CSE216
Programming Abstraction

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Homework 03

Exercise 1 (points = 20)

Consider the lambda term (λa.aλb.ab)λa.a

1. Draw a parse tree

2. Reduces it to a normal form

A term is said to be in "normal form" if no beta reductions are possible. This means you've simplified the term as much as you can, and there are no more beta reductions to perform.

Exercise 2 (points = 20)

Consider the lambda term $(\lambda x.\lambda w.xwz)y\ x$

1. Draw a parse tree
2. Reduces it to a normal form

Exercise 3 (points = 35)

In the following, we write TRUE as an alias for $\lambda x.\lambda y.x$, and FALSE as an alias for $\lambda x.\lambda y.y$.

Now, simplify the following lambda terms to their normal form using beta reduction. You don't need to show intermediate results.

1. TRUE TRUE TRUE
2. TRUE TRUE FALSE
3. TRUE FALSE TRUE
4. TRUE FALSE FALSE
5. FALSE TRUE TRUE
6. FALSE FALSE TRUE
7. FALSE TRUE FALSE

Hint: To complete this exercise, you you can either mechanically use beta reductions, or you can attempt to deduce the meanings of TRUE and FALSE.

Exercise 4 (points = 25)

Simplify the following lambda terms to their normal form using beta reduction. You don't need to show intermediate results.

1. (λx. λy. x y) a b

2. (λx. x (λy. y x)) (λz. z)

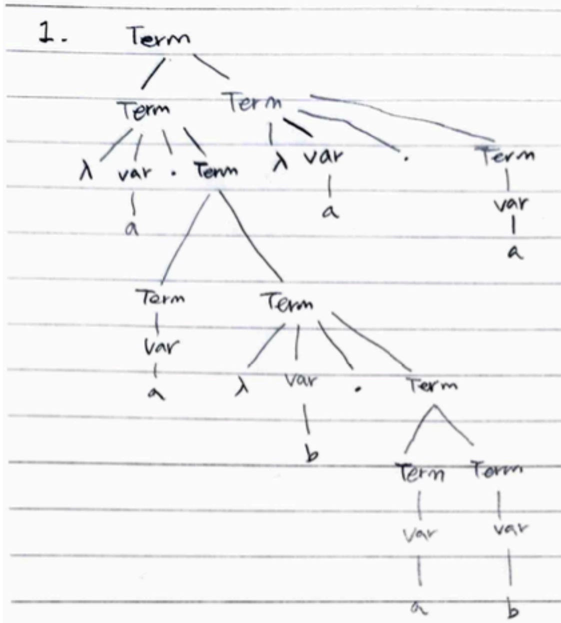
3. (λx. λy. x (λz. y z)) a b

4. (λx. λy. λz. x z (y z)) a b c

5. (λx. x (λy. y)) (λx. x)

Exercise 1

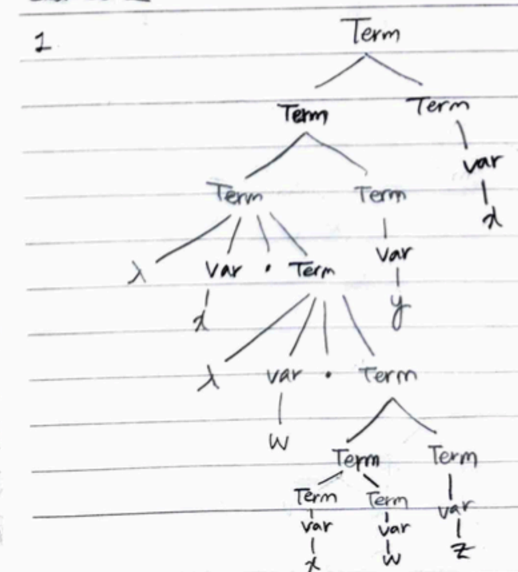
1.



2. $(\lambda a.a\lambda b.ab)\lambda a.a$
$\rightarrow (\lambda a.a)(\lambda b.(\lambda a.a)b)$
$\rightarrow \lambda b.(\lambda a.a)b$
$\rightarrow \lambda b.b$

Exercise 2

1



2. $(\lambda x.\lambda w.xwz)yx$
$\rightarrow (\lambda w.ywz)x$
$\rightarrow yxz$

Exercise 3

1. TRUE TRUE TRUE
$\rightarrow (\lambda x.\lambda y.x)(\lambda x.\lambda y.x)(\lambda x.\lambda y.x)$
$\rightarrow \lambda x.\lambda y.x$
$\rightarrow$ TRUE

2. TRUE TRUE FALSE
$\rightarrow (\lambda x.\lambda y.x)(\lambda x.\lambda y.x)(\lambda x.\lambda y.y)$
$\rightarrow \lambda x.\lambda y.x$
$\rightarrow$ TRUE

