Guideline

Homework for week 04.

Please submit your solutions in a single PDF on Brightspace.

If you are writing by hand, please ensure your handwriting is legible.

Multiple submissions are possible before the due time; the last submission will be graded.

Key points for proof-writing

For the purpose of clarity, start your proof with "Proof" and end it with "QED". For a direct proof for a "for all x, P(x) -> Q(x)" question, clearly state your assumption (which is the P part), and end with a conclusion.

Your proof may have significant points deducted for the following reasons:

- If your proof is unclear or structurally unacceptable. This includes proofs that deviate significantly from an essay-like structure or contain fragmented sentences, impeding comprehension.
- If your proof is conceptually confusing. An example would be accepting an assumption as a fact, or accepting the conclusion as an assumption.

Exercise 1 (points = 40)

Determine whether the statements below are true or false. You do not need to explain.

1. 119 is a prime number.

2. 161 is a prime number.

3. 42k is an even number for any integer k.

4. For each integer n with 2 <= n <= 6, $n^2 - n + 11$ is a prime number.

5. The average of any two odd integers is odd.

6. For any real number x, if x * x >= 4, then x >= 2.

7. For any real numbers x and y, $x^2 - 2xy + y^2 >= 0$.

8. There exists an integer x, such that $(2x + 1)^2$ is even.

Exercise 2 (points = 30)

Prove the following proposition: An even number multiplied by an integer is an even number.

Check "Key points for proof-writing" above.

Exercise 3 (points = 30)

Suppose a ∈ Z. Prove: If a is an odd integer, then $a^2 + 3a + 5$ is odd.

Check "Key points for proof-writing" above.