CSE215
Foundations of Computer Science

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Homework week 02

Exercise 1 (points = 10)

Prove that the following are equivalent using truth tables:

1. **Distributive Law**
    - $p \wedge (q \vee r)$
    - $(p \wedge q) \vee (p \wedge r)$
2. **Absorption Law**
    - $p \vee (p \wedge q)$
    - p

Exercise 2 (score = 20)

Determine whether the arguments below are valid. Explanation is needed based on the truth table.

(1)

- premises: p -> q, q
- conclusion: p

(2)

- premises: p-> q, ~p
- conclusion: ~q

(3)

- premises: p-> q, p
- conclusion: q

(4)

- premises: p-> q, ~q
- conclusion: ~p

Exercise 3 (score = 10)

Prove the two statement forms below are logically equivalent using equivalence laws only. State at each step the law used. Be patient because it may involve much calculation.

- $(p \to q) \land (q \to r) \land (r \to p)$
- $(\sim p \land \sim q \land \sim r) \lor (p \land q \land r)$

Exercise 4 (score = 60)

Use truth tables to determine whether the arguments below are valid. Explanation is needed (e.g. based on the truth table).

(1)

- Premises: $p \to q$, $\sim p \to \sim q$
- Conclusion: $p \vee q$

(2)

- Premises: $p \vee q$, $p \to \sim q$, $\sim r \to \sim p$
- Conclusion: r

(3)

- Premises: p, $\sim q \to \sim p$, $\sim q \vee r$
- Conclusion r

(4)

- Premises: $p \wedge q \to \sim r$, $p \vee \sim q$, $\sim q \to p$
- Conclusion: $\sim r$

(5)

- Premises: $p \to r$, $q \to r$
- Conclusion: $(p \vee q) \to r$

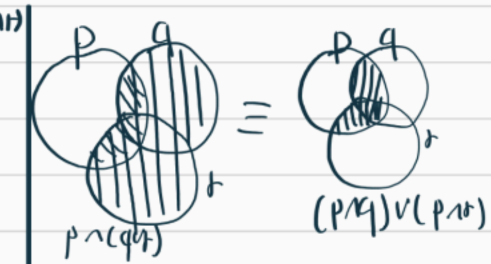
(6)

- Premises: $p \to (q \vee r)$, $\sim q \vee \sim r$
- Conclusion: $\sim p \vee \sim r$

Exercise 1

1.

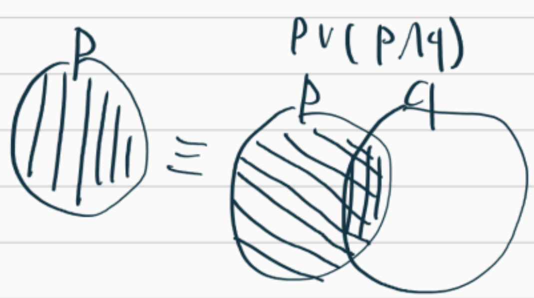
p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$(p \wedge q)$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F



According to the truth table, $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$ are equivalent. Using distributive laws, $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

2.

p	q	$p \wedge q$	$p \vee (p \wedge q)$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F



According to the truth table, p and $p \vee (p \wedge q)$ are equivalent.

All good; Diagram is not necessary

Exercise 2

(1) Premises: $p \to q$, q / conclusion: p

p	q	p→q	
T	T	T	···· critical row
T	F	F	
F	T	T	···· critical row
F	F	T	

but,

q	p→q	p	
T	T	T	
F	F	T	
T	T	F	← conclusion is F,
F	T	F	

So, this argument is invalid.

(2) premises: $p \to q$, $\sim p$ / conclusion: $\sim q$

p	q	p→q	~p	
T	T	T	F	
T	F	F	F	
F	T	T	T	— critical row
F	F	T	T	— critical row

~q	
F	
T	
F	← conclusion is F,
T	

So this argument is invalid.

(3) premises: $p \to q$, p / conclusion: q

p	q	p→q	
T	T	T	— c.r
T	F	F	
F	T	T	
F	F	T	

q	
T	← conclusion is T,
F	
T	
F	

So this argument is valid.

(4) premises: $p \to q$, $\sim q$ / conclusion: $\sim p$

p	q	p→q	~q	
T	T	T	F	
T	F	F	T	
F	T	T	F	
F	F	T	T	— c.r

~p	
F	
F	
T	
T	← conclusion is T,

So this argument is valid.

