CSE215
Foundations of Computer Science

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Plan

- Valid arguments

Valid arguments

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Problem 1. [5 points]
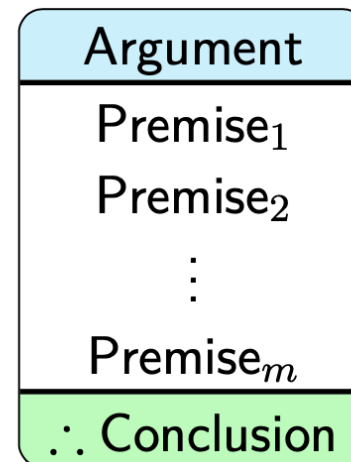Determine if the following deduction rule is valid.

$p \to (q \lor r)$

$\sim (p \to q)$

$\therefore r$

What is a logical argument?

Definitions

- **Logical argument.** Sequence of statements aimed at demonstrating the truth of an assertion
- **Conclusion.** Last statement in an argument
- **Premises.** Last-but-one statements in an argument

Argument
Premise$_1$
Premise$_2$
$\vdots$
Premise$_m$
$\therefore$ Conclusion

If Premise$_1$ and Premise$_2$ and $\cdots$ and Premise$_m$, then Conclusion.

What is a valid argument?

Definition

- An argument is valid if the conclusion follows necessarily from the premises

- Every person will die

- Socrates is a person

- So, Socrates will die

- Congressmen/women own classified info

- Investors can take profit from classified info

- So, congressmen/women should not be allowed to actively do investment

Exercise: Valid or Not?

- All cups are blue

- Socrates is cup

- So, Socrates is blue

Valid or not?

Examples

Valid

- If Socrates is a man, then Socrates is mortal.
  Socrates is a man.
  Therefore, Socrates is mortal.

  If p, then q.
  p.
  Therefore, q.

Invalid

- If Socrates is a man, then Socrates is mortal.
  Socrates is mortal.
  Therefore, Socrates is a man.

  If p, then q.
  q.
  Therefore, p.

Valid

- If Socrates is a man, then Socrates is mortal.
  Socrates is not mortal.
  Therefore, Socrates is not a man.

  If p, then q.
  $\sim q$.
  Therefore, $\sim p$.

Invalid

- If Socrates is a man, then Socrates is mortal.
  Socrates is not a man.
  Therefore, Socrates is not mortal.

  If p, then q.
  $\sim p$.
  Therefore, $\sim q$.

How to mathematically check if an argument is valid?

- Truth table

- Inference rules

Method 1: Truth table

1. Identify the premises and conclusion
2. Construct a truth table for premises and conclusion
3. A row of the truth table in which all the premises are true is called a critical row.
   If there is a critical row in which the conclusion is false, then the argument is invalid. If the conclusion in every critical row is true, then the argument is valid.

Importantly, if there are no critical rows, then the arguments is considered valid

Example

Problem

- Determine the validity of the argument:

$p \rightarrow q \lor \sim r$

$q \rightarrow p \land r$

$\therefore p \rightarrow r$

p	q	r	$\sim r$	$q \lor \sim r$	$p \land r$	$p \rightarrow q \lor \sim r$	$q \rightarrow p \land r$	$p \rightarrow r$
T	T	T	F	T	T	T	T	T
T	T	F	T	T	F	T	F	
T	F	T	F	F	T	F	T	
T	F	F	T	T	F	T	T	F
F	T	T	F	T	F	T	F	
F	T	F	T	T	F	T	F	
F	F	T	F	F	F	T	T	T
F	F	F	T	T	F	T	T	T

Exercise

Problem 1. [5 points]
Determine if the following deduction rule is valid.

$p \to (q \lor r)$

$\sim (p \to q)$

$\therefore r$

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