Guideline

Homework for Week 03

Please submit your solutions as a single PDF on Brightspace.

Multiple submissions are allowed before the due time; the last submission will be graded.

Exercise 1 (points = 8)

Consider the lambda term (λa.aλb.ab)λa.a

1. Draw a parse tree based on the following grammar of lambda calculus:

```
Term -> Var | λ Var . Term | Term Term
```

1. Reduces it to a normal form. Remind that A term is said to be in "normal form" if no beta reductions are possible. This means you've simplified the term as much as you can, and there are no more beta reductions to perform.

Exercise 2 (points = 8)

Consider the lambda term (λx.λw.xwz)y x

1. Draw a parse tree
2. Reduces it to a normal form

Exercise 3 (points = 28)

In the following, we write TRUE as an alias for λx.λy.x, and FALSE as an alias for λx.λy.y.

Now, simplify the following lambda terms to their normal form using beta reduction. You don't need to show intermediate results.

1. TRUE TRUE TRUE
2. TRUE TRUE FALSE
3. TRUE FALSE TRUE
4. TRUE FALSE FALSE
5. FALSE TRUE TRUE
6. FALSE FALSE TRUE
7. FALSE TRUE FALSE

Hint: To complete this exercise, you you can either mechanically use beta reductions, or you can attempt to deduce the meanings of TRUE and FALSE.

Exercise 4 (points = 20)

Simplify the following lambda terms to their normal form using beta reduction. You don't need to show intermediate results.

1. (λx. λy. x y) a b
2. (λx. x (λy. y x)) (λz. z)
3. (λx. λy. x (λz. y z)) a b
4. (λx. λy. λz. x z (y z)) a b c
5. (λx. x (λy. y)) (λx. x)

Exercise 5 (points = 16)

Check if the following is alpha equivalent

1. (λx.xy) vs. (λx.yx)
2. (λx.xu) vs. (λx.xv)
3. (λa.λb.abc) vs. (λx.λy.xyz)
4. (λx.λy.xyz) vs. (λy.λx.yxz)

Exercise 6 (points = 4)

True or False?

- The lambda expression ((λx. λy.y) y) (λx.x z) reduces to z as the normal form.

If false, provide the correct normal form.

Exercise 7 (points = 16)

Reduce the lambda term below until you get a normal form

1. (λx.λy.xyz)y x
2. (λx.λf.λy.fxy)(fy)
3. (λx.λy.yx)(λx.xx)y
4. (λx.λy.xyy)(λy.y)z