

Exercise 1 (points = 40)

1. How many elements are there in the powerset of $\{1, 2, 3, 42\}$? Your answer must be an integer.
2. Let $A = \{1, 2, 3\}$, $B = \{x, y\}$, $C = \{\text{"hello"}, \text{"world"}\}$. How many elements are there in $A \times B \times C$? Your answer must be an integer.
3. Which of the following is correct? **Choose a single correct answer.**
 - (a) $\{0\} = \emptyset$.
 - (b) $\{1\} \subseteq \mathbb{Z}$.
 - (c) $\{2n - 2 \mid n \in \mathbb{Z}\} \neq \mathbb{Z}$
 - (d) $\{5n + 6m \mid n \in \mathbb{Z}, m \in \mathbb{Z}\} \neq \mathbb{Z}$.
4. Let \mathbb{N} denote the set of natural numbers $\{1, 2, \dots\}$. For each $n \in \mathbb{N}$, let $A_n = \{-2n, 0, 2n\}$. Calculate the following two expressions involving A_n :
 - (a) $\bigcup_{i \in \mathbb{N}} A_i$
 - (b) $\bigcap_{i \in \mathbb{N}} A_i$

Exercise 2 (points = 60)

Use set identities to prove the following set properties, clearly stating which laws you are applying at each step. You can optionally follow the following pattern in writing,

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LHS = ... // by (set identity name)
     = ... // by (set identity name)
     = RHS
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1. $B \cup (\emptyset \cap A) = B$
2. $(A \cap B) \cup (A \cap B') = A$
3. $(A \cap B) \cup (A \cup B')' = B$
4. $A \cap (A \cup B) = A$