Chapter *in* American Journal of Physics · May 2003 DOI: 10.1119/1.1564612 CITATIONS READS 122 421 2 authors: Marc Lefranc Robert Gilmore Drexel University Université de Lille 113 PUBLICATIONS 1,049 CITATIONS 31 PUBLICATIONS 535 CITATIONS SEE PROFILE SEE PROFILE Some of the authors of this publication are also working on these related projects: Ostreococcus Circadian clock View project Coupling the mammalian circadian clock to metabolism: mathematical modeling View project

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The Topology of Chaos

Robert Gilmore

Physics Department Drexel University Philadelphia, PA 19104 robert.gilmore@drexel.edu

Colloquium, Physics Department University of Florida, Gainesville, FL

October 17, 2008

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The Topology of Chaos



Robert Gilmore
Physics Department
Drexel University
Philadelphia, PA 19104



robert.gilmore@drexel.edu

Colloquium, Physics Department University of Florida, Gainesville, FL October 23, 2008

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- Overview
- 2 Experimental Challenge
- Topology of Orbits
- Topological Analysis Program
- Basis Sets of Orbits
- Quantizing Chaos
- Summary

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J. R. Tredicce

Can you explain my data?

I dare you to explain my data!

Motivation

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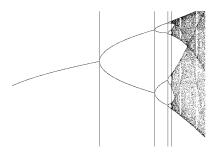
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Where is Tredicce coming from?



Feigenbaum:

$$\alpha = 4.66920 \ 16091 \dots$$

$$\delta = -2.50290\ 78750\$$

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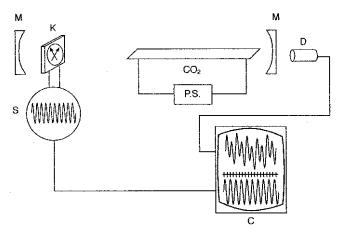
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Laser with Modulated Losses Experimental Arrangement



Our Hope

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Original Objectives

Construct a simple, algorithmic procedure for:

- Classifying strange attractors
- Extracting classification information

from experimental signals.

Our Result

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Result

There is now a classification theory.

- It is topological
- ② It has a hierarchy of 4 levels
- Each is discrete
- There is rigidity and degrees of freedom
- **5** It is applicable to R^3 only for now

Topology Enters the Picture

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The 4 Levels of Structure

- Basis Sets of Orbits
- Branched Manifolds
- Bounding Tori
- Extrinsic Embeddings

Experimental Schematic

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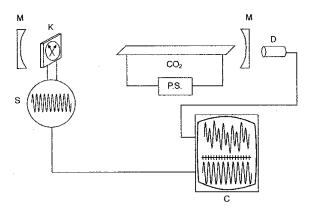
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Laser Experimental Arrangement

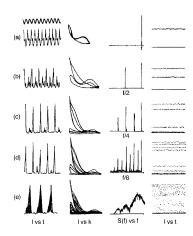


Experimental Motivation

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Oscilloscope Traces



Results, Single Experiment

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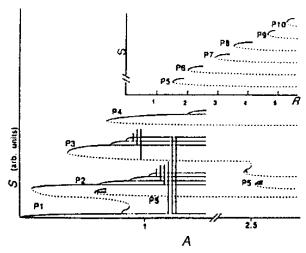
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Bifurcation Schematics



Some Attractors

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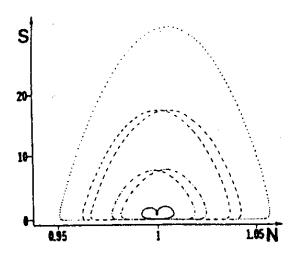
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Coexisting Basins of Attraction



Many Experiments

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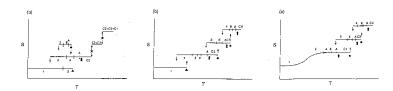
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Bifurcation Perestroikas



Real Data

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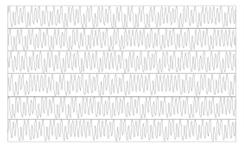
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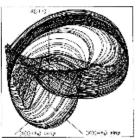
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Experimental Data: LSA





