CSE215
Foundations of Computer Science

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March 15, 2022

Info on Midterm

- For all students: Online through blackboard; open-book

- Score contribution is (midterm1 + midterm2) / 2 * 20%

Today

Homework 1
Homework 2

Break (5 mins)

Midterm exam preparation exercises

- To finish by 4h25

Homework 1

```
* Exercise 1 (score = 10)
Construct the truth table for the following statement forms:

1. false \/ p
2. True /\ p
```

p	false \/ p
true	true
false	false

p	true \/ p
true	true
false	false

1. false $\vee$ p

p	false	false $\vee$ P
T	F	T
F	F	F

2. True $\wedge$ p

p	True	True $\wedge$ p
T	T	T
F	T	F

```
* Exercise 2 (score = 30)

Write truth tables for the following statement  forms.

1. p -> q
2. ~p \/ q
3. q -> p
4. ~q \/ p
5. ~q -> ~p
6. ~p -> ~q

* Exercise 3   (score = 25)

Among the six statement forms in Exercise 2, find at least five pairs
that are equivalent? For example, if you believe statement forms 1 and
2 in Exercise 2 are equivalent, you have found a pair (p -> q, ~p \/
q).
```

- Keys: A->B = ~A ∨ B; comm., double neg. laws.
- 1, 2, 5 are equivalent
- 3, 4, 6 are equivalent

* Exercise 4 (score = 10)

Let x be a particular integer. Use De Morgan’s laws to write negations for the following

1. 0 < x < 10
2. x < 2 or x > 5
3. x <= 0 or x >=1
4. 0 > x >= -7

- 0 < x < 10 mean x > 0 and x < 10.

- Use De morgen gets x <= 0 or x >= 10

* Exercise 5 (score = 15)

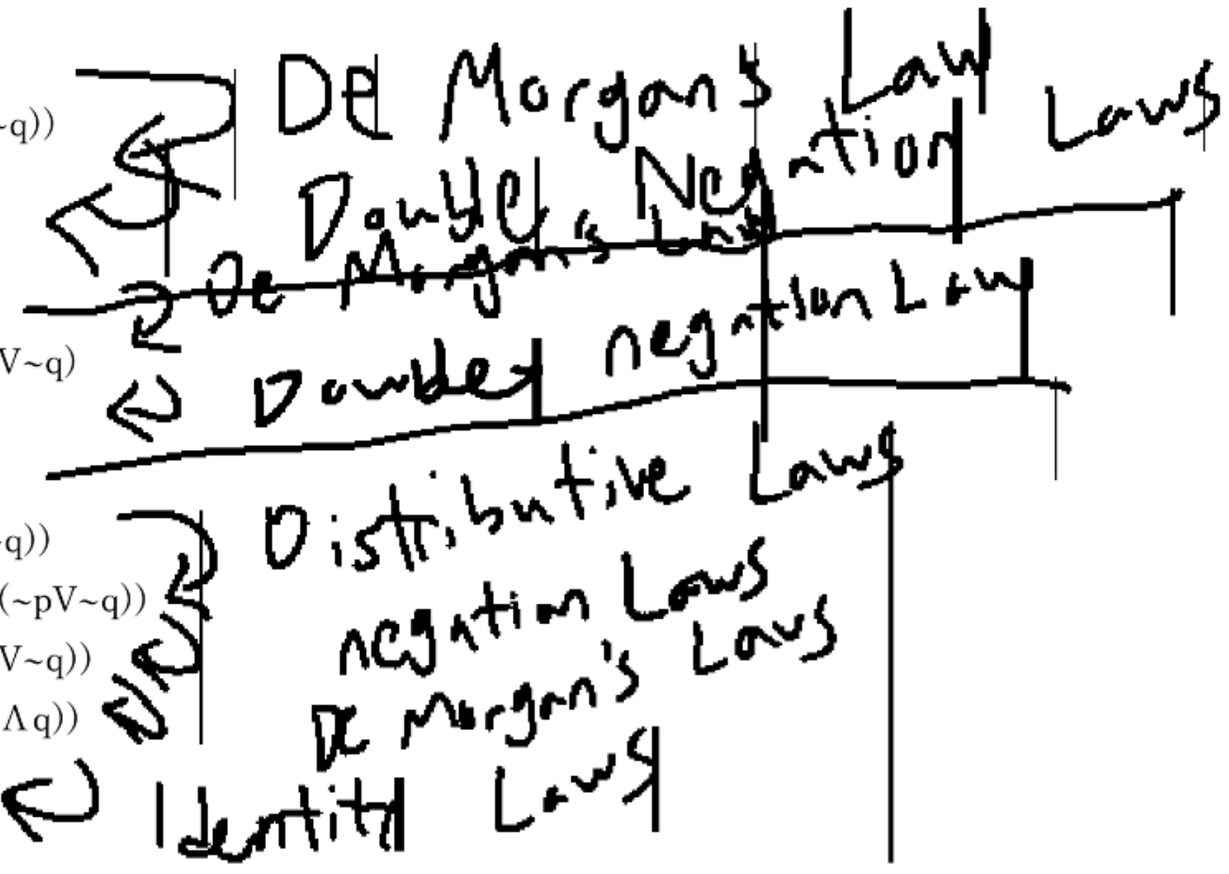
For each statement form below, use laws of logical equivalence to determine if it is a tautology or contradiction. Explain which laws you are applying to get your results.

