CSE215
Foundations of Computer Science

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Midterm 1 Class Statistics

Number of submitted grades: 31 / 32

Minimum: 49 %

Maximum: 97 %

Average: 81 %

Mode: 85 %, 96 %

Median: 84 %

Standard Deviation: 12.55 %

“Don't let a grade define you: It tells where you are, not where you can go”

Problem 1. Propositional statements [points = 12]

Determine if the following statements are true or false. No explanation is needed.

1. For every integer y, if y < 0 , then $y^2 \geq 1$.
2. For every real number y, there exists a real number x such that $xy = 1$.
3. For every positive integer m, the integer $3m^2 + 5m + 11$ is prime.
4. There exists a real number b for which b + x = x for every real number x.

Problem 2. Negation [points = 12]

Negate the following statements.

1. For any integer a, if a is even, then a^2 is even.
2. For every positive integer p, there is interger q such that q > p.
3. If x is prime, then x is not a multiple of 2.
4. "If the rate of the confirmed covid cases surpasses 10% of the university community, all classes will be provided online."

Problem 3. Inference rule [points = 12]

1. Consider the following argument:

   ```
   If it is Wednesday, then Homeplus will be crowded. It is Wednesday.
   Thus,  Homeplus is crowded.
   ```

   Which of the following is correct:

   (A) The argument is invalid.

   (B) The argument is valid based on Modus tollens.

   (C) The argument is valid based on Modus ponens.

   (D) The argument is valid based on Specialization.

2. Consider the following argument:

   ```
   We love Pizza. Thus, the everybody  loves pizza.
   ```

   Which of the following is correct:

   (A) The argument is invalid.

   (B) The argument is valid based on Specialization.

   (C) The argument is valid based on Conjuction.

   (D) The argument is valid based on Generalization.

3. Consider the following argument:

```
If it rains, then the ground will be wet.
The ground is not wet. Therefore, it does not rain.
```

   Which of the following is correct:

   (A) The argument is invalid.

   (B) The argument is valid based on Modus tollens.

   (C) The argument is valid based on Modus ponens.

   (D) The argument is valid based on Specialization.

4. You are a detective investigating a crime scene with Mr. Sherlock. You come across a note that says: "If the butler did it, then the knife will be found in the kitchen."

   Mr. Sherlock responds, "Well, we just found the knife in the kitchen. Therefore, the butler did it."

Which of the following is correct:

(A) Sherlock made an invalid argument.

(B) Sherlock made a valid argument based on Modus tollens.

(C) Sherlock made a valid argument based on Modus ponens.

(D) Sherlock made a valid argument based on Specialization.

Problem 4. Truth tables [points = 12]

Determine if the following statement is True or False. Do not explain.

1. The two formula are equivalent: $p \wedge true$ and $p \leftrightarrow true$.
2. The two formula are equivalent: $p \wedge true$ and $p \vee false$.
3. The two formula are equivalent: $p \wedge true$ and $p \oplus true$, where $\oplus$ refers to exclusive or.
4. The two formula are equivalent: $p \wedge true$ and $true \rightarrow p$.

Problem 5. Proof concepts (20)

1. Consider the following proof problem: Suppose a is an integer. $a^2 = a$ if and only if a = 0 or a = 1.

   Our proof objective will be:

   (A) $a^2 = a$ implies a =0 or a = 1.

   (B) a is odd implies $a^3 + a^2 + a + 1$ is odd

   (C) A and B

   (D) A or B

2. Consider the following proof problem: $\sqrt{3}$ is irrational.

   If we want to prove this statement using proof by contradiction, we should get started with:

   (A) Assuming $\sqrt{3}$ is rational

   (B) Assuming $\sqrt{3}$ is irrational

   (A) Recognizing $\sqrt{3}$ is rational

   (B) Recognizing $\sqrt{3}$ is irrational

3. Consider the following proof problem: Suppose x is a real number. If |x| < 1 then $x^2 < 1$.

   If we want to prove this statement using proof by contraposition, we will need to prove the contraposition of the original proof objective. Which of the following is the contraposition:

   (A) If |x| < 1 then $x^2 \geq 1$

   (B) If $x^2 < 1$ then |x| < 1

   (C) If $x^2 \geq 1$ then $|x| \geq 1$

   (D) |x| < 1 and $x^2 \geq 1$

4. Consider the following proof problem: Suppose x is a real number. If |x| < 1 then $x^2 < 1$.

   If we want to prove this statement using proof by contradiction, we will start by assuming:

   (A) If |x| < 1 then $x^2 \geq 1$

   (B) If $x^2 < 1$ then |x| < 1

   (C) If $x^2 \geq 1$ then $|x| \geq 1$

   (D) |x| < 1 and $x^2 \geq 1$

Problem 6. Validity [points = 12]

Is the following argument valid or invalid? Please explain why.