Lefranc - Cargese

Real Data

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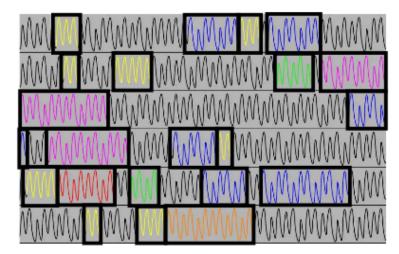
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Mechanism

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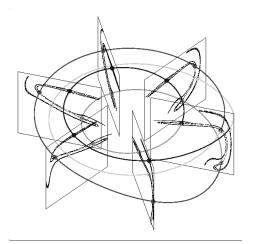
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Stretching & Squeezing in a Torus



Time Evolution

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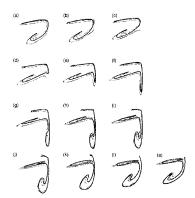
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Rotating the Poincaré Section around the axis of the torus



Time Evolution

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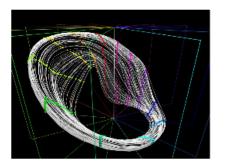
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Rotating the Poincaré Section around the axis of the torus



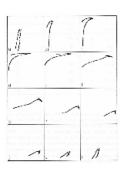


Figure 2. Left: Intersections of a chaotic attractor with a series of section planes are computed. Right: Their evolution from plane to plane shows the interplay of the stretching and squeezing mechanisms.

Experimental Schematic

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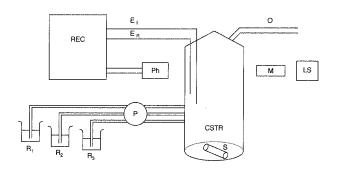
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Experimenta 02 A Chemical Experiment

The Belousov-Zhabotinskii Reaction



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Chaos

Motion that is

- **Deterministic:** $\frac{dx}{dt} = f(x)$
- Recurrent
- Non Periodic
- Sensitive to Initial Conditions

Strange Attractor

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Strange Attractor

The Ω limit set of the flow. There are unstable periodic orbits "in" the strange attractor. They are

- "Abundant"
- Outline the Strange Attractor
- Are the Skeleton of the Strange Attractor

Skeletons

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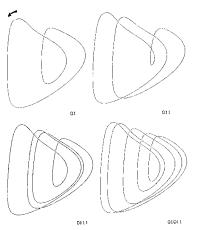
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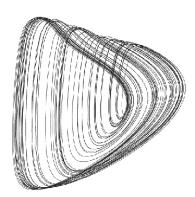
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UPOs Outline Strange attractors





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UPOs Outline Strange attractors

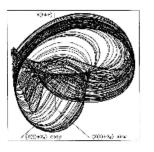




Figure 5. Left: a chaotic attractor reconstructed from a time series from a chaotic laser; Right: Superposition of 12 periodic orbits of periods from 1 to 10.

Dynamics and Topology

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Organization of UPOs in R³: Gauss Linking Number

$$LN(A,B) = \frac{1}{4\pi} \oint \oint \frac{(\mathbf{r}_A - \mathbf{r}_B) \cdot d\mathbf{r}_A \times d\mathbf{r}_B}{|\mathbf{r}_A - \mathbf{r}_B|^3}$$

Interpretations of LN $\simeq \#$ Mathematicians in World

Linking Numbers

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Lefranc - Cargese

Linking Number of Two UPOs

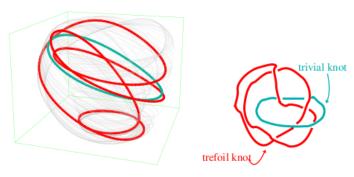
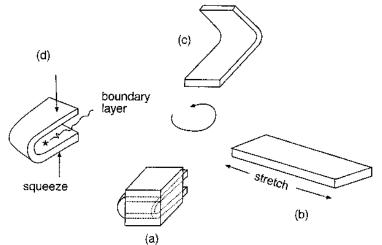


Figure 6. Left: two periodic orbits of periods 1 and 4 embedded in a strange attractor; Right: a link of two knots that is equivalent to the pair of periodic orbits up to continuous deformations without crossings.

