Firm Intangible Capital, Loan Contracts, and Monetary Policy

Renbin Zhang
Shandong University

Weimin Zhou
Peking University

Oct, 2023

Introduction

- How do loan contracts and firms' capital structure affect monetary policy transmission to firm-level investment

- Do firms' capital structure play a role in choosing the contract type? What's the mechanism behind?

- This paper: empirical evidence + quantitative analysis

What We Do

- Empirically:
  - firm-level data: Dealscan + Compustat
  - for firms with loan contracts: investment of firms with high intangibles react more to MP shocks
  - decompose contract types into cash flow-based and asset-based: cash flow-based borrowers drive this pattern
  - Why? : intangible investment accounts for half to the rate of sensitivities for cash flow-based borrowers; but negligible contribution for asset-based borrowers.
- Quantitatively:
  - A tractable GE model with heterogeneous firms: endogeneous choice of contract types + tangible & intangible investment decision.
  - An interest rate change: affect the extensive margin for high-INT firms within cash flow-based borrowers
  - able to explain the observed facts

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Related Literature

- A rising importance of intangible capital (Crouzet and Eberly (2018))
  - weaker response to MP for high-INT firms: cash holding channel (Döttling and Ratnovski (2023), Caggese and Pérez-Orive (2022), Li (2022), Falato, Kadyrzhanova, Sim, and Steri (2022))
  - This paper: focus on firms with loan contracts: stronger response to MP for high-INT firms, driven by cash flow-based borrowers
  - a model to further rationalize this finding

- Role of financial frictions and debt covenants
  - prevalence of cash flow-based covenants (Greenwald et al. (2019), Lian and Ma (2021), Drechsel (2023), Öztürk (2023))
  - firm-level heterogeneous sensitivity: liquidity (Jeenas (2019)), age/dividend (Cloyne, Ferreira, Froemel, and Surico (2018)), leverage/credit spread (Cesa-Bianchi and Sokol (2021)), and distance to default (Ottonello and Winberry (2020)).
  - This paper: provide new evidence: intangible investment + contract types.

Data

- US non-financial firms quarterly data: Compustat (balance sheet info) + Dealscan (loan covenants); Sample: 1992Q1 - 2017Q2;

- cash-flow based loans: loans secured with "all assets", follwing Lian and Ma (2021)
  - creditors make sure that there are further growing cash inflows due to past financial statements
  - perform detailed cash flow analyses, and monitor earnings extensively

- asset-based: loans backed by specific assets, following Drechsel (2023) Details
  - borrowers pledge the assets of the organisation
  - creditors evalute the market value of teh collateral

- intangible investment: sum of R&D expense, 30% SG&A expense, following Peters and Taylor (2017)

- contractionary MP shock: proxy SVAR following Gertler and Karadi (2015) Details

Summary Statistics: Asset-based vs. Cash flow-based

	mean	sd	p50	p25	p75
Panel A: Asset-based Firms (N = 4,213)					
Physical capital assets	1031.579	5548.842	104.948	20.031	483.009
Total intangible assets	249.240	1233.115	21.242	4.221	107.103
Size	6.022	1.983	6.069	4.682	7.383
Leverage	0.315	0.830	0.259	0.100	0.424
Market-book ratio	1.650	17.141	1.073	0.770	1.629
Sales growth	0.018	0.319	0.018	-0.065	0.105
Liquidity	0.486	0.233	0.487	0.304	0.669
Intangible ratio	0.284	0.258	0.204	0.062	0.463
Panel B: Cash flow-based Firms (N = 10,340)					
Physical capital assets	1128.599	7038.389	104.423	21.437	457.900
Total intangible assets	517.092	2855.040	53.141	12.405	232.739
Size	6.198	1.895	6.268	4.946	7.448
Leverage	0.361	2.237	0.268	0.103	0.454
Market-book ratio	1.776	18.097	1.116	0.783	1.695
Sales growth	0.010	0.308	0.011	-0.064	0.087
Liquidity	0.458	0.244	0.445	0.259	0.642
Intangible ratio	0.423	0.286	0.400	0.161	0.671

