CSE215
Foundations of Computer Science

State University of New York, Korea

News:

Cardinality will not be a part of this final

But it is still an essential part for your education

The mock exam is designed to help you review the topics.

It may differ significantly from the real exam.

While we will discuss the solutions to the mock exam, they will not be provided in a copy-paste format.

Part 1. Propositions and Predicates (Points = 24)

Please select a single answer for multi-choice questions.

(1) We write $\mathbb{N}$ for the set of natural numbers, namely, integers starting from 1. Which one of the following propositions is false?

- A. For any n $\in \mathbb{N}$, there exists an m $\in \mathbb{N}$ such that m = n * 7.
- B. There exists an m $\in \mathbb{N}$ such that for any n $\in \mathbb{N}$, m = n * 7.
- C. For any x $\in \mathbb{R}$, if $x^7 + 5x^5 > 0$, then $x > 0$.
- D. There exist two positive irrational numbers a and b, such that a * b is rational.

(2) A function $f : \mathbb{R} \to \mathbb{R}$ is said to be *uniformly continuous* on a set $\mathbb{R} \subseteq \mathbb{R}$ if for every $\epsilon > 0$, there exists a $\delta > 0$ such that for all $x, y \in \mathbb{R}$, if $|x - y| < \delta$, then $|f(x) - f(y)| < \epsilon$. Which of the below corresponds to describing a function being *not* uniformly continuous.

- A. For every $\epsilon > 0$, there exists a $\delta > 0$ such that for all $x, y \in \mathbb{R}$, if $|x - y| < \delta$, then $|f(x) - f(y)| \geq \epsilon$.
- B. There exists $\epsilon > 0$, for all $\delta > 0$, there exists $x, y \in \mathbb{R}$, $|x - y| < \delta$ and $|f(x) - f(y)| \geq \epsilon$.
- C. There exists $\epsilon > 0$, for all $\delta > 0$, there exists $x, y \in \mathbb{R}$, if $|x - y| < \delta$, then $|f(x) - f(y)| \geq \epsilon$.
- D. There exists $\epsilon > 0$, for all $\delta > 0$, there exists $x, y \in \mathbb{R}$, if $|x - y| \geq \delta$, then $|f(x) - f(y)| \geq \epsilon$.

(3) Which of the following is a tautology?

- A. $p \wedge (p \to q)$
- B. $(p \to q) \vee (q \to p)$
- C. $(p \to q) \wedge (q \to p)$
- D. $(p \to q) \wedge (\sim p \to \sim q)$

(4) Consider the argument: "I loves pizza. Thus, everyone love pizza." Which of the following is correct?

- A. The argument is invalid.
- B. The argument is valid based on Specialization.
- C. The argument is valid based on Conjunction.
- D. The argument is valid based on Generalization.

(5) Consider the argument: "If it rains, the ground will be wet. It does not rain. Therefore, the ground will not be wet." Which of the following is correct?

- A. The argument is invalid.
- B. The argument is valid based on Modus tollens.
- C. The argument is valid based on Modus ponens.
- D. The argument is valid based on Specialization.

(6) Which of the following statements about logical equivalence is false?

- A. The formulas $p \wedge (p \vee q)$ and p are equivalent.
- B. The formulas $p \vee (p \wedge q)$ and $p \rightarrow q$ are equivalent.
- C. The formulas $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are equivalent.
- D. The formulas $p \rightarrow (q \wedge \neg q)$ and $\neg p$ are equivalent.

Part 2. Proof Concepts and Sequences (Points = 16)

Please select a single answer for multi-choice questions.

(7) When we use proof by contrapositive to prove "if n^3 is odd, then n is odd", we prove its equivalent form konwn as contrapositive. What is the contrapositive of this statement?

- A. If n^3 is not odd, then n is not odd.
- B. If n^3 is odd, then n is not odd.
- C. If n is not odd, then n^3 is not odd.
- D. If n^3 is even, then n is even.

(8) Consider the statement: There exisits integers m and n such that $3m + 6n = 12$. What will be a correct procedure of proving it?

- A. Assume that there exist integers m and n such that $3m + 6n = 12$. Then show the assumption is true.
- B. Assume that there exist integers m and n such that $3m + 6n = 12$. Then show the assumption is false.
- C. Find two specific integers m and n such that $3m + 6n = 12$.
- D. Prove that for any integers m and n, $3m + 6n = 12$ cannot be true.

(9) Consider the recursive sequence: $f_0 = 2$ and $f_k = f_{k-1} + 2$ for all integers $k \geq 1$. Based on this definition, what is the explicit form of the sequence?

- A. $f_k = 2 * k$
- B. $f_k = 2^k$
- C. $f_k = 2 * (k - 1)$
- D. $f_k = 2 * (k + 1)$

(10) Consider the proposition: The multiplcation of any four consecutive integers is sa multiple of 4. Namely, $\forall n \in \mathbb{Z}, 4|n(n+1)(n+2)(n+3)$. Which of the following options uses proof by dividing into cases to prove this proposition?

