CSE215: Lecture 04
Foundations of Computer Science

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State University of New York, Korea

February 28, 2022

Course materials and Info available here:
https://github.com/zhoulaifu/22_cse215_spring

Class policy effective from this week

- Attendance check from Wednesday (we are still having new students enrolled today)
- In-person only unless covid
- Per-week homework —> Schedule and weights of grading have changed accordingly. See course website on GitHub.

Two important skills from previous lectures

- Truth table (= evaluate a logical expression)
- Logical equivalence laws
- Useful for checking logical equivalence

SBU 2021 Midterm-1

Problem 2. [5 points]

Check the logical equivalence of $((p \wedge q) \rightarrow r)$ **and** $((p \rightarrow r) \vee (q \rightarrow r))$.

Today’s plan

- Strengthening last lecture with emphasis on **intuition**
- Quiz
- Exam problems

Strengthening our last lecture with more intuition

Statement form

Definition

- Statement form or propositional form is a compound statement with propositional variables (such as p, q, r) and logical connectives (such as $\sim, \wedge, \vee$).

Examples

- $(p \vee q) \wedge \sim (\sim p \wedge r)$
- $(\sim p \wedge q \wedge r) \vee (q \vee \sim r)$

Biconditional statement $(p \leftrightarrow q)$

Definitions

- The biconditional of p and q is of the form "p if and only if q" and is denoted by $p \leftrightarrow q$. It is true when p and q have the same truth value, and it is false, otherwise.
- $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Examples

- Assume x and y are real numbers.
  "$x^2 + y^2 = 0$ if and only if $x = 0$ and $y = 0$."

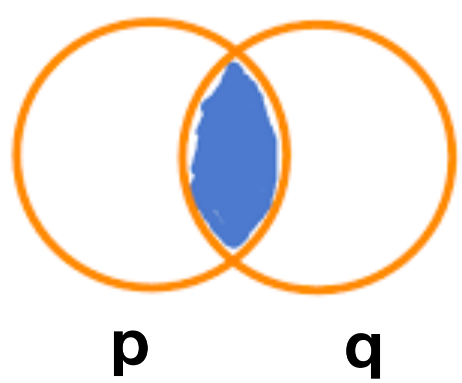
Questions: x * y = 0 if and only if _____?

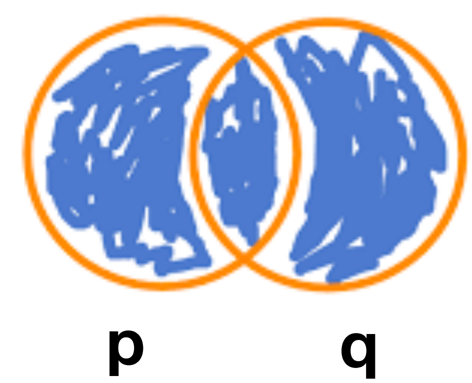
Logical equivalence

Laws	Formula	Formula
Commutative laws	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
Associative laws	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
Distributive laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
Identity laws	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
Negation laws	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
Double neg. law	$\sim (\sim p) \equiv p$	
Idempotent laws	$p \wedge p \equiv p$	$p \vee p \equiv p$
Uni. bound laws	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$
De Morgan's laws	$\sim (p \wedge q) \equiv \sim p \vee \sim q$	$\sim (p \vee q) \equiv \sim p \wedge \sim q$
Absorption laws	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Negations	$\sim \mathbf{t} \equiv \mathbf{c}$	$\sim \mathbf{c} \equiv \mathbf{t}$

Commutative Law

Laws	Formula	Formula
Commutative laws	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$

