

Semantic Web

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About me

- ▶ 2008 PhD in Mathematics, Peking University, China
- ▶ 2008-2012 Postdoc in Computer Science, Université Paris Nord, France
- ▶ 2012-2014 Research Assistant in Computer Science, TU Dresden, Germany
- ▶ since September 2014 : Associate Professor in Computer Science, University Paris Sud, France

Research topics :

Semantic Web, Ontology, Automated Reasoning

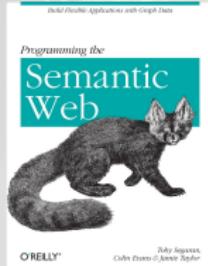
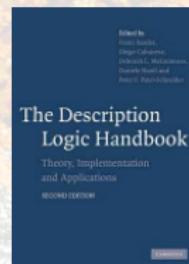
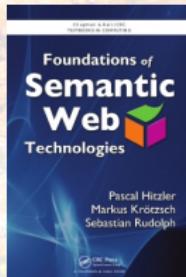
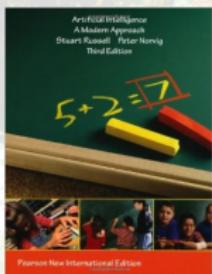
My expectation :

give you a solid foundation on Semantic Web
help your D2K career

General Information

- ▶ This lecture is inspired by the following courses :
 - Knowledge Representation for the Semantic Web
Pascal Hitzler, Wright State University, US
 - Foundations of Semantic Web Technologies
Sebastian Rudolph, TU Dresden, Germany
 - Introduction à l'intelligence artificielle
Laurent Simon, Polytech Paris Sud

- ▶ Reference books :



General Information

- ▶ Course materials : www.lri.fr/~ma/M2DK
- ▶ Organizational matters :
 - ▶ 4.5 lectures + exercices, 3.5 labs
 - ▶ Initial plans :
 - ▶ 1st week : introduction+ first deductive reasoning
 - ▶ 2nd week : implementation of the deductive reasoning
 - ▶ 3rd week : semantic web ontology standards, description logics (DLs)
 - ▶ 4th week : exercises on DLs + building your first ontology
 - ▶ 5th week : more DLs
 - ▶ 6th week : working with ontology reasoners
 - ▶ 7th week : student oral presentations
- ▶ Grading : 50% Project + 30% Presentation + 20% Exam
 - ▶ Project : implementation of a reasoning system
 - ▶ Presentation : an oral presentation on a topic you choose
 - ▶ Exam : written

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 - ▶ 7th week : student oral presentations
- ▶ Grading : 30% Project + 30% Presentation + 40% Exam
 - ▶ Project : implementation of a reasoning algorithm
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Outline

Why Semantic Web ?

Semantics and Knowledge Representation

Principle

Syntax and Semantics

Propositional (logic) language

Inference Algorithms for a special PL

A special PL sub-language : Horn rules

Forward Chaining

Backforward Chaining

The Current Web

- ▶ WWW is penetrating our society (immensely successful)
- ▶ Huge amounts of data
- ▶ Syntax standards for transfer of structured data
- ▶ Machine-processable, human-readable documents

BUT

- ▶ Content/knowledge cannot be accessed by machines.
- ▶ Meaning (semantics) of transferred data is not accessible.

Limits of the current Web

- ▶ Too much information with too little structure made for human consumption
 - ▶ Content search is very simplistic
 - ▶ future requires better methods
- ▶ Web content is heterogeneous
 - ▶ in terms of content
 - ▶ in terms of structure
 - ▶ in terms of character encoding
 - ▶ future requires intelligent information integration
- ▶ Humans can derive new (implicit) information from given pieces of information but on the current Web we can only deal with syntax.
 - ▶ requires automated reasoning techniques

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Examples

- ▶ Find that landmark article on data integration written by an Indian researcher in the 1990s.

[How can you get the answer ?]

- ▶ Are lobsters spiders ?

[This is getting easier these days, but was impossible a few years ago. It still needs finding and integrating over different websites, as well as some background knowledge.]

- ▶ Which car is called a “duck” in German ?

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Another Example

Identify congress members, who have voted “No” on pro environmental legislation in the past four years, with high-pollution industry in their congressional districts.

In principle, all the required knowledge is on the Web – most of it even in machine-readable form.

However, without automated processing and reasoning we cannot obtain a useful answer.

