CSE216
Programming Abstractions

State University of New York, Korea

Agenda

- Regular expression
- context free grammar

Regular Expressions

What is a program?

- Let us consider an expression x + y*10

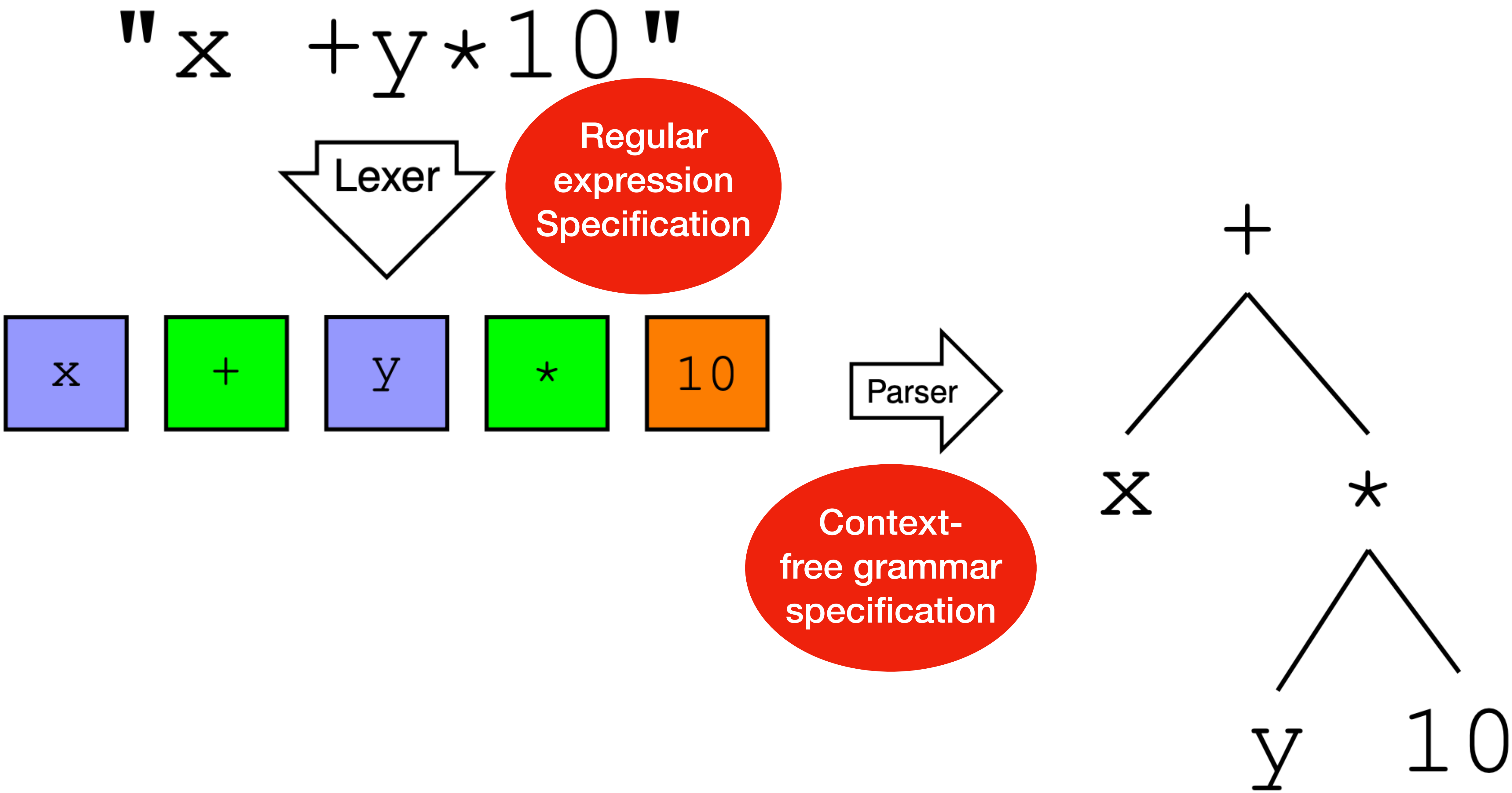
- Think of this expression as a program in a programming language

- This is actually a program written in a programming language used by a calculator

- Today we will analyze the syntax of a general program — Syntax analysis

- Syntax analysis can take a whole semester to learn; we will touch only the surface

How to specify program syntax?



