

# **CSE215**

# **Foundations of Computer Science**

**Instructor: Zhoulai Fu**

**State University of New York, Korea**

# Jinho Kang



Suny Korea, CS Major

Office Hours:

Monday (7pm – 9pm), Tuesday (7pm – 10pm),  
Wednesday (7pm – 10pm)

# CSE 215 : Undergraduate TA

## Gyujeong Park

- SUNY Korea, AMS major
- Office hours:

**Monday**(12 pm - 1pm, 5pm - 6:30 pm), **Tuesday**( 3:30 pm - 5 pm),  
**Wednesday**(12 pm - 1pm, 3 pm - 5 pm), **Thursday**(3:30 pm - 5 pm)

- Email: [gyujeong.park@stonybrook.edu](mailto:gyujeong.park@stonybrook.edu)



# Previous lecture

- Truth table
- How to use the truth table to determine equivalence

**Midterm Exam I (March 10, 2021, 08:30 am - 09:55 am)**  
**CSE 215: Foundations of Computer Science**

**Problem 1. [5 points]**

Construct a truth table for the following statement form:  $(p \wedge (q \vee r)) \rightarrow (p \wedge r)$ .

**Problem 2. [5 points]**

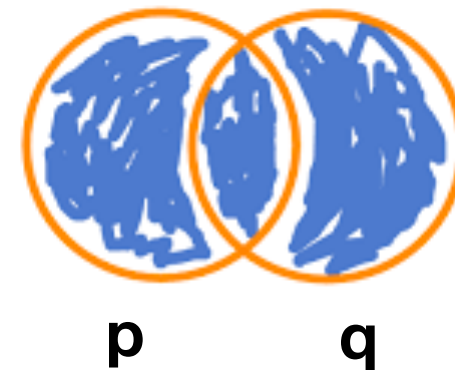
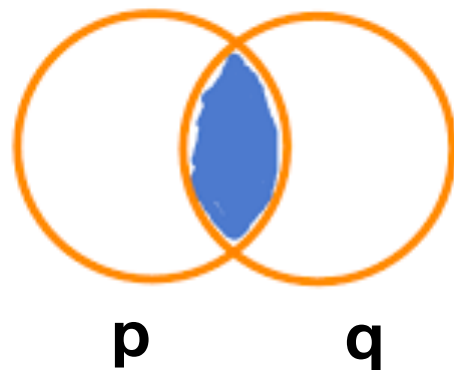
Check the logical equivalence of  $((p \wedge q) \rightarrow r)$  and  $((p \rightarrow r) \vee (q \rightarrow r))$ .

# Today's plan

- Equivalence laws

# Commutative Law

Laws	Formula	Formula
Commutative laws	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$



- Give some equivalent statement forms for  $(p \wedge q) \vee (s \vee t)$

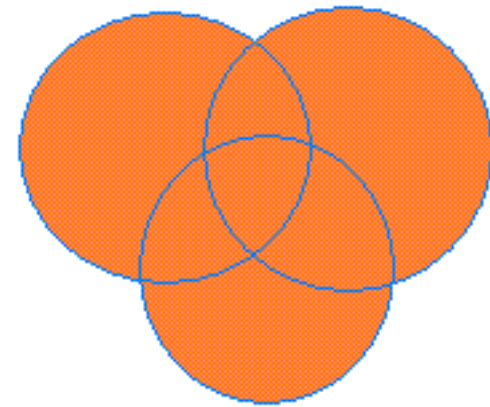
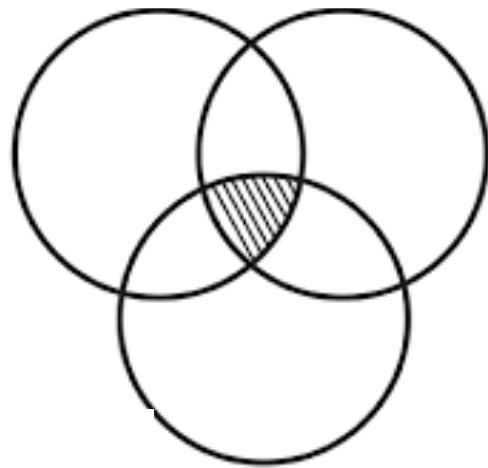


