Scattering Coefficient and Kirchhoff Approximation

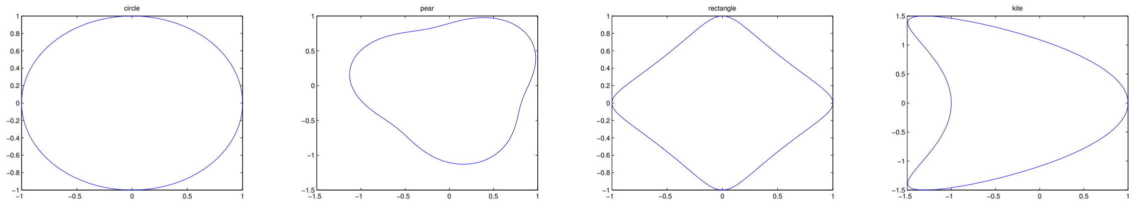

Figure 1.

1. Introdution

we introduce the concept of the scattering coefficient for incident plane waves.

Definition 1.1 *For any unit vector $\eta \in \mathbb{R}^2$, let $v^i = e^{\mathbf{i}kx\cdot\eta}$ be the incident wave and $v^s = v^s(x, \eta)$ be the radiation solution of the Helmholtz equation:*

$$\Delta v^s + k^2 v^s = 0 \quad \text{in } \mathbb{R}^2 \backslash \bar{D}, \quad v^s = -e^{\mathbf{i}kx\cdot\eta} \quad \text{on } \Gamma_D.$$

The scattering coefficient $R(x, \eta)$ for $x \in \Gamma_D$ is defined by the relation

$$\frac{\partial(v^s + v^i)}{\partial\nu} = \mathbf{i}kR(x, \eta)e^{\mathbf{i}kx\cdot\eta} \qquad \text{on } \Gamma_D.$$

In the case of Kirchhoff high frequency approximation and the mathematical justification for strictly convex obstacles, the scattering coefficient can be approximated by

$$R(x, \eta) = \begin{cases} 2\nu(x)\cdot\eta & \text{If } x \in \partial D_\eta^- := \{x \in \Gamma_D : \nu(x)\cdot\eta < 0\}, \\ 0 & \text{If } x \in \partial D_\eta^+ := \{x \in \Gamma_D : \nu(x)\cdot\eta > 0\}. \end{cases}$$

Here ∂D_η^- and ∂D_η^+ are respectively the illuminating and shadow region for the incident wave $e^{\mathbf{i}kx\cdot\eta}$.

