Earnings-based Constraint in a Real Business Cycle Model

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Abstract

This paper studies the macroeconomic implications of firm's earnings-based borrowing constraints, using a real business cycle (RBC) framework. Compared with the model under collateral constraints, a benchmark RBC model with earnings-based constraints predicts different sign of firm's debt dynamics. The paper empirically verifies these model predictions using US aggregate data. The positive response of aggregate debt to investment shocks in the data supports the relevance of earnings-based constraints. Finally, theoretical predictions from an extended model under earnings-based constraints better fit with the empirical evidence, further support the relevance of firm's earnings-based constraints.

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1 Introduction

Borrowing constraints are nowadays a key amplification in macroeconomic dynamics and one of the central mechanisms by which asset prices interact with macroeconomic dynamics. Recent research has focused on linking asset prices and business cycle with borrowing constraints, such as collateral constraints. This paper studies macroeconomic implications of firm's earnings-based constraints. The focus on firm's earnings-based constraint is motivated by stylized evidence on the importance of firms' current earnings flows for their access to debt.[2] Based on this, I show that model with an earnings-based constraint in real business cycle analysis is able to correctly capture aggregate macro dynamics compared with collateral constraint.

Since firms' debt access is mostly affected by their current earning flows instead of their assets, and given that earnings-based constraints and collateral constraints generally imply different debt dynamics. Based on this fact, this paper investigates the macreconomic dynamics by comparing firm's earnings-based debt borrowing constraint with a typical asset-based collateral constraint, which has become a standard building block in macroeconomics. I also study how alternative borrowing constraint affects the propagation and amplification of macroeonomic shocks. To understand these salient features of such mechanism, I first compare these two borrowing constraints in a standard real business cycle model, which is a generalization of Kiyotaki and Moore (1997), extended by incorporating firms' borrowing constraints, either an earnings-based constraint or a collateral constraint. I begin by investigating how corporate borrowing constraints transmit shocks through the economy in a standard real business cycle model. To demonstrate this, I build a theoretical model in which firm's debt can be restricted either by firm's current earnings or by a fraction of the expected value of its capital as collateralized assets. Model predictions reconfirm that firm's debt dynamics behave differently under alternative borrowing constraints. Given a permanent investment shock, firm debt increase under an earnings-based constraint while decrease if firm uses its assets as collateral to borrow from the lender. Financial shocks are amplified in a larger effect under collateral constraint, however, firm is less vulnerable to collateral damage from asset price declines in the form of operating earnings under an earnings-based constraint. However, the baseline model fails to predict the positive

[2]The motivating evidence builds on existing studies, in particular Lian and Ma (2019) stress the pervasive of firm loan covenant and the importance of earnings in the loan covenant, conclude that earnings are a key indicator that determines the extent to which firms have access to debt, as for US non-financial firms, 80% of debt is based predominantly on cash flows from firms' operations ("cash flow-based lending") while only 20% of debt by value is collateralized by physical assets ("asset-based lending"); Drechsel (2019) points out that earnings-based covenants dominate within overall covenants than collaterized assets-based covenants in micro data, and by estimating a RBC model, he conclude that firm debt dynamics implied by earnings-based constriants are supported by empirical evidence.

co-movement between hours worked and output. Apart from the firm's debt IRFs, the baseline model behaves extremely similar under the two constraints.

To verify the benchmark model predictions under alternative borrowing constraint, I estimate a structural vector-autoregressive (SVAR) model on aggregate US data to study the dynamics of output, firm's earnings, capital, investment, and debt following technology shocks and investment shocks.[3] The debt dynamics in aggregate data support the relevance of firm's earnings-based borrowing constraints, which is consistent with the conclusion from Drechsel (2019). However, the benchmark model fails to capture the macro dynamics of hours worked and firm's debt in response to TFP shocks, and the reponses of hours worked, earnings to IST shocks.

Finally, to improve model predictions in the sense of capturing the macro dynamics implied by empirical evidence, I extend the model to incorporate three key features: a non-separable preference of utility with consumption habit formation, an equity payout cost function and a combined borrowing constraints with working capital formation. The impulse response functions implied by extended model under earnings-based constraint are further supported by empirical evidence while model under collateral constriant are not. Specifically, model under earnings-based constraint captures the correct dynamics of hours worked and output, the overshoot features in reponse to TFP shocks and significant positive reponses to IST shocks. Besides, financial shocks imply a larger amplification effects under collateral constraints.

