CSE216
Programming Abstraction

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Today

- Capture avoiding substitution in beta reduction

- Extending lambda calculus core

Capture avoiding substitutions

- The specific names of bound variables in the lambda calculus are meaningless

- λ x. x same as λ y. y

- (λx.(λy.yx)) is equivalent to (λa.(λb.ba))

- Lambda terms that differ only by bound variable names are called alpha equivalent

Exercise: alpha equivalence?

- λ x. xy ? λ z. zy
- λ x. xy ? λ z. xz
- λ x. x λ y. y ? λ z. z λ p. p
- λ x. x λ y. y ? λ y. y λ y. y
- λ x. x λ y. x y ? λ y. y λ y. y y

Exercises: beta reduction

- (λ x.λ y. x)y

Exercises: beta reduction

- (λ x. x λ y.x)y

Exercises: beta reduction

- (λ x.λ y. x y) y

Exercises: beta reduction

- $(\lambda x.\lambda y.xy)(\lambda x.\lambda y.xy)$

Extending lambda calculus core

“extending lambda calculus core”?

- The core has only three constructs
- It does not have numbers, conditions, logic, loops
- These things can all be encoded by the core
- We will write **‖<languaage syntax>‖=<lambda-term>** for the encoding
- "*The integers were created by God, everything else is the work of man”*

TRUE, FALSE, and IF

- IF c e1 e2 returns e1 if c is TRUE, or e2 if c is FALSE
- So we encode TRUE by λ x. λ y. x
- We encode FALSE by λ x. λ y. x
- We encode IF by λ c. λ x. λ y. c x y

- Examples
  - if true then b else c = (λx.λy.x) b c → (λy.b) c → b
  - if false then b else c = (λx.λy.y) b c → (λy.y) c → c

Logic AND

- ||AND x y|| = ||IF x y FALSE|| = x y FALSE = x y x
- Thus ||AND|| = λ x. λ y. x y x
- Exercise: ||OR||=?
- Exercise: ||NOT||=?

Logic OR

- Exercise: ||OR||=?

Logic NOT

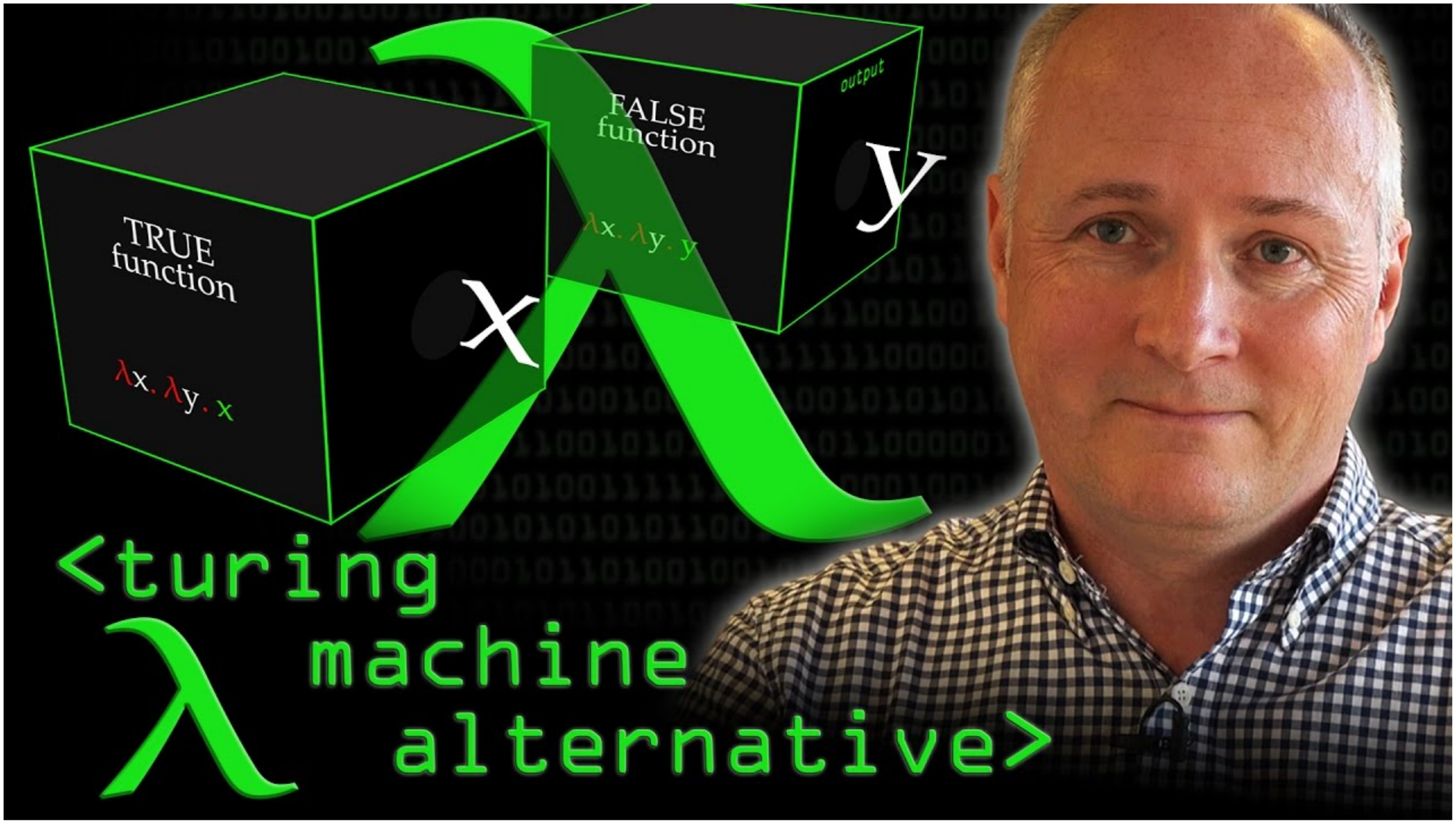
- Exercise: ||NOT||=?

Numbers

- Any counting system that makes sense would work
- We want n f x = f(f(f(f(… f(x)))))
- Since 0 f x = x, we have ||0|| = λ f. λ x. x
- Since 1 f x = f x, we have ||1|| = λ f. λ x. f x
- … (called Church numerals)

Things important in CS but not essential for next parts

- SUM
- PROD
- ....
- Recursion



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