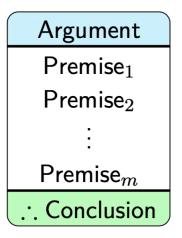
CSE215 Foundations of Computer Science

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Previous lecture



- Valid argument
- Check validity using truth table

Today

Check validity using inference rules

Inference rules

Definition

 A rule of inference is a valid argument form that can be used to establish logical deductions

Modus Ponens

Definition

It has the form:

If p, then q

p

 $\therefore q$

• The term modus ponens in Latin means "method of affirming"

\cline{p}	q	p o q	p	q
Т	Т	Т	Т	Т
Т	F	F	Т	
F	Т	Т	F	
F	F	Т	F	

What is wrong?

- You are a detective investigating a crime scene with your partner,
 Sherlock. You come across a note that says: "If the butler did it, then the knife will be found in the kitchen."
- Sherlock responds, "Well, we just found the knife in the kitchen.
 Therefore, the butler did it."

Modus Tollens

Definition

• It has the form:

If p, then q

 $\sim q$

 $\sim p$

• The term modus tollens in Latin means "method of denying"

$\int p$	q	p o q	$\sim q$	$\sim p$
Т	Т	Т	F	
Т	F	F	Т	
F	Т	Т	F	
F	F	Т	Т	Т

- If it rains, then the ground will be wet."
- The ground is not wet
- ==> ?

Generalization

Definition

• It has the form:

p

 $\therefore p \lor q$

p	q	p	$\boxed{p \lor q}$
Т	Т	Н	Т
Т	F	Η	Т
F	Т	F	
F	F	F	

Example

- 35 is odd.
 - .: (more generally) 35 is odd or 35 is even.

What is wrong?

- a friend who tells you, "I really like pizza." Based on this statement, you can apply the Generalization inference rule to make the following conclusion:
- "All people like pizza."

Specialization

Definition

• It has the form:

$$p \wedge q$$

 $\therefore p$

$\int p$	q	$p \wedge q$	p
Т	Τ	Т	Н
Т	F	F	
F	Т	F	
F	F	F	

Example

- Ana knows numerical analysis and Ana knows graph algorithms.
 - .: (in particular) Ana knows graph algorithms

Conjunction

Definition

• It has the form:

p

 \boldsymbol{q}

 $\therefore p \land q$

for p	q	$p \wedge q$
Т	H	Т
Т	F	
F	Т	
F	F	

Example

• Lily loves mathematics.

Lily loves algorithms.

... Lily loves both mathematics and algorithms.

Elimination

Definition

• It has the form:

$$p \lor q$$

$$\sim q$$

$$\therefore p$$

• Intuition: When you have only two possibilities and you can rule one out, the other must be the case

p	q	$p \lor q$	$\sim q$	$oxed{p}$
Т	Т	Т	F	Τ
Т	F	Т	Т	Т
F	Т	Т	F	F
F	F	F	Т	F

Example

• Suppose x - 3 = 0 or x + 2 = 0.

Also, suppose x is nonnegative.

$$\therefore x = 3.$$

Transitivity

Definition

• It has the form:

$$p \to q$$

$$q \to r$$

$$\therefore p \to r$$

• Can be generalized to a chain with any number of conditionals

Example

- If 18,486 is divisible by 18, then 18,486 is divisible by 9. If 18,486 is divisible by 9, then the sum of the digits of 18,486 is divisible by 9.
 - ... If 18,486 is divisible by 18, then the sum of the digits of 18,486 is divisible by 9.

Division into cases

Definition

• It has the form:

```
\begin{array}{l} p \lor q \\ p \to r \\ q \to r \\ \therefore r \end{array}
```

Example

ullet x is positive or x is negative.

```
If x is positive, then x^2 > 0.
```

If x is negative, then $x^2 > 0$.

$$\therefore x^2 > 0.$$

Summary of Inference rules

Name	Rule	Name	Rule	
Modus Ponens	$p \rightarrow q$	Elimination	$p \lor q$	$p \vee q$
	p		$\sim q$	$\sim p$
	$\therefore q$		$\therefore p$	$\therefore q$
Modus Tollens	$p \to q$	Transitivity	p o q	
	$\sim q$		$q \rightarrow r$	
	$\therefore \sim p$		$\therefore p \rightarrow r$	
Proof by division	$p \lor q$	Generalization	p	q
into cases	$p \rightarrow r$		$\therefore p \lor q$	$\therefore p \lor q$
	$q \rightarrow r$	Specialization	$p \wedge q$	$p \wedge q$
	∴ <i>r</i>		∴ p	$\therefore q$
Conjunction	p	Contradiction	$\sim p \to c$	
	q		$\therefore p$	
	$\therefore p \land q$			

Some wrong inference

Definition

A fallacy is an error in reasoning that results in an invalid argument

Fallacy: Converse error

Definition

• It has the form:

$$p \rightarrow q$$

 \boldsymbol{q}

 $\therefore p$

• Superficially resembles modus ponens but is invalid

p	q	p o q	q	p
Т	Т	Т	H	Н
Т	F	F	F	Т
F	Т	Т	Т	F
F	F	Т	F	F

Example

• If x > 2, then $x^2 > 4$.

$$x^2 > 4$$
.

