CSE215
Foundations of Computer Science

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Agenda

- Set identities

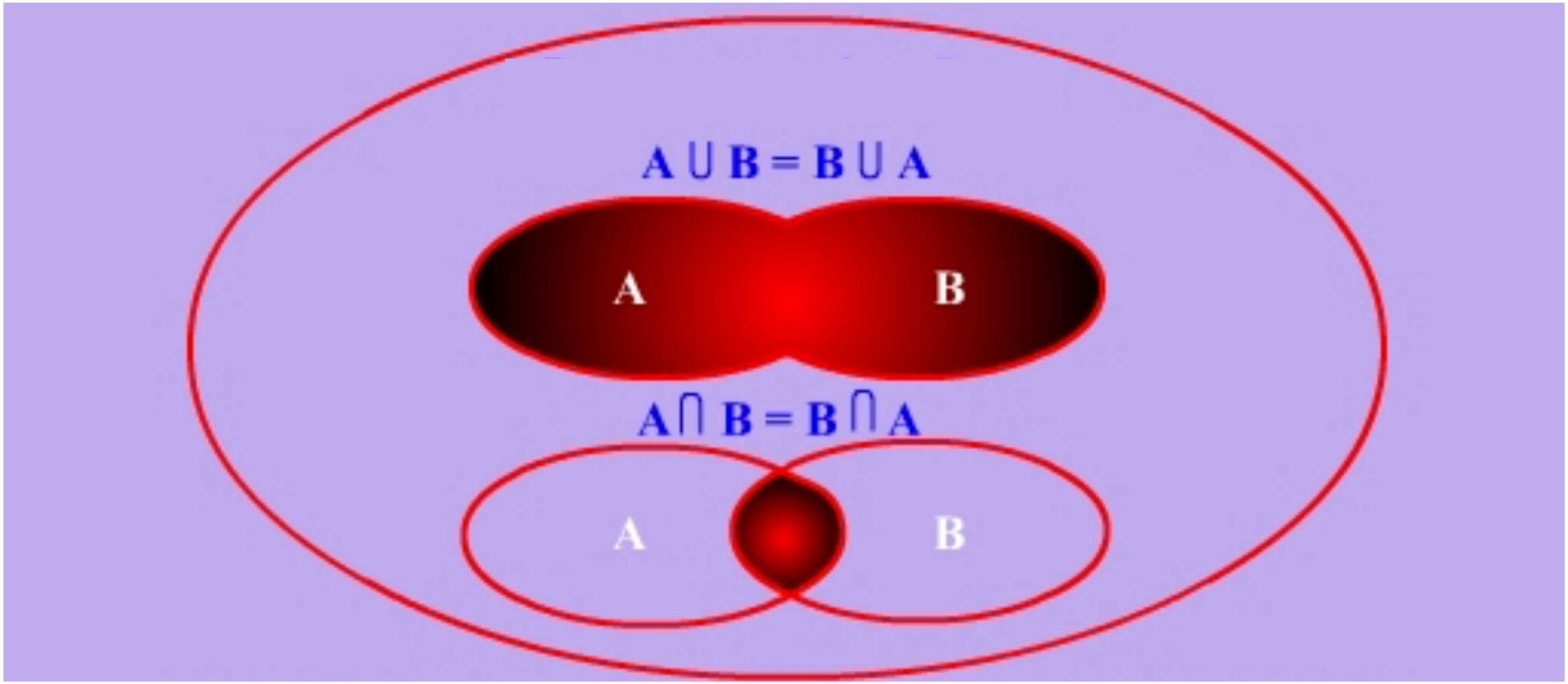
Set identities

Laws	Formula	Formula
Commutative laws	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Associative laws	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
Distributive laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identity laws	$A \cup \phi = A$	$A \cap U = A$
Complement laws	$A \cup A' = U$	$A \cap A' = \phi$
Double comp. law	$(A')' = A$	
Idempotent laws	$A \cup A = A$	$A \cap A = A$
Uni. bound laws	$A \cup U = U$	$A \cap \phi = \phi$
De Morgan's laws	$(A \cup B)' = A' \cap B'$	$(A \cap B)' = A' \cup B'$
Absorption laws	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$
Complements	$U' = \phi$	$\phi' = U$
Set diff. laws	$A - B = A \cap B'$	

Comparison: Logical laws

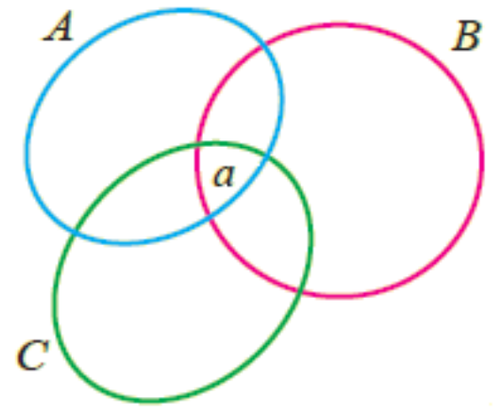
Laws	Formula	Formula
Commutative laws	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
Associative laws	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
Distributive laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
Identity laws	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
Negation laws	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
Double neg. law	$\sim (\sim p) \equiv p$	
Idempotent laws	$p \wedge p \equiv p$	$p \vee p \equiv p$
Uni. bound laws	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$
De Morgan's laws	$\sim (p \wedge q) \equiv \sim p \vee \sim q$	$\sim (p \vee q) \equiv \sim p \wedge \sim q$
Absorption laws	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Negations	$\sim \mathbf{t} \equiv \mathbf{c}$	$\sim \mathbf{c} \equiv \mathbf{t}$

