CSE215
Foundations of Computer Science

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Today’s plan

- Equivalence laws
- Valid arguments

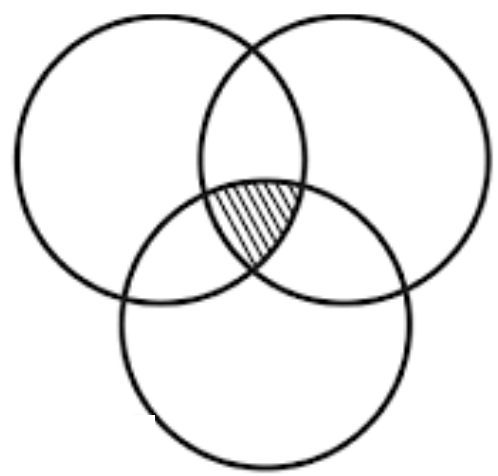
Commutative Law

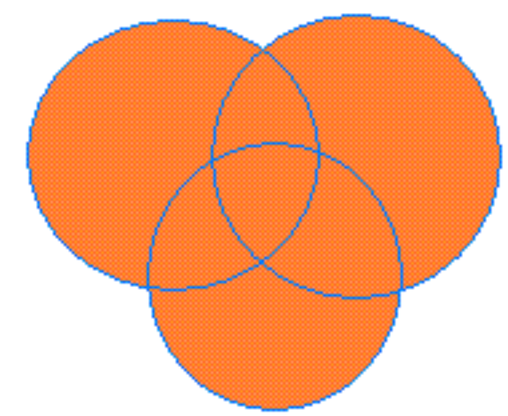
Laws	Formula	Formula
Commutative laws	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$

- Give some equivalent statement forms for $(p \wedge q) \vee (s \vee t)$

Associative Law

Associative laws	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$





- Think about an equivalent forms for $(p \wedge q) \vee (s \vee t)$

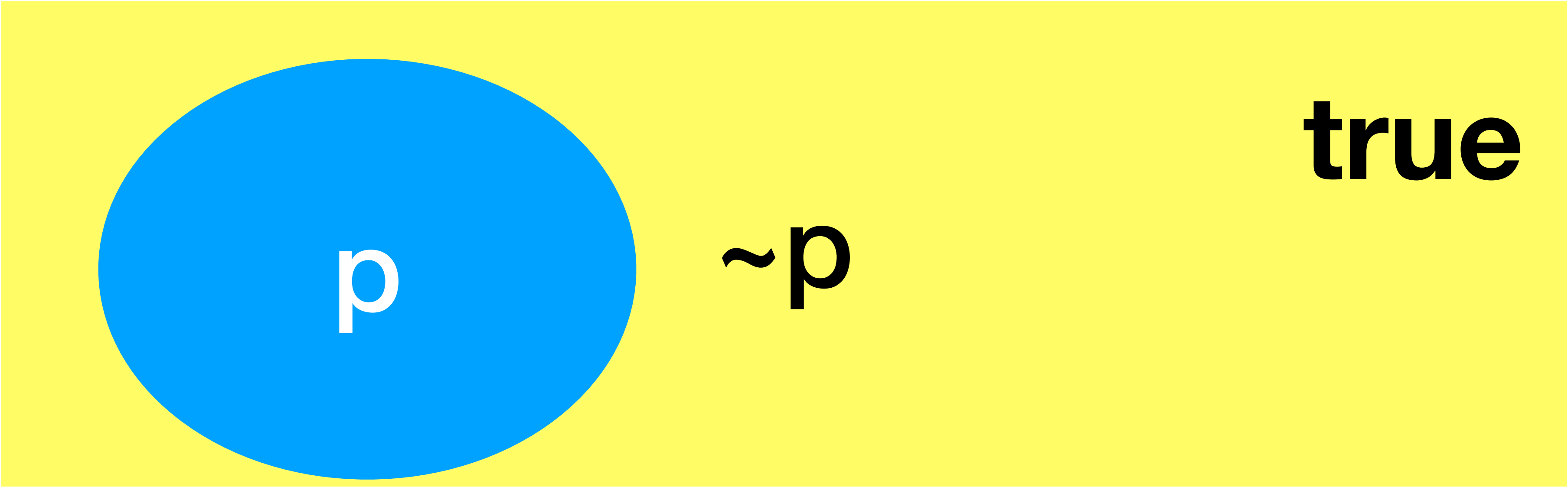
Distributive Law

Distributive laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

- A bit like a * (b + c) = a * b + a * c
- Think about an equivalent forms for $(p \wedge q) \vee (s \vee t)$

