

Firm Intangible Capital, Loan Contracts, and Monetary Policy

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Oct, 2023

Introduction

- How do loan contracts and firms' capital structure affect monetary policy transmission to firm-level investment
- Do firms' capital structure play a role in choosing the contract type? What's the mechanism behind?
- This paper: empirical evidence + quantitative analysis

What We Do

- Empirically:
 - ▶ firm-level data: Dealscan + Compustat
 - ▶ for firms with loan contracts: investment of firms with high intangibles react more to MP shocks
 - ▶ decompose contract types into cash flow-based and asset-based: cash flow-based borrowers drive this pattern
 - ▶ Why? : intangible investment accounts for half to the rate of sensitivities for cash flow-based borrowers; but negligible contribution for asset-based borrowers.
- Quantitatively:
 - ▶ A tractable GE model with heterogeneous firms: endogenous choice of contract types + tangible & intangible investment decision.
 - ▶ An interest rate change: affect the extensive margin for high-INT firms within cash flow-based borrowers
 - ▶ able to explain the observed facts

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Related Literature

- A rising importance of intangible capital ([Crouzet and Eberly \(2018\)](#))
 - ▶ weaker response to MP for high-INT firms: cash holding channel ([Döttling and Ratnovski \(2023\)](#), [Caggese and Pérez-Orive \(2022\)](#), [Li \(2022\)](#), [Falato, Kadyrzhanova, Sim, and Steri \(2022\)](#))
 - ▶ This paper: focus on firms with loan contracts: stronger response to MP for high-INT firms, driven by cash flow-based borrowers
 - ▶ a model to further rationalize this finding
- Role of financial frictions and debt covenants
 - ▶ prevalence of cash flow-based covenants ([Greenwald et al. \(2019\)](#), [Lian and Ma \(2021\)](#), [Drechsel \(2023\)](#), [Öztürk \(2023\)](#))
 - ▶ firm-level heterogeneous sensitivity: liquidity ([Jeenas \(2019\)](#)), age/dividend ([Cloyne, Ferreira, Froemel, and Surico \(2018\)](#)), leverage/credit spread ([Cesa-Bianchi and Sokol \(2021\)](#)), and distance to default ([Ottonello and Winberry \(2020\)](#)).
 - ▶ This paper: provide new evidence: intangible investment + contract types.

Data

- US non-financial firms quarterly data: Compustat (balance sheet info) + Dealscan (loan covenants); Sample: 1992Q1 - 2017Q2;
- cash-flow based loans: loans secured with “all assets”, following [Lian \(2021\)](#)
 - ▶ creditors make sure that there are further growing cash inflows due to past financial statements
 - ▶ perform detailed cash flow analyses, and monitor earnings extensively
- asset-based: loans backed by specific assets, following [Drechsel \(2023\)](#)
 - Details
 - ▶ borrowers pledge the assets of the organisation
 - ▶ creditors evaluate the market value of the collateral
- intangible investment: sum of R&D expense, 30% SG&A expense, following [Peters and Taylor \(2017\)](#)
- contractionary MP shock: proxy SVAR following [Gertler and Karadi \(2015\)](#)
 - Details

Summary Statistics: Asset-based vs. Cash flow-based

	mean	sd	p50	p25	p75
Panel A: Asset-based Firms (N = 4,213)					
Physical capital assets	1031.579	5548.842	104.948	20.031	483.009
Total intangible assets	249.240	1233.115	21.242	4.221	107.103
Size	6.022	1.983	6.069	4.682	7.383
Leverage	0.315	0.830	0.259	0.100	0.424
Market-book ratio	1.650	17.141	1.073	0.770	1.629
Sales growth	0.018	0.319	0.018	-0.065	0.105
Liquidity	0.486	0.233	0.487	0.304	0.669
Intangible ratio	0.284	0.258	0.204	0.062	0.463
Panel B: Cash flow-based Firms (N = 10,340)					
Physical capital assets	1128.599	7038.389	104.423	21.437	457.900
Total intangible assets	517.092	2855.040	53.141	12.405	232.739
Size	6.198	1.895	6.268	4.946	7.448
Leverage	0.361	2.237	0.268	0.103	0.454
Market-book ratio	1.776	18.097	1.116	0.783	1.695
Sales growth	0.010	0.308	0.011	-0.064	0.087
Liquidity	0.458	0.244	0.445	0.259	0.642
Intangible ratio	0.423	0.286	0.400	0.161	0.671