# Associative Law

Associative laws

$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$

$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$



- Think about an equivalent form for  $(p \wedge q) \vee (s \vee t)$

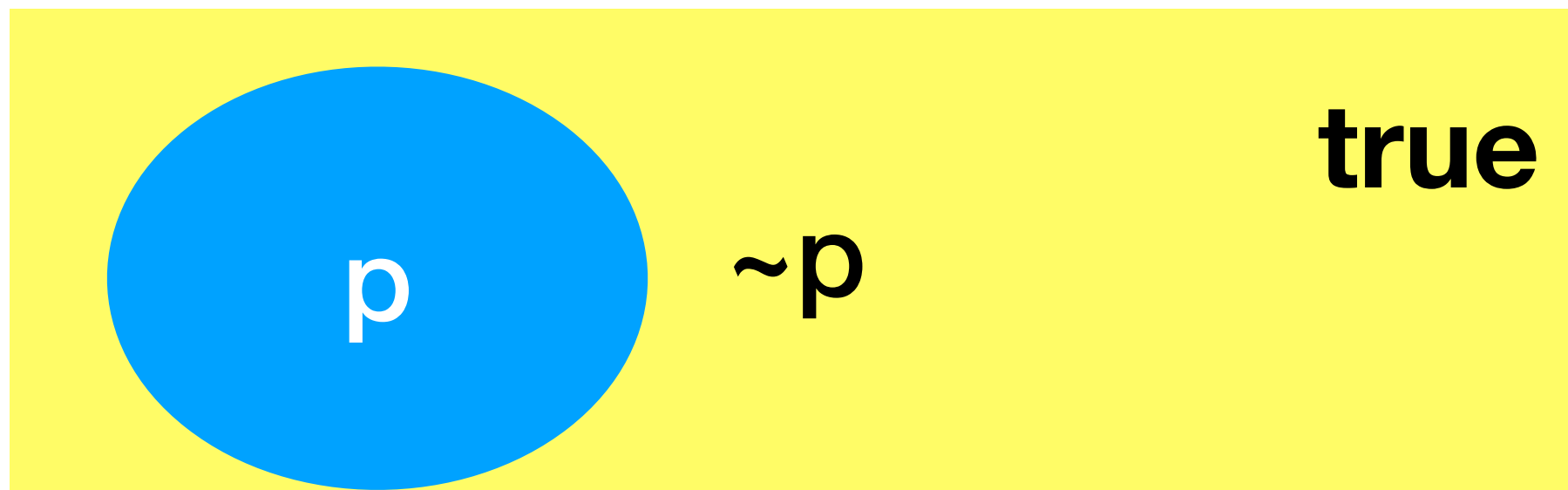
# Distributive Law

Distributive laws  $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$   $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

- A bit like  $a * (b + c) = a * b + a * c$
- Think about an equivalent forms for  $(p \wedge q) \vee (s \vee t)$

