CSE216
Programming Abstraction

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Regarding quiz

- No more quiz: Seems it was more scary than supportive.
- First quiz results will be retained as-is.
- Overall grading formula remains the same

Plan

- Today: More on beta reduction

Sharing A good thing on lambda calculus

- I find this 3-page notes from Yale particularly clear:

- https://www.cs.yale.edu/homes/hudak/CS201S08/lambda.pdf

How beta reduction works

- Identify a redex (λ x.M)N
- Reduce the redex by substitution M [x:=N]
- Continue until no redex found

Redex

- Reducible expression
- A redex is formed when a lambda definition is immediately followed by an argument, indicating that the function can be applied to that argument

- (λ x.M)N

Exercise: Identify redex

- (λ x. λ y. x y) (λ s. s)
- (λ x. x) (λ y. y)
- (λ x. x x) (λ x. x x)

Substitution

- (λ x.M)N -> M [x := N]
- For all occurrence of x in M:
    - If x is bound by the formal parameter “x” in (λ x.M)
    - the replace x by N in M
- We will consider capture-avoiding situations later

Normal form

- No redex

Exercise: Normal form or not

- λ x.λ y. xy
- λ x. x λ y.x y
- (λ x.λ y. x)y
- (λ x. x λ y.x)y

Exercise: beta reduction

- (λ x. λ y. x y) (λ s. s)
- (λ x. x) (λ y. y)
- (λ x. x x) (λ x. x x)

Does beta reduction order matter?

- Let D be the lambda term (λ x. xx). Reduce
  - (λ x.y)(D D)

Does beta reduction order matter?

- (λ x.y)(D D) -> (λ x.y)(D D) -> …
- (λ x.y)(D D) -> y
- The second one is lazy evaluation
- The first one is eager evaluation

Church-Rosser Theorem

- A lambda term can never have two different normal forms