# Numerical Tests of RTM for Locally Perturbed Two-layers Media

## 1. Introdution

$$
\begin{aligned}
&\Delta u + k(x)^2 u = 0 \quad \text{ in } \mathbb{R}^2, \\
&\Delta u_0 + k_0(x)^2 u = 0 \quad \text{ in } \mathbb{R}^2, \\
&u(x) = u^s(x) + u_0(x) \\
&u_0(x) = u^i(x) + u^r(x) \\
&u^i(x) = e^{-\mathbf{i}k_1 d\cdot x}
\end{aligned}
$$

where

$$
k_0(x) = \begin{cases} k_1 & \text{in } \mathbb{R}^2_+ \\ k_2 & \text{in } \mathbb{R}^2_- \end{cases}
$$

$$
k(x) = \begin{cases} k_1 & \text{in } \mathbb{R}^2_+ \backslash \bar{D} \\ k_2 & \text{in } \mathbb{R}^2_- \backslash \bar{D} \\ k_3 & \text{in } D \end{cases}
$$

and

$$
d = (\cos\theta, \sin\theta), \quad \theta \in (0, \pi)
$$

$$
u^r(x,d) == \begin{cases} \frac{k_1 \sin\theta - k_2 \sin\phi}{k_1 \sin\theta + k_2 \sin\phi} e^{-\mathbf{i}k_1(\cos\theta x_1 - \sin\theta x_2)} & \text{in } \mathbb{R}^2_+ \\ \frac{2k_1 \sin\theta}{k_1 \sin\theta + k_2 \sin\phi} e^{-\mathbf{i}k_2(\cos\phi x_1 + \sin\phi x_2)} & \text{in } \mathbb{R}^2_- \end{cases}
$$

$$
k_1 \cos\theta = k_2 \cos\phi
$$

Imaging Condition:

$$
I(z) = \operatorname{Im} \sum_{j=1}^{N_d} \sum_{r=1}^{N_r} u^{d_j}(z) [\frac{\partial \Phi(k_1, x_r, z)}{\partial x_r(x_2)} \overline{(u^s(x_r, d_j) + u^r(x_r, d_j))}] \, ds(x_r) ds(x_s).
$$

where

$$
\begin{aligned}
&d_j = (\cos\theta_j, \sin\theta_j), \quad \theta_j = \frac{j\pi}{N_d + 1} \\
&x_r = (\frac{2(r-1)d}{N_r - 1} - d, h) \\
&\Phi(k,x,y) \ \text{ is the fundamental solution of Helmholtz equation.}
\end{aligned}
$$

## 2. Numerical Test

Parameter setting:

$$
N_d = 201, \ N_r = 401, \ d = 200, \ h = 2, k_1 = 1\omega, \ k_2 = 1.5\omega, \ k_3 = 2\omega
$$

Figure1,Figure5 : $\omega = 1$
Figure2,Figure6 : $\omega = 1.5$
Figure3,Figure7 : $\omega = 2$

