CSE215
Foundations of Computer Science

Instructor: Zhoulai Fu

State University of New York, Korea

Previous lecture

Argument
Premise_1
Premise_2
$\vdots$
Premise_m
$\therefore$ Conclusion

- Valid argument
- Check validity using truth table

Today

- Check validity using inference rules

Inference rules

Definition

- A rule of inference is a valid argument form that can be used to establish logical deductions

Modus Ponens

Definition

- It has the form:
  If p, then q
  p
  $\therefore q$
- The term *modus ponens* in Latin means "method of affirming"

p	q	$p \to q$	p	q
T	T	T	T	T
T	F	F	T	
F	T	T	F	
F	F	T	F	

What is wrong?

- You are a detective investigating a crime scene with your partner, Sherlock. You come across a note that says: "If the butler did it, then the knife will be found in the kitchen."

- Sherlock responds, "Well, we just found the knife in the kitchen. Therefore, the butler did it."

Modus Tollens

Definition

- It has the form:
  If p, then q
  $\sim q$
  $\therefore \sim p$
- The term *modus tollens* in Latin means "method of denying"

p	q	$p \to q$	$\sim q$	$\sim p$
T	T	T	F	
T	F	F	T	
F	T	T	F	
F	F	T	T	T

- If it rains, then the ground will be wet."
- The ground is not wet
- ==> ?

Generalization

Definition

- It has the form:

  p

  $\therefore p \vee q$

p	q	p	$p \vee q$
T	T	T	T
T	F	T	T
F	T	F	
F	F	F	

Example

- 35 is odd.

  $\therefore$ (more generally) 35 is odd or 35 is even.

What is wrong?

- a friend who tells you, "I really like pizza." Based on this statement, you can apply the Generalization inference rule to make the following conclusion:
- "All people like pizza."

Specialization

Definition

- It has the form:
  $p \wedge q$
  $\therefore p$

p	q	$p \wedge q$	p
T	T	T	T
T	F	F	
F	T	F	
F	F	F	

Example

- Ana knows numerical analysis and Ana knows graph algorithms.
  $\therefore$ (in particular) Ana knows graph algorithms

Conjunction

Definition

- It has the form:

  p

  q

  $\therefore p \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	
F	T	
F	F	

Example

- Lily loves mathematics.

  Lily loves algorithms.

  $\therefore$ Lily loves both mathematics and algorithms.

Elimination

Definition

- It has the form:
  $p \vee q$
  $\sim q$
  $\therefore p$
- Intuition: When you have only two possibilities and you can rule one out, the other must be the case

p	q	$p \vee q$	$\sim q$	p
T	T	T	F	T
T	F	T	T	T
F	T	T	F	F
F	F	F	T	F

Example

- Suppose $x - 3 = 0$ or $x + 2 = 0$.
  Also, suppose x is nonnegative.
  $\therefore x = 3$.

Transitivity

Definition

- It has the form:

  $p \to q$

  $q \to r$

  $\therefore p \to r$
- Can be generalized to a chain with any number of conditionals

Example

- If 18,486 is divisible by 18, then 18,486 is divisible by 9.

  If 18,486 is divisible by 9, then the sum of the digits of 18,486 is divisible by 9.

  $\therefore$ If 18,486 is divisible by 18, then the sum of the digits of 18,486 is divisible by 9.

Division into cases

Definition

- It has the form:
  $p \lor q$
  $p \to r$
  $q \to r$
  $\therefore r$

Example

- x is positive or x is negative.
  If x is positive, then $x^2 > 0$.
  If x is negative, then $x^2 > 0$.
  $\therefore x^2 > 0$.

Summary of Inference rules

Name	Rule	Name	Rule	
Modus Ponens	$p \to q$ p $\therefore q$	Elimination	$p \vee q$ $\sim q$ $\therefore p$	$p \vee q$ $\sim p$ $\therefore q$
Modus Tollens	$p \to q$ $\sim q$ $\therefore \sim p$	Transitivity	$p \to q$ $q \to r$ $\therefore p \to r$	
Proof by division into cases	$p \vee q$ $p \to r$ $q \to r$ $\therefore r$	Generalization	p $\therefore p \vee q$	q $\therefore p \vee q$
		Specialization	$p \wedge q$ $\therefore p$	$p \wedge q$ $\therefore q$
Conjunction	p q $\therefore p \wedge q$	Contradiction	$\sim p \to c$ $\therefore p$	

Some wrong inference

Definition

- A fallacy is an error in reasoning that results in an invalid argument

Fallacy: Converse error

Definition

- It has the form:

  $p \to q$

  q

  $\therefore p$

- Superficially resembles modus ponens but is invalid

p	q	$p \to q$	q	p
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	F

Example

- If $x > 2$, then $x^2 > 4$.