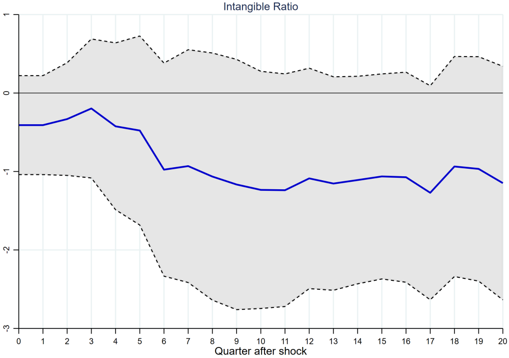
- intangible ratio defined as intangible assets / total assets
- prevalence of cash flow-based; higher intangible capital

Dynamic Effects on Investment

- estimate the differential pass-through of the exogenous shocks ε_t to firm's investment, conditional on initial intangible ratio:

$$\log y_{j,t+h} - \log y_{j,t-1} = \beta_h \times \varepsilon_t \times \mathrm{IR}_{j,t-1} + \Gamma_h' Z_{j,t-1} + \alpha_{j,h} + \alpha_{st,h} + e_{j,t+h}, \tag{1}$$

- subsample of firms with loan contracts:



- robustness checks: alt mp shocks intan inv. intan inv. + alt. shocks

Decompose the Dynamic Effects

- 90% confidence bands std error using Newey and West (1987).
- strong response for high-INT firms: driven by cash flow-based borrowers
- robustness checks: alt mp shocks intan inv. intan inv. + alt. shocks

Check: Contemporaneous Effect

- Contemporaneous effects:

$$\begin{aligned}\log y_{j,t} - \log y_{j,t-1} = \beta^a \mathrm{IR}_{j,t-1} \times \mathbb{I}^{Asset}_{j,t-1} \times \varepsilon_t + \beta^c \mathrm{IR}_{j,t-1} \times \mathbb{I}^{Cash}_{j,t-1} \times \varepsilon_t \\ + \gamma^a \mathrm{IR}_{j,t-1} \times \mathbb{I}^{Asset}_{j,t-1} + \gamma^c \mathrm{IR}_{j,t-1} \times \mathbb{I}^{Cash}_{j,t-1} \\ + \alpha^a \mathbb{I}^{Asset}_{j,t-1} \times \varepsilon_t + \alpha^c \mathbb{I}^{Cash}_{j,t-1} \times \varepsilon_t + \Gamma' Z_{j,t-1} + \alpha_j + \alpha_{st} + e_{j,t},\end{aligned} \tag{2}$$

- all single terms included in $Z_{j,t-1}$
- firm-level controls: market-book ratio, leverage, size, liquidity ratio, and fiscal quarters dummies;
- sector-quarter FE and firm FE

	(1) Total Investment	(2) Intangible Investment	(3) Physical Investment
$\text{IR}_{j,t-1} \times \mathbb{I}^{\text{Cash}}_{j,t-1} \times \epsilon_t$	-6.702**	-18.559***	-4.687*
	(3.143)	(4.683)	(2.760)
$\text{IR}_{j,t-1} \times \mathbb{I}^{\text{Asset}}_{j,t-1} \times \epsilon_t$	4.111	1.505	1.831
	(3.068)	(4.563)	(2.697)
$\text{IR}_{j,t-1}$	-94.335***	-143.010***	-26.407***
	(5.913)	(8.757)	(5.152)
$\text{IR}_{j,t-1} \times \mathbb{I}^{\text{Cash}}_{j,t-1}$	14.786***	7.009	10.526***
	(4.025)	(5.910)	(3.501)
$\text{IR}_{j,t-1} \times \mathbb{I}^{\text{Asset}}_{j,t-1}$	3.816	6.797	6.374*
	(3.957)	(5.815)	(3.445)
$\mathbb{I}^{\text{Cash}}_{j,t-1} \times \epsilon_t$	0.922	3.305***	0.480
	(0.619)	(0.930)	(0.541)
$\mathbb{I}^{\text{Asset}}_{j,t-1} \times \epsilon_t$	-0.825	-0.758	0.030
	(0.585)	(0.877)	(0.512)
Observations	14041	13974	14169
Firm Controls	Yes	Yes	Yes
Single Term	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes
Sector-Quarter FE	Yes	Yes	Yes

