Guideline

Homework for week 03. Due: 23h59 on Thursday, 2023-09-21.

Please submit your solutions in a single PDF on Brightspace.

Typewriting is preferred; If you're writing by hand, please ensure your handwriting is legible.

Multiple submissions are possible before the due time; the last submission will be graded.

Exercise 1 (points = 36)

Below are some arguments. For each argument try to determine whether or not it is valid. If it is valid, you answer needs to be "Valid (<specify-inference-rule>)". If it is invalid, your answer needs to be "Invalid", for which you do not need to explain.

For example, your answer to the following arguments should be "Valid (Modus Ponens)".

- If it rains, the ground is wet.
- It rains.
- Therefore, the ground is wet.

A.

1. If Jane has a cat, then Jane has a pet
2. Jane has a cat
3. Therefore, Jane has a pet

B.

1. If Jane has a cat, then Jane has a pet
2. Jane has a pet
3. Therefore, Jane has a cat

C.

1. If Jane has a cat, then Jane has a pet
2. It is not the case that Jane has a pet
3. Therefore, it is not the case that Jane has a cat

D.

1. If Jane has a cat, then Jane has a pet
2. It is not the case that Jane has a cat
3. Therefore, it is not the case that Jane has a pet

E.

1. If pigs fly, then hell has frozen over

2. Pigs fly
3. Therefore, hell has frozen over

F.

1. It is not the case that Yoda is green
2. If Darth Vader is Luke's Dad, then Yoda is green
3. Therefore, it is not the case that Darth Vader is Luke's dad

G.

1. If Professor is sick, the class will be cancelled
2. If the class is cancelled, the students will be happy
3. Therefore, if Professor is sick, students will be happy

H.

1. If Rufus is a human being, then Rufus has a right to life
2. It is not the case that Rufus is a human being
3. Therefore, it is not the case that Rufus has a right to life

I.

1. Amy joins the Army, or Mary joins the Marines
2. It is not the case that Mary joins the Marines
3. Therefore, Amy joins the Army

J.

1. Ariel joins the Air Force or Nancy joins the Navy
2. Nancy joins the Navy
3. Therefore, Ariel joins the Air Force

K.

1. I like chocolates
2. Therefore, We like chocolates

L.

1. I like Bulgogi and Bibimbap
2. Therefore, I like Bibimbap

Exercise 2 (points = 16)

Use inference rules to show the following argument is valid. To assist your writing, you can provide a list of sentences that look like this: `From "...", we have "..." following "..."`. Example: From premises "p" and "p->q", we have "q" following the inference rule "Modus Ponens".

Premises

- p ∨ q
- q -> r
- p ∧ s -> t
- ~r
- ~q → u ∧ s

Conclusion

- t

Exercise 3 (points = 30)

Rewrite the statements below using quantifiers and variables. For example, a statement like "Even numbers are divisible by 2" becomes: "for each even number n, n is divisible by 2", or "for each number n, if n is an even number, then n is divisible by 2". You do not necessarily need to use the exact words or patterns as above.

1. No two leaves are alike.
2. Even integers equals twice some integer.
3. The sum of two positive integers is a positive number.
4. Everyone loves ice cream.
5. At least one student has finished the homework.
6. No cats are reptiles.
7. There exists a number which is both even and prime.
8. There's no place like home.
9. All that glitters is not gold.
10. All men are mortal.

Exercise 5 (points = 18)

Determine whether the statements below are true or false. You do not need to give the reasons.

1. 42k is an even number for any integer k.

2. For each integer n with 2 <= n <= 6, n^2 – n + 11 is a prime number.

3. The average of any two odd integers is odd.

4. For any real number x, if x * x >= 4, then x >= 2.

5. For any real numbers x and y, x^2 - 2xy + y^2 >= 0.

6. There exists an integer x, such that $(2x + 1)^2$ is even.