CSE215
Foundations of Computer Science

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Zoom is needed for today

This week

- Some other proof tech (cont.)
- Organizing thoughts into clear writings
- SBU exam exercises
- Mock exam will be announced as ungraded homework.
- Use it for review; midterm 1 can be different.

Some other proof tech.
(cont)

Non constructive proof

Proposition There exist irrational numbers x, y for which x^y is rational.

Solution

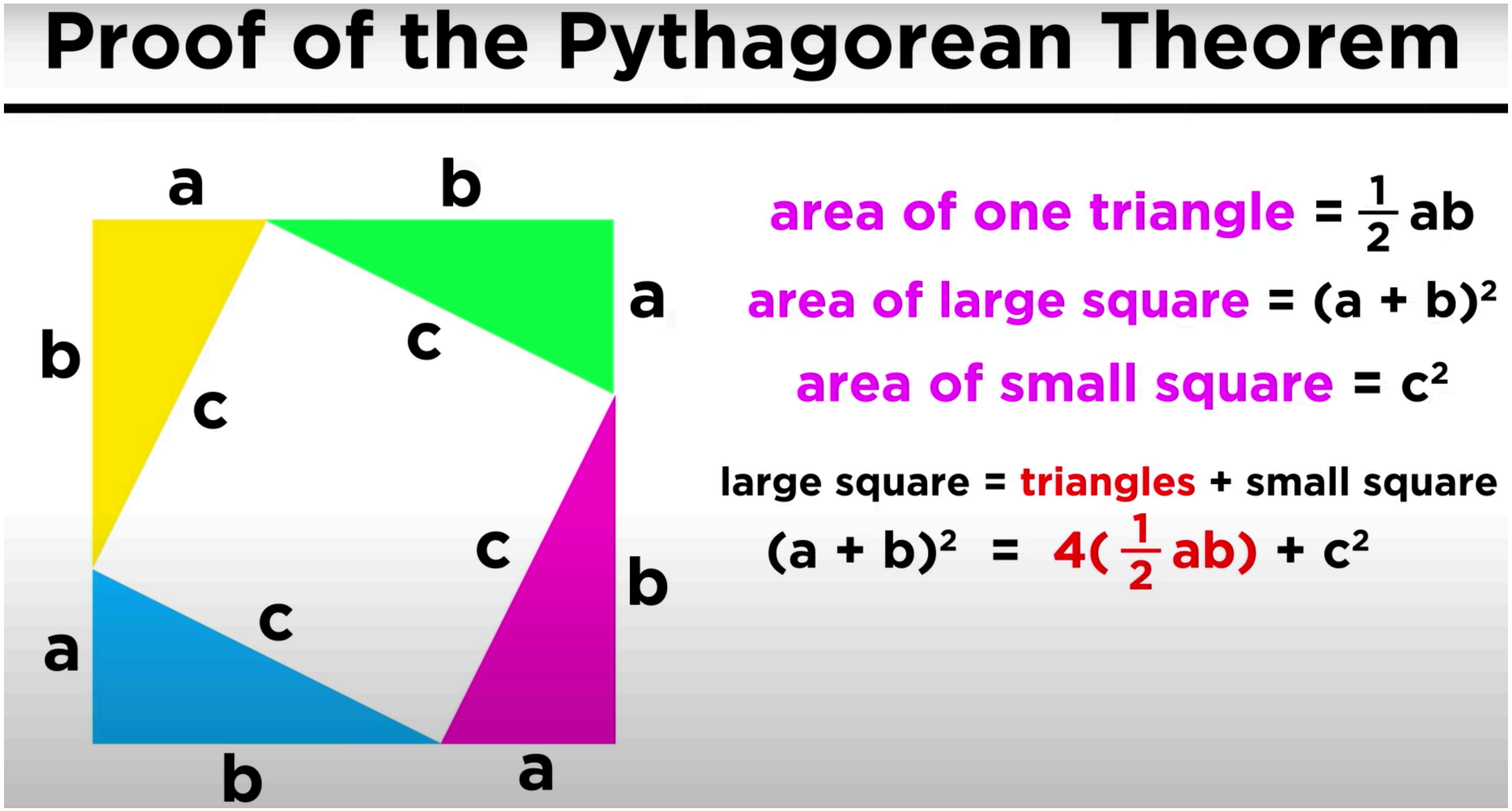
Proposition There exist irrational numbers x, y for which x^y is rational.

