

# **CSE215**

# **Foundations of Computer Science**

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# Agenda

- Attendance check
- WHILE (response rate  $< 80\%$ )
  - { do course evaluation }
- (IF  $t < 3h45$ ) Extra problems for midterm 2
- Mock Final

**Zoom on today!**

**if  $t < 3\text{h}45$**

**Extra problems midterm 2**

## Problem 1 (points = 3)

Let  $A_1, A_2, A_3, \dots$  be a sequence of sets. Prove  $(A_1 \cup A_2 \dots \cup A_n)' = A_1' \cap A_2' \dots \cap A_n'$  for any integer  $n \geq 1$ . [Hint: Mathematical induction.]

# Solution

Ex1

Let  $P(k): (A_1 \cup A_2 \dots \cup A_k)' = A_1' \cap A_2' \dots \cap A_k'$

Assume  $P(k)$  holds

In order for  $P(n)$  to hold,  $P(k+1)$  must hold  
with the assumption that  $P(k)$  holds

$P(k+1): (A_1 \cup A_2 \dots \cup A_{k+1})' = A_1' \cap A_2' \dots \cap A_{k+1}'$

$$\begin{aligned}(A_1 \cup A_2 \dots \cup A_{k+1})' &= ((A_1 \cup A_2 \dots \cup A_k) \cup A_{k+1})' \\ &= (A_1 \cup A_2 \dots \cup A_k)' \cap A_{k+1}' \\ &= A_1' \cap A_2' \dots \cap A_k' \cap A_{k+1}'\end{aligned}$$

$\therefore P(k+1)$  holds with the assumption  $P(k)$  holds

QED

## Problem 10 (points = 6)

1. Prove that for any integer  $n$ , if  $3 \mid n^2$ , then  $3 \mid n$ . [Hint: You may use the following fact: An integer is not a multiple of 3 if and only if it can be written as  $3k + 1$  or  $3k + 2$  for some integer  $k$ .]
2. Prove that  $\sqrt{3}$  is irrational. [Hint: You may use the fact proven above to prove this one.]

# Solution

1.  $n$  can be represented as  $3k, 3k+1$ , or  $3k+2$   $k \in \mathbb{Z}$

$$(3k)^2 = 9k^2 = 3(3k^2)$$
$$(3k+1)^2 = 9k^2 + 6k + 1 = 3(3k^2 + 2k) + 1$$
$$(3k+2)^2 = 9k^2 + 12k + 4 = 3(3k^2 + 4k) + 4$$

For  $3|n^2$  to hold  $n$  must be  $3k$   
 $3|3k$  holds

2.  $\sqrt{3}$  is rational

$$\sqrt{3} = \frac{p}{q} \quad p, q \in \mathbb{Z}$$

$$3 = \frac{p^2}{q^2}$$

$$3q^2 = p^2 \quad \text{Since } 3|p^2 \text{ holds, } p = 3k \quad k \in \mathbb{Z}$$

$$3q^2 = 9k^2$$

$$q^2 = 3k^2 \quad \text{Since } 3|q^2 \text{ holds, } q = 3n \quad n \in \mathbb{Z}$$

Since  $p$  and  $q$  are both multiples of 3,  
 $\sqrt{3}$  cannot be rational. Hence,  $\sqrt{3}$  is irrational  
QED

# Different styles for designing the final

- ? to make everyone pass
- ? to make someone suffer
- ? to make the score curve looks like Gaussian
- ? to provide a fair evaluation

# Good designing in my humble opinion

- Fairness, fairness, and fairness!
- Comprehensive
  - covering most points, with well-distributed weights
- Adequate problems for each covered point
  - Quantity & difficulty

# Outcome

- Problem 1 Propositional statements
- Problem 2 Negation
- Problem 3: Inference rules
- Problem 4: Truth tables
- Problem 5: Proof by dividing into cases
- Problem 6: Direct proof
- Problem 7: Proof by contraposition/counterexamples
- Problem 8: Mathematical induction
- Problem 9: Proof on set properties
- Problem 10: Cardinality of infinite sets
- Problem 11: Relations
- Problem 12: Functions

## \* Mock Problem 1 Propositional statements

True or false

1.  $\forall x \in \mathbb{Z}, x^2 \geq 1$  (For every integer  $x$ ,  $x^2 \geq 1$ .)
2.  $\forall x \in \mathbb{R}, \exists n \in \mathbb{N}, x^n \geq 0$  (For every real number  $x$ , there exists a natural number  $n$  such that  $x^n \geq 0$ .)