Basic ingredients for the Semantic Web

- ▶ Open Standards for describing information on the Web
- ▶ Methods for obtaining further information from such descriptions

We'll talk about these matters in this course.

Basic ingredients for the Semantic Web

- ▶ Open Standards for describing information on the Web
- ▶ Methods for obtaining further information from such descriptions

Main approach : Logical deduction (aka automated reasoning)

E.g.,

D.C. is a capital
Every capital is a city

Hence : D.C. is a city

Based on **predicate logic**. – it needs to be specified which conclusions are **valid**. Plus, we need algorithms for these.

A photograph of a forest during autumn. The trees are heavily laden with leaves in shades of yellow, orange, and red, with some green leaves interspersed. The branches are intricate and tangled, creating a complex web against a bright, slightly overexposed sky.

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Les principaux problèmes

We need to store/represent information :

- ▶ for a domain of interest
- ▶ to be possibly queried by users

Inference algorithms

Answers to queries should be computed by the algorithm independent on the application domain :

- ▶ It takes as input a knowledge base K and a query ;
- ▶ It can correctly answer the query according to the information given in K .

Main studies on :

- ▶ the expressivity of a formal language
- ▶ the soundness / completeness / complexity of inference algorithms for a chosen representation language

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Formal Knowledge Representation Language

To be processable by machines, the language needs to be

- ▶ Formal (syntactically well defined)
- ▶ Precise (semantically well defined)
- ▶ like C, Java, Python, ...
- ▶ unlike ... ?

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Syntax and Semantics

Syntax

is to represent the knowledge and queries in a machine readable format (similar to sentences and words in a book, but it's the object and variable symbols in the computer memory.)

Semantics

is to link the symbols to the real world, without which syntax is meaningless.

In short

- ▶ Computers manipulate the syntactical objects (i.e., the process of automatic inference)
- ▶ The manipulation is meaningful because of the meaning associated to the syntax.

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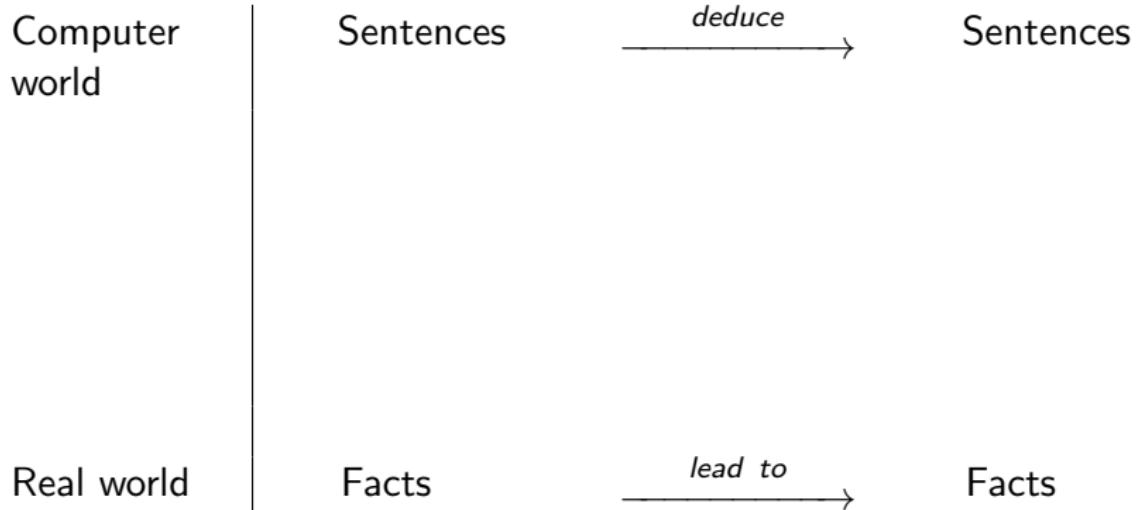
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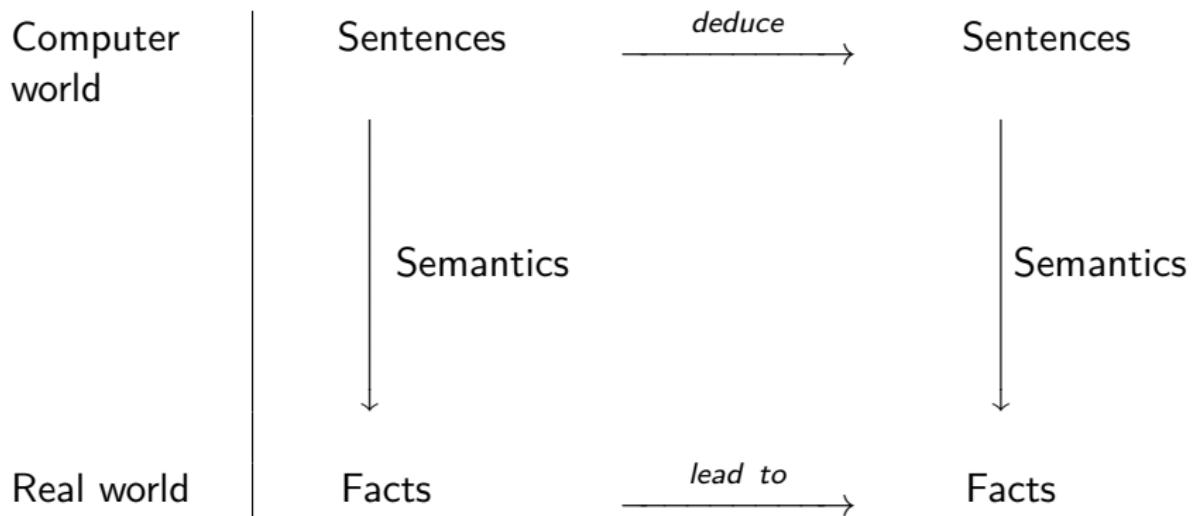
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Syntax / Semantics / Inference



