CSE215
Foundations of Computer Science

State University of New York, Korea

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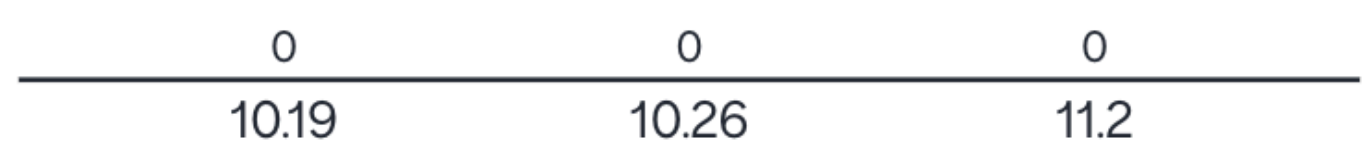
Great time for Midterm 1



Agenda

- Proof by contradiction review
- Proof by contraposition
- Proof by dividing into cases

Proof by contradiction review

Proof by contradiction

- Your objective is to prove P. E.g., \sqrt(2) is irrational.
- You go by proving ~P is False.
- You assume ~P, and then get a contradiction

$\sqrt{2}$ is irrational

- Proof.
  - We use proof by contradiction.
  - Assume sqrt(2) is a rational number, namely:
    - (A) there exists two integers m, n such that sqrt(2)=m/n, and m and n have no common factors.
  - Thus m^2 = 2 n^2. Thus, m^2 is even. Thus m must be even (otherwise m^2 becomes odd).
  - Thus m = 2k for some integer k. Thus, n ^2= 2 k^2. Thus n^2 is even and therefore n must be even.
  - But the fact that m and n are both even contradicts with (A)
  - Therefore sqrt(2) must be irrational.
- QED.

Proof by contraposition

Proof by contraposition

- You are asked to prove P -> Q and you feel ~Q -> ~P is easier to prove
- Proof
  - We use proof by contraposition to proceed
  - We will prove: ___
  - Suppose ____
  - Therefore ____
- QED.

n^2 is even $\implies$ n is even

Prove:

Suppose n is an integer. If n^2 is even, then n is even

Exercise 1: Prove the following

Suppose $x \in \mathbb{R}$. If $x^2 + 5x < 0$ then $x < 0$.

Exercise 2: Prove the following

Suppose $x \in \mathbb{R}$. If $x^3 - x > 0$ then $x > -1$.

Proof by dividing into cases

Proof by dividing into cases

- You are asked to prove Q and you consider an **exhaustive** list cases. For each case, Q is true.
    - We use proof by dividing into cases to proceed
    - We consider __N__ cases.
    - Case 1: ____….. Therefore Q
    - Case 2: ____….. Therefore Q
    - …
    - Case N: ____….. Therefore Q
- QED.

$n^2 + 3n + 2$ is even

$n^2 + 3n + 2$ is even

- Proof. We consider two cases.
  - Case 1: n is even. n^2 + 3n + 2 is even + even + even, therefore even.
  - Case 2: n is odd. n^2 + 3n + 2 is odd + odd + even, therefore even.
- QED.

Example: Prove the following statement

Proposition If $n \in \mathbb{N}$, then $1+(-1)^n(2n-1)$ is a multiple of 4.

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Proof. Suppose $n \in \mathbb{N}$.
Then n is either even or odd. Let's consider these two cases separately.

Case 1. Suppose n is even. Then $n = 2k$ for some $k \in \mathbb{Z}$, and $(-1)^n = 1$.
Thus $1+(-1)^n(2n-1) = 1+(1)(2\cdot 2k-1) = 4k$, which is a multiple of 4.

Case 2. Suppose n is odd. Then $n = 2k+1$ for some $k \in \mathbb{Z}$, and $(-1)^n = -1$.
Thus $1+(-1)^n(2n-1) = 1-(2(2k+1)-1) = -4k$, which is a multiple of 4.

These cases show that $1+(-1)^n(2n-1)$ is always a multiple of 4. ■

Exercise: Prove the following statement

- If two integers have opposite parity, then their sum is odd

Proof. Suppose m and n are two integers with opposite parity.
We need to show that $m+n$ is odd. This is done in two cases, as follows.
Case 1. Suppose m is even and n is odd. Thus $m = 2a$ and $n = 2b+1$ for some integers a and b. Therefore $m+n = 2a+2b+1 = 2(a+b)+1$, which is odd (by Definition 8.2).
Case 2. Suppose m is odd and n is even. Thus $m = 2a+1$ and $n = 2b$ for some integers a and b. Therefore $m+n = 2a+1+2b = 2(a+b)+1$, which is odd (by Definition 8.2).

In either case, $m+n$ is odd. ■

Summary

- proof by contradiction
- Proof by contraposition
- proof by dividing into cases