CSE215
Foundations of Computer Science

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Today’s objectives

To understand

- What is propositional logic and scope of our study
- Truth table
- Logical Equivalence

Proposition

Definition

- A statement or proposition is a sentence for which a truth value (either true or false) can be assigned

True or False?

- The atomic number of Oxygen is 8
- $1 + 1 = 3$
- (Judge asking Witness) The man chased the thief until he fell.
- My mom never made cakes, which we hate.
- There exists life in other planets.
- If earth is round, I can return to where I am by traveling toward a certain direction.
- If Luna drops to 0 won, I will go bankruptcy.
- $a \land b \rightarrow a$
- $(a \land \sim a)$

Scope of our study

- Mathematical logic, not ambiguous English
- Compound statements, not unit statements
- So, we will check if a proposition like (p -> q) -> (q -> p) is true of false

Why logic?

Artificial Intelligence 47 (1991) 31–56
Elsevier

Logic and artificial intelligence

Nils J. Nilsson
Computer Science Department, Stanford University, Stanford, CA 94305, USA

Received February 1989

Abstract

Nilsson, N.J., Logic and artificial intelligence, Artificial Intelligence 47 (1990) 31–56.

The theoretical foundations of the logical approach to artificial intelligence are presented.

- Quote: “Logic provides the vocabulary and many of the techniques needed both for analyzing the processes of representation and reasoning and for synthesizing machines that represent and reason.”

Example: Software techniques used at FAANG

Question: Simplify this code

```
5    int x = 0;
6    while (x < 10){
7        x = x + 1;
8    }
```

- Answer: x must equals to 10. Following three facts
  - x<11 at Line 6 (before entering the loop)
  - x>=10 after the loop
  - x is an integer

How to check truthfulness of propositions?

Compound statements

Definition

- A compound statement is a complex sentence that is obtained by joining propositional variables using logical connectives

Logical operator	Notation	Read as
Negation	$\sim p$	not p
Conjunction	$p \wedge q$	p and q
Disjunction	$p \vee q$	p or q
Conditional	$p \rightarrow q$	p implies q if p, then q p only if q q if p q, provided that p
Biconditional	$p \leftrightarrow q$	p if and only if q
Logical equivalence	$p \equiv q$	p logically equivalent to q

Examples

- $(p \vee q) \wedge \sim (\sim p \wedge r)$
- $(\sim p \wedge q \wedge r) \vee (q \vee \sim r)$

Truthfulness of compound statements

Negation $(\sim p)$

Definition

- Negation of a statement p, denoted by $\sim p$, is a statement obtained by changing the truth value of p.

p	$\sim p$
T	F
F	T

Truthfulness of compound statements

Conjunction $(p \wedge q)$

Definition

- Conjunction of statements p and q, denoted by $p \wedge q$, is a statement such that it is true if both p and q are true and it is false, otherwise.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Truthfulness of compound statements

Disjunction $(p \vee q)$

Definition

- Disjunction of statements p and q, denoted by $p \vee q$, is a statement such that it is false if both p and q are false and it is true, otherwise.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Truthfulness of compound statements

Exclusive or $(p \oplus q)$

Definition

- Exclusive or of statements p and q, denoted by $p \oplus q$, is defined as p or q but not both. It is computed as $(p \vee q) \wedge \sim (p \wedge q)$

p	q	$p \vee q$	$p \wedge q$	$\sim (p \wedge q)$	$(p \vee q) \wedge \sim (p \wedge q)$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F

Example: Do you want Kimchi, or do you want Gimbap?

Truthfulness of compound statements

Definition

- Conditional or implication is a compound statement of the form "if p, then q". It is denoted by $p \to q$ and read as "p implies q". It is false when p is true and q is false, and it is true, otherwise.

p ->q seen as
~p ⋁ q

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Examples: False -> Anything is true!