- A. Prove $4|n(n+1)(n+2)(n+3)$ assuming n is odd.
- B. Prove $4|n(n+1)(n+2)(n+3)$ assuming n is even.
- C. Divide into two cases: n is even and n is odd, and show $4|n(n+1)(n+2)(n+3)$ in both cases.
- D. Divide into two cases: n is even and n is odd, and show $4|n(n+1)(n+2)(n+3)$ holds in if either of the two cases occurs.

Part 3. Sets (Points = 12)

Please select a single answer for multi-choice questions.

(11) Which of the following propositions regarding sets is true:

- A. The set $\{x \in \mathbb{R} | x^5 + 1 = 0\}$ is a subset of $\{x \in \mathbb{R} | x^4 - 1 = 0\}$.
- B. For any sets A and B, $(A \cup B) - B = A$.
- C. There exists sets A, B, and C such that $(A \cup B) \cap C \neq A \cup (B \cap C)$.
- D. Consider sets $A = \{1, 3, 4, 5\}$ and $B = \{2, 3, 4, 5\}$. Then the powerset of $(A - B) \times (B - A)$ contains four elements.

(12) Which of the following statementd is true?

- A. The empty set has no elements. Thus, it equals $\{0\}$.
- B. The powerset of the empty set is an empty set.
- C. The powerset of the empty set includes two element.
- D. The intersection of any set with the empty set is an empty set.

(13) In a city, 60% of the population travels by car, 50% by bus, and 20% use both modes of transport. What percentage of the population uses neither car nor bus?

- A. 10%
- B. 20%
- C. 30%
- D. 40%

Part 4. Functions and Relations(Points = 28)

Please select a single answer for multi-choice questions.

(14) What is the result of $f \circ f$ for the function $f : \mathbb{R}^2 \to \mathbb{R}^2$, defined by $f(x, y) = (xy^2, x^3)$?

- A. $(f \circ f)(x, y) = (x^4y^2, x^3)$
- B. $(f \circ f)(x, y) = (x^7y^2, x^3y^6)$
- C. $(f \circ f)(x, y) = (x^7y^2, x^6)$
- D. $(f \circ f)(x, y) = (x^4y, x^3)$

(15) Consider the sets $A = \mathbb{R}$ and $B = (-\infty, 0)$, namely, the interval of reals from $-\infty$ to 0. Which one of the following functions is a bijection from A to B?

- A. $f(x) = - |x|
- B. $f(x) = -e^x$
- C. $f(x) = -x^2$
- D. $f(x) = -\log(|x|)$

(16) The ReLU (Rectified Linear Unit) function is widely used in machine learning for adding non-slinearity in the constructed models. The function $ReLu : \mathbb{R} \to \mathbb{R}$ is explicitly defined as $ReLu(x) = \max(0, x)$. In other words,

- $ReLu(x) = 0$ if $x \leq 0$
- $ReLu(x) = x$ if $x > 0$

Considering its definition, determine whether the ReLU function is injective (one-to-one) and/or surjective (onto).

- A. ReLu is injective but not surjective.
- B. ReLu is surjective but not injective.
- C. ReLu is neither injective nor surjective.
- D. ReLu is both injective and surjective.

(17) Consider the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ defined as $f(m, n) = 4n + 6m$

- A. f is injective but not surjective.
- B. f is not injective but surjective
- C. f is neither injective nor surjective.
- D. f is both injective and surjective.

(18) Consider the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ defined as $f(m, n) = 4n + 7m$.

- A. f is injective but not surjective.
- B. f is surjective but not injective.
- C. f is neither injective nor surjective.
- D. f is both injective and surjective.

(19) In set theory, we say two sets have the same *cardinality* if there is a bijective function (one-to-one and onto mapping) between them. For example, the sets $\{1, 2, 3\}$ and $\{a, b, c\}$ have the same cardinality because we can establish a bijective function such as $f(1) = a$, $f(2) = b$, and $f(3) = c$. The set of even numbers E and odd numbers O have the same cardinality since we can establish a bijective function $f : E \to O$ as $f(n) = n + 1$.

Now, consider the two sets $S = \{x \in \mathbb{R} \mid 0 < x < 4\}$ and $U = \{x \in \mathbb{R} \mid 0 < x < 8\}$.

Which of the following statements is true regarding the cardinality of sets S and U?

- A. S and U have the same cardinality because one can establish a bijective function from S to U: $f(x) = 2x$.
- B. S and U have the same cardinality because one can establish a bijective function from S to U: $f(x) = x + 4$.
- C. S and U do not have the same cardinality since $S \subseteq U$.
- D. S and U do not have the same cardinality since there is no way to establish a bijective function between them.

(20) Let $S = \{a, b, c, d\}$ and consider the relation R on S defined by the set of ordered pairs $R = \{(a, a), (b, b), (c, c), (d, d), (a, b), (b, a), (c, d), (d, c)\}$. Which of the following statements about R is true?

- A. R is reflexive, symmetric, and transitive.
- B. R is reflexive and symmetric, but not transitive.
- C. R is reflexive and transitive, but not symmetric.
- D. R is symmetric and transitive, but not reflexive.

Part 5: Proof (points = 20)

(21) Prove that there exist integers a and b such that $4a + 10b = 32$.

(22) Prove that $24|(5^{2n} - 1)$ for every integer $n \geq 1$.