Premises:

1. $u \rightarrow r$
2. $(r \wedge s) \rightarrow (p \vee q)$
3. $u \wedge s$
4. $\sim q$

Conclusion: p

** Note: Explanation is needed for this problem. **

Problem 7. Proof [points = 20]

Consider the following statement:

```
A rational number multiplied by an irrational number must be irrational.
```

1. (points = 5) Determine if the statement is true or not.

2. (points = 15) Then, if it is true, prove it is true; if it is false, disprove it (in other words, prove its negation is true).

** Note: Explanation is needed for this problem's second part. **

Solution

Problem 1. Propositional statements [points = 12]

Determine if the following statements are true or false. No explanation is needed.

1. For every integer y, if y < 0 , then $y^2 \geq 1$. $\forall y \in \mathbb{Z}, y<0 \to y^2 \geq 1$ T
2. For every real number y, there exists a real number x such that $xy = 1$. F when $y = 0$
3. For every positive integer m, the integer $3m^2 + 5m + 11$ is prime. F when $m = 11$
4. There exists a real number b for which b + x = x for every real number x. T

Problem 2. Negation [points = 12]

Negate the following statements.

1. For any integer a, if a is even, then a^2 is even.
2. For every positive integer p, there is interger q such that q > p.
3. If x is prime, then x is not a multiple of 2.
4. "If (the rate of the confirmed covid cases surpasses 10% of the university community) (all classes will be provided online.")

p q

$\sim(p \to q) \equiv p \land \sim q$

1. $\sim(\forall a \in \mathbb{Z},\ a \text{ even} \to a^2 \text{ even}) \equiv \exists a \in \mathbb{Z},\ a \text{ even} \land a^2 \text{ odd}.$

2. $\sim(\forall p \in \text{positive int},\ \exists q \in \mathbb{Z},\ q > p)$

$\equiv \exists p \in \text{Positive int},\ \forall q \in \mathbb{Z},\ q \leq p$

3. $\sim(x \text{ is prime} \to 2 \nmid x) \equiv x \text{ is prime} \land 2 \mid x$

4. (rate of the confirmed Covid cases surpasses 10% of the university community)

$\land$ (Some classes will be provided offline.)

Problem 3. Inference rule [points = 12]

1. Consider the following argument:

$p \rightarrow q \quad p$

```
If it is Wednesday, then Homeplus will be crowded. It is Wednesday.
Thus,  Homeplus is crowded.
```

q

Which of the following is correct:

(A) The argument is invalid. X

(B) The argument is valid based on Modus tollens. X

(C) The argument is valid based on Modus ponens. O

(D) The argument is valid based on Specialization. X

2. Consider the following argument:

```
We love Pizza. Thus, the everybody  loves pizza.
```

Which of the following is correct:

(A) The argument is invalid.

(B) The argument is valid based on Specialization.

(C) The argument is valid based on Conjuction.

(D) The argument is valid based on Generalization.

3. Consider the following argument:

$p \rightarrow q$

```
If it rains, then the ground will be wet.
The ground is not wet. Therefore, it does not rain.
```

$\sim q \quad \sim p$

Which of the following is correct:

(A) The argument is invalid. X

(B) The argument is valid based on Modus tollens. O

(C) The argument is valid based on Modus ponens. X

(D) The argument is valid based on Specialization. X

4. You are a detective investigating a crime scene with Mr. Sherlock. You come across a note that says: "If the butler did it, then the knife will be found in the kitchen."

$p \rightarrow q$

Mr. Sherlock responds, "Well, we just found the knife in the kitchen. Therefore, the butler did it."

$q \quad p$

Which of the following is correct:

(A) Sherlock made an invalid argument.

(B) Sherlock made a valid argument based on Modus tollens. X

(C) Sherlock made a valid argument based on Modus ponens. X

(D) Sherlock made a valid argument based on Specialization. X

Problem 4. Truth tables [points = 12]

Determine if the following statement is True or False. Do not explain.

