CSE215: Lecture 08
Foundations of Computer Science

Instructor: Zhoulai Fu

State University of New York, Korea

March 14, 2022

Many slides taken from Prof. Pramod Ganapathi (SBU). Thanks!

News

- Gradings for homework-1 are available.
- Fairness is our no. 1 priority. If you think grading is unfair, let me know tomorrow when we go through the exercises.
- I observed good homework performance from most students.
- About three students performed less. All the three students had to stay home due to COVID. I am so sorry! Please ask me or TA while catching up. We want to help.
- Midterm exam next Monday. Hybrid exam?

Let us review last lecture by doing some exercises

Today’s topic

Propositional statements with multiple quantifiers

- Try to finish around 4h40

Propositions with multiple quantifiers

$$\forall x \in D, \exists y \in E, \text{such that } p(x, y)$$

$$\exists x \in D, \forall y \in E, \text{such that } p(x, y)$$

$$\forall x \in D, \forall y \in E, \text{such that } p(x, y)$$

$$\exists x \in D, \exists y \in E, \text{such that } p(x, y)$$

Translation English to Statements with multiple quantifiers

- “There is a person supervising every detail of the production process.”
- $\exists$ person p such that $\forall$ detail d, p supervises d

Try to translate

Every nonzero real number has a reciprocal

The reciprocal of 4 is 1/4 (namely 0.25)

- for any nonzero real number r, r has a reciprocal
- for any nonzero real number r, there exists a real number s, such that r * s = 1

Negation of propositions with multiple quantifiers

$$\sim (\forall x \text{ in } D, \exists y \text{ in } E \text{ such that } P(x,y))$$
$$\equiv \exists x \text{ in } D \text{ such that} \sim (\exists y \text{ in } E \text{ such that } P(x,y))$$
$$\equiv \exists x \text{ in } D \text{ such that } \forall y \text{ in } E, \sim P(x,y)$$

$$\sim (\exists x \text{ in } D \text{ such that } \forall y \text{ in } E, P(x,y))$$
$$\equiv \forall x \text{ in } D, \sim (\forall y \text{ in } E, P(x,y))$$
$$\equiv \forall x \text{ in } D, \exists y \text{ in } E \text{ such that} \sim P(x,y)$$

Note:
the order of quantifiers matter

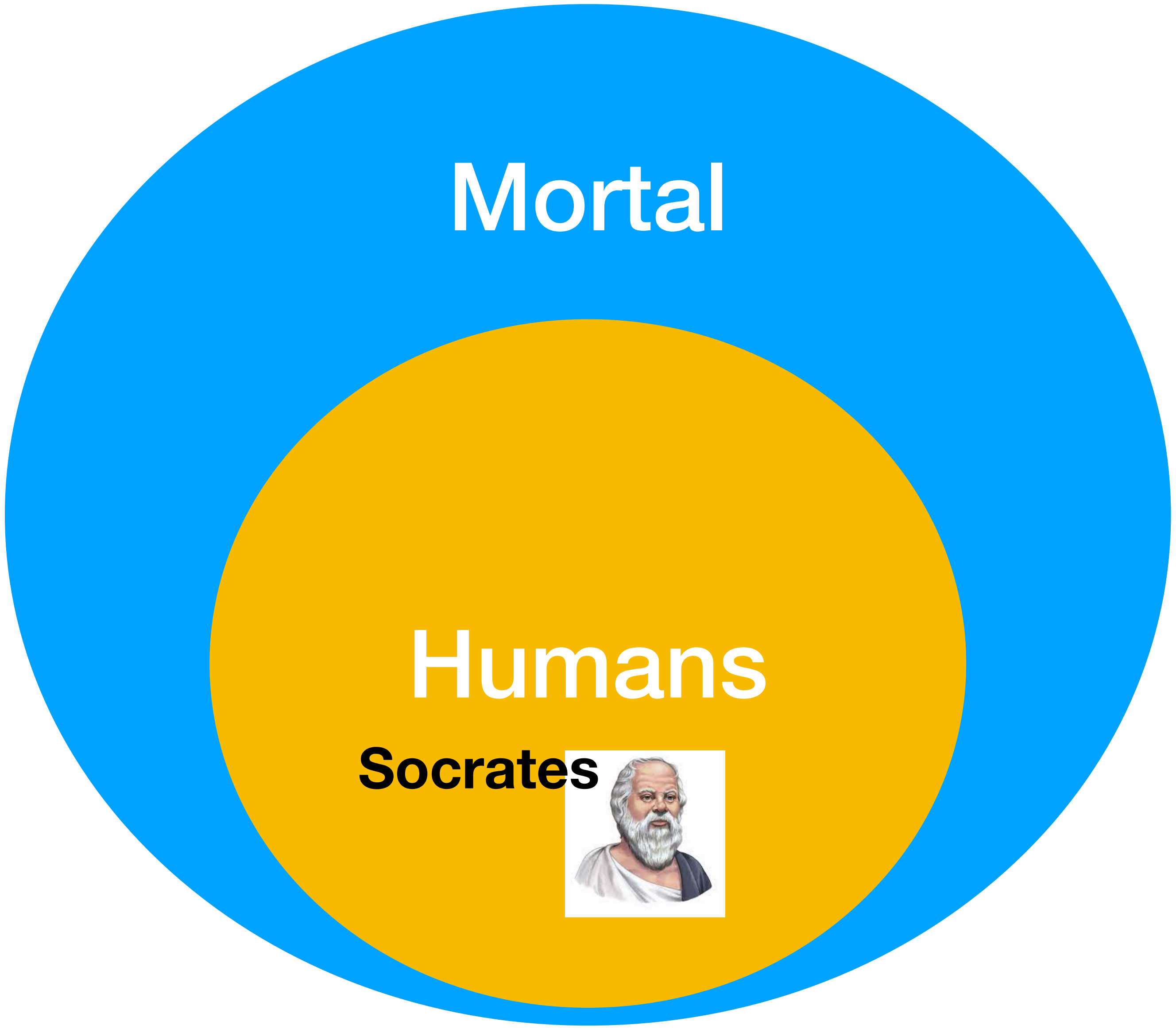
- Every lock has a key
- For any lock L, there exists a key K, such that K can unlock L.
- There is a key for every lock
- There exists a key K, such that for any lock L, K can unlock L.

