CSE215
Foundations of Computer Science

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Plan

- Midterm 2
- Break + Course evaluation
- Homework 13
- Recitation: Ungraded homework

Midterm 2

Problem 1 (12 points)

Evaluate the following statements as true or false.

1. Let $B = \{x \in \mathbf{Z} | -3 \leq x \leq 3\}$. Then B is a subset of the set of the positive numbers.

2. $\{x \in \mathbf{R} | x^2 = 4\} = \{x \in \mathbf{R} | x^3 = 8\}$.

3. For any real number y, if $y^2 - 2y > 0$, then $y > 2$.

4. If c and d are rational numbers, then $c - d$ must be rational.

Problem 2 (21 points)

Consider sets $C = \{1, 3, 5\}$, $D = \{2, 3, 4, 5\}$, and the universal set $V = \{1, 2, 3, 4, 5\}$. Determine the following.

1. Number of elements in C'. Your answer should be an integer.

2. Number of elements in D'. Your answer should be an integer.

3. Number of elements in $C \cap C'$. Your answer should be an integer.

4. Number of elements in $C \cup C'$. Your answer should be an integer.

5. Number of elements in $C - D$. Your answer should be an integer.

6. Number of elements in $D - C$. Your answer should be an integer.

7. Number of elements in the powerset of $(C - D) \times (D - C)$. Your answer should be an integer.

Problem 3 (15 points)

Let E be the set $\{\{1\},\{1,10\},1\}$. Decide if the following statements are true or false.

1. $1 \in E$

2. $\{1\} \in E$

3. $\{1,10\} \in E$

4. $\{1,10\} \subseteq E$

5. $\{1\} \subseteq E$

Problem 4 (20 points)

Determine if the following is true or false.

1. For any sets X and Y, $(X \cup Y) - Y = X$.

2. For any sets X, Y, and V, $(X \cup Y) \cap V = X \cup (Y \cap V)$.

3. Let $\mathbf{Z}$ denote the set of integers.
   Let $S = \{9a + 10b \mid a \in \mathbf{Z}, b \in \mathbf{Z}\}$. Then $S = \mathbf{Z}$

4. For any sets X, Y, $(X \cup (X \cup Y)) \cap Y = Y$.

5. There exists a set X such that $X \cap \emptyset \neq \emptyset$.

Problem 5 (16 points)

1. Prove that no integers m and n satisfy $6m + 7 = 4n$.

2. Prove that there exist integers m and n such that $7m + 6 = 4n$.

Problem 6 (16 points)

1. Prove that

$$\Sigma_{i=1}^{n} \frac{i}{(i+1)!} = 1 - \frac{1}{(n+1)!}$$

for every integer $n \geq 1$.

1. Prove that $8|(3^{2n} - 1)$ for every integer $n \geq 1$.

Course evaluation

- While (rate of participance < 90%){
  - Do course evaluation;
- }
- Do other things

Homework 13

Exercise 1.(points = 10)

Let A = {1,2,3,4} and B = {a,b,c}. Give an example of a function f: A -> B that is neither one-to-one nor onto. Diagrams will suffice.

Exercise 2. (points = 18)

Consider the sine function sin: R -> [-1, 1]. Determine

1. whether this function is one-to-one and
2. whether it is onto.

Explanation is not needed.

Exercise 3.(points = 18)

A function $f : Z \to Z$ is defined as $f(n) = 2n + 1$. Determine

1. whether this function is one-to-one and
2. whether it is onto.

Explanation is not needed.

Exercise 4. (points = 18)

A function f: Z → Z × Z is defined as f(n) = (2n, n + 3). Determine

1. whether this function is one-to-one and
2. whether it is onto.

[Hint: a pair (p,q) is equal to another pair (p',q') if and only if p = p' and q = q'.]

Explanation is not needed.

Exercise 5.(points = 18)

A function $f : Z \times Z \to Z$ is defined as $f(m,n) = 2n - 4m$. Determine

1. whether this function is one-to-one, and

2. whether it is onto.

Explanation is not needed.

Exercise 6.(points = 18)

A function $f : Z \times Z \to Z$ is defined as $f(m,n) = 3n - 4m$. Determine

1. whether this function is one-to-one and
2. whether it is onto.

Explanation is not needed.