Guideline

Due Date: Thursday, 2023-10-12, by 23:59.

Upload your answers as a singular PDF to Brightspace.

If you're writing by hand, please ensure your handwriting is legible.

Multiple submissions are possible before the due time; the last submission will be graded.

Exercise 1(points = 20)

Suppose a and b are integers. Prove $a^2 - 4b \neq 2$.

Exercise 2 (points = 20)

Suppose x is a real number. Prove that if $x^3 - x > 0$ then $x > -1$.

Exercise 3 (points = 80)

For each statement below: (1) determine if the statement is true or false. (2) Prove it. Namely, if true, prove it is true; if false, prove it is false.

Hint: To demonstrate a statement is false, we need to establish the truth of its negation. For example, if we are to prove that "for any integer x, $x^2 > 0$" is false, we need to prove "there exists an integer x such that $x^2 <= 0$", and for the latter, we will show that choosing x = 0 makes $x^2 = 0$ and therefore $x^2 <= 0$ by generalization.

1. rational/irrational is irrational.
2. Irrational*irrational is irrational.
3. The sum of any two positive irrational numbers is irrational.
4. The square root of any rational number is irrational.