The rest of the paper is organized in the following manner. Section 2 discusses related literature. Section 3 introduces a standard business cycle model that features an earnings-based constraint and discusses the resulting assets price and macroeconomic dynamics in comparison to a collateral constraint. Section 4 presents empirical evidence by estimating a SVAR model on US aggregate data. Section 5 extends the baseline model to validate the importance of earnings-based constraint in connection with the empirical evidence. Section 6 concludes.

2 Literature Review

First and foremost, given the stylized fact that a very large fraction of firm financing comes in the form of conventional debt.[4] There are two popular ways to throw a wrench into the use of debt to finance operations. Literature builds on the seminal works by Kiyotaki and Moore (1997) and Bernanke, Gertler, and Gilchrist (1999)

[3]The focus on technology shocks and investment shocks is because their importance to business cycle fluctuations and the convenience of identification in the data. To identify investment shock, I follow Fisher (2006), imposing long-run restrictions.

[4]See Drechsel (2019), Lian and Ma (2019).

(henceforth, BGG) which provide a direct link between firms' assets and investment spending is known as "costly enforcement". Another popular way of debt access to finance operations is based on "costly state verification" (CSV) problems in the spirit of Townsend (1979). This has been brought to the fore in macroeconomic models in Gertler and Bernanke (1995), Carlstrom and Fuerst (1997) and Bernanke, Gertler, and Gilchrist (1999).

Following this, there's a burgeoning literature studying financial frictions, with a special focus on borrowing constraints in macroeconomics (examples include Cooley, Marimon, and Quadrini (2004), Cordoba and Ripoll (2004a), Gertler, Gilchrist, and Natalucci (2007), Jermann and Quadrini (2012), Liu, Wang, and Zha (2013), He, Wright, and Zhu (2015), Azariadis, Kaas, and Wen (2016), Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017) and Cao and Nie (2017), Kiyotaki and Moore (2018)). Most of the literature usually considers collateral constraint as a standard building block in macroeconomics which is based on limited contract enforcement, and currently a convenient way to model financial frictions in real business cycle models. This paper relates to the literature on the role of amplification effect and borrowing constraint. Specifically, Kocherlakota et al. (2000) and Cordoba and Ripoll (2004b) investigate the quantitative importance of collateral constraints. Jermann and Quadrini (2012) and Krishnamurthy (2003) find sizable amplification effect in response to shocks on collateral constraints.

More recently, Lian and Ma (2019) proposes a strong micro evidence on earnings-based constraints which consists of 80% debt by value whereas only 20% debt is collateralized by physical assets. Drechsel (2019) further formalizes this link through an earnings-based constraint on firm borrowing and studies its macroeconomic implications on firm's debt dynamics. In spirit of Kehoe, Midrigan, and Pastorino (2018) who highlight the importance of disciplining macro models with direct micro evidence, following Drechsel (2019), my paper builds upon such a micro evidence on firm borrowing to study the macroeconomic implications of earnings-based constraints in comparison with collateral constriants.

This paper belongs to two parts of the literature related to the role of different borrowing constraints and comparisons in a business cycle framework. First, there is a strand of literature on the comparisons of financial frictions on standard DSGE models, Christensen and Dib (2008), Kamber, Smith, and Thoenissen (2015), and Merola (2015) focus on comparing the model with and without financial frictions, find out the financial accelerator mechanisms do exist. Brzoza-Brzezina, Kolasa, and Makarski (2013) compares collateral constraint and "costly state verification" (CSV) version of financial friction in a New Keynesian model with respect to their impulse responses to different shocks and variance decompositions. It's also worth to notice that there's another bulk of literature on cash-flow based credit constraints but mainly focusing on housing market. The housholds have another type of borrowing constraint, typically refer to Debt-Service-to-Income (DTI) constraints as opposed to the Loan-to-Value (LTV) constraints. The DTI constraint is a generalization of the natural borrowing limit in Aiyagari (1994). On comparison of cash flow income-based constraint and collateral constraint, Greenwald (2018) research on the housing mortgage credit channel in comparison of Payment-to-Income (PTI) constaints and Loan-to-Value(LTV) constraints. Ingholt (2019) further extend to compare two occasionally binding constraints: a Loan-to-Value(LTV) constraint and a Debt-Service-to-Income(DTI) constraint, find out DTI constraint is relative more efficient for preventing the boom while LTV limit could contrarily not, since soaring house prices slackened this constraint. With similar intention, I compare an earnings-based constraint and a collateral constraint with a focus on corporate debt rather than housing mortages.