Evolution in Phase Space

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One Stretch-&-Squeeze Mechanism



Motion of Blobs in Phase Space

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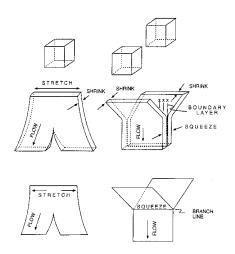
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Stretching — Squeezing



Collapse Along the Stable Manifold

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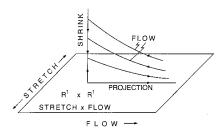
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Birman - Williams Projection

Identify x and y if

$$\lim_{t \to \infty} |x(t) - y(t)| \to 0$$



Fundamental Theorem

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Birman - Williams Theorem

If:

Then:

Fundamental Theorem

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Birman - Williams Theorem

If: Certain Assumptions

Then:

Fundamental Theorem

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Birman - Williams Theorem

If: Certain Assumptions

Then: Specific Conclusions

Birman-Williams Theorem

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Assumptions, B-W Theorem

A flow $\Phi_t(x)$

- on R^n is dissipative, $\underline{n=3}$, so that $\lambda_1 > 0, \lambda_2 = 0, \lambda_3 < 0$.
- Generates a <u>hyperbolic</u> strange attractor SA

IMPORTANT: The underlined assumptions can be relaxed.

Birman-Williams Theorem

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Conclusions, B-W Theorem

- The projection maps the strange attractor \mathcal{SA} onto a 2-dimensional branched manifold \mathcal{BM} and the flow $\Phi_t(x)$ on \mathcal{SA} to a semiflow $\overline{\Phi}(x)_t$ on \mathcal{BM} .
- UPOs of $\Phi_t(x)$ on \mathcal{SA} are in 1-1 correspondence with UPOs of $\overline{\Phi}(x)_t$ on \mathcal{BM} . Moreover, every link of UPOs of $(\Phi_t(x), \mathcal{SA})$ is isotopic to the correspond link of UPOs of $(\overline{\Phi}(x)_t, \mathcal{BM})$.

Remark: "One of the few theorems useful to experimentalists."

A Very Common Mechanism

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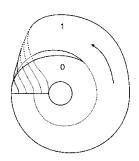
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Rössler:

Attractor Branched Manifold





A Mechanism with Symmetry

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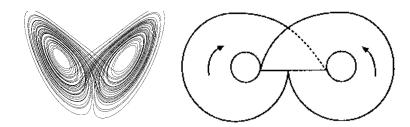
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Lorenz:

Attractor Branched Manifold



Examples of Branched Manifolds

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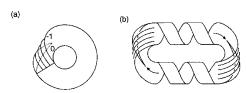
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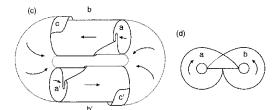
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Inequivalent Branched Manifolds





Aufbau Princip for Branched Manifolds

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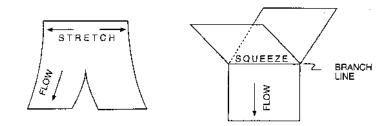
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Any branched manifold can be built up from stretching and squeezing units



subject to the conditions:

- Outputs to Inputs
- No Free Ends



Dynamics and Topology

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Rossler System



 $\frac{dx}{dt} = -y - \epsilon$

 $\frac{dy}{di} = x + ay$

 $\frac{dz}{dt}=b+z(z-a)$





(f)

 $\begin{bmatrix}
 -1 & 0 \\
 0 & 0
 \end{bmatrix}$

 $\left[\begin{array}{cc} 0 & \pm 1 \end{array}\right]$





(d)



Dynamics and Topology

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Lorenz System

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\alpha x + \alpha y$$

$$\frac{dy}{dt} = Rx \cdot y \cdot xz$$

$$\frac{dz}{dt} = -bz + xy$$

$$\left(+1,-1\right)$$

(b)















