CSE215
Foundations of Computer Science

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Plan

- Today1: Review with mock exam
- Today2: Unfinished exercises from last time
- Today3: Review with Homework 07
- Thursday: Midterm 1

Mock Midterm

Problem 1. Propositional statements [points = 12]

Determine if the following statements are true or false. No explanation is needed.

1. For every integer x, if x > 0, then x^2 >= 1.
2. For every real number x, there exists a natural number n such that x^n >= 0.
3. For every natural number n, the integer $2n^2 - 4n + 31$ is prime.
4. There exists a real number a for which a * x = x for every real number x.

Problem 2. Negation [points = 16]

Negate the following statements.

1. For any integer a, if a is odd then a^2 is odd.
2. For every prime number p, there is another prime number q such that $q > p$.
3. If x is prime, then x is not a multiple of 2.
4. For every positive number ε, there is a positive number M, such that $|f(x) - b| < \varepsilon$ whenever $x > M$.

[Hint: "whenever" here has the same meaning as "if".]

Problem 3. Inference rule [points = 10]

Fill in the missing "- - - -" parts following the inference rule mentioned in the text.

a.

1. $(p \wedge r) \rightarrow \sim q$ Premise
2. $\sim q \rightarrow r$ Premise
3. $p \wedge r$ Premise
4. - - - - Transitivity with 1, 2
5. - - - - Modus Ponens with 3, 4

b.

1. $(p \wedge q) \rightarrow r$ Premise
2. $\sim r$ Premise
3. p Premise
4. - - - - Modus Tollens with 1, 2
5. - - - - De Morgan with 4
6. - - - - Double negation with 3 and then elimination with 4

Problem 4. Validity [points = 6]

Which arguments below are valid? There can be zero, one, or more than one choice. You do not need to explain. All points will be lost if there is an error in your answer.

(1)

Premises:

- If Adam is innocent, Adam will not be punished.
- Adam is innocent.

Conclusion: Adam will not be punished.

(2)

Premises:

- If Adam is innocent, Adam will not be punished.
- Adam will be punished.

Conclusion: Adam is not innocent

(3)

Premises:

- If Adam is innocent, Adam will not be punished.
- Adam is not innocent.

Conclusion: Adam will be punished.

(4)

Premises:

- If Adam is innocent, Adam will not be punished.
- Adam will not be punished.

Conclusion: Adam is innocent.

Problem 5. Truth tables [points = 24]

1. Determine whether $(p \wedge (p \oplus q)) \rightarrow \sim q$ is a tautology or not. Please justify your answer with explanations.

2. Determine if this argument below is valid or not. Please justify your answer with explanations.

    - premmise $p \rightarrow r$
    - premise $q \rightarrow r$
    - conclusion $(p \vee q) \rightarrow r$

3. Determine if $P \rightarrow Q$ and $(P \wedge \sim Q) \rightarrow (Q \wedge \sim Q)$ are equivalent or not. Please justify your answer with explanations.

Problem 6. Direct proof [points = 8]

Suppose a is an integer. Prove that if $7 \mid 4a$, then $7 \mid a$.

Problem 7. Proof by dividing into cases [points = 8]

Prove: If $n \in Z$, then $5n^2 + 3n + 7$ is odd.

Problem 8. Proof by contraposition and contradiction [points = 8]

Suppose a, b, c are integers. If a does not divide b * c, then a does not divide b.

Problem 9. Application [points = 8]

Consider 3 premises:

- “I am going to hike this weekend.”
- “I will not go hiking on Saturday.”
- “If I go hiking on Sunday, I will be tired on Monday.”

In the first premise, "weekend" means Saturday or Sunday. Can the following conclusion be inferred from the premises in a valid argument? Please justify your answers with explanations with inference rules.

- I will be tired on Monday

Unfinished exercises from last time

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Problem 8. [5 points]

Prove by contradiction that there are no integers x and y such that $x^2 = 4y + 2$.

- Proof.
  - (formal statement of proof objectives)
  - (our proof strategy, derived proof objectives, assumptions)
  - (core proof)
- QED.

Solution

- Proof.
  - **(formal statement of proof objectives)** We want to prove: ~(\exists x,y\in Z, such that x^2 = 4 y + 2)
  - **(proof strategy, derived proof objectives, assumptions)** We use proof by contradiction. Assume
    - (A) \exists x, y\in Z, such that x^2 = 4 y + 2
  - **(core proof)**
    - x^2 must be even since x^2 = 4 y + 2. Thus x must be even. Let x = 2k for some integer k.
    - Then 4 k^2 = 4 y + 2. Thus, 2 * k^2 = 2y + 1.
    - This is a contradiction, since 2 * k^2 is even and 2y + 1 is odd. Therefore (A) must be false.
- QED.

- Prove: Given an integer a, then $a^3 + a^2 + a$ is even if and only if a is even.

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Problem 6. [5 points]

Let $a_1, a_2, \ldots, a_n$ be real numbers for $n \geq 1$. Prove that at least one of these numbers is greater or equal to the average of the numbers.

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Problem 7. [5 points]

Prove or disprove the following statement. If x and y are rational, then x^y is rational.