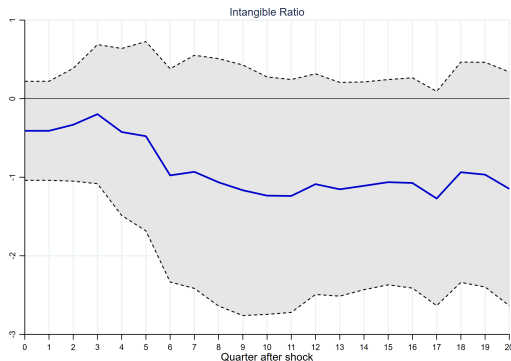
- intangible ratio defined as intangible assets / total assets
- prevalence of cash flow-based; higher intangible capital

Dynamic Effects on Investment

- estimate the differential pass-through of the exogenous shocks ε_t to firm's investment, conditional on initial intangible ratio:

$$\log y_{j,t+h} - \log y_{j,t-1} = \beta_h \times \varepsilon_t \times \text{IR}_{j,t-1} + \Gamma'_h Z_{j,t-1} + \alpha_{j,h} + \alpha_{st,h} + e_{j,t+h}, \quad (1)$$

- subsample of firms with debt contracts:



- robustness checks: alt mp shocks intan inv. intan inv. + alt. shocks

Decompose the Dynamic Effects



- 90% confidence bands std error using [Newey and West \(1987\)](#).
- strong response for high-INT firms: driven by cash flow-based borrowers
- robustness checks: [alt mp shocks](#) [intan inv.](#) [intan inv. + alt. shocks](#)

Check: Contemporaneous Effect

- Contemporaneous effects:

$$\begin{aligned}\log y_{j,t} - \log y_{j,t-1} = & \beta^a \text{IR}_{j,t-1} \times \mathbb{I}_{j,t-1}^{\text{Asset}} \times \varepsilon_t + \beta^c \text{IR}_{j,t-1} \times \mathbb{I}_{j,t-1}^{\text{Cash}} \times \varepsilon_t \\ & + \gamma^a \text{IR}_{j,t-1} \times \mathbb{I}_{j,t-1}^{\text{Asset}} + \gamma^c \text{IR}_{j,t-1} \times \mathbb{I}_{j,t-1}^{\text{Cash}} \\ & + \alpha^a \mathbb{I}_{j,t-1}^{\text{Asset}} \times \varepsilon_t + \alpha^c \mathbb{I}_{j,t-1}^{\text{Cash}} \times \varepsilon_t + \Gamma' Z_{j,t-1} + \alpha_j + \alpha_{st} + e_{j,t},\end{aligned}\quad (2)$$

- all single terms included in $Z_{j,t-1}$
- firm-level controls: market-book ratio, leverage, size, liquidity ratio, and fiscal quarters dummies;
- sector-quarter FE and firm FE

	(1)	(2)	(3)
	Total Investment	Intangible Investment	Physical Investment
$IR_{j,t-1} \times \mathbb{I}_{j,t-1}^{\text{Cash}} \times \epsilon_t$	-6.702** (3.143)	-18.559*** (4.683)	-4.687* (2.760)
$IR_{j,t-1} \times \mathbb{I}_{j,t-1}^{\text{Asset}} \times \epsilon_t$	4.111 (3.068)	1.505 (4.563)	1.831 (2.697)
$IR_{j,t-1}$	-94.335*** (5.913)	-143.010*** (8.757)	-26.407*** (5.152)
$IR_{j,t-1} \times \mathbb{I}_{j,t-1}^{\text{Cash}}$	14.786*** (4.025)	7.009 (5.910)	10.526*** (3.501)
$IR_{j,t-1} \times \mathbb{I}_{j,t-1}^{\text{Asset}}$	3.816 (3.957)	6.797 (5.815)	6.374* (3.445)
$\mathbb{I}_{j,t-1}^{\text{Cash}} \times \epsilon_t$	0.922 (0.619)	3.305*** (0.930)	0.480 (0.541)
$\mathbb{I}_{j,t-1}^{\text{Asset}} \times \epsilon_t$	-0.825 (0.585)	-0.758 (0.877)	0.030 (0.512)
Observations	14041	13974	14169
Firm Controls	Yes	Yes	Yes
Single Term	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes
Sector-Quarter FE	Yes	Yes	Yes

- In response to a contractionary mp shock: strong response for high-INT firms conditional on cash flow-based.

