CSE215
Foundations of Computer Science

Instructor: Zhoulai Fu

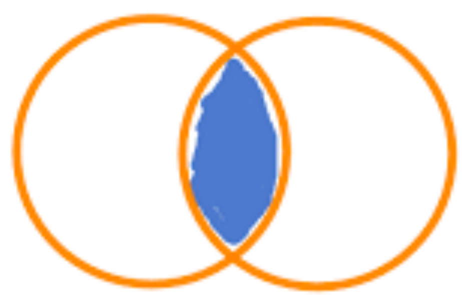
State University of New York, Korea

Today's plan

- Equivalence laws
- Valid arguments

Commutative Law

Laws	Formula	Formula
Commutative laws	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$



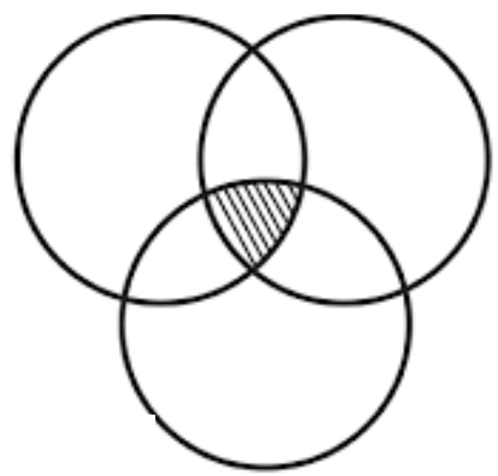
p q

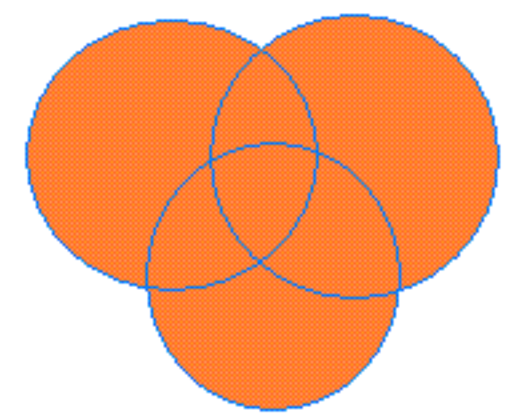
p q

- Give some equivalent statement forms for $(p \wedge q) \vee (s \vee t)$

Associative Law

Associative laws	$(p \land q) \land r \equiv p \land (q \land r)$	$(p \lor q) \lor r \equiv p \lor (q \lor r)$





