General Grading Criteria

- Grading will be *highly strict*, with little tolerance for mistakes.
- Factual errors in grading will be corrected upon review, but partial grading will not be reconsidered.
- If a submission is plagiarized or appears AI-generated, the issue will be reported to the instructor for further investigation.

Grading Criteria for Proof Questions

Critical errors that will result in point deductions include:

- Unclear reasoning due to ambiguous language, broken English, or lack of explanation.
- Missing assumptions necessary for valid proof.
- Logical errors or invalid arguments that do not follow from the premises.
- Confusion in the proof structure, such as misidentifying the proof target, assumptions, or key facts.

Exercise 1 (points = 30)

Prove: If n is odd, then $8|(n^2-1)$.

[Hint: direct proof]

Exercise 2 (points = 30)

Prove the following using the proof by contradiction strategy.

There does not exist the smallest integer.

Exercise 3 (points = 40)

The quotient remainder theorem says: Given any integer A and a positive integer B, there exist unique integers Q and R such that A = B * Q + R where 0 ≤ R < B. Examples:

- A = 7, B = 2: 7 = 2 * 3 + 1
- A = 8, B = 4: 8 = 4 * 2 + 0
- A = 13, B = 5: 13 = 5 * 2 + 3
- A = -16, B = 26 :-16 = 26 * (-1) + 10

From the quotient-remainder theorem, we know that any integer divided by a positive integer will have a set number of remainders and thus a set number of representations. For example, any integer divided by 4 will produce a remainder between 0 and 3 (inclusive). So every integer n can be represented by one of the 4 forms: 4q, 4q + 1, 4q + 2, 4q + 3 (where q is an integer). Similarly, any integer divided by 3 will produce a remainder between 0 and 2 (inclusive).

Now prove the following proposition.

- The product of any four consecutive integers is a multiple of 8.

Hint: Perhaps you can use the quotient remainder theorem when applying a proof of dividing into cases. You can also prove the proposition without it.