Another strand of seminal work belongs to investigate the role of financial frictions in a business cycle framework. Kamber, Smith, and Thoenissen (2015) find out the importance of investment shocks is greatly diminished with collateral constraints.[5] Apart from investment shocks since they fail to generate the comovement between consumption and output under a model with collateral constraints, literature proposes many other shocks instead which could capture correct comovement among macro variables and assets price or explain better fluctuations in business cycle models. For example, Christiano and Fisher (2003) explain the pro-cyclical of stock market prices and countercyclical of prices of investment goods in a business cycle model as well as accounts for other co-movements. Miao, Wang, and Xu (2013) come up with a sentiment shock which is found out to drive the movements of assets price bubbles and is transmitted to the real economy through endogenous credit constraints. Liu, Wang, and Zha (2013) propose a shock to land price in housing sector that could explain interesting land dynamics while considering land as a collateral asset in firms' credit constraints. Recently, a pioneering work by Drechsel (2019) suggests an investment shock captures correct dynamics of firm's debt via earnings-based constraints instead of collateral constraints and explains more fluctuations in the business cycle frequencies under model with earnings-based constraints. Christiano, Motto, and Rostagno (2014) stress the importance of risk shocks in a business cycle model with BGG mechanisms as financial frictions. This directly triggers the interests about what's the role of the

[5]Research related to investment shocks keynoted by Justiniano, Primiceri, and Tambalotti (2010) who contributes by estimating a DSGE model with stochastic volatility and finding that reduced investment-specific technology shocks play a very important role in the Great Moderation of US business cycles seen from 1984 to 2007 and it is likely to proxy for more fundamental disturbances to the functioning of the financial sector. Curatola, Donadelli, Grüning, and Meinerding (2017) show that investment shock captures correct co-movements among macro variables and asset prices in a NK model.

firm's earnings-based borrowing constraints relative to different structural shocks on assets price and macro dynamics.

Finally, the empirical verification belongs to the literature on investment shocks in business cycle. Since Greenwood, Hercowitz, and Krusell (2000) discover that investment specific technical (IST hereafter) change is a more important source of growth than growth in the neutral technology. And this strongly suggests it is also important for the business cycle. Fisher (2006) re-ignited the interest by identifying the IST shocks using SVAR, finds that combining IST shock with the TFP(Total Factor Productivity) shock accounts for about 40-60% of the fluctuations in output and hours worked at business cycle frequencies. This results have been followed up by Justiniano and Primiceri (2008) in an assessment of the Great Moderation.

3 The Baseline Model

To examine how these two borrowing constraints differentially affects the transmission of shocks, this section considers a standard real business cycle model with a firm subject to either collateral constraints or earnings-based constraints. To derive sharp differential predictions, the characterization of the model dynamics focuses on a structural shock that moves earnings and the value of collateral in opposite directions: the investment shock, thus make the results comparable and in line with Drechsel (2019). Section 5 extends the baseline model to capture better macro dynamics and highlights

First, this section considers a discrete infinite-horizon economy that consists of a continuum of identical firms of measure unity and a continuum of identical households of measure unity. The frequency is quarterly. Households are risk-averse, trade debt claims on firm and receive dividend and interest from them. Households supply labor to firms, deposit funds in competitive financial intermediaries, and trade firm shares in a stock market. Here firm issues debt, which is either constrained by current earnings or the value of collateral. Firm owner maximizes its discounted sum of dividend. Firms and households can save in competitive financial intermediaries, which make one-period loans to borrowers.

As a starting point, this baseline model features only borrowing constraints and investment adjustment cost.[6]

[6] Modern adjustment cost models follow Abel (1981)and Hayashi (1982), and are widely used starting from Smets and Wouters (2007) and Christiano, Eichenbaum, and Evans (2005) which suggest that investment adjustment cost is able to generate the sort of inertia in investment observed in the aggregate data.

