Numerical Tests of RTM for Locally Perturbed Two-layers Media

1. Introdution

$$\begin{aligned}
&\Delta u + k(x)^2 u = 0 \quad \text{ in } \mathbb{R}^2,\\
&\Delta u_0 + k_0(x)^2 u = 0 \quad \text{ in } \mathbb{R}^2,\\
&u(x) = u^s(x) + u_0(x)\\
&u_0(x) = u^i(x) + u^r(x)\\
&u^i(x) = e^{-\mathbf{i}k_1 d\cdot x}
\end{aligned}$$

where

$$k_0(x) = \begin{cases} k_1 & \text{in } \mathbb{R}^2_+ \\ k_2 & \text{in } \mathbb{R}^2_- \end{cases}$$

$$k(x) = \begin{cases} k_1 & \text{in } \mathbb{R}^2_+ \backslash \bar{D} \\ k_2 & \text{in } \mathbb{R}^2_- \backslash \bar{D} \\ k_3 & \text{in } D \end{cases}$$

and

$$\begin{aligned}
&d = (\cos\theta, \sin\theta), \quad \theta \in (0, \pi)\\
&u^r(x, d) == \begin{cases} \frac{k_1 \sin\theta - k_2 \sin\phi}{k_1 \sin\theta + k_2 \sin\phi} e^{-\mathbf{i}k_1(\cos\theta x_1 - \sin\theta x_2)} & \text{in } \mathbb{R}^2_+ \\ \frac{2k_1 \sin\theta}{k_1 \sin\theta + k_2 \sin\phi} e^{-\mathbf{i}k_2(\cos\phi x_1 + \sin\phi x_2)} & \text{in } \mathbb{R}^2_- \end{cases}\\
&k_1 \cos\theta = k_2 \cos\phi
\end{aligned}$$

Imaging Condition:

$$I(z) = \operatorname{Im} \sum_{j=1}^{N_d} \sum_{r=1}^{N_r} u^{d_j}(z)[\frac{\partial \Phi(k_1, x_r, z)}{\partial x_r(x_2)} \overline{(u^s(x_r, d_j) + u^r(x_r, d_j))}]\, ds(x_r) ds(x_s).$$

where

$$\begin{aligned}
&d_j = (\cos\theta_j, \sin\theta_j), \quad \theta_j = \frac{j\pi}{N_d + 1}\\
&x_r = (\frac{2(r-1)d}{N_r - 1} - d, h)\\
&\Phi(k, x, y) \ \text{ is the fundamental solution of Helmholtz equation.}
\end{aligned}$$

2. Numerical Test

Parameter setting:

$$N_d = 201,\ N_r = 401,\ d = 200,\ h = 2, k_1 = 1\omega,\ k_2 = 1.5\omega,\ k_3 = 2\omega$$

The first row shows the imaginary part of I(z), and the second shows the real part of I(z).

Figure1,Figure5 : $\omega = 1$

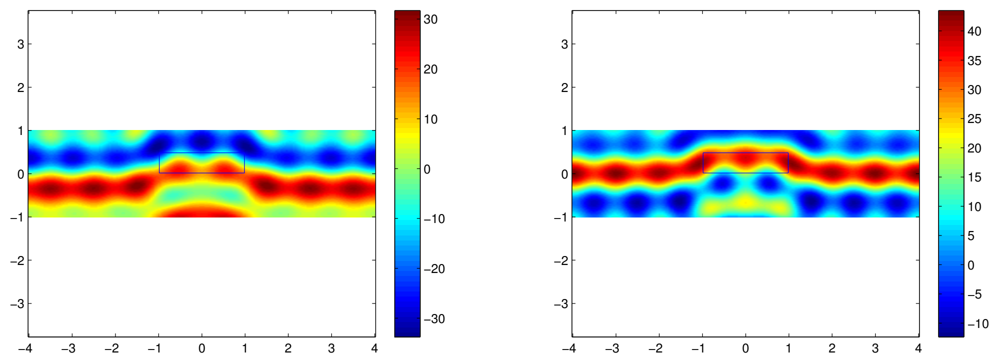


Figure 1.

