CSE215
Foundations of Computer Science

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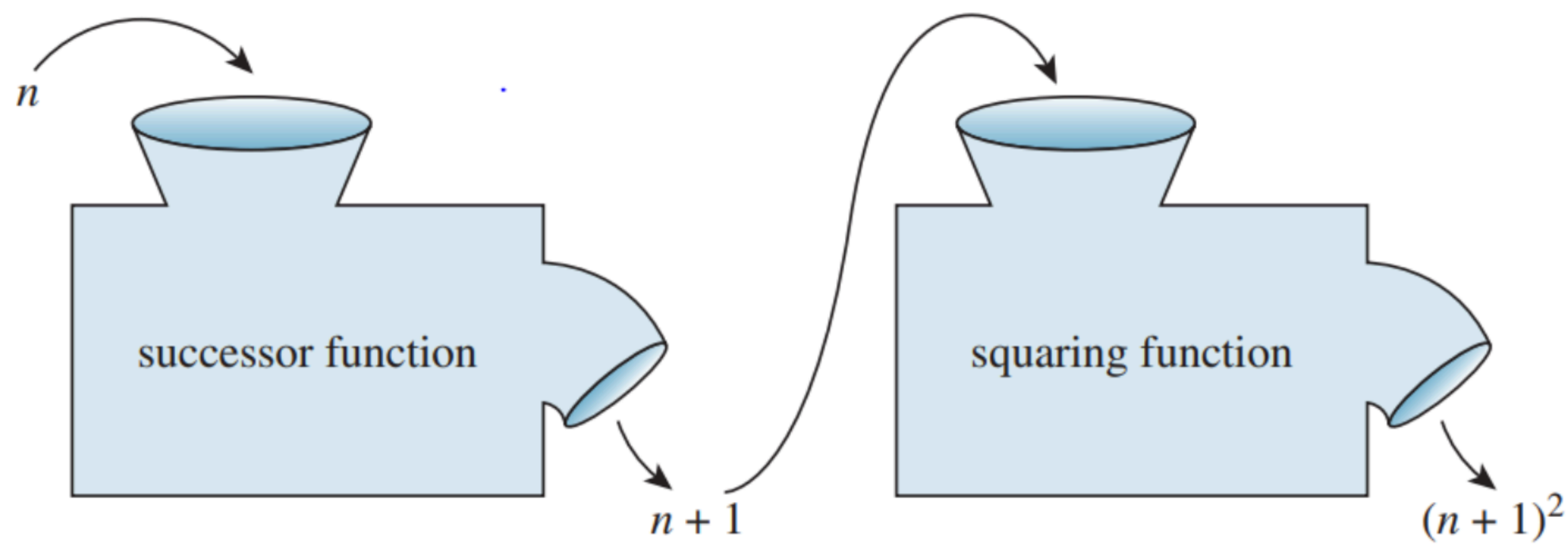
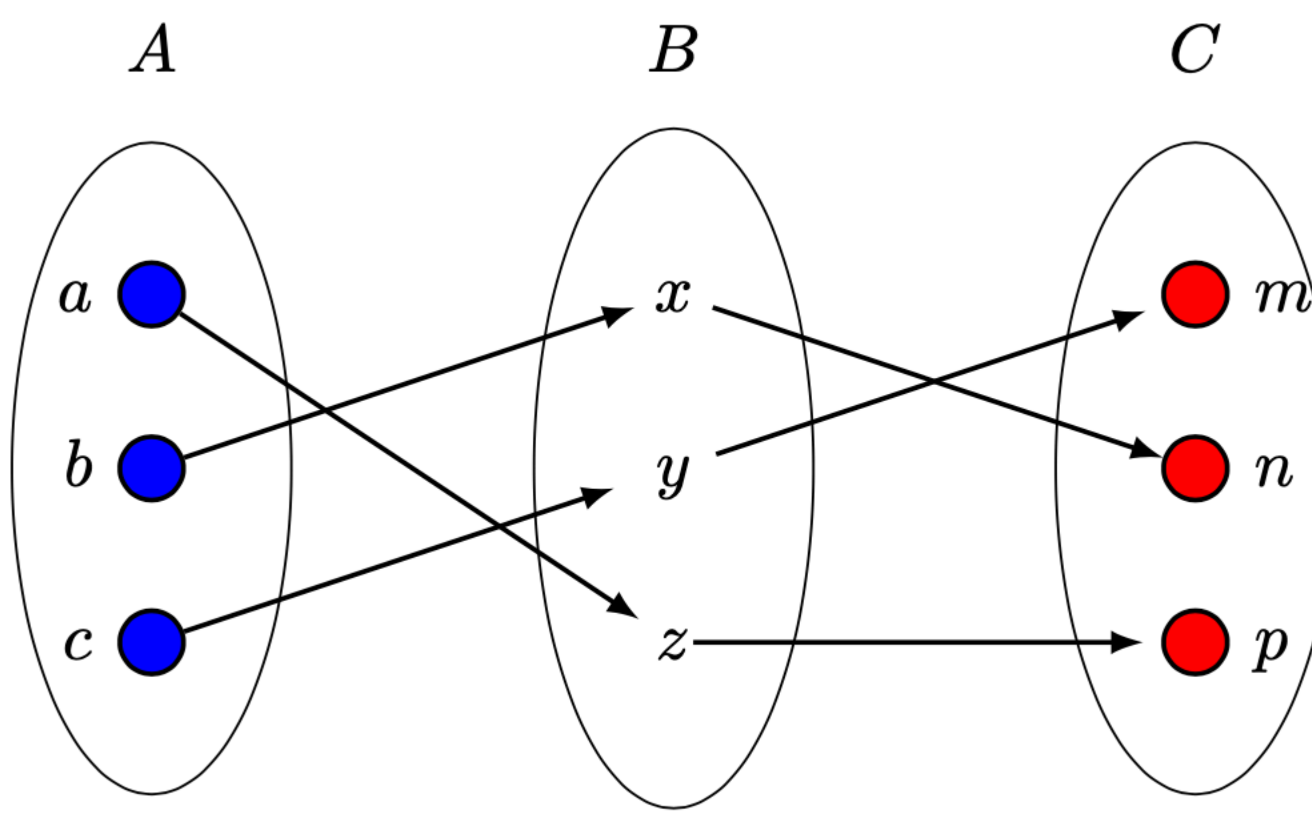
- Regular lecture for this recitation
- Function composition

