CSE215: Lecture 06
Foundations of Computer Science

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Course materials and Info available here:
https://github.com/zhoulaifu/22_cse215_spring

Previous lectures

Argument
Premise_1
Premise_2
$\vdots$
Premise_m
$\therefore$ Conclusion

- use truth table to check if a logic argument is valid

Problem 1. [5 points]

Determine if the following deduction rule is valid.

$p \to (q \vee r)$

$\sim (p \to q)$

$\therefore\ r$

Final, 2020-1

Today

- Use inference rules to prove an argument is valid
- Examples of wrong inferences
- Exercises

Inference rules

Definition

- A rule of inference is a valid argument form that can be used to establish logical deductions

Name	Rule		Name	Rule	
Modus Ponens	$p \to q$ p $\therefore q$		Elimination	$p \vee q$ $\sim q$ $\therefore p$	$p \vee q$ $\sim p$ $\therefore q$
Modus Tollens	$p \to q$ $\sim q$ $\therefore \sim p$		Transitivity	$p \to q$ $q \to r$ $\therefore p \to r$	
Proof by division into cases	$p \vee q$ $p \to r$ $q \to r$ $\therefore r$		Generalization	p $\therefore p \vee q$	q $\therefore p \vee q$
			Specialization	$p \wedge q$ $\therefore p$	$p \wedge q$ $\therefore q$
Conjunction	p q $\therefore p \wedge q$		Contradiction	$\sim p \to c$ $\therefore p$	

Modus Ponens

Definition

- It has the form:
  If p, then q
  p
  $\therefore q$
- The term *modus ponens* in Latin means "method of affirming"

p	q	$p \to q$	p	q
T	T	T	T	T
T	F	F	T	
F	T	T	F	
F	F	T	F	

Example

- If the sum of the digits of 371,487 is divisible by 3, then 371,487 is divisible by 3.
  The sum of the digits of 371,487 is divisible by 3.
  $\therefore$ 371,487 is divisible by 3.

Modus Tollens

Definition

- It has the form:
  If p, then q
  $\sim q$
  $\therefore \sim p$
- The term *modus tollens* in Latin means "method of denying"

p	q	$p \to q$	$\sim q$	$\sim p$
T	T	T	F	
T	F	F	T	
F	T	T	F	
F	F	T	T	T

Example

- If Zeus is human, then Zeus is mortal.
  Zeus is not mortal.
  $\therefore$ Zeus is not human.

Generalization

Definition

- It has the form:

  p

  $\therefore p \vee q$

p	q	p	$p \vee q$
T	T	T	T
T	F	T	T
F	T	F	
F	F	F	

Example

- 35 is odd.

  $\therefore$ (more generally) 35 is odd or 35 is even.

Specialization

Definition

- It has the form:

  $p \wedge q$

  $\therefore p$

p	q	$p \wedge q$	p
T	T	T	T
T	F	F	
F	T	F	
F	F	F	

Example

- Ana knows numerical analysis and Ana knows graph algorithms.

  $\therefore$ (in particular) Ana knows graph algorithms

Conjunction

Definition

- It has the form:

  p

  q

  $\therefore p \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	
F	T	
F	F	

Example

- Lily loves mathematics.

  Lily loves algorithms.

  $\therefore$ Lily loves both mathematics and algorithms.

Elimination

Definition

- It has the form:
  $p \vee q$
  $\sim q$
  $\therefore p$
- Intuition: When you have only two possibilities and you can rule one out, the other must be the case

p	q	$p \vee q$	$\sim q$	p
T	T	T	F	T
T	F	T	T	T
F	T	T	F	F
F	F	F	T	F

Example

- Suppose $x - 3 = 0$ or $x + 2 = 0$.
  Also, suppose x is nonnegative.
  $\therefore x = 3$.

Transitivity

Definition

- It has the form:
  $p \rightarrow q$
  $q \rightarrow r$
  $\therefore p \rightarrow r$
- Can be generalized to a chain with any number of conditionals

Example

- If 18,486 is divisible by 18, then 18,486 is divisible by 9.
  If 18,486 is divisible by 9, then the sum of the digits of 18,486 is divisible by 9.
  $\therefore$ If 18,486 is divisible by 18, then the sum of the digits of 18,486 is divisible by 9.

Division into cases

Definition

- It has the form:
  $p \vee q$
  $p \rightarrow r$
  $q \rightarrow r$
  $\therefore r$

Example

- x is positive or x is negative.
  If x is positive, then $x^2 > 0$.
  If x is negative, then $x^2 > 0$.
  $\therefore x^2 > 0$.