Mechanism behind: estimating the change

- quantify the changing shares for the transmission of MP
- estimated peak value of investment responses \times its share
- = implied contribution to the rate sensitivity

		2013Q1 - 2017Q2		Full Sample	
	IRF (h=12)	Share	Contribution	Share	Contribution
Asset-based borrowers					
Intangible Investment	0.166	0.292	0.0484	0.0345	0.00572
Physical Investment	2.596	0.708	1.837	0.965	2.507
Cash flow-based borrowers					
Intangible Investment	0.762	0.385	0.293	0.184	0.140
Physical Investment	0.346	0.615	0.213	0.816	0.283

- larger response of intangible investment for cash flow-based borrowers, account over half to the rate of sensitivities
- negligible contribution of intangibles for asset-based borrowers

Taking stock of empirical evidence

- Unlike [Döttling and Ratnovski \(2023\)](#): less response of investment to mp for high-INT firms \implies cash holding + unconstrained
- our results: stronger response of investment to mp for high-INT firms if focusing on firms with debt contracts
 - ▶ cash flow-based borrowers drive this pattern
 - ▶ mechanisms: high contribution, high variation of intangible investment to firm production for cash flow-based borrowers
- Next: heterogeneous firms with iid productivity on intangible capital, choosing debt contracts to borrow and invest
- able to explain the empirical facts

Static Model: Setup

- A continuum of firms with unit measure, endowed with e net worth, obtain b amount of bank loans at loan rate R^b
- flow of funds: $e + b = k^T + k^I$
- Two types of investment:
 - ▶ invest k^T on tangible assets with rate of return R^k
 - ▶ invest k^I on intangible assets with rate $\varepsilon\phi R^k$
 - ▶ $\phi > 1$, ε is idiosyncratic productivity shock with i.i.d. CDF $F(\cdot)$, following [Gourio \(2013\)](#) and [Allub, Ferriere, Franjo, and Zheng \(2023\)](#)
- borrow via loan covenants:
 - ▶ risk-neutral bank offers loan covenants to prevent default and is indifferent between two offers:
 - ▶ asset-based: $b \leq \theta^1 Q k^T$; cash flow-based: $b \leq \theta^2 R^k (k^T + \varepsilon\phi k^I)$

Firm's Problem

- firms draw productivity shock on intangible capital
- decide investment (k^T, k^I) with endogenous choice of debt contracts:

$$\begin{aligned}\max \varphi(\varepsilon) &= R^k f(k^T, k^I) = R^k(k^T + \varepsilon \phi k^I) + Qk^T - R^b b \\ \text{s.t. } b + e &= k^T + k^I \\ \min\{k^T, k^I\} &\geq \rho e \\ b &\leq \max\{\theta^1 Qk^T, \theta^2 R^k(k^T + \varepsilon \phi k^I)\}\end{aligned}$$

Investment Decisions and Choice of Loan Contracts

- optimal investment decisions: Proof

$$(k^T, k^I) = \begin{cases} (\rho e, \frac{\theta^2 R^k \rho e + (1-\rho)e}{1-\theta^2 R^k \epsilon \phi}), & \text{if } \epsilon \geq \epsilon^{**}, \text{ cash flow-based} \\ (\frac{\theta^2 R^k \epsilon \phi \rho e + (1-\rho)e}{1-\theta^2 R^k}, \rho e), & \text{if } \epsilon^c \leq \epsilon < \epsilon^{**}, \text{ cash flow-based} \\ (\frac{(1-\rho)e}{1-\theta^1 Q}, \rho e), & \text{if } \epsilon < \epsilon^c, \text{ asset-based} \end{cases} \quad (3)$$