1. The two formula are equivalent: $p \wedge true$ and $p \leftrightarrow true$. True
2. The two formula are equivalent: $p \wedge true$ and $p \vee false$. True
3. The two formula are equivalent: $p \wedge true$ and $p \oplus true$, where $\oplus$ refers to exclusive or. False
4. The two formula are equivalent: $p \wedge true$ and $true \rightarrow p$. True

1.

p	t	$p \wedge t$	$p \rightarrow t$	$t \rightarrow p$	$p \leftrightarrow t$
T	T	T	T	T	T
F	T	F	T	F	F

2.

p	t	c	$p \wedge t$	$p \vee c$
T	T	F	T	T
F	T	F	F	F

3.

p	t	$p \wedge t$	$p \oplus t$
T	T	T	F
F	T	F	T

Problem 5. Proof concepts (20)

1. Consider the following proof problem: Suppose a is an integer. $a^2 = a$ if and only if a = 0 or a = 1.

   Our proof objective will be:

   (A) $a^2 = a$ implies a =0 or a = 1.

   (B) ~~a is odd implies $a^3 + a^2 + a + 1$ is odd~~ $a=0$ or $a=1$ $\rightarrow$ $a^2 = a$

   (C) A and B

   (D) A or B

2. Consider the following proof problem: $\sqrt{3}$ is irrational.

   If we want to prove this statement using proof by contradiction, we should get started with:

   (A) Assuming $\sqrt{3}$ is rational

   (B) Assuming $\sqrt{3}$ is irrational

   (A) Recognizing $\sqrt{3}$ is rational

   (B) Recognizing $\sqrt{3}$ is irrational

$\forall x \in \mathbb{R}, |x|<1 \rightarrow x^2<1$

3. Consider the following proof problem: Suppose x is a real number. If |x| < 1 then $x^2 < 1$.

   If we want to prove this statement using proof by contraposition, we will need to prove the contraposition of the original proof objective. Which of the following is the contraposition:

   (A) If |x| < 1 then $x^2 \geq 1$

   $\forall x \in \mathbb{R}, x^2 \geq 1 \rightarrow |x| \geq 1$

   (B) If $x^2 < 1$ then |x| < 1

   (C) If $x^2 \geq 1$ then $|x| \geq 1$

   (D) |x| < 1 and $x^2 \geq 1$

4. Consider the following proof problem: Suppose x is a real number. If |x| < 1 then $x^2 < 1$.

   If we want to prove this statement using proof by contradiction, we will start by assuming:

   (A) If |x| < 1 then $x^2 \geq 1$

   $\exists x \in \mathbb{R}, |x|<1 \wedge x^2 \geq 1$

   (B) If $x^2 < 1$ then |x| < 1

   (C) If $x^2 \geq 1$ then $|x| \geq 1$

   (D) |x| < 1 and $x^2 \geq 1$

Problem 6. Validity [points = 12]

Is the following argument valid or invalid? Please explain why.

Premises:

1. $u \rightarrow r$
2. $(r \wedge s) \rightarrow (p \vee q)$
3. $u \wedge s$
4. $\sim q$

Conclusion: p

** Note: Explanation is needed for this problem. **

- From $u \wedge s$ we have u, s followed by Specialization.
- From u, $u \rightarrow r$ we have r followed by Modus Ponens
- From s, r, $(r \wedge s) \rightarrow (p \vee q)$ we have $p \vee q$ followed by Conjunction and Modus Ponens
- From $p \vee q$, $\sim q$ we have p followed by Elimination

$\therefore$ The argument is valid.

Problem 7. Proof [points = 20]

Consider the following statement:

There exists a rational that is rational.

```
A rational number multiplied by an irrational number must be irrational.
```

1. (points = 5) Determine if the statement is true or not. False
2. (points = 15) Then, if it is true, prove it is true; if it is false, disprove it (in other words, prove its negation is true).

** Note: Explanation is needed for this problem's second part. **

Disproof. In order to disprove the following statement, we prove its negation to be true. Therefore, we must prove that there exists a rational number multiplied by an irrational number that is rational. Let 0 be the rational number and $\sqrt{2}$ be the irrational number. $0 \times \sqrt{2} = 0$, which is rational. Thus, there exists a rational number multiplied by an irrational number that is rational.

Therefore, the statement above is disproved.