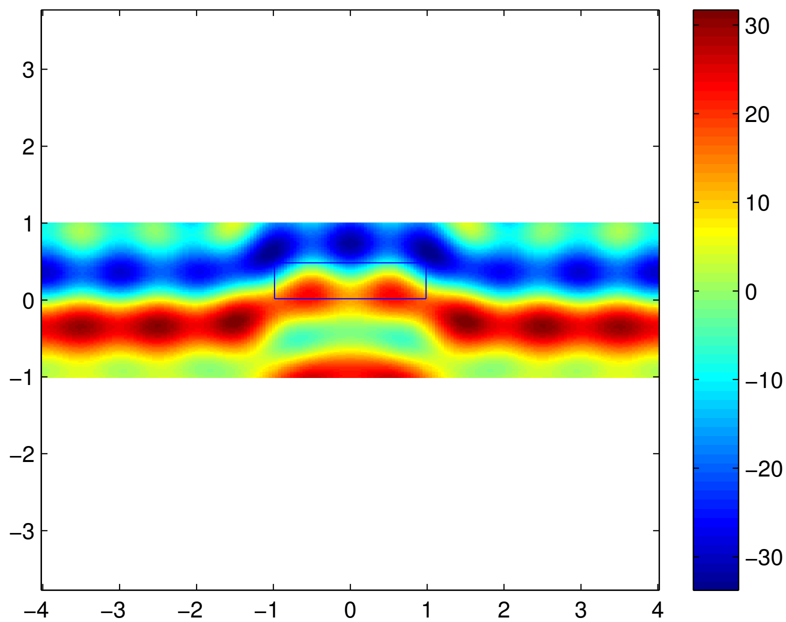


**Figure 1.** square

Figure4,Figure8 : $\omega = [1, 1.5, 2]$
Figure1-Figure4 : the shape of D is retangle: $[-1, 1] \times [0, 0.5]$
Figure5-Figure8 : the shape of D is hemicircle: $x_1^2 + (2x_2)^2 = 1,\ x_2 > 0$
Imaging domain is $[-4, 4] \times [-1, 1]$