- where:

$$\epsilon^{**} \equiv \frac{R^k + Q - R^b \theta^2 R^k}{R^k \phi (1 - \theta^2 R^b)}, \quad \epsilon^c \equiv \frac{(\theta^1 Q - \theta^2 R^k) \frac{1-\rho}{\rho}}{\theta^2 R^k \phi (1 - \theta^1 Q)}$$

- for asset-based borrowers: identical intangible ratio, affected by asset price Q
- for cash flow-based borrowers (PE):
 - ▶ extensive margin: an interest rate hike \implies share of high-INT group \downarrow ; share of low-INT group \uparrow

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Full-Fledged Model

- following [Dong, Guo, Peng, and Xu \(2022\)](#)
- at t : ε_t realized; purchase k_t^T at nominal price Q_t^k and decide k_t^I
- at $t + 1$: hire labor n_{t+1} at nominal wage W_{t+1} , produce and sell capital, earn profit, exit market.
- production function: $y_{t+1} = A_{t+1}f(k_t^T, k_t^I)^\alpha (n_{t+1})^{1-\alpha}$
- profit: $\pi_{jt+1} = R_{t+1}^k f(k_t^T, k_t^I)$, where $R_{t+1}^k = \alpha(P_{t+1}^m A_{t+1} / W_{t+1})^{\frac{1}{\alpha}} W_{t+1}^{\frac{\alpha-1}{\alpha}}$
- Similar to the static model, at equilibrium:

$$K_t^{I,c} = \int_{\varepsilon_t^{**}} \frac{\theta^2 R_{t+1}^k \rho + (1 - \rho)}{1 - \theta^2 R_{t+1}^k \varepsilon_t \phi} dF(\varepsilon_t) E_t + (F(\varepsilon_t^{**}) - F(\varepsilon_t^c)) \rho E_t$$

$$K_t^{T,c} = [1 - F(\varepsilon_t^{**})] \rho E_t + E_t \int_{\varepsilon_t^c}^{\varepsilon_t^{**}} \frac{\theta^2 R_{t+1}^k \varepsilon_t \phi \rho + (1 - \rho)}{1 - \theta^2 R_{t+1}^k} dF(\varepsilon_t)$$

$$K_t^{I,a} = F(\varepsilon_t^c) \rho E_t, \quad K_t^{T,a} = F(\varepsilon_t^c) \frac{(1 - \rho)}{1 - \theta^1 Q_{t+1}^k} E_t$$

- assume θ fraction of profit is consumed when exit:

$$E_t = (1 - \theta) E_{t-1} + (1 - \theta) \int \pi_t(\varepsilon_t) dF(\varepsilon_t)$$

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The Rest

- **HH:** consumes C_t , provides labor N_t , and save deposits S_t at rate R_t
 - ▶ $\max U(C_t, N_t), \text{ s.t., } P_t C_t + S_t = W_t N_t + (1 + R_t) S_{t-1} + T_t$
- **Capital goods producers:** $\max E_0 \sum \beta^t \Lambda_t D_t^k$, where

$$D_t^k = Q_t^k I_t^T - v_t \left[1 + \frac{\Omega^k}{2} \left(\frac{I_t^T}{I_{t-1}^T} - 1 \right)^2 \right] I_t^T$$

- **Banks:** $V_t = \max_{B_{t+1}} \left[\Pi_t^B + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1} \right]$, where

$$\Pi_t^B = \left(1 + R_{t-1}^b \right) B_{t-1} - P_t \Psi(B_t/P_t) - B_t + S_t - (1 + R_{t-1}) S_{t-1}$$

- $\Psi(b_t) = \xi_1 / (1 + \xi_2) (b_t / \bar{b})^{1 + \bar{\xi}_2} \bar{b}$
- **NK block:** monopolistic retail sector with Calvo pricing
- **MP rule:** $\frac{1+R_t}{1+R} = \left(\frac{\pi_t}{\pi} \right)^{\varphi_\pi} \left(\frac{Y_t}{Y} \right)^{\varphi_y}$, for $\varphi_\pi, \varphi_y > 0$
- **Market Clears:** $C_t + \frac{1}{P_t} \left[1 + \frac{\Omega^k}{2} \left(\frac{I_t^T}{I_{t-1}^T} - 1 \right)^2 \right] I_t^T + I_t^I + \Psi(B_t/P_t) = Y_t$

Next Step

- consider quantitative effects under general equilibrium (calibrate the model and continue)
- examine the macro-prudential policies regulating firms' leverage under alternative debt contracts.
- empirical part: robustness check, replace continuous variables to dummies (high-INT, low-INT) as interaction term to mp shocks.