- In response to a contractionary mp shock: strong response for high-INT firms conditional on cash flow-based only.

Mechanism behind: estimating the change

- quantify the changing shares for the transmission of MP
- estimated peak value of investment responses $\times$ its share
- = implied contribution to the rate sensitivity

		2013Q1 - 2017Q2		Full Sample	
	IRF (h=12)	Share	Contribution	Share	Contribution
Asset-based borrowers					
Intangible Investment	0.166	0.292	0.0484	0.0345	0.00572
Physical Investment	2.596	0.708	1.837	0.965	2.507
Cash flow-based borrowers					
Intangible Investment	0.762	0.385	0.293	0.184	0.140
Physical Investment	0.346	0.615	0.213	0.816	0.283

- larger response of intangible investment for cash flow-based borrowers, account over half to the rate of sensitivites
- negligible contribution of intangibles for asset-based borrowers

Taking stock of empirical evidence

- Unlike Döttling and Ratnovski (2023): less response of investment to mp for high-INT firms $\implies$ cash holding + unconstrained
- our results: stronger response of investment to mp for high-INT firms if focusing on firms with loan contracts
  - cash flow-based borrowers drive this pattern
  - mechanisms: high contribution, high variation of intangible investment to firm production for cash flow-based borrowers
- Next: heterogeneous firms with iid productivity on intangible capital, choosing loan contracts to borrow and invest
- able to explain the empirical facts

Static Model: Setup

- A continuum of firms with unit measure, endowed with e net worth, obtain b amount of bank loans at loan rate R^b
- flow of funds: $e + b = k^T + k^I$
- Two types of investment:
  - invest k^T on tangible assets with rate of return R^k
  - invest k^I on intangible assets with rate $\varepsilon\phi R^k$
    - $\phi > 1$, ε is idiosyncratic productivity shock with i.i.d. CDF $F(\cdot)$, following Gourio (2013) and Allub, Ferriere, Franjo, and Zheng (2023)
- borrow via loan covenants:
  - risk-neutral bank offers loan covenants to prevent default and is indifferent between two offers:
  - asset-based: $b \leq \theta^1 Q k^T$; cash flow-based: $b \leq \theta^2 R^k (k^T + \varepsilon\phi k^I)$

Firm's Problem

- firms draw productivity shock on intangible capital
- decide investment (k^T, k^I) with endogeneous choice of debt contracts:

$$\max \varphi(\varepsilon) = R^k f(k^T, k^I) = R^k(k^T + \varepsilon\phi k^I) + Qk^T - R^b b$$
$$\text{s.t. } b + e = k^T + k^I$$
$$\min\{k^T, k^I\} \geq \rho e$$
$$b \leq \max\{\theta^1 Q k^T, \theta^2 R^k (k^T + \varepsilon\phi k^I)\}$$

Investment Decisions and Choice of Loan Contracts

- optimal investment decisions: Proof

$$
(k^T, k^I) = \begin{cases} \left(\rho e, \frac{\theta^2 R^k \rho e + (1-\rho)e}{1-\theta^2 R^k \varepsilon \phi}\right), & \text{if } \varepsilon \geq \varepsilon^{**} \text{ , cash flow-based} \\ \left(\frac{\theta^2 R^k \varepsilon \phi \rho e + (1-\rho)e}{1-\theta^2 R^k}, \rho e\right), & \text{if } \varepsilon^c \leq \varepsilon < \varepsilon^{**} \text{ , cash flow-based} \\ \left(\frac{(1-\rho)e}{1-\theta^1 Q}, \rho e\right), & \text{if } \varepsilon < \varepsilon^c \text{ , asset-based} \end{cases} \quad (3)
$$