Syntax / Semantics / Inference



Propositional Language (Syntax)

Let $\mathcal{V} = \{x_1, \dots, x_n\}$ be a set of (propositional) variables, the language \mathcal{LP} of **propositions** is constructed inductively by applying the following rules :

- ▶ $\top, \perp \in \mathcal{LP}$;
- ▶ $x \in \mathcal{LP}$ for all $x \in \mathcal{V}$;
- ▶ $\neg f \in \mathcal{LP}$ for all $f \in \mathcal{LP}$;
- ▶ $f \vee g, f \wedge g, f \rightarrow g \in \mathcal{LP}$ for all $f, g \in \mathcal{LP}$.

An equivalent notation :

The propositional formulas φ are built by the following grammar :

$$\varphi ::= p \mid \neg \varphi \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \varphi \rightarrow \varphi \quad \text{with } p \in \mathcal{V}$$

Example

- ▶ $x_1 : Capital, x_2 : City$ in \mathcal{V} , so

$$Capital \rightarrow City \in \mathcal{LP}$$

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$$(((v \vee w) \vee \neg x_1) \wedge (v \vee \neg x_1 \vee \neg y_1)) \rightarrow (((y_1 \wedge y_2) \wedge \neg w) \wedge \neg x_2) \in \mathcal{LP}$$

Propositional Knowledge Bases

A propositional knowledge base K is a set of propositional formulae.

Example. $K = \{Paris \rightarrow Capital, Capital \rightarrow City\}$

Semantics for the propositional language

- ▶ The semantics : if $x = \text{"Paris is a capital"}$, x is a true fact.
- ▶ Interpretations of a formula : assigning true/false to each variable

A formula with n variables, we obtain 2^n possible Interpretations, which can be generated by truth tables.

The truth tables (Semantics)

A propositional valuation/interpretation $I: \Pi \rightarrow \{0, 1\}$ is function mapping a propositional variable $p \in \Pi$ to a truth value $I(p) \in \{0, 1\}$. Let p and q be two proposition letters, the semantics of a propositional formula is defined inductively by the following tables :

		Conjonction		Disjonction	
		p	q	$p \wedge q$	$p \vee q$
Négation		vrai	vrai	vrai	vrai
$\neg p$		vrai	faux	faux	faux
p		faux	vrai	faux	vrai
$\neg p$		faux	faux	faux	faux
		Implication		Equivalence	
		p	q	$p \rightarrow q$	$p \leftrightarrow q$
$\neg p$		vrai	vrai	vrai	vrai
$\neg p$		vrai	faux	faux	faux
$\neg p$		faux	vrai	vrai	faux
$\neg p$		faux	faux	vrai	vrai

Example : all interpretations of $(p \rightarrow (q \wedge r))$

interpretations	p	q	r	$(q \wedge r)$	$(p \rightarrow (q \wedge r))$
I_1	true	true	true	true	true
I_2	true	true	false	false	false
I_3	true	false	true	false	false
I_4	true	false	false	false	false
I_5	false	true	true	true	true
I_6	false	true	false	false	true
I_7	false	false	true	false	true
I_8	false	false	false	false	true

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Model of a formula

I is a model if I makes the formula α true, i.e. $I(\alpha) = \text{true}$.

