# Exercise 1 (points = 60)

Consider the following functions. For each function, determine whether it is injective (one-to-one) and whether it is surjective (onto). An explanation is not needed.

| | Function Definition | Injective (one-to-one) | Surjective (onto) |
|---|---|---|---|
| 1 | $f: \mathbb{R} \to [-1, 1]$, $f(x) = \sin(x)$ | | |
| 2 | $f: \mathbb{R} \to \mathbb{R}$, $f(x) = \sin(x)$ | | |
| 3 | $f: \mathbb{Z} \to \mathbb{Z}$, $f(n) = 2n + 1$ | | |
| 4 | $f: \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$, $f(n) = (2n, n + 3)$ | | |
| 5 | $f: \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$, $f(m, n) = 2n - 4m$ | | |
| 6 | $f: \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$, $f(m, n) = 3n - 4m$ | | |

# Exercise 2 (points = 40)

1. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined by the formula:

$$f(x, y) = (xy, x^3)$$

Find $f \circ f$ explicitly. For example, if $f : \mathbb{Z} \to \mathbb{Z}$ is a cubic function defined as $f(n) = n^3$, then you can answer: $f \circ f$ is a function of type $\mathbb{Z} \to \mathbb{Z}$ defined as $(f \circ f)(n) = n^9$.

2. Consider the functions $f, g : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ defined as $f(m, n) = (mn, m^2)$ and $g(m, n) = (m + 1, m + n)$. Find (1) $g \circ f$ and (2) $f \circ g$.

*Hint: If you are confused, just follow the definition of function composition, that is* $f \circ g(a) = f(g(a))$. *Here "a" can be a 2-D element such as in this exercise.*