2. Numerical Tests

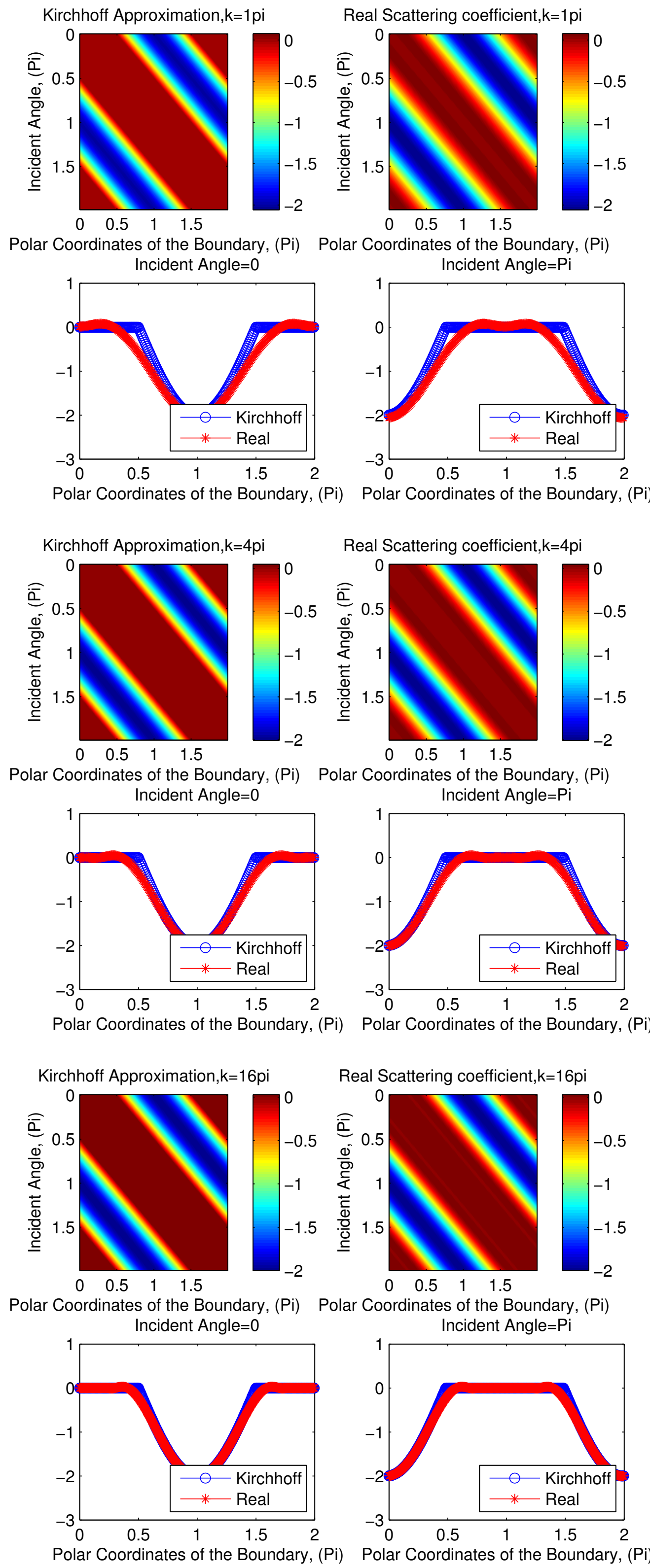
Kirchhoff Approximation,k=1pi
Real Scattering coefficient,k=1pi
Incident Angle, (Pi)
Polar Coordinates of the Boundary, (Pi)
Incident Angle=0
Incident Angle=Pi
Kirchhoff
Real
Kirchhoff Approximation,k=4pi
Real Scattering coefficient,k=4pi
Kirchhoff Approximation,k=16pi
Real Scattering coefficient,k=16pi

Figure 2.

