

CSE215

Foundations of Computer Science

State University of New York, Korea

**Propositional
Logic**

**Predicate
Logic**

Proof

CSE215: Foundations of Computer Science

Sequences

Sets

Functions

Relations

Today's objectives

Know a list of **key things**
that will be covered in the
exams

Today's work

book chapter	Topics	Exam problems
2	Propositional logic	2021-final, pb 1
3	Predicate logic	2021-midterm1, pb3
4	Proof	2021-final, pb4
5	Sequences	2021-final, pb7
6	Sets	2021-midterm2, pb2
7	Functions	2021-final, pb9
8	Relations	2021-final, pb11

How we proceed next:

- We first go over the exam problems, emphasizing “key” concepts.
- Expectation: get an intuition. Do not expect to understand all.

Propositional Logic

Final 2021

Problem 1. [5 points]

Construct a truth table for the following statement form: $p \wedge (q \vee r) \leftrightarrow p \wedge (q \wedge r)$.

Key: Truth Table

Truth table for $p \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Predicate Logic — Midterm 1, 2021

Problem 3. [10 points]

Give negations of the following statements. Reasoning is not required.

(f) [1 point] $\forall x, \forall y$ such that $p(x, y)$

(g) [1 point] $\forall x, \exists y$ such that $p(x, y)$

Key: Negation on quantifiers

$$\blacksquare \sim(\forall \mathbf{x}, P(\mathbf{x})) \equiv \exists \mathbf{x}, \sim P(\mathbf{x})$$

$$\blacksquare \sim(\exists \mathbf{x}, P(\mathbf{x})) \equiv \forall \mathbf{x}, \sim P(\mathbf{x})$$

Proof — Final 2021

Problem 4. [5 points]

Prove that the sum of the squares of any two consecutive odd integers is even.

Key: Prove things about integers from basic facts

Example of basic facts: an even integer can be written as $2n$; or $(x+y)^2 = x^2 + 2xy + y^2$

Sequences - Final 2021

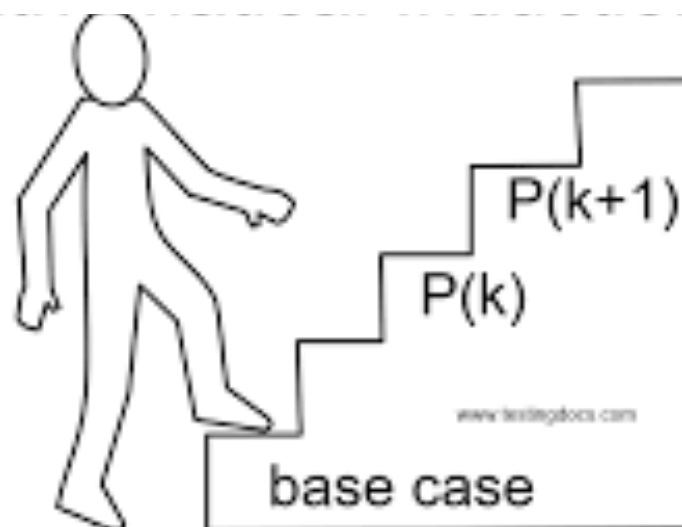
Problem 7. [10 points]

Use mathematical induction to prove the following identities.

