1. RTM phaseless: elastic; 03.15

The RTM imaging function studied in [1] for reconstructing extended targets is

$$I_1(z) = -\omega^2 \mathrm{Im} \sum_{q=e_1,e_2} \int_{\Gamma_s} \int_{\Gamma_r} \Big(c_p G_p(z, x_r s) q + c_s G_s(z, x_s) q \Big)$$
$$\cdot \Big(c_p G_p(z, x_r) + c_s G_s(z, x_r) \Big) \overline{u_q^s(x_r, x_s)} ds(x_r) ds(x_s)$$

For vector $x = (x_1, x_2)^T$, we introduce tow unit vectors $\hat{x} = x/|x| := (\hat{x}_1, \hat{x}_2)^T$ and $\tilde{x} = (-\hat{x}_2, \hat{x}_1)$. We define $A(x) = \hat{x}\hat{x}^T$ and $B(x) = \tilde{x}\tilde{x}^T$

$$I_2(z) = -\omega^2 \mathrm{Im} \sum_{q=e_1,e_2} \int_{\Gamma_s} \int_{\Gamma_r} \Big(k_p g_p(z, x_r s) A(x_s) q + k_s g_s(z, x_s) B(x_s) q \Big)$$
$$\cdot \Big(k_p g_p(z, x_r) A(x_r) + k_s g_s(z, x_r) B(x_r) \Big) \overline{u_q^s(x_r, x_s)} ds(x_r) ds(x_s)$$

or

$$I_2(z) = -\omega^2 \mathrm{Im} \sum_{q=e_1,e_2} \int_{\Gamma_s} \int_{\Gamma_r} \Big(c_p G_p(z, x_r s) q + c_s G_s(z, x_s) q \Big)$$
$$\cdot \Big(k_p g_p(z, x_r) A(x_r) + k_s g_s(z, x_r) B(x_r) \Big) \overline{u_q^s(x_r, x_s)} ds(x_r) ds(x_s)$$

and

$$I_3(z) = -\omega^2 \mathrm{Im} \sum_{q=e_1,e_2} \int_{\Gamma_s} \int_{\Gamma_r} \Big(k_p g_p(z, x_r s) A(x_s) q + k_s g_s(z, x_s) B(x_s) q \Big)$$
$$\cdot \Big(k_p g_p(z, x_r) \hat{x_r} D_p(x_r, x_s) + k_s g_s(z, x_r) \tilde{x_r} D_s(x_r, x_s) \Big) ds(x_r) ds(x_s)$$

or

$$I_3(z) = -\omega^2 \mathrm{Im} \sum_{q=e_1,e_2} \int_{\Gamma_s} \int_{\Gamma_r} \Big(c_p G_p(z, x_r s) q + c_s G_s(z, x_s) q \Big)$$
$$\cdot \Big(k_p g_p(z, x_r) \hat{x_r} D_p(x_r, x_s) + k_s g_s(z, x_r) \tilde{x_r} D_s(x_r, x_s) \Big) ds(x_r) ds(x_s)$$

where

$$D_p(x_r, x_s) = \frac{|\hat{x_r}^T u_q(x_r, x_s)|^2 - |\hat{x_r}^T u_q^i(x_r, x_s))|^2}{\hat{x_r}^T u_q^i(x_r, x_s))}$$
$$D_s(x_r, x_s) = \frac{|\tilde{x_r}^T u_q(x_r, x_s)|^2 - |\tilde{x_r}^T u_q^i(x_r, x_s))|^2}{\tilde{x_r}^T u_q^i(x_r, x_s))}$$

Conjecture

$$|I_1(z) - I_2(z)| \le C \frac{1}{k_p R_s}, \quad |I_2(z) - I_3(z)| \le C \frac{1}{k_p R_s}$$

Lemma 1.1 *We have*

$$k_p \int_{|x|=R} g_p(z, x) A(x) \overline{G(x, y)} ds(x) = \mathrm{Im}\, G_p(z, y) + W_p(y, z)$$
$$k_s \int_{|x|=R} g_s(z, x) B(x) \overline{G(x, y)} ds(x) = \mathrm{Im}\, G_s(z, y) + W_s(y, z)$$

where $|W_\alpha^{ij}(z,y)| + k_\alpha^{-1}|\nabla_z W_\alpha^{ij}(z,y)| \le C_\alpha R^{-1}$ *for some constanc* C_α *depending on* $k_\alpha|z|, k_\alpha|y|$, $\alpha \in \{p,s\}$.