Commutative laws



Associative laws

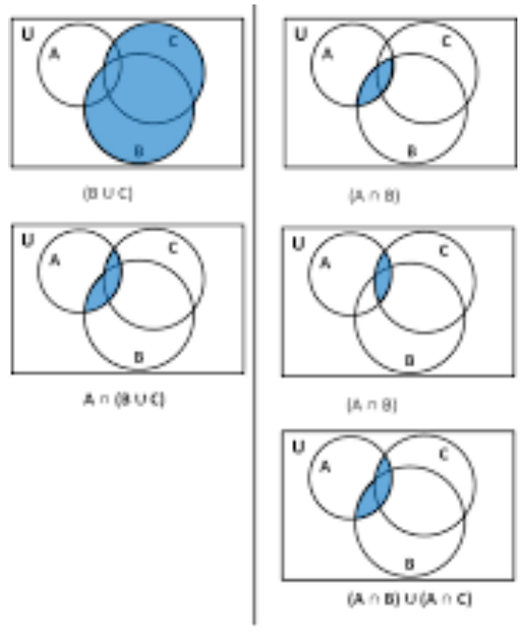
$$(A \cup B) \cup C = A \cup (B \cup C) \qquad (A \cap B) \cap C = A \cap (B \cap C)$$



Distributive laws

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$



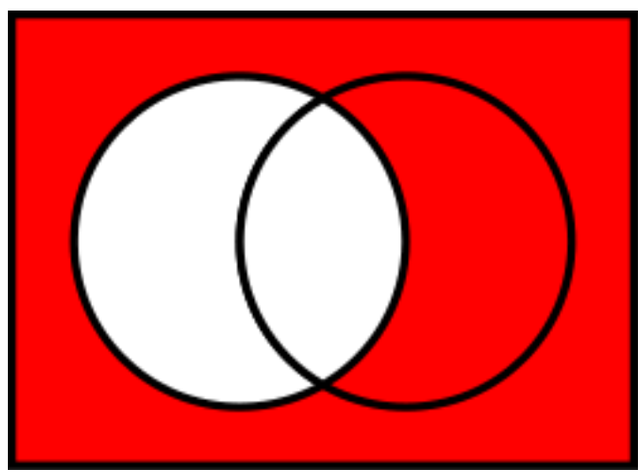
Identity, Idempotent & Universal Bound laws

Identity laws	$A \cup \phi = A$	$A \cap U = A$
Idempotent laws	$A \cup A = A$	$A \cap A = A$
Uni. bound laws	$A \cup U = U$	$A \cap \phi = \phi$

Complement & Double complement laws

$A \cup A' = U$	$A \cap A' = \phi$

$$(A')' = A$$



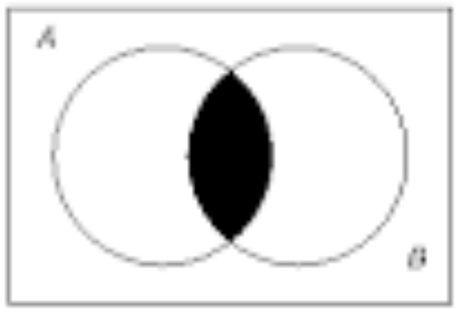
De Morgen Laws

$$(A \cup B)' = A' \cap B' \qquad (A \cap B)' = A' \cup B'$$

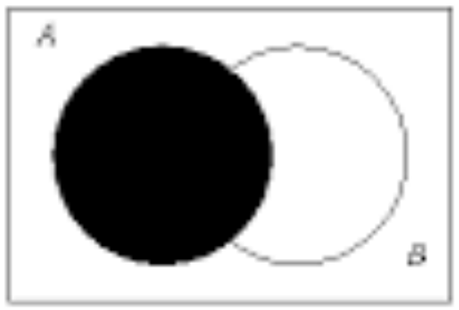
Absorption laws

Absorption laws $A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$

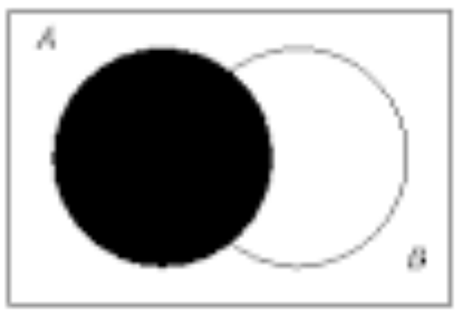
$A \cap B$



A



$A \cup (A \cap B)$



Exercises

1. Prove that for any sets A and B, $A \cup (A' \cap B) = A \cup B$. [Hint: Use set identities]
2. Prove that for any sets A, B and C, $(A - B) - (B - C) = A - B$. [Hint: Use set identities]
3. Prove that for any sets A, B, and C, $(A - C) \cap (C - B) = \emptyset$. [Hint: Use set identities]