  $x^2 > 4$.

  $\therefore x > 2$.

Fallacy: Inverse error

Definition

- It has the form:

  $p \to q$

  $\sim p$

  $\therefore \sim q$

- Superficially resembles modus tollens but is invalid

p	q	$p \to q$	$\sim p$	$\sim q$
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Example

- If $x > 2$, then $x^2 > 4$.

  $x \leq 2$.

  $\therefore x^2 \leq 4$.

Break;

Exercises

1.

Supply the missing statements using inference rules

1.	1. $p \to \sim q$	Premise	**2.**	1. $\sim p \to q$	Premise
	2. p	Premise		2. $\sim p$	Premise
	3. - - - -	1,2 Modus Ponens		3. - - - -	1,2 Modus Ponens
3.	1. $(\sim p \vee q) \to \sim(q \wedge r)$	Premise	**4.**	1. $(\sim p \wedge q) \to (q \wedge \sim r)$	Premise
	2. $\sim p \vee q$	Premise		2. $\sim p \wedge q$	Premise
	3. - - - -	1,2 Modus Ponens		3. - - - -	1,2 Modus Ponens
5.	1. $(\sim p \vee q) \to \sim(q \wedge r)$	Premise	**6.**	1. $(\sim p \wedge q) \to (q \wedge \sim r)$	Premise
	2. $q \wedge r$	Premise		2. $\sim(q \wedge \sim r)$	Premise
	3. - - - -	1,2 Modus Tollens		3. - - - -	1,2 Modus Tollens
7.	1. $\sim(\sim p \vee q)$	Premise	**8.**	$\sim(p \wedge \sim q)$	Premise
	2. - - - -	De Morgan		2. - - - -	De Morgan
9.	1. $(p \wedge r) \to \sim q$	Premise	**10.**	1. $(\sim p \wedge q) \to (q \wedge \sim r)$	Premise
	2. $\sim q \to r$	Premise		2. $(q \wedge \sim r) \to s$	Premise
	3. - - - -	1,2 Transitive Law		3. - - - -	1,2 Transitive Law

2.

1. $(p \wedge r) \rightarrow \sim q$	Premise
2. $\sim q \rightarrow r$	Premise
3. $\sim r$	Premise
4. - - - -	1,2 Transitive Law
5. - - - -	3,4 Modus Tollens

12.

1. $(\sim p \wedge q) \rightarrow (q \wedge \sim r)$	Premise
2. $(q \wedge \sim r) \rightarrow s$	Premise
3. $\sim s$	Premise
4. - - - -	1,2 Transitive Law
5. - - - -	3,4 Modus Tollens

15.

1. $p \rightarrow (r \wedge q)$	Premise
2. $\sim r$	Premise
3. - - - -	2, Addition of $\sim q$
4. - - - -	3, De Morgan
5. - - - -	1,4 Modus Tollens

17.

1. $(p \wedge q) \rightarrow r$	Premise
2. q	Premise
3. p	Premise
4. - - - -	3,2 Rule C
5. - - - -	1,4 Modus Ponens

18.

1. $p \rightarrow r$	Premise
2. p	Premise
3. s	Premise
4. - - - -	1,2 Modus Ponens
5. - - - -	3,4 Rule C

These great exercises are taken from
https://www.zweigmedia.com/RealWorld/logic/logicex5.html

3.

Prove the following is valid using logical inference

Problem 1. [5 points]

~~Determine if the following deduction rule is valid.~~

$p \to (q \vee r)$

$\sim (p \to q)$

$\therefore\ r$

Problem of truth tellers and liars

Problem

- There is an island containing two types of people: truth tellers who always tell the truth and liars who always lie. You visit the island and are approached by two natives who speak to you as follows:
  A says: B is a truth teller.
  B says: A and I are of opposite type.
- What are A and B?

Education is what remains after one has forgotten what one has learned in school. — Albert Einstein

Summary

- Prove validity using inference rules