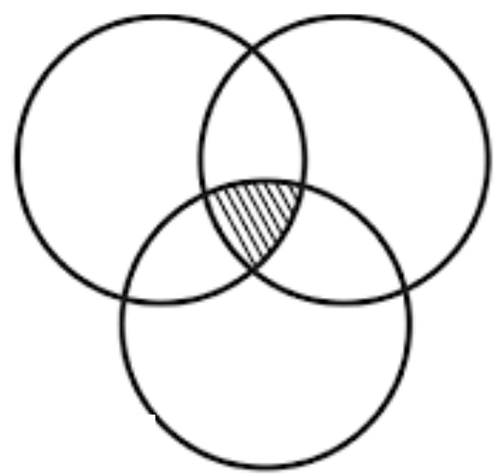


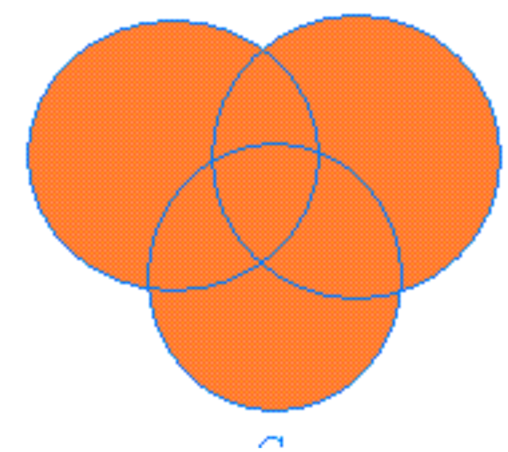


- Give some equivalent statement forms for (p∧q) ∨ (s∨t)

Associative Law

Associative laws	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$





- Think about an equivalent forms for (p/\q) \/ (s\/t)