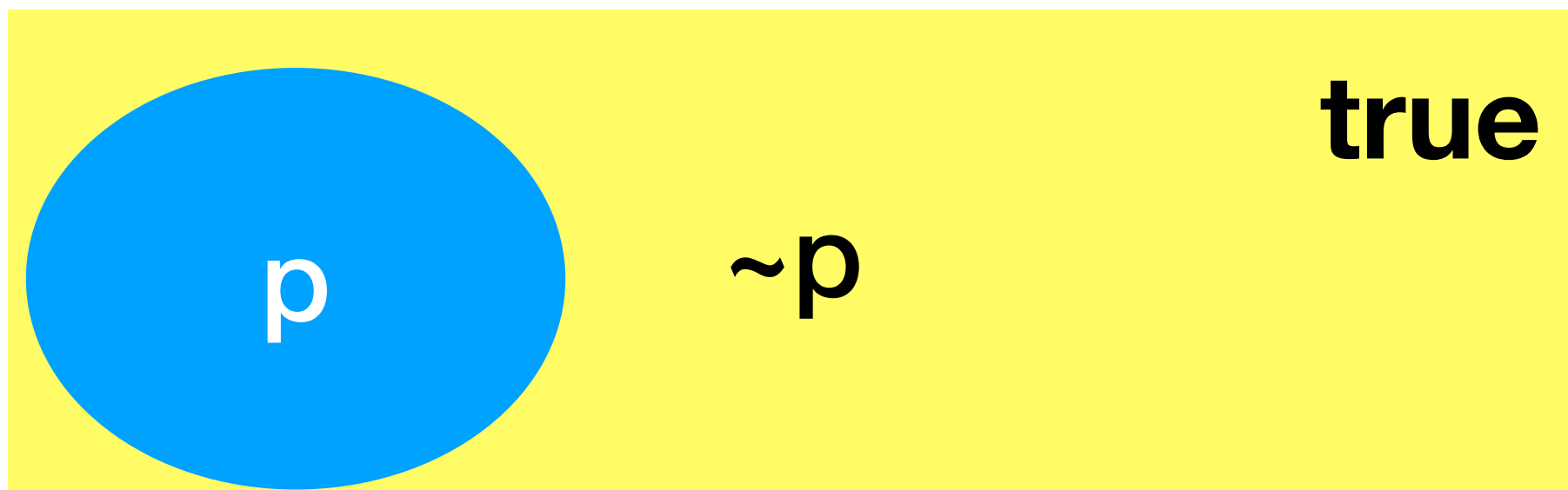
# Laws with “true” and “false”

Identity laws	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
Negation laws	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
Uni. bound laws	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$



# Double-negation Law

Double neg. law	$\sim (\sim p) \equiv p$
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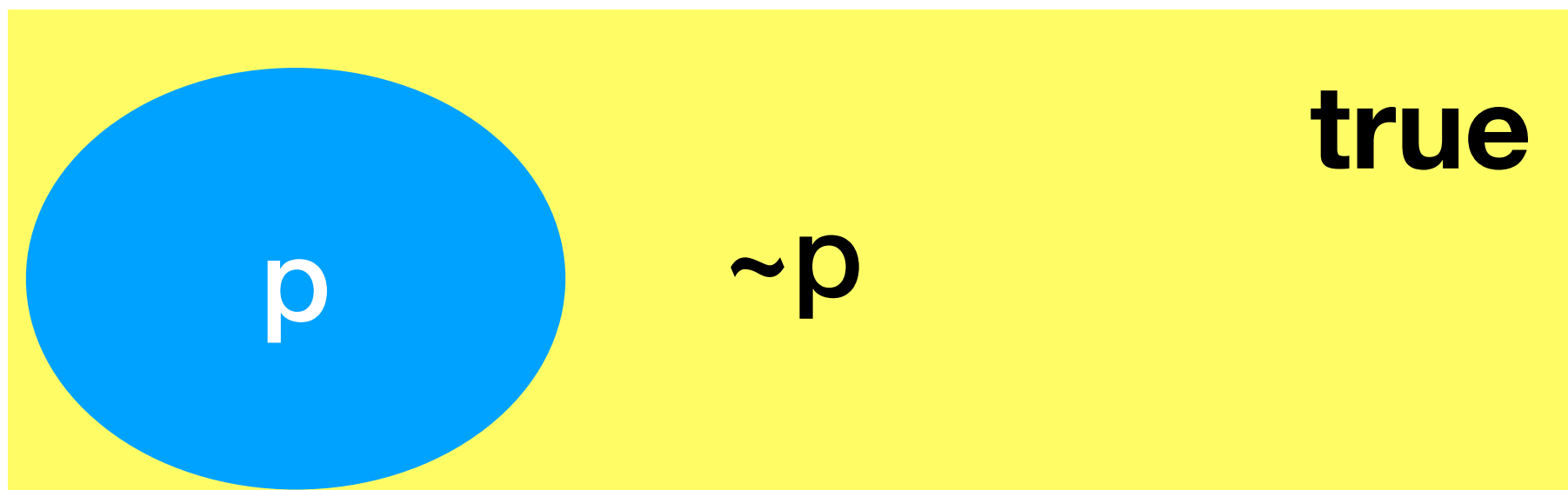