Proof. We first recall the following estimate for the first Hankel function in [2, p.197], for any $t>0$, we have

$$H_0^{(1)}(t) = \left(\frac{2}{\pi t}\right)^{1/2} e^{\mathbf{i}(t-\pi/4)} + R_0(t), \quad H_1^{(1)}(t) = \left(\frac{2}{\pi t}\right)^{1/2} e^{\mathbf{i}(t-3\pi/4)} + R_1(t),$$

where $|R_j(t)| \le Ct^{-3/2}$, $j=0,1$, for some constant $C>0$ independent of t. By the defination of Green Tensor, we have

$$G_p(x,y) = \frac{\mathbf{i}}{\sqrt{8\pi}(\lambda+2\mu)} A(x-y)\frac{1}{(k_p|x-y|)^{1/2}} e^{\mathbf{i}k_p|x-y|-\mathbf{i}\frac{\pi}{4}} + O(\frac{1}{(k_p|x-y|)^{3/2}})$$
$$G_s(x,y) = \frac{\mathbf{i}}{\sqrt{8\pi}\mu} B(x-y)\frac{1}{(k_s|x-y|)^{1/2}} e^{\mathbf{i}k_p|x-y|-\mathbf{i}\frac{\pi}{4}} + O(\frac{1}{(k_s|x-y|)^{3/2}})$$

Some simple manipulation yields:

$$|A(x-y)-A(x)| \le C_1/|x|, \quad |B(x-y)-B(x)| \le C_2/|x|$$
$$|\frac{1}{|x-y|} - \frac{1}{|x|}| \le C_3/|x|^2, \quad ||x-y| - (|x| - \hat{x}\cdot y)| \le C_4/|x|$$

where C_i, i=1,2,3,4 depend on $|y|$.

$$G_p(x,y) = \frac{\mathbf{i}}{\sqrt{8\pi}(\lambda+2\mu)} A(x)\frac{1}{(k_p|x|)^{1/2}} e^{\mathbf{i}k_p(|x|-\hat{x}\cdot y)-\mathbf{i}\frac{\pi}{4}} + \gamma_p(x,y)$$
$$G_s(x,y) = \frac{\mathbf{i}}{\sqrt{8\pi}\mu} B(x)\frac{1}{(k_s|x|)^{1/2}} e^{\mathbf{i}k_s(|x|-\hat{x}\cdot y)-\mathbf{i}\frac{\pi}{4}} + \gamma_s(x,y)$$
$$g_\alpha(x,y) = \frac{\mathbf{i}}{\sqrt{8\pi}\mu} \frac{1}{(k_\alpha|x|)^{1/2}} e^{\mathbf{i}k_s(|x|-\hat{x}\cdot y)-\mathbf{i}\frac{\pi}{4}} + \gamma(x,y)$$

where $|\gamma_\alpha(x,y)| \le C(k_\alpha|x|)^{-3/2}$ for some constant C depending on $k_\alpha|y|$, $\alpha\in\{p,s\}$. □

Now we turn to the analysisi of the imaging function $I_3(z)$. We first observe that:

$$D_p(x_r,x_s) = \hat{x_r}^T\overline{u_q^s} + \frac{|\hat{x_r}^T u_q^s(x_r,x_s)|^2}{\hat{x_r}^T u_q^i(x_r,x_s))} + \frac{(\hat{x_r}^T u_q^s(x_r,x_s))(\hat{x_r}^T\overline{u_q^i(x_r,x_s)})}{\hat{x_r}^T u_q^i(x_r,x_s))}$$
$$D_s(x_r,x_s) = \tilde{x_r}^T\overline{u_q^s} + \frac{|\tilde{x_r}^T u_q^s(x_r,x_s)|^2}{\tilde{x_r}^T u_q^i(x_r,x_s))} + \frac{(\tilde{x_r}^T u_q^s(x_r,x_s))(\tilde{x_r}^T\overline{u_q^i(x_r,x_s)})}{\tilde{x_r}^T u_q^i(x_r,x_s))}$$

Lemma 1.2 *We have* $|u_q^s(x_r,x_s)| \le C(1+\|T\|)(k_pR_r)^{-1/2}(k_pR_s)^{-1/2}$ *for any* $x_r \in \Gamma_r, x_s \in \Gamma_s$, *where the constant* C *may depend on* kd_D *bu is independent of* k_p, k_s, d_D, R_r, R_s.

2. Numerical Experiment

References

[1] Zhiming CHEN and GuangHui HUANG. Reverse time migration for extended obstacles: Elastic waves. *SCIENTIA SINICA Mathematica*, 45(8):1103–1114, 2015.

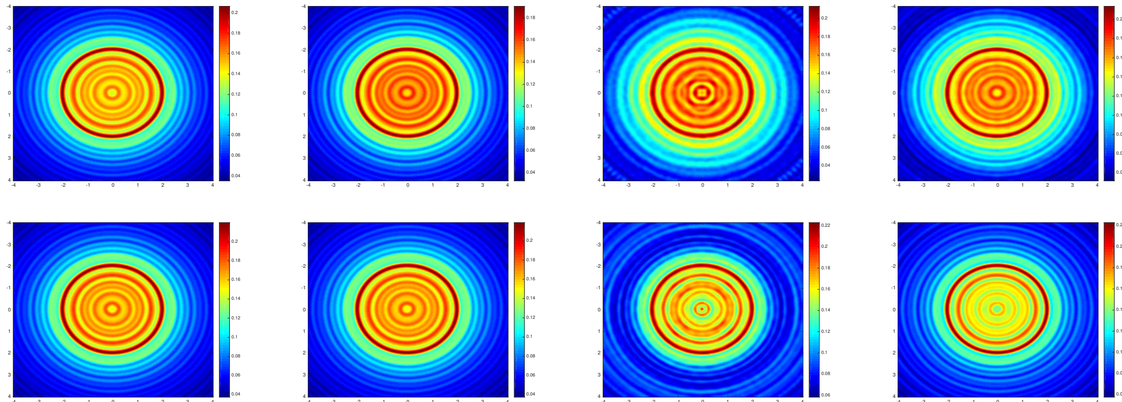

Figure 1. Circle;From left to right: vector imaging, scalar imaging, phaseless imaging128, phaseless imaging512; From up to down: R=10, R=100