- where:

$$
\varepsilon^{**} \equiv \frac{R^k + Q - R^b \theta^2 R^k}{R^k \phi (1 - \theta^2 R^b)}, \quad \varepsilon^c \equiv \frac{(\theta^1 Q - \theta^2 R^k) \frac{1-\rho}{\rho}}{\theta^2 R^k \phi (1 - \theta^1 Q)}
$$

- for asset-based borrowers: identical intangible ratio, affected by asset price Q
- for cash flow-based borrowers (PE):
  - extensive margin: an interest rate hike $\Longrightarrow$ share of high-INT group $\downarrow$; share of low-INT group $\uparrow$

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Full-Fledged Model

- following Dong, Guo, Peng, and Xu (2022)
- at t: ε_t realized; purchase k_t^T at nominal price Q_t^k and decide k_t^I
- at $t+1$: hire labor n_{t+1} at nominal wage W_{t+1}, produce and sell capital, earn profit, exit market.
- production function: $y_{t+1} = A_{t+1} f(k_t^T, k_t^I)^\alpha (n_{t+1})^{1-\alpha}$
- profit: $\pi_{jt+1} = R_{t+1}^k f(k_t^T, k_t^I)$, where $R_{t+1}^k = \alpha (P_{t+1} A_{t+1} / W_{t+1})^{\frac{1}{\alpha}} W_{t+1}^{\frac{\alpha-1}{\alpha}}$
- Similar to the static model, at equilibrium:

$$
\begin{aligned}
K_t^{I,c} &= \int_{\varepsilon_t^{**}} \frac{\theta^2 R_{t+1}^k \rho + (1-\rho)}{1 - \theta^2 R_{t+1}^k \varepsilon_t \phi} dF(\varepsilon_t) E_t + (F(\varepsilon_t^{**}) - F(\varepsilon_t^c)) \rho E_t \\
K_t^{T,c} &= [1 - F(\varepsilon_t^{**})] \rho E_t + E_t \int_{\varepsilon_t^c}^{\varepsilon_t^{**}} \frac{\theta^2 R_{t+1}^k \varepsilon_t \phi \rho + (1-\rho)}{1 - \theta^2 R_{t+1}^k} dF(\varepsilon_t) \\
K_t^{I,a} &= F(\varepsilon_t^c) \rho E_t, \quad K_t^{T,a} = F(\varepsilon_t^c) \frac{(1-\rho)}{1 - \theta^1 Q_{t+1}^k} E_t
\end{aligned}
$$

- assume θ fraction of profit is consumed when exit:

$$
E_t = (1-\theta) E_{t-1} + (1-\theta) \int \pi_t(\varepsilon_t) dF(\varepsilon_t)
$$

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The Rest

- **HH**: consumes C_t, provides labor N_t, and save deposits S_t at rate R_t
  - $\max U(C_t, N_t)$, s.t., $P_t C_t + S_t = W_t N_t + (1 + R_t) S_{t-1} + T_t$
- **Capital goods producers**: $\max E_0 \sum \beta^t \Lambda_t D_t^k$, where

$$D_t^k = Q_t^k I_t^T - \nu_t \left[1 + \frac{\Omega^k}{2} \left(\frac{I_t^T}{I_{t-1}^T} - 1 \right)^2 \right] I_t^T$$

- **Banks**: $V_t = \max_{B_{t+1}} \left[\Pi_t^B + \beta \mathbf{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1} \right]$, where

$$\Pi_t^B = \left(1 + R_{t-1}^b \right) B_{t-1} - P_t \Psi \left(B_t / P_t \right) - B_t + S_t - \left(1 + R_{t-1} \right) S_{t-1}$$

