CSE215
Foundations of Computer Science

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Today

- Direct proof exercises and revision

Direct proof

- How to prove “If A, then B”
  - Suppose A, … Therefore B.
- How to prove “for all real number x, P(x)”
  - Let x be a real number. …Therefore P(x).
- How to prove “for all real number x, P(x) -> Q(x) ”
  - Let x be a real number. Suppose P(X). … Therefore Q(x).
- How to prove “there exist x, P(x)”
  - Let x be <something you choose>. …. We have P(x) holds.

Problem 4. [5 points]

Prove that the sum of the squares of any two consecutive odd integers is even.

- Proof.
  - We need to prove the following:
    - for any integer n, $(2n+1)^2 + (2n+3)^2$ is even.
  - Let n be an arbitrary integer.
  - We have $(2n+1)^2 + (2n+3)^2 = 8n^2 + 20n + 2 = 2(4n^2 + 10n+1)$ following algebraic Identities.
  - Therefore, $(2n+1)^2 + (2n+3)^2$ is even.
- QED.

Problem 5. [5 points]

Suppose that x and y are real numbers. Prove that $x = y$ if and only if $xy = (x + y)^2/4$.

- Proof.
    - Let x and y be two real numbers.
    - We need to prove
        - (1) $x = y \rightarrow xy = (x+y)^2/4$
        - (2) $xy = (x+y)^2/4 \rightarrow xy$
    - We first prove (1):
        - Suppose $x = y$
        - Therefore $xy = (x+y)^2/4$ following algebraic Identities
        - We have proved (1)
    - We then prove (2)
        - Suppose $xy=(x+y)^2/4$.
        - We have $x^2 - 2xy + y^2 = 0$, namely $(x-y)^2=0$, following algebraic Identities
        - Therefore $x=y$
        - We have proved (2)
- QED

Problem 5. Direct proof (points = 5)

Suppose a, b and c are integers. If $a^2|b$ and $b^3|c$, then $a^6|c$.

- Proof.
  - Let a, b, and c be three integers.
  - Suppose $a^2 \mid b$ and $b^3 \mid c$
  - By definition, we have $b = k a^2$ for some integer k, and $c = k' b^3$ for some integer k'.
  - Thus, $c = (k' k^3) a^6$
  - Therefore $a^6 \mid c$.
- QED

Additional Exercises on Direct Proof

Review exercise 1

Prove: For any natural number n, $n^2 + 3n + 2$ is composite

Review exercise 2

For any integer x, y, if x is even, then xy is even.

Review exercise 3

Prove: there exist two irrational number r1, r2, such that r1*r2 is a rational number.

Review exercise 4

Prove: Suppose a is an integer. If $7|4a$, then $7|a$.

That is all for today

- Direct proof exercises and revision
- Practice, practice, and practice

Thank you!