CSE215
Foundations of Computer Science

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Agenda

- Attendance check
- WHILE (response rate < 80%)
    - { do course evaluation}
- (IF t < 3h45) Extra problems for midterm 2
- Mock Final

Zoom on today!

if t < 3h45

Extra problems midterm 2

Problem 1 (points = 3)

Let $A_1, A_2, A_3, \ldots$ be a sequence of sets. Prove $(A_1 \cup A_2 \ldots \cup A_n)' = A_1' \cap A_2' \ldots \cap A_n'$ for any integer $n \geq 1$. [Hint: Mathematical induction.]

Solution

Ex1

Let $P(k): (A_1 \cup A_2 \ldots \cup A_k)' = A_1' \cap A_2' \ldots \cap A_k'$

Assume $P(k)$ holds

In order for $P(n)$ to hold, $P(k+1)$ must hold with the assumption that $P(k)$ holds

$P(k+1): (A_1 \cup A_2 \ldots \cup A_{k+1})' = A_1' \cap A_2' \ldots \cap A_{k+1}'$

$$\begin{aligned}(A_1 \cup A_2 \ldots \cup A_{k+1})' &= ((A_1 \cup A_2 \ldots A_k) \cup A_{k+1})' \\ &= (A_1 \cup A_2 \ldots A_k)' \cap A_{k+1}' \\ &= A_1' \cap A_2' \ldots \cap A_k' \cap A_{k+1}'\end{aligned}$$

$\therefore$ $P(k+1)$ holds with the assumption $P(k)$ holds

QED

Problem 10 (points = 6)

1. Prove that for any integer n, if $3 \mid n^2$, then $3 \mid n$. [Hint: You may use the following fact: An integer is not a multiple of 3 if and only if it can be written as 3k + 1 or 3k + 2 for some integer k.]
2. Prove that $\sqrt{3}$ is irrational. [Hint: You may use the fact proven above to prove this one.]

Solution

1. n can be represented as $3k$, $3k+1$, or $3k+2$ $k \in Z$
$(3k)^2 = 9k^2 = 3(3k^2)$
$(3k+1)^2 = 9k^2 + 6k + 1 = 3(3k^2 + 2k) + 1$
$(3k+2)^2 = 9k^2 + 12k + 4 = 3(3k^2 + 4k) + 4$
For $3 | n^2$ to hold n must be $3k$
$3 | 3k$ holds

2. $\sqrt{3}$ is rational
$\sqrt{3} = \frac{p}{q}$ $p, q \in Z$
$3 = \frac{p^2}{q^2}$
$3q^2 = p^2$ Since $3 | p^2$ holds, $p = 3k$ $k \in Z$
$3q^2 = 9k^2$
$q^2 = 3k^2$ Since $3 | q^2$ holds, $q = 3n$ $n \in Z$
Since p and q are both multiples of 3,
$\sqrt{3}$ cannot be rational, Hence, $\sqrt{3}$ is irrational
QED

Different styles for designing the final

- ? to make everyone pass
- ? to make someone suffer
- ? to make the score curve looks like Gaussian
- ? to provide a fair evaluation

Good designing
in my humble opinion

- Faireness, faireness, and fairness!
- Comprehensive
  - covering most points, with well-distributed weights
- Adequate problems for each covered point
  - Quantity & difficulty

Outcome

- Problem 1 Propositional statements
- Problem 2 Negation
- Problem 3:Inference rules
- Problem 4: Truth tables
- Problem 5: Proof by dividing into cases
- Problem 6: Direct proof
- Problem 7: Proof by contraposition/counterexamples
- Problem 8: Mathematical induction
- Problem 9: Proof on set properties
- Problem 10: Cardinality of infinite sets
- Problem 11: Relations
- Problem 12: Functions

* Mock Problem 1 Propositional statements

True or false

1. $\forall x \in Z, x^2 >= 1$ (For every integer x, $x^2 >= 1$.)

2. $\forall x \in R, \exists n \in N, x^n \geq 0$ (For every real number x, there exists a natural number n such that $xn >= 0$.)

