CSE215
Foundations of Computer Science

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- Plan:
  - Proof by contradiction
  - Proof by contraposition

Contradiction

Proof by contradiction

You are asked to prove P and you feel ~P is ridicules

Prove: There is no greatest integer

- Proof.
  - We use proof by contradiction.
  - Assume there exists a greatest integer.
  - Let G denote the greatest integer. We have
    - (A): for any integer n, n <=G.
  - But G + 1 is an integer satisfying G+1>G. This contradicts with (A)
  - Therefore, there does not exist a greatest integer
- QED.

$\sqrt{2}$ is irrational

- Proof.
  - We use proof by contradiction.
  - Assume sqrt(2) is a rational number, namely:
    - (A) there exists two integers m, n such that sqrt(2)=m/n, and m and n have no common factors.
  - Thus m^2 = 2 n^2. Thus, m^2 is even. Thus m must be even (otherwise m^2 becomes odd).
  - Thus m = 2k for some integer k. Thus, n ^2= 2 k^2. Thus n^2 is even and therefore n must be even.
  - But the fact that m and n are both even contradicts with (A)
  - Therefore sqrt(2) must be irrational.
- QED.

Prove: For any prime number p and natural number n,

If $p|n$**, then** $p \nmid (n+1)$**.**

- Proof.
  - We use proof by contradiction
  - Assume:
    - (A) there exists a prime p and a natural number n, such that $p \mid n$ and $p \mid (n+1)$
  - Since $p \mid n$, $n = pk$ for some integer k
  - Since $p \mid (n+1)$, $n+1=pk'$ for some integer k'
  - Thus $1 = p(k' - k)$. Thus $p = 1$ which contradicts with the fact p is a prime.
  - Therefore (A) is false
- QED

Summary so far

- To prove P is true, we can prove ~P -> False
- In other words, we assume ~P and try to derive a contradiction

Break

Exercises

Exercise 1

If $a, b \in \mathbb{Z}$, then $a^2 - 4b - 3 \neq 0$.

Exercise 2: Prove the following

Suppose $x \in \mathbb{R}$. If $x^2 + 5x < 0$ then $x < 0$.

Exercise 3: Prove the following

Suppose $x \in \mathbb{R}$. If $x^3 - x > 0$ then $x > -1$.

Exercise 4

Prove . If $a, b \in \mathbb{Z}$, then $a^2 - 4b \neq 2$.

Proof by contraposition

You are asked to prove $P \rightarrow Q$ and you feel $\sim Q \rightarrow \sim P$ is easier to prove

n^2 is even $\implies$ n is even

Prove:

Suppose n is an integer. If n^2 is even, then n is even

Idea:
proof by contraposition

- Proposition, for all integer n, n^2 even -> n even
- Equivalently, for all integer n, n is odd -> n^2 is odd

Exercises 1-4 again, using proof by contraposition

Summary

- proof by contradiction
- Proof by contraposition