Thank you!

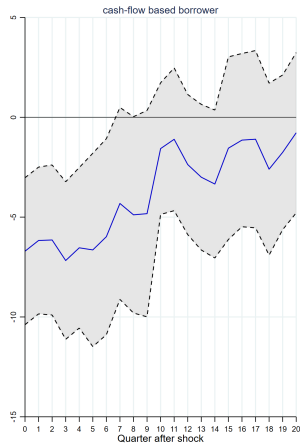
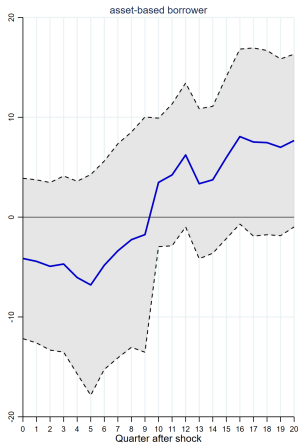
Details on Loan Classification

- [Lian and Ma \(2021\)](#) classify facilities secured by all assets as cash-flow based loans, because the value of this form of collateral in the event of bankruptcy is calculated based on the cash flow value from continuing operations.
- Cash flow-based: loan facilities secured by “all”
- Asset-based:
 - ▶ loans pledged by specific assets (excluded those that are backed by “all”)
 - ▶ secured revolving line of credit: asset-based loans [◀ Go back](#)

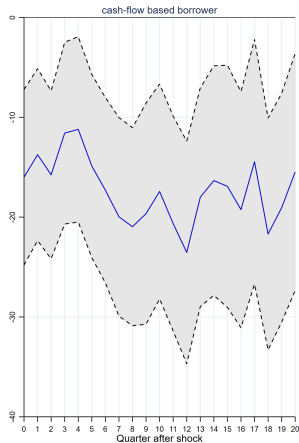
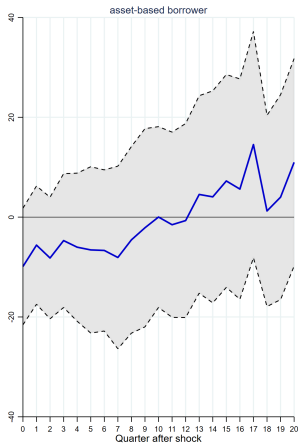
Details on MP Shock Identification

- Identification: proxy SVAR with monthly data: $\log(\text{INDPRO})$, $\log(\text{CPIAUCSL})$, $\log(\text{GS1})$, EBP
- instrument 1 z_t : HF FF4 surprises
- instrument 2 z_t^* : orthogonal known macro predictor prior to the announcement date, following [Bauer and Swanson \(2023\)](#)
 - ▶ $z_t = \alpha + \beta' X_{t-} + u_t$, X_{t-} : 6 macro predictors prior to announcement t : Nonfarm payrolls surprise; Employment growth; S&P 500; Yield curve slope; Commodity prices; Treasury skewness
- 2SLS for:
$$Y_t = \alpha + B(L)Y_{t-1} + s_1 Y_t^{2y} + \tilde{u}_t$$
- where $SS' = \text{Var}[u_t] \equiv \Omega$, $u_t = S\varepsilon_t$ [◀ Go back](#)

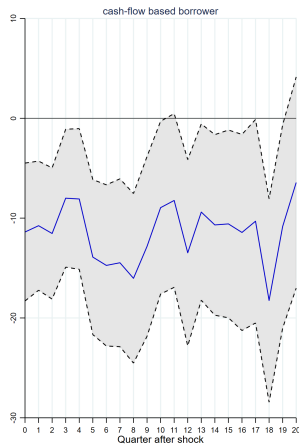
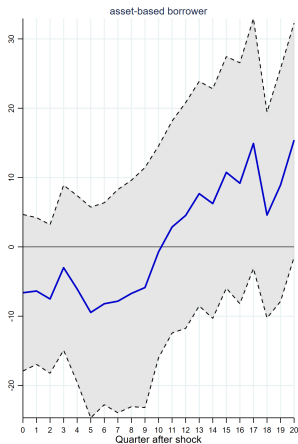
Differential IRFs of Investment to Monetary Shocks (orthogonal)