- 1. False
- 2. True

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* Mock Problem 2. Negation

1. If a is odd then a^2 is odd.

2. For every prime number p, there is another prime number q with q > p.
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- a is odd, and a^2 is even
- There exists a prime number p, such that for any prime number q, q<=p

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*  Mock problem 3. Inference rule

1. (~p\/q)->  ~(q/\r)
2. ~p\/q
3  -----  Modus Ponens with 1,2,

1. (p/\r)->~q Premise
2.  ~q->r Premise
3. ----   Transitivity with 1,2


1. (p\/q)->r    Premise
2.  ~r          Premise
3. ----         Modus Tollens with 1,2
4. ----         De Morgan with 3
5. ----         Specification, using the form "A /\ B infers A".
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- ~(q/\r)
- (p/\r) -> r
- ~(p\/q), ~p /\ ~q, ~p

* Moxk problem 4 truth table

1. Build a truth table of $(p \wedge (p \oplus q)) \rightarrow \neg q$ to show this statement is a tautology.

2. use truth table to determine validity

- premmise p -> r
- premise q -> r
- conclusion p \/ q -> r

- Tautology: show it is true for each row
- Validity: Check critical rows — conclusion must be true whenever premises are true

* Mock problem 5. proof by cases

- 14. IfnϵZ,then 5n^2+3n+7 is odd.

- If n is odd
- If n is even

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* Mock problem 6  direct proof

-  If a is an integer and a^2 |a, then a∈ {-1,0,1} .
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- Suppose a is an integer and a^2 |a
- We have a = a^2 * k for some integer k
- Thus, a(1-ak)=0
- Thus a =0 or ak=1
- Since a and k are integers, a =0 or a = 1, or a = -1

* Mock problem 7. Proof by contraposition / contradiction

Suppose $x, y \in R$. If $y^3 + yx^2 \leq x^3 + xy^2$, then $y \leq x$.

- If $y > x$ …

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* Mock problem 8.  MI

 If n is a non-negative integer, then 5 | (n^5 - n).
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- Let P(n) be 5 | n^5 - n
- P(0) is clearly true
- Suppose P(n) is true, we want to show P(n+1) is true
- P(n+1) is Let P(n) be 5 | (n+1)^5 - (n+1)
- (n+1)^5 - (n+1) = n^5 + 5n^4+10n^3 + 10 n^2 + 5n - (n+1)
- So, (n+1)^5 - (n+1) is n^5 -n + 5n + 5n^4+10n^3 + 10 n^2 + 5n
- Using the assumption P(n), we have 5 | (n+1)^5 - (n+1)

* Mock problmem 9 proof of sets

1. If A,B and C are sets, then A×(B−C)=(A×B)−(A×C).

- Left is subset of right. Suppose (x,y) is an element of LHS, then x \in A and x\in B and x \not\in C. Thus (x,y) \in A x B and (x,y)\not\in A x C

- Right is sestet of left. Suppose (x,y) is an element of RHS, then (x,y) \in A x B and (x,y) \not\in A x C. Thus x \in A, y \in B and y \not \in C. Thus (x,y) is an element of LHS.

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* Mock problem 10 set sizes

1. Prove (2,3) and (6, 10) is of the same cardinality.
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- $f(x) = (x-2)*4+6$ namely $4x-2$
- We show f is one-to-one function
- We show f is onto function

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* Mock problem 11. Relation

1.  Define a relation R on Z as xRy if and only if 4|(x+3y). Prove R is an
equivalence relation.
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- Reflexivity. That is to show $4 \mid x + 3x$ for all x
- Symmetry. That is to show $4 \mid x + 3y$ implies $4 \mid y + 3x$
- Transitivity. That is to show $4 \mid x + 3y$ and $4 \mid y + 3z$ implies $4 \mid x + 3z$

* Mock problem 12. Function

- 7. This question concerns functions f : {A,B,C,D,E,F,G} → {1,2}. How many such functions are there? How many of these functions are injective? How many are surjective? How many are bijective?

- 2^7 functions
- 0 injective
- 2^7 - 2 surjective
- 0 bijective