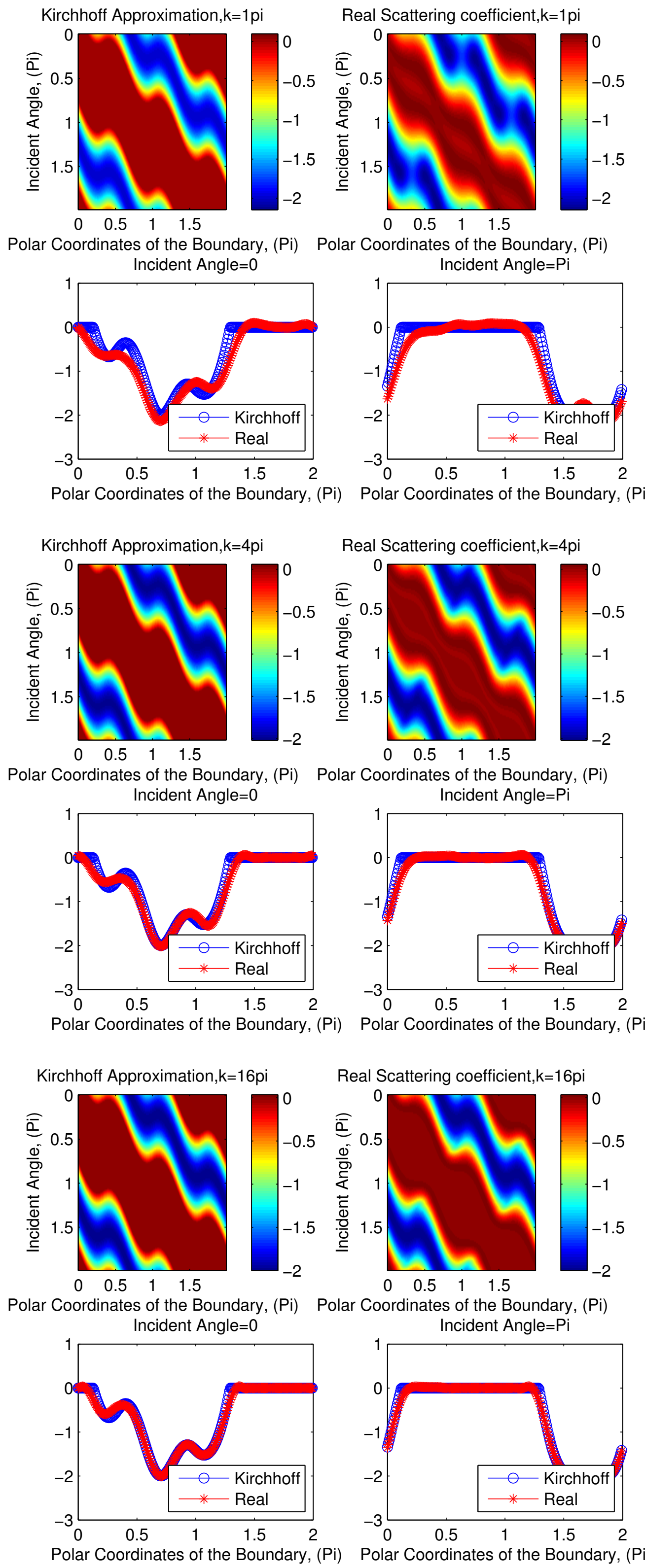

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Figure 3.

