Guideline

Due Date: Thursday, 2023-10-19, by 23:59.

Upload your answers as a singular PDF to Brightspace.

If you're writing by hand, please ensure your handwriting is legible.

Multiple submissions are possible before the due time; the last submission will be graded.

Exercise 1 (points = 40)

Prove the following using the proof by contraposition strategy.

1. Suppose a ∈ Z. If a^2 is not divisible by 4, then a is odd.
2. Suppose x ∈ R. If $x^2 + 7x < 0$, then x < 0.

Exercise 2 (points = 20)

Prove: If n is odd, then $8|(n^2-1)$.

Exercise 3 (points = 20)

The quotient remainder theorem says: Given any integer A and a positive integer B, there exist unique integers Q and R such that A = B * Q + R where 0 ≤ R < B. Examples:

- A = 7, B = 2: 7 = 2 * 3 + 1
- A = 8, B = 4: 8 = 4 * 2 + 0
- A = 13, B = 5: 13 = 5 * 2 + 3
- A = -16, B = 26 :-16 = 26 * (-1) + 10

From the quotient-remainder theorem, we know that any integer divided by a positive integer will have a set number of remainders and thus a set number of representations. For example, any integer divided by 4 will produce a remainder between 0 and 3 (inclusive). So every integer n can be represented by one of the 4 forms: 4q, 4q + 1, 4q + 2, 4q + 3 (where q is an integer). Similarly, any integer divided by 3 will produce a remainder between 0 and 2 (inclusive).

Now prove the following proposition.

- The product of any four consecutive integers is a multiple of 8.

Hint: Perhaps you can use the quotient remainder theorem when applying a proof of dividing into cases.

Exercise 4 (points = 20)

Fermat's Last Theorem states that no three positive integers a, b, and c satisfy the equation $a^n + b^n = c^n$ for any integer value of n greater than 2.

Now, prove:

- The cube root of 2 is irrational. (The cube root of a number a is a number b such that b*b*b=a.)

This statement can be proven with either Fermat's Last Theorem or without it.