

CSE215

Foundations of Computer Science

State University of New York, Korea

Instructor: Zhoulai Fu

- Plan:
 - Proof by contradiction
 - Proof by contraposition

Contradiction



My spear can pierce
through all shields

My shield can block
all spears

Proof by contradiction

You are asked to prove P and you feel $\sim P$
is ridiculous

Prove: There is no greatest integer

- Proof.
 - We use proof by contradiction.
 - Assume there exists a greatest integer.
 - Let G denote the greatest integer. We have
 - (A): for any integer n , $n \leq G$.
 - But $G + 1$ is an integer satisfying $G + 1 > G$. This contradicts with (A)
 - Therefore, there does not exist a greatest integer
- QED.

$\sqrt{2}$ is irrational

- Proof.
 - We use proof by contradiction.
 - Assume $\sqrt{2}$ is a rational number, namely:
 - (A) there exists two integers m, n such that $\sqrt{2}=m/n$, and m and n have no common factors.
 - Thus $m^2 = 2 n^2$. Thus, m^2 is even. Thus m must be even (otherwise m^2 becomes odd).
 - Thus $m = 2k$ for some integer k . Thus, $n^2 = 2 k^2$. Thus n^2 is even and therefore n must be even.
 - But the fact that m and n are both even contradicts with (A)
 - Therefore $\sqrt{2}$ must be irrational.
- QED.

Prove: For any prime number p and natural number n ,

If $p|n$, then $p \nmid (n + 1)$.

- Proof.
 - We use proof by contradiction
 - Assume:
 - (A) there exists a prime p and a natural number n , such that $p \mid n$ and $p \mid (n+1)$
 - Since $p \mid n$, $n = pk$ for some integer k
 - Since $p \mid (n+1)$, $n+1 = pk'$ for some integer k'
 - Thus $1 = p(k' - k)$. Thus $p = 1$ which contradicts with the fact p is a prime.
 - Therefore (A) is false
- QED

Summary so far

- To prove P is true, we can prove $\sim P \rightarrow \text{False}$
- In other words, we assume $\sim P$ and try to derive a contradiction

Break

Exercises

Exercise 1

If $a, b \in \mathbb{Z}$, then $a^2 - 4b - 3 \neq 0$.

Exercise 2: Prove the following

Suppose $x \in \mathbb{R}$. If $x^2 + 5x < 0$ then $x < 0$.

Exercise 3: Prove the following

Suppose $x \in \mathbb{R}$. If $x^3 - x > 0$ then $x > -1$.

Exercise 4

Prove . If $a, b \in \mathbb{Z}$, then $a^2 - 4b \neq 2$.