1. (~p\/q)\/(p/\~q)
2. (p/\~q)∧(~p\/q)
3. (p/\q)\/(~p\/(p/\~q))

① $(\sim pVq)V(p\Lambda\sim q)$
$(\sim pVq)V\sim(\sim pV\sim(\sim q))$
$(\sim pVq)V\sim(\sim pVq)$
= tautology

② $(p\Lambda\sim q)\Lambda(\sim pVq)$
$(p\Lambda\sim q)\Lambda\sim(\sim(\sim p)V\sim q)$
$(p\Lambda\sim q)\Lambda\sim(pV\sim q)$
= contradiction

③ $(p\Lambda q)V(\sim pV(p\Lambda\sim q))$
$(p\Lambda q)V((\sim pVp)\Lambda(\sim pV\sim q))$
$(p\Lambda q)V(true\Lambda(\sim pV\sim q))$
$(p\Lambda q)V(true\Lambda\sim(p\Lambda q))$
$(p\Lambda q)V\sim(p\Lambda q)$
= tautology

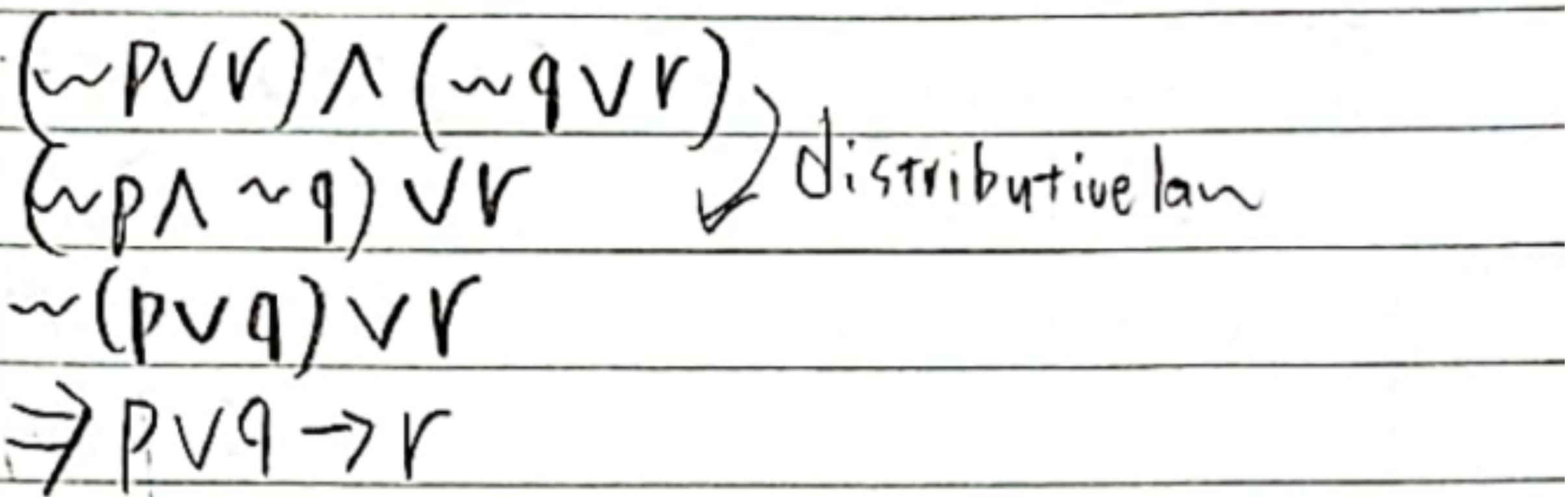


- Correct, including details. But hard to read.

```
* Exercise 6. (score = 10)

  Check if the two statement forms below are logically equivalent and
  argue why

-  p \/ q -> r
- (p -> r) /\ (q ->r)
```



- Correct. More details will be appreciated.

Homework 2

* Exercise 1 (score = 20)

Use truth tables to determine whether the argument form below is valid

(1)

- premises: p -> q, q
- conclusion: p

(2)

- premises: p-> q, ~p
- conclusion: ~q

(3)

- premises: p-> q, p
- conclusion: q

(4)

- premises: p-> q, ~q
- conclusion: ~p

Solution 1.1

(1)

- premises: p -> q, q
- conclusion: p
- **Not valid**

p	q	p->q	q	p
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	F

Solution 1.2

(2)

- premises: p-> q, ~p
- conclusion: ~q
- **Not valid**

p	q	p->q	~p	~q
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Solution 1.3

(3)

