CSE215
Foundations of Computer Science

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Today

- Revision on predicates
- Negation

Exercise: Translate to formal logic

- Given:
  - S: set of students
  - P(s): s passed the exam.
  - W(s): s worked hard.
  - C(s): s is in Professor Cho’s class.
- **Every student in Professor Cho’s class passed the exam**

Exercise: Translate to formal logic

- Given:
    - S: set of students
    - P(s): s passed the exam.
    - W(s): s worked hard.
    - C(s): s is in Professor Cho’s class.
- **Some students studied hard but did not pass the exam**

Exercise: Translate to formal logic

- Given:
    - S: set of students
    - P(s): s passed the exam.
    - W(s): s worked hard.
    - C(s): s is in Professor Cho’s class.
- **There are students who did not study hard but passed the exam**

Exercise: Translate to formal logic

- Given:
    - S: set of students
    - P(s): s passed the exam.
    - W(s): s worked hard.
    - C(s): s is in Professor Cho’s class.
- **All students who studied hard passed the exam.**

Exercise: Translate to formal logic

- Given:
  - S: set of students
  - P(s): s passed the exam.
  - W(s): s worked hard.
  - C(s): s is in Professor Cho's class.
- **No student in Professor Cho's class failed the exam.**

Exercise: Translate to formal logic

- Given:
    - S: set of students
    - P(s): s passed the exam.
    - W(s): s worked hard.
    - C(s): s is in Professor Cho’s class.
- **There are no students in Professor Cho’s class who did not study hard but still passed the exam.**

Exercise: Translate to formal logic

- Given:
    - L: set of locks
    - K set of keys
    - unlock (k,l): k can unlock l
- **Every lock has a key**

Exercise: Translate to formal logic

- Given:
  - L: set of locks
  - K set of keys
  - unlock (k,l): k can unlock l
- **There is a key for all locks**

Exercise: Translate to formal logic

- Given:
    - L: set of locks
    - K set of keys
    - unlock (k,l): k can unlock l
- **Some lock has no keys**

Exercise: Translate to formal logic

- Given:
    - L: set of locks
    - K set of keys
    - unlock (k,l): k can unlock l
- **Some keys cannot unlock any lock.**

Exercise:
translate to formal logic

Every nonzero real number has a reciprocal

The reciprocal of 4 is 1/4 (namely 0.25)

- for any nonzero real number r, r has a reciprocal
- for any nonzero real number r, there exists a real number s, such that r * s = 1

Exercise: translate to formal logic

- “There is a person supervising every detail of the production process.”

- $\exists$ person p such that $\forall$ detail d, p supervises d

Negation

Negation of quantified statements ($\sim$)

Definition

- Formally,

$$\sim (\forall x \in D, p(x)) \equiv \exists x \in D, \sim p(x)$$
$$\sim (\exists x \in D, p(x)) \equiv \forall x \in D, \sim p(x)$$

- Negation of a universal statement ("all are") is logically equivalent to an existential statement ("there is at least one that is not")
  Negation of an existential statement ("some are") is logically equivalent to a universal statement ("all are not")

Negation of quantified statements ($\sim$)

Examples

- All mathematicians wear glasses
  Negation (incorrect): No mathematician wears glasses

  Negation (correct): There is at least one mathematician who does not wear glasses
- Some snowflakes are the same
  Negation (incorrect):: Some snowflakes are different
  Negation (correct):: All snowflakes are different

Negation of quantified statements ($\sim$)

Examples

- $\forall$ primes p, p is odd

Negation of quantified statements ($\sim$)

Examples

- $\forall$ primes p, p is odd
  Negation: $\exists$ primes p, p is even
- $\exists$ triangle T, sum of angles of T equals 200°

Negation of quantified statements ($\sim$)

Examples

- $\forall$ primes p, p is odd
  Negation: $\exists$ primes p, p is even
- $\exists$ triangle T, sum of angles of T equals $200°$
  $\forall$ triangles T, sum of angles of T does not equal $200°$
- No politicians are honest

Negation of quantified statements ($\sim$)

Examples

- $\forall$ primes p, p is odd
  Negation: $\exists$ primes p, p is even
- $\exists$ triangle T, sum of angles of T equals $200°$
  $\forall$ triangles T, sum of angles of T does not equal $200°$
- No politicians are honest
  Formal statement: $\forall$ politicians x, x is not honest
  Formal negation: $\exists$ politician x, x is honest
  Informal negation: Some politicians are honest
- 1357 is not divisible by any integer between 1 and 37

Negation of quantified statements ($\sim$)

Examples

- $\forall$ primes p, p is odd
  Negation: $\exists$ primes p, p is even
- $\exists$ triangle T, sum of angles of T equals $200°$
  $\forall$ triangles T, sum of angles of T does not equal $200°$
- No politicians are honest
  Formal statement: $\forall$ politicians x, x is not honest
  Formal negation: $\exists$ politician x, x is honest
  Informal negation: Some politicians are honest
- 1357 is not divisible by any integer between 1 and 37
  Formal statement: $\forall n \in [1, 37]$, 1357 is not divisible by n
  Formal negation: $\exists n \in [1, 37]$, 1357 is divisible by n
  Informal negation: 1357 is divisible by some integer between 1 and 37

