CSE215
Foundations of Computer Science

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Homework 06

Exercise 1(points = 20)

Suppose a and b are integers. Prove $a^2 - 4b \neq 2$.

Issue

Exercise 1

Suppose a and b are integers. Prove $a^2-4b \neq 2$.

We use proof by contradiction,
we need to prove $a^2-4b=2$. **<——wrong logic**

$a^2-4b=2$

$a^2=4b+2$

$a^2=\underbrace{2(2b+1)}$

this is even form, a^2 must be even, $a=2k$.

$4k^2=2(2b+1)$

$2k^2=2b+1$

left is even, right is odd.
It is contradiction.

QED.

Exercise 2 (points = 20)

Suppose x is a real number. Prove that if $x^3 - x > 0$ then $x > -1$.

Issue

Ex-2.(20p)

Proof.

We need to prove that supposing x is a real number, if $x^3 - x > 0$, then $x > -1$.

We use a proof by contradiction.

Assume:

Wrong logic —> A) $x^3 - x \geqslant 0 \Rightarrow x \leqslant -1$

Then, the solution set for $x^3 - x \geqslant 0$ is $[-1, 0]$ and $[1, 0]$, which contradicts the proposition A that $x \leqslant -1$.

Q.E.D. ■

Issue

Exercise 2

Proof.

- We use proof by contraposition.
- We will prove for real number x, if $x^3 - x > 0$, then $x > -1$.

wrong logic ->

- Suppose if $x \leq -1$, then $x^3 - x \leq 0$.
- Thus, $x^3 - x = x(x - 1)(x + 1)$.
- Since $x \leq -1$, there will be $x \leq -1$, $x - 1 \leq -2$, $x + 1 \leq 0$.
- Thus, $x(x - 1)(x + 1)$ will always be 0 or negative.
- Therefore, the statement of if $x^3 - x > 0$, then $x > -1$ is true because contraposition of it is true.

QED.

Exercise 3 (points = 80)

For each statement below: (1) determine if the statement is true or false. (2) Prove it. Namely, if true, prove it is true; if false, prove it is false.

Hint: To demonstrate a statement is false, we need to establish the truth of its negation. For example, if we are to prove that "for any integer x, $x^2 > 0$" is false, we need to prove "there exists an integer x such that $x^2 <= 0$", and for the latter, we will show that choosing x = 0 makes $x^2 = 0$ and therefore $x^2 <= 0$ by generalization.

1. rational/irrational is irrational.
2. Irrational*irrational is irrational.
3. The sum of any two positive irrational numbers is irrational.
4. The square root of any rational number is irrational.

Solutions

#1 ## Exercise 1(points = 20)

Suppose a and b are integers. Prove $a^2 - 4b \neq 2$.

proof.

we use proof by contradiction

Assume that there are integers a and b, and $a^2-4b=2$.

$a^2-4b=2$

we can rearrange the equation, $a^2=2+4b$

Now, we can recognize that LHS (left-hand side), which is a^2 is always positive integer

but not always even number. (e.g. $7^2=49$ ← not even)

How about RHS (right-hand side) it is always even integer because $2+4b=2(1+2b)$

Thus, square integer does not mean always same as even integer.

Hence, assumption, which is $a^2-4b=2$ is false.

QED

#2 Exercise 2 (points = 20)

Suppose x is a real number. Prove that if $x^3 - x > 0$ then $x > -1$.

proof.

we use proof by contraposition to proceed.

we need to prove $x \leq -1$, then $x^3 - x \leq 0$

suppose $x \leq -1$

and we know that $x^3 - x = x(x^2 - 1) = x((x+1)(x-1)$ by simplification

Since $x \leq -1$, then $x < 0$,

$x+1 \leq 0$

$x-1 < 0$

Therefore $x^3 - x \leq 0$

QED

Ex3

Minor error in 3.3: 2-pi should have been 20 - pi

1. rational/irrational is irrational

(1) this statement is false

(2) To demonstrate a statement is false, we need to establish the truth of negation.

The negation of this statement is as following and we need to prove:

There exists a rational number and an irrational number such that rational/irrational is not irrational,

and for the latter, we will show that example is 0 for rational number.

Their devision is 0/irrational, and that is always 0 which is rational number.

Negation of this statement is true.

Thus this demonstrate that the original statement is false

2. irrational × irrational is irrational.

(1) This statement is false.

(2) To demonstrate this statement is false, we need to establish the truth of negation.

The negation is as following and we need to prove:

There exists two irrational number K and m such that $K \cdot m$ is not irrational,

and for the latter, we will show that choosing $K = \sqrt{2}$, and $m = \sqrt{2}$

when we multiply K and m, $K \cdot m = \sqrt{2} \cdot \sqrt{2} = 2$, thus, rational number.

this means the negation statement is true, at the same time, this indicate original statement is

3. The sum of any two positive irrational number is irrational.

(1) This statement is false

(2) To demonstrate this statement is false, we need to establish the truth of negation.

The negation is as following:

There exist two positive irrational number x and y such that $x+y$ is rational number,

and for the latter, we will show that choosing $x = 2-\pi$ and $y = \pi$

makes $x+y = (2-\pi)+\pi = 2$, therefore sum of two positive irrational number is rational

by generalization. Negation of this statement is true.

Hence original statement is false.

4. The square root of rational number is irrational.

(1) This statement is false.

(2) To demonstrate this statement is false, we need to establish the truth of negation.

The negation of statement is as following:

There exists some rational number x, such that $\sqrt{x}$ (square root of x) is rational,

and for the latter, we will show that choosing $x = \frac{4}{9}$ makes $\sqrt{x} = \sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$.

which is rational number.

Therefore the square root of rational number is rational.

Negation of this statement is true.

Hence, original statement is false.