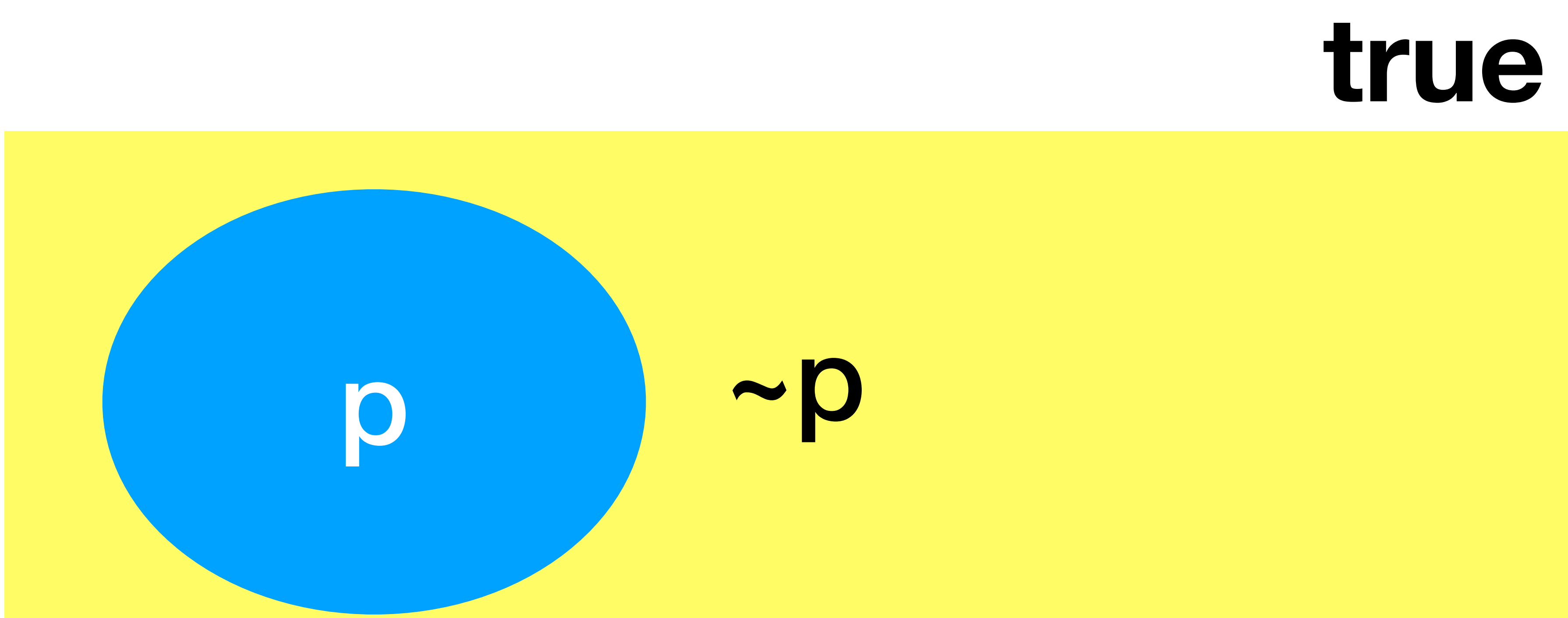
Distributive Law

Distributive laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

- A bit like a * (b + c) = a * b + a * c
- Think about an equivalent forms for $(p \wedge q) \vee (s \vee t)$

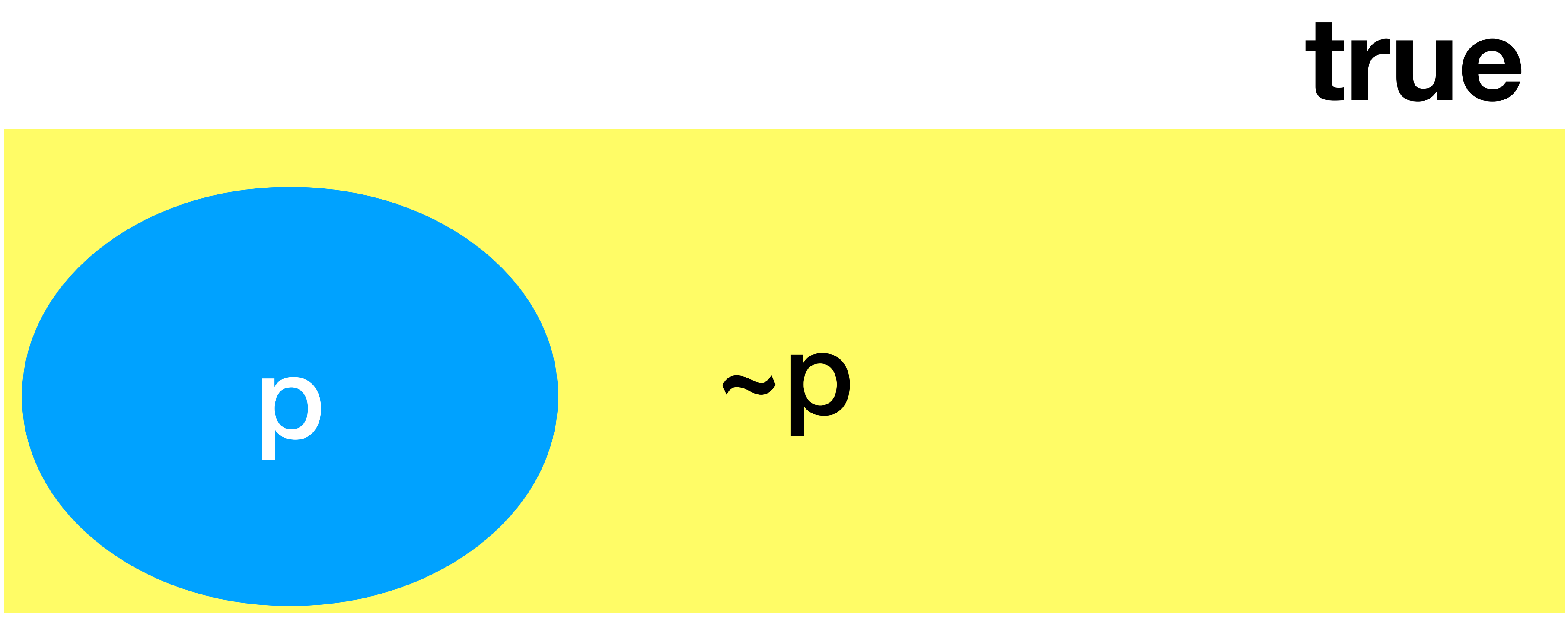
Laws with “true” and “false”

Identity laws	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
Negation laws	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
Uni. bound laws	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$



