CSE215
Foundations of Computer Science

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Midterm2

- We will strategically use it for final review
- So, results to be announced next week

Plan ahead

~~11.14 Tu:~~	Sets	- REC on Sets; no homework review in REC
~~11.16~~	Review Midterm 2	No homework to be announced
~~11.21 Tu~~	**Midterm 2**	- REC on Functions
~~11.23~~	Function	Homework to be announced
11.28 Tu	Function	**- Regular lecture in REC**
11.30	Function	Ungraded homework to be announced
12.05 Tu	Final Review	- **Reviewing with Midterm 2**
12.07	Final Review	No homework to be announced
12.12 Tue	**Final**	-

Agenda

- Review on one-to-one and onto functions
- Inverse functions

Summary of last class: one-to-one and onto functions

How to show a function $f : A \to B$ is injective:

Contraposive approach:
Suppose $x, y \in A$ and $f(x) = f(y)$.

$\vdots$

Therefore $x = y$.

How to show a function $f : A \to B$ is surjective:

Suppose $b \in B$.
[Prove there exists $a \in A$ for which $f(a) = b$.]

Exercise 1

SBU 2021 Final

Problem 9. [5 points]

Write and fill the table with ✓ or ✗. If a function is one-to-one or onto, then use ✓. On the other hand, if a function is not one-to-one or not onto, then use ✗.

Function	Domains	One-to-one function?	Onto function?
$f(x) = 3x$	$f : \mathbb{Z} \to \mathbb{Z}$		
$f(x) = 3x$	$f : \mathbb{R} \to \mathbb{R}$		
$f(x) = 3x^2$	$f : \mathbb{Z} \to \mathbb{Z}$		
$f(x) = 3x^2$	$f : \mathbb{R} \to \mathbb{R}$		

Exercise 2

True or false?

the function $f : \mathbb{R} - \{1\} \to \mathbb{R} - \{1\}$ defined by $f(x) = \left(\dfrac{x+1}{x-1}\right)^3$ is bijective.

Solution

- We first prove f is injective

We want to prove $\forall x, y \in R-\{1\}.\ f(x)=f(y) \to x=y$

Suppose $x, y \in R-\{1\}$ and $f(x)=f(y)$

We have $\left(\frac{x+1}{x-1}\right)^3 = \left(\frac{y+1}{y-1}\right)^3$ Thus $\frac{x+1}{x-1} = \frac{y+1}{y-1}$

Thus $1+\frac{2}{x-1} = 1+\frac{2}{y-1}$. Thus $\frac{2}{x-1} = \frac{2}{y-1}$

Thus $2(y-1)=2(x-1)$ Thus $x=y$

We have proved $\forall x, y \in R-\{1\}\ f(x)=f(y) \to x=y$

- We then prove f is surjective

We want to prove $\forall y \in R-\{1\},\ \exists x \in R-\{1\}.\ f(x)=y$

Suppose $y \in R-\{1\}$.

We choose $x = \frac{2}{\sqrt[3]{y}-1}+1$

We have $f(x)=y$

We have proved $\forall y \in R-\{1\}\ \exists x \in R-\{1\}\ f(x)=y$

Exercise 3

- Consider the function $\theta : \{0,1\} \times \mathbb{N} \to \mathbb{Z}$ defined as $\theta(a,b) = (-1)^a b$. Is θ injective? Is it surjective? Explain.

Solution

First we show that θ is injective. Suppose $\theta(a,b)=\theta(c,d)$. Then $(-1)^a b=(-1)^c d$. As b and d are both in $\mathbb{N}$, they are both positive. Then because $(-1)^a b=(-1)^c d$, it follows that $(-1)^a$ and $(-1)^c$ have the same sign. Since each of $(-1)^a$ and $(-1)^c$ equals ± 1, we have $(-1)^a=(-1)^c$, so then $(-1)^a b=(-1)^c d$ implies $b=d$. But also $(-1)^a=(-1)^c$ means a and c have the same parity, and because $a,c\in\{0,1\}$, it follows $a=c$. Thus $(a,b)=(c,d)$, so θ is injective.
Next note that θ **is not surjective** because $\theta(a,b)=(-1)^a b$ is either positive or negative, but never zero. Therefore there exist no element $(a,b)\in\{0,1\}\times\mathbb{N}$ for which $\theta(a,b)=0\in\mathbb{Z}$.