Argue with multiple quantifiers

Universal Instantiation

- All humans are mortal
- Socrates is a human
- Thus, Socrates is mortal

$$\forall x \in D, P(x)$$
$$d \in D$$
$$------$$
$$P(d)$$



Universal modus ponens

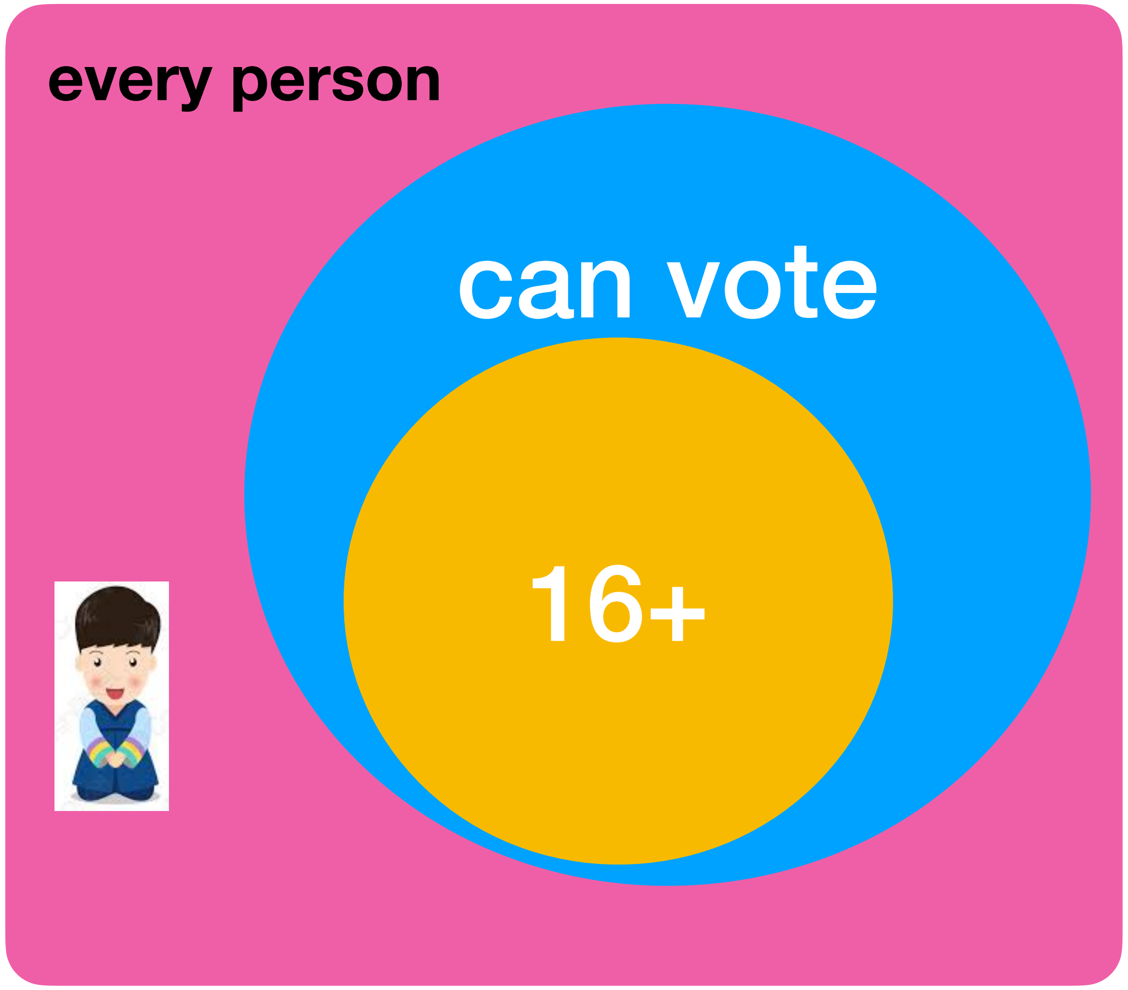
$$\forall x, P(x) \to Q(x)$$
$$P(d)$$
$$------$$
$$Q(d)$$

- For any integer x, if x is even, then x * x is even
- The integer 1024 is even
- Thus, 1024 * 1024 is even

Universal modus tollens

- For every person, if the person is 16+, he/she can vote
- Tom cannot vote
- Thus, Tom is not 16+.

$$\forall x, P(x) \to Q(x)$$
$$\sim Q(d)$$
$$------$$
$$\sim P(d)$$



Summary so far

- Proposition with multiple quantifiers
- Translation
- Negation
- Arguments

Time for exercises!