Different Imaging Functions and their Numerical Tests

1. Imaging Functions

$$I_1(z) = \mathrm{Im} \sum_{q=e_1,e_2} \int_{\Gamma_0^d} \int_{\Gamma_0^d} [\mathbb{T}_D(x_s,z)^T q] \cdot [\mathbb{T}_D(x_r,z)^T \overline{u_q^s(x_r,x_s)}]\, ds(x_r) ds(x_s).$$

$$I_2(z) = \mathrm{Im} \sum_{q=e_1,e_2} \int_{\Gamma_0^d} \int_{\Gamma_0^d} \mathbf{i}[\frac{\partial \Phi^s(x_s,z)}{\partial x_2(x_s)}] \nabla_z \times [\mathbb{T}_D(x_r,z)^T \overline{u_{e_2}^s(x_r,x_s)}]\, ds(x_r) ds(x_s).$$

$$I_3(z) = \mathrm{Im} \sum_{q=e_1,e_2} \int_{\Gamma_0^d} \int_{\Gamma_0^d} \mathbf{i}[\frac{\partial \Phi^p(x_s,z)}{\partial x_2(x_s)}] \nabla_z \times [\mathbb{T}_D(x_r,z)^T \overline{u_{e_2}^s(x_r,x_s)}]\, ds(x_r) ds(x_s).$$

$$I_4(z) = \mathrm{Im} \sum_{q=e_1,e_2} \int_{\Gamma_0^d} \int_{\Gamma_0^d} \mathbf{i}[\frac{\partial \Phi^s(x_s,z)}{\partial x_2(x_s)}] \nabla_z \cdot [\mathbb{T}_D(x_r,z)^T \overline{u_{e_2}^s(x_r,x_s)}]\, ds(x_r) ds(x_s).$$

$$I_5(z) = \mathrm{Im} \sum_{q=e_1,e_2} \int_{\Gamma_0^d} \int_{\Gamma_0^d} \mathbf{i}[\frac{\partial \Phi^p(x_s,z)}{\partial x_2(x_s)}] \nabla_z \cdot [\mathbb{T}_D(x_r,z)^T \overline{u_{e_2}^s(x_r,x_s)}]\, ds(x_r) ds(x_s).$$

$$I_6(z) = \mathrm{Im} \sum_{q=e_1,e_2} \int_{\Gamma_0^d} \int_{\Gamma_0^d} \mathbf{i}[\frac{\partial \Phi^s(x_s,z)}{\partial x_2(x_s)}] \nabla_z \times [\mathbb{T}_D(x_r,z)^T \overline{u_{e_1}^s(x_r,x_s)}]\, ds(x_r) ds(x_s).$$

$$I_7(z) = \mathrm{Im} \sum_{q=e_1,e_2} \int_{\Gamma_0^d} \int_{\Gamma_0^d} \mathbf{i}[\frac{\partial \Phi^s(x_s,z)}{\partial x_2(x_s)}] \nabla_z \times [\mathbb{T}_D(x_r,z)^T \overline{u_{e_1}^s(x_r,x_s)}]\, ds(x_r) ds(x_s).$$

$$I_8(z) = \mathrm{Im} \sum_{q=e_1,e_2} \int_{\Gamma_0^d} \int_{\Gamma_0^d} \mathbf{i}[\frac{\partial \Phi^s(x_s,z)}{\partial x_2(x_s)} + \frac{\partial \Phi^p(x_s,z)}{\partial x_2(x_s)}] \nabla_z \cdot [\mathbb{T}_D(x_r,z)^T \overline{u_{e_2}^s(x_r,x_s)}]\, ds(x_r) ds(x_s).$$

$$I_9(z) = \mathrm{Im} \sum_{q=e_1,e_2} \int_{\Gamma_0^d} \int_{\Gamma_0^d} \mathbf{i}[\frac{\partial \Phi^s(x_s,z)}{\partial x_2(x_s)} + \frac{\partial \Phi^p(x_s,z)}{\partial x_2(x_s)}] \nabla_z \times [\mathbb{T}_D(x_r,z)^T \overline{u_{e_1}^s(x_r,x_s)}]\, ds(x_r) ds(x_s).$$