CSE215
Foundations of Computer Science

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Plan

- New: Valid arguments

- Reinforcement on tautology/contradiction

Valid arguments

Final, 2020-1

Problem 1. [5 points]

Determine if the following deduction rule is valid.

$p \to (q \vee r)$

$\sim (p \to q)$

$\therefore\ r$

What is a logical argument?

Definitions

- Logical argument. Sequence of statements aimed at demonstrating the truth of an assertion
- Conclusion. Last statement in an argument
- Premises. Last-but-one statements in an argument

Argument
Premise_1
Premise_2
$\vdots$
Premise_m
$\therefore$ Conclusion

If Premise_1 and Premise_2 and $\cdots$ and Premise_m, then Conclusion.

What is a valid argument?

Definition

- An argument is valid if the conclusion follows necessarily from the premises

https://youglish.com/pronounce/%22Valid%20Argument%22/english/us?

- Every person will die
- Socrates is a person
- So, Socrates will die

- Congressmen/women own classified info
- Investors can take profit from classified info
- So, congressmen/women should not be allowed to actively do investment

Exercise: Valid or Not?

- All cups are blue
- Socrates is cup
- So, Socrates is blue

Valid or not?

Examples

Valid

- If Socrates is a man, then Socrates is mortal.
  Socrates is a man.
  Therefore, Socrates is mortal.

  If p, then q.
  p.
  Therefore, q.

Invalid

- If Socrates is a man, then Socrates is mortal.
  Socrates is mortal.
  Therefore, Socrates is a man.

  If p, then q.
  q.
  Therefore, p.

Valid

- If Socrates is a man, then Socrates is mortal.
  Socrates is not mortal.
  Therefore, Socrates is not a man.

  If p, then q.
  $\sim q$.
  Therefore, $\sim p$.

Invalid

- If Socrates is a man, then Socrates is mortal.
  Socrates is not a man.
  Therefore, Socrates is not mortal.

  If p, then q.
  $\sim p$.
  Therefore, $\sim q$.

How to mathematically check if an argument is valid?

- Truth table
- Inference rules

Method 1: Truth table

1. Identify the premises and conclusion
2. Construct a truth table for premises and conclusion
3. A row of the truth table in which all the premises are true is called a critical row.
   If there is a critical row in which the conclusion is false, then the argument is invalid. If the conclusion in every critical row is true, then the argument is valid.

Importantly, if there are no critical rows, then the arguments is considered valid

Example

Problem

- Determine the validity of the argument:

  $p \to q \vee \sim r$

  $q \to p \wedge r$

  $\therefore p \to r$

p	q	r	$\sim r$	$q \vee \sim r$	$p \wedge r$	$p \to q \vee \sim r$	$q \to p \wedge r$	$p \to r$
T	T	T	F	T	T	T	T	T
T	T	F	T	T	F	T	F	
T	F	T	F	F	T	F	T	
T	F	F	T	T	F	T	T	F
F	T	T	F	T	F	T	F	
F	T	F	T	T	F	T	F	
F	F	T	F	F	F	T	T	T
F	F	F	T	T	F	T	T	T

Exercise

Problem 1. [5 points]

Determine if the following deduction rule is valid.

$p \to (q \vee r)$

$\sim (p \to q)$

$\therefore\ r$

2020 Final-1

Reinforcement: Tautology and Contradiction

Two special logical equivalence: Tautology and contradiction

Definitions

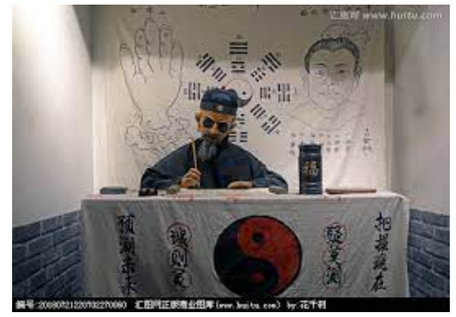
- A tautology is a statement form that is always true regardless of the truth values of the individual statements substituted for its statement variables.
- A contradication is a statement form that is always false regardless of the truth values of the individual statements substituted for its statement variables.

