

1. RTM phaseless: elastic; 03.15

The RTM imaging function studied in [1] for reconstructing extended targets is

$$I_1(z) = -\omega^2 \text{Im} \sum_{q=e_1, e_2} \int_{\Gamma_s} \int_{\Gamma_r} \left(c_p G_p(z, x_r s) q + c_s G_s(z, x_s) q \right) \cdot \left(c_p G_p(z, x_r) + c_s G_s(z, x_r) \right) \overline{u_q^s(x_r, x_s)} ds(x_r) ds(x_s)$$

For vector $x = (x_1, x_2)^T$, we introduce tow unit vectors $\hat{x} = x/|x| := (\hat{x}_1, \hat{x}_2)^T$ and $\tilde{x} = (-\hat{x}_2, \hat{x}_1)$. We define $A(x) = \hat{x} \hat{x}^T$ and $B(x) = \tilde{x} \tilde{x}^T$

$$I_2(z) = -\omega^2 \text{Im} \sum_{q=e_1, e_2} \int_{\Gamma_s} \int_{\Gamma_r} \left(k_p g_p(z, x_r s) A(x_s) q + k_s g_s(z, x_s) B(x_s) q \right) \cdot \left(k_p g_p(z, x_r) A(x_r) + k_s g_s(z, x_r) B(x_r) \right) \overline{u_q^s(x_r, x_s)} ds(x_r) ds(x_s)$$

or

$$I_2(z) = -\omega^2 \text{Im} \sum_{q=e_1, e_2} \int_{\Gamma_s} \int_{\Gamma_r} \left(c_p G_p(z, x_r s) q + c_s G_s(z, x_s) q \right) \cdot \left(k_p g_p(z, x_r) A(x_r) + k_s g_s(z, x_r) B(x_r) \right) \overline{u_q^s(x_r, x_s)} ds(x_r) ds(x_s)$$

and

$$I_3(z) = -\omega^2 \text{Im} \sum_{q=e_1, e_2} \int_{\Gamma_s} \int_{\Gamma_r} \left(k_p g_p(z, x_r s) A(x_s) q + k_s g_s(z, x_s) B(x_s) q \right) \cdot \left(k_p g_p(z, x_r) \hat{x}_r D_p(x_r, x_s) + k_s g_s(z, x_r) \tilde{x}_r D_s(x_r, x_s) \right) ds(x_r) ds(x_s)$$

or

$$I_3(z) = -\omega^2 \text{Im} \sum_{q=e_1, e_2} \int_{\Gamma_s} \int_{\Gamma_r} \left(c_p G_p(z, x_r s) q + c_s G_s(z, x_s) q \right) \cdot \left(k_p g_p(z, x_r) \hat{x}_r D_p(x_r, x_s) + k_s g_s(z, x_r) \tilde{x}_r D_s(x_r, x_s) \right) ds(x_r) ds(x_s)$$

where

$$D_p(x_r, x_s) = \frac{|\hat{x}_r^T u_q(x_r, x_s)|^2 - |\hat{x}_r^T u_q^i(x_r, x_s)|^2}{\hat{x}_r^T u_q^i(x_r, x_s)}$$

$$D_s(x_r, x_s) = \frac{|\tilde{x}_r^T u_q(x_r, x_s)|^2 - |\tilde{x}_r^T u_q^i(x_r, x_s)|^2}{\tilde{x}_r^T u_q^i(x_r, x_s)}$$

Conjecture

$$|I_1(z) - I_2(z)| \leq C \frac{1}{k_p R_s}, \quad |I_2(z) - I_3(z)| \leq C \frac{1}{k_p R_s}$$

Lemma 1.1 *We have*

$$k_p \int_{|x|=R} g_p(z, x) A(x) \overline{G(x, y)} ds(x) = \text{Im} G_p(z, y) + W_p(y, z)$$

$$k_s \int_{|x|=R} g_s(z, x) B(x) \overline{G(x, y)} ds(x) = \text{Im} G_s(z, y) + W_s(y, z)$$

where $|W_\alpha^{ij}(z, y)| + k_\alpha^{-1} |\nabla_z W_\alpha^{ij}(z, y)| \leq C_\alpha R^{-1}$ for some constant C_α depending on $k_\alpha |z|, k_\alpha |y|$, $\alpha \in \{p, s\}$.