- $\Psi(b_t) = \xi_1 / \left(1 + \xi_2 \right) \left(b_t / \bar{b} \right)^{1+\bar{\zeta}_2} \bar{b}$
- **NK block**: monopolistic retail sector with Calvo pricing
- **MP rule**: $\frac{1+R_t}{1+R} = \left(\frac{\pi_t}{\pi} \right)^{\varphi_\pi} \left(\frac{Y_t}{\bar{Y}} \right)^{\varphi_y}$, for $\varphi_\pi, \varphi_y > 0$
- Market Clears: $C_t + \frac{1}{P_t} \left[1 + \frac{\Omega^k}{2} \left(\frac{I_t^T}{I_{t-1}^T} - 1 \right)^2 \right] I_t^T + I_t^I + \Psi(B_t / P_t) = Y_t$

Next Step

- consider quantitative effects under general equilibrium (calibrate the model and continue)
- examine the macro-prudential policies regulating firms' leverage under alternative debt contracts.
- empirical part: robustness check, replace continuous variables to dummies (high-INT, low-INT) as interaction term to mp shocks.

Thank you!

Details on Loan Classification

- Lian and Ma (2021) classify facilities secured by all assets as cash-flow based loans, because the value of this form of collateral in the event of bankruptcy is calculated based on the cash flow value from continuing operations.
- Cash flow-based: loan facilities secured by "all"
- Asset-based:
  - loans pledged by specific assets (exclued thoses that are backed by "all")
  - secured revolving line of credit: asset-based loans Go back

Details on MP Shock Identification

- Identification: proxy SVAR with monthly data: log(INDPRO), log(CPIAUCSL), log(GS1), EBP
- instrument 1 z_t: HF FF4 surprises
- instrument 2 z_t^*: orthogonal known macro predictor prior to the announcement date, following Bauer and Swanson (2023)
  - $z_t = \alpha + \beta' X_{t-} + u_t$, X_{t-}: 6 macro predictors prior to announcement t: Nonfarm payrolls surprise; Employment growth; S&P 500; Yield curve slope; Commodity prices; Treasury skewness
- 2SLS for:

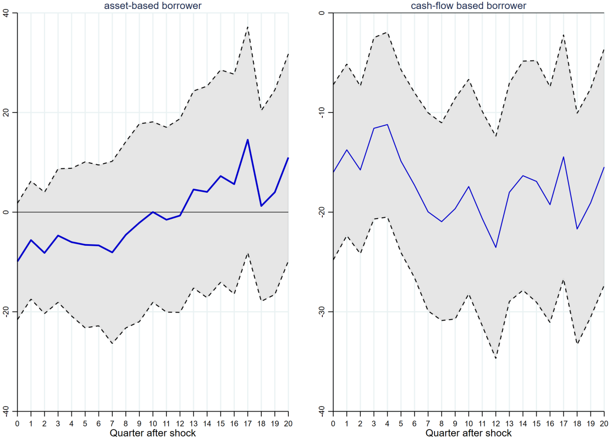
$$Y_t = \alpha + B(L)Y_{t-1} + s_1 Y_t^{2y} + \tilde{u}_t$$

- where $SS' = \mathrm{Var}[u_t] \equiv \Omega$, $u_t = S\varepsilon_t$ Go back

Differential IRFs of Investment to Monetary Shocks (orthogonal)

Go back

Differential IRFs of Intangible Investment to Monetary Shocks



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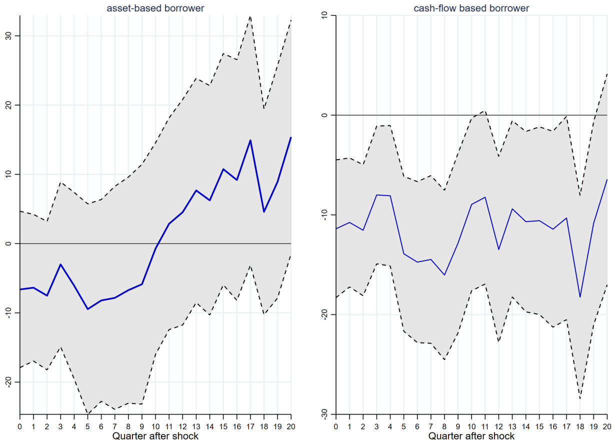
Differential IRFs of Intangible Investment to Monetary Shocks (orthogonal)