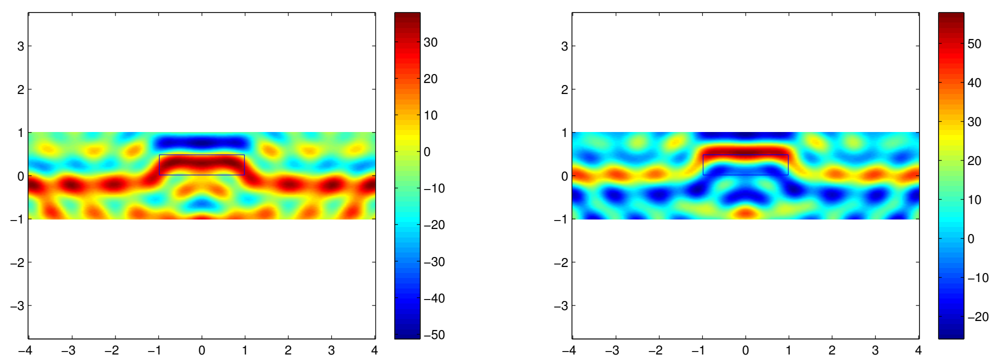


Figure 2.

Figure2,Figure6 : $\omega = 1.5$
Figure3,Figure7 : $\omega = 2$
Figure4,Figure8 : $\omega = [1, 1.5, 2]$
Figure1-Figure4 : the shape of D is retangle: $[-1, 1] \times [0, 0.5]$
Figure5-Figure8 : the shape of D is hemicircle: $x_1^2 + (2x_2)^2 = 1,\ x_2 > 0$
Imaging domain is $[-4, 4] \times [-1, 1]$

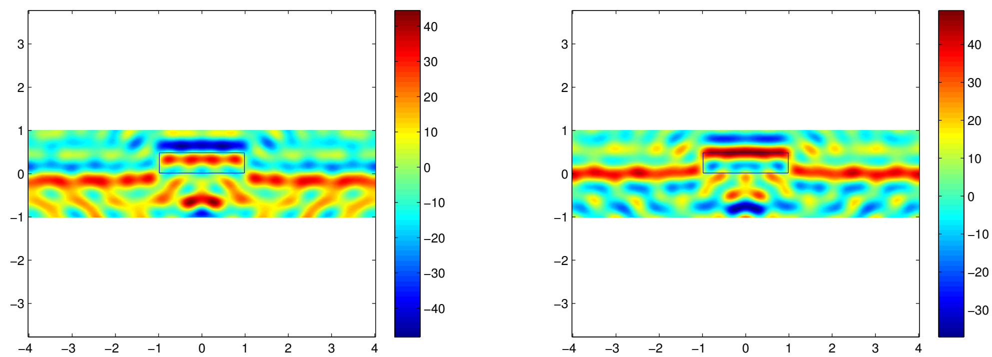


Figure 3.

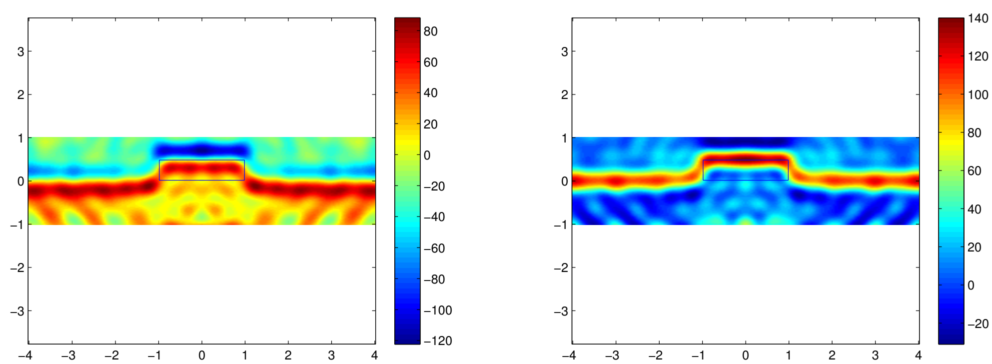


Figure 4.

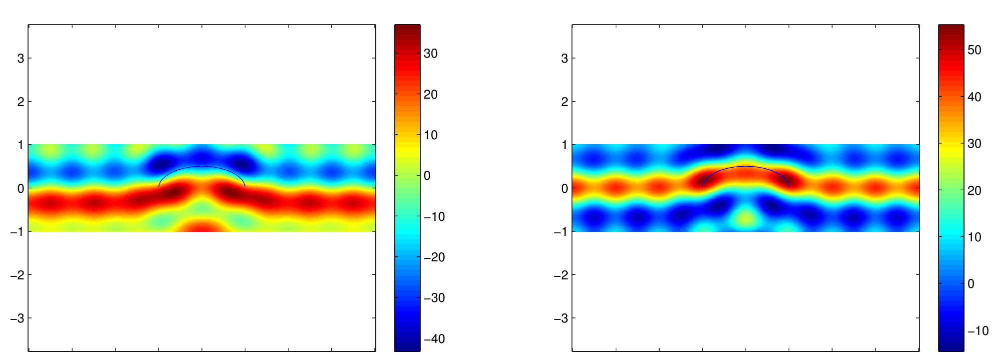


Figure 5.

