

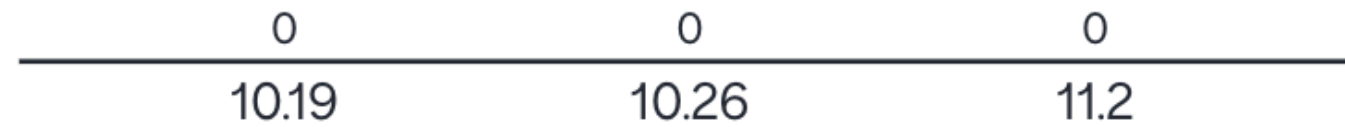
CSE215

Foundations of Computer Science

State University of New York, Korea

Instructor: Zhoulai Fu

Great time for Midterm 1



<https://www.menti.com/alw2z47gb6sp>

Agenda

- Proof by contradiction review
- Proof by contraposition
- Proof by dividing into cases

Proof by contradiction review

Proof by contradiction

- Your objective is to prove P . E.g., $\sqrt{2}$ is irrational.
- You go by proving $\sim P$ is False.
- You assume $\sim P$, and then get a contradiction

$\sqrt{2}$ is irrational

- Proof.
 - We use proof by contradiction.
 - Assume $\sqrt{2}$ is a rational number, namely:
 - (A) there exists two integers m, n such that $\sqrt{2}=m/n$, and m and n have no common factors.
 - Thus $m^2 = 2 n^2$. Thus, m^2 is even. Thus m must be even (otherwise m^2 becomes odd).
 - Thus $m = 2k$ for some integer k . Thus, $n^2 = 2 k^2$. Thus n^2 is even and therefore n must be even.
 - But the fact that m and n are both even contradicts with (A)
 - Therefore $\sqrt{2}$ must be irrational.
- QED.

Proof by contraposition

Proof by contraposition

- You are asked to prove $P \rightarrow Q$ and you feel $\sim Q \rightarrow \sim P$ is easier to prove
- Proof
 - We use proof by contraposition to proceed
 - We will prove: ____
 - Suppose ____
 - Therefore ____
- QED.

n^2 is even $\implies n$ is even

Prove:

Suppose n is an integer. If n^2 is even, then n is even

Exercise 1: Prove the following

Suppose $x \in \mathbb{R}$. If $x^2 + 5x < 0$ then $x < 0$.

Exercise 2: Prove the following

Suppose $x \in \mathbb{R}$. If $x^3 - x > 0$ then $x > -1$.

Proof by dividing into cases

Proof by dividing into cases

- You are asked to prove Q and you consider an **exhaustive** list cases. For each case, Q is true.
 - We use proof by dividing into cases to proceed
 - We consider __N__ cases.
 - Case 1: ____..... Therefore Q
 - Case 2: ____..... Therefore Q
 - ...
 - Case N: ____..... Therefore Q
- QED.

$n^2 + 3n + 2$ is even

$$n^2 + 3n + 2 \text{ is even}$$

- Proof. We consider two cases.
 - Case 1: n is even. $n^2 + 3n + 2$ is even + even + even, therefore even.
 - Case 2: n is odd. $n^2 + 3n + 2$ is odd + odd + even, therefore even.
- QED.

Example: Prove the following statement

Proposition If $n \in \mathbb{N}$, then $1 + (-1)^n(2n - 1)$ is a multiple of 4.

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Proof. Suppose $n \in \mathbb{N}$.

Then n is either even or odd. Let's consider these two cases separately.

Case 1. Suppose n is even. Then $n = 2k$ for some $k \in \mathbb{Z}$, and $(-1)^n = 1$.

Thus $1 + (-1)^n(2n - 1) = 1 + (1)(2 \cdot 2k - 1) = 4k$, which is a multiple of 4.

Case 2. Suppose n is odd. Then $n = 2k + 1$ for some $k \in \mathbb{Z}$, and $(-1)^n = -1$.

Thus $1 + (-1)^n(2n - 1) = 1 - (2(2k + 1) - 1) = -4k$, which is a multiple of 4.

These cases show that $1 + (-1)^n(2n - 1)$ is always a multiple of 4. ■

Exercise: Prove the following statement

- If two integers have opposite parity, then their sum is odd

Proof. Suppose m and n are two integers with opposite parity.

We need to show that $m + n$ is odd. This is done in two cases, as follows.

Case 1. Suppose m is even and n is odd. Thus $m = 2a$ and $n = 2b + 1$ for some integers a and b . Therefore $m + n = 2a + 2b + 1 = 2(a + b) + 1$, which is odd.

Case 2. Suppose m is odd and n is even. Thus $m = 2a + 1$ and $n = 2b$ for some integers a and b . Therefore $m + n = 2a + 1 + 2b = 2(a + b) + 1$, which is odd (by Definition 8.2).

In either case, $m + n$ is odd.



Summary

- proof by contradiction
- Proof by contraposition
- proof by dividing into cases