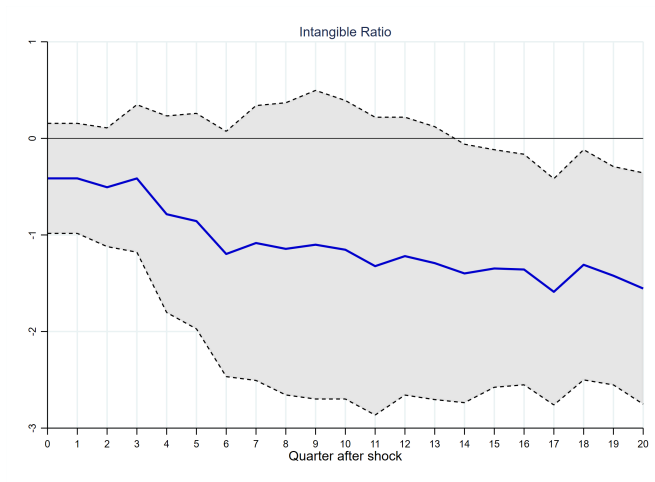
Differential IRFs of Intangible Investment to Monetary Shocks



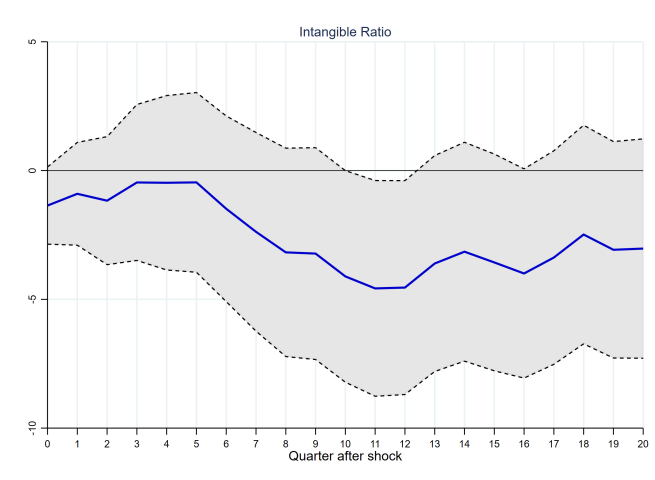
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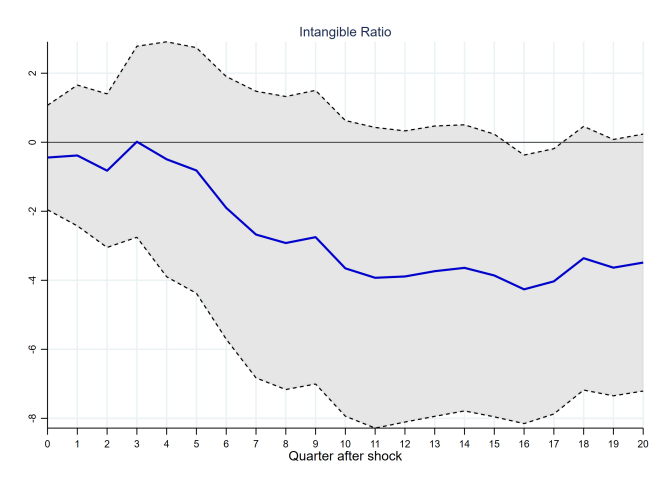
Differential IRFs of Total Investment to Monetary Shocks (orthogonal)



Differential IRFs of Intangible Investment to Monetary Shocks



Differential IRFs of Intangible Investment to Monetary Shocks (orthogonal)



Timing of the model

The timing for an individual firm is as follows:

1. firms choose labor supply n_{jt} given a competitive wage rate w_t ;
2. firms choose how much investment allocated to tangible and intangible part conditional on prepayment $b_{jt-1}R_{t-1}$.
3. firms commit to a feasible borrowing $b_{jt}^*(\varepsilon_{jt}; s_t)$ for each possible realization of ε_{jt} .
4. firms draw ε_{jt} and choose their new loan size b_{jt}^* subject to their loan contracts.
5. firms realize insurance claims and buy new Arrow securities, and choose consumption.