Composition of functions



Composition of functions



Composition of functions

Definition

- Let $f : X \to Y$ and $g : Y \to Z$. Let the range of f is a subset of the domain of g.
- Define a new composition function $g \circ f : X \to Z$ as follows:

$$(g \circ f)(x) = g(f(x)) \text{ for all } x \in X,$$

The notation g ∘ f is read as "g of f ", "g after f ", "g circle f ", "g round f ", "g about f ", "g composed with f ", "g following f ", "f then g", or "g on f ", or "the composition of g and f ".

A missing slide about function equality

- Let f and g two functions A -> B
- We say f = g if for any a in A, f(a) = g(a)
- We say f!=g if there exists a in A, f(a) != g(a)
- For example,
  - If f (x) = (x+1)^2, and g (x) = x^2 + 2 x + 1. Then f = g.
  - If f (x) = (x+1)^2, and g (x) = x^2 + 1. Then f != g.

Composition of functions: Example 1

Problem

- Let $f : \mathbb{Z} \to \mathbb{Z}$ be the successor function and let $g : \mathbb{Z} \to \mathbb{Z}$ be the squaring function. Then $f(n) = n + 1$ for all $n \in \mathbb{Z}$ and $g(n) = n^2$ for all $n \in \mathbb{Z}$. Find $g \circ f$. Find $f \circ g$. Is $g \circ f = f \circ g$?

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Solution

- $g \circ f$.
  $(g \circ f)(n) = g(f(n)) = g(n + 1) = (n + 1)^2$ for all $n \in \mathbb{Z}$.
- $f \circ g$.
  $(f \circ g)(n) = f(g(n)) = f(n^2) = n^2 + 1$ for all $n \in \mathbb{Z}$.
- $g \circ f \neq f \circ g$.
  E.g. $(g \circ f)(1) = 4$ and $(f \circ g)(1) = 2$

Exercises

Exercise 1

. Define $L: \mathbf{Z} \to \mathbf{Z}$ and $M: \mathbf{Z} \to \mathbf{Z}$ by the rules $L(a) = a^2$ and $M(a) = a \; mod \; 5$ for all integers a.

a. Find $(L \circ M)(12)$, $(M \circ L)(12)$, $(L \circ M)(9)$, and $(M \circ L)(9)$.

b. Is $L \circ M = M \circ L$?

Exercise 2

- An identity function I is a function that always returns itself: I(a) = a for any a of the domain of I.

Prove the following

Theorem

- If f is a function from a set X to a set Y, and I_X is the identity function on X, and I_Y is the identity function on Y, then $f \circ I_X = f$ and $I_Y \circ f = f$.

Exercise 3

Consider two functions f and g, both mapping real numbers to real numbers ($f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$). If both the functions are injective (one-to-one), is the function f + g also injective? To clarify, the function f + g is defined such that it maps any real number x to the sum of f(x) and g(x).

Exercise 4

- Now, if the functions f and g are surjective (onto), does this guarantee that the function f + g (defined in the same way as above) is also surjective?