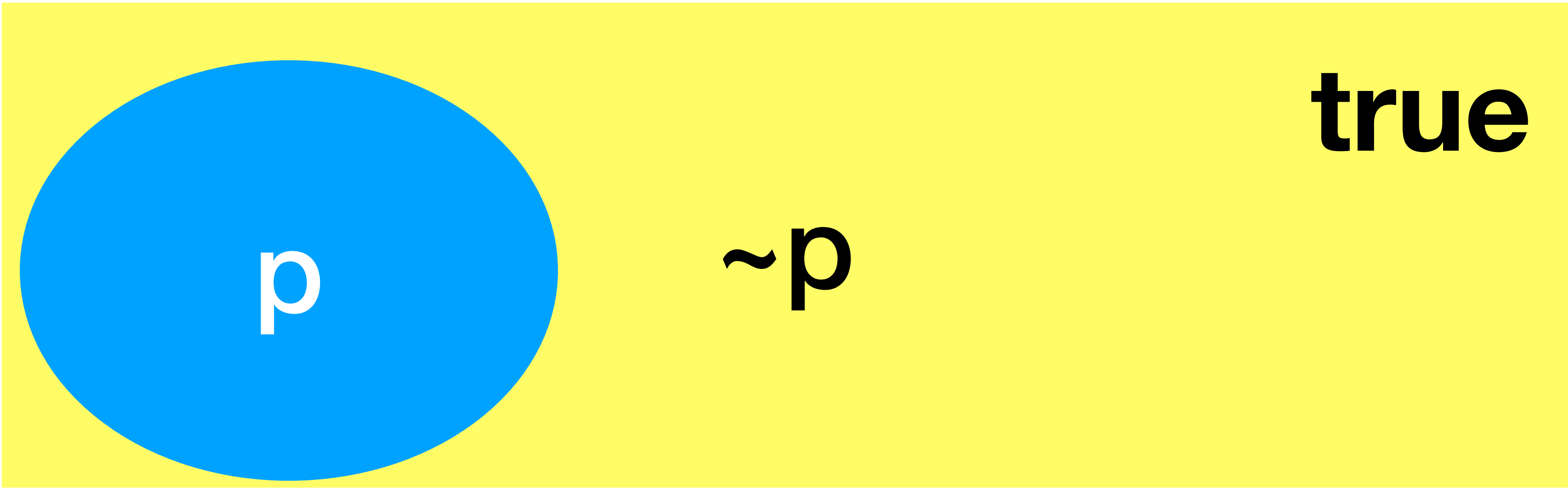
Laws with "true" and "false"

Identity laws	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
Negation laws	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
Uni. bound laws	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$



