

Contents

	List of Portraits		page ix	
	Pref	face	xi	
Pa	rt I	Different classes of solitons		
	Intro	oduction	1	
1	Non	topological solitons: the Korteweg-de Vries equation	7	
	1.1	The discovery	7	
	1.2	The solutions of the KdV equation	17	
	1.3	Conservation rules	23	
	1.4	Nonlinear electrical lines	25	
	1.5	Blood pressure waves	33	
	1.6	Internal waves in oceanography	38	
	1.7	Generality of the KdV equation	39	
2	Topological solitons: the sine-Gordon equation			
	2.1	A simple mechanical example: the chain of coupled pendula	42	
	2.2	Solutions of the sine-Gordon equation	44	
	2.3	Long Josephson junctions	60	
	2.4	Other examples of topological solitons	70	
3	Enve	elope solitons and nonlinear localisation: the nonlinear		
	So	chrödinger equation	75	
	3.1	Nonlinear waves in the pendulum chain: the NLS equation	75	
	3.2	Properties of the nonlinear Schrödinger equation	80	
	3.3	Conservation laws	90	
	3.4	Næther's theorem	94	
	3.5	Nonlinear electrical lines	96	
	3.6	Solitons in optical fibres	98	
	3.7	Self-focusing in optics: the NLS equation in two space dimension	ns 112	
	3.8	Conclusion	116	



vi		Contents	
4	The modelling process: ion acoustic waves in a plasma		
	4.1	Introduction	117
	4.2	The plasma	118
	4.3	Study of the linear dynamics	122
	4.4	Nonlinear study	124
	4.5	Derivation of the nonlinear Schrödinger equation	127
	4.6	Experimental observations	130
	4.7	Discussion	133
Pa	rt II	Mathematical methods for the study of solitons	
	Introd	luction	139
5	Linea	risation around the soliton solution	141
	5.1	Spectrum of the excitations around a sine-Gordon soliton	141
	5.2	Application: perturbation of a soliton	144
	5.3	Spectrum of the excitations around a ϕ^4 soliton	150
6	Colle	ctive coordinate method	156
	6.1	sine-Gordon soliton interacting with an impurity: effective	
		Lagrangian method	156
	6.2	Improving the method with a second collective coordinate	160
7	The inverse-scattering transform		165
	7.1	Inverse scattering transform for the KdV equation	165
	7.2	The inverse scattering transform: a 'nonlinear Fourier analysis'	175
Pa	rt III	Examples in solid state and atomic physics	
	Introduction		185
8	The F	Fermi–Pasta–Ulam problem	187
	8.1	The physical question	187
	8.2	Fermi, Pasta and Ulam: the characters	189
	8.3	The solution of the FPU problem	192
	8.4	Kruskal and Zabusky: the pioneers	195
	8.5	FPU and the Japanese School	197
9	A simple model for dislocations in crystals		199
	9.1	Plastic deformation of crystals	199
	9.2	A one-dimensional model: the Frenkel-Kontorova model	202
	9.3	Continuum limit approximation: the sine-Gordon equation	204
	9.4	Are dislocations solitons?	205
	9.5	Applications	208
10	Ferro	electric domain walls	211
	10.1	Ferroelectric materials	211
	10.2	Δ one-dimensional ferroelectric model	214



		Contents	vii
	10.3	Structure of the domain walls in the continuum limit	
		approximation	216
	10.4	Dielectric response of a ferroelectric material	220
	10.5	Thermodynamics of a nonlinear system	223
11	Incom	mensurate phases	235
	11.1	Examples in solid state physics	235
	11.2	A one-dimensional model for incommensurate phases: the	
		Frenkel-Kontorova model	236
	11.3	Commensurate phases	237
	11.4	The commensurate-incommensurate transition	238
	11.5	Structure of the incommensurate phase	239
	11.6	Calculation of δ_c	240
	11.7	Phase diagram	243
	11.8	Dynamics of the incommensurate phase	245
	11.9	Formation of the discommensurations	247
	11.10	Conclusion	250
12	Solito	ns in magnetic systems	252
	12.1	Ferromagnetism and antiferromagnetism	252
		Equations for the dynamics of a spin chain	254
	12.3	Magnons and solitons	257
	12.4	Validity of the sine-Gordon approximation	261
	12.5	Solitons in antiferromagnetic spin chains	267
	12.6	Conclusion	268
13		ns in conducting polymers	269
	13.1	Materials	269
	13.2	The physical model of polyacetylene	272
	13.3	The ground state of polyacetylene	274
	13.4	The excited state of polyacetylene: the soliton solution	283
	13.5	Mechanism of electric conduction in conducting polyacetylene	287
	13.6	An experimental test for the presence of solitons	292
	13.7	The other nonlinear excitations of polyacetylene	295
14		ns in Bose–Einstein condensates	297
	14.1	Introduction	297
		Theoretical description of a condensate	299
		Magnetic traps	305
	14.4	Dynamic properties	306
	14.5	Soliton solutions	312
Pa	rt IV	Nonlinear excitations in biological molecules	
		uction	323



VII	1	Contents	
15	Energ	gy localisation and transfer in proteins	326
	15.1	The mechanism proposed by Davydov	326
	15.2	The Davydov equations	341
	15.3	Does the Davydov soliton exist?	343
	15.4	A model physical system: the acetanilide crystal	345
16	6 Nonlinear dynamics and statistical physics of DNA		351
	16.1	A simple DNA model	351
	16.2	Nonlinear dynamics of DNA	362
	16.3	Statistical physics of DNA thermal denaturation	370
	16.4	Stability of a domain wall: another approach to denaturation	377
Co	onclus	ion Physical solitons: do they exist?	389
Pa	rt V	Appendices	
A	Deriv	ration of the KdV equation for surface hydrodynamic waves	395
	A.1	Basic equations and boundary conditions	395
	A.2	Mathematical formulation of the problem	397
	A.3	The linear limit	401
	A.4	The nonlinear equation in shallow water	402
В		nanics of a continuous medium	405
В		nanics of a continuous medium Lagrangian formalism	405 405
В	Mech		
В	Mech B.1 B.2	Lagrangian formalism	405
	Mech B.1 B.2 Cohe	Lagrangian formalism Hamiltonian formalism	405 407