CSE215
Foundations of Computer Science

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Reminder

- Final exam: Dec 12 Tuesday 12:30 - 15:00 pm at B207

- Unlimited physical notes possible

- As usual, submit a copy in person and a digital copy to BrightSpace

Oops…

- I just noticed HW 11 is not yet graded. Sorry!
- Will grade them asap
- Meanwhile, you can find the solution online

Let’s make grading transparent

- Numerical grades
- Letter grades: 50% A-ish
- Assignments after solution disclosure has never been accepted
- Participation: +1 for highly frequent contributors, 0.5 for less-than-frequent contributors
- New: The lowest-score homework will be excluded

Today

- Ungraded homework 14

Ungraded homework 14

Exercise 1 (20 points)

The *sigmoid* function plays a pivotal role in machine learning. Particularly, it's instrumental in classification problems where we map predicted values to probabilities. The sigmoid function can squish any real-valued number into the range between 0 and 1, making it extremely useful for converting values into probabilities.

The sigmoid function S: $\mathbb{R} \to (0, 1)$, is defined as:

$$S(x) = \frac{1}{1 + e^{-x}}$$

Your task in this exercise is to check if this sigmoid function S is bijective. In other words, determine whether it's both injective (or one-to-one) and surjective (or onto).

As a reminder:

- Injectivity: Show that if S(x) = S(y), then x = y. This means that no two different inputs will yield the same output.
- Surjectivity: Show that for any number y in the range (0, 1), there is an x in the domain of real numbers such that S(x) = y. This means that every possible output is produced by some input.

Exercise 2 (20 points)

1. We are considering the function $f : \mathbb{R}^2 \to \mathbb{R}^2$, defined by the equation $f(x, y) = (xy, x^3)$. Your task is to determine $f \circ f$, the composition of f with itself.

   Hint: Function composition is essentially feeding the output of one function into the other. In this case, you're feeding the output of f back into itself.

2. Now, consider the functions $f, g : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$, which are defined as $f(m, n) = (mn, m^2)$ and $g(m, n) = (m + 1, m + n)$. The task here is to compute (a) $g \circ f$, and (b) $f \circ g$.

Exercise 3. (points = 10)

Let $S = \{x \in \mathbb{R} \mid 0 < x < 1\}$

1. Let $U = {x ∈ R | 0 < x <2}. Prove that S and U have the same cardinality.
2. Let V = {x ∈ R | 2 < x <5}. Prove that S and V have the same cardinality.

Exercise 4. (points = 50)

For each pair of sets given below, establish their cardinality equality by explicitly defining a bijection between them. There's no need to formally prove your function is bijective; simply provide the function's definition. For this, you can use the following template: "Let $f : A \to B$ be defined by $f(x) = \ldots$".

1. $\mathbb{R}$ and $(0, \infty)$ (Hint: Try using the exponential function in some way.)
2. $\mathbb{R}$ and $(\sqrt{2}, \infty)$
3. $\mathbb{R}$ and $(0, 1)$

4. $\mathbb{Z}$ and S where $S = \{\ldots, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, 16, \ldots\}$

5. $A = \{3k : k \in \mathbb{Z}\}$ and $B = \{7k : k \in \mathbb{Z}\}$

6. $A = \{(5n, -3n) : n \in \mathbb{Z}\}$ and $\mathbb{Z}$

7. The set of even integers, denoted by E, and the set of odd integers, denoted by O.

8. $\mathbb{Z}$ and S where $S = x \in \mathbb{R} : \sin(x) = 1$

9. $\{0, 1\} \times \mathbb{N}$ and $\mathbb{N}$

10. $\mathbb{N}$ and $\mathbb{Z}$ (Hint: create a function that interleaves positive and negative numbers as shown in our slides)

Our paths have crossed for a reason...

Your career and us

- Our school wants to see you succeed
- You may hope to find a good job after graduation, or
- You may hope to find a good Master/Phd graduate school
- In either case, you need a resume that looks unique
- I am co-leading Software and Artificial Intelligence Lab (SAIL). With some brilliant students, we are working on machine learning, scientific computing, software engineering, and programming language techniques
- Together with you, we hope to produce great research work to make your resume shine
- Method 1. CSE487: Get credits, but can only do twice
- Method 2. RA: Try and get pay
- Prerequisite: U3/U4, strong motivation, and A-ish in most CS courses

Three advices before we depart

1

2

3

"Be yourself; everyone else is already taken."

Oscar Wilde

- While (evaluation_participation_rate < 90%){
    - do_course_evaluation();
- }