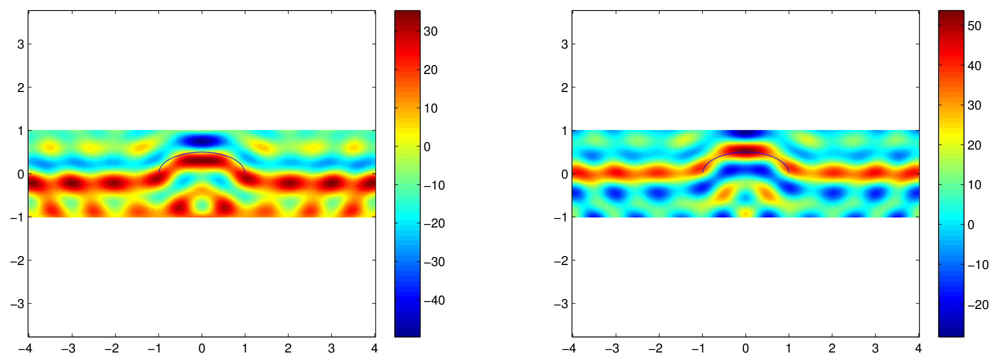
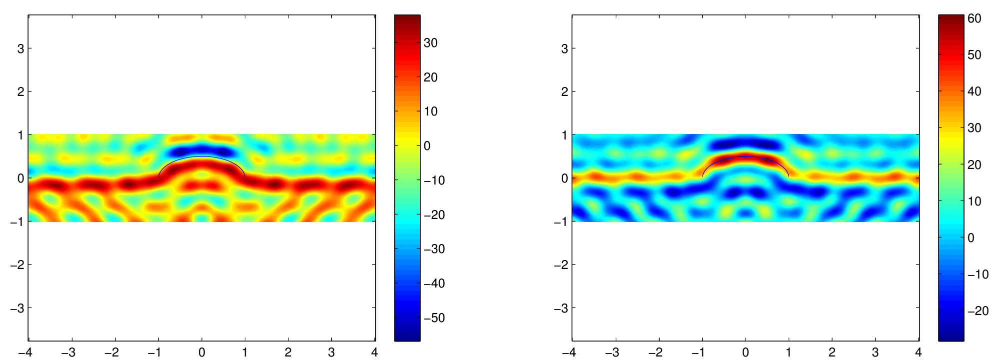


Figure 6.



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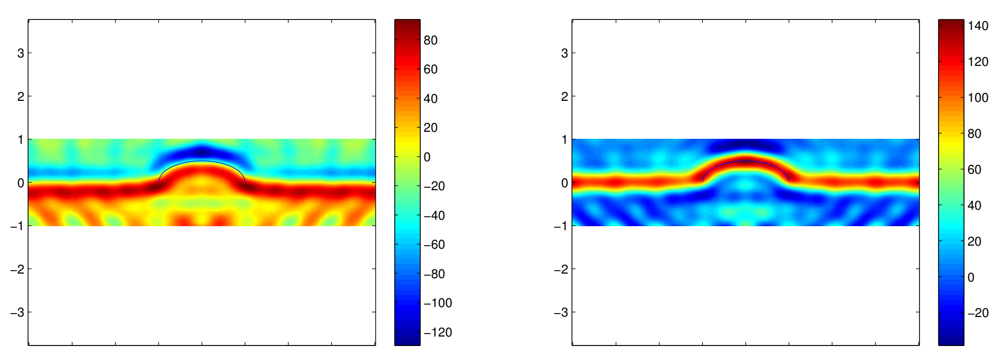


Figure 8.