 $\therefore x > 2$.

Fallacy: Inverse error

Definition

• It has the form:

$$p \rightarrow q$$

$$\sim p$$

$$\therefore \sim q$$

• Superficially resembles modus tollens but is invalid

p	q	p o q	$\sim p$	$\sim q$
Т	Т	Т	F	F
Т	F	F	F	Т
F	Т	Т	Т	F
F	F	Т	Т	Т

Example

• If x > 2, then $x^2 > 4$.

$$x \leq 2$$
.

$$\therefore x^2 \leq 4$$
.

Break;

Exercises

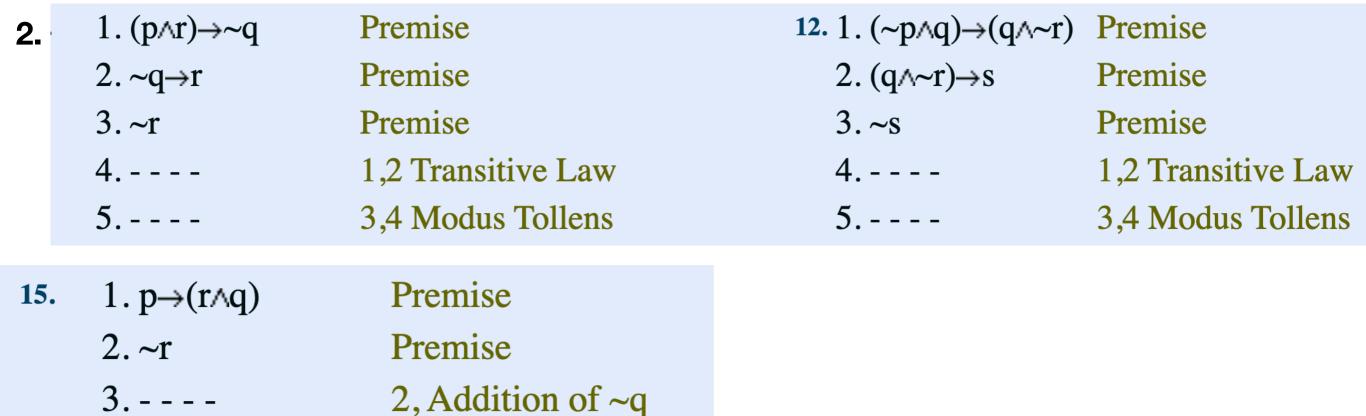
1. Supply the missing statements using inference rules

1.	1. p→~q	Premise	2. 1.~p→q	Premise
	2. p	Premise	2. ~p	Premise
	3	1,2 Modus Ponens	3	1,2 Modus Ponens
3.	1. $(\sim p \vee a) \rightarrow \sim (a \wedge r)$) Premise	4. 1. (~p∧a)→(a∧	r) Premise

3. 1.
$$(\sim p \lor q) \rightarrow \sim (q \land r)$$
 Premise
2. $\sim p \lor q$ Premise
3. - - - 1,2 Modus Ponens
4. 1. $(\sim p \land q) \rightarrow (q \land \sim r)$ Premise
2. $\sim p \land q$ Premise
3. - - - 1,2 Modus Ponens

1.
$$(\sim p \lor q) \rightarrow \sim (q \land r)$$
 Premise
2. $q \land r$ Premise
2. $q \land r$ Premise
2. $\sim (q \land \sim r)$ Premise

9. 1.
$$(p \land r) \rightarrow \sim q$$
 Premise 10. 1. $(\sim p \land q) \rightarrow (q \land \sim r)$ Premise 2. $(q \land \sim r) \rightarrow s$ Premise 3. - - - 1,2 Transitive Law 3. - - - 1,2 Transitive Law



17.	1. (p∧q)→r	Premise	18. 1. $p\rightarrow r$	Premise
	2. q	Premise	2. p	Premise
	3. p	Premise	3. s	Premise
	4	3,2 Rule C	4	1,2 Modus Ponens
	5	1,4 Modus Ponens	5	3,4 Rule C

3, De Morgan

1,4 Modus Tollens

4. - - - -

5. - - - -

These great exercises are taken from https://www.zweigmedia.com/RealWorld/logic/logicex5.html

3.

Prove the following is valid using logical inference

Problem 1. [5 points]

Determine if the following deduction rule is valid.

$$\begin{array}{l} p \to (q \lor r) \\ \sim (p \to q) \\ \therefore r \end{array}$$

Problem of truth tellers and liars

Problem

• There is an island containing two types of people: truth tellers who always tell the truth and liars who always lie. You visit the island and are approached by two natives who speak to you as follows:

A says: B is a truth teller.

B says: A and I are of opposite type.

• What are A and B?

Education is what remains after one has forgotten what one has learned in school. — Albert Einstein

Summary

Prove validity using inference rules