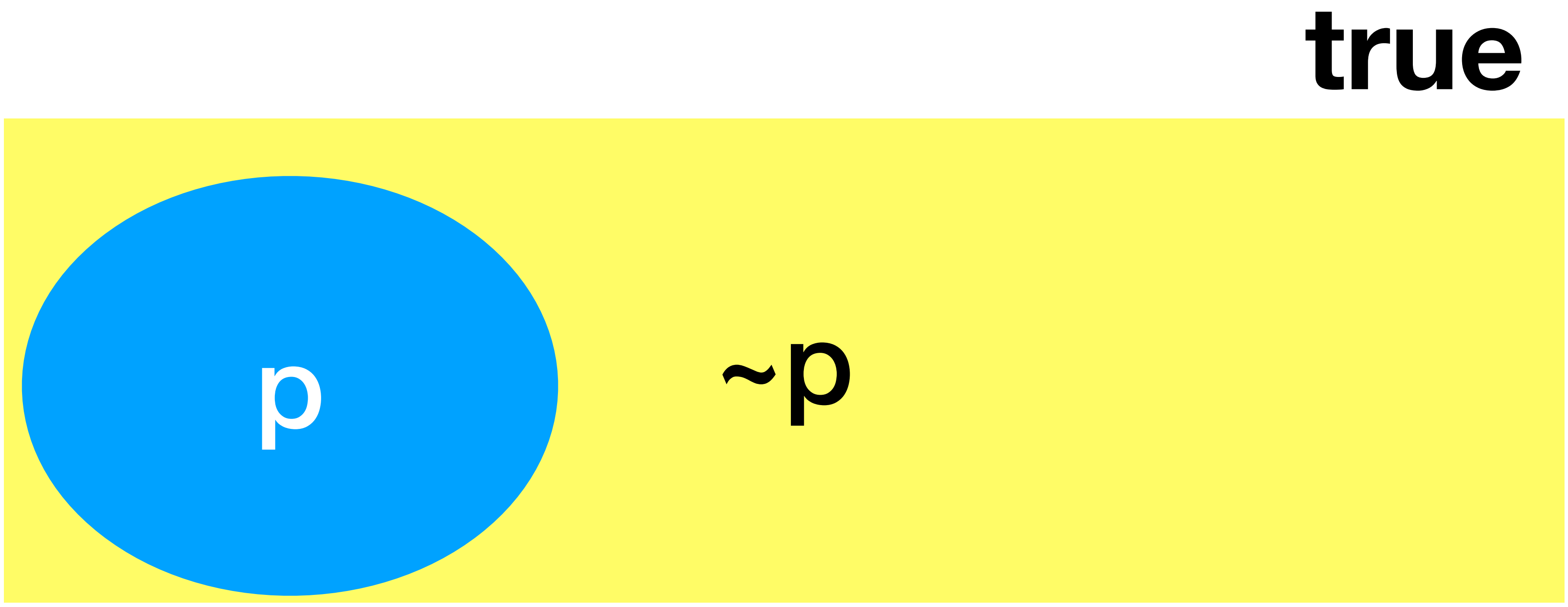
Double-negation Law

Double neg. law	$\sim(\sim p) \equiv p$



Idempotent Law

Idempotent laws $p \wedge p \equiv p$ $p \vee p \equiv p$



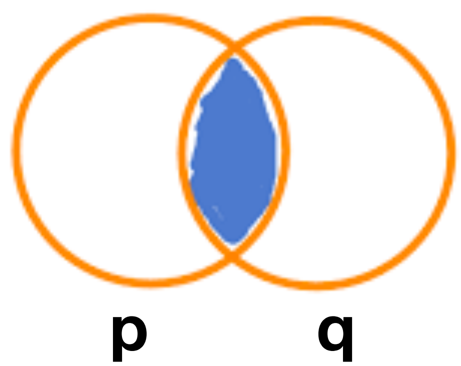
De Morgen Law

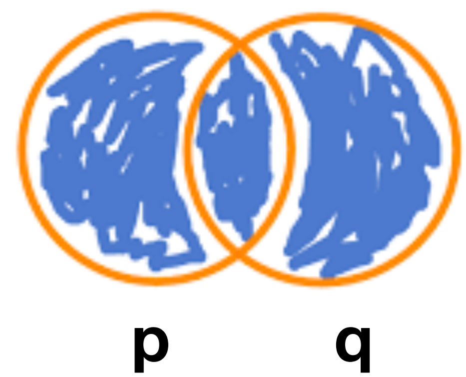
De Morgan's laws	$\sim(p \wedge q) \equiv \sim p \vee \sim q$	$\sim(p \vee q) \equiv \sim p \wedge \sim q$

