CSE215
Foundations of Computer Science

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News

- Online 2 weeks (at least)
- Continue reporting covid. Provost: “we still need to collect the data and report cases to covid19report@sunykorea.ac.kr as before.”

Anonymous poll: Midterm exam feelings

Problems are too easy

- True
- False

Problems are too few

- True
- False

Problems are unclear

- True
- False

Do you have other comments?

Dear students, please fill in this form by today: https://forms.gle/ddi1TBYqNkdyqJ7v6

Today

Midterm exam problems
Homework-3 (predicate) problems

5-10 mins break for questions

Look ahead: Proof problems

- To finish by 4h25

Midterm exam problems

Problem 1 [points = 10]

Write the negation of the following proposition:

"If the rate of the confirmed covid cases surpasses 10% of the university community, all classes will be provided online."

Answer

The rate of the confirmed covid cases surpasses 10% of the university community, and all classes will not be provided online….

But more precisely,

The rate of the confirmed covid cases surpasses 10% of the university community, and there exists a class that will not be provided online.

Problem 2 [points = 20]

Which arguments below are valid? There can be zero, one, or more than one choice. You do not need to explain.

(1)

Premises:

- If Adam is innocent, Adam will not be punished.
- Adam is innocent.

Conclusion: Adam will not be punished.

(2)

Premises:

- If Adam is innocent, Adam will not be punished.
- Adam will be punished.

Conclusion: Adam is not innocent

(3)

Premises:

- If Adam is innocent, Adam will not be punished.
- Adam is not innocent.

Conclusion: Adam will be punished.

(4)

Premises:

- If Adam is innocent, Adam will not be punished.
- Adam will not be punished.

Conclusion: Adam is innocent.

Answer

- 1 and 2

Problem 3 [points = 20]

Which propositions below are logically equivalent with $p \wedge true$? There can be zero, one, or more than one choice. You do not need to explain.

1. $p \leftrightarrow true$
2. $p \vee false$
3. $p \oplus true$
4. $true \rightarrow p$

Answer

- 1,2,4

Problem 4 [points = 20]

If the proposition $q \wedge r$ is true, find all combinations of truth values for p and s such that $(q \rightarrow (\sim p \vee s)) \wedge (\sim s \rightarrow r)$ is true. Please explain.

Answer

($q \wedge r$ is true so q is true and r is true through specialization.)

p	q	r	s	~s	~p	~p ∨ s	q→(~p ∨ s)	~s→r	(q→(~p ∨ s)) ∧ (~s→r)
T	T	T	T	F	F	T	T	T	T
T	T	T	F	T	F	F	F	T	F
F	T	T	T	F	T	T	T	T	T
F	T	T	F	T	T	T	T	T	T

- Thus, (1) p = true, s = true, (2) p = false, s = true, and (3) p = false, s = false

Problem 5 [points = 20]

Is the following argument valid or invalid? Please explain.

Premises:

1. $u \rightarrow r$
2. $(r \wedge s) \rightarrow (p \vee t)$
3. $u \wedge s$
4. $\sim t$

Conclusion: p

Answer

- u is true through specialization ([3] u $\wedge$ s).
- s is true through specialization ([3] u $\wedge$ s).
- u is true, then r is true through Modus Ponens ([1] u $\rightarrow$ r).
- (r $\wedge$ s) in [2] is true through conjunction (r = true, s = true).
- (p $\vee$ t) in [2] is true through Modus Ponens ([2] (r $\wedge$ s) $\rightarrow$ (p $\vee$ t)).
- p (conclusion) is true through elimination (p $\vee$ t is true and [4] ~t).
- It is a valid argument because the conclusion is true if all the premises are true. The premises imply the conclusion.