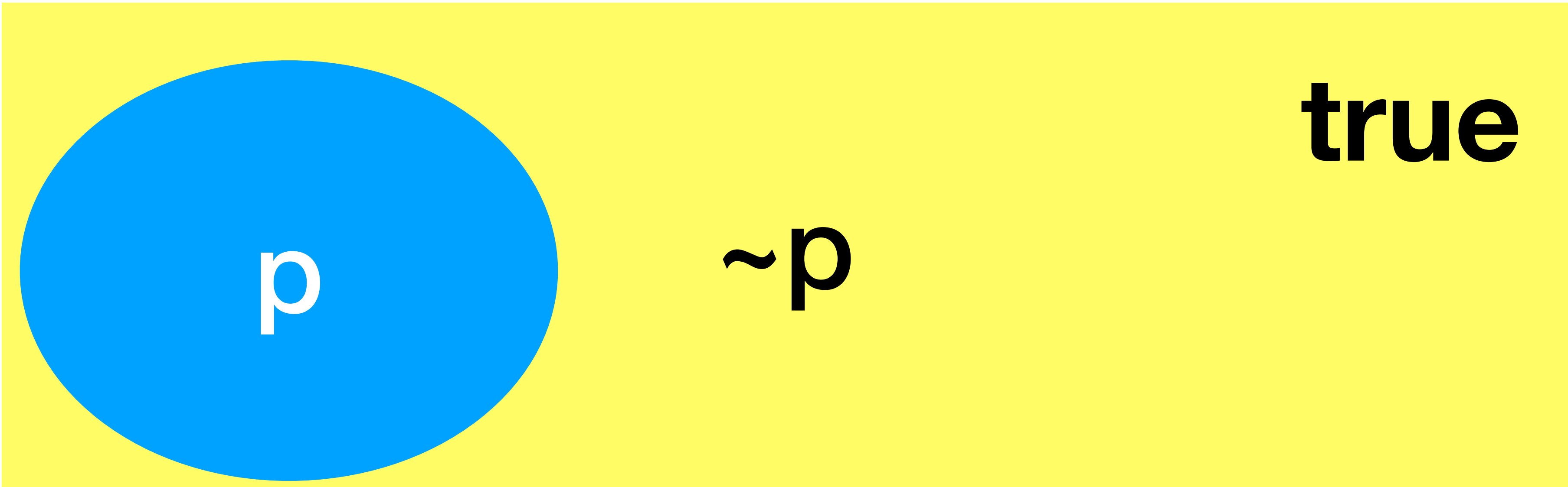
Double-negation Law

Double neg. law	$\sim(\sim p) \equiv p$



Idempotent Law

Idempotent laws $p \wedge p \equiv p$ $p \vee p \equiv p$



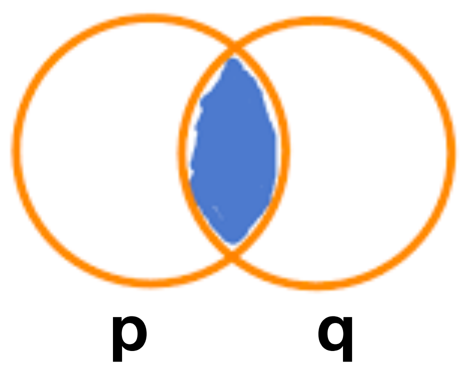
De Morgen Law

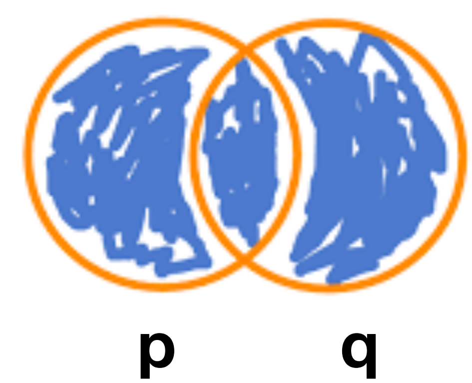
De Morgan's laws	$\sim (p \wedge q) \equiv \sim p \vee \sim q$	$\sim (p \vee q) \equiv \sim p \wedge \sim q$

- p = student A is from Korea
- q = student B is from Korea
- $p \wedge q$ = Both student A and B are from Korea
- $\sim(p \wedge q)$ = Either A is not from Korea, or B is not from Korea
- $p \vee q$ = student A or student B is from Korea
- $\sim(p \vee q)$ = ____

Absorption Law

Absorption laws	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$





Equivalence laws

Laws	Formula	Formula
Commutative laws	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
Associative laws	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
Distributive laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
Identity laws	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
Negation laws	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
Double neg. law	$\sim (\sim p) \equiv p$	
Idempotent laws	$p \wedge p \equiv p$	$p \vee p \equiv p$
Uni. bound laws	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$
De Morgan's laws	$\sim (p \wedge q) \equiv \sim p \vee \sim q$	$\sim (p \vee q) \equiv \sim p \wedge \sim q$
Absorption laws	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Negations	$\sim \mathbf{t} \equiv \mathbf{c}$	$\sim \mathbf{c} \equiv \mathbf{t}$

Exercise

Problem 3. [5 points]

Mention whether the following statements are true or false. Reasons are not needed.