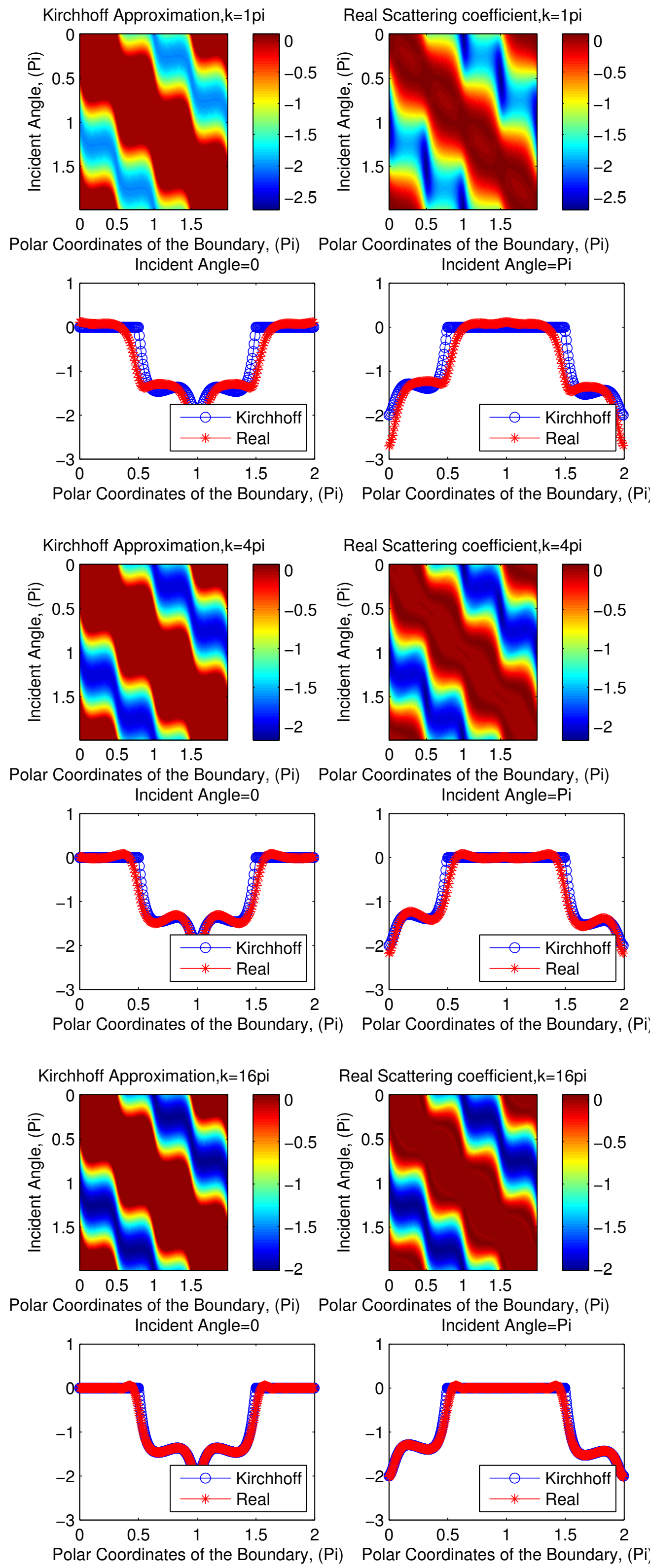

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Figure 4.

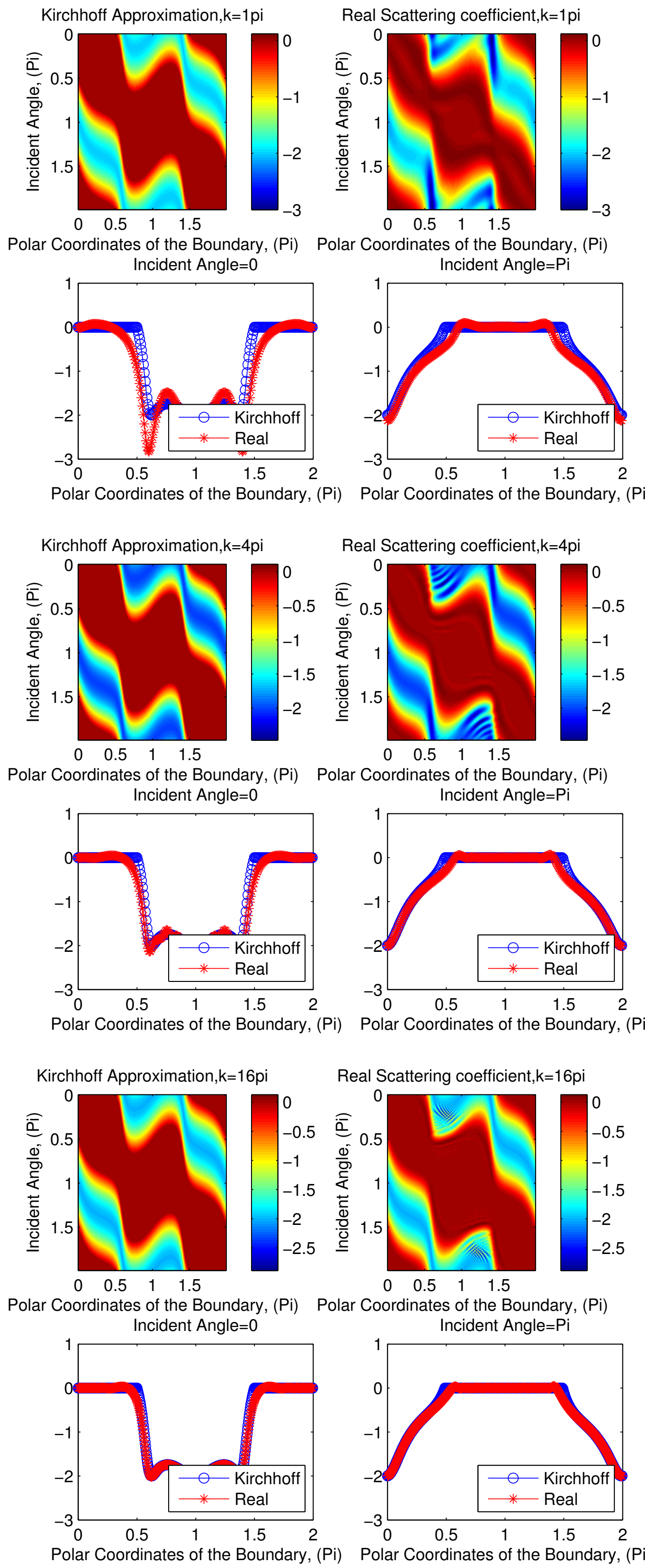

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Figure 5.