Examples

- $p \vee \sim p$ ▷ Tautology
- $p \wedge \sim p$ ▷ Contradiction

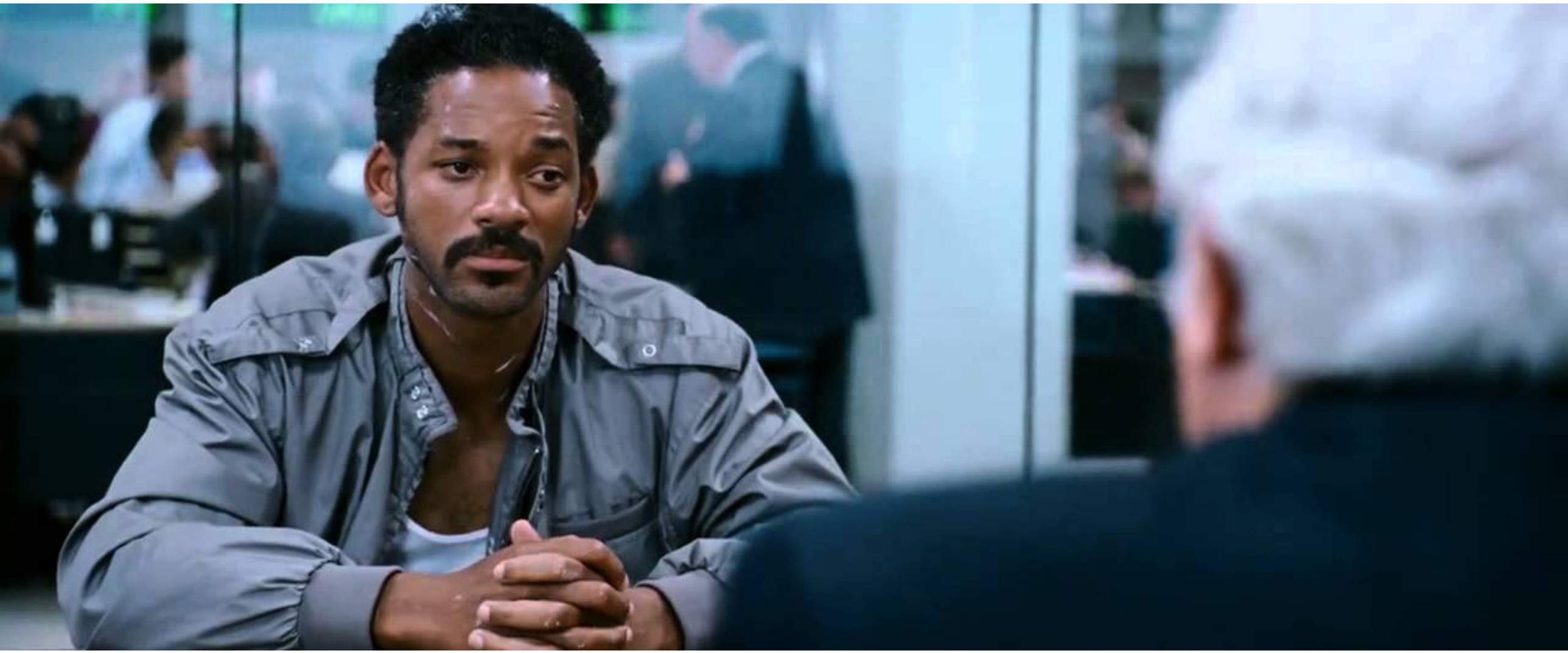
The secret of a fortune teller

- Three students ask a fortune teller if they got an “A” in the exam
- The fortune teller says nothing but shows 1 finger
- If they all got A —> 1 is right



- If they all failed to get A —> 1 is right
- If one students get A —> 1 is right
- If two students get A (meaning one does not) —> 1 is right
- **The fortune teller will always be right, since he said a tautology.**

See how logic saved Chris Gardner



https://www.youtube.com/watch?v=W2r4BUB-Rsc

- Interviewer (giving a proposition): What would you say, if a guy walked in for an interview with such a bad T-shirt, and I hired him?

- Chris Gardner (thinking about logic): He must have really nice pants.

What would you say if a person with such a T-shirt walking into the interview, and I hired him

—> Get Hired

- Interviewer's **proposition**: Bad-T-shirt /\ Get-hired
- **Common-sense:** Bad-T-shirt —> ~ Get-hired
- If Chris follows common-sense and interview's proposition, he will obtain ~Get-hired /\ Get-hired. That means **contradiction**.
- Never tell interviewers that they say a contradiction.
- So, Chris has to challenge the common-sense, to argue Bad-T-shirt —> ~Get-hired is **false.**
- Chris knows that "Bad-T-shirt —> ~Get-hired" and "Get-hired -> ~Bad-T-shirt" are **equivalent**
- So, Chris is now thinking what to imply from Get-hired?
- Since Get-hired means there must be some extraordinary quality. Chris thinks of two things: Get-hired -> Nice-T-shirt \/ Nice-Pants
- But Nice-T-shirt contradicts with Interviewer's proposition, so Christ concludes "Nice-Pants"