Battery Charging and Discharging Optimisation

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1 Problem statement

This exercise is the build an optimisation model to charge/discharge the battery over the time period provided (2018-2020) in order to maximise profits. The battery is a price-taker and does not affect the market prices.

There are three electricity markets:

Market	Trading frequency
1	half hourly
2	half hourly
3	daily

2 Model design

2.1 Model horizon

Although we are given the retrospective prices at any time between 2018 - 2020, in real world operation, it is not possible for the operator to have an accurate estimate far into the future. Hence in this model, we need to define a time horizon in which we have certain confidence of the price predictions and can make optimisation decisions based on that information.

We consider a 72 hours planning horizon, where we assume we have perfect knowledge of the prices within this horizon. The plan for charging and discharging is then computed with the price data. The plan, however will only be executed up to 24 hours. This is because the plan in the later part of the horizon will likely be suboptimal as we do not have information beyond the horizon. The planning exercise will be conducted every 24 hours with the next 72 hour data.

2.2 Objective function

As we are optimising for profit, the objective function represents the profit itself. We define the following variables:

- Π: profit (£)
- t: time unit
- Δt: the actual time in hours of each time unit (h)
- $P_{m,t}$: price of electricity in market m at time t (£/MWh)
- V_t: volume of battery at time t (MWh)
- S_t: state of charge of battery at time t (MWh)
- $c_{m,t}$: charge power from market m at time t (MW)
- $d_{m,t}$: discharge power to market m at time t (MW)
- η_c: charging efficiency

- η_d: discharging efficiency
- f: battery degradation per cycle

We want to optimize for Π over a time period T, where

$$\Pi = \text{revenue} - \text{cost}$$

Revenue is the money gained by selling electricity on the market

$$\text{revenue} = \sum_{t=1}^{T} \sum_{m \in \{1,2,3\}} P_{m,t} d_{m,t} \Delta t$$

Cost can be split into direct cost, which is the cost used in buying electricity from the market to charge the battery, and indirect cost, which is the cost of degradation of the battery over time.

Direct cost is the money used to buy electricity on the market

$$\text{direct cost} = \sum_{t=1}^{T} \sum_{m \in \{1,2,3\}} P_{m,t} c_{m,t} \Delta t$$

Indirect cost comes from the degradation of the battery due to charging and discharging over time. The battery has a fixed lifetime of 10 years and a maximum charge, n_{max} of 5000. Let's assume a linear depreciation over the lifetime and denote the remaining battery value as γ_H at any the start of some time horizon H. The cost per cycle of charge can then be defined as the remaining battery value divided by remaining number of charges. We further define the following variables

- γ_H: remaining battery value at start of horizon H (£)
- V_t: volume of battery at time t
- $\tilde{V}_H$: volume of battery at start of horizon H (MWh), use a constant volume over the current horizon for simplification
- $\tilde{n}_H$: number of charge cycles completed at start of horizon H, use a constant value over the current horizon for simplification

The number of cycles from charge n_t at any time t in the current horizon is

$$n_t = \frac{\sum_{m \in \{1,2,3\}} c_{m,t} \Delta t}{V_{t-1}} \approx \frac{\sum_{m \in \{1,2,3\}} c_{m,t} \Delta t}{\tilde{V}_H}$$

The cost per cycle of charge at any time t in the current horizon is

$$\text{cost per cycle}_t = \frac{(\text{remaining battery value})_t}{(\text{remaining number of charges})_t} \approx \frac{\gamma_H}{n_{max} - \tilde{n}_H}$$

The cost of degradation, i.e. the indirect cost incurred in charging the battery is

$$\text{indirect cost} = \sum_{t=1}^{T} (\text{cost per cycle}_t \times n_t)$$

$$\text{indirect cost} = \sum_{t=1}^{T} \sum_{m \in \{1,2,3\}} \frac{\gamma_H}{(n_{max} - \tilde{n}_H)\tilde{V}_H} c_{m,t}$$

The final objective function of profit over the horizon H consisting of $t = 1...T$ time steps is

$$\max_{d_{m,t},\ c_{m,t}} \Pi_H = \sum_{t=1}^{T} \sum_{m \in \{1,2,3\}} \left[P_{m,t} d_{m,t} - \left(P_{m,t} + \frac{\gamma_H}{(n_{max} - \tilde{n}_H)\tilde{V}_H} \right) c_{m,t} \right] \Delta t$$

2.3 Constraints

There are a total of 3 sets of constraints:

- **Charging and discharging constraints:** Charging and discharging cannot be negative and do not exceed max charging/discharging capacity. Charging and discharging do not occur simultaneously
- **State of charge constraints:** State of charge of battery cannot be negative and does not exceed the battery volume
- **Trading frequency constraints:** Trading in market needs to follow market trading frequency based on trading frequency of a market, the battery must export/import a constant level of power for the full period

