# CSE216 Foundations of Computer Science

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## Higher-order functions

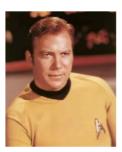
Functions are values

• Can use them **anywhere** we use values

- Functions can **take** functions as arguments
- Functions can **return** functions as results
   ...functions can be *higher-order*

## Map

map shirt\_color [







= [gold, blue, red]

## Map Implementation

```
let rec map f xs =
   match xs with
   [] -> []
   | x::xs' -> (f x)::(map f xs')
```

```
map : ('a -> 'b) -> 'a list -> 'b list
```

Implemented in List.map;

can be used in many data structures like queue, stack

What is value of lst after this code?

```
let is_even x = (x mod 2 = 0)
let lst = map is_even [1;2;3;4]
```

- A. [1;2;3;4]
- B. [2;4]
- C. [false; true; false; true]
- D. false

- Write a function square\_list to convert a list of integers to a list of their squares.
  - 1. First, do not use List.map
  - 2. Then use **List.map**

#### **Example:**

Input: [1; 2; 3; 4]

Output: [1; 4; 9; 16]

## Filter

filter is\_vulcan [







= [



## Filter (2)

```
filter: ('a -> bool) -> 'a list -> 'a list
```

#### Filter is also HUGE

– In library: List.filter

What is value of 1st after this code?

```
let is_even x = (x mod 2 = 0)
let lst = filter is_even [1;2;3;4]
```

- A. [1;2;3;4]
- B. [2;4]
- C. [false; true; false; true]
- D. false

- Write a function positive\_list to get from a list of integers to a list of its positive integers.
  - 1. First, do not use **List.map**
  - 2. Then use **List.map**

#### **Example:**

Input: [1; 2; -3; 4]

Output: [1; 2; 4]

#### Iterators

- Map and filter are iterators
  - Not built-in to the language, an idiom
- Benefit of iterators: separate recursive traversal from data processing
  - Can reuse same traversal for different data processing
  - leads to modular, maintainable, beautiful code!
- So far: iterators that change or omit data
  - what about combining data?
  - e.g., sum all elements of list

## Folding v1.0

Idea: stick an operator between every element of list

```
folding [1;2;3] with (+)
becomes
1+2+3
-->
```

But list could have 1 element, so need an initial value

## Folding v2.0

```
folding [1;2;3] with 0 and (+)
becomes
0+1+2+3
-->
6
```

#### Or list could be empty; just return initial value

```
folding [] with 0 and (+)
becomes
0
```

### Question

What should the result of folding [1;2;3;4] with 1 and ( \* ) be?

A. 1

B. 24

C. 10

D. 0

## Implementation details

iterate left-to-right or right-to-left?

```
folding [1;2;3] with 0 and (+)
```

left to right becomes: ((0+1)+2)+3

right to left becomes: 1+(2+(3+0))

Both evaluate to 6; does it matter?

Yes: not all operators are associative, e.g. subtraction, division, exponentiation, ...

#### Fold in Ocaml

Two versions in OCaml library:

```
List.fold_left
: ('a -> 'b -> 'a) -> 'a -> 'b list -> 'a
```

```
List.fold_right
: ('a -> 'b -> 'b) -> 'a list -> 'b -> 'b
```

### fold\_left

```
let rec fold_left f acc xs =
   match xs with
   [] -> acc
   | x::xs' -> fold_left f (f acc x) xs'
```

#### Accumulates an answer by

- repeatedly applying f to "answer so far",
- starting with initial value **acc**,
- folding "from the left"

```
fold_left f acc [a;b;c]
computes
f (f (f acc a) b) c
```

## Google's Map-Reduce

- Fold has many synonyms/cousins in various functional languages, including scan and reduce
- Google organizes large-scale data-parallel computations with Map-Reduce
  - open source implementation by Apache called Hadoop

"[Google's Map-Reduce] abstraction is inspired by the map and reduce primitives present in Lisp and many other functional languages. We realized that most of our computations involved applying a map operation to each logical record in our input in order to compute a set of intermediate key/value pairs, and then applying a reduce operation to all the values that shared the same key in order to combine the derived data appropriately."

[Dean and Ghemawat, 2008]

## Fold is general

Implement so many other functions with fold!

```
let rev xs = fold_left (fun xs x -> x::xs) [] xs
let length xs = fold_left (fun a _ -> a+1) 0 xs
let map f xs = fold_right
  (fun x a -> (f x)::a) xs []
let filter f xs = fold_right
  (fun x a -> if f x then x::a else a) xs []
```

- List.fold\_left (+) 0 [1;2;3;4]
- List.fold\_left (\*) 1 [4;6;8]

List.fold\_left (fun xs x -> x:: xs) [] [1;2;3;4]

List.fold\_left (fun a \_ -> a+1) 0 [1;2;3;4]