CSE215
Foundations of Computer Science

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Homework 04

Exercise 1 (points = 32)

Determine whether the statements below are true or false. Do not explain.

1. 119 is a prime number.
2. 161 is a prime number.
3. 42k is an even number for any integer k.
4. For each integer n with 2 <= n <= 6, $n^2 - n + 11$ is a prime number.
5. The average of any two odd integers is odd.
6. For any real number x, if x * x >= 4, then x >= 2.
7. For any real numbers x and y, $x^2 - 2xy + y^2 >= 0$.

8. There exists an integer x, such that $(2x + 1)^2$ is even.

Exercise 2. (points = 8)

- Conduct a bit of research: Describe the formal definition of continuity for a real-valued function f at the point x.
- Provide the formal definition for f being discontinuous at x.

issue

Exercise 2

To be continuous on point x, the function $f(x)$ must be defined at $x=a$, $f(a)$, also limit must be existed $\lim_{x\to a} f(x)$ = some value, $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$, and $\lim_{x\to a} f(x) = f(a)$.

To be discontinuous at x, $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$, and

$\lim_{x\to a} f(x) \neq f(a)$. limit of $f(x)$ is not defined.

Issue

Exercise 2

- any $\epsilon > 0$, there exists $\delta > 0$ such that for all x, if $|x - c| < \delta$, then $|f(x) - f(c)| < \epsilon$
- There exists $\epsilon < 0$, any $\delta \le 0$ such that for some x, $|x - c| < \delta$ and $|f(x) - f(c)| \ge \epsilon$

Exercise 3 (points = 15)

Prove the following proposition: An even number multiplied by an integer is an even number.

Check "Key points for proof-writing" above.

Exercise 4 (points = 15)

Prove the following proposition: An odd number multiplied by an odd number is an odd number.

Check "Key points for proof-writing" above.

Issue

Exercise 4

$(2n+1)^2$

Proof

suppose we have two odd number $(2x+1)$. $(2x+1)^2$ is equal to $4x^2+4x+1$. By definition, $4x^2$, $4x$ is an even number and by adding 1, it will be an odd number.

QED.

Exercise 5 (points = 15)

We say an integer is a perfect square if it can be expressed as a square of some integer. For example, 81 is a perfect square; 80 is not.

Prove the following statement: there is a perfect square that can be written as a sum of two other perfect squares.

Check "Key points for proof-writing" above.

Issue

Exercise 5

1. proof : for some k, r from integer, assume there is a perfect square k^2 and r^2. Then the sum of two perfect squares is k^2 + r^2. We can find this structure from Pythagorean Theorem. In Pythagorean Theorem, a^2 + b^2 = c^2, so there exists a perfect square that can be written as a sum of two other perfect squares.
   Q.E.D

Issue

Exercise 5)

prove the following statement : there is a perfect square that can be written as a sum of two other perfect squares..

· proof.

- suppose n is a perfect square, j is a perfect square, and m is a perfect square, we need to show $n = m + j$
- Since n is a perfect square, $n = a^2$ for some real number a
- Since j is a perfect square, $j = b^2$ for some real number b
- Since m is a perfect square, $m = c^2$ for some real number c
- If $a=5$, $b=3$, and $c=4$ then we note

$$a^2 = 5^2 = 25 = 3^2 + 4^2 = 9 + 16 = b^2 + c^2 = n = m + j$$

- We note that $a^2 = b^2 + c^2$ holds in this case
- Thus, there must be at least one perfect square that can be written as a sum of two other perfect squares.

·QED

Exercise 6 (points = 15)

Suppose a $\in$ Z. Prove: If a is an odd integer, then $a^2 + 3a + 5$ is odd.

Check "Key points for proof-writing" above.

Issue

Exercise 6 (points = 15)

Suppose a ∈ Z. Prove: If a is an odd integer, then a^2 + 3a + 5 is odd.
Check "Key points for proof-writing" above.

Answer:

- Let a be an odd integer.
- 3 and 5 are odd integers.
- The sum of three odd integers is always odd, we can conclude that a^2 + 3a + 5 is odd.

Issue

Exercise 6.

proof.

- Suppose a is an odd integer, then $a^2 + 3a + 5$ is odd.
- Since $a = 2k+1$ for some integer k.
- $(2k+1)^2+3(2k+1)+5 = 4k^2 + 4k + 1+6k+3+5 = 4k^2 + 10k+9 = 2(2k^2+5k+4)+1$, which shows that if a is an odd integer, then $a^2 + 3a + 5$ is odd.
- QED.

Solution

#1

(1) False

(2) False

(3) True

(4) True

(5) False

(6) False

(7) True

(8) False

f is continuous at x if

$\forall \epsilon > 0, \exists \delta > 0, \forall y, |y - x| < \delta \to |f(y) - f(x)| < \epsilon$

f is discontinuous at x if

$\exists \epsilon > 0, \forall \delta > 0, \exists y, |y - x| < \delta \land |f(y) - f(x)| \geq \epsilon$

Exercise 3

Proof

Suppose n is an even number and k an integer.

Since n is even, $n = 2k'$ for some integer k'.

Thus, $n \cdot k = 2k'k$

Thus, $n \cdot k$ is even

QED

Exercise 4

Proof

Suppose n and m are odd numbers.

Since n and m are odd, $n = 2k+1$ and $m = 2k'+1$ for some integers k, k'

$$n \cdot m = (2k+1)(2k'+1) = 4kk' + 2k + 2k' + 1$$

$$= 2(2kk' + k + k') + 1$$

Thus, $n \cdot m$ is odd

QED

Exercise 5

Proof

Let $a = 9$ and $b = 16$

Then, a and b are two perfect squares

$a+b = 25$ is also a perfect square

Thus, there exists a perfect square that can be written as a sum of two other perfect squares.

QED

Exercise 6

Proof

Suppose a is an odd integer.

$a = 2k+1$ for some integer k.

$a^2 + 3a + 5 = (2k+1)^2 + 3(2k+1) + 5 = 4k^2 + 4k + 1 + 6k + 3 + 5$

$= 4k^2 + 10k + 9 = 2(2k^2 + 5k + 4) + 1$

Thus, $a^2 + 3a + 5$ is odd

QED.