Exercise 4

SBU 2022 Midterm

Problem 4. [5 points]

Let A and B be finite sets where $|A| = |B|$. Is it possible to define a function $f : A \to B$ that is one-to-one but not onto? Is it possible to define a function $g : A \to B$ that is onto but not one-to-one?

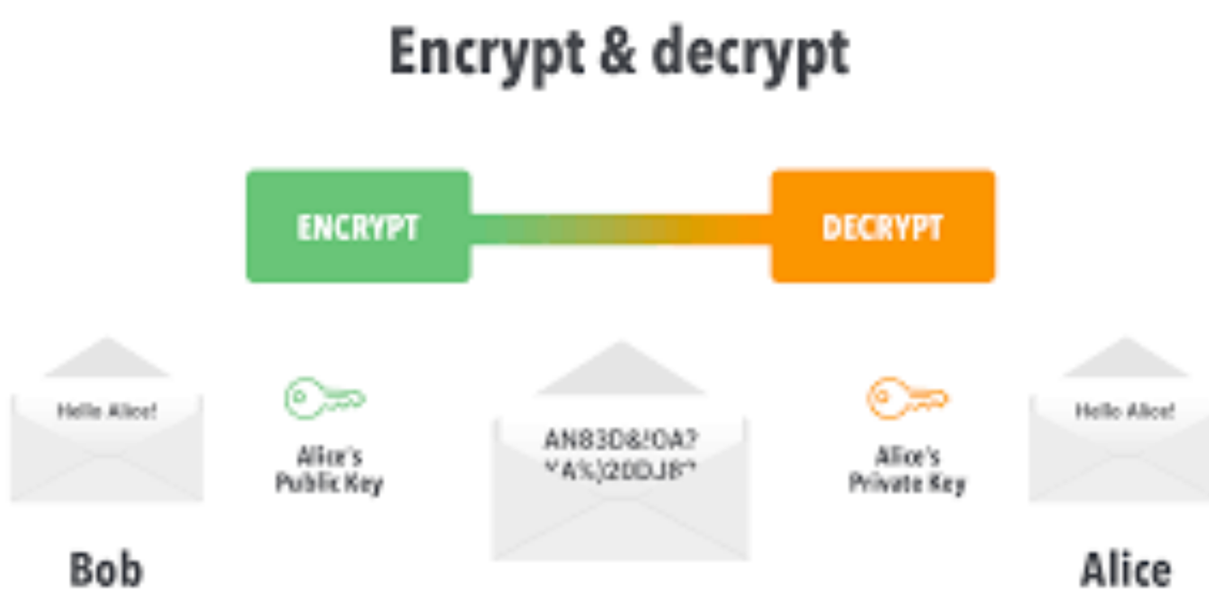
Inverse functions

Inverse functions

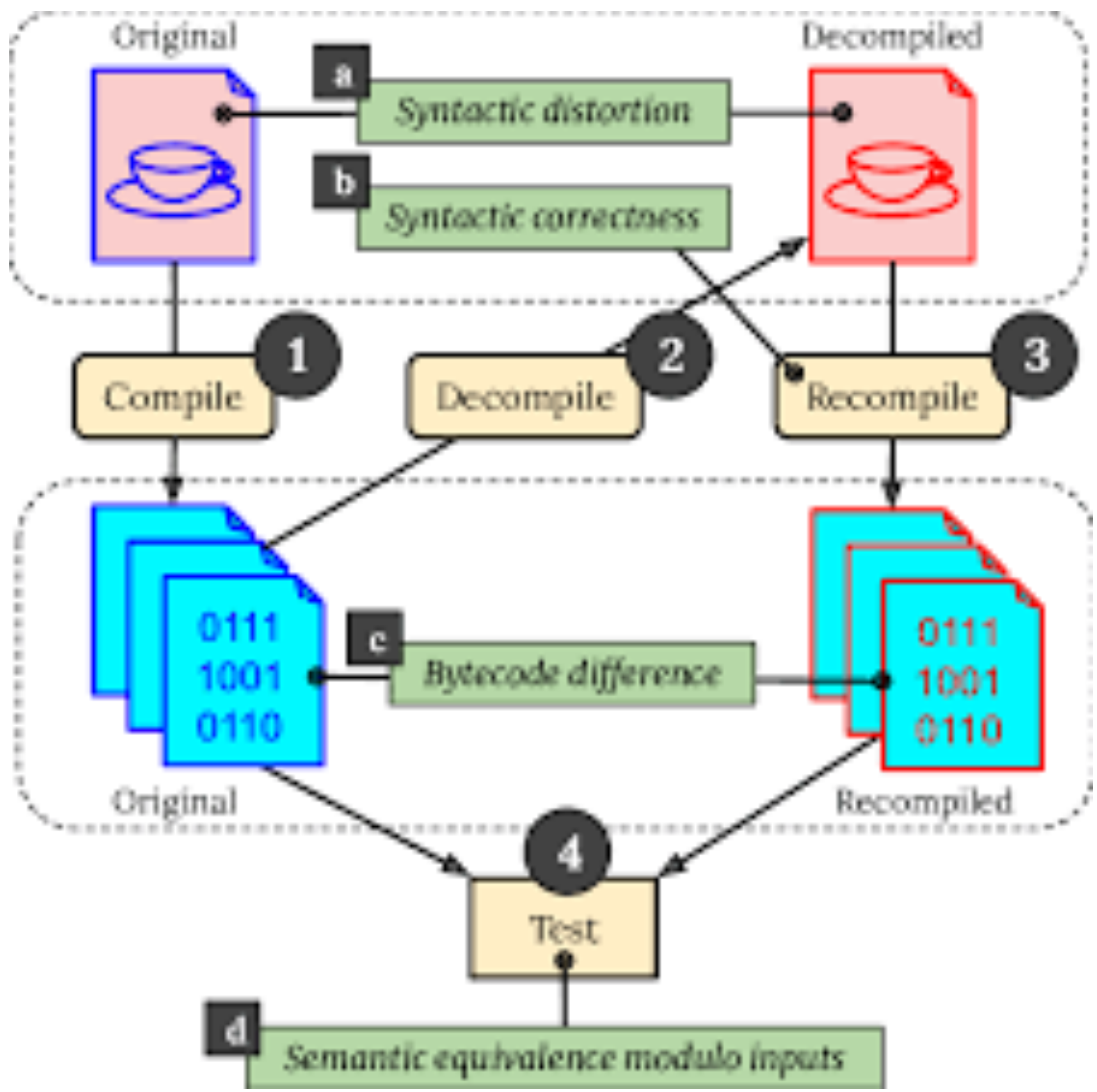
Definition

- Suppose $F : X \to Y$ is a one-to-one correspondence. Then, the inverse function $F^{-1} : Y \to X$ is defined as follows: Given any element y in Y, $F^{-1}(y) =$ that unique element x in X such that $F(x) = y$.
- $F^{-1}(y) = x \Leftrightarrow y = F(x)$.

Does encryption have an inverse function?



Does Java compilation have an inverse function?



Example

Problem

- Define $f : \mathbb{R} \to \mathbb{R}$ by the rule $f(x) = 4x - 1$ for all $x \in \mathbb{R}$. Find its inverse function.

Proof

For any y in R, by definition of f^{-1}

- $f^{-1} =$ unique number x such that $f(x) = y$

  Consider $f(x) = y$