2.3.1 Charging and discharging constraints

The total charging and discharging at any time period cannot exceed the maximum charging and discharging rates respectively, and must also be positive. We defined the max charging and discharging rate as

- κ_c: max charging rate
- κ_d: max discharging rate

This two constraints are specified as such

$$\sum_{m \in \{1,2,3\}} c_{m,t} \in [0, \kappa_c]$$

$$\sum_{m \in \{1,2,3\}} d_{m,t} \in [0, \kappa_d]$$

To enforce that charging and discharging do not happen simultaneously (not possible for batteries in general, otherwise we can take advantage of instantaneous price differences between markets), requires the use of logical constraint, which means the problem becomes a mixed-integer programming problem.

Introduce two binary variables $M_{c,t}$ and $M_{d,t}$ to indicate if the battery is in charging or discharging mode of operation respectively. Then by using the Big-M method, we can enforce both single mode of operation and max charging/discharging constraints together by using a Big-M value equal to the max charging/discharging rate.

This constraint ensures that both variables cannot be 1:

$$M_{c,t} + M_{d,t} \leq 0$$

These two constraints ensure that if mode is charging, the charge value cannot exceed κ_c, and similarly for discharging:

$$\sum_{m \in \{1,2,3\}} c_{m,t} \leq \kappa_c M_{c,t}$$

$$\sum_{m \in \{1,2,3\}} d_{m,t} \leq \kappa_d M_{d,t}$$

2.3.2 State of charge constraints

The next constraint to impose is on the battery volume. The amount of charging and discharging to the battery must not exceed the capacity of the battery

$$S_t \in [0, V_t]$$

The state of charge of the battery S_t at the end of time t is the state of charge at the end of the previous time $t-1$, plus the change due to charging and discharging during the current time t.

$$S_t = S_{t-1} + \eta_c \sum_{m \in \{1,2,3\}} c_{m,t} \Delta t - \frac{1}{\eta_d} \sum_{m \in \{1,2,3\}} d_{m,t} \Delta t$$

$$S_t = S_0 + \eta_c \sum_{t=1}^{T} \sum_{m\in\{1,2,3\}} c_{m,t}\Delta t - \frac{1}{\eta_d} \sum_{t=1}^{T} \sum_{m\in\{1,2,3\}} d_{m,t}\Delta t$$

Due to storage volume degradation, V_t is not constant but decreases with charging. The relationship between V_t and number of charging cycles is

$$V_t = V_0(1-f)^{cycles} = V_0(1-f)^{\sum_1^t \frac{C_t \Delta t}{V_{t-1}}}$$

This is a non-linear relationship, however since f is a relatively small value (in this case 1e-5, we may approximate V_t with a linear function $\tilde{V_t}$.

$$V_t \approx \tilde{V_t} = \left(1 - f\frac{\sum_{m\in\{1,2,3\}} c_{m,t}\Delta t}{V_{t-1}}\right) V_{t-1} = V_{t-1} - f \sum_{m\in\{1,2,3\}} c_{m,t}\Delta t$$

$$V_t \approx \tilde{V_t} = V_0 - f \sum_{t=1}^{T} \sum_{m\in\{1,2,3\}} c_{m,t}\Delta t$$

Therefore the constraint for state of charge can be rewritten as

$$S_0 - V_0 + (\eta_c + f)\Delta t \sum_{t=1}^{T} \sum_{m\in\{1,2,3\}} c_{m,t} - \frac{1}{\eta_d}\Delta t \sum_{t=1}^{T} \sum_{m\in\{1,2,3\}} d_{m,t} \leq 0$$

$$S_0 + \eta_c\Delta t \sum_{t=1}^{T} \sum_{m\in\{1,2,3\}} c_{m,t} - \frac{1}{\eta_d}\Delta t \sum_{t=1}^{T} \sum_{m\in\{1,2,3\}} d_{m,t} \geq 0$$

2.3.3 Trading frequency constraints

When the market does not trade at frequency $= t$, additional constraint is required to ensure that a constant charge or discharge rate is to be maintained for the entirety of the fixed-price period. For example, market 3 which trades at 1 day frequency (48 half-hour timesteps), the charge or discharge rate at any time after 00:00, should be the same value as was committed at 00:00 until the following day. Thus, when $(t-1) \mod 48 \neq 0$, the charge or discharge rate is equal to that at $\lfloor(\frac{t-1}{48})\rfloor \times 48$

this can be enforced by

$$c_{m,t} = c_{m,t'}$$

$$d_{m,t} = d_{m,t'}$$

where $t' = \lfloor(\frac{t-1}{T_{trade}})\rfloor \times T_{trade}$ and T_{trade} is the fixed-price trading period