\{z \in N \mid 3z = n^2, \text{ for a natural number } n\} \Rightarrow \{3, 12, 27\}$

Exercise 3

1. $X \times Y = \{a,a\}, \{a,b\}, \{a,e\}, \{a,f\}, \{c,a\}, \{c,b\}, \{c,e\}, \{c,f\}$
2. $Y \times X = \{a,a\}, \{a,c\}, \{b,a\}, \{b,c\}, \{e,a\}, \{e,c\}, \{f,a\}, \{f,c\}$
3. $X \times X = \{a,a\}, \{a,c\}, \{c,a\}. \{c,c\}$

Exercise 4

1.


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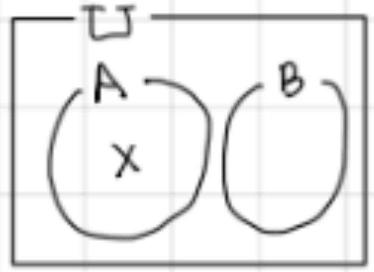

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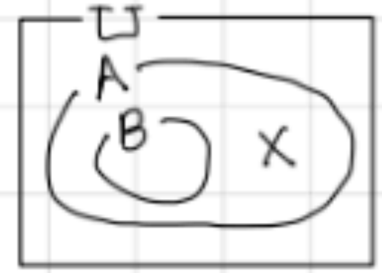

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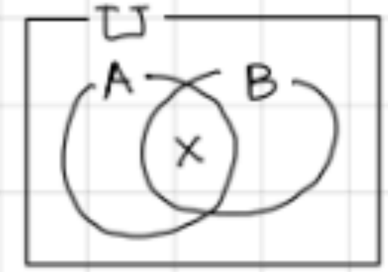
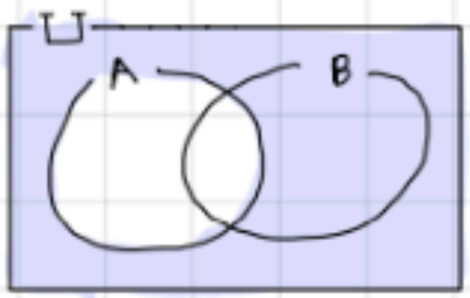


Exercise 5

1.


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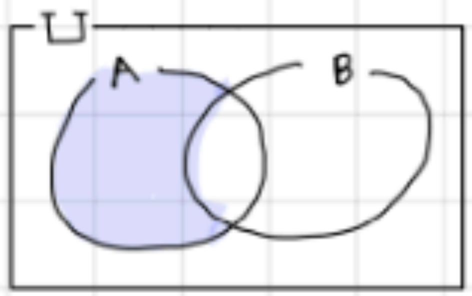

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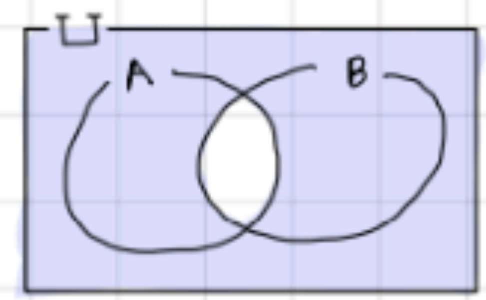

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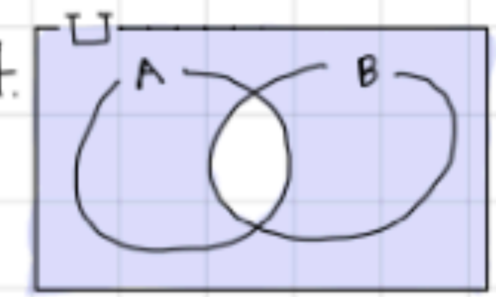

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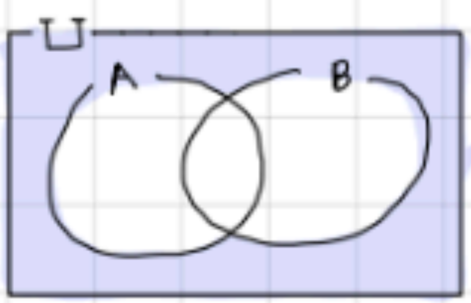

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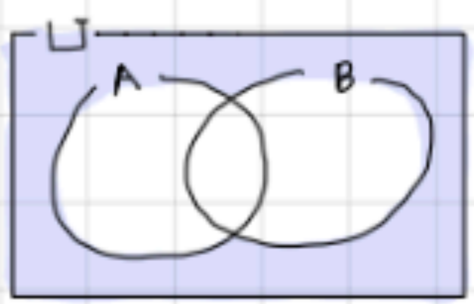

Exercise 6.

1. $B \cup (\varnothing \cap A) = B$

Distributive

$(B \cup \varnothing) \cap (B \cup A)$

Identity

$B \cap (B \cup A)$

Absorption

2. $(A' \cap U)' = A$

Identity

$(A')'$

Double Comp.

3. $(C \cup A) \cap (B \cup A) = A \cup (B \cap C)$

Distributive

$((C \cup A) \cap B) \cup ((C \cup A) \cap A)$

Distributive Absorption

$((B \cap C) \cup (B \cap A)) \cup A$

Associative

$(A \cup (B \cap A)) \cup (B \cap C)$

Absorption

$A \cup (B \cap C)$

5. $(A \cap B) \cup (A \cup B')' = B$

Demorgan

$(A \cap B) \cup (A' \cap B)$

distributive

$((A \cap B) \cup A') \cap ((A \cap B) \cup B)$

distributive Absorption

$(A' \cup A) \cap (A \cup B)$

Complement

$U \cap (A \cup B)$

Identity

$(A \cup B) \cap B$

Absorption

6. $A \cap (A \cup B) = A$

Absorption