

# **CSE215**

# **Foundations of Computer Science**

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# Reminder

- Final exam: Dec 12 Tuesday 12:30 - 15:00 pm at B207
- Unlimited physical notes possible
- As usual, submit a copy in person and a digital copy to BrightSpace

# Oops...

- I just noticed HW 11 is not yet graded. Sorry!
- Will grade them asap
- Meanwhile, you can find the solution online

# Let's make grading transparent

- Numerical grades
- Letter grades: 50% A-ish
- Assignments after solution disclosure has never been accepted
- Participation: +1 for highly frequent contributors, 0.5 for less-than-frequent contributors
- New: The lowest-score homework will be excluded

# Today

- Ungraded homework 14

# Ungraded homework 14

# Exercise 1 (20 points)

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The *sigmoid* function plays a pivotal role in machine learning. Particularly, it's instrumental in classification problems where we map predicted values to probabilities. The sigmoid function can squish any real-valued number into the range between 0 and 1, making it extremely useful for converting values into probabilities.

The sigmoid function  $S: \mathbb{R} \rightarrow (0, 1)$ , is defined as:

$$S(x) = \frac{1}{1 + e^{-x}}$$

Your task in this exercise is to check if this sigmoid function  $S$  is bijective. In other words, determine whether it's both injective (or one-to-one) and surjective (or onto).

As a reminder:

- Injectivity: Show that if  $S(x) = S(y)$ , then  $x = y$ . This means that no two different inputs will yield the same output.
- Surjectivity: Show that for any number  $y$  in the range  $(0, 1)$ , there is an  $x$  in the domain of real numbers such that  $S(x) = y$ . This means that every possible output is produced by some input.

## Exercise 2 (20 points)

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1. We are considering the function  $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ , defined by the equation  $f(x, y) = (xy, x^3)$ . Your task is to determine  $f \circ f$ , the composition of  $f$  with itself.

Hint: Function composition is essentially feeding the output of one function into the other. In this case, you're feeding the output of  $f$  back into itself.



2. Now, consider the functions  $f, g : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z} \times \mathbb{Z}$ , which are defined as  $f(m, n) = (mn, m^2)$  and  $g(m, n) = (m + 1, m + n)$ . The task here is to compute (a)  $g \circ f$ , and (b)  $f \circ g$ .

## Exercise 3. (points = 10)

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Let  $S = \{x \in \mathbb{R} \mid 0 < x < 1\}$

1. Let  $U = \{x \in \mathbb{R} \mid 0 < x < 2\}$ . Prove that  $S$  and  $U$  have the same cardinality.
2. Let  $V = \{x \in \mathbb{R} \mid 2 < x < 5\}$ . Prove that  $S$  and  $V$  have the same cardinality.

## Exercise 4. (points = 50)

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For each pair of sets given below, establish their cardinality equality by explicitly defining a bijection between them. There's no need to formally prove your function is bijective; simply provide the function's definition. For this, you can use the following template: "Let  $f : A \rightarrow B$  be defined by  $f(x) = \dots$ ".

1.  $\mathbb{R}$  and  $(0, \infty)$  (Hint: Try using the exponential function in some way.)
2.  $\mathbb{R}$  and  $(\sqrt{2}, \infty)$
3.  $\mathbb{R}$  and  $(0, 1)$

4.  $\mathbb{Z}$  and  $S$  where  $S = \{\dots, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, 16, \dots\}$

5.  $A = \{3k : k \in \mathbb{Z}\}$  and  $B = \{7k : k \in \mathbb{Z}\}$

6.  $A = \{(5n, -3n) : n \in \mathbb{Z}\}$  and  $\mathbb{Z}$

7. The set of even integers, denoted by  $E$ , and the set of odd integers, denoted by  $O$ .

8.  $\mathbb{Z}$  and  $S$  where  $S = \{x \in \mathbb{R} : \sin(x) = 1\}$

9.  $\{0, 1\} \times \mathbb{N}$  and  $\mathbb{N}$

10.  $\mathbb{N}$  and  $\mathbb{Z}$  (Hint: create a function that interleaves positive and negative numbers as shown in our slides)



Our paths have  
crossed for a  
reason...

**Your career  
and us**

- Our school wants to see you succeed
- You may hope to find a good job after graduation, or
- You may hope to find a good Master/Phd graduate school
- In either case, you need a resume that looks unique
- I am co-leading Software and Artificial Intelligence Lab (SAIL). With some brilliant students, we are working on machine learning, scientific computing, software engineering, and programming language techniques
- Together with you, we hope to produce great research work to make your resume shine
- Method 1. CSE487: Get credits, but can only do twice
- Method 2. RA: Try and get pay
- Prerequisite: U3/U4, strong motivation, and A-level CSE216



# Three advices before we depart

1



2



3

"Be yourself; everyone else is already taken."

*Oscar Wilde*

- While (evaluation\_participation\_rate < 90%) {
  - do\_course\_evaluation();
- }