Guideline

Homework for Week 03

Please submit your solutions as a single PDF on Brightspace.

Multiple submissions are allowed before the due time; the last submission will be graded.

Grading Criteria

- Grading will be *highly strict*, with minimal tolerance for mistakes or misconduct.
- Factual grading errors will be corrected, but partial grading decisions will not be negotiable.
- If the solution appears to be plagiarized or AI-generated, the issue will be reported to the instructor.

Exercise 1 (points = 30)

Rewrite the statements below using quantifiers and variables. For example, a statement like "Even numbers are divisible by 2" becomes: "for each even number n, n is divisible by 2", or "for each number n, if n is an even number, then n is divisible by 2". You do not necessarily need to use the exact words or patterns as above but you do need to figure out the hidden quantifiers from the English meaning of each sentence.

1. No two leaves are alike.
2. Even integers equals twice some integer.
3. The sum of two positive integers is a positive number.
4. Everyone loves ice cream.
5. All that glitters is not gold.

Exercise 2 (points = 40)

Determine whether the statements below are true or false. You do not need to explain.

1. 119 is a prime number.
2. 161 is a prime number.
3. 42k is an even number for any integer k.
4. For each integer n with 2 <= n <= 6, n^2 – n + 11 is a prime number.
5. The average of any two odd integers is odd.
6. For any real number x, if x * x >= 4, then x >= 2.
7. For any real numbers x and y, x^2 - 2xy + y^2 >= 0.
8. There exists an integer x, such that $(2x + 1)^2$ is even.

Exercise 3 (points = 30)

Translate each statement into formal logic and then negate them.

Given:

- (P(x)): (x) is a person.
- (R(x, y)): (x) likes (y).

1. Everyone likes at least one person.

2. Some people don't like themselves.