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Poincaré Smiles at Us in R³

- Determine organization of UPOs \Rightarrow
- Determine branched manifold ⇒
- Determine equivalence class of SA

Topological Analysis Program

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Topological Analysis Program

Locate Periodic Orbits

Create an Embedding

Determine Topological Invariants (LN)

Identify a Branched Manifold

Verify the Branched Manifold

Model the Dynamics

Validate the Model



Locate UPOs

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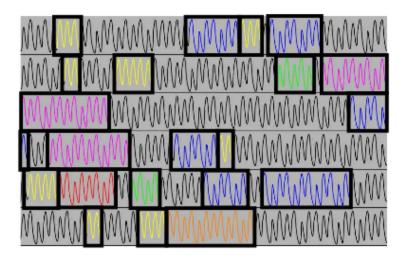
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Method of Close Returns



Embeddings

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Embeddings

Many Methods: Time Delay, Differential, Hilbert Transforms, SVD, Mixtures, ...

Tests for Embeddings: Geometric, Dynamic, Topological[†]

None Good

We Demand a 3 Dimensional Embedding

Locate UPOs

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An Embedding and Periodic Orbits

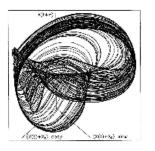




Figure 5. Left: a chaotic attractor reconstructed from a time series from a chaotic laser; Right: Superposition of 12 periodic orbits of periods from 1 to 10.

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Linking Number of Orbit Pairs

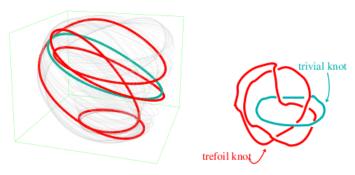


Figure 6. Left: two periodic orbits of periods 1 and 4 embedded in a strange attractor; Right: a link of two knots that is equivalent to the pair of periodic orbits up to continuous deformations without crossings.

Lefranc - Cargese

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Compute Table of Expt'l LN

Table 7.2 Linking numbers for all the surrogate periodic orbits, to period 8, extracted from Belousov-Zhabotinskii data^a

Orbit	Symbolics	1	2	3	4	5	6	7	8a	8Ь
1	1	0	1	1	2	2	2	3	4	3
2	01	1	1	2	3	4	4	5	6	6
3	011	1	2	2	4	5	6	7	8	8
4	0111	2	3	4	5	8	8	11	13	12
5	01 011	2	4	5	8	8	10	13	16	15
6	011 0M1	2	4	6	8	10	9	14	16	16
7	01 01 011	3	5	7	11	13	14	16	21	21
8a	01 01 0111	4	6	8	13	16	16	21	23	24
8Ь	01 011 011	3	6	8	12	15	16	21	24	21

All indices are negative.

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Compare w. LN From Various βM

Table 2.1 Linking numbers for orbits to period five in Smale horseshoe dynamics.

	19	1 <i>f</i>	21	3 f	39	41	$4_{2}f$	$4_{2}9$	5 ₃ f	538	5_2f	529	5_1f	51
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	1	2	1	1	1	1	2	2	2	2
01	0	1	1	2	2	3	2	2	2	2	3	3	4	4
001	0	1	2	2	3	4	3	3	3	3	4	4	5	5
011	0	1	2	3	2	4	3	3	3	3	5	5	5	5
0111	0	2	3	4	4	5	4	4	4	4	7	7	8	8
0001	0	1	2	3	3	4	3	4	4	4	5	5	5	5
0011	0	1	2	3	3	4	4	3	4	4	5	5	5	5
00001	0	1	2	3	3	4	4	4	4	5	5	5	5	5
00011	0	1	2	3	3	4	4	4	5	4	5	5	5	5
00111	0	2	3	4	5	7	5	5	5	5	6	7	8	9
00101	0	2	3	4	5	7	5	5	5	5	7	6	8	9
01101	0	2	4	5	3	8	5	5	5	5	8	8	8	10
01111	0	2	4	5	5	8	5	5	5	5	9	9	10	8

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Guess Branched Manifold

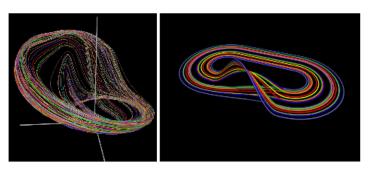


Figure 7. "Combing" the intertwined periodic orbits (left) reveals their systematic organization (right) created by the stretching and squeezing mechanisms.

