CSE215
Foundations of Computer Science

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News

- Instructor will check Zoom chats more frequently.
- Students who need a break can do so while Zoom chats are being checked, and questions are being answered.
- For efficiency, instructor will use Keynote instead Adobe PDF, meaning that on-class demo are not recorded. Please use screenshot/recording from your end if needed.
- Everyone is encouraged to participate in the class by unmuting him/herself.

Clarification on prime/ composite definitions (last time)

- An integer n is composite <=> There exists integers a and b such that: n = a * b, and a !=1 and b != 1

- n is prime is equivalent to say: n is not composite

- Thus, an integer n is prime <=> for all integers a and b, n != a * b, or a =1 or b = 1

- <=> for all integers a and b, n = a * b -> (a =1 or b = 1)

Agenda

- Examples of wrong proof
- Exam-level problems

Finish around 4h45

Some wrong “proof”

To finish around 4h45

Theorem: For all integers k, if $k > 0$ then $k^2 + 2k + 1$ is composite.

"**Proof:** For $k = 2$, $k^2 + 2k + 1 = 2^2 + 2 \cdot 2 + 1 = 9$. But $9 = 3 \cdot 3$, and so 9 is composite. Hence the theorem is true."

Theorem: The sum of any two even integers equals $4k$ for some integer k.

"**Proof:** Suppose m and n are any two even integers. By definition of even, $m = 2k$ for some integer k and $n = 2k$ for some integer k. By substitution,

$$m + n = 2k + 2k = 4k.$$

This is what was to be shown."

Theorem: The difference between any odd integer and any even integer is odd.

"**Proof:** Suppose n is any odd integer, and m is any even integer. By definition of odd, $n = 2k + 1$ where k is an integer, and by definition of even, $m = 2k$ where k is an integer. Then

$$n - m = (2k + 1) - 2k = 1.$$

But 1 is odd. Therefore, the difference between any odd integer and any even integer is odd."

Exam problems

To finish around 4h45

SBU 2021 Final

Problem 6. [5 points]

Prove that if $n^2 + 8n + 20$ is odd, then n is odd for natural numbers n.

- Proof.
  - We want to prove: for all natural numbers n, n^2+8n+20 is odd -> n is odd.
  - We use proof by contradiction to prove the statement above.
  - That is, we assume there exists a natural number n such that n^2+8n+20 is odd, and n is even.
  - From “n is even”, we know n^2 must be even, and 8n must be even
  - Therefore n^2+8n+20 must be even, which contradicts with the assumption above.
- QED.

SBU 2022 Midterm

Problem 6. [5 points]

Let $a_1, a_2, \ldots, a_n$ be real numbers for $n \geq 1$. Prove that at least one of these numbers is greater or equal to the average of the numbers.

- Proof.
  - We want to prove: for any real numbers a1,…a_n, there exists an a_i where 1<=i<=n, such that a_i>= (a1+a2+…a_n)/n.
  - We use proof by contradiction to prove the statement above.
  - That is, we assume: there exists some real numbers a1,…a_n, such that for all a_i, 1<=i<=n, a_i<(a1+a2+…a_n)/n.
  - From this assumption, we know (a1+a2+…a_n) < n * (a1+a2+…a_n)/n, which is a contradiction.
- QED.

SBU 2021 Final

Problem 4. [5 points]
Prove that $n^2+9n+27$ is odd for all natural numbers n. You can use any proof technique.

- Proof.
  - We want to prove: for all natural numbers n, n^2+9n+27 is odd.
  - Let n be an arbitrary natural number.
  - We know n^2+9n = n(n+9).
  - We consider two cases depending on whether on is even or odd.
    - Case 1: If n is even, n(n+9) is even. (since even * odd = even)
    - Case 2: If n is odd, n(n+9) is even. (since odd * even = even)
  - Thus n^2+9n must be even, and n^2+9n+27 must be odd.
- QED.

SBU 2020 Midterm

Problem 8. [5 points]

Prove that for all integers a, if a^3 is even, then a is even.

- Proof.
  - We want to prove ________________
  - We use **proof by contradiction** to prove the statement above.
  - That is, __________
  - From this assumption, we know…………, which is a contradiction.
- QED.

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Problem 8. [5 points]

Prove that for any two integers a and b, if ab is odd, then a and b are both odd.

- Proof.
  - We want to prove ________________
  - We use **proof by contradiction** to prove the statement above.
  - That is, __________
  - From this assumption, we know…………, which is a contradiction.
- QED.

Problem 6. [5 points]

Prove that the product of any four consecutive integers is a multiple of 4.

- Proof.
  - We want to prove ________________
  - We prove the statement by **division into cases.**
  - Case 1: ____
  - Case 2: _____
  - Thus, ___
- QED.

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Problem 8. [5 points]

Prove by contradiction that there are no integers x and y such that $x^2 = 4y + 2$.

- Proof.
  - ___
  - ___
  - ___
- QED.

That is all for today

- Wrong proofs
- Exam problem 1 - 3: Proof by contradiction; proof by division into cases
- Example problem 4 - 7: Your work

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Problem 7. [5 points]

Prove or disprove the following statement. If x and y are rational, then x^y is rational.