- premises: p-> q, p
- conclusion: q
- **Valid**

p	q	p->q	p	q
T	T	T	T	T
T	F	F	T	F
F	T	T	F	T
F	F	T	F	F

Solution 1.4

(4)

- premises: p-> q, ~q
- conclusion: ~p
- **Valid**

p	q	p->q	~q	~p
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

* Exercise 2 (score = 60)

Use truth tables to determine whether the argument form below is valid

(1)
- Premises: p -> q, ~p -> ~q
- Conclusion: p \/ q

(2)
- Premises: p \/ q, p -> ~q, ~r -> ~p
- Conclusion: r

(3)
- Premises: p, ~q -> ~p, ~q \/ r
- Conclusion r

(4)
- Premises: p/\q -> ~r, p \/ ~q, ~q -> p
- Conclusion: ~r

(5)
- Premises: p -> r, q -> r
- Conclusion: (p \/ q) -> r

(6)
- Premises: p -> (q \/ r), ~q \/ ~r
- Conclusion: ~p \ / ~r

Solution 2.1

(1)

- Premises: p -> q, ~p -> ~q
- Conclusion: p \/ q
- **Not valid**

p	q	~p	~q	p->q	~p->~q	p \/ q
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	T
F	F	T	T	T	T	F

Solution 2.2

(2)

- Premises: p ∨ q, p -> ~q, ~r -> ~p
- Conclusion: r
- **Not valid**

p	q	r	~p	~q	~r	p ∨ q	p->~q	~r->~p	r
T	T	T	F	F	F	T	F	T	T
T	T	F	F	F	T	T	F	F	F
T	F	T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T	F	F
F	T	T	T	F	F	T	T	T	T
F	T	F	T	F	T	T	T	T	F
F	F	T	T	T	F	F	T	T	T
F	F	F	T	T	T	F	T	T	F

Solution 2.3

(3)

- Premises: p, ~q -> ~p, ~q ∨ r
- Conclusion: r
- **Valid**

p	q	r	~q	~p	~q->~p	~q ∨ r	p	r
T	T	T	F	F	T	T	T	T
T	T	F	F	F	T	F	T	F
T	F	T	T	F	F	T	T	T
T	F	F	T	F	F	T	T	F
F	T	T	F	T	T	T	F	T
F	T	F	F	T	T	F	F	F
F	F	T	T	T	T	T	F	T
F	F	F	T	T	T	T	F	F

Solution 2.4

(4)

- Premises: p/\q -> ~r, p \/ ~q, ~q -> p
- Conclusion: ~r
- **Not valid**

p	q	r	~q	~r	p /\ q	p/\q ->~r	p\/~q	~q->p	~r
T	T	T	F	F	T	F	T	T	F
T	T	F	F	T	T	T	T	T	T
T	F	T	T	F	F	T	T	T	F
T	F	F	T	T	F	T	T	T	T
F	T	T	F	F	F	T	F	T	F
F	T	F	F	T	F	T	F	T	T
F	F	T	T	F	F	T	T	F	F
F	F	F	T	T	F	T	T	F	T

Solution 2.5

(5)

- Premises: p -> r, q -> r
- Conclusion: (p ∨ q) -> r
- **Valid**

p	q	r	p ∨ q	p -> r	q -> r	p ∨ q -> r
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	T	T	T	T
T	F	F	T	F	T	F
F	T	T	T	T	T	T
F	T	F	T	T	F	F
F	F	T	F	T	T	T
F	F	F	F	T	T	T

Solution 2.6

(6)

- Premises: p -> (q ∨ r), ~q ∨ ~r
- Conclusion: ~p ∨ ~r
- **Not valid**

p	q	r	~p	~q	~r	q ∨ r	p->(q∨r)	~q ∨ ~r	~p ∨ ~r
T	T	T	F	F	F	T	T	F	F
T	T	F	F	F	T	T	T	T	T
T	F	T	F	T	F	T	T	T	F
T	F	F	F	T	T	F	F	T	T
F	T	T	T	F	F	T	T	F	T
F	T	F	T	F	T	T	T	T	T
F	F	T	T	T	F	T	T	T	T
F	F	F	T	T	T	F	T	T	T

```
* Exercise 3 (score = 20)

  Check if the two statement form are equivalent, and expaian why:

  - (p -> q)/\(q -> r)/\(r -> p)
  - p /\ q /\ r
```

- Intuition: #1 = p, q, r have the same truth value, either true, or false

Solution 3

p	q	r	~p	~q	~r	p->q	q->r	r->p	(p->q)/\(q->r)/\(r->p)	p /\ q /\ r
T	T	T	F	F	F	T	T	T	T	T
T	T	F	F	F	T	T	F	T	F	F
T	F	T	F	T	F	F	T	T	F	F
T	F	F	F	T	T	F	T	T	F	F
F	T	T	T	F	F	T	T	F	F	F
F	T	F	T	F	T	T	F	T	F	F
F	F	T	T	T	F	T	T	F	F	F
F	F	F	T	T	T	T	T	T	T	F

Break ~5 min.

Mid-term exam exercises

- To finish by 4h25