[go back](#)

Proof of Aggregation

Firm's Lagrangian:

$$\begin{aligned}\mathcal{L} = & \sum_{t=0}^{\infty} \sum_{s^t} \beta_f^t \int_{\varepsilon_{jt}} \left\{ u(c_{jt}^f) + \lambda_{jt} \left[R_t^k (k_{jt-1}^T + \varepsilon_{jt} \phi k_{jt-1}^I) \right. \right. \\ & \left. \left. + P_t^k ((1 - \delta) k_{jt-1} - k_{jt}) \right. \right. \\ & \left. \left. + b_{jt}^* - R_{t-1} b_{jt-1}^* + a_{jt-1}(s_t) + \sum_{s_{t+1}|s^t} p_t^a(s_{t+1}) a_{jt}(s_{t+1}) \right. \right. \\ & \left. \left. + \psi_{jt} \left(\max \left\{ \bar{b}_{jt}^{cash}, \bar{b}_{jt}^{asset} \right\} - b_{jt}^* \right) \right] \right\} dF(\varepsilon_{jt})\end{aligned}$$

[go back](#)

Proof of Static Model: cash flow-based

- if firms choose cash flow-based covenants:

$$\theta^1 Q k^T \leq \theta^2 R^k (k^T + \varepsilon \phi k^I) \implies \varepsilon \geq \frac{(\theta^1 Q - \theta^2 R^k) k^T}{\theta^2 R^k \phi k^I} \equiv \varepsilon^c(k^T, k^I) \quad (4)$$

- then, the PMP becomes: $\left[R^k + Q - R^b \theta^2 R^k \right] k^T + \left[R^k \varepsilon \phi (1 - R^b \theta^2) \right] k^I$
- the firm decision of investment is:

$$(k^T, k^I) = \begin{cases} (\rho e, \frac{\theta^2 R^k \rho e + (1-\rho)e}{1 - \theta^2 R^k \varepsilon \phi}), & \text{if } \varepsilon > \varepsilon^{**} \equiv \frac{R^k + Q - R^b \theta^2 R^k}{R^k \phi (1 - \theta^2 R^b)} \\ (\frac{\theta^2 R^k \varepsilon \phi \rho e + (1-\rho)e}{1 - \theta^2 R^k}, \rho e), & \text{if } \varepsilon < \varepsilon^{**} \end{cases} \quad (5)$$

- assume $1 - \theta^2 R^k \varepsilon \phi > 0$. Given above capital allocation, backout ε^c :
- for the capital allocation with $\varepsilon > \varepsilon^{**}$:

$$(\theta^1 Q - \theta^2 R^k) \leq \theta^2 R^k \varepsilon \phi \frac{k^I}{k^T} \implies \varepsilon > \frac{\theta^1 Q - \theta^2 R^k}{(\theta^1 Q + (1-\rho)/\rho) \theta^2 R^k \phi} \equiv \varepsilon_1^c \quad (6)$$

- compare ε_1^c in (6) and ε^{**} in (5), assume $\theta^1 Q - \theta^2 R^k > 0$:

$$\frac{\varepsilon^{**}}{\varepsilon_1^c} = \underbrace{\frac{1 + Q/R^k - \theta^2 R^b}{1 - \theta^2 R^b}}_{>1} \underbrace{\frac{1 + \frac{1-\rho}{\rho} \frac{1}{\theta^1 Q}}{\frac{1}{\theta^2 R^k} - \frac{1}{\theta^1 Q}}}_{>1} \geq 1$$

- $\varepsilon > \varepsilon^{**}$: invest more intangibles + cash flow-based covenants. [go back](#)

