CSE216
Foundations of Computer Science

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Part 1. Regular Expressions (20)

1. Give a regular expression to match strings over {a, b} that has aabb as a substring.

2. Write a regular expression to match valid hexadecimal numbers, which start with 0x followed by one or more hexadecimal digits. Hexadecimal digits can be any combination of the numbers 0-9 and the letters A-F (both uppercase and lowercase).

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   Example valid inputs: 0x3A, 0xA2F, 0xFF, 0xaF

   Example invalid inputs: 3A, 0xG5, 0x
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   Your task is to construct the regular expression pattern to accurately match valid hexadecimal numbers in the given format.

3. Give a regular expression for positive integers, not including ones with leading 0s such as 07 or 007. Examples: 12345, 7070.

4. Give a regular expression for the language over {0,1} of strings that contains at least two occurrences of 001. Examples: 001001, 001100100.

Part 2. Context-free Grammar (20)

Consider the following grammar where {a, b} are terminals, {S, A, B} are non-terminals, and S is the starting symbol. Find the languages generated by each grammar. Example: If you are asked to find the language for grammar "S → AB, A → a | aa, B → b", your answer will be {ab,aab}.

    1. S → AB, A → ab, B → bb

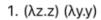   2. S → AB, A → a | aa, B → b | bb

    Make a context-free grammar generating

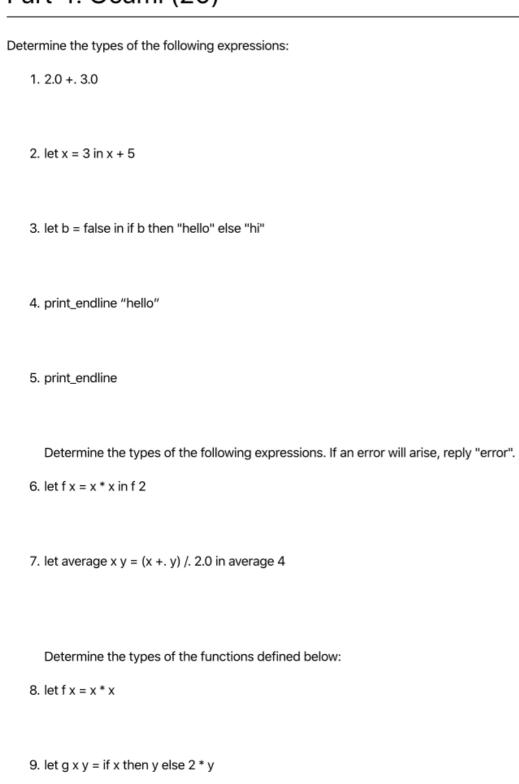   3. $\{a^n b^{3n} : n \geq 0\}$. Examples: abbb, aabbbbbb, empty string.

    4. Which of the following context-free grammars generates the language over {a,b} consisting of the strings that read the same forward and backward, e.g. aba, baab, abaaba, b?

A. $S \rightarrow aSa|bSb|a|b|\epsilon$

B. $S \rightarrow aS|bS|a|b$

C. $S \rightarrow aAa|bBb|\epsilon, A \rightarrow a|b, B \rightarrow a|b$

D. $S \rightarrow Aa|Bb, A \rightarrow aSa|bB|\epsilon, B \rightarrow bSb|aA|\epsilon$

Part 3. Lambda Calculus (40)

Apply β-reduction to the following λ-terms as much as possible

1. (λz.z) (λy.y)

2. (λz.z) (λy.y y)

3. (λz.z) (λy.y y) (λx.x a)

4. (λz.z) (λz.z z) (λz.z y)

5. (λx.λy.x y y) (λa.a) b

6. (λx.λy.x y y) (λy.y) y

7. (λx.x x) (λy.y x) z

8. (λx. (λy. x y) y) z

9. ((λx.x x) λy.y) (λy.y)

10. ((λx. λy.x y)λy.y) w

Part 4. Ocaml (20)

Determine the types of the following expressions:

1. 2.0 +. 3.0

2. let x = 3 in x + 5

3. let b = false in if b then "hello" else "hi"

4. print_endline "hello"

5. print_endline

Determine the types of the following expressions. If an error will arise, reply "error".

