CSE215
Foundations of Computer Science

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Plan

- Homework 01

Avoid overlapping q & a

Consider the proposition ~P ⋀ (Q -> P). What can you conclude about P and Q if you know the statement is true? For this exercise, the explanation is not needed. Just show your results concisely.

p → false Q → false

Exercise 5 (points = 20)

A tautology and a contradiction are terms used in logic to describe specific types of propositions.

- A *tautology* is a proposition that is always true, regardless of the truth values of its constituent parts. In other words, a proposition is a tautology if it is true in every row of a truth table. For example, p ⋁ ~p. An example in plain language: "It will either rain today or it won't."
- A *contradiction* is a proposition that is always false, regardless of the truth values of its constituent parts. In other words, a proposition is a contradiction if it is false in every row of a truth table. For example, p ⋀ ~p. An example in plain language: "I will finish the homework today and I will not."

For each statement form below, determine if it is a tautology, contradiction, or neither. For this exercise, the explanation is not needed. Just show your results concisely.

1. (~p ⋁ q) ⋁ (p ⋀ ~q) neither
2. (p ⋀ ~q) ⋀ (~p ⋁ q) contradiction
3. (p ⋀ q) ⋁ (~p ⋁ (p ⋀ ~q))

tautology

Exercise 6 (points = 20)

Verifying Simple Code with Hoare Logic

Exercise 1. (points = 10)

Check if the two statement forms below are logically equivalent. Produce a truth table and make a conclusion.

- $p \vee q \rightarrow r$
- $(p \rightarrow r) \wedge (q \rightarrow r)$

#1

i) $p \vee q \to r$

p	q	r	$p \vee q$	$p \vee q \to r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	T
F	F	F	F	T

ii) $(p \to r) \wedge (q \to r)$

p	q	r	$p \to r$	$q \to r$	$(p \to r) \wedge (q \to r)$
T	T	T	T	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

$\therefore$ $p \vee q \to r$ and $(p \to r) \wedge (q \to r)$ are equivalent.

Exercise 2 (points = 10)

Check if the two statement forms below are equivalent. Produce a truth table and make a conclusion.

- (p -> q) $\wedge$ (q -> r) $\wedge$ (r -> p)
- p $\wedge$ q $\wedge$ r

#2

i) $(p \to q) \wedge (q \to r) \wedge (r \to p)$

p	q	r	$p \to q$	$q \to r$	$r \to p$	$(p \to q) \wedge (q \to r)$	$(p \to q) \wedge (q \to r) \wedge (r \to p)$
T	T	T	T	T	T	T	T
T	T	F	T	F	T	F	F
T	F	T	F	T	T	F	F
T	F	F	F	T	T	F	F
F	T	T	T	T	F	T	F
F	T	F	T	F	T	F	F
F	F	T	T	T	F	T	F
F	F	F	T	T	T	T	T

ii) $p \wedge q \wedge r$

p	q	r	$p \wedge q$	$p \wedge q \wedge r$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	F
T	F	F	F	F
F	T	T	F	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

$\therefore$ $(p \to q) \wedge (q \to r) \wedge (r \to p)$ and $p \wedge q \wedge r$ are not equivalent.

Exercise 3 (points = 30)

Consider six statement forms (a-f):

- (a) p -> q
- (b) q -> p
- (c) ~p \/ q
- (d) ~q \/ p
- (e) ~q -> ~p
- (f) ~p -> ~q

1. Find all statement forms that are equivalent to (a), except (a) itself.
2. Find all statement forms that are equivalent to (b), except (b) itself.

For this exercise, the explanation is not needed. Just show your results concisely.

#3

1. (a) $p \to q$

$$= \sim p \vee q$$

$$= q \vee \sim p$$

$$= \sim q \to \sim p$$

$\therefore$ (a) is equivalent to (c), (e)

2. (b) $q \to p$

$$= \sim q \vee p$$

$$= p \vee \sim q$$

$$= \sim p \to \sim q$$

$\therefore$ (b) is equivalent to (d), (f)

Exercise 4 (points = 10)

Consider the proposition ~P /\ (Q -> P). What can you conclude about P and Q if you know the statement is true? For this exercise, the explanation is not needed. Just show your results concisely.

#4

$\sim P \wedge (Q \to P)$

P	Q	$\sim P$	$Q \to P$	$\sim P \wedge (Q \to P)$
T	T	F	T	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

If the statement $\sim P \wedge (Q \to P)$ is true, it shows that both P and Q are false.

Exercise 5 (points = 20)

A tautology and a contradiction are terms used in logic to describe specific types of propositions.

- A *tautology* is a proposition that is always true, regardless of the truth values of its constituent parts. In other words, a proposition is a tautology if it is true in every row of a truth table. For example, $p \lor \sim p$. An example in plain language: "It will either rain today or it won't."
- A *contradiction* is a proposition that is always false, regardless of the truth values of its constituent parts. In other words, a proposition is a contradiction if it is false in every row of a truth table. For example, $p \land \sim p$. An example in plain language: "I will finish the homework today and I will not."

For each statement form below, determine if it is a tautology, contradiction, or neither. For this exercise, the explanation is not needed. Just show your results concisely.

1. $(\sim p \lor q) \lor (p \land \sim q)$
2. $(p \land \sim q) \land (\sim p \lor q)$
3. $(p \land q) \lor (\sim p \lor (p \land \sim q))$

#5

1. $(\sim p \vee q) \vee (p \wedge \sim q)$

p	q	$\sim p$	$\sim q$	$\sim p \vee q$	$p \wedge \sim q$	$(\sim p \vee q) \vee (p \wedge \sim q)$
T	T	F	F	T	F	T
T	F	F	T	F	T	T
F	T	T	F	T	F	T
F	F	T	T	T	F	T

Since last column contains all T's, it is tautology.

2. $(p \wedge \sim q) \wedge (\sim p \vee q)$

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \vee q$	$(p \wedge \sim q) \wedge (\sim p \vee q)$
T	T	F	F	F	T	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

Since last column contains all F's, it is contradiction.

3. $(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$

p	q	$\sim p$	$\sim q$	$p \wedge q$	$p \wedge \sim q$	$\sim p \vee (p \wedge \sim q)$	$(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$
T	T	F	F	T	F	F	T
T	F	F	T	F	T	T	T
F	T	T	F	F	F	T	T
F	F	T	T	F	F	T	T

Since last column contains all T's, it is tautology.

Exercise 6 (points = 20)

Verifying Simple Code with Hoare Logic

1. $\{x = 3 \wedge y = 2\}$ `Code` $\{ x = 2 \wedge y = 3\}$, where "Code" refers to the following

```
x := x + y
y := x - y
x := x - y
```

2. $\{x > 0\}$ `while (x<=10) {x = x + 1}` $\{x > 10\}$

#6

1. $\{x=3 \wedge y=2\}\ x := x+y$

$y := x-y$

$x := x-y\ \{x=2 \wedge y=3\}$

$x = 3+2 = 5$

$y = 5-2 = 3$

$x = 5-3 = 2$

$\therefore$ correct

2. $\{x>0\}$ while $(x<=10)$ $\{x = x+1\}$ $\{x>10\}$

$x=1 \Rightarrow x=11$

$x=0.1 \Rightarrow x=10.1$

no counterexample

$\therefore$ correct