Contradiction rule

Definition

- It has the form:

  $\sim p \to \mathbf{c}$

  $\therefore p$
- Intuition: If an assumption leads to a contradiction, then that assumption must be false
- Extensively used in a proof technique called proof by contradiction

p	$\sim p$	$\mathbf{c}$	$\sim p \to \mathbf{c}$	p
T	F	F	T	T
F	T	F	F	

Wrong inference

What is a fallacy?

Definition

- A fallacy is an error in reasoning that results in an invalid argument

Types

1. Using ambiguous premises, and treating them as if they were unambiguous
2. Circular reasoning (assuming what is to be proved without having derived it from the premises)
3. Jumping to a conclusion (without adequate grounds)
4. Converse error
5. Inverse error

Fallacy: Converse error

Definition

- It has the form:

  $p \to q$

  q

  $\therefore p$
- Intuition: A conditional is not equivalent to its converse
- Also known as the *fallacy of affirming the consequent*
- Superficially resembles modus ponens but is invalid

p	q	$p \to q$	q	p
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	F

Example

- If $x > 2$, then $x^2 > 4$.

  $x^2 > 4$.

  $\therefore x > 2$.

Fallacy: Inverse error

Definition

- It has the form:

  $p \to q$

  $\sim p$

  $\therefore \sim q$
- Intuition: A conditional is not equivalent to its inverse
- Also known as the *fallacy of denying the antecedent*
- Superficially resembles modus tollens but is invalid

p	q	$p \to q$	$\sim p$	$\sim q$
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Example

- If $x > 2$, then $x^2 > 4$.

  $x \leq 2$.

  $\therefore x^2 \leq 4$.

Validity $\neq$ Truthfulness; Invalidity $\neq$ Falsity

Types

- A valid argument can have a false conclusion
- An invalid argument can have a true conclusion

Examples

- Valid argument with false conclusion (Modus ponens)
  If Isaac Newton was a scientist, then Albert Einstein was not a scientist.
  Isaac Newton was a scientist.
  $\therefore$ Albert Einstein was not a scientist.
- Invalid argument with true conclusion (Converse error)
  If New York is a big city, then New York has tall buildings.
  New York has tall buildings.
  $\therefore$ New York is a big city.

What is a sound argument?

Definition

- A sound argument is an argument that is valid and has true premises
- Validity shows that an argument is logical
  Soundness shows that an argument is truthful

Examples

- Valid argument with true premises (Modus ponens)
  If Isaac Newton was a scientist, then Albert Einstein was a scientist.
  Isaac Newton was a scientist.
  $\therefore$ Albert Einstein was a scientist.

Exercises

Adapted from 2020 Final-1

Prove the following is valid using logical inference

Problem 1. [5 points]

~~Determine if the following deduction rule is valid.~~

$p \to (q \vee r)$

$\sim (p \to q)$

$\therefore\ r$

$$p \to (q \vee r)$$
$$\sim (p \to q)$$
$$\therefore\ r$$

- We have p -> (q ⋁ r), ~(p -> q) as premises
- Thus, p -> (q ⋁ r), ~(~p ⋁ q) hold // p->q = ~p ⋁ q
- Thus, p -> (q ⋁ r), p ⋀ ~q hold //doub. neg., De Morg. laws
- Thus, p -> (q ⋁ r), p, ~q hold //specification rule
- Thus, q ⋁ r, ~q hold // modus ponens
- Thus, ~q ->r, ~q hold // ~q ->r = q ⋁ r
- Thus, r holds // modus ponens

2020 Mid-exam-2

Problem 9. [5 points]

A set of premises and a conclusion are given. Use the valid arguments forms to deduce the conclusion from the premises, giving a reason for each step. Assume all variables are statement variables.

1. $b \vee \sim a \to c$
2. $\sim b \vee d$
3. $\sim e$
4. $c \wedge \sim a \to \sim d$
5. $a \to e$
6. $\therefore \sim b$

Problem of truth tellers and liars

Problem

- There is an island containing two types of people: truth tellers who always tell the truth and liars who always lie. You visit the island and are approached by two natives who speak to you as follows:
  A says: B is a truth teller.
  B says: A and I are of opposite type.
- What are A and B?

Education is what remains after one has forgotten what one has learned in school. — Albert Einstein

Summary

- Prove validity using inference rules
- Examples of wrong inference
- Quiz and exam problems

Thank you!