1 Flooding algorithm

For Equations

$$m_1 x + m_2 y + m_3 z = \alpha \tag{1}$$

Supposed it ranks as $m_1 < m_2 < m_3$.

Three intercepts on the coordcates are

$$m_1 x + m_2 y + m_3 z = \alpha \tag{2}$$

$$h_1 = \frac{\alpha}{m_1} h_2 = \frac{\alpha}{m_2} h_3 = \frac{\alpha}{m_3} \tag{3}$$

where $h_1 < h_2 < h_3$.

1.1 Cut 1

$$V = \frac{1}{6} h_1 h_2 h_3 = \frac{\alpha}{6 m_1 m_2 m_3} \tag{4}$$

$$c_{x1} = \frac{1}{4} m_1 c_{x2} = \frac{1}{4} m_2 c_{x3} = \frac{1}{4} m_3 \tag{5}$$

2 Advection of Volume function

Use a simple 3 stencil 1D grid as an example:

2.1 Weymouth-Yue

Original volume plus the boundary flux (Eulerian).

$$\tilde{f_c} = f_c + VOF_c^1 - VOF_c^3 - VOF_r^1 + VOF_l^3 \tag{6}$$

2.2 CIAM

Backward lagrangian of the grid face and find the intersection between two faces (Lagrangian).

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$$\tilde{f_c} = VOF_c^2 + VOF_r^1 + VOF_l^3 \tag{7}$$

Compared with W-Y advection, we obtain

$$VOF2_c = f_c - 2VOF_r^1 - VOF_c^1 - VOF_c^3 \tag{8}$$