CSE215
Foundations of Computer Science

State University of New York, Korea

News:

Cardinality will not be a part of this final

But it is still an essential part for your education

Part 1. Propositions and Predicates (Points = 24)

Please select a single answer for multi-choice questions.

(1) We write $\mathbb{N}$ for the set of natural numbers, namely, integers starting from 1. Which one of the following propositions is false?

- A. For any n $\in \mathbb{N}$, there exists an m $\in \mathbb{N}$ such that m = n * 7.
- B. There exists an m $\in \mathbb{N}$ such that for any n $\in \mathbb{N}$, m = n * 7.
- C. For any x $\in \mathbb{R}$, if $x^7 + 5x^5 > 0$, then $x > 0$.
- D. There exist two positive irrational numbers a and b, such that a * b is rational.

(2) A function $f : \mathbb{R} \to \mathbb{R}$ is said to be *uniformly continuous* on a set $\mathbb{R} \subseteq \mathbb{R}$ if for every $\epsilon > 0$, there exists a $\delta > 0$ such that for all $x, y \in \mathbb{R}$, if $|x - y| < \delta$, then $|f(x) - f(y)| < \epsilon$. Which of the below corresponds to describing a function being *not* uniformly continuous.

- A. For every $\epsilon > 0$, there exists a $\delta > 0$ such that for all $x, y \in \mathbb{R}$, if $|x - y| < \delta$, then $|f(x) - f(y)| \geq \epsilon$.
- B. There exists $\epsilon > 0$, for all $\delta > 0$, there exists $x, y \in \mathbb{R}$, $|x - y| < \delta$ and $|f(x) - f(y)| \geq \epsilon$.
- C. There exists $\epsilon > 0$, for all $\delta > 0$, there exists $x, y \in \mathbb{R}$, if $|x - y| < \delta$, then $|f(x) - f(y)| \geq \epsilon$.
- D. There exists $\epsilon > 0$, for all $\delta > 0$, there exists $x, y \in \mathbb{R}$, if $|x - y| \geq \delta$, then $|f(x) - f(y)| \geq \epsilon$.

(3) Which of the following is a tautology?

- A. $p \wedge (p \rightarrow q)$
- B. $(p \rightarrow q) \vee (q \rightarrow p)$
- C. $(p \rightarrow q) \wedge (q \rightarrow p)$
- D. $(p \rightarrow q) \wedge (\sim p \rightarrow \sim q)$

(4) Consider the argument: "I loves pizza. Thus, everyone love pizza." Which of the following is correct?

- A. The argument is invalid.
- B. The argument is valid based on Specialization.
- C. The argument is valid based on Conjunction.
- D. The argument is valid based on Generalization.

(5) Consider the argument: "If it rains, the ground will be wet. It does not rain. Therefore, the ground will not be wet." Which of the following is correct?

- A. The argument is invalid.
- B. The argument is valid based on Modus tollens.
- C. The argument is valid based on Modus ponens.
- D. The argument is valid based on Specialization.

(6) Which of the following statements about logical equivalence is false?

- A. The formulas $p \wedge (p \vee q)$ and p are equivalent.
- B. The formulas $p \vee (p \wedge q)$ and $p \rightarrow q$ are equivalent.
- C. The formulas $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are equivalent.
- D. The formulas $p \rightarrow (q \wedge \neg q)$ and $\neg p$ are equivalent.

Part 2. Proof Concepts and Sequences (Points = 16)

Please select a single answer for multi-choice questions.

(7) When we use proof by contrapositive to prove "if n^3 is odd, then n is odd", we prove its equivalent form konwn as contrapositive. What is the contrapositive of this statement?

- A. If n^3 is not odd, then n is not odd.
- B. If n^3 is odd, then n is not odd.
- C. If n is not odd, then n^3 is not odd.
- D. If n^3 is even, then n is even.

(8) Consider the statement: There exisits integers m and n such that $3m + 6n = 12$. What will be a correct procedure of proving it?

- A. Assume that there exist integers m and n such that $3m + 6n = 12$. Then show the assumption is true.
- B. Assume that there exist integers m and n such that $3m + 6n = 12$. Then show the assumption is false.
- C. Find two specific integers m and n such that $3m + 6n = 12$.
- D. Prove that for any integers m and n, $3m + 6n = 12$ cannot be true.

(9) Consider the recursive sequence: $f_0 = 2$ and $f_k = f_{k-1} + 2$ for all integers $k \geq 1$. Based on this definition, what is the explicit form of the sequence?

- A. $f_k = 2 * k$
- B. $f_k = 2^k$
- C. $f_k = 2 * (k - 1)$
- D. $f_k = 2 * (k + 1)$

(10) Consider the proposition: The multiplcation of any four consecutive integers is sa multiple of 4. Namely, $\forall n \in \mathbb{Z}, 4|n(n+1)(n+2)(n+3)$. Which of the following options uses proof by dividing into cases to prove this proposition?