Negation of universal conditional statements

Definition

- Formally,

$$\begin{aligned} \sim (\forall x, p(x) \to q(x)) &\equiv \exists x, \sim (p(x) \to q(x)) \\ &\equiv \exists x, (p(x) \wedge \sim q(x)) \end{aligned}$$

Negation of universal conditional statements

Definition

- Formally,

$$\begin{aligned} \sim (\forall x, p(x) \to q(x)) &\equiv \exists x, \sim (p(x) \to q(x)) \\ &\equiv \exists x, (p(x) \wedge \sim q(x)) \end{aligned}$$

Examples

- $\forall$ real x, if $x > 10$, then $x^2 > 100$.
  Negation: $\exists$ real x such that $x > 10$ and $x^2 \leq 100$.
- If a computer program has more than 100,000 lines, then it contains a bug.
  Negation: There is at least one computer program that has more than 100,000 lines and does not contain a bug.

Negation of propositions with multiple quantifiers

$$
\begin{aligned}
&\sim (\forall x \text{ in } D,\ \exists y \text{ in } E \text{ such that } P(x,y)) \\
&\equiv \exists x \text{ in } D \text{ such that } \sim (\exists y \text{ in } E \text{ such that } P(x,y)) \\
&\equiv \exists x \text{ in } D \text{ such that } \forall y \text{ in } E,\ \sim P(x,y)
\end{aligned}
$$

$$
\begin{aligned}
&\sim (\exists x \text{ in } D \text{ such that } \forall y \text{ in } E,\ P(x,y)) \\
&\equiv \forall x \text{ in } D,\ \sim (\forall y \text{ in } E,\ P(x,y)) \\
&\equiv \forall x \text{ in } D,\ \exists y \text{ in } E \text{ such that } \sim P(x,y)
\end{aligned}
$$

Exercise

- Do some research: Formal definition of continuity of a real-valued function f on a point x

- Give a formal definition of f being discontinuous on x

More Exercises

Exercise: 2021 midterm-1

Problem 3. [10 points]

Give negations of the following statements. Reasoning is not required.

(a) [1 point] $p \wedge q$

(b) [1 point] $p \vee q$

(c) [1 point] $p \oplus q$

(d) [1 point] $p \rightarrow q$

(e) [1 point] $p \leftrightarrow q$

(f) [1 point] $\forall x, \forall y$ such that $p(x, y)$

(g) [1 point] $\forall x, \exists y$ such that $p(x, y)$

(h) [1 point] $\exists x, \forall y$ such that $p(x, y)$

(i) [1 point] $\exists x, \exists y$ such that $p(x, y)$

(j) [1 point] $\exists x, \forall y, \exists z$ such that $p(x, y, z)$

Final 2021

Problem 6. [5 points]

Prove that if $n^2 + 8n + 20$ is odd, then n is odd for natural numbers n.

- Express the propositions we need to prove here

Final 2021

Problem 5. [5 points]

Prove using contradiction that the cube root of an irrational number is irrational.

- Express the propositions we need to prove here

Exercise: Translate to formal logic, and then negate

- Given:
  - $P(x)$: x is a doctor.
  - $Q(x)$: x is busy.
  - $R(x, y)$: x likes y.
- **All doctors are busy.**

Exercise: Translate to formal logic, and then negate

- Given:
    - $P(x)$: x is a doctor.
    - $Q(x)$: x is busy.
    - $R(x, y)$: x likes y.
- **Some doctors are not busy.**

Exercise: Translate to formal logic, and then negate

- Given:
  - $P(x)$: x is a doctor.
  - $Q(x)$: x is busy.
  - $R(x, y)$: x likes y.
- **Every person likes themselves.**

Exercise: Translate to formal logic, and then negate

- Given:
  - $P(x)$: x is a doctor.
  - $Q(x)$: x is busy.
  - $R(x, y)$: x likes y.
- **There's someone who doesn't like themselves.**

Exercise: Translate to formal logic, and then negate

- Given:
  - $P(x)$: x is a doctor.
  - $Q(x)$: x is busy.
  - $R(x, y)$: x likes y.
- **There's at least one doctor that everyone likes.**

Exercise: Translate to formal logic, and then negate

- Given:
  - $P(x)$: x is a doctor.
  - $Q(x)$: x is busy.
  - $R(x, y)$: x likes y.
- **Everyone likes at least one doctor.**

Exercise: Translate to formal logic, and then negate

- Given:
  - $P(x)$: x is a doctor.
  - $Q(x)$: x is busy.
  - $R(x, y)$: x likes y.
- **Some doctors don't like themselves.**