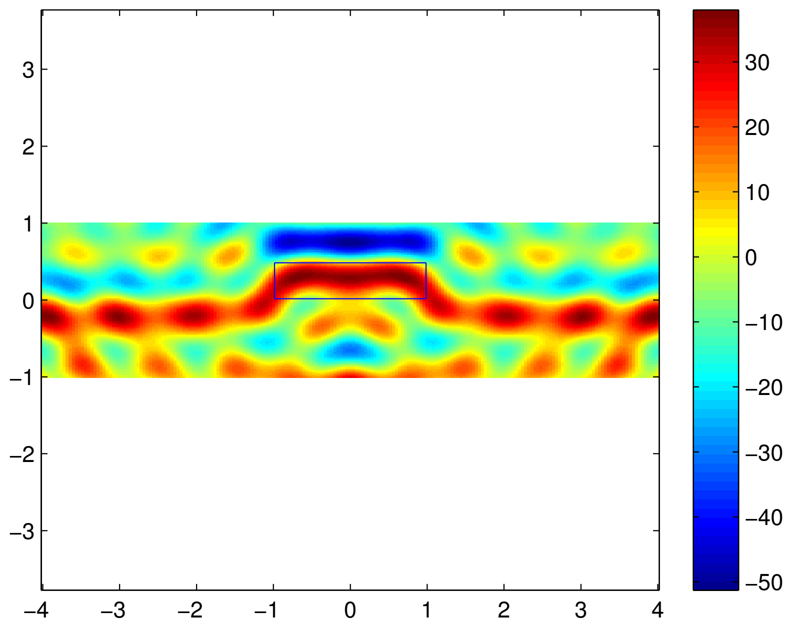


**Figure 2.** square

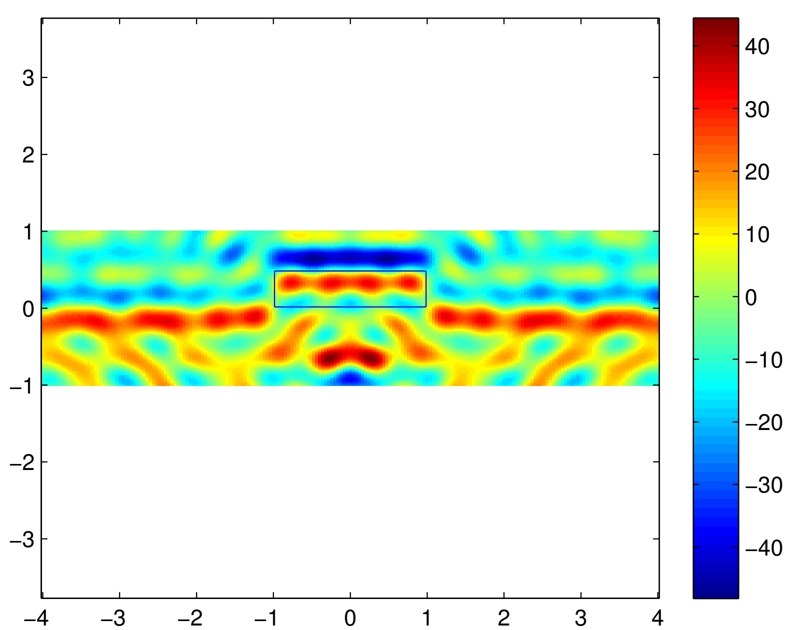


**Figure 3.** square

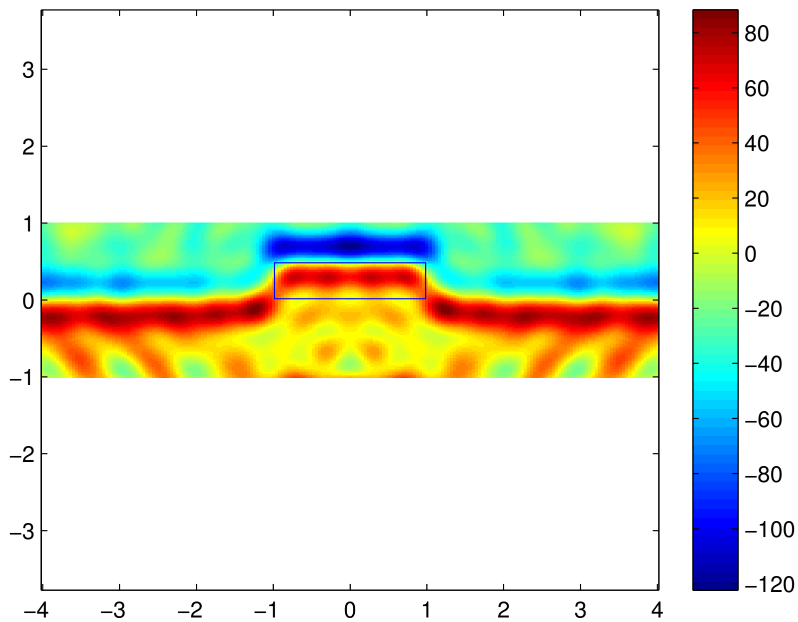