(a) [5 points] For all integers $n \geq 1$,

$$\sum_{i=1}^n i(i!) = (n+1)! - 1.$$

Key: Use Mathematical Induction to show facts about integers



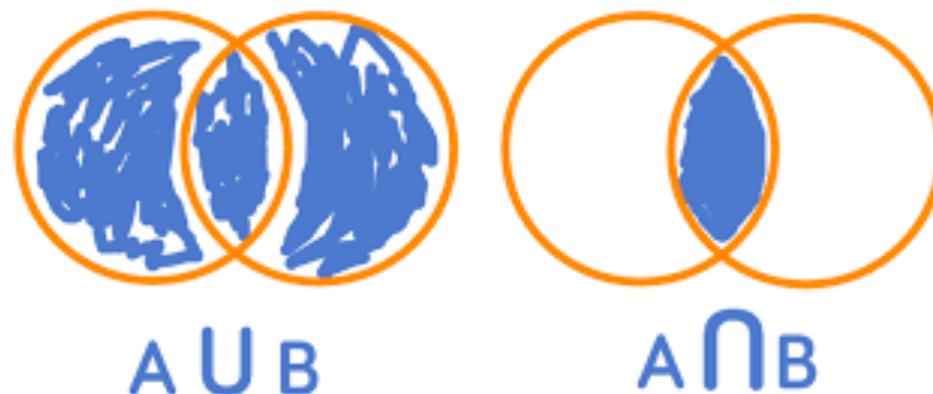
Sets — Midterm 2, 2021

Problem 2. [5 points]

Mention whether the following statements are true or false without giving any reasons. Assume all sets are subsets of a universal set U .

(a) [1 point] $(A \cap B) \cap (A \cap C) = A \cap (B \cup C)$

Key: Union and intersection on Sets



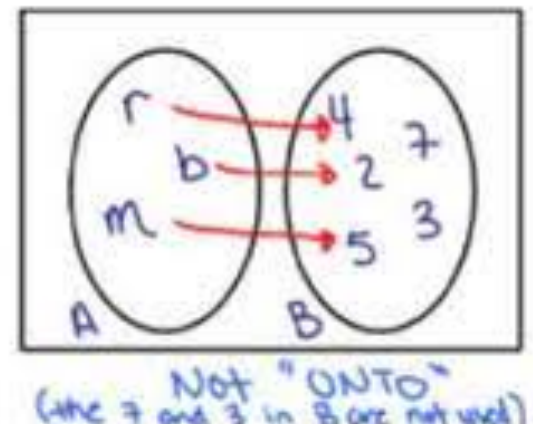
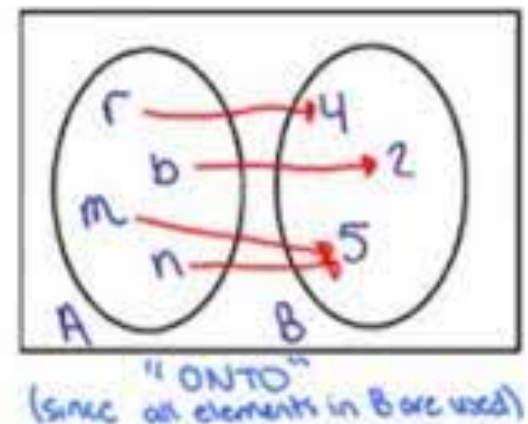
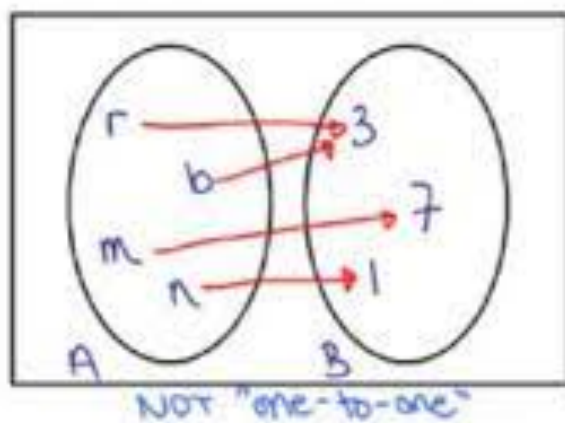
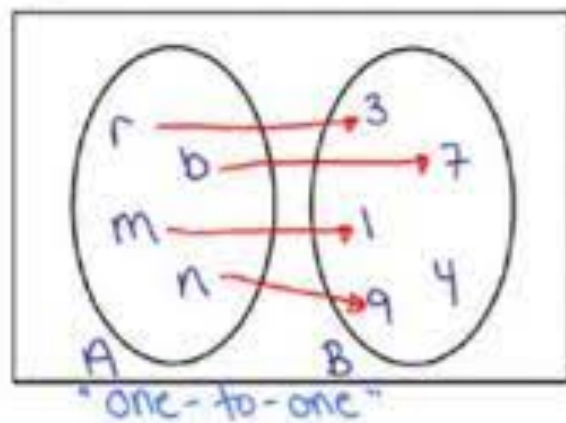
Functions — Final 2021

Problem 9. [5 points]

Write and fill the table with \checkmark or \times . If a function is one-to-one or onto, then use \checkmark . On the other hand, if a function is not one-to-one or not onto, then use \times .