3.1 Households

Households have time-separable preferences of consumption and leisure. Each household maximizes expected discounted lifetime utility as follows:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} + \chi \log(1-N_t) \right],$$

where $\beta \in (0,1)$ is the discount factor, C_t and N_t denote consumption and labor respectively. σ and χ are risk aversion and relative utility of leisure.

The representative household's budget constraint is given by:

$$\text{s.t. } C_t + \frac{b_t}{1+r_t} + P_t S_t + T_t = w_t N_t + b_{t-1} + S_{t-1}(P_t + D_t),$$

where $S_t, P_t, D_t, R_t, b_t, w_t, T_t$ denote share holdings, the aggregate stock (equity) price of all firms, the aggregate dividend, the bond interest rate, bond holdings, the wage rate, and the lump-sum tax respectively. Hence, the household's optimality conditions with respect to n_t, b_t, s_t are:

$$C_t^{-\sigma} w_t = \frac{\chi}{(1-N_t)} \tag{1}$$

$$C_t^{-\sigma} = \beta(1+r_t)\mathbb{E}_t C_{t+1}^{-\sigma} \tag{2}$$

$$P_t = \beta \mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} (P_{t+1} + D_{t+1}) \tag{3}$$

3.2 Firm

The representative firm produces a final consumption good using capital, which it owns and accumulates, and labor, which it hires on a competitive labor market taking the wage rate w_t as given. Denote the consumption good produced with a Cobb-Douglas production function as:

$$y_t = z_t k_{t-1}^{\alpha} N_t^{1-\alpha}, \tag{4}$$

where $\alpha \in (0,1)$, and the output price is normalized to 1, z_t is total factor productivity (TFP) stochastic shocks, n_t is the quantity of labor input, k_{t-1} is the predetermined capital at the beginning of the period.

The firm's capital stock k_t accumulates over time subject to an investment adjustment costs, its accumulation mechanism follows:

$$k_t = (1-\delta)k_{t-1} + v_t i_t \left[1 - \Phi\left(\frac{i_t}{i_{t-1}} \right) \right], \tag{5}$$

where δ is the depreciation rate of the capital stock, shock to the process of v_t is "investment shock", since the production of capital and investment goods are not disaggregated into different sectors, the investment shock represents both the "investment-specific technology" (IST) and "marginal efficiency of investment"(MEI) shock. IST shock is a shock which affects the transformation of consumption goods into investment goods, whereas the MEI shock impacts the transformation of investment goods into capital goods.[7] The interpretation of the Lagrange multiplier for equation (5) is denoted by $Q_{k,t}$, is the extra value to the firm of an additional unit of new productive capital, k_t, therefore has the interpretation as the price of new capital in terms of goods, thus reflects the market price of capital.[8] Both the presence of investment adjustment costs as well as v_t will lead to variation in the market value of capital. The investment adjustment cost with property $\Phi_t(1) = 0$, $\Phi_t'(1) = 0$, and $\Phi_t''(1) = \phi_t > 0$, following, Christiano, Eichenbaum, and Evans (2005), is specified below:

$$\Phi_t\left(\frac{i_t}{i_{t-1}}\right) = \frac{\phi_t}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2.$$

The dividend to shareholders is equal to:

$$D_t = y_t - w_t N_t - i_t + \frac{b_t}{R_t} - b_{t-1}, \tag{6}$$

where R_t denotes the effective gross interest rate faced by firms, and is subject to a tax advantage, captured by τ, of the following form:

$$R_t = 1 + r_t(1 - \tau), \tag{7}$$

where r_t is the market interest rates received by lenders. According to Hennessy and Whited (2005) and Drechsel (2019), such specification creates a preference for debt over equity and makes the firm want to borrow up to its constraints. This further ensures the borrowing constraints to be always binding. To propose earnings-based constraint, following Lian and Ma (2019) and Drechsel (2019), I define firm's current earnings π_t, as profits net of labor costs, but before subtracting investment, interest and taxes:

$$\pi_t \equiv y_t - w_t N_t. \tag{8}$$

Firm's debt financing b_t, is undertaken in the form of one-period risk-free bond, the gross interest rates faced by firms are $1 + r_t$. Thus, firm borrows up to its constraints either based on collateralized assets or based on current flow of earnings:

[7] In the empirical part, I narrow it down to IST shocks since the convenience of collecting data.
[8] See Hayashi (1982)

$$\frac{b_t}{1+r_t} \leq \xi_t \theta^k \mathbb{E}_t Q_{t+1}(1-\delta)k_t, \tag{9}$$

$$\frac{b_t}{1+r_t} \leq \xi_t \theta^\pi \pi_t. \tag{10}$$

Due to the tax advantage in Equation (7), we stay in the always binding borrowing constraints. In the collateral constraint, $Q_{k,t+1}$ reflects the market value of net capital stock next period, and also the Lagrange multiplier on capital accumulation equation (5), $(1-\delta)k_t$ is how much capital the firm has, and ξ_t is a financial (credit) shock which shifts the borrowing limits imposed by both constraints. $\theta^\pi > 1$ represents the debt borrowing up to a multiple of current earnings, while $\theta^k < 1$ represents the debt borrowing is limited to a fraction of collateralized assets.[9]

Each firm maximizes the present discounted value of dividends in the following manner:

$$\max \mathbb{E}_k \sum_{k=0}^{\infty} \frac{\beta^k \Lambda_{t+k}}{\Lambda_t} D_{t+k},$$

subject to equations (4), (5), (6), where $\frac{\beta^k \Lambda_{t+k}}{\Lambda_t}$ is a stochastic discount factor. Firm's first order conditions with respect to N_t, b_t, i_t, k_t are:[10]

$$w_t = F_{n,t}, \tag{11}$$

$$R_t \mathbb{E}_t \left\{ \frac{\beta \Lambda_{t+1}}{\Lambda_t} \right\} + \mathbb{1}_{\mu_t^k \neq 0}(\mu_t^k \frac{R_t}{1+r_t}) + \mathbb{1}_{\mu_t^\pi \neq 0}(\mu_t^\pi \frac{R_t}{1+r_t}) = 1, \tag{12}$$

$$Q_t v_t (1 - \Phi_t' i_t - \Phi_t) + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} Q_{t+1} v_{t+1} i_{t+1} \Phi_{t+1}' = 1, \tag{13}$$

$$Q_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t}((1 + \mu_{\pi,t}\xi_t\theta^\pi)F_{k,t+1} + Q_{t+1}(1-\delta)) + \mathbb{E}_t \mu_{k,t}\xi_t\theta^k(1-\delta)Q_{t+1}, \tag{14}$$

where $F_{n,t} = (1-\alpha)\frac{y_t}{n_t}$ and $F_{k,t+1} = \alpha\frac{y_{t+1}}{k_t}$ denote the marginal products of labor and capital, respectively.

To close the economy, resource constraint should be satisfied ($y_t = c_t + i_t$). And for government sector, the lump sum tax of household T_t is required to finance the tax

[9] θ^k, θ^π also capture the exogenous tightness of the constraints for the relative borrowing constraint. The differences among constraint on b_t, $\frac{b_t}{1+r_t}$, and $r_t b_t$ would not change the macro dynamics of the model in a meaningful way, hence, I choose the debt-to-earnings formulation, the corresponding covenant in firm's micro data which is shown by Drechsel (2019) to be the most frequently used version.

[10] For ease of notation, I put first order conditions of alternative borrowing constraints together, for model under specific borrowing constraint, we can shut down the Lagrange multiplier of another constraint by letting $\mu^j = 0$.

advantage of debt that is given to the firm, and government does not save or borrow, hence a balance budget should be:

$$T_t = \frac{b_t}{R_t} - \frac{b_t}{1+r_t}$$

3.3 Stochastic Processes

$$\begin{aligned}
\log(z_t) &= (1-\rho_z)\log(\bar{z}) + \rho_z \log(z_{t-1}) + \varepsilon_t^z, \\
\log(v_t) &= (1-\rho_v)\log(\bar{v}) + \rho_v \log(v_{t-1}) + \varepsilon_t^v, \\
\log(\phi_t) &= (1-\rho_\phi)\log(\bar{\phi}) + \rho_\phi \log(\phi_{t-1}) + \varepsilon_t^\phi, \\
\log(\xi_t) &= (1-\rho_\xi)\log(\bar{\xi}) + \rho_\xi \log(\xi_{t-1}) + \varepsilon_t^\xi.
\end{aligned}$$

All uncorrelated structural shocks $\{\varepsilon_t^z, \varepsilon_t^v, \varepsilon_t^\phi, \varepsilon_t^\xi\}$ are normally independent and identically distributed random variables with standard deviations $\{\sigma_z, \sigma_v, \sigma_\phi, \sigma_\xi\}$

3.4 Model Parameterization

The specification of investment adjustment costs gives a steady state market value of capital 1. In steady state, $\Phi''(1) = \bar{\phi}$. parameter values are selected in order to keep in line with Drechsel (2019). Table 1 reports the assigned parameter values and relative explanations.