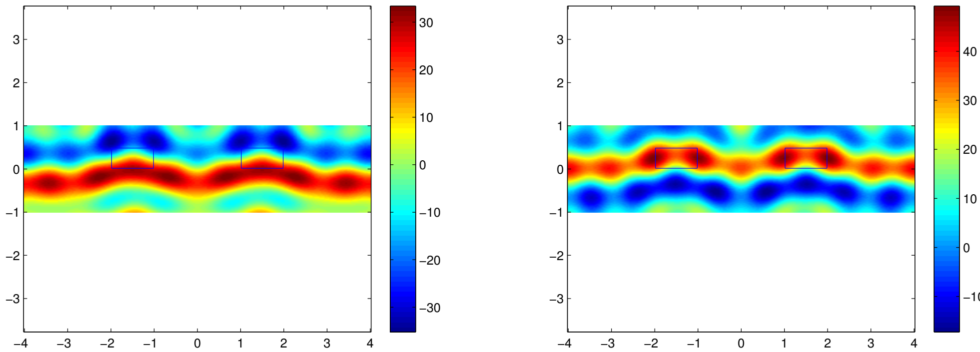


Figure 9.

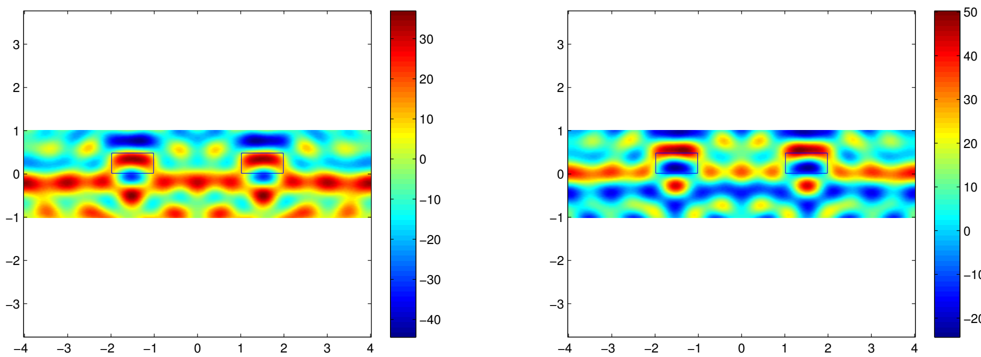


Figure 10.

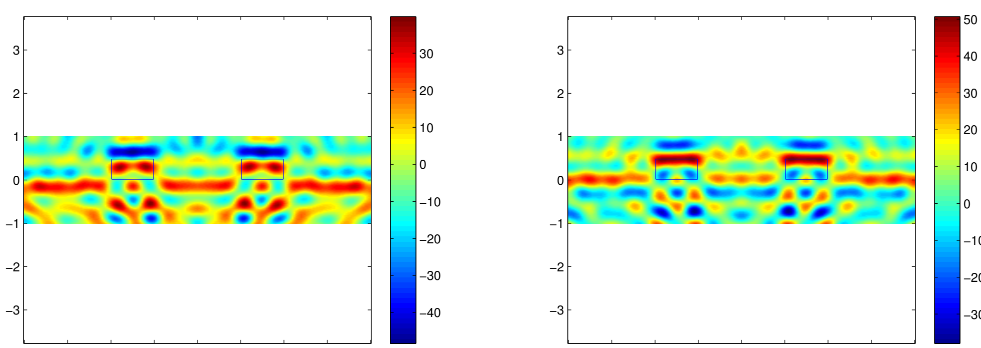
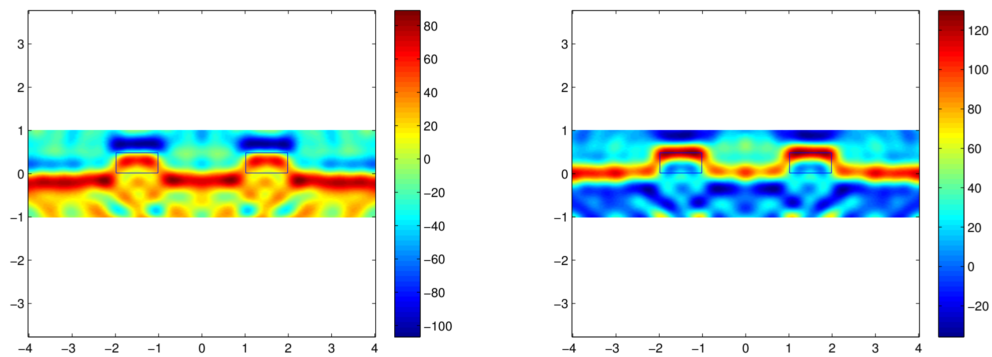


Figure 11.


3
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−2
−3
−4
−3
−2
−1
0
1
2
3
4
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20
0
−20
−40
−60
−80
−100
120
100
80
60
40
20
0
−20

Figure 12.