Regular expression specification looks like this in Ocaml

```
rule Token = parse
  | [' ' '\t' '\n' '\r']  { Token lexbuf }
  | ['0'-'9']+            { CSTINT (...) }
  | ['a'-'z''A'-'Z']['a'-'z''A'-'Z''0'-'9']*
                          { keyword (...) }
  | '+'                   { PLUS  }
  | '-'                   { MINUS }
  | '*'                   { TIMES }
  | '('                   { LPAR  }
  | ')'                   { RPAR  }
  | eof                   { EOF   }
  | _                     { lexerError lexbuf "Bad char" }
```

Regular expressions

r	Meaning	Language $\mathcal{L}(r)$
`a`	Character `a`	$\{$"`a`"$\}$
ε	Empty string	$\{$""$\}$

Regular expressions

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$\texttt{a}$	Character $\texttt{a}$	$\{"\texttt{a}"\}$
ε	Empty string	$\{""\}$
$r_1 r_2$	r_1 followed by r_2	$\{s_1 s_2 \mid s_1 \in \mathcal{L}(r_1), s_2 \in \mathcal{L}(r_2)\}$

Regular expressions

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$r*$	Zero or more r	$\{s_1 \ldots s_n \mid s_i \in \mathcal{L}(r), n \geq 0\}$

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$r*$	Zero or more r	$\{s_1 \ldots s_n \mid s_i \in \mathcal{L}(r), n \geq 0\}$
$r_1 \| r_2$	Either r_1 or r_2	$\mathcal{L}(r_1) \cup \mathcal{L}(r_2)$

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$r_1 r_2$	r_1 followed by r_2	$\{s_1 s_2 \mid s_1 \in \mathcal{L}(r_1), s_2 \in \mathcal{L}(r_2)\}$
r^*	Zero or more r	$\{s_1 \dots s_n \mid s_i \in \mathcal{L}(r), n \geq 0\}$
$r_1 \mid r_2$	Either r_1 or r_2	$\mathcal{L}(r_1) \cup \mathcal{L}(r_2)$

Examples

$\texttt{ab}^*$ represents $\{"\texttt{a}", "\texttt{ab}", "\texttt{abb}", \dots\}$

$(\texttt{ab})^*$ represents $\{"", "\texttt{ab}", "\texttt{abab}", \dots\}$

$(\texttt{a}|\texttt{b})^*$ represents $\{"", "\texttt{a}", "\texttt{b}", "\texttt{aa}", "\texttt{ab}", "\texttt{ba}", \dots\}$

Regular expressions

r	Meaning	Language $\mathcal{L}(r)$
$\texttt{a}$	Character $\texttt{a}$	$\{"\texttt{a}"\}$
ε	Empty string	$\{""\}$
$r_1 r_2$	r_1 followed by r_2	$\{s_1 s_2 \mid s_1 \in \mathcal{L}(r_1), s_2 \in \mathcal{L}(r_2)\}$
r^*	Zero or more r	$\{s_1 \dots s_n \mid s_i \in \mathcal{L}(r), n \geq 0\}$
$r_1 \vert r_2$	Either r_1 or r_2	$\mathcal{L}(r_1) \cup \mathcal{L}(r_2)$

Examples

$\texttt{ab}^*$ represents $\{"\texttt{a}", "\texttt{ab}", "\texttt{abb}", \dots\}$

$(\texttt{ab})^*$ represents $\{"", "\texttt{ab}", "\texttt{abab}", \dots\}$

$(\texttt{a}|\texttt{b})^*$ represents $\{"", "\texttt{a}", "\texttt{b}", "\texttt{aa}", "\texttt{ab}", "\texttt{ba}", \dots\}$

Exercise

What does $(\texttt{a}|\texttt{b})\texttt{c}^*$ represent?

Regular expression abbreviations

Abbrev.	Meaning	Expansion
[aeiuo]	Set	a|e|i|o|u
[0–9]	Range	0|1|...|8|9
[0–9a–Z]	Ranges	0|1|...|8|9|a|b|...|y|z
$r?$	Zero or one r	$r\|\varepsilon$
r^+	One or more r	$r\,r^*$

Exercises:
Regular Expression

Exercise 1

Write regular expressions for:

- Non-negative integer constants

Demo: https://regex101.com/

Exercise 2

Write regular expressions for:

- Integer constants

Exercise 3

Write regular expressions for:

- Floating-point constants:
  - `3.14`
  - `3E8`
  - `+6.02E23`
  - **3E+08**
  - **4.6E-09**

Exercise 4

Write regular expressions for:

- Java variable names:
  - `xy`
  - `x12`
  - `_x`

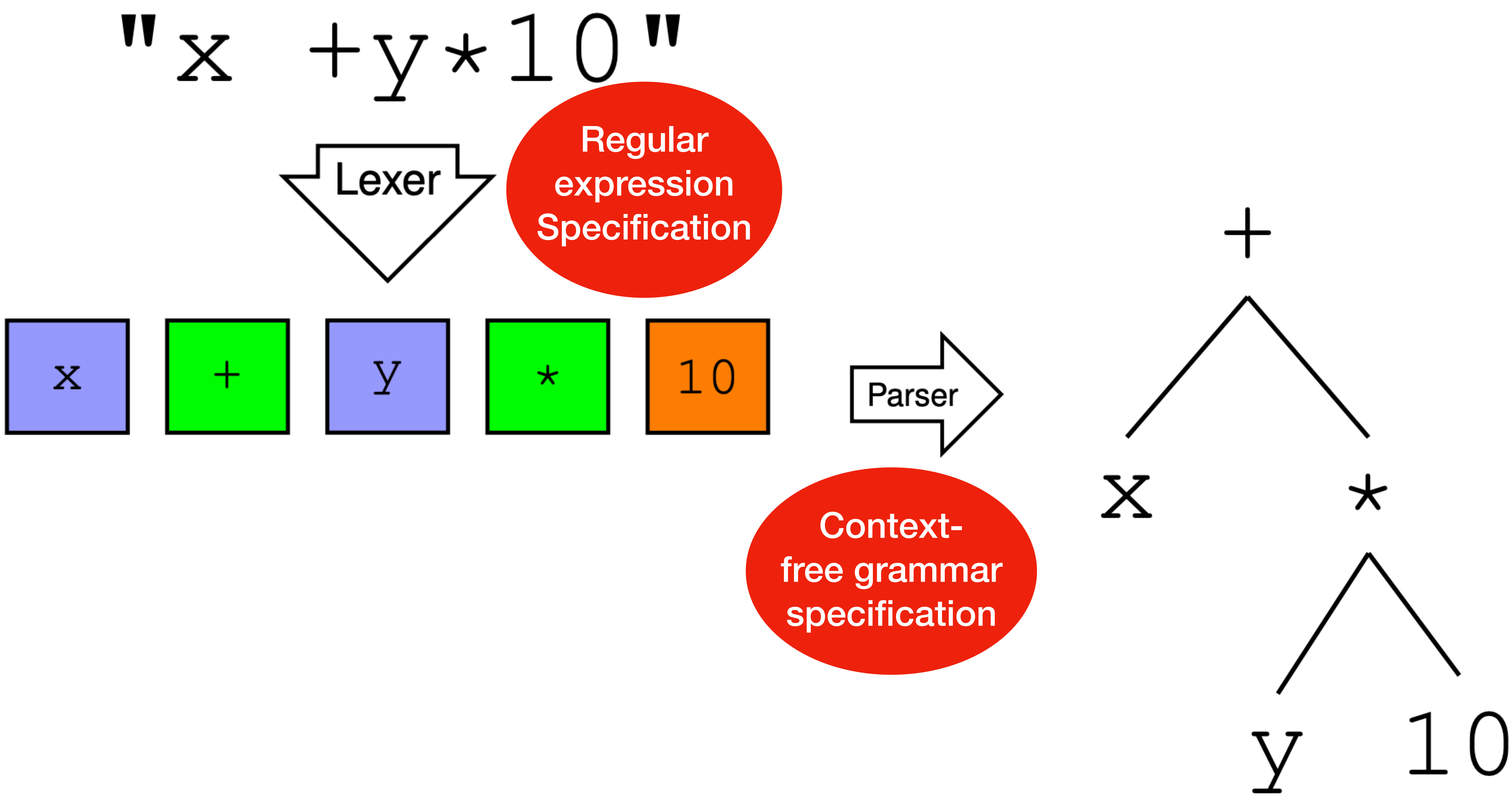
Exercise 5

Give a RE for: $L = \{0^i 1^j \mid i \text{ is even and } j \text{ is odd } \}$

0^i above refers to repeating “0” i times. E.g. 0^4 means “0000”

Context-free grammar

Parsing



"x +y*10"
Lexer
Regular expression Specification
x
+
y
*
10
Parser
Context-free grammar specification
+
x
*
y
10

The need for a grammar

"Afternoon good, I'd room a like."

Mr. Men and Little Miss Series

Context-Free Grammar

- A notation for describing languages.
- More powerful than regular expressions.
- It still cannot define all possible languages.
- Useful for recursive structures, e.g., most today's programming languages.

Basic idea

- A language can be decomposed to smaller parts
- Each part can be defined recursively
- Use **production rules** to generate the language

Example 1

```
Program ::= Stmt Program
          | ""
Stmt    ::= Var = AExpr
          | if ( BExpr ) Stmt else Stmt
          | while ( BExpr ) Stmt
AExpr   ::= AExpr + AExpr
          | AExpr - AExpr
          | AExpr * AExpr
          | AExpr / AExpr
          | Var
          | Number
BExpr   ::= AExpr == AExpr
          | AExpr < AExpr
          | not (BExpr)
          | BExpr and BExpr
          ...
Var    ::= x | y | z ...
Number ::= 0 | 1 | 2 ...
```

Demo: Parse tree for x = 2

Example 2

- Production rule:
  - S → 01
  - S → 0S1
- Basis: 01 is in the language.
- Induction: If w is in the language, then so is 0w1.
- The generated language is {0^n 1^n, n>=1}

Overview

- Use **terminal** symbols a, b, c, d… for the alphabet of a language.
- Use **nonterminal** symbols A, B, C, D, recursively
- **Starting symbol** is a special nonterminal
- Use **production rules** to generate the language

Production

- A production has the form
  variable $\longrightarrow$ string of variables and terminals
- Convention
  - A, B, C,. . . are variables.
  - a, b, c,. . . are terminals.

