

# **CSE215**

# **Foundations of Computer Science**

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**April 27, 2022**

# Agenda

- Midterm 2 info
- Mock exam

**Finish around 4h45**

# **Midterm exam 2 info**

**To finish around 4h45**

- Format – **Online**
- Content: Proof techniques, sequences, sets
- Difficulty ~ Homework

# Summary on Sequences

- Sum
- Multiple of n
- Sequence explicit forms

# Sequence 2021 SBU

## Problem 1. [20 points]

Use mathematical induction to prove the following identities.

- (a) [5 points] For all integers  $n \geq 1$ ,

$$1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \cdots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$

- (b) [5 points] For all integers  $n \geq 1$ ,  $n(n^2 + 5)$  is a multiple of 6.

- (c) [5 points] For all integers  $n \geq 0$ ,

$$1 + \frac{2}{3} + \frac{4}{9} + \cdots + \left(\frac{2}{3}\right)^n = 3 \left[1 - \left(\frac{2}{3}\right)^{n+1}\right]$$

- (d) [5 points] Suppose that  $c_1, c_2, c_3, \dots$  is a sequence defined as follows:

$$c_1 = 3, c_2 = -9$$

$$c_k = 7c_{k-1} - 10c_{k-2} \quad \text{for all integers } k \geq 3$$

Prove that  $c_n = 4 \cdot 2^n - 5^n$  for all integers  $n \geq 1$ .

# Sequences 2020 SBU

## Problem 1. [20 points]

Use mathematical induction to prove the following identities.

(a) [5 points] For integers  $n \geq 1$ ,

$$\sum_{i=1}^n \frac{1}{(2i-1)(2i+1)} = \frac{n}{2n+1}$$

(b) [5 points] For integers  $n \geq 1$ ,

$$1^2 - 2^2 + 3^2 - \cdots + (-1)^{n-1} n^2 = (-1)^{n-1} \frac{n(n+1)}{2}$$

(c) [5 points]  $9^n + 3$  is divisible by 4 for integers  $n \geq 1$ .

(d) [5 points] Suppose that  $g_1, g_2, g_3, \dots$  is a sequence defined as follows:

$$g_1 = 3, g_2 = 5$$

$$g_k = 3g_{k-1} - 2g_{k-2} \quad \text{for all integers } k \geq 3.$$

Prove that  $g_n = 2^n + 1$  for all integers  $n \geq 1$ .

# Summary on Sets

- Venn diagram
- Prove set properties with set identities
- Prove set properties with element argument

# Sets SBU 2021

## Problem 2. [5 points]

Mention whether the following statements are true or false without giving any reasons.  
Assume all sets are subsets of a universal set  $U$ .

- (a) [1 point]  $(A \cap B) \cap (A \cap C) = A \cap (B \cup C)$
- (b) [1 point]  $A = A \cup (A \cap B)$
- (c) [1 point]  $A \subseteq A \cup B$
- (d) [1 point]  $A \cap (A \cup B) = A \cap B$
- (e) [1 point]  $A \subseteq B$  if and only if  $A \cup B = B$

# Sets SBU 2020

## Problem 2. [5 points]

Find if the statement is true or false. If the statement is true, prove it. If the statement is false, find a counterexample. Assume all sets are subsets of a universal set  $U$ . Let set  $A'$  be the complement of set  $A$ .

For all sets  $A$  and  $B$ , if  $A' \subseteq B$ , then  $A \cup B = U$ .

# Summary of summary

- integer properties – even, odd, prime, composite
- sequence properties – sum, multiple of  $n$ , explicit form
- set properties – subsets, equality

# **Mock exam**

**To finish around 4h45**

# Problem 1 (points=20)

Let  $B = \{2, \{2\}, \{3\}\}$ . Which of the following are true

- $2 \in B$
- $\{2\} \in B$
- $\{2\} \subseteq B$
- $\{\{2\}\} \subseteq B$

How many subsets of B are there?

# Problem 2 (points=10)

Determine all the elements of

$$\{1 + (-1)^n \mid n \in N\}$$

$$\{n^2 + n^3 \mid n \in \{1, 2, 3\}\}$$

# Problem 3 (points=10)

Let  $A, B, C$  be three sets. Prove that if  $A \subseteq B$  and  $B \subseteq C$ . then  $A \subseteq C$ .

# Problem 4 (points=15)

Prove:

For any integers  $x$  and  $y$ , if  $x$  is even, then  $xy$  is even.

# Problem 5 (points=15)

Prove:

$$\text{If } n \in \mathbb{N}, \text{ then } 1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + 4 \cdot 6 + \cdots + n(n+2) = \frac{n(n+1)(2n+7)}{6}.$$

# Problem 6 (points=15)

True or False. Prove your conclusion.

If  $n \in \mathbb{Z}$ , then  $n^2 + 3n + 4$  is even.

# Problem 7 (points=15)

Prove that  $\sqrt[3]{2}$  is irrational.