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Example.

- ▶ I_5, I_6, I_7, I_8 are the models of $(p \rightarrow (q \wedge r))$.
- ▶ Let $K = \{x, \neg x\}$. Is K satisfiable ?

Semantic Notions

- ▶ **Inconsistency** : a formula A is called inconsistent or unsatisfiable if A does not have a model.

Example : $p \wedge \neg p$ is inconsistent

- ▶ **Consistency** : a formula A is called consistent or satisfiable if the formula A has un model.

Example : $p \vee q$ is a consistent formula.

- ▶ **Tautology** : a formula A is called valid (tautology) if A is true for all interpretations of their variables. We denote $\models A$

Example : $(p \vee \neg p)$ is valid.

- ▶ **logically equivalence** : α is logically equivalent to β iff.?

Example : $(p \rightarrow q)$ and $(\neg p \vee q)$ are logically equivalent.
(Check by yourself)

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Semantic Notions

Semantic entailment

A propositional knowledge base K implies a formula ϕ , written
 $K \models \phi$, iff. each model of K is a model of ϕ .

$K_1 = \{x_1 \rightarrow x_2, x_2 \rightarrow x_3\}$, does $K \models x_3$?

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$K2 = \{x1 \rightarrow x2, x2 \wedge x3 \wedge \neg x1 \rightarrow x4, x4 \rightarrow \neg x1, x1\dots\}$, does
 $K2 \models x4$?

How to check $K \models \phi$?

- ▶ Using truth table (by definition)
- ▶ Using syntactic inference systems (Resolution principle, Nature deduction system, ...).

We write $K \vdash \phi$ if ϕ follows from K syntactically.

- ▶ \vdash is called **sound** : if $K \vdash \phi$ then $K \models \phi$.
- ▶ \vdash is called **complete** : if $K \models \phi$ $K \vdash \phi$.

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- ▶ Using syntactic inference systems (Resolution principle, Nature deduction system, ...).

We write $K \vdash \phi$ if ϕ follows from K syntactically.

- ▶ \vdash is called **sound** : if $K \vdash \phi$ then $K \models \phi$.
- ▶ \vdash is called **complete** : if $K \models \phi$ $K \vdash \phi$.

Tutorial 1 - ex1

Importance in computational complexity theory

Definition (Litteral, Clause)

- ▶ A **litteral** is either x (positive) or $\neg x$ (negative litteral) with $x \in \mathcal{V}$. x and $\neg x$ are called **complementary**.
- ▶ A **clause** is a disjunction of litterals.

Example (Clause)

Sont des clauses :

- ▶ $a \vee \neg b \vee c$
- ▶ $(a \vee \neg b \vee c) \wedge (\neg a \vee \neg b \vee c)$

SAT problem : Given a propositional CNF formula, decide if this formula is satisfiable (i.e., has a model).

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NP-complete Problems : we don't know any polynomial algorithm to solve it.

3-SAT and 2-SAT

Complexities :

- ▶ **NP-completeness of 3-SAT** : 3-SAT and general SAT are NP-C problems by Cook Theorem
- ▶ **2-SAT is in the complexity class \mathcal{P}** : There are several polynomial algorithms to solve the 2-SAT problem.

Every decision problem in the complexity class NP can be reduced to the SAT problem !

Many problems are proven to be NP-hard due to a polynomial deduction from SAT problem (i.e., deciding whether a given graph has a 3-coloring)

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Solving puzzles by propositional logics

Given the following facts :

- ▶ Jean says : If Bernard is guilty, Sophie is too.
- ▶ Bernard says : Jean is guilty and Sophie is not.
- ▶ Sophie says : she is not guilty but at least one of the others are

We assume that each one lies if and only if he/she is guilty.