**Figure 4.** square

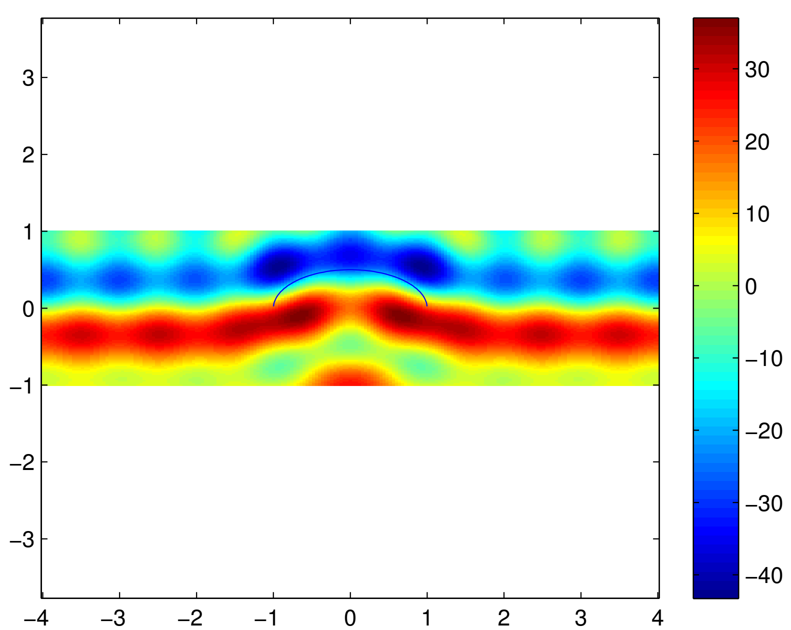


**Figure 5.** square

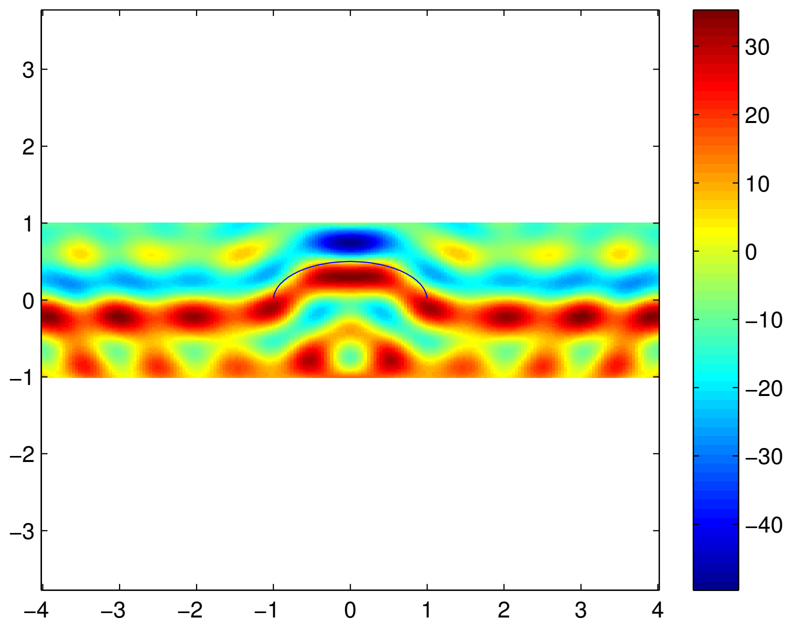


**Figure 6.** square

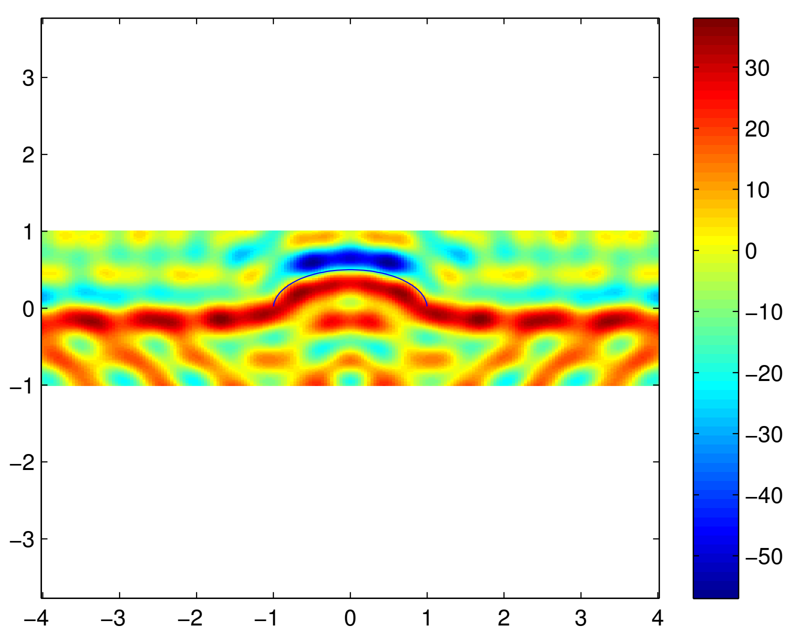