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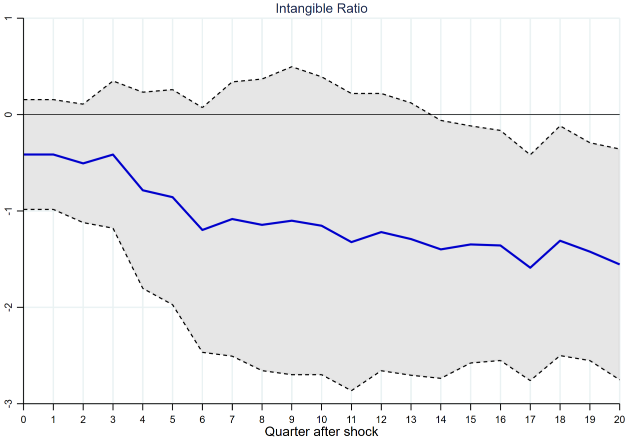
Differential IRFs of Total Investment to Monetary Shocks (orthogonal)



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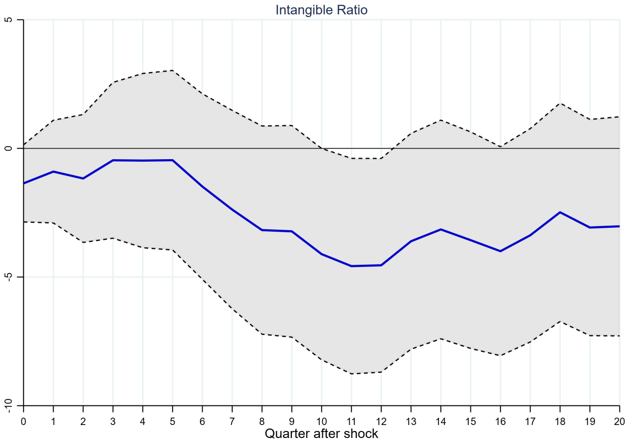
Differential IRFs of Intangible Investment to Monetary Shocks



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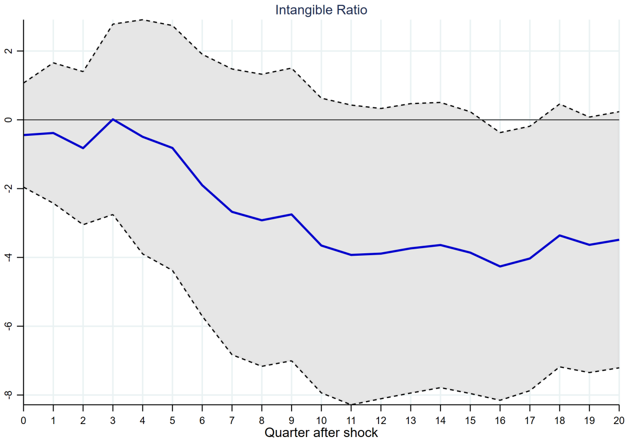
Differential IRFs of Intangible Investment to Monetary Shocks (orthogonal)



◂ Go back

Timing of the model

The timing for an individual firm is as follows:

1. firms choose labor supply n_{jt} given a competitive wage rate w_t;
2. firms choose how much investment allocated to tangible and intangible part conditional on prepayment $b_{jt-1}R_{t-1}$.
3. firms commit to a feasible borrowing $b^*_{jt}(\varepsilon_{jt}; s_t)$ for each possible realization of ε_{jt}.
4. firms draw ε_{jt} and choose their new loan size b^*_{jt} subject to their loan contracts.
5. firms realize insurance claims and buy new Arrow securities, and choose consumption.