- Think about an equivalent forms for $(p \land q) \lor (s \lor t)$

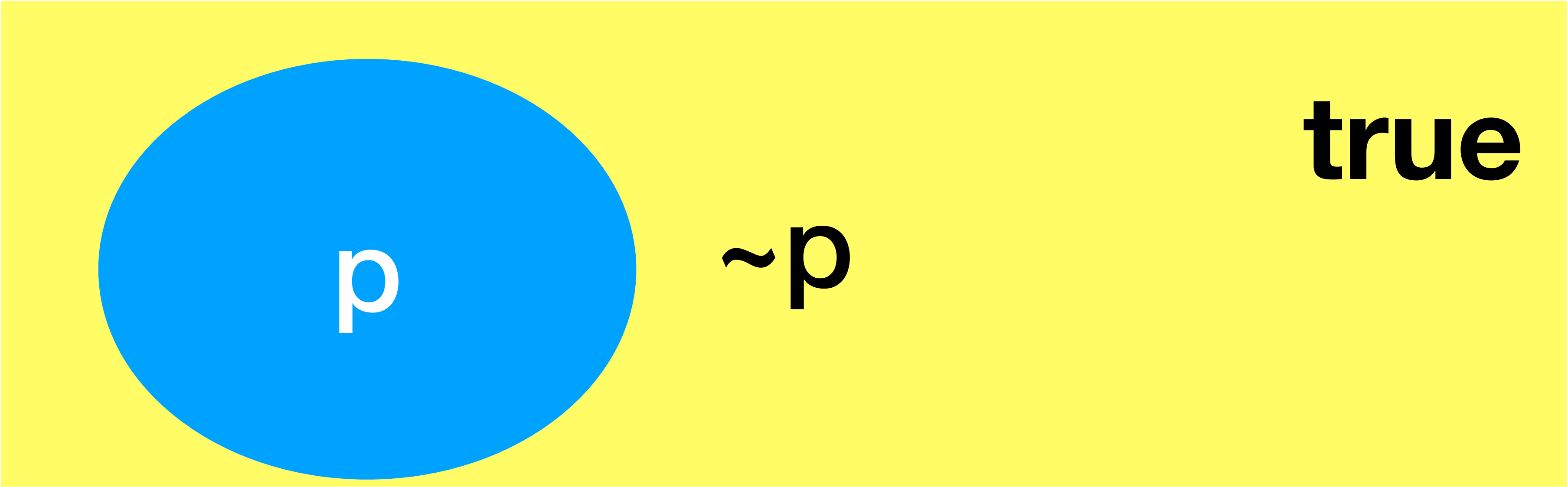
Distributive Law

Distributive laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

- A bit like a * (b + c) = a * b + a * c
- Think about an equivalent forms for $(p \wedge q) \vee (s \vee t)$

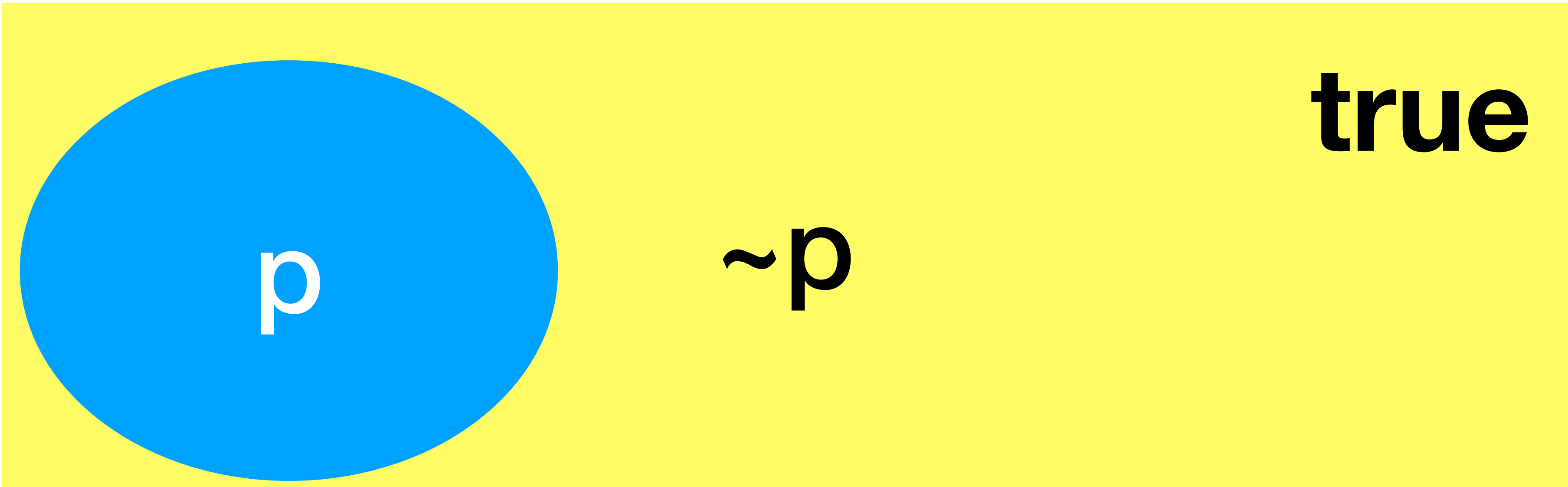
Laws with “true” and “false”

Identity laws	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
Negation laws	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
Uni. bound laws	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$