**Figure 7.** square

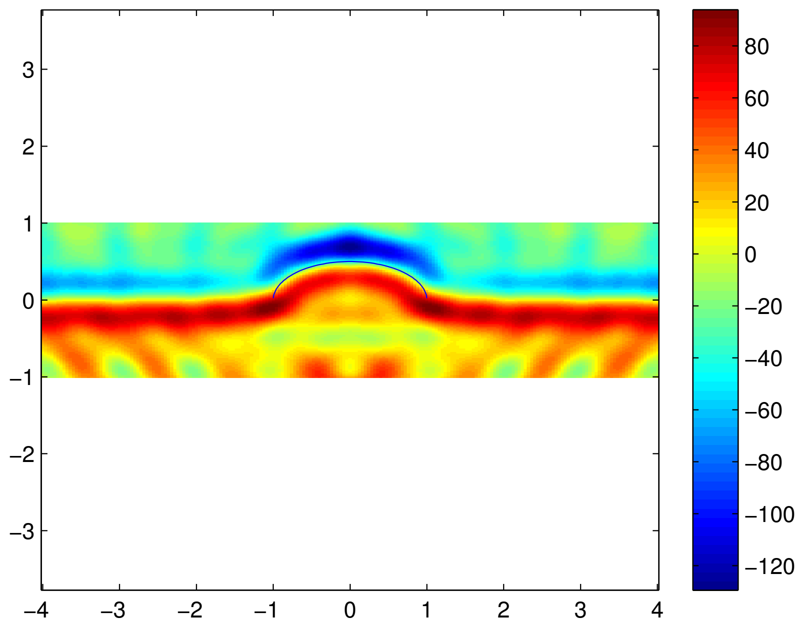


**Figure 8.** square

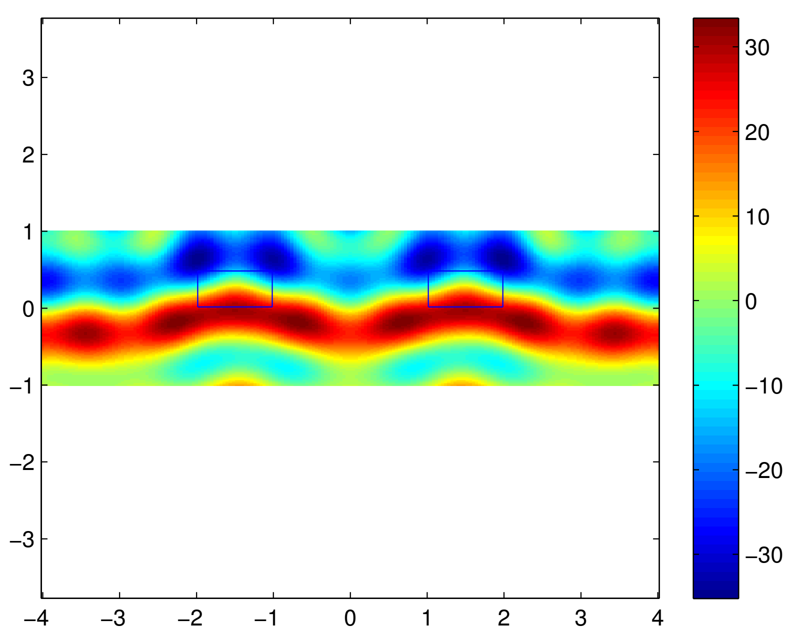


**Figure 9.** square

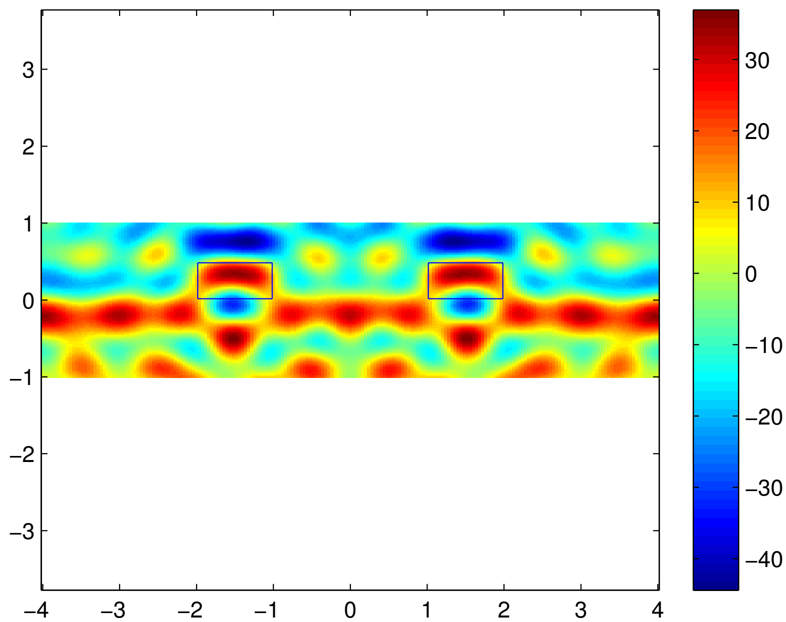