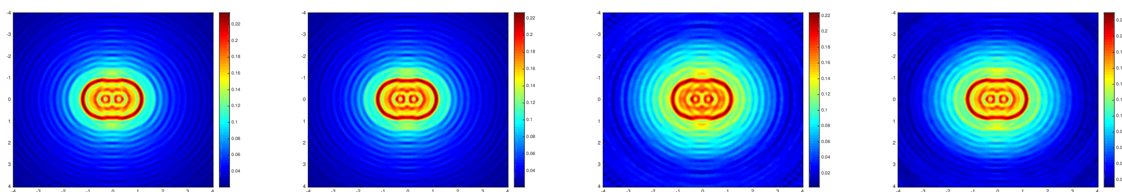

Figure 2. Peanut;From left to right: vector imaging, scalar imaging, phaseless imaging128, phaseless imaging512;

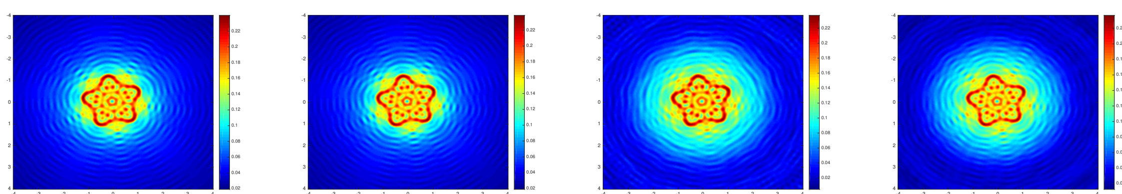

Figure 3. Peanut;From left to right: vector imaging, scalar imaging, phaseless imaging128, phaseless imaging512;

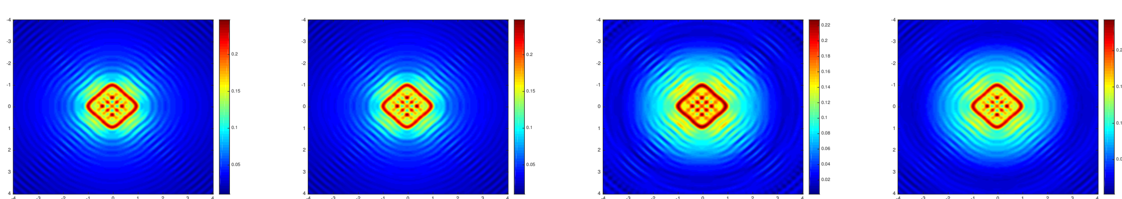

Figure 4. Peanut;From left to right: vector imaging, scalar imaging, phaseless imaging128, phaseless imaging512;

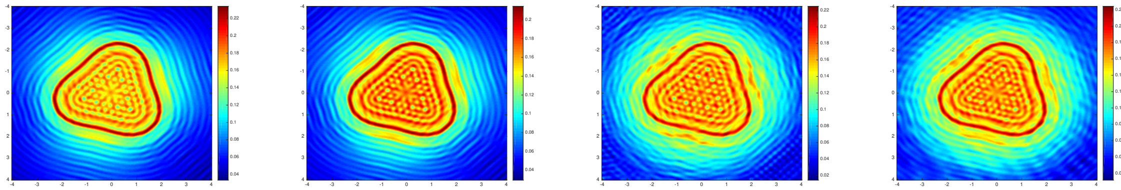

Figure 5. Peanut;From left to right: vector imaging, scalar imaging, phaseless imaging128, phaseless imaging512;

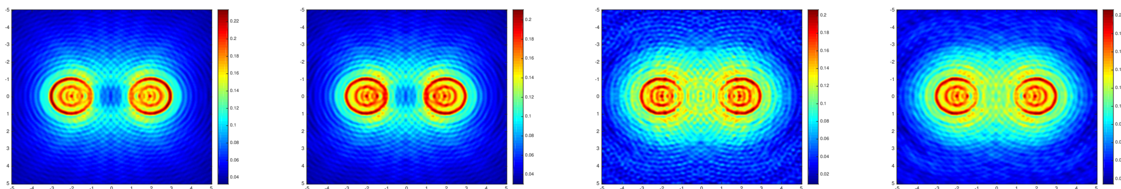

Figure 6. Circle;From left to right: vector imaging, scalar imaging, phaseless imaging128, phaseless imaging512; From up to down: R=10, R=100

[2] George Neville Watson. *A treatise on the theory of Bessel functions.* Cambridge university press, 1995.