go back

Proof of Aggregation

Firm's Lagrangian:

$$\mathcal{L} = \sum_{t=0}^{\infty} \sum_{s^t} \beta_f^t \int_{\varepsilon_{jt}} \Big\{ u(c_{jt}^f) + \lambda_{jt} \Big[R_t^k (k_{jt-1}^T + \varepsilon_{jt} \phi k_{jt-1}^I)$$

$$+ P_t^k ((1-\delta) k_{jt-1} - k_{jt})$$

$$+ b_{jt}^* - R_{t-1} b_{jt-1}^* + a_{jt-1}(s_t) + \sum_{s_{t+1}|s^t} p_t^a (s_{t+1}) a_{jt}(s_{t+1})$$

$$+ \psi_{jt} \left(\max \left\{ \bar{b}_{jt}^{cash}, \bar{b}_{jt}^{asset} \right\} - b_{jt}^* \right) \Big] \Big\} dF(\varepsilon_{jt})$$

go back

Proof of Static Model: cash flow-based

- if firms choose cash flow-based covenants:

$$\theta^1 Q k^T \leq \theta^2 R^k (k^T + \varepsilon \phi k^I) \implies \varepsilon \geq \frac{(\theta^1 Q - \theta^2 R^k) k^T}{\theta^2 R^k \phi k^I} \equiv \varepsilon^c (k^T, k^I) \qquad (4)$$

- then, the PMP becomes: $\left[R^k + Q - R^b \theta^2 R^k\right] k^T + \left[R^k \varepsilon \phi (1 - R^b \theta^2)\right] k^I$
- the firm decision of investment is:

$$(k^T, k^I) = \begin{cases} (\rho e, \frac{\theta^2 R^k \rho e + (1-\rho) e}{1 - \theta^2 R^k \varepsilon \phi}), & \text{if } \varepsilon > \varepsilon^{**} \equiv \frac{R^k + Q - R^b \theta^2 R^k}{R^k \phi (1 - \theta^2 R^b)} \\ (\frac{\theta^2 R^k \varepsilon \phi \rho e + (1-\rho) e}{1 - \theta^2 R^k}, \rho e), & \text{if } \varepsilon < \varepsilon^{**} \end{cases} \qquad (5)$$

- assume $1 - \theta^2 R^k \bar{\varepsilon} \phi > 0$. Given above capital allocation, backout ε^c:
- for the capital allocation with $\varepsilon > \varepsilon^{**}$:

$$(\theta^1 Q - \theta^2 R^k) \leq \theta^2 R^k \varepsilon \phi \frac{k^I}{k^T} \implies \varepsilon > \frac{\theta^1 Q - \theta^2 R^k}{(\theta^1 Q + (1-\rho)/\rho) \theta^2 R^k \phi} \equiv \varepsilon_1^c \quad (6)$$

- compare ε_1^c in (6) and ε^{**} in (5), assume $\theta^1 Q - \theta^2 R^k > 0$:

$$\frac{\varepsilon^{**}}{\varepsilon_1^c} = \underbrace{\frac{1 + Q/R^k - \theta^2 R^b}{1 - \theta^2 R^b}}_{>1} \underbrace{\frac{1 + \frac{1-\rho}{\rho} \frac{1}{\theta^1 Q}}{\frac{1}{\theta^2 R^k} - \frac{1}{\theta^1 Q}}}_{>1} \geq 1$$

- $\varepsilon > \varepsilon^{**}$: invest more intangibles + cash flow-based covenants. go back

Proof of Static Model: cash flow-based

- for the capital allocation with $\varepsilon < \varepsilon^{**}$:

$$(\theta^1 Q - \theta^2 R^k) \leq \theta^2 R^k \varepsilon \phi \frac{k^I}{k^T} \implies \varepsilon > \frac{(\theta^1 Q - \theta^2 R^k)\frac{1-\rho}{\rho}}{\theta^2 R^k \phi (1 - \theta^1 Q)} \equiv \varepsilon_2^c \qquad (7)$$

