Guideline

Due Date: Thursday, 2023-09-28, by 23:59.

Upload your answers as a singular PDF to Brightspace.

If you're writing by hand, please ensure your handwriting is legible.

Multiple submissions are possible before the due time; the last submission will be graded.

Key points for proof-writing

- Understand the statement
- Choose a proof method
- Structure your proof
    - Start with `Proof.`
    - State assumptions clearly
    - Proceed Step-by-Step: Each step should follow logically from the previous one. Every claim you make should either be self-evident, previously proven, or proven within your proof.
    - End with `QED.`
- Read again your proof. Make it read like an essay.

To guide you in crafting proofs with rigor, clarity, and conciseness, we outline two scenarios in which significant points may be deducted from your proof exercises:

- If your proof is unclear or incoherent. This includes proofs that deviate significantly from an essay-like structure or contain fragmented sentences, impeding comprehension.
- If your proof, while comprehensible, is incorrect. An example would be accepting an assumption as a fact, or accepting the conclusion as an assumption.

Exercise 1 (points = 32)

Determine whether the statements below are true or false. Do not explain.

1. 119 is a prime number.
2. 161 is a prime number.
3. 42k is an even number for any integer k.
4. For each integer n with 2 <= n <= 6, $n^2 - n + 11$ is a prime number.
5. The average of any two odd integers is odd.
6. For any real number x, if x * x >= 4, then x >= 2.
7. For any real numbers x and y, $x^2 - 2xy + y^2 >= 0$.

8. There exists an integer x, such that $(2x + 1)^2$ is even.

Exercise 2. (points = 8)

- Conduct a bit of research: Describe the formal definition of continuity for a real-valued function f at the point x.
- Provide the formal definition for f being discontinuous at x.

Exercise 3 (points = 15)

Prove the following proposition: An even number multiplied by an integer is an even number.

Check "Key points for proof-writing" above.

Exercise 4 (points = 15)

Prove the following proposition: An odd number multiplied by an odd number is an odd number.

Check "Key points for proof-writing" above.

Exercise 5 (points = 15)

We say an integer is a perfect square if it can be expressed as a square of some integer. For example, 81 is a perfect square; 80 is not.

Prove the following statement: there is a perfect square that can be written as a sum of two other perfect squares.

Check "Key points for proof-writing" above.

Exercise 6 (points = 15)

Suppose a ∈ Z. Prove: If a is an odd integer, then $a^2 + 3a + 5$ is odd.

Check "Key points for proof-writing" above.