Lefranc - Cargese

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Identification & 'Confirmation'

- ullet \mathcal{BM} Identified by LN of small number of orbits
- Table of LN GROSSLY overdetermined
- Predict LN of additional orbits
- Rejection criterion

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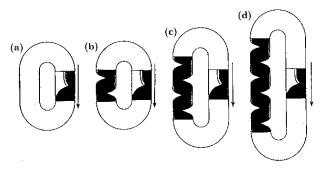
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What Do We Learn?

- BM Depends on Embedding
- Some things depend on embedding, some don't
- Depends on Embedding: Global Torsion, Parity, ..
- Independent of Embedding: Mechanism



Perestroikas of Strange Attractors

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Evolution Under Parameter Change

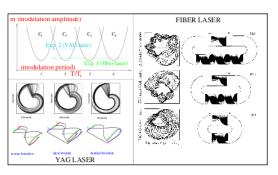


Figure 11. Various templates observed in two laser experiments. Top left: schematic representation of the parameter space of forced nonlinear oscillators showing resonance tongues. Right: templates observed in the fiber laser experiment; global torsion increases systematically from one tongue to the next [40]. Bottom left: templates observed in the YAG laser experiment (only the branches are shown); there is a variation in the topological organization across one chaotic tongue [39, 41].

Perestroikas of Strange Attractors

The Topology of Chaos

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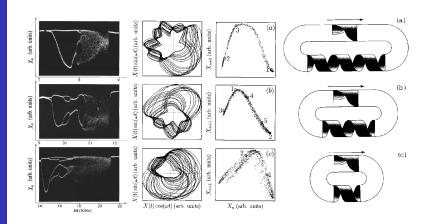
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Evolution Under Parameter Change



Lefranc - Cargese



An Unexpected Benefit

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Analysis of Nonstationary Data

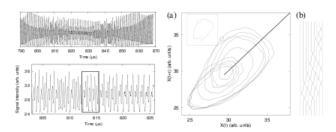


Figure 16. Top left: time series from an optical parametric oscillator showing a burst of irregular behavior. Bottom left: segment of the time series containing a periodic orbit of period 9. Right: embedding of the periodic orbit in a reconstructed phase space and representation of the braid realized by the orbit. The braid entropy is $h_T = 0.377$, showing that the underlying dynamics is chaotic. Reprinted from [61].

Lefranc - Cargese

Our Hope \rightarrow Now a Result

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Compare with Original Objectives

Construct a simple, algorithmic procedure for:

- Classifying strange attractors
- Extracting classification information

from experimental signals.

Orbits Can be "Pruned"

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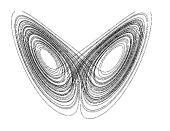
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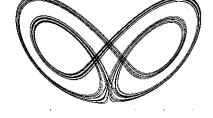
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There Are Some Missing Orbits





Lorenz

Shimizu-Morioka



Linking Numbers, Relative Rotation Rates, Braids

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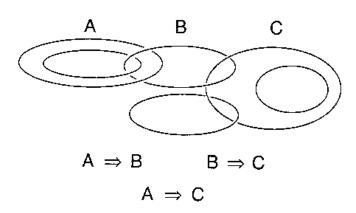
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Orbit Forcing



An Ongoing Problem

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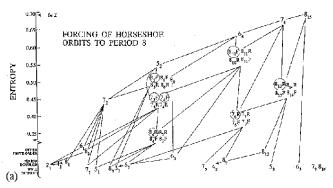
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Forcing Diagram - Horseshoe



u - SEQUENCE ORDER



An Ongoing Problem

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Status of Problem

- Horseshoe organization active
- More folding barely begun
- Circle forcing even less known
- Higher genus new ideas required