Proof. We first recall the following estimate for the first Hankel function in [2, p.197], for any $t > 0$, we have

$$H_0^{(1)}(t) = \left(\frac{2}{\pi t}\right)^{1/2} e^{i(t-\pi/4)} + R_0(t), \quad H_1^{(1)}(t) = \left(\frac{2}{\pi t}\right)^{1/2} e^{i(t-3\pi/4)} + R_1(t),$$

where $|R_j(t)| \leq Ct^{-3/2}$, $j = 0, 1$, for some constant $C > 0$ independent of t . By the definition of Green Tensor, we have

$$G_p(x, y) = \frac{\mathbf{i}}{\sqrt{8\pi}(\lambda + 2\mu)} A(x - y) \frac{1}{(k_p|x - y|)^{1/2}} e^{ik_p|x - y| - i\frac{\pi}{4}} + O\left(\frac{1}{(k_p|x - y|)^{3/2}}\right)$$

$$G_s(x, y) = \frac{\mathbf{i}}{\sqrt{8\pi}\mu} B(x - y) \frac{1}{(k_s|x - y|)^{1/2}} e^{ik_s|x - y| - i\frac{\pi}{4}} + O\left(\frac{1}{(k_s|x - y|)^{3/2}}\right)$$

Some simple manipulation yields:

$$|A(x - y) - A(x)| \leq C_1/|x|, \quad |B(x - y) - B(x)| \leq C_2/|x|$$

$$\left|\frac{1}{|x - y|} - \frac{1}{|x|}\right| \leq C_3/|x|^2, \quad ||x - y| - (|x| - \hat{x} \cdot y)| \leq C_4/|x|$$

where C_i , $i=1,2,3,4$ depend on $|y|$. □

Now we turn to the analysis of the imaging function $I_3(z)$. We first observe that:

$$D_p(x_r, x_s) = \hat{x}_r^T \overline{u_q^s} + \frac{|\hat{x}_r^T u_q^s(x_r, x_s)|^2}{\hat{x}_r^T u_q^i(x_r, x_s)} + \frac{(\hat{x}_r^T u_q^s(x_r, x_s))(\hat{x}_r^T \overline{u_q^i(x_r, x_s)})}{\hat{x}_r^T u_q^i(x_r, x_s)}$$

$$D_s(x_r, x_s) = \tilde{x}_r^T \overline{u_q^s} + \frac{|\tilde{x}_r^T u_q^s(x_r, x_s)|^2}{\tilde{x}_r^T u_q^i(x_r, x_s)} + \frac{(\tilde{x}_r^T u_q^s(x_r, x_s))(\tilde{x}_r^T \overline{u_q^i(x_r, x_s)})}{\tilde{x}_r^T u_q^i(x_r, x_s)}$$

2. Numerical Experiment

References

- [1] Zhiming CHEN and GuangHui HUANG. Reverse time migration for extended obstacles: Elastic waves. *SCIENTIA SINICA Mathematica*, 45(8):1103–1114, 2015.
- [2] George Neville Watson. *A treatise on the theory of Bessel functions*. Cambridge university press, 1995.

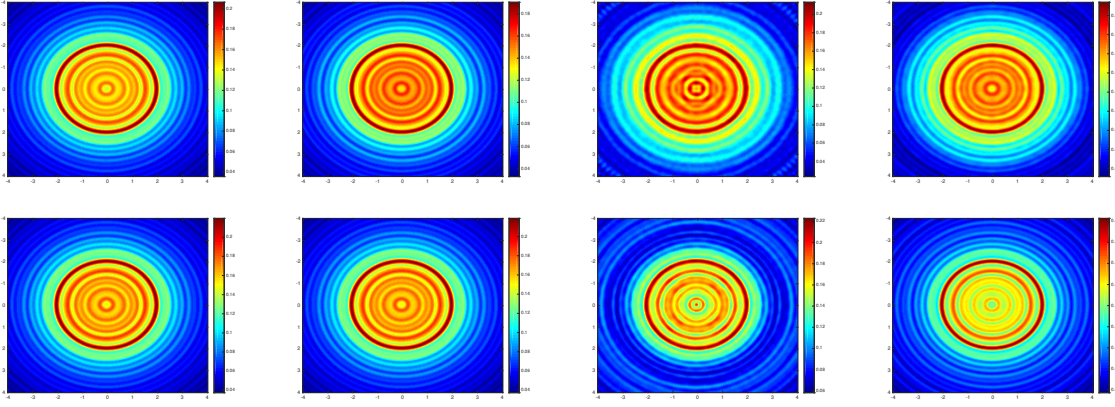


Figure 1. Circle; From left to right: vector imaging, scalar imaging, phaseless imaging128, phaseless imaging512; From up to down: $R=10$, $R=100$

