CSE215
Foundations of Computer Science

State University of New York, Korea

Review
Infinite union/intersection

Notations

$$\bigcup_{i=1}^{\infty} A_i \;=\; A_1 \cup A_2 \cup A_3 \cup \cdots \;=\; \{x : x \in A_i \text{ for at least one set } A_i \text{ with } 1 \le i\}.$$

$$\bigcap_{i=1}^{\infty} A_i \;=\; A_1 \cap A_2 \cap A_3 \cap \cdots \;=\; \{x : x \in A_i \text{ for every set } A_i \text{ with } 1 \le i\}.$$

Example

$$A_1 = \{-1,0,1\},\quad A_2 = \{-2,0,2\},\quad A_3 = \{-3,0,3\},\quad \cdots,\quad A_i = \{-i,0,i\},\quad \cdots$$

Observe that $\displaystyle\bigcup_{i=1}^{\infty} A_i = \mathbb{Z}$, **and** $\displaystyle\bigcap_{i=1}^{\infty} A_i = \{0\}$.

Extended Notation

Here is a useful twist on our new notation. We can write

$$\bigcup_{i=1}^{3} A_i \quad = \quad \bigcup_{i\in\{1,2,3\}} A_i,$$

as this takes the union of the sets A_i for $i = 1, 2, 3$. Likewise:

$$\begin{aligned}
\bigcap_{i=1}^{3} A_i &= \bigcap_{i\in\{1,2,3\}} A_i \\
\bigcup_{i=1}^{\infty} A_i &= \bigcup_{i\in\mathbb{N}} A_i \\
\bigcap_{i=1}^{\infty} A_i &= \bigcap_{i\in\mathbb{N}} A_i
\end{aligned}$$

Extended Notations

Here we are taking the union or intersection of a collection of sets A_i where i is an element of some set, be it $\{1,2,3\}$ or $\mathbb{N}$. In general, the way this works is that we will have a collection of sets A_i for $i \in I$, where I is the set of possible subscripts. The set I is called an **index set**.

If A_α is a set for every α in some index set I, then

$$\begin{aligned}
\bigcup_{\alpha \in I} A_\alpha &= \{x : x \in A_\alpha \text{ for at least one set } A_\alpha \text{ with } \alpha \in I\} \\
\bigcap_{\alpha \in I} A_\alpha &= \{x : x \in A_\alpha \text{ for every set } A_\alpha \text{ with } \alpha \in I\}.
\end{aligned}$$

Exercise 1

(a) $\bigcup_{i \in \mathbb{N}} [0, i+1] =$

(b) $\bigcap_{i \in \mathbb{N}} [0, i+1] =$

Solution

- [0, infinity)
- [0, 2]

Review
Questions on A = B?

Exercises

- Let Z be the set of integers
- Let A be the set of {7a + 8b | a \in Z and b \in Z}
- Is Z = A?
- No need to write a proof/disproof for Z=A for

Solution

- Proof.
  - A is clearly a subset of Z
  - So we only need to prove Z is a subset of A
  - Suppose n is arbitrary element of Z, n can be written as 7 (-n) + 8 (n), Therefore Z is a subset of A
- QED.

Exercises - True or False

- $Z = \{7a + 3b \mid a \in Z \text{ and } b \in Z\}$
- $Z = \{7a + 2b \mid a \in Z \text{ and } b \in Z\}$
- $Z = \{8a + 3b \mid a \in Z \text{ and } b \in Z\}$
- $Z = \{8a + 4b \mid a \in Z \text{ and } b \in Z\}$

Review
Proof by contradiction

1

Prove that $\sqrt[3]{2}$ is irrational.

2

Prove that $\sqrt{6}$ is irrational.

3

Prove that $\sqrt{3}$ is irrational.

4

Suppose $a, b, c \in \mathbb{Z}$. If $a^2 + b^2 = c^2$, then a or b is even.

5

If a and b are positive real numbers, then $a+b \geq 2\sqrt{ab}$.

6

There exist no integers a and b for which $21a + 30b = 1$.

7

There exist no integers a and b for which $18a + 6b = 1$.

8

For every $x \in [\pi/2, \pi]$, $\sin x - \cos x \geq 1$.

Hint: Use sin(x) ^2 + cos(x) ^2 = 1