(a) [1 point] $p \vee \sim p \equiv \mathbf{c}$

(b) [1 point] $p \vee (p \wedge q) \equiv p \wedge (p \vee q)$

(c) [1 point] $\mathbf{c} \equiv p \vee \mathbf{t}$

(d) [1 point] $p \wedge p \equiv p \vee p$

(e) [1 point] $p \wedge \mathbf{c} \equiv \sim \mathbf{t}$

2021-Final

Exercise

Problem 2. [5 points]
Is conditional operator $\rightarrow$ an associative operator? That is, is $(p \rightarrow q) \rightarrow r$ logically equivalent to $p \rightarrow (q \rightarrow r)$? Prove your answer.

- (p->q)->r = (~p∨q)->r = ~(~p∨q)∨r = (p ∧~q)∨r
- p ->(q ->r) = ~p∨(q->r) = ~p∨(~q∨r)=(~p∨~q)∨r
- To show the two differ, consider r=false, ~q=false, p = false
- Alternatively, we could use a truth table

Break?
Part 2. Valid arguments

Final, 2020-1

Problem 1. [5 points]

Determine if the following deduction rule is valid.

$p \to (q \vee r)$

$\sim (p \to q)$

$\therefore\ r$

What is a logical argument?

Definitions

- Logical argument. Sequence of statements aimed at demonstrating the truth of an assertion
- Conclusion. Last statement in an argument
- Premises. Last-but-one statements in an argument

Argument
Premise_1
Premise_2
$\vdots$
Premise_m
$\therefore$ Conclusion

If Premise_1 and Premise_2 and $\cdots$ and Premise_m, then Conclusion.

What is a valid argument?

Definition

- An argument is valid if the conclusion follows necessarily from the premises

https://youglish.com/pronounce/%22Valid%20Argument%22/english/us?

- Every person will die
- Socrates is a person
- So, Socrates will die

- Congressmen/women own classified info
- Investors can take profit from classified info
- So, congressmen/women should not be allowed to actively do investment

Exercise: Valid or Not?

- All cups are blue
- Socrates is cup
- So, Socrates is blue

Valid or not?

Examples

- **Valid** If Socrates is a man, then Socrates is mortal.
  Socrates is a man.
  Therefore, Socrates is mortal.

  If p, then q.
  p.
  Therefore, q.

- **Invalid** If Socrates is a man, then Socrates is mortal.
  Socrates is mortal.
  Therefore, Socrates is a man.

  If p, then q.
  q.
  Therefore, p.

- **Valid** If Socrates is a man, then Socrates is mortal.
  Socrates is not mortal.
  Therefore, Socrates is not a man.

  If p, then q.
  $\sim q$.
  Therefore, $\sim p$.

- **Invalid** If Socrates is a man, then Socrates is mortal.
  Socrates is not a man.
  Therefore, Socrates is not mortal.

  If p, then q.
  $\sim p$.
  Therefore, $\sim q$.

How to mathematically check if an argument is valid?

- Truth table
- Inference rules

Method 1: Truth table

1. Identify the premises and conclusion
2. Construct a truth table for premises and conclusion
3. A row of the truth table in which all the premises are true is called a critical row.
   If there is a critical row in which the conclusion is false, then the argument is invalid. If the conclusion in every critical row is true, then the argument is valid.

Importantly, if there are no critical rows, then the arguments is considered valid

Example

Problem

- Determine the validity of the argument:

  $p \to q \vee \sim r$

  $q \to p \wedge r$

  $\therefore p \to r$

p	q	r	$\sim r$	$q \vee \sim r$	$p \wedge r$	$p \to q \vee \sim r$	$q \to p \wedge r$	$p \to r$
T	T	T	F	T	T	T	T	T
T	T	F	T	T	F	T	F	
T	F	T	F	F	T	F	T	
T	F	F	T	T	F	T	T	F
F	T	T	F	T	F	T	F	
F	T	F	T	T	F	T	F	
F	F	T	F	F	F	T	T	T
F	F	F	T	T	F	T	T	T

Exercise

Problem 1. [5 points]

Determine if the following deduction rule is valid.

$p \to (q \vee r)$

$\sim (p \to q)$

$\therefore\ r$

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Summary

- Logical equivalence
- Valid arguments
- Check validity using truth tables