**Figure 10.** square

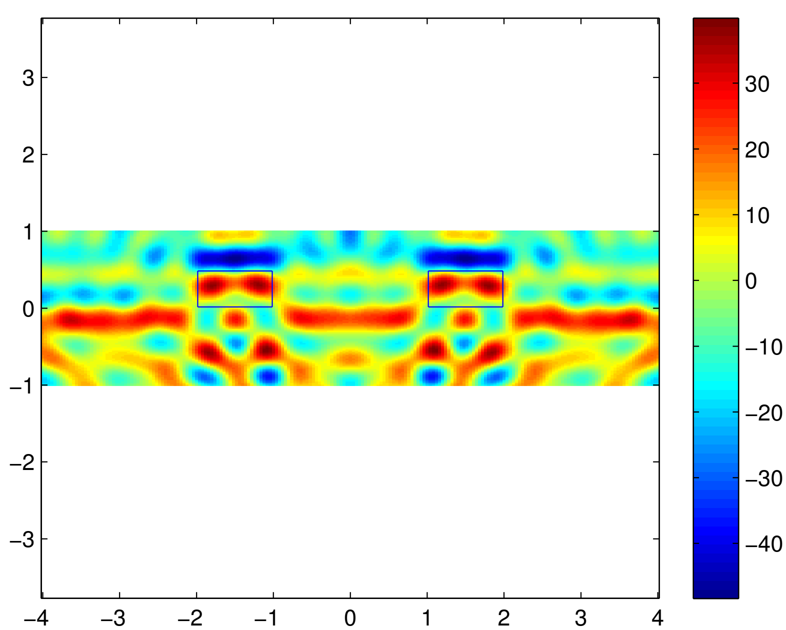


**Figure 11.** square

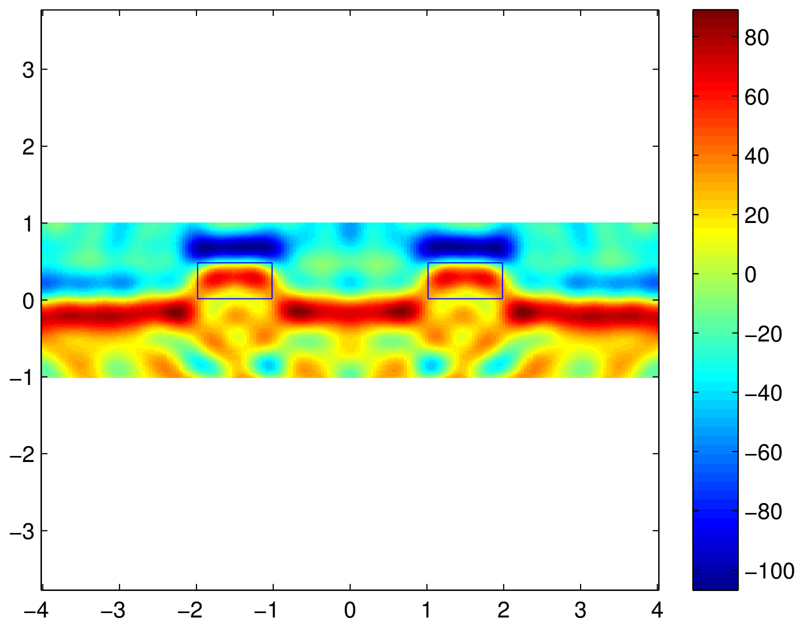


**Figure 12.** square