3. TRUE FALSE TRUE
$\rightarrow (\lambda x.\lambda y.x)(\lambda x.\lambda y.y)(\lambda x.\lambda y.x)$
$\rightarrow \lambda x.\lambda y.y$
$\rightarrow$ FALSE

4. TRUE FALSE FALSE
$\rightarrow (\lambda x.\lambda y.x)(\lambda x.\lambda y.y)(\lambda x.\lambda y.y)$
$\rightarrow \lambda x.\lambda y.y$
$\rightarrow$ FALSE

5. FALSE TRUE TRUE
$\rightarrow (\lambda x.\lambda y.y)(\lambda x.\lambda y.x)(\lambda x.\lambda y.x)$
$\rightarrow \lambda x.\lambda y.x$
$\rightarrow$ TRUE

6. FALSE FALSE TRUE
$\rightarrow (\lambda x.\lambda y.y)(\lambda x.\lambda y.y)(\lambda x.\lambda y.x)$
$\rightarrow \lambda x.\lambda y.x$
$\rightarrow$ TRUE

7. FALSE TRUE FALSE
$\rightarrow (\lambda x.\lambda y.y)(\lambda x.\lambda y.x)(\lambda x.\lambda y.y)$
$\rightarrow \lambda x.\lambda y.y$
$\rightarrow$ FALSE

Exercise 4

1. ab
2. $\lambda y.y(\lambda z.z)$
3. $a(\lambda z.bz)$
4. $ac(bc)$
5. $\lambda y.y$

Homework 04

Exercise 1 (points = 40)

Check if the following is alpha equivalent

- $(\lambda x.xy)$ vs. $(\lambda x.yx)$
- $(\lambda x.xu)$ vs. $(\lambda x.xv)$
- $(\lambda a.\lambda b.abc)$ vs. $(\lambda x.\lambda y.xyz)$
- $(\lambda x.\lambda y.xyz)$ vs. $(\lambda y.\lambda x.yxz)$

Exercise 2 (points = 10)

True or False?

- The lambda expression ((λx. λy.y) y) (λx.x z) reduces to z as the normal form.

If false, provide the correct normal form.

Exercise 3 (points = 40)

Reduce the lambda term below until you get a normal form

- (λx.λy.xyz)y x
- (λx.λf.λy.fxy)(fy)
- (λx.λy.yx)(λx.xx)y
- (λx.λy.xyy)(λy.y)z

Exercise 4 (points = 10)

Let f be a function of the type `A -> B`, and g be a function of the type `B -> C`. The composition of f and g, will be a function of the type `A -> C`. This composition is read as "g of f". For example, if we let f denote the function that increments an integer by 1, and g denote division by 2, then the composition g of f applied to 41 will yield 21 (i.e., 41+1, then divided by 2).

Find out the lambda expression "compose" that takes g and f as input, and produce the composition "g of f" as explained above. In other words, you need construct a lambda term, denoted by `compose` such that `compose`

`g f x` will reduce to `g (f x)`.

Exercise 1)

- (λx.xy) vs. (λx.yx) Not equivalent
- (λx.xu) vs. (λx.xv) Not equivalent
- (λa.λb.abc) vs. (λx.λy.xyz) Not equivalent
- (λx.λy.xyz) vs. (λy.λx.yxz) Equivalent

Exercise 2)

True or False?

- The lambda expression ((λx. λy.y) y) (λx.x z) reduces to z as the normal form.

If false, provide the correct normal form.

False;

$((\lambda x.\lambda y.y)y)(\lambda x.x\ z)$

$=((\lambda x.\lambda \bar{y}.\bar{y})y)(\lambda x.x\ z)$

$=(\lambda \bar{y}.y)(\lambda x.x\ z)$

$=\lambda x.x\ z$

Exercise 3)

- (λx.λy.xyz)y x

$=(\lambda x.\lambda \bar{y}.x\bar{y}z)y\ x$

$=(\lambda \bar{y}.y\bar{y}z)x$

$=y\ x\ z$

- (λx.λf.λy.fxy)(fy)

$=(\lambda x.\lambda \bar{f}.\lambda \bar{y}.\bar{f}\ x\ \bar{y})(f\ y)$

$=\lambda \bar{f}.\lambda \bar{y}.\bar{f}\ (f\ y)\ \bar{y}$

- (λx.λy.yx)(λx.xx)y

$=(\lambda y.y\ (\lambda x.x\ x))y$

$=(\lambda \bar{y}.\bar{y}\ (\lambda x.x\ x))y$

$=y\ (\lambda x.x\ x)$

- (λx.λy.xyy)(λy.y)z

$=(\lambda x.\lambda \bar{y}.x\bar{y}\bar{y})(\lambda y.y)z$

$=(\lambda \bar{y}.(\lambda y.y)\bar{y}\bar{y})z$

$=(\lambda y.y)z\ z$

$=z\ z$

Exercise 4

answer: $(\lambda a.\lambda b.\lambda c.\ a\ (b\ c))$