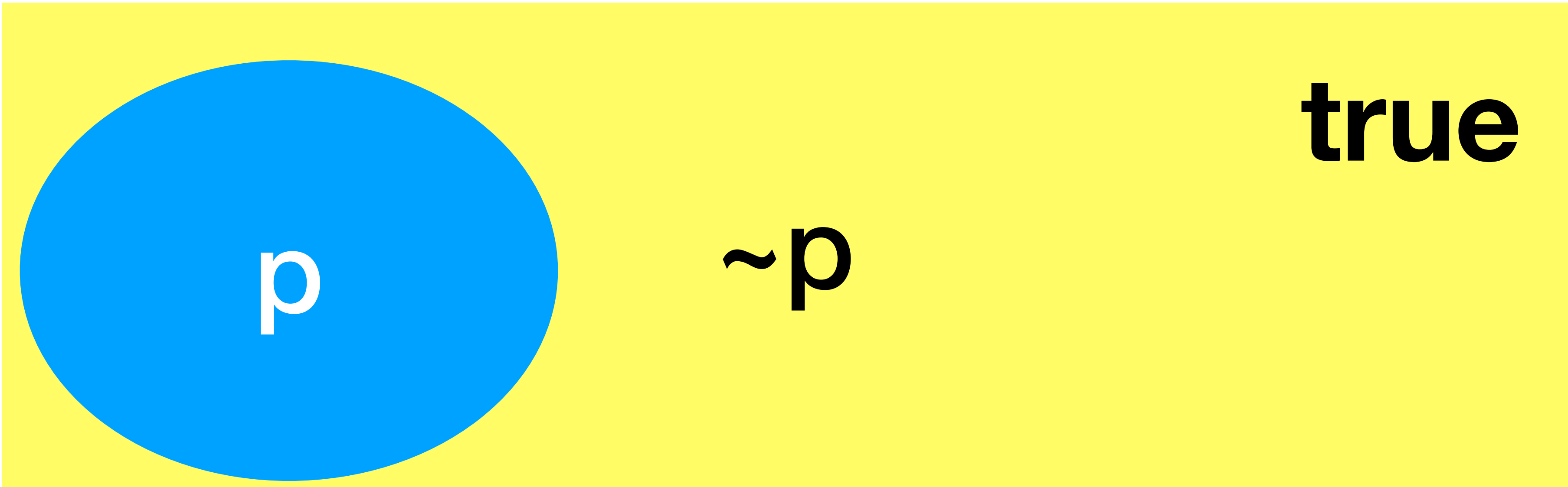
Double-negation Law

Double neg. law	$\sim(\sim p) \equiv p$



Idempotent Law

Idempotent laws $p \wedge p \equiv p$ $p \vee p \equiv p$



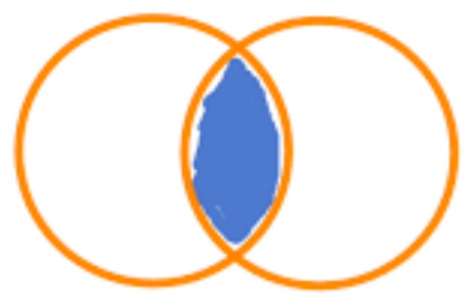
De Morgen Law

De Morgan's laws	$\sim (p \wedge q) \equiv \sim p \vee \sim q$	$\sim (p \vee q) \equiv \sim p \wedge \sim q$

- p = student A is from Korea
- q = student B is from Korea
- $p \wedge q$ = Both student A and B are from Korea
- $\sim(p \wedge q)$ = Either A is not from Korea, or B is not from Korea
- $p \vee q$ = student A or student B is from Korea
- $\sim(p \vee q)$ = ____

Absorption Law

Absorption laws	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$



p q

p q

Equivalence laws

Laws	Formula	Formula
Commutative laws	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
Associative laws	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
Distributive laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
Identity laws	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
Negation laws	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
Double neg. law	$\sim (\sim p) \equiv p$	
Idempotent laws	$p \wedge p \equiv p$	$p \vee p \equiv p$
Uni. bound laws	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$
De Morgan's laws	$\sim (p \wedge q) \equiv \sim p \vee \sim q$	$\sim (p \vee q) \equiv \sim p \wedge \sim q$
Absorption laws	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Negations	$\sim \mathbf{t} \equiv \mathbf{c}$	$\sim \mathbf{c} \equiv \mathbf{t}$

Exercise

Problem 3. [5 points]

Mention whether the following statements are true or false. Reasons are not needed.

(a) [1 point] $p \vee \sim p \equiv \mathbf{c}$

(b) [1 point] $p \vee (p \wedge q) \equiv p \wedge (p \vee q)$

(c) [1 point] $\mathbf{c} \equiv p \vee \mathbf{t}$

(d) [1 point] $p \wedge p \equiv p \vee p$

(e) [1 point] $p \wedge \mathbf{c} \equiv \sim \mathbf{t}$

2021-Final

Exercise

Problem 2. [5 points]

Is conditional operator $\to$ an associative operator? That is, is $(p \to q) \to r$ logically equivalent to $p \to (q \to r)$? Prove your answer.

- (p->q)->r = (~p∨q)->r = ~(~p∨q)∨r = (p ∧~q)∨r
- p ->(q ->r) = ~p∨(q->r) = ~p∨(~q∨r)=(~p∨~q)∨r
- To show the two differ, consider r=false, ~q=false, p = false
- Alternatively, we could use a truth table