Function	Domains	One-to-one function?	Onto function?
$f(x) = 3x$	$f : \mathbb{Z} \rightarrow \mathbb{Z}$		

Key: One-to-one and onto functions



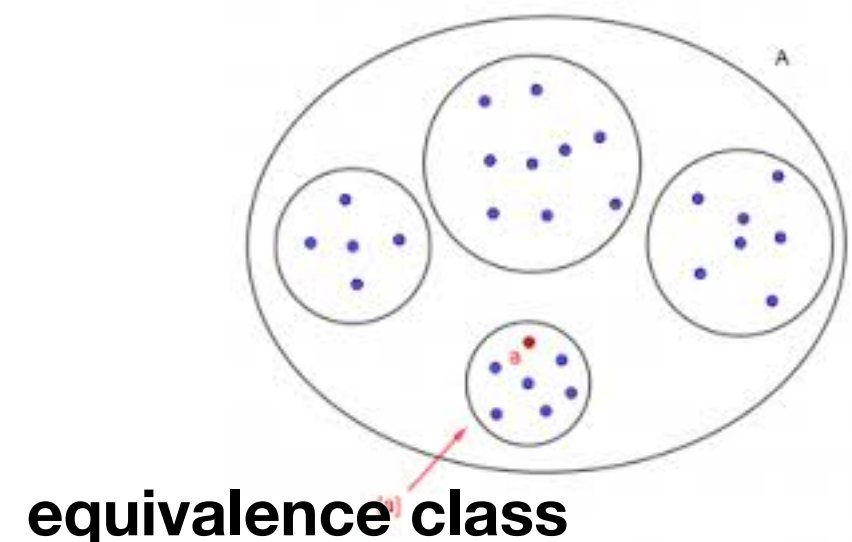
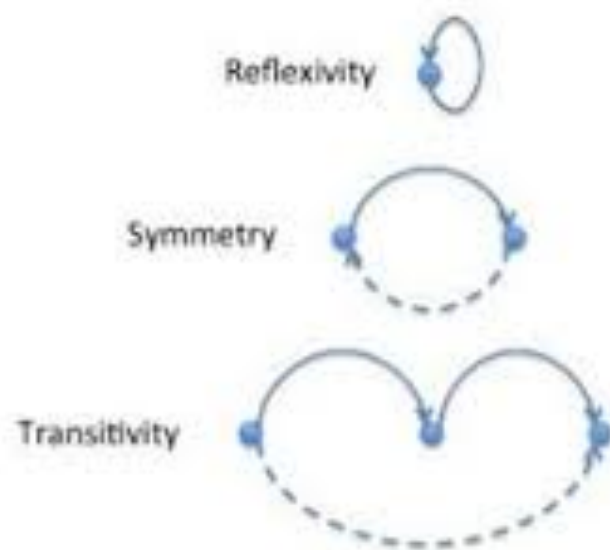
Relations - Final 2021

Problem 11. [5 points]

Let A be the set of all people. Let R be the relation defined on A as follows: For persons p and q in A , we have $p R q \Leftrightarrow p$ has the same birthday as q .

Is R an equivalence relation? Prove your answer. If R is an equivalence relation, what are the distinct equivalence classes of the relation?

**Key: Equivalence relations and
Equivalence classes**



Today's take-away

book chapter	Topics	Exam problems	Key
2	Propositional logic	2021-final, pb 1	truth table
3	Predicate logic	2021-midterm1, pb3	negation on quantifiers
4	Proof	2021-final, pb4	facts about integers
5	Sequences	2021-final, pb7	math induction
6	Sets	2021-midterm2, pb2	unions and intersections
7	Functions	2021-final, pb9	1-1 and onto
8	Relations	2021-final, pb11	equiv. rel. and classes

Thank you for your attention!