The truth table agrees with it. When all four premises are true, then the conclusion is true.

p	r	s	u	t	r $\wedge$ s	p $\vee$ t	u $\rightarrow$ r	(r $\wedge$ s) $\rightarrow$ (p $\vee$ t)	u $\wedge$ s	~t	p
T	T	T	T	F	T	T	T	T	T	T	T
T	T	T	F	F	T	T	T	T	F	T	T
T	T	F	T	F	F	T	T	T	F	T	T
T	T	F	F	F	F	T	T	T	F	T	T

(cont)

T	F	T	T	F	F	T	F	T	T	T	T
T	F	T	F	F	F	T	T	T	F	T	T
T	F	F	T	F	F	T	F	T	F	T	T
T	F	F	F	F	F	T	T	T	F	T	T
F	T	T	T	F	T	F	T	F	T	T	F
F	T	T	F	F	T	F	T	F	F	T	F
F	T	F	T	F	F	F	T	T	F	T	F
F	T	F	F	F	F	F	T	T	F	T	F
F	F	T	T	F	F	F	F	T	T	T	F
F	F	T	F	F	F	F	T	T	F	T	F
F	F	F	T	F	F	F	F	T	F	T	F
F	F	F	F	F	F	F	T	T	F	T	F

Problem 6 [points = 10]

In logic and computer science, we say a proposition is *satisfiable* if at least an assignment of truth values exists for the variables that make the proposition true. For example,

- $p \wedge q$ is satisfiable. This is because the proposition becomes true if we assign "true" to p and q.
- $(p \wedge q) \vee (\sim q \vee \sim q)$ is not satisfiable. This is because there are no possible truth values for p, q for which the proposition is true.

In this problem, you are to determine whether $(\sim p \vee q) \wedge (q \rightarrow \sim r \wedge \sim p) \wedge (p \vee r)$ is satisfiable or not. Please explain.

Answer

$(\sim p \vee q) \wedge (q \rightarrow \sim r \wedge \sim p) \wedge (p \vee r)$ is satisfiable because there are possible combinations of the true value of p, q, and r for which the proposition is true (p = false, q = false, r = true).

p	q	r	$\sim p$	$\sim r$	$\sim r \wedge \sim p$	$\sim p \vee q$	$q \rightarrow \sim r \wedge \sim p$	$p \vee r$	$(\sim p \vee q) \wedge (q \rightarrow \sim r \wedge \sim p) \wedge (p \vee r)$
T	T	T	F	F	F	T	F	T	F
T	T	F	F	T	F	T	F	T	F
T	F	T	F	F	F	F	T	T	F
T	F	F	F	T	F	F	T	T	F
F	T	T	T	F	F	T	F	T	F
F	T	F	T	T	T	T	T	F	F
F	F	T	T	F	F	T	T	T	T
F	F	F	T	T	T	T	T	F	F

Homework-03 (predicate) exercises

Exercise 1 (points = 24)

Rewrite the statements below using quantifiers and variables. For example, "for each even number n, n is divisible by 2", or "for each number n, if n is a need to use the exact words or patterns as above.

1. No two leaves are alike.
2. Even integers equals twice some integer.
3. The sum of two positive integers is a positive number.

Exercise 2 (points = 56)

Write a negation for each statement. For example, the negation of
real number x, x * x > 4 and x <= 2".

7. If the square of an integer is odd, then the integer is odd.

Break 5-10 min.

Then, look ahead for proof problems

- To finish by 4h25

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Problem 4. [5 points]

Prove that the sum of the squares of any two consecutive odd integers is even.

Problem 5. [5 points]

Suppose that x and y are real numbers. Prove that $x = y$ if and only if $xy = (x + y)^2/4$.

Problem 6. [5 points]

Use algebra to prove that $0.3181818\ldots \times 0.888\ldots = 28/99$. The first number has 18 repeated and the second number has 8 repeated for an infinite number of times.

Problem 7. [10 points]

Use mathematical induction to prove the following identities.

(a) [5 points] For all integers $n \geq 1$,

$$\sum_{i=1}^{n} i(i!) = (n + 1)! - 1.$$

(b) [5 points] Consider the Fibonacci sequence: $f_0 = 0$, $f_1 = 1$, and $f_n = f_{n-1} + f_{n-2}$, for $n \geq 2$. For all integers $n \geq 2$,

$$f_{n+2} = 1 + \sum_{i=0}^{n} f_i.$$

Problem 4. [5 points]

Prove that the sum of the squares of any two consecutive odd integers is even.

Problem 5. [5 points]

Suppose that x and y are real numbers. Prove that $x = y$ if and only if $xy = (x+y)^2/4$.

Problem 7. [10 points]

Use mathematical induction to prove the following identities.

(a) [5 points] For all integers $n \geq 1$,

$$\sum_{i=1}^{n} i(i!) = (n+1)! - 1.$$

That is all for today

- Midterm exam 1
- Homework 03
- First step in proof: Formalization

Thank you!