  $\implies 4x - 1 = y \qquad (\because$ Defn. of $f)$

  $\implies x = \frac{y+1}{4} \qquad (\because$ Simplify)
- Hence, $f^{-1}(y) = \frac{y+1}{4}$ is the inverse function.

Exercise 0

- Check If the function $f : Z \rightarrow Z$ defined by $f(n) = 6-n$ is one-to-one correspondence. If yes, compute its inverse.

Exercise 1

- Check if the function $f : R \rightarrow R$ defined as $f(x) = x^3 + 1$ is a one-t-one correspondence. If yes, find its inverse.

Exercise 2

- The function $f : R - \{2\} \rightarrow R - \{5\}$ defined by $f(x) = (5x+1)/(x-2)$ is bijective. Find its inverse.

Solution

- Let y be an element of R - {5}. We have y = f(x) if and only if x = 11/(y-5)+2.
- Thus f^{-1} (y) = 11/(y-5)+2

Exercise 3

- The function $g: Z \times Z \rightarrow Z \times Z$ defined by the formula $g(m,n) = (m+n, m+2n)$ is a one-to-one correspondence. Find its inverse.

Solution

- Let (u,v) be an arbitrary element if Z x Z. Then f(m,n) = (u,v) if and only if m = 2u-v and n = v-u.

- Thus, f^{-1) (u,v) = (2u-v, v-u).