Disclaimer

We will dedicate time next week to carefully clarify the concepts. Don't worry!

Exercise 1 (Points = 88)

Show that each pair of the two given sets has equal cardinality by describing an explicit function that is a bijection from one to the other. You do not need to prove the function is bijective. Consider using the template: *"Let f: A -> B be defined as f(x) = ..."*.

[Hint for 1-3: use exponential functions in some way.]

1. $\mathbb{R}$ and $(0, \infty)$
2. $\mathbb{R}$ and $(\sqrt{2}, \infty)$
3. $\mathbb{R}$ and $(0, 1)$
4. $\mathbb{Z}$ and S, where $S = \{\ldots, 1/8, 1/4, 1/2, 1, 2, 4, 8, 16, \ldots\}$
5. $A = \{3k : k \in \mathbb{Z}\}$ and $B = \{7k : k \in \mathbb{Z}\}$
6. $A = \{(5n, -3n) : n \in \mathbb{Z}\}$ and $\mathbb{Z}$
7. The set of even integers and the set of odd integers
8. $\mathbb{Z}$ and S, where $S = \{x \in \mathbb{R} : \sin(x) = 1\}$
9. $\{0, 1\} \times \mathbb{N}$ and $\mathbb{N}$
10. $\mathbb{N}$ and $\mathbb{Z}$
11. $P(\mathbb{N})$ and $P(\mathbb{Z})$, where P refers to the power set

Exercise 3 (Points = 12)

Let $S = \{x \in \mathbb{R} \mid 0 < x < 1\}$.

1. Let $U = \{x \in \mathbb{R} \mid 0 < x < 2\}$. Prove that S and U have the same cardinality.
   Reminder: Two sets S and U are said to have the same cardinality if a bijective function can be produced from S to U (or from U to S). In this exercise, you do not need to prove that your function is bijective. Instead, simply find a function between S and U that you believe to be bijective.

2. Let $V = \{x \in \mathbb{R} \mid 2 < x < 5\}$. Prove that S and V have the same cardinality.