- p = student A is from Korea
- q = student B is from Korea
- $p \wedge q$ = Both student A and B are from Korea
- $\sim(p \wedge q)$ = Either A is not from Korea, or B is not from Korea
- $p \vee q$ = ____
- $\sim(p \vee q)$ = ____

Absorption Law

Absorption laws	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$





Quiz &
Exam problems

Final Exam (May 18, 2021, 08:00 - 10:45 am)
CSE 215: Foundations of Computer Science
State University of New York at Stony Brook, Spring 2021
Instructor: Prof. Pramod Ganapathi

Total points = 60. Total questions = 11. Total pages = 2.

Problem 1. [5 points]

Construct a truth table for the following statement form: $p \wedge (q \vee r) \leftrightarrow p \wedge (q \wedge r)$.

Problem 2. [5 points]

Construct a truth table for the following statement form: $(p \to q) \vee ((q \oplus r) \to \sim p)$.

Problem 3. [5 points]

Mention whether the following statements are true or false. Reasons are not needed.

(a) [1 point] $p \vee \sim p \equiv \mathbf{c}$

(b) [1 point] $p \vee (p \wedge q) \equiv p \wedge (p \vee q)$

(c) [1 point] $\mathbf{c} \equiv p \vee \mathbf{t}$

(d) [1 point] $p \wedge p \equiv p \vee p$

(e) [1 point] $p \wedge \mathbf{c} \equiv \sim \mathbf{t}$

Final Exam (December 17, 2020, 08:00 - 10:35 am)
CSE 215: Foundations of Computer Science
State University of New York at Stony Brook, Fall 2020
Instructor: Prof. Pramod Ganapathi

Total points = 60. Total questions = 11. Total pages = 2.

- Please write your full name and SBU student ID on the answer sheet.
- Please include the following integrity statement on your answer sheet:
  "Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that any suspected instance of academic dishonesty will be reported to the appropriate office and that I will be subjected to the maximum possible penalty permitted under University guidelines."

Problem 1. [5 points]

Determine if the following deduction rule is valid.

$p \to (q \vee r)$
$\sim (p \to q)$
$\therefore\ r$

Problem 2. [5 points]

Suppose p and q are propositional statements. Prove that p and q are logically equivalent if and only if $p \leftrightarrow q$ is a tautology.

Problem 3. [5 points]

Verify using truth tables if the following two logical expressions are equivalent.

$(p \to q) \wedge (\sim p \to \sim q)$ and $\sim p \leftrightarrow \sim q$

Final Exam (December 17, 2020, 11:15 am - 01:50 pm)
CSE 215: Foundations of Computer Science
State University of New York at Stony Brook, Fall 2020
Instructor: Prof. Pramod Ganapathi

Total points = 60. Total questions = 11. Total pages = 2.

- Please write your full name and SBU student ID on the answer sheet.
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Problem 1. [5 points]

Determine if the following deduction rule is valid.

$(p \wedge q) \rightarrow r$
$\sim p \vee \sim q$
$\therefore \quad \sim r$

Problem 2. [5 points]

Is conditional operator $\rightarrow$ an associative operator? That is, is $(p \rightarrow q) \rightarrow r$ logically equivalent to $p \rightarrow (q \rightarrow r)$? Prove your answer.

Problem 3. [5 points]

Verify using truth tables if the following two logical expressions are equivalent.
$\sim p \leftrightarrow \sim q$ and $\sim (p \oplus q)$

Summary

- Intuition on logical equivalence
- Quiz & Exam problems: **truth tables, and logical equivalence**

Thank you for your attention!