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Exercise 3

- (p->r)∩(q->r)∩(r->p)
- (~p∩~q∩~r)∪(p∩q∩r)

$$(p\to q)\wedge(q\to r)\wedge(r\to p)$$

Conditional statements written as a Disjunction

$$=(\sim p\vee q)\wedge(\sim q\vee r)\wedge(\sim r\vee p)$$

Distributive Law

$$=\big[((\sim p\vee q)\wedge\sim q)\vee((\sim p\vee q)\wedge r)\big]\wedge(\sim r\vee p)$$

Distributive Law

$$=\big[((\sim p\wedge\sim q)\vee(\cancelto{\text{False}}{q\wedge\sim q}))\vee((\sim p\vee q)\wedge r)\big]\wedge(\sim r\vee p)$$

Negation Law + Identity Law

$$=\big[(\sim p\wedge\sim q)\vee((\sim p\vee q)\wedge r)\big]\wedge(\sim r\vee p)$$

Distributive Law

$$=((\sim p\wedge\sim q)\wedge(\sim r\vee p))\vee((\sim p\vee q)\wedge r)\wedge(\sim r\vee p)$$

Distributive Law

$$=((\sim p\wedge\sim q\wedge\sim r)\vee(\cancelto{\text{False}}{\sim p\wedge\sim q\wedge p}))\vee((\sim p\vee q)\wedge r)\wedge(\sim r\vee p)$$

Associative Law + Negation Law + Identity Law

$$=(\sim p\wedge\sim q\wedge\sim r)\vee(\sim p\vee q)\wedge(r\wedge(\sim r\vee p))$$

Distributive Law

$$=(\sim p\wedge\sim q\wedge\sim r)\vee(\sim p\vee q)\wedge((\cancelto{\text{False}}{r\wedge\sim r})\vee(r\wedge p))$$

Negation Law + Identity Law

$$=(\sim p\wedge\sim q\wedge\sim r)\vee(\sim p\vee q)\wedge(r\wedge p)$$

Distributive Law

$$=(\sim p\wedge\sim q\wedge\sim r)\vee(\sim p\wedge(r\wedge p))\vee(q\wedge(r\wedge p))$$

Associative Law

$$=(\sim p\wedge\sim q\wedge\sim r)\vee(\cancelto{\text{False}}{\sim p\wedge p\wedge r})\vee(p\wedge q\wedge r)$$

Negation Law + Identity Law

$$=(\sim p\wedge\sim q\wedge\sim r)\vee(p\wedge q\wedge r)$$

Exercise 4
(1)

p	q	~p	~q	p→q	~p→~q	p v q (conclusion)
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	T
F	F	T	T	T	T	F

Since the last conclusion is false in the critical row, this argument is invalid.

(2)

p	q	r	~p	~q	~r	p v q	p→~q	~r→~p	r (conclusion)
T	T	T	F	F	F	T	F	T	T
T	T	F	F	F	T	T	F	F	F
T	F	T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T	F	F
F	T	T	T	F	F	T	T	T	T
F	T	F	T	F	T	T	T	T	F
F	F	T	T	T	F	F	T	T	T
F	F	F	T	T	T	F	T	T	F

[redacted]

This argument is invalid because the conclusion of the 6th row and the last row is false in the critical row.

(3)

p	q	r	~q	~p	p	~q→~p	~q v r	r (conclusion)
T	T	T	F	F	T	T	T	T
T	T	F	F	F	T	T	F	F
T	F	T	T	F	T	F	T	T
T	F	F	T	F	T	F	T	F
F	T	T	F	T	F	T	T	T
F	T	F	F	T	F	T	F	F
F	F	T	T	T	F	T	T	T
F	F	F	T	T	F	T	T	F

This argument is valid because the conclusion is true in the critical row (premises) on the first row.

(4)

p	q	r	~r	~q	p ∧ q	p∧q→~r	p v ~q	~q→p	~r (conclusion)
T	T	T	F	F	T	F	T	T	F
T	T	F	T	F	T	T	T	T	T
T	F	T	F	T	F	T	T	T	F
T	F	F	T	T	F	T	T	T	T
F	T	T	F	F	F	T	F	T	F
F	T	F	T	F	F	T	F	T	T
F	F	T	F	T	F	T	T	F	F
F	F	F	T	T	F	T	T	F	T

This argument is invalid because the third conclusion is false in the critical row.

4.cont.

(5)

p	q	r	p v q	p→r	q→r	(p v q)-->r (conclusion)
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	T	T	T	T
T	F	F	T	F	T	F
F	T	T	T	T	T	T
F	T	F	T	T	F	F
F	F	T	F	T	T	T
F	F	F	F	T	T	T

This argument is valid because the conclusions are all true in the critical rows.

(6)

p	q	r	~p	~q	~r	q v r	p→(q v r)	~q v ~r	~p v ~r (conclusion)
T	T	T	F	F	F	T	T	F	F
T	T	F	F	F	T	T	T	T	T
T	F	T	F	T	F	T	T	T	F
T	F	F	F	T	T	F	F	T	T
F	T	T	T	F	F	T	T	F	T
F	T	F	T	F	T	T	T	T	T
F	F	T	T	T	F	T	T	T	T
F	F	F	T	T	T	F	T	T	T

This argument is invalid because the conclusion of the third row is false in the critical row. Even though there are many cases where the conclusions are true in the critical rows, it is invalid since there is one case of the false condition in the critical row which is the third row.