# Idempotent Law

Idempotent laws

$$p \wedge p \equiv p$$

$$p \vee p \equiv p$$



# De Morgan Law

De Morgan's laws	$\sim (p \wedge q) \equiv \sim p \vee \sim q$	$\sim (p \vee q) \equiv \sim p \wedge \sim q$
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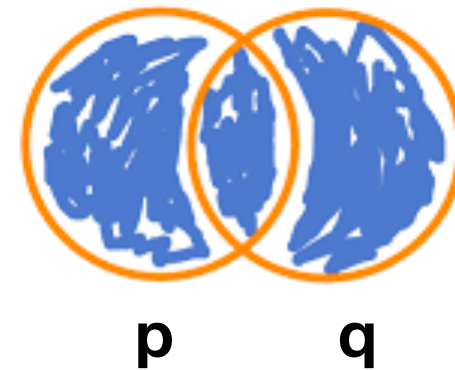
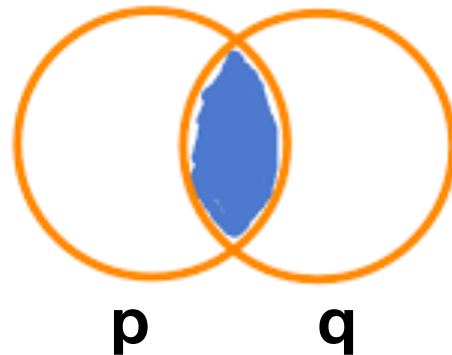
- $p$  = student A is from Korea
- $q$  = student B is from Korea
- $p \wedge q$  = Both student A and B are from Korea
- $\sim(p \wedge q)$  = Either A is not from Korea, or B is not from Korea
- $p \vee q$  = student A or student B is from Korea
- $\sim(p \vee q)$  = \_\_\_\_\_

# Absorption Law

Absorption laws

$$p \vee (p \wedge q) \equiv p$$

$$p \wedge (p \vee q) \equiv p$$



# Equivalence laws

Laws	Formula	Formula
Commutative laws	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
Associative laws	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
Distributive laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
Identity laws	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
Negation laws	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
Double neg. law	$\sim(\sim p) \equiv p$	
Idempotent laws	$p \wedge p \equiv p$	$p \vee p \equiv p$
Uni. bound laws	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$
De Morgan's laws	$\sim(p \wedge q) \equiv \sim p \vee \sim q$	$\sim(p \vee q) \equiv \sim p \wedge \sim q$
Absorption laws	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Negations	$\sim \mathbf{t} \equiv \mathbf{c}$	$\sim \mathbf{c} \equiv \mathbf{t}$



# Exercise

## Problem 3. [5 points]

Mention whether the following statements are true or false. Reasons are not needed.

- (a) [1 point]  $p \vee \sim p \equiv \mathbf{c}$
- (b) [1 point]  $p \vee (p \wedge q) \equiv p \wedge (p \vee q)$
- (c) [1 point]  $\mathbf{c} \equiv p \vee \mathbf{t}$

Notation: **c** means contradiction, or False  
**t** means True

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# Cont.

(c) [1 point]  $\mathbf{c} \equiv p \vee \mathbf{t}$

(d) [1 point]  $p \wedge p \equiv p \vee p$

(e) [1 point]  $p \wedge \mathbf{c} \equiv \sim \mathbf{t}$

# Exercise

**Problem 2. [5 points]**

Is conditional operator  $\rightarrow$  an associative operator? That is, is  $(p \rightarrow q) \rightarrow r$  logically equivalent to  $p \rightarrow (q \rightarrow r)$ ? Prove your answer.

# A solution for #2

## Problem 2. [5 points]

Is conditional operator  $\rightarrow$  an associative operator? That is, is  $(p \rightarrow q) \rightarrow r$  logically equivalent to  $p \rightarrow (q \rightarrow r)$ ? Prove your answer.

- $(p \rightarrow q) \rightarrow r = (\sim p \vee q) \rightarrow r = \sim(\sim p \vee q) \vee r = (p \wedge \sim q) \vee r$
- $p \rightarrow (q \rightarrow r) = \sim p \vee (q \rightarrow r) = \sim p \vee (\sim q \vee r) = (\sim p \vee \sim q) \vee r$
- To show the two differ, consider  $r = \text{false}$ ,  $\sim q = \text{false}$ ,  $p = \text{false}$
- Alternatively, we could use a truth table