CSE215
Foundations of Computer Science

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Plan

- Revision: tautology/contradiction
- Revision: Equivalence laws
- Revision: Truth tables for validity and equivalence

Revision:
Tautology and Contradiction

Two special logical equivalence: Tautology and contradiction

Definitions

- A tautology is a statement form that is always true regardless of the truth values of the individual statements substituted for its statement variables.
- A contradication is a statement form that is always false regardless of the truth values of the individual statements substituted for its statement variables.

Examples

- $p \vee \sim p$ ▷ Tautology
- $p \wedge \sim p$ ▷ Contradiction

The secret of a fortune teller

- Three students ask a fortune teller if they got an “A” in the exam
- The fortune teller says nothing but shows 1 finger

- If they all got A —> 1 is right
- If they all failed to get A —> 1 is right
- If one students get A —> 1 is right
- If two students get A (meaning one does not) —> 1 is right
- **The fortune teller will always be right, since he said a tautology.**

See how logic saved Chris Gardner

https://www.youtube.com/watch?v=W2r4BUB-Rsc

- Interviewer (giving a proposition): What would you say, if a guy walked in for an interview with such a bad T-shirt, and I hired him?

- Chris Gardner (thinking about logic): He must have really nice pants.

What would you say if a person with such a T-shirt walking into the interview, and I hired him

—> Get Hired

- Interviewer's **proposition**: Bad-T-shirt $\wedge$ Get-hired
- **Common-sense:** Bad-T-shirt —> ~ Get-hired
- If Chris follows common-sense and interview's proposition, he will obtain ~Get-hired $\wedge$ Get-hired. That means **contradiction**.
- Never tell interviewers that they say a contradiction.
- So, Chris has to challenge the common-sense, to argue Bad-T-shirt —> ~Get-hired is **false.**
- Chris knows that "Bad-T-shirt —> ~Get-hired" and "Get-hired -> ~Bad-T-shirt" are **equivalent**
- So, Chris is now thinking what to imply from Get-hired?
- Since Get-hired means there must be some extraordinary quality. Chris thinks of two things: Get-hired -> Nice-T-shirt $\vee$ Nice-Pants
- But Nice-T-shirt contradicts with Interviewer's proposition, so Christ concludes "Nice-Pants"

Revision:
Equivalence Laws

Equivalence laws

Laws	Formula	Formula
Commutative laws	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
Associative laws	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
Distributive laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
Identity laws	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
Negation laws	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
Double neg. law	$\sim (\sim p) \equiv p$	
Idempotent laws	$p \wedge p \equiv p$	$p \vee p \equiv p$
Uni. bound laws	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$
De Morgan's laws	$\sim (p \wedge q) \equiv \sim p \vee \sim q$	$\sim (p \vee q) \equiv \sim p \wedge \sim q$
Absorption laws	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Negations	$\sim \mathbf{t} \equiv \mathbf{c}$	$\sim \mathbf{c} \equiv \mathbf{t}$

Credits: https://cs.gmu.edu/~carlotta/teaching/INFS-501/logic1.pdf

Use the laws of logical equivalences discussed in class (e.g., Commutative laws, Associative laws, etc..) to verify the following logical equivalence. Supply a reason for each step.

$$\sim (p\vee \sim q) \vee (\sim p\wedge \sim q) \equiv \sim p$$

Use the laws of logical equivalences discussed in class (e.g., Commutative laws, Associative laws, etc..) to verify the following logical equivalence. Supply a reason for each step.

$$(p \wedge (\sim (\sim p \vee q))) \vee (p \wedge q) \equiv p$$

(a) Show that the following statement forms are all logically equivalent by using known logical equivalences. Do not use truth tables.

$$p \to (q \vee r)$$

$$(p \wedge \sim q) \to r$$

$$(p \wedge \sim r) \to q$$

(b) Use the logical equivalences established in part (a) to rewrite the following sentence in two different ways in English. (Assume n represents a fixed integer.)

If n is prime, then n is odd or n is 2.

Revision:
Truth tables for validity, equivalence

2021 Final

Problem 1. [5 points]

Construct a truth table for the following statement form: $p \wedge (q \vee r) \leftrightarrow p \wedge (q \wedge r)$.

Solution

p	q	r	p AND (q OR r)	p AND (q AND r)	p AND (q OR r) <==> p AND (q AND r)
T	T	T	T	T	T
T	T	F	T	T	T
T	F	F	F	F	T
T	F	T	T	F	F
F	T	T	F	F	T
F	T	F	F	F	T
F	F	F	F	F	T
F	F	T	F	F	T

Errata: row #2 (T T F row), column “p AND (q AND r)” should be F, and the last column should be F too.

2021 Final

Problem 2. [5 points]

Construct a truth table for the following statement form: $(p \to q) \vee ((q \oplus r) \to \sim p)$.

Solution

p	q	r	p->q	q xor r	(q xor r) -> ~p	(p -> q) \/ ((q xor r) -> ~p)
t	t	t	t	f	t	t
t	t	f	t			t
t	f	t	f	t	f	f
t	f	f	f	f	t	t
f	t	t	t			t
f	t	f	t			t
f	f	t	t			t
f	f	f	t			t

2021 Final

Problem 3. [5 points]

Verify using truth tables if the following two logical expressions are equivalent.
$(p \to q) \wedge (\sim p \to \sim q)$ and $\sim p \leftrightarrow \sim q$

Solution

p	q	p->q	~p -> ~q	(p->q) /\ (~p ->~q)	~p <-> ~q
t	t	t	t	t	t
t	f	f	t	f	f
f	t	t	f	f	f
f	f	t	t	t	t

Therefore, the two are equivalent since both logical expressions in question yield the same results in the truth table.

2020 Final-a

Problem 1. [5 points]

Determine if the following deduction rule is valid.

$p \to (q \vee r)$

$\sim (p \to q)$

$\therefore\ r$

Solution

p, q, r	p -> (q \/ r)	~ (p -> q)	r
t t t	t	f	t
t t f	t	f	f
t f t	t	t	t
t f f	f	t	f
f t t	t	f	t
f t f	t	f	f
f f t	t	f	t
f f f	t	f	f

Hence, it is valid, with the critical row (only #3) showing both the premises and the conclusion are true.

2020-final-b

Problem 3. [5 points]

Verify using truth tables if the following two logical expressions are equivalent.

$\sim p \leftrightarrow \sim q$ and $\sim (p \oplus q)$

Solution

p	q	~p <-> ~q	~(p xor q)
t	t	t	t
t	f	f	f
f	t	f	f
f	f	t	t

Therefore, the two are equivalent since both logical expressions in question yield the same results in the truth table.