CSE215
Foundations of Computer Science

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Agenda

- Exercises on numbers
- A proof exercise
- Break 5 min
- Exam-level problems

Finish around 4h20

Exercises on numbers

Define numbers precisely

- A number n is an odd if ___
- A number r is rational if ___
- A number r is irrational if ___
- A number p is prime if ___
- A number p is composite if ___

expand-factorize

- expand $(x+1)^2$
- expand $(x-1)^2$
- expand $(x+y)^2$
- expand $(x+a)(x+b)$
- factorize x^2+2x+1
- factorize x^2+3x+2
- factorize x^2+4x+3
- Solve this equation of real numbers: $x^2- 2x + 1 = 0$
- Solve this equation of real numbers: $x^2 - 3x + 2 = 0$

odd-even-prime-composite

- if a and b are integers, is $10a^2+64b+7$ is odd
- if a and b are integers, is $a^2+64b+7$ is even
- Write the first five prime numbers
- Write the first composite numbers
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True or false

- If a is any odd integer and b is any even integer, then, $2a+3b$ is even.
- If a is any odd integer and b is any even integer, then, $2a+3b$ is even.
- For all real numbers $x>0$, $x^2 > x$.
- For all integers m and n, if $mn=1$ then $m=n=1$.
- For all integers m and n,if $m+n$ is odd then m and n are both odd.
- There exists an integer n such that $6n^2+27$ is prime
- There exists an integer $m>=3$ such that m^2-1 is prime
- composite + composite = composite

A proof exercise

Prove: rational + rational = rational

- Ask yourself, is it right? Check. What would be the intuition for the statement being true/false.
- Start by writing “Proof“
- Copy the statement in a formal way
- Which type is the statement — existential, universal?
- If existential, try to find an example; if universal, prove by first picking an arbitrary element in the domain set
- End by “QED.”

Break 5 min

Exam problems

To finish around 4h20

SBU 2021 Final

Problem 4. [5 points]

Prove that $n^2+9n+27$ is odd for all natural numbers n. You can use any proof technique.

SBU 2021 Final

Problem 6. [5 points]

Prove that if $n^2 + 8n + 20$ is odd, then n is odd for natural numbers n.

SBU 2022 Midterm

Problem 6. [5 points]

Let $a_1, a_2, \ldots, a_n$ be real numbers for $n \geq 1$. Prove that at least one of these numbers is greater or equal to the average of the numbers.

SBU 2022 Midterm

Problem 7. [5 points]

Prove or disprove the following statement. If x and y are rational, then x^y is rational.

SBU 2022 Midterm

Problem 8. [5 points]

Prove that for any two integers a and b, if ab is odd, then a and b are both odd.

That is all for today

- proof by contradiction
- Proof by division
- Practice, practice, and practice

Thank you!