

Guideline

Due Date: Thursday, 2023-11-16, by 23:59.

Upload your answers as a singular PDF to Brightspace.

If you're writing by hand, please ensure your handwriting is legible.

Multiple submissions are possible before the due time; the last submission will be graded.

Exercise 1 (points = 10)

1. What is the number of subsets of the set $\{1, 2, 3\}$?
2. Let $A = \{1, 2, 3\}$, $B = \{x, y\}$, $C = \{\text{"hello"}, \text{"world"}\}$. What is the number of elements in $A \times B \times C$?
3. In a city, 20 percent of the population travels by car, 50 percent travels by bus and 10 percent travels by both car and bus. Then what is the percentage of population traveling by car or bus?
4. Which of the following is the empty set?
 - (a) $\{x : x \text{ is a real number and } x^2 - 1 = 0\}$
 - (b) $\{x : x \text{ is a real number and } x^2 + 1 = 0\}$
 - (c) $\{x : x \text{ is a real number and } x^2 + 10x + 25 = 0\}$
 - (d) $\{x : x \text{ is a real number and } x^2 = x + 2\}$

Exercise 2 (points = 10)

Let N denote the set of natural numbers $\{1, 2, \dots\}$. For each $n \in N$, let $A_n = \{-2n, 0, 2n\}$. Calculate the following:

1. $\bigcup_{i \in N} A_i$
2. $\bigcap_{i \in N} A_i$

Exercise 3 (10 points)

Prove that $n^3 + 2n$ is divisible by 3 for all integer $n \geq 1$.

Exercise 4 (points = 60)

Use set identities to prove the following, stating which laws you are using at each step.

1. $B \cup (\emptyset \cap A) = B$
2. $(A' \cap U)' = A$
3. $(C \cup A) \cap (B \cup A) = A \cup (B \cap C)$

4. $(A \cap B) \cup (A \cap B') = A$

5. $(A \cap B) \cup (A \cup B')' = B$

6. $A \cap (A \cup B) = A$

Exercise 5 (points = 10)

Suppose A, B and C are sets. Prove $A \times (B - C) = (A \times B) - (A \times C)$, where "x" means set cartesian product.