CSE215
Foundations of Computer Science

State University of New York, Korea

1.

Determine if the following statements are true or false. You do not need to explain the reasons.

1. Let $A = \{1, 2, 3, 4, 5\}$ and $\mathbf{Z}$ be the set of integers. Then A is a proper subset of $\mathbf{Z}$.
2. $\{x \in \mathbf{Z} \mid x^3 - 1 = 0\} = \{x \in \mathbf{Z} \mid x^2 - x = 0\}$.
3. Suppose x is a real number. If $x^{100} + 5x^9 < 0$ then $x < 0$.
4. Suppose a and b are two real numbers. If $a - b$ is irrational and a is rational, then b is irrational.

2.

Let $A = \{0, 2, 4\}, B = \{1, 2, 3, 4\}$ and the universal set $U = \{0, 1, 2, 3, 4\}$. Find:

1. A'
2. B'
3. $A \cap A'$
4. $A \cup A'$
5. $A - B$
6. $B - A$
7. $(A - B) \times (B - A)$

3.

1. Prove that for any sets A and B, $A \cup (A' \cap B) = A \cup B$. [Hint: Use set identities]
2. Prove that for any sets A and B, $A - (A - B) = A \cap B$. [Hint: Use set identities]

4.

1. Prove that there are no integers a and b such that $10a + 1 = 6b$.
2. Prove that there exist integers a and b such that $10a + 2 = 6b$.

5.

3. Prove that $9 \mid (4^{3n}+8)$ for every integer $n \geq 0$.

6.

For the following statement, first determine whether it is true or false. If it is true, provide a proof; if it is false, disprove it.

There exist no integers a and b for which $21a + 30b = 1$.

7.

For the following statement, first determine whether it is true or false. If it is true, provide a proof; if it is false, disprove it.

There exist no integers a and b for which $18a + 6b = 1$.

8.

For the following statement, first determine whether it is true or false. If it is true, provide a proof; if it is false, disprove it.

For every $x \in [\pi/2, \pi]$, $\sin x - \cos x \geq 1$.

Hint: Use sin(x) ^2 + cos(x) ^2 = 1