Creating New Attractors

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Rotating the Attractor

$$\frac{d}{dt} \left[\begin{array}{c} X \\ Y \end{array} \right] = \left[\begin{array}{c} F_1(X,Y) \\ F_2(X,Y) \end{array} \right] + \left[\begin{array}{c} a_1 \sin(\omega_d t + \phi_1) \\ a_2 \sin(\omega_d t + \phi_2) \end{array} \right]$$

$$\left[\begin{array}{c} u(t) \\ v(t) \end{array}\right] = \left[\begin{array}{cc} \cos \Omega t & -\sin \Omega t \\ \sin \Omega t & \cos \Omega t \end{array}\right] \left[\begin{array}{c} X(t) \\ Y(t) \end{array}\right]$$

$$\frac{d}{dt} \begin{bmatrix} u \\ v \end{bmatrix} = R\mathbf{F}(R^{-1}\mathbf{u}) + R\mathbf{t} + \Omega \begin{bmatrix} -v \\ +u \end{bmatrix}$$

$$\Omega = n \ \omega_d$$

$$q \Omega = p \omega_d$$

Global Diffeomorphisms

Local Diffeomorphisms (p-fold covers)

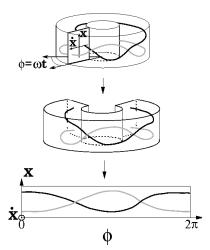




Another Visualization

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Cutting Open a Torus



Satisfying Boundary Conditions

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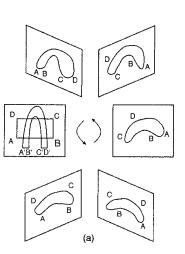
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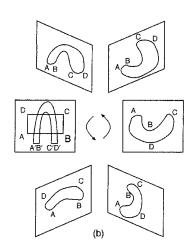
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Global Torsion





Two Phase Spaces: R^3 and $D^2 \times S^1$

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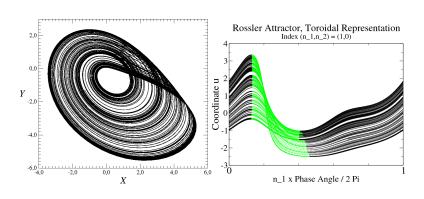
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Rossler Attractor: Two Representations





Other Diffeomorphic Attractors

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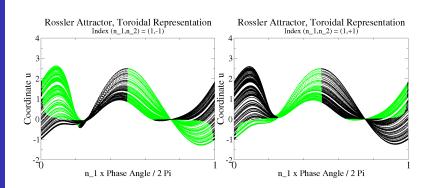
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Rossler Attractor:

Two More Representations with $n = \pm 1$



Subharmonic, Locally Diffeomorphic Attractors

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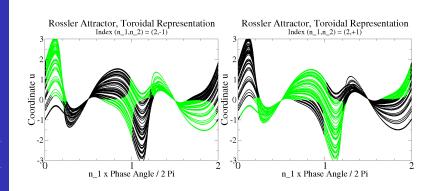
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Rossler Attractor:

Two Two-Fold Covers with $p/q = \pm 1/2$



Subharmonic, Locally Diffeomorphic Attractors

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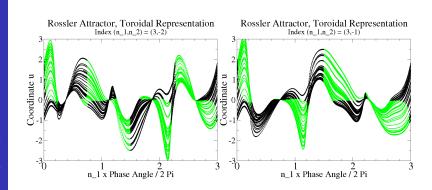
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Rossler Attractor:

Two Three-Fold Covers with p/q = -2/3, -1/3



Subharmonic, Locally Diffeomorphic Attractors

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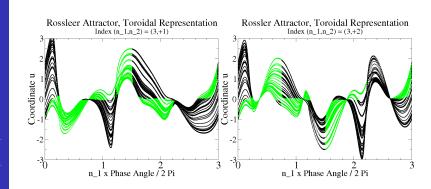
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Rossler Attractor:

And Even More Covers (with p/q = +1/3, +2/3)



