General Grading Criteria

- Grading will be *highly strict*, with little tolerance for mistakes.
- Factual errors in grading will be corrected upon review, but partial grading will not be reconsidered.
- If a submission is plagiarized or appears AI-generated, the issue will be reported to the instructor for further investigation.

Grading Criteria for Proof Questions

Critical errors that will result in point deductions include:

- Unclear reasoning due to ambiguous language, broken English, or lack of explanation.
- Missing assumptions necessary for a valid proof.
- Logical errors or invalid arguments that do not follow from the premises.
- Confusion in the proof structure, such as misidentifying the proof target, assumptions, or key facts.

Exercise 1 (points = 25)

Fermat's Last Theorem states that no three positive integers a, b, and c satisfy the equation $a^n + b^n = c^n$ for any integer value of n greater than 2.

Now, prove:

- The cube root of 2 is irrational. (The cube root of a number a is a number b such that b*b*b=a.)

This statement can be proven with either Fermat's Last Theorem or without it.

Writing

You can optionally follow the template below when writing your proof using mathematical induction.

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Proof.
Let P(n) denote ______. We want to prove P(n) is true for ______
(something like "n>=0").
We use mathematical induction to proceed.
Base Step: We want to prove _______.
________ (put your proof of the above goal in the base step here)

Inductive Step: We want to prove ______.
________ (put your proof of the above goal in the inductive step here)
QED.
```

You can get full credit if the underlined parts above are correctly and clearly stated in your proof.

Exercise 2 (25 points)

Use mathematical induction to prove that for any integer $n \geq 1$, the expression

$$3^{2n+2} - 8n - 9$$

is divisible by 64.

Exercise 3 (25 points)

Suppose p is a real number satisfying $p > -1$. Prove

$$(1+p)^n \geq 1 + np$$

for any integer $n \geq 1$.

Exercise 4 (25 points)

The triangle inequality states that for all real numbers a and b, |a + b| <= |a| + |b|. Use the triangle inequality and mathematical induction to prove:

For any n real numbers a_1, a_2, ..., and a_n,

$$|\Sigma_{i=1}^{n} a_i| \leq \Sigma_{i=1}^{n} |a_i|$$