Proof of Static Model: cash flow-based

- for the capital allocation with $\varepsilon < \varepsilon^{**}$:

$$(\theta^1 Q - \theta^2 R^k) \leq \theta^2 R^k \varepsilon \phi \frac{k^I}{k^T} \implies \varepsilon > \frac{(\theta^1 Q - \theta^2 R^k) \frac{1-\rho}{\rho}}{\theta^2 R^k \phi (1 - \theta^1 Q)} \equiv \varepsilon_2^c \quad (7)$$

- under certain parameter assumptions:

$$\frac{\varepsilon^{**}}{\varepsilon_2^c} \geq 1 \implies \frac{1}{\theta^1 Q + \frac{1-\rho}{\rho}} \frac{1 - \theta^1 Q}{\frac{1-\rho}{\rho}} > 1$$

- $\varepsilon_2^c < \varepsilon < \varepsilon^{**}$: invest more tangibles + cash flow-based covenants.

go back

Proof of Static Model: asset-based

- if firms choose asset-based covenants, the PMP becomes:

$$\max \left[R^k + Q - R^b \theta^1 Q \right] k^T + \left[R^k \varepsilon \phi \right] k^I$$

- firm investment is:

$$(k^T, k^I) = \begin{cases} (\rho e, \theta^1 Q \rho e + (1 - \rho)e), & \text{if } \varepsilon > \varepsilon^* \equiv \frac{R^k + (1 - \theta^1 R^b)Q}{R^k \phi} \\ \left(\frac{(1 - \rho)e}{1 - \theta^1 Q}, \rho e \right), & \text{if } \varepsilon < \varepsilon^* \end{cases} \quad (8)$$

- for the capital allocation with $\varepsilon > \varepsilon^*$:

$$(\theta^1 Q - \theta^2 R^k) > \theta^2 R^k \varepsilon \phi \frac{k^I}{k^T} \implies \varepsilon < \frac{\theta^1 Q - \theta^2 R^k}{\theta^2 R^k \phi} \frac{1}{\theta^1 Q + (1 - \rho)/\rho} \equiv \varepsilon_1^a$$

- this requires $\varepsilon^* < \varepsilon_1^a$:

$$\frac{\theta^1 Q - \theta^2 R^k}{(\theta^1 Q + \frac{1 - \rho}{\rho})(\theta^2 R^k + (1 - \theta^1 R^b)\theta^2 Q)} > 1$$

- which fails to hold. [go back](#)

Proof of Static Model: asset-based

- for the capital allocation with $\varepsilon < \varepsilon^*$:

$$(\theta^1 Q - \theta^2 R^k) > \theta^2 R^k \varepsilon \phi \frac{k^I}{k^T} \implies \varepsilon < \frac{\theta^1 Q - \theta^2 R^k}{\theta^2 R^k \phi (1 - \theta^1 Q)} \frac{1 - \rho}{\rho} \equiv \varepsilon_2^a \quad (9)$$

- we can show that:

$$\frac{\varepsilon_2^a}{\varepsilon^*} = \frac{\theta^1 R^k - \theta^2 R^k \frac{1-\rho}{\rho}}{\theta^2 [R^k + (1 - \theta^1 R^b)Q] (1 - \theta^1 Q)} < 1$$

- therefore, firms with $\varepsilon < \varepsilon^*$: invest more tangibles + asset-based covenants [go back](#)

Aggregation for Static Model

- for asset-based borrower: identical share of intangible; only affected by asset-price channel:

$$K_a^I = \rho F(\varepsilon^c), \quad K_a^T = \frac{1 - \rho}{1 - \theta^1 Q} F(\varepsilon^c)$$

- cash flow-based, high-INT group:

$$K_1^I/E = \int_{\varepsilon^{**}} \frac{\theta^2 R^k \rho + (1 - \rho)}{1 - \theta^2 R^k \varepsilon \phi} dF(\varepsilon), \quad K_1^T/E = \rho(1 - F(\varepsilon^{**}))$$

- cash flow-based, low-INT group:

$$K_2^I/E = \rho(F(\varepsilon^{**}) - F(\varepsilon^c)), \quad K_2^T/E = \int_{\varepsilon^c}^{\varepsilon^{**}} \frac{\theta^2 R^k \varepsilon \phi \rho + (1 - \rho)}{1 - \theta^2 R^k} dF(\varepsilon)$$

- [go back](#)