- If 1+1 = 3, then 1 = 0
- If the earth is plat, I am walking on the moon

Truthfulness of compound statements

Biconditional statement $(p \leftrightarrow q)$

Definitions

- The biconditional of p and q is of the form "p if and only if q" and is denoted by $p \leftrightarrow q$. It is true when p and q have the same truth value, and it is false, otherwise.
- $p \leftrightarrow q \equiv (p \to q) \land (q \to p)$

p	q	$p \to q$	$q \to p$	$(p \to q) \land (q \to p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Examples

- Assume x and y are real numbers.
  "$x^2 + y^2 = 0$ if and only if $x = 0$ and $y = 0$."

Precedence of Logical Operators

Priority	Operator	Comments
1	$\sim$	Evaluate $\sim$ first
2	$\wedge$ $\vee$	Evaluate $\wedge$ and $\vee$ next; Use parenthesis to avoid ambiguity
3	$\rightarrow$ $\leftrightarrow$	Evaluate $\rightarrow$ and $\leftrightarrow$ next; Use parenthesis to avoid ambiguity
4	$\equiv$	Evaluate $\equiv$ last

- p ⋁ q ⋀ r reads as …
- ~ p -> q reads as …
- p -> q ⋀ q -> p reads as …

Exercise 1: check truthfulness of (p -> q) -> (q -> p) with a truth table

Break;

Logical Equivalence

Logic equivalence

Definition

- Two statement forms p and q are logically equivalent, denoted by $p \equiv q$, if and only if they have the same truth values for all possible combination of truth values for the propositional variables

Checking logical equivalence

1. Construct and compare truth tables (most powerful)
2. Use logical equivalence laws

Logical equivalence: Example

Problem

- Show that $p \wedge (q \vee r) \not\equiv (p \wedge q) \vee r$

p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$p \wedge q$	$(p \wedge q) \vee r$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	F	T
T	F	F	F	F	F	F
F	T	T	T	F	F	T
F	T	F	T	F	F	F
F	F	T	T	F	F	T
F	F	F	F	F	F	F

Exercise 2: check the logical equivalence between (p->q) and (~q ->~p)

Two special logical equivalence: Tautology and contradiction

Definitions

- A tautology is a statement form that is always true regardless of the truth values of the individual statements substituted for its statement variables.
- A contradication is a statement form that is always false regardless of the truth values of the individual statements substituted for its statement variables.

Examples

- $p \vee \sim p$ ▷ Tautology
- $p \wedge \sim p$ ▷ Contradiction

The secret of a fortune teller

- Three students ask a fortune teller if they got an “A” in the exam
- The fortune teller says nothing but shows 1 finger

- If they all got A —> 1 is right
- If they all failed to get A —> 1 is right
- If one students get A —> 1 is right
- If two students get A (meaning one does not) —> 1 is right
- **The fortune teller will always be right, since he said a tautology.**

See how logic saved Chris Gardner

https://www.youtube.com/watch?v=W2r4BUB-Rsc

- Interviewer (giving a proposition): What would you say, if a guy walked in for an interview with such a bad T-shirt, and I hired him?

- Chris Gardner (thinking about logic): He must have really nice pants.

What would you say if a person with such a T-shirt walking into the interview, and I hired him

—> Get Hired

- Interviewer's **proposition**: Bad-T-shirt $\land$ Get-hired

- **Common-sense:** Bad-T-shirt —> ~ Get-hired

- If Chris follows common-sense and interview's proposition, he will obtain ~Get-hired $\land$ Get-hired. That means **contradiction**.

- Never tell interviewers that they say a contradiction.

- So, Chris has to challenge the common-sense, to argue Bad-T-shirt —> ~Get-hired is **false.**

- Chris knows that "Bad-T-shirt —> ~Get-hired" and "Get-hired -> ~Bad-T-shirt" are **equivalent**

- So, Chris is now thinking what to imply from Get-hired?

- Since Get-hired means there must be some extraordinary quality. Chris thinks of two things: Get-hired -> Nice-T-shirt $\lor$ Nice-Pants

- But Nice-T-shirt contradicts with Interviewer's proposition, so Christ concludes "Nice-Pants"

Let's call it a day!

- Propositional logic.
- Truth Table.
- Logical Equivalence.
- Tautology and Contradiction.

Thank you for your attention!