- 1. False
- 2. True

## \* Mock Problem 2. Negation

1. If  $a$  is odd then  $a^2$  is odd.
2. For every prime number  $p$ , there is another prime number  $q$  with  $q > p$ .

- $a$  is odd, and  $a^2$  is even
- There exists a prime number  $p$ , such that for any prime number  $q$ ,  $q \leq p$

## \* Mock problem 3. Inference rule

1.  $(\sim p \vee q) \rightarrow \sim(q \wedge r)$   
2.  $\sim p \vee q$   
3. ----- Modus Ponens with 1,2,

1.  $(p \wedge r) \rightarrow \sim q$  Premise  
2.  $\sim q \rightarrow r$  Premise  
3. ----- Transitivity with 1,2

1.  $(p \vee q) \rightarrow r$  Premise  
2.  $\sim r$  Premise  
3. ----- Modus Tollens with 1,2  
4. ----- De Morgan with 3  
5. ----- Specification, using the form " $A \wedge B$  infers  $A$ ".

- $\sim(q \wedge r)$
- $r$
- $\sim(p \vee q), \sim p \wedge \sim q, \sim p$

## \* Moxk problem 4 truth table

1. Build a truth table of  $(p \wedge (p \oplus q)) \rightarrow \neg q$  to show this statement is a tautology.

2. use truth table to determine validity

- premmise  $p \rightarrow r$
- premise  $q \rightarrow r$
- conclusion  $p \vee q \rightarrow r$

- Tautology: show it is true for each row
- Validity: Check critical rows — conclusion must be true whenever premises are true

\* Mock problem 5. proof by cases

– 14. If  $n \in \mathbb{Z}$ , then  $5n^2 + 3n + 7$  is odd.

- If  $n$  is odd
- If  $n$  is even

\* Mock problem 6 direct proof

– If  $a$  is an integer and  $a^2 \mid a$ , then  $a \in \{-1, 0, 1\}$  .

- Suppose  $a$  is an integer and  $a^2 \mid a$
- We have  $a = a^2 \cdot k$  for some integer  $k$
- Thus,  $a(1 - ak) = 0$
- Thus  $a = 0$  or  $ak = 1$
- Since  $a$  and  $k$  are integers,  $a = 0$  or  $a = 1$ , or  $a = -1$

\* Mock problem 7. Proof by contraposition / contradiction

Suppose  $x, y \in \mathbb{R}$ . If  $y^3 + yx^2 \leq x^3 + xy^2$ , then  $y \leq x$ .

- If  $y > x$  ...

\* Mock problem 8. MI

If  $n$  is a non-negative integer, then  $5 \mid (n^5 - n)$ .

\* Mock problem 9 proof of sets

1. If  $A, B$  and  $C$  are sets, then  $A \times (B - C) = (A \times B) - (A \times C)$ .

\* Mock problem 10 set sizes

1. Prove  $(2,3)$  and  $(6, 10)$  is of the same cardinality.

\* Mock problem 11. Relation

1. Define a relation  $R$  on  $\mathbb{Z}$  as  $xRy$  if and only if  $4|(x+3y)$ . Prove  $R$  is an equivalence relation.

## \* Mock problem 12. Function

– 7. This question concerns functions  $f : \{A, B, C, D, E, F, G\} \rightarrow \{1, 2\}$ . How many such functions are there? How many of these functions are injective? How many are surjective? How many are bijective?