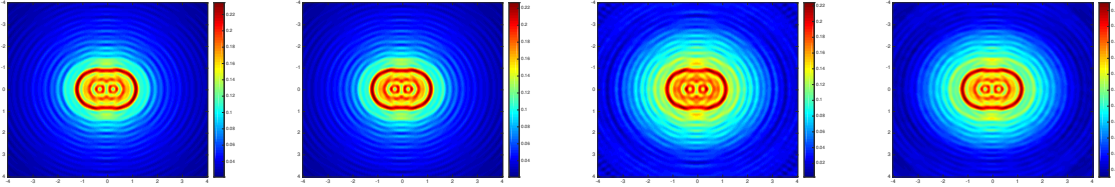


Figure 2. Peanut; From left to right: vector imaging, scalar imaging, phaseless imaging128, phaseless imaging512;

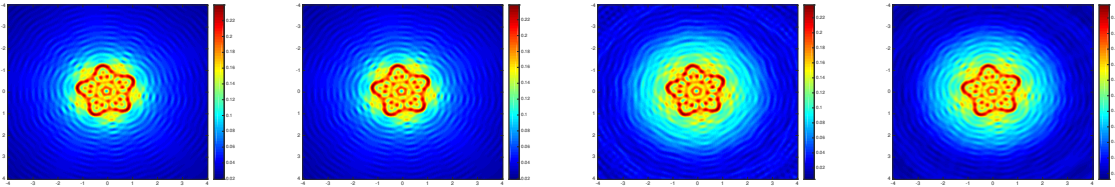


Figure 3. Peanut; From left to right: vector imaging, scalar imaging, phaseless imaging128, phaseless imaging512;

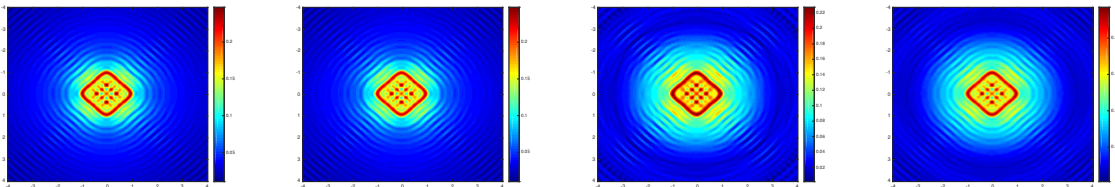


Figure 4. Peanut; From left to right: vector imaging, scalar imaging, phaseless imaging128, phaseless imaging512;

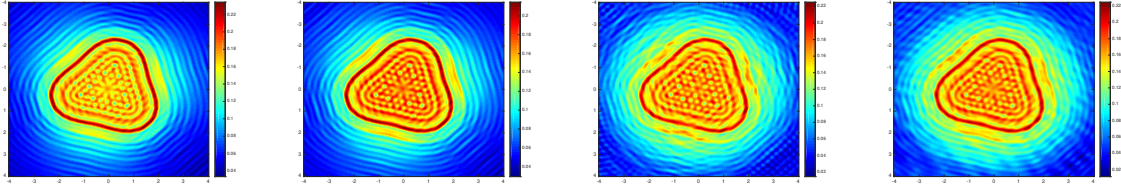


Figure 5. Peanut; From left to right: vector imaging, scalar imaging, phaseless imaging128, phaseless imaging512;

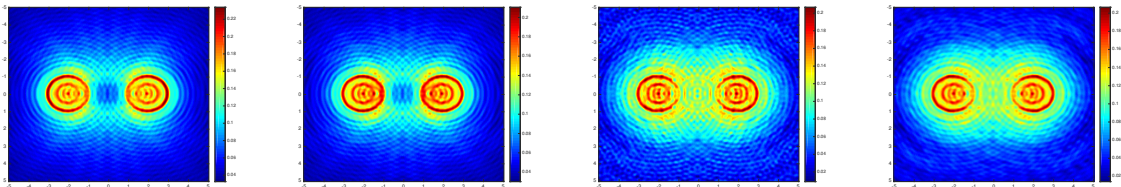


Figure 6. Circle; From left to right: vector imaging, scalar imaging, phaseless imaging128, phaseless imaging512; From up to down: $R=10$, $R=100$