3.5 Macro dynamics implied by baseline model

The model is solved using first order perturbation techniques, and is approximated about a point where the borrowing constraint binds, and then implicitly ignore any possibility of the constraint perhaps not binding at some point in the future. Details about the calculation of the steady state values can be found in Appendix A.1. It is worth to notice that apart from the debt IRFs the model behaves extremely similar under the two always binding borrowing constraints. This is because only firm's debt dynamics are tied directly to the borrowing constraint formulation and are not driven by further modeling choices on the structure in which they operate. Always binding firm's debt borrowing constraints themselves typically do not have strong effects on the model's overall dynamics, Cordoba and Ripoll (2004b) provide a detailed exploration of this insight. Based on this intention, I extend the model which allow for dividend rigidities affecting the substitution between debt and equity and also a working capital

Table 1: Model parameterizations and calibration targets

Parameter	Value	Source/target
α	0.33	Capital share of output of 1/3
β	0.9752	Steady state annualized interest rate of 6.6%
δ	0.025	2.5% per quarter
$\bar{\phi}$	4	Prior of Smets and Wouters (2007)
χ	3.44	Target $n = 0.3$ in steady state
τ	0.35	Following Hennessy and Whited (2005)
σ	1.5	Exogenously chosen
θ^{π}	4.6/4	Mean of financial shock to earnings-based constraint[a]
θ^{k}	0.0589	Mean of financial shock to collateral constraint[b]
$\sigma_z, \sigma_v, \sigma_\phi, \sigma_\xi$	0.05	Prior of Drechsel (2019)
ρ_z, ρ_v	1	Perform long run shocks in baseline model
ρ_ϕ, ρ_ξ	0.95	Perform persistent shocks

[a]Following Drechsel (2019), it represents average value of debt-to-EBITDA covenants quarterly.
[b]Calibrated by the same steady state debt as earnings-based constraint case.

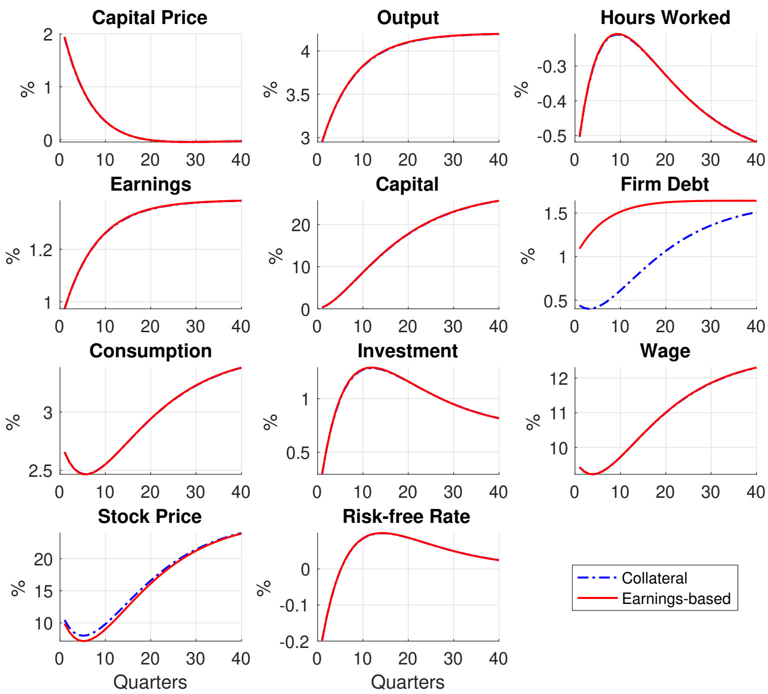
formulation of borrowing constraints in the next