CSE215
Foundations of Computer Science

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Today

- Negation

Negation of a universal statement

$$\sim (\forall x \in D, p(x)) \equiv \exists x \in D, \sim p(x)$$

Example: ”All swans are white.”

Negation: "There exists at least one swan that is not white."

Exercise

- Negate "Every phone on the table is turned off."

Negation of an existential statement

$$\sim (\exists x \in D, p(x)) \equiv \forall x \in D, \sim p(x)$$

Example: ”There is a car in the parking lot that is electric.”

Negation: “No car in the parking lot is electric”, or “For every car c in the parking lot, c is not electric”

Exercise

- Negate "There is a person in the village who speaks Italian."

Summary

$$\sim (\forall x \in D, p(x)) \equiv \exists x \in D, \sim p(x)$$
$$\sim (\exists x \in D, p(x)) \equiv \forall x \in D, \sim p(x)$$

- Negation of a universal statement (“all are”) is logically equivalent to an existential statement (“there is at least one that is not”)
  Negation of an existential statement (“some are”) is logically equivalent to a universal statement (“all are not”)

Example

- $\forall$ primes p, p is odd

Example

- $\exists$ triangle T, sum of angles of T equals 200°

Example

- No child is left behind

Common mistakes

Examples

- All mathematicians wear glasses
  Negation (incorrect): No mathematician wears glasses

  Negation (correct): There is at least one mathematician who does not wear glasses
- Some snowflakes are the same
  Negation (incorrect):: Some snowflakes are different
  Negation (correct):: All snowflakes are different

Negation of universal conditional statements

Definition

- Formally,

$$\begin{aligned}\sim(\forall x, p(x) \to q(x)) &\equiv \exists x, \sim(p(x) \to q(x)) \\ &\equiv \exists x, (p(x) \wedge \sim q(x))\end{aligned}$$

Negation of universal conditional statements

Definition

- Formally,

$$\begin{aligned} \sim (\forall x, p(x) \to q(x)) &\equiv \exists x, \sim (p(x) \to q(x)) \\ &\equiv \exists x, (p(x) \wedge \sim q(x)) \end{aligned}$$

Examples

- $\forall$ real x, if $x > 10$, then $x^2 > 100$.
  Negation: $\exists$ real x such that $x > 10$ and $x^2 \leq 100$.
- If a computer program has more than 100,000 lines, then it contains a bug.
  Negation: There is at least one computer program that has more than 100,000 lines and does not contain a bug.

Negation of propositions with multiple quantifiers

$$\begin{aligned}
&\sim (\forall x \text{ in } D,\ \exists y \text{ in } E \text{ such that } P(x,y)) \\
&\equiv \exists x \text{ in } D \text{ such that } \sim (\exists y \text{ in } E \text{ such that } P(x,y)) \\
&\equiv \exists x \text{ in } D \text{ such that } \forall y \text{ in } E,\ \sim P(x,y)
\end{aligned}$$

$$\begin{aligned}
&\sim (\exists x \text{ in } D \text{ such that } \forall y \text{ in } E,\ P(x,y)) \\
&\equiv \forall x \text{ in } D,\ \sim (\forall y \text{ in } E,\ P(x,y)) \\
&\equiv \forall x \text{ in } D,\ \exists y \text{ in } E \text{ such that } \sim P(x,y)
\end{aligned}$$

Exercise

- Do some research: Formal definition of continuity of a real-valued function f on a point x
- Give a formal definition of f being discontinuous on x

More Exercises

Exercise: Translate to formal logic, and then negate

- Given:
    - $P(x)$: x is a person.
    - $Q(x)$: x is busy.
    - $R(x, y)$: x likes y.
- **All people are busy.**

Exercise: Translate to formal logic, and then negate

- Given:
    - $P(x)$: x is a person.
    - $Q(x)$: x is busy.
    - $R(x, y)$: x likes y.
- **Some people are not busy.**

Exercise: Translate to formal logic, and then negate

- Given:
    - $P(x)$: x is a person.
    - $Q(x)$: x is busy.
    - $R(x, y)$: x likes y.
- **Every person likes themselves.**

Exercise: Translate to formal logic, and then negate

- Given:
  - $P(x)$: x is a person.
  - $Q(x)$: x is busy.
  - $R(x, y)$: x likes y.
- **There's someone who doesn't like themselves.**

Exercise: Translate to formal logic, and then negate

- Given:
  - P(x): x is a person.
  - Q(x): x is busy.
  - R(x, y): x likes y.
- **There's at least one person that everyone likes.**

Exercise: Translate to formal logic, and then negate

- Given:
  - $P(x)$: x is a person.
  - $Q(x)$: x is busy.
  - $R(x, y)$: x likes y.
- **Everyone likes at least one person.**

Exercise: Translate to formal logic, and then negate

- Given:
  - $P(x)$: x is a doctor.
  - $Q(x)$: x is busy.
  - $R(x, y)$: x likes y.
- **Some people don't like themselves.**