New Measures

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Angular Momentum and Energy

$$L(0) = \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^{\tau} X dY - Y dX \qquad K(0) = \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^{\tau} \frac{1}{2} (\dot{X}^2 + \dot{Y}^2) dt$$

$$L(\Omega) = \langle u\dot{v} - v\dot{u}\rangle$$
 $K(\Omega) = \langle \frac{1}{2}(\dot{u}^2 + \dot{v}^2)\rangle$

$$= L(0) + \Omega \langle R^2 \rangle$$

$$= K(0) + \Omega L(0) + \frac{1}{2} \Omega^2 \langle R^2 \rangle$$

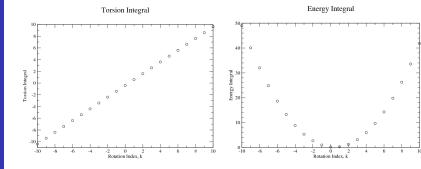
$$\langle R^2 \rangle = \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^{\tau} (X^2 + Y^2) dt = \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^{\tau} (u^2 + v^2) dt$$

New Measures, Diffeomorphic Attractors

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Energy and Angular Momentum

Diffeomorphic, Quantum Number n





New Measures, Subharmonic Covering Attractors

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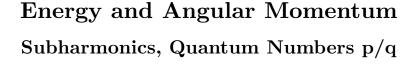
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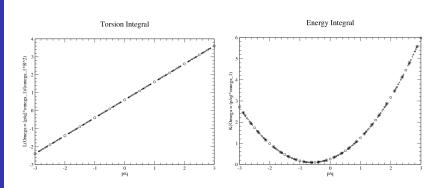
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Summary

1 Question Answered ⇒2 Questions Raised

We must be on the right track!

Our Hope

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Original Objectives Achieved

There is now a simple, algorithmic procedure for:

- Classifying strange attractors
- Extracting classification information

from experimental signals.

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Result

There is now a classification theory for low-dimensional strange attractors.

- 1 It is topological
- 2 It has a hierarchy of 4 levels
- Each is discrete
- There is rigidity and degrees of freedom
- **1** It is applicable to R^3 only for now

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The Classification Theory has 4 Levels of Structure

Basis Sets of Orbits



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- Basis Sets of Orbits
- ② Branched Manifolds

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- Basis Sets of Orbits
- ② Branched Manifolds
- 8 Bounding Tori

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- Basis Sets of Orbits
- ② Branched Manifolds
- Bounding Tori
- Extrinsic Embeddings

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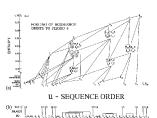
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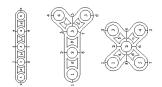
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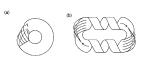
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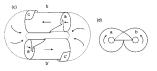
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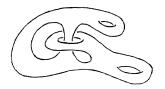


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Topological Components

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Poetic Organization

organize
BOUNDING TORI
organize
BRANCHED MANIFOLDS
organize
LINKS OF PERIODIC ORBITS



Answered Questions

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Some Unexpected Results

- Perestroikas of orbits constrained by branched manifolds
- Routes to Chaos = Paths through orbit forcing diagram
- Perestroikas of branched manifolds constrained by bounding tori
- Global Poincaré section = union of g-1 disks
- Systematic methods for cover image relations
- Existence of topological indices (cover/image)
- Universal image dynamical systems
- NLD version of Cartan's Theorem for Lie Groups
- Topological Continuation Group Continuuation
- Cauchy-Riemann symmetries
- Quantizing Chaos
- Representation labels for inequivalent embeddings
- Representation Theory for Strange Attractors



Unanswered Questions

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We hope to find:

- Robust topological invariants for \mathbb{R}^N , N>3
- A Birman-Williams type theorem for higher dimensions
- An algorithm for irreducible embeddings
- Embeddings: better methods and tests
- Analog of χ^2 test for NLD
- Better forcing results: Smale horseshoe, $D^2 \to D^2$, $n \times D^2 \to n \times D^2$ (e.g., Lorenz), $D^N \to D^N$, N>2
- Representation theory: complete
- Singularity Theory: Branched manifolds, splitting points (0 dim.), branch lines (1 dim).
- Singularities as obstructions to isotopy