- under certain parameter assumptions:

$$\frac{\varepsilon^{**}}{\varepsilon_2^c} \geq 1 \implies \frac{1}{\theta^1 Q + \frac{1-\rho}{\rho}} \frac{1 - \theta^1 Q}{\frac{1-\rho}{\rho}} > 1$$

- $\varepsilon_2^c < \varepsilon < \varepsilon^{**}$: invest more tangibles + cash flow-based covenants.

go back

Proof of Static Model: asset-based

- if firms choose asset-based covenants, the PMP becomes:

$$\max \left[R^k + Q - R^b\theta^1 Q\right] k^T + \left[R^k \varepsilon\phi\right] k^I$$

- firm investment is:

$$(k^T, k^I) = \begin{cases} (\rho e, \theta^1 Q\rho e + (1-\rho)e), & \text{if } \varepsilon > \varepsilon^* \equiv \frac{R^k + (1-\theta^1 R^b)Q}{R^k\phi} \\ (\frac{(1-\rho)e}{1-\theta^1 Q}, \rho e), & \text{if } \varepsilon < \varepsilon^* \end{cases} \tag{8}$$

- for the capital allocation with $\varepsilon > \varepsilon^*$:

$$(\theta^1 Q - \theta^2 R^k) > \theta^2 R^k \varepsilon\phi \frac{k^I}{k^T} \implies \varepsilon < \frac{\theta^1 Q - \theta^2 R^k}{\theta^2 R^k \phi} \frac{1}{\theta^1 Q + (1-\rho)/\rho} \equiv \varepsilon_1^a$$

- this requires $\varepsilon^* < \varepsilon_1^a$:

$$\frac{\theta^1 Q - \theta^2 R^k}{(\theta^1 Q + \frac{1-\rho}{\rho})(\theta^2 R^k + (1-\theta^1 R^b)\theta^2 Q)} > 1$$

- which fails to hold. go back

Proof of Static Model: asset-based

- for the capital allocation with $\varepsilon < \varepsilon^*$:

$$(\theta^1 Q - \theta^2 R^k) > \theta^2 R^k \varepsilon \phi \frac{k^I}{k^T} \implies \varepsilon < \frac{\theta^1 Q - \theta^2 R^k}{\theta^2 R^k \phi (1 - \theta^1 Q)} \frac{1-\rho}{\rho} \equiv \varepsilon_2^a \qquad (9)$$

- we can show that:

$$\frac{\varepsilon_2^a}{\varepsilon^*} = \frac{\theta^1 R^k - \theta^2 R^k \frac{1-\rho}{\rho}}{\theta^2 \left[R^k + (1 - \theta^1 R^b) Q\right] (1 - \theta^1 Q)} < 1$$

- therefore, firms with $\varepsilon < \varepsilon^*$: invest more tangibles + asset-based covenants go back

Aggregation for Static Model

- for asset-based borrower: identical share of intangible; only affected by asset-price channel:

$$K_a^I = \rho F(\varepsilon^c), \quad K_a^T = \frac{1-\rho}{1-\theta^1 Q} F(\varepsilon^c)$$

- cash flow-based, high-INT group:

$$K_1^I / E = \int_{\varepsilon^{**}} \frac{\theta^2 R^k \rho + (1-\rho)}{1-\theta^2 R^k \varepsilon \phi} dF(\varepsilon), \quad K_1^T / E = \rho(1 - F(\varepsilon^* *))$$

- cash flow-based, low-INT group:

$$K_2^I / E = \rho(F(\varepsilon^* *) - F(\varepsilon^c)), \quad K_2^T / E = \int_{\varepsilon^c}^{\varepsilon^{**}} \frac{\theta^2 R^k \varepsilon \phi \rho + (1-\rho)}{1-\theta^2 R^k} dF(\varepsilon)$$

- go back