

CSE215

Foundations of Computer Science

Instructor: Zhoulai Fu

State University of New York, Korea

May 31, 2022

Agenda

- Attendance check
- WHILE (response rate < 80%)
 - { do course evaluation}
- (IF $t < 3h45$) Extra problems for midterm 2
- Mock Final

Zoom on today!

if $t < 3h45$

Extra problems midterm 2

Problem 1 (points = 3)

Let A_1, A_2, A_3, \dots be a sequence of sets. Prove $(A_1 \cup A_2 \dots \cup A_n)' = A'_1 \cap A'_2 \dots \cap A'_n$ for any integer $n \geq 1$. [Hint: Mathematical induction.]

Solution

Ex1

Let $P(k): (A_1 \cup A_2 \dots \cup A_k)' = A_1' \cap A_2' \dots \cap A_k'$

Assume $P(k)$ holds

In order for $P(n)$ to hold, $P(k+1)$ must hold

with the assumption that $P(k)$ holds

$P(k+1): (A_1 \cup A_2 \dots \cup A_{k+1})' = A_1' \cap A_2' \dots \cap A_{k+1}'$

$(A_1 \cup A_2 \dots \cup A_{k+1})' = ((A_1 \cup A_2 \dots \cup A_k) \cup A_{k+1})'$

$= (A_1 \cup A_2 \dots \cup A_k)' \cap A_{k+1}'$

$= A_1' \cap A_2' \dots \cap A_k' \cap A_{k+1}'$

$\therefore P(k+1)$ holds with the assumption $P(k)$ holds

QED

Problem 10 (points = 6)

1. Prove that for any integer n , if $3 \mid n^2$, then $3 \mid n$. [Hint: You may use the following fact: An integer is not a multiple of 3 if and only if it can be written as $3k + 1$ or $3k + 2$ for some integer k .]
2. Prove that $\sqrt{3}$ is irrational. [Hint: You may use the fact proven above to prove this one.]

Solution

1. n can be represented as $3k, 3k+1, \text{ or } 3k+2 \ k \in \mathbb{Z}$

$$(3k)^2 = 9k^2 = 3(3k^2)$$

$$(3k+1)^2 = 9k^2 + 6k + 1 = 3(3k^2 + 2k) + 1$$

$$(3k+2)^2 = 9k^2 + 12k + 4 = 3(3k^2 + 4k) + 4$$

For $3|n^2$ to hold n must be $3k$
 $3|3k$ holds

2. $\sqrt{3}$ is rational

$$\sqrt{3} = \frac{p}{q} \quad p, q \in \mathbb{Z}$$

$$3 = \frac{p^2}{q^2}$$

$3q^2 = p^2$ Since $3|p^2$ holds, $p = 3k \ k \in \mathbb{Z}$

$$3q^2 = q^2 k^2$$

$q^2 = 3k^2$ Since $3|q^2$ holds, $q = 3n \ n \in \mathbb{Z}$

Since p and q are both multiples of 3,

$\sqrt{3}$ cannot be rational. Hence, $\sqrt{3}$ is irrational

QED

Different styles for designing the final

- ? to make everyone pass
- ? to make someone suffer
- ? to make the score curve looks like Gaussian
- ? to provide a fair evaluation

Good designing in my humble opinion

- Fairness, fairness, and fairness!
- Comprehensive
 - covering most points, with well-distributed weights
- Adequate problems for each covered point
 - Quantity & difficulty

Outcome

- Problem 1 Propositional statements
- Problem 2 Negation
- Problem 3: Inference rules
- Problem 4: Truth tables
- Problem 5: Proof by dividing into cases
- Problem 6: Direct proof
- Problem 7: Proof by contraposition/counterexamples
- Problem 8: Mathematical induction
- Problem 9: Proof on set properties
- Problem 10: Cardinality of infinite sets
- Problem 11: Relations
- Problem 12: Functions

* Mock Problem 1 Propositional statements

True or false

1. $\forall x \in \mathbb{Z}, x^2 \geq 1$ (For every integer x , $x^2 \geq 1$.)
2. $\forall x \in \mathbb{R}, \exists n \in \mathbb{N}, x^n \geq 0$ (For every real number x , there exists a natural number n such that $x^n \geq 0$.)

- 1. False
- 2. True

* Mock Problem 2. Negation

1. If a is odd then a^2 is odd.
2. For every prime number p , there is another prime number q with $q > p$.

- a is odd, and a^2 is even
- There exists a prime number p , such that for any prime number q , $q \leq p$

* Mock problem 3. Inference rule

1. $(\sim p \vee q) \rightarrow \sim(q \wedge r)$
2. $\sim p \vee q$
3. ----- Modus Ponens with 1,2,

1. $(p \wedge r) \rightarrow \sim q$ Premise
2. $\sim q \rightarrow r$ Premise
3. ----- Transitivity with 1,2

1. $(p \vee q) \rightarrow r$ Premise
2. $\sim r$ Premise
3. ----- Modus Tollens with 1,2
4. ----- De Morgan with 3
5. ----- Specification, using the form "A \wedge B infers A".

- $\sim(q \wedge r)$
- r
- $\sim(p \vee q), \sim p \wedge \sim q, \sim p$

* Moxk problem 4 truth table

1. Build a truth table of $(p \wedge (p \oplus q)) \rightarrow \neg q$ to show this statement is a tautology.
 2. use truth table to determine validity
 - premmise $p \rightarrow r$
 - premise $q \rightarrow r$
 - conclusion $p \vee q \rightarrow r$
-
- Tautology: show it is true for each row
 - Validity: Check critical rows – conclusion must be true whenever premises are true

* Mock problem 5. proof by cases

- 14. If $n \in \mathbb{Z}$, then $5n^2 + 3n + 7$ is odd.

- If n is odd
- If n is even

* Mock problem 6 direct proof

- If a is an integer and $a^2 \mid a$, then $a \in \{-1, 0, 1\}$.

- Suppose a is an integer and $a^2 \mid a$
- We have $a = a^2 * k$ for some integer k
- Thus, $a(1-ak)=0$
- Thus $a=0$ or $ak=1$
- Since a and k are integers, $a=0$ or $a=1$, or $a=-1$

* Mock problem 7. Proof by contraposition / contradiction

Suppose $x, y \in \mathbb{R}$. If $y^3 + yx^2 \leq x^3 + xy^2$, then $y \leq x$.

- If $y > x \dots$

* Mock problem 8. MI

If n is a non-negative integer, then $5 \mid (n^5 - n)$.

* Mock problem 9 proof of sets

1. If A, B and C are sets, then $A \times (B - C) = (A \times B) - (A \times C)$.

* Mock problem 10 set sizes

1. Prove $(2, 3)$ and $(6, 10)$ is of the same cardinality.

* Mock problem 11. Relation

1. Define a relation R on \mathbb{Z} as xRy if and only if $4|(x+3y)$. Prove R is an equivalence relation.

* Mock problem 12. Function

- 7. This question concerns functions $f : \{A, B, C, D, E, F, G\} \rightarrow \{1, 2\}$. How many such functions are there? How many of these functions are injective? How many are surjective? How many are bijective?