- ▶ Modeling the knowledge in propositional logic.
- ▶ Can we deduce who is guilty ?

Try yourselves.

Outline

Why Semantic Web ?

Semantics and Knowledge Representation

Principle

Syntax and Semantics

Propositional (logic) language

Inference Algorithms for a special PL

A special PL sub-language : Horn rules

Forward Chaining

Backforward Chaining

Inference Algorithms

General algorithm for PL inference can be as expensive as truth table enumeration ! (exponential)

Syntax restrictions of formulae, Horn

Only formulae of the form are allowed in a knowledge base :

$$l_1 \wedge l_2 \wedge l_3 \wedge \dots \wedge l_n \rightarrow l$$

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Chaining from two sides

Two chaining algorithms

Forward Chaining

Principles : starting from a base of initial facts, deduce new facts according to rules in the base

- ▶ **Releasable rule** : all of its conditions are satisfiable
- ▶ **New facts** : the conclusions of releasable rules until saturation or the queried goal is achieved

Backward Chaining

Principles : starting from the goal, replace the goal by sub-goals.

Sub-goals : the conditions of a rule whose conclusion is a goal until that all the sub-goals are either facts or there is no sub-goal can be replaceable (not a conclusion of any rules)

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Forward Chaining Algorithm

Iterative Evaluation of Rules over a base of facts

FB : Fact base ; RB : Rule base

$ForwardChaining(FB_{init}, RB)$ computes the smallest fix point of the operator $Cons$ of immediate consequences from FB_{init}

- ▶ $Cons$ is applied over FB if there exists a releasable rule over FB
- ▶ The result $Cons(FB)$ is obtained by adding FB the conclusion of the rule

Evaluation of a rule

Let FB be a fact base, and the rule $R : \text{if } l_1 \text{ and } \dots \text{ and } l_n \text{ then } //$.

- ▶ $match(l, FB) = \text{true}$ iff $l \in FB$
- ▶ $\text{eval}(R, FB) = \text{true}$ iff for all i , $match(l_i, FB) = \text{true}$

Forward Chaining (ctd.)

```
1 : function ForwardChaining( $FB_{init}$ ,  $RB$ )
2 :      $s(FB) \leftarrow FB_{init}$ 
3 :     repeat
4 :          $FB \leftarrow s(FB)$ 
5 :         for all Rule  $R$  is not yet applied do
6 :             if eval( $R, FB$ ) then  $s(FB) \leftarrow s(FB) \cup conclusions(R)$ 
7 :             end if
8 :         end for
9 :         until  $s(FB) = FB$ 
10 :        return  $s(FB)$ 
11 : end function
```

Forward Chaining (ctd.)

Evaluation of a query Q by forward chaining

Q is satisfiable iff Q appears in the result of

$\text{ForwardChaining}(FB_{init}, RB)$

Result : the algorithm $\text{ForwardChaining}(FB_{init}, RB)$ is polynomial w.r.t. the size of sFB_{init} , the size of RB , and the size of a rule.

Forward Chaining (example)

Tutorial 1, ex2-Q1

Optimisation of the forward chaining algorithm

Idea : don't repeat the operation of making correspondences between facts and conditions

- ▶ from one rule to another
- ▶ from one iteration to another

We proceed with the propagation of facts over the rules where the facts appear as condition.

- ▶ need indexing of atom-conditions regarding rules :
 $InRuleConditions(I) = \{R \mid I \text{ is an atom appearing in the condition of } R\}$
- ▶ for each rule, we keep memorizing the conditions satisfied in previous iterations.

We update the rules by keeping only their conditions that are not yet satisfied while the propagation of a fact f :

$$conditions(R) \leftarrow conditions(R) - f$$

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Algorithm of Forward Chaining with Propagation

```
function ForwardChainingBis( $FB_{init}$ ,  $RB$ )
     $FB \leftarrow FB_{init}$ 
    for all fact  $F \in FB_{init}$  do
         $FB \leftarrow FB \cup \text{Propagate}(F, RB)$ 
    end for
    return  $FB$ 
end function

function Propagate( $F, RB$ )
     $\Delta F \leftarrow \emptyset$ 
    for all rule  $R \in \text{InRuleConditions}(F)$  do
        Delete from  $\text{conditions}(R)$  the atom  $I$  such that  $\text{match}(I, \text{condition}(R))$ 
        if  $\text{conditions}(R) = \emptyset$  then
             $RB \leftarrow RB \setminus \{R\}$ 
             $\Delta F \leftarrow \Delta F \cup \text{conclusions}(R)$ 
        end if
    end for
     $FB \leftarrow \Delta F$ 
    for all fact  $F \in \Delta F$  do
         $FB \leftarrow FB \cup \text{Propagate}(F, RB)$ 
    end for
    return  $FB$ 
end function
```

