CSE215
Foundations of Computer Science

State University of New York, Korea

Disproof

Disproof

dis·proof | dis'pro͞of |

noun

a set of facts that prove that something is untrue: *the theory also provides a disproof of the principle of closure*.

- **the action of proving that something is untrue:** *considerations that are subject to scientific verification or disproof*.

https://youglish.com/pronounce/%22disprove%22/english/us?

True or False

- If x, y $\in$ R, then |x+y| = |x| + |y|.
- For every natural number n, 2n^2 – 4n + 31 is prime.
- If a,b $\in$ N, then a + b < ab
- Every odd integer is the sum of three odd integers.
- Rational + Irrational = Irrational
- Rational * Irrational = Irrational

Principle of disproving

- to disprove a statement P = to prove that P is false
- to prove that P is false = to prove ~P is true

How to disprove P: Prove $\sim P$.

Question 1:
How to Disprove “for all x, P(x)”?

How to disprove $\forall x \in S, P(x)$.

Produce an example of an $x \in S$ that makes $P(x)$ false.

Question 2:
How to disprove “there exists x, P(x)”?

- To disprove it, we prove its negation
  - $\sim (\exists x \in S, P(x)) = \forall x \in S, \sim P(x)$.

Question 3:
How to disprove “for all x, P(x)-> Q(x)” ?

How to disprove $P(x) \Rightarrow Q(x)$.

Produce an example of an x that makes $P(x)$ true and $Q(x)$ false.

Example: Prove or disprove the following conjecture

Conjecture: For every $n \in \mathbb{Z}$, the integer $f(n) = n^2 - n + 11$ is prime.

n	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
$f(n)$	23	17	13	11	11	13	17	23	31	41	53	67	83	101

Disproof. The statement "*For every $n \in \mathbb{Z}$, the integer $f(n) = n^2 - n + 11$ is prime,*" is **false**. For a counterexample, note that for $n = 11$, the integer $f(11) = 121 = 11 \cdot 11$ is not prime. ■

Break

Exercises

To disprove

- If $x, y \in R$, then $|x+y| = |x| + |y|$.

To disprove

- For every natural number n, the integer $2n^2 – 4n + 31$ is prime.

To disprove

- Rational * Irrational = Irrational

Summary

- How to prove something is false