6. let f x = x * x in f 2

7. let average x y = (x +. y) /. 2.0 in average 4

Determine the types of the functions defined below:

8. let f x = x * x

9. let g x y = if x then y else 2 * y

Evaluate the expression below. What is the value of this expression?

10. let x= 3 in (let y= x+2 in y) - (let z = x * x * x in z)

Solution

1. Give a regular expression to match strings over {a, b} that has aabb as a substring.

$$(a|b)^* aabb (a|b)^*$$

2. Write a regular expression to match valid hexadecimal numbers, which start with 0x followed by one or more hexadecimal digits. Hexadecimal digits can be any combination of the numbers 0-9 and the letters A-F (both uppercase and lowercase).

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Example valid inputs: 0x3A, 0xA2F, 0xFF, 0xaF

Example invalid inputs: 3A, 0xG5, 0x
```

Your task is to construct the regular expression pattern to accurately match valid hexadecimal numbers in the given format.

$$0x [0-9a-f A-F]^+$$

3. Give a regular expression for positive integers, not including ones with leading 0s such as 07 or 007.
   Examples: 12345, 7070.

$$[1-9][0-9]^*$$

4. Give a regular expression for the language over {0,1} of strings that contains at least two occurrences of 001. Examples: 001001, 001100100.

$$(0|1)^* 001 (0|1)^* 001 (0|1)^*$$

Part 2. Context-free Grammar (20)

Consider the following grammar where {a, b} are terminals, {S, A, B} are non-terminals, and S is the starting symbol. Find the languages generated by each grammar. Example: If you are asked to find the language for grammar "S → AB, A → a | aa, B → b", your answer will be {ab,aab}.

1. S → AB, A → ab, B → bb

$$\{abbb\}$$

2. S → AB, A → a | aa, B → b | bb

$$\{ab, abb, aab, aabb\}$$

Make a context-free grammar generating

3. $\{a^n b^{3n} : n \geq 0\}$. Examples: abbb, aabbbbbb, empty string.

$$S \rightarrow aSbbb \mid \varepsilon$$

4. Which of the following context-free grammars generates the language over {a,b} consisting of the strings that read the same forward and backward, e.g. aba, baab, abaaba, b?

A. $S \rightarrow aSa|bSb|a|b|\epsilon$

B. $S \rightarrow aS|bS|a|b$

C. $S \rightarrow aAa|bBb|\epsilon, A \rightarrow a|b, B \rightarrow a|b$

D. $S \rightarrow Aa|Bb, A \rightarrow aSa|bB|\epsilon, B \rightarrow bSb|aA|\epsilon$

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Part 3. Lambda Calculus (40)

Apply β-reduction to the following λ-terms as much as possible

1. (λz.z) (λy.y)

$$\lambda y.y$$

2. (λz.z) (λy.y y)

$$\lambda y.yy$$

3. (λz.z) (λy.y y) (λx.x a)

$$aa$$

4. (λz.z) (λz.z z) (λz.z y)

$$yy$$

5. (λx.λy.x y y) (λa.a) b

$$bb$$

6. (λx.λy.x y y) (λy.y) y

$$yy$$

7. (λx.x x) (λy.y x) z

$$xxz$$

8. (λx. (λy. x y) y) z

$$zy$$

9. ((λx.x x) λy.y) (λy.y)

$$\lambda y.y$$

10. ((λx. λy.x y)λy.y) w

$$w$$

Part 4. Ocaml (20)

Determine the types of the following expressions:

1. 2.0 +. 3.0

    float

2. let x = 3 in x + 5

    int

3. let b = false in if b then "hello" else "hi"

    string

4. print_endline "hello"

    unit

5. print_endline

    string → unit

Determine the types of the following expressions. If an error will arise, reply "error".

6. let f x = x * x in f 2

    int

7. let average x y = (x +. y) /. 2.0 in average 4

    error

Determine the types of the functions defined below:

8. let f x = x * x

    int → int

9. let g x y = if x then y else 2 * y

    bool → int → int

Evaluate the expression below. What is the value of this expression?

10. let x= 3 in (let y= x+2 in y) - (let z = x * x * x in z)

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