CSE215
Foundations of Computer Science

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Homework week 01

Homework policy reminder

- Assignments **submitted after the deadline are typically not acceptable.** Should you have a valid reason for a late submission, it is essential to communicate with the course instructor promptly, providing all necessary supporting documents.

- Please note, **once solutions to the homework exercises have been disclosed, no submissions can be accepted under any circumstances**.

Email policy

- Write [CSE215] at the beginning of the subject line.

- (In this way, your question or request will be well stored).

Exercise 1. (points = 10)

Check if the two statement forms below are logically equivalent. Explanation is needed based on the truth table.

- p ⋁ q -> r
- (p -> r) ⋀ (q -> r)

Exercise 2 (points = 10)

Check if the two statement forms below are equivalent. Explanation is needed based on the truth table.

- (p -> q) /\ (q -> r) /\ (r -> p)
- p /\ q /\ r

Exercise 3 (points = 30)

For this exercise, explanation is needed based on the truth table.

Consider six statement forms (a-f):

- (a) p -> q
- (b) q -> p
- (c) ~p ⋁ q
- (d) ~q ⋁ p
- (e) ~q -> ~p
- (f) ~p -> ~q

1. Find all statement forms that are equivalent to (a), except (a) itself.
2. Find all statement forms that are equivalent to (b), except (b) itself.

Exercise 4 (points = 5)

Consider the proposition ~P /\ (Q -> P). What can you conclude about P and Q if you know the statement is true? Explanation is needed based on the truth table.

Exercise 5 (points = 15)

A tautology and a contradiction are terms used in logic to describe specific types of propositions.

- A *tautology* is a proposition that is always true, regardless of the truth values of its constituent parts. In other words, a proposition is a tautology if it is true in every row of a truth table. For example, p ⋁ ~p. An example in plain language: "It will either rain today or it won't."
- A *contradiction* is a proposition that is always false, regardless of the truth values of its constituent parts. In other words, a proposition is a contradiction if it is false in every row of a truth table. For example, p ⋀ ~p. An example in plain language: "I will finish the homework today and I will not."

For each statement form below, determine if it is a tautology, contradiction, or neither. Explanation is needed based on the truth tables.

1. (~p ⋁ q) ⋁ (p ⋀ ~q)
2. (p ⋀ ~q) ⋀ (~p ⋁ q)
3. (p ⋀ q) ⋁ (~p ⋁ (p ⋀ ~q))

Exercise 6. (points = 5)

Check if the two statement forms below are logically equivalent and explain with truth tables.

- p ⋁ (q -> r)
- (p -> r) ⋀ (q ->r)

Exercise 7. (points = 15)

Smart Home Security System

Consider a house that is equipped with a smart security system. The system uses a combination of a facial recognition camera and a door sensor to control the main entrance lock.

- The facial recognition camera detects if a family member's face is recognized.
- The door sensor detects if the door is closed.

The main entrance lock will unlock if: A family member's face is recognized *AND* the door is closed.

Let $Face$ represent the facial recognition result, where $Face$ = T if a family member's face is recognized and $Face$ = F if not.

Let $Door$ represent the door sensor where $Door$ = T if the door is open and $Door$ = F if the door is closed.

Let $Lock$ represent the main entrance lock, where $Lock$ = T if the lock is locked and $Lock$ = F if the lock is not locked.

Questions:

1. Construct a truth table based on the above conditions and symbols.

   Hint: This truth table should have three columns of $Face, $Door$, and $Lock$.

2. Analyze the table to determine under what conditions the door will unlock.

3. As an extension, think of real-world scenarios where it might be beneficial to add more conditions (e.g., time of day, alarm set status) and describe how they might affect the logic of the security system. Explain in plain words, and maybe update your truth table.

Exercise 8 (points = 10)

Questions: Determine whether the following Hoare triple is correct or not:

1. {x = 3 /\ y = 2} `Code` { x = 2 /\ y = 3}, where "Code" refers to the following

```
x := x + y
y := x - y
x := x - y
```

2. {x > 0} `while (x<=10) {x = x + 1}` {x > 10}

1.

Exercise 1

p	**q**	**r**	**p V q → r**	**(p → r) ∧ (q → r)**
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	F	F
F	F	T	T	T
F	F	F	T	T

Since the result rows from each truth table are same, p V q → r and (p → r) ∧ (q → r) are **logically equivalent.**

2.

Exercise 2

p	q	r	$(p \rightarrow q) \wedge (q \rightarrow r) \wedge (r \rightarrow p)$	$p \wedge q \wedge r$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	F	F
F	T	F	F	F
F	F	T	F	F
F	F	F	T	F

Since result rows from truth table in where the p, q and r all F are different each other, $(p \rightarrow q) \wedge (q \rightarrow r) \wedge (r \rightarrow p)$ and $p \wedge q \wedge r$ are **not logically equivalent.**

Exercise 3

p	q	(a) p→q	(b) q→p	(c) ~pVq	(d) ~qVp	(e) ~q→~p	(f) ~p→~q
T	T	T	T	T	T	T	T
T	F	F	T	F	T	F	T
F	T	T	F	T	F	T	F
F	F	T	T	T	T	T	T

1. (c) and (e) are equivalent to (a), because the truth table low from p→q is same as truth table rows that (c) and (e) have.
2. (d) and (f) are equivalent to (b), because the truth table low from q→p is same as truth table rows that (d) and (f) have.

Exercise 4

P	Q	~P∧(Q→P)
T	T	F
T	F	F
F	T	F
F	F	T

We can conclude that if ~P∧(Q→P) is true, P and Q will always be F for both of them based on the truth table because table shows only one true in a row that ~P∧(Q→P) has which was P and Q were both False.

Exercise 5

1. (~pVq)V(p∧~q)

p	q	(~pVq)V(p∧~q)
T	T	T
T	F	T
F	T	T
F	F	T

Since the values in the truth table are all trues, (~pVq)V(p∧~q) is a **tautology.**

2. (p∧~q)∧(~pVq)

p	q	(p∧~q)∧(~pVq)
T	T	F
T	F	F
F	T	F
F	F	F

Since the values in the truth table are all falses, (~pVq)V(p∧~q) is a **contradiction.**

3. (p∧q)V(~pV(p∧~q))

p	q	(p∧q)V(~pV(p∧~q))
T	T	T
T	F	T
F	T	T
F	F	T

Since the values in the truth table are all trues, (p∧q)V(~pV(p∧~q)) is a **tautology.**

Exercise 6

p	q	r	p V (q → r)	(p → r) ∧ (q → r)
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	T	T
F	F	F	T	T

Since the result rows from each truth table are different each other, p V (q → r) and (p → r) ∧ (q → r) are **not logically equivalent.**

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Exercise 7 $\sim F \wedge \sim D$

(1)

$ Face $	$ Door $	$ Lock $
T	T	F
T	F	T
F	T	F
F	F	F

(2)

Under conditions that facial recognition is true, and door is closed.

(3) Let's say $ Time added to the conditions.

$ Face $	$ Door $	$ Time $	$ Lock $
T	T	T	F
T	T	F	F
T	F	T	T ✓
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

* The time will be True if time T is $6:00 \leq T \leq 6:00$

It will be unlocked only if

- face is recognized
- door is closed
- Time is $6:00 \leq T \leq 6:00$

8.1 Correct
8.2 Correct