Guideline

Homework for week 4. Due: 11:59 pm on Thursday, March 30, 2022.

Submit your solutions in a single PDF on Brightspace. Multiple submissions are possible before the due time; the last submission will be graded.

Format requirement:

- Write your answers following the same order in the original order.
- If you opt for handwriting, ensure your writing is legible.
- Avoid drawing randomly inside the homework 😀
- Follow the proof templates.

Exercise 1 (points = 32)

Determine whether the statements below are true or false. Do not explain.

1. 119 is a prime number.
2. 161 is a prime number.
3. 42k is an even number for any integer k.
4. For each integer n with 2 <= n <= 6, $n^2 - n + 11$ is a prime number.
5. The average of any two odd integers is odd.
6. For any real number x, if x * x >= 4, then x >= 2.
7. For any real numbers x and y, $x^2 - 2xy + y^2 >= 0$.
8. There exists an integer x, such that $(2x + 1)^2$ is even.

Exercise 2. (points = 8)

- Conduct a bit of research: Describe the formal definition of continuity for a real-valued function f at the point x.
- Provide the formal definition for f being discontinuous at x.

Exercise 3 (points = 15)

Prove the following proposition: An even number multiplied by an integer is an even number.

Exercise 4 (points = 15)

Prove the following proposition: An odd number multiplied by an odd number is an odd number.

Exercise 5 (points = 15)

We say an integer is a perfect square if it can be expressed as a square of some integer. For example, 81 is a perfect square; 80 is not.

Prove the following statement: there is a perfect square that can be written as a sum of two other perfect squares.

Exercise 6 (points = 15)

Suppose a ∈ Z. Prove: If a is an odd integer, then $a^2 +3a+5$ is odd.