Put Everything Together

- Here is a formal CFG for $\{0^n 1^n \mid n \geq 1\}$.
- Terminals $= \{0,1\}$.
- Variables $= \{S\}$.
- Start symbol $= S$.
- Productions $=$
  $S \rightarrow 01$
  $S \rightarrow 0S1$

Notation

- Symbol ::= is often used for $\to$.
- Symbol $|$ is used for or.
  - A shorthand for a list of productions with the same left side.

Example: $S \to 0S1 \mid 01$ is a shorthand for
$S \to 0S1$ and $S \to 01$.

Exercise 1: Construct a parse tree

```
Program ::= Stmt Program
        | ""
Stmt    ::= Var = AExpr
        | if ( BExpr ) Stmt else Stmt
        | while ( BExpr ) Stmt
AExpr   ::= AExpr + AExpr
        | AExpr − AExpr
        | AExpr ∗ AExpr
        | AExpr / AExpr
        | Var
        | Number
BExpr   ::= AExpr == AExpr
        | AExpr < AExpr
        | not (BExpr)
        | BExpr and BExpr
        ...
Var    ::= x | y | z ...
Number ::= 0 | 1 | 2 ...
```

while (x<10) x = x + 1

Exercise 2: Construct a parse tree

```
AExpr   ::= AExpr + AExpr
          | AExpr − AExpr
          | AExpr ∗ AExpr
          | AExpr / AExpr
          | Var
          | Number
BExpr   ::= AExpr == AExpr
          | AExpr < AExpr
          | not (BExpr)
          | BExpr and BExpr
          ...
Var    ::= x | y | z ...
Number ::= 0 | 1 | 2 ...
```

1 + 0 * 2

Exercise 3: Construct a parse tree

```
expr   ::= term
         |  expr + term
         |  expr - term

term   ::= factor
         |  term * factor
         |  term / factor

factor ::= NUMBER
         |  NAME
         |  ( expr )
```

1 + 0 * 2

Ambiguous grammar

- An ambiguous grammar is a formal grammar that can produce **multiple parse trees** or interpretations for the same input sentence or sequence of symbols.

- This can be problematic in various contexts because it can **make it difficult to determine the correct meaning** or parse tree of a sentence or sequence of symbols.

- To avoid ambiguity, it is often necessary to **use unambiguous grammars** or to add rules or constraints to the ambiguous grammar to disambiguate the interpretations.

Summary

- Context free grammar concepts
- Parse tree
- Ambiguous grammar
- Grammar -> Language
- Language -> Grammar

Exercises:
Context-Free Grammar

Exercise 1

- Construct a parse tree for 000111 for this grammar:

$$S \rightarrow 01$$
$$S \rightarrow 0S1$$

Exercise 2

- Construct a parse tree for (())() for this grammar:

$$S \rightarrow SS \mid (S) \mid ()$$

Exercise 3

Given the grammar G with the following productions:

- $S \to aSb$
- $S \to \epsilon$

Determine the language $L(G)$ generated by G.

Exercise 4

Given the grammar G with the productions:

- $S \to aSa$
- $S \to bSb$
- $S \to \epsilon$

What is the language $L(G)$?

Exercise 5

Consider the grammar G defined as:

- $S \rightarrow aS$
- $S \rightarrow Sb$
- $S \rightarrow \epsilon$

Define the language $L(G)$.

Exercise 6

Create a grammar that generates the language of all strings of the form:

- a language containing only the words "dog", "cat", and “fish”.

Exercise 7

Create a grammar that generates the language of all strings of the form:

- "a^n", where n ≥ 0.

Exercise 8

Create a grammar that generates the language of all strings of the form:

- "a^n b^m", where n, m ≥ 0.

Exercise 9

Create a grammar that generates the language of all strings of the form:

- "a^n b^n", where n ≥ 0.

Exercise 10

Create a grammar that generates the language of all strings of the form:

- all strings over {a, b} that start with 'a' and end with 'b'.

Exercise 11

- Create a grammar that generates all valid sequences of balanced parentheses, e.g., " ", "()", "(())", "()()", but not "(()", ")(", or "())(").