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        end if
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     $FB \leftarrow \Delta F$ 
    for all fact  $F \in \Delta F$  do
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    end for
    return  $FB$ 
end function
```

Example

Toturail 1, ex2-Q2/3

Backforward Chaining

Principle

Algorithm of replacing a goal by sub-goals, by matching a current goal (the user's query at beginning) with the conclusion atoms of rules.

When to use it ?

To guide the questions proposed by users in the case of incomplete information from FB to satisfy the query.

Generally, it is combined with the forward chaining :

- ▶ forward chaining to saturate the FB
- ▶ backward chaining to guide the interaction with users of a AND/OR tree.

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Backward Chaining Algorithm

```
1 : function BackwardChaining( $Q, FB_{init}, RB$ )
2 :      $FB \leftarrow FB_{init}$ 
3 :     if  $match(Q, FB)$  then
4 :         return true
5 :     else
6 :         for all Rule  $r$  : If  $I_1$  And  $I_2$  And ... And  $I_n$  Then Conc t.q.
7 :              $match(Q, Conc)$  do
8 :                  $bool \leftarrow true; i \leftarrow 1$ 
9 :                 while  $bool$  And  $i \leq n$  do
10 :                      $bool \leftarrow BackwardChaining(I_i, FB, RB)$ 
11 :                      $i \leftarrow i + 1$ 
12 :                 end while
13 :                 if  $bool$  then return true
14 :                 end if
15 :             end for
16 :             return false
17 :     end if
18 : end function
```

Backward Chaining with questions to users

Principles of the questions

To decide the questions that can be asked to users.

- ▶ *demandable(I)* : we can ask a question over *I*. Generally, if *I* doesn't appear in conclusions of rules, it is demandable.
- ▶ *question(I)* : boolean returning the answer of the user to the question : if *I* is *true*.

Modification Principles : If we cannot find a fact, we can ask the user as last resort.

Backward Chaining with questions to users

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Backward Chaining with interactive questions

```
function BackwardChaining2(Q, FBinit, RB)
    FB ← FBinit
    if match(Q, FB) then
        return true
    else
        for all rgle r : if l1 and l2 and ... and ln then Conc t.q. match(Q, Conc)
    do
        bool ← true; i ← 1
        while bool Et i ≤ n do
            bool ← BackwardChaining(li, FB, RB)
            i ← i + 1
        end while
        if bool then return true
        end if
    end for
    if demandable(Q) then return question(Q)
    else return false
    end if
end if
end function
```

Exemple

Tutorial 1, ex3.

Recursive Rules

Problem of loopback risk

Solution : don't re-apply the algorithm over the sub-goals that have been examined.

```
function BackwardChaining3(Q, FBinit, RB)
    FB ← FBinit
    if match(Q, FB) then
        return true
    else
        AlreadyTried ← ∅
        for all Rule r : if I1 and I2 and ... and In then Conc t.q. match(Q, Conc) do
            bool ← true; i ← 1
            while bool And i ≤ n do
                if Ii ∉ AlreadyTried then
                    AlreadyTried ← AlreadyTried ∪ {Ii}
                    bool ← BackwardChaining(Ii, FB, RB)
                end if
                i ← i + 1
            end while
            if bool then return true
        end if
    end for
    if demandable(Q) then return question(Q)
    else return false
end if
end function
```

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```
function BackwardChaining3(Q, FBinit, RB)
    FB ← FBinit
    if match(Q, FB) then
        return true
    else
        AlreadyTried ← ∅
        for all Rule r : if I1 and I2 and ... and In then Conc t.q. match(Q, Conc) do
            bool ← true; i ← 1
            while bool And i ≤ n do
                if Ii ∉ AlreadyTried then
                    AlreadyTried ← AlreadyTried ∪ {Ii}
                    bool ← BackwardChaining(Ii, FB, RB)
                end if
                i ← i + 1
            end while
            if bool then return true
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```