

# **CSE216**

# **Foundations of Computer Science**

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Many slides taken from Cornell's CS3110. Thanks!  
[https://www.cs.cornell.edu/courses/cs3110/2014fa/lecture\\_notes.php](https://www.cs.cornell.edu/courses/cs3110/2014fa/lecture_notes.php)

# Higher-order functions

- Functions are values
- Can use them **anywhere** we use values
  - Functions can **take** functions as arguments
  - Functions can **return** functions as results
    - ...functions can be *higher-order*

# Map

```
map shirt_color [
```



```
]
```

```
= [gold, blue, red]
```

# Map Implementation

```
let rec map f xs =  
  match xs with  
    [] -> []  
  | x::xs' -> (f x) :: (map f xs')
```

`map : ('a -> 'b) -> 'a list -> 'b list`

Implemented in List.map;

can be used in many data structures like queue, stack

# Exercise

What is value of `lst` after this code?

```
let is_even x = (x mod 2 = 0)  
let lst = map is_even [1;2;3;4]
```

- A. `[1;2;3;4]`
- B. `[2;4]`
- C. `[false; true; false; true]`
- D. `false`

# Exercise

- Write a function **square\_list** to convert a list of integers to a list of their squares.
  - 1. First, do not use **List.map**
  - 2. Then use **List.map**

**Example:**

**Input: [1; 2; 3; 4]**

**Output: [1; 4; 9; 16]**

# Filter

`filter is_vulcan [`



`]`

`= [`



`]`



# Filter (2)

```
let filter f xs =  
  match xs with  
    [] -> []  
  | x::xs' -> if f x  
                then x::(filter f xs')  
                else filter f xs'
```

**filter : ('a -> bool) -> 'a list -> 'a list**

Filter is also HUGE

– In library: **List.filter**

# Exercise

What is value of `lst` after this code?

```
let is_even x = (x mod 2 = 0)
let lst = filter is_even [1;2;3;4]
```

- A. [1;2;3;4]
- B. [2;4]
- C. [false; true; false; true]
- D. false

# Exercise

- Write a function **positive\_list** to get from a list of integers to a list of its positive integers.
  - 1. First, do not use **List.map**
  - 2. Then use **List.map**

**Example:**

**Input: [1; 2; -3; 4]**

**Output: [1; 2; 4]**

# Iterators

- Map and filter are *iterators*
  - Not built-in to the language, an idiom
- Benefit of iterators: separate recursive traversal from data processing
  - Can reuse same traversal for different data processing
  - leads to modular, maintainable, beautiful code!
- So far: iterators that change or omit data
  - what about combining data?
  - e.g., sum all elements of list

# Folding v1.0

*Idea: stick an operator between every element of list*

folding [ 1 ; 2 ; 3 ] with (+)

becomes

1+2+3

-->

6

*But list could have 1 element, so need an initial value*

# Folding v2.0

folding **[ 1 ; 2 ; 3 ]** with **0** and **(+)**

becomes

**0+1+2+3**

**-->**

**6**

*Or list could be empty; just return initial value*

folding [ ] with **0** and **(+)**

becomes

**0**

# Question

What should the result of folding  $[1; 2; 3; 4]$  with  $1$  and  $( \ * \ )$  be?

- A. 1
- B. 24
- C. 10
- D. 0



# Implementation details

iterate left-to-right or right-to-left?

folding [ 1 ; 2 ; 3 ] with 0 and (+)

**left to right becomes:**  $((0+1)+2)+3$

**right to left becomes:**  $1+(2+(3+0))$

Both evaluate to 6; does it matter?

Yes: not all operators are associative, e.g. subtraction, division, exponentiation, ...

# Fold in Ocaml

Two versions in OCaml library:

**List.fold\_left**

**: ('a -> 'b -> 'a) -> 'a -> 'b list -> 'a**

**List.fold\_right**

**: ('a -> 'b -> 'b) -> 'a list -> 'b -> 'b**

# fold\_left

```
let rec fold_left f acc xs =  
  match xs with  
    []          -> acc  
  | x::xs'     -> fold_left f (f acc x) xs'
```

Accumulates an answer by

- repeatedly applying **f** to “answer so far”,
- starting with initial value **acc**,
- folding “from the left”

**fold\_left f acc [a;b;c]**

computes

**f (f (f acc a) b) c**

# Google's Map-Reduce

- Fold has many synonyms/cousins in various functional languages, including **scan** and **reduce**
- Google organizes large-scale data-parallel computations with Map-Reduce
  - open source implementation by Apache called Hadoop

*"[Google's Map-Reduce] abstraction is inspired by the map and reduce primitives present in Lisp and many other functional languages. We realized that most of our computations involved applying a map operation to each logical record in our input in order to compute a set of intermediate key/value pairs, and then applying a reduce operation to all the values that shared the same key in order to combine the derived data appropriately."*

[Dean and Ghemawat, 2008]

# Fold is general

Implement so many other functions with fold!

```
let rev xs = fold_left (fun xs x -> x::xs) [] xs
let length xs = fold_left (fun a _ -> a+1) 0 xs
let map f xs = fold_right
  (fun x a -> (f x)::a) xs []
let filter f xs = fold_right
  (fun x a -> if f x then x::a else a) xs []
```

# Exercise

- `List.fold_left (+) 0 [1;2;3;4]`
- `List.fold_left ( *) 1 [4;6;8]`

# Exercise

- `List.fold_left (fun xs x -> x:: xs) [] [1;2;3;4]`

# Exercise

- `List.fold_left (fun a _ -> a+1) 0 [1;2;3;4]`