Proof. Let $x = \sqrt{2}^{\sqrt{2}}$ and $y = \sqrt{2}$. We know y is irrational, but it is not clear whether x is rational or irrational. On one hand, if x is irrational, then we have an irrational number to an irrational power that is rational:

$$x^y = \left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = \sqrt{2}^{\sqrt{2}\sqrt{2}} = \sqrt{2}^2 = 2.$$

On the other hand, if x is rational, then $y^y = \sqrt{2}^{\sqrt{2}} = x$ is rational. Either way, we have a irrational number to an irrational power that is rational. ■

Non-analytical, geometry-based proof



Organizing thoughts into clear writings
(1) math writing tips (2) structures

#1 thanks to:

- Richard Hammack @ Virginia Commonwealth
- Donald Knuth @Stanford

Math Writing tip 1

Clarity is the gold standard of mathematical writing.

Math Writing tip 2

Don't start a sentence with a symbol.

Bad: $x^n - a$ has n distinct zeroes.

Good: The polynomial $x^n - a$ has n distinct zeroes.

Math Writing tip 3

Symbols in different formulas must be separated by words.

Bad: Consider S_q, $q < p$.

Good: Consider S_q, where $q < p$.

Math Writing tip 4

- **Use the first person plural.** In mathematical writing, it is common to use the words “we” and “us” rather than “I,” “you” or “me.” It is as if the reader and writer are having a conversation, with the writer guiding the reader through the details of the proof.

Math Writing tip 5

Explain each new symbol. In writing a proof, you must explain the meaning of every new symbol you introduce. Failure to do this can lead to ambiguity, misunderstanding and mistakes. For example, consider the following two possibilities for a sentence in a proof, where a and b have been introduced on a previous line.

Since $a \mid b$, it follows that $b = ac$. ×

Since $a \mid b$, it follows that $b = ac$ for some integer c. ✓

If you use the first form, then a reader who has been carefully following your proof may momentarily scan backwards looking for where the c entered into the picture, not realizing at first that it came from the definition of divides.

Problem 8. [5 points]

Prove that for all integers a, if a^3 is even, then a is even.

- Proof.
  - We want to prove ________________
  - We use **proof by contraposition** to prove the statement above.
  - That is, we want to prove __________
  - Assume ______
  - _______
  - Therefore _______
- QED.

SBU 2022 Midterm

Problem 8. [5 points]

Prove that for any two integers a and b, if ab is odd, then a and b are both odd.

- Proof.
  - We want to prove ________________
  - We use **proof by contradiction** to prove the statement above.
  - That is, we assume __________ (A)
  - From this assumption, ______
  - So, we get a contradiction with (A)
- QED.

SBU 2020 Midterm

Problem 6. [5 points]
Prove that the product of any four consecutive integers is a multiple of 4.

- Proof.
  - Suppose ____. We want to prove ________________ (Q)
  - We prove the statement by **division into cases.**
  - Case 1: _____. In this case, we have ____.
  - Case 2: _____. In this case, we have ____.
  - Thus, (Q) holds in either case.
- QED.

SBU 2021 Midterm

Problem 8. [5 points]

Prove by contradiction that there are no integers x and y such that $x^2 = 4y + 2$.

- Proof.
  - (formal statement of proof objectives)
  - (our proof strategy, derived proof objectives, assumptions)
  - (core proof)
- QED.

Solution

- Proof.
  - **(formal statement of proof objectives)** We want to prove: ~(\exists x,y\in Z, such that x^2 = 4 y + 2)
  - **(proof strategy, derived proof objectives, assumptions)** We use proof by contradiction. Assume
    - (A) \exists x, y\in Z, such that x^2 = 4 y + 2
  - **(core proof)**
    - x^2 must be even since x^2 = 4 y + 2. Thus x must be even. Let x = 2k for some integer k.
    - Then 4 k^2 = 4 y + 2. Thus, 2 * k^2 = 2y + 1.
    - This is a contradiction, since 2 * k^2 is even and 2y + 1 is odd. Therefore (A) must be false.
- QED.

Solution

- The statement is false.
- To disprove it, we choose x=2, y=1/2. Then both x and y are rational, but x^y is irrational.

Three SBU exam problems

- Prove: Given an integer a, then $a^3 + a^2 + a$ is even if and only if a is even.

SBU 2022 Midterm

Problem 6. [5 points]

Let $a_1, a_2, \ldots, a_n$ be real numbers for $n \geq 1$. Prove that at least one of these numbers is greater or equal to the average of the numbers.

SBU 2022 Midterm

Problem 7. [5 points]

Prove or disprove the following statement. If x and y are rational, then x^y is rational.