- A. Prove $4|n(n+1)(n+2)(n+3)$ assuming n is odd.
- B. Prove $4|n(n+1)(n+2)(n+3)$ assuming n is even.
- C. Divide into two cases: n is even and n is odd, and show $4|n(n+1)(n+2)(n+3)$ in both cases.
- D. Divide into two cases: n is even and n is odd, and show $4|n(n+1)(n+2)(n+3)$ holds in if either of the two cases occurs.

Part 3. Sets (Points = 12)

Please select a single answer for multi-choice questions.

(11) Which of the following propositions regarding sets is true:

- A. The set $\{x \in \mathbb{R} | x^5 + 1 = 0\}$ is a subset of $\{x \in \mathbb{R} | x^4 - 1 = 0\}$.
- B. For any sets A and B, $(A \cup B) - B = A$.
- C. There exists sets A, B, and C such that $(A \cup B) \cap C \neq A \cup (B \cap C)$.
- D. Consider sets $A = \{1, 3, 4, 5\}$ and $B = \{2, 3, 4, 5\}$. Then the powerset of $(A - B) \times (B - A)$ contains four elements.

(12) Which of the following statementd is true?

- A. The empty set has no elements. Thus, it equals $\{0\}$.
- B. The powerset of the empty set is an empty set.
- C. The powerset of the empty set includes two element.
- D. The intersection of any set with the empty set is an empty set.

(13) In a city, 60% of the population travels by car, 50% by bus, and 20% use both modes of transport. What percentage of the population uses neither car nor bus?

- A. 10%
- B. 20%
- C. 30%
- D. 40%

Part 4. Functions and Relations(Points = 28)

Please select a single answer for multi-choice questions.

(14) What is the result of $f \circ f$ for the function $f : \mathbb{R}^2 \to \mathbb{R}^2$, defined by $f(x, y) = (xy^2, x^3)$?

- A. $(f \circ f)(x, y) = (x^4y^2, x^3)$
- B. $(f \circ f)(x, y) = (x^7y^2, x^3y^6)$
- C. $(f \circ f)(x, y) = (x^7y^2, x^6)$
- D. $(f \circ f)(x, y) = (x^4y, x^3)$

(15) Consider the sets $A = \mathbb{R}$ and $B = (-\infty, 0)$, namely, the interval of reals from $-\infty$ to 0. Which one of the following functions is a bijection from A to B?

- A. $f(x) = - |x|
- B. $f(x) = -e^x$
- C. $f(x) = -x^2$
- D. $f(x) = -\log(|x|)$

(16) The ReLU (Rectified Linear Unit) function is widely used in machine learning for adding non-slinearity in the constructed models. The function $ReLu : \mathbb{R} \to \mathbb{R}$ is explicitly defined as $ReLu(x) = \max(0, x)$. In other words,

- $ReLu(x) = 0$ if $x \leq 0$
- $ReLu(x) = x$ if $x > 0$

Considering its definition, determine whether the ReLU function is injective (one-to-one) and/or surjective (onto).

- A. ReLu is injective but not surjective.
- B. ReLu is surjective but not injective.
- C. ReLu is neither injective nor surjective.
- D. ReLu is both injective and surjective.

(17) Consider the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ defined as $f(m, n) = 4n + 6m$

- A. f is injective but not surjective.
- B. f is not injective but surjective
- C. f is neither injective nor surjective.
- D. f is both injective and surjective.

(18) Consider the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ defined as $f(m, n) = 4n + 7m$.

- A. f is injective but not surjective.
- B. f is surjective but not injective.
- C. f is neither injective nor surjective.
- D. f is both injective and surjective.

(19) In set theory, we say two sets have the same *cardinality* if there is a bijective function (one-to-one and onto mapping) between them. For example, the sets $\{1, 2, 3\}$ and $\{a, b, c\}$ have the same cardinality because we can establish a bijective function such as $f(1) = a$, $f(2) = b$, and $f(3) = c$. The set of even numbers E and odd numbers O have the same cardinality since we can establish a bijective function $f : E \to O$ as $f(n) = n + 1$.

Now, consider the two sets $S = \{x \in \mathbb{R} \mid 0 < x < 4\}$ and $U = \{x \in \mathbb{R} \mid 0 < x < 8\}$.

Which of the following statements is true regarding the cardinality of sets S and U?

- A. S and U have the same cardinality because one can establish a bijective function from S to U: $f(x) = 2x$.
- B. S and U have the same cardinality because one can establish a bijective function from S to U: $f(x) = x + 4$.
- C. S and U do not have the same cardinality since $S \subseteq U$.
- D. S and U do not have the same cardinality since there is no way to establish a bijective function between them.

(20) Let $S = \{a, b, c, d\}$ and consider the relation R on S defined by the set of ordered pairs $R = \{(a, a), (b, b), (c, c), (d, d), (a, b), (b, a), (c, d), (d, c)\}$. Which of the following statements about R is true?

- A. R is reflexive, symmetric, and transitive.
- B. R is reflexive and symmetric, but not transitive.
- C. R is reflexive and transitive, but not symmetric.
- D. R is symmetric and transitive, but not reflexive.

Part 5: Proof (points = 20)

(21) Prove that there exist integers a and b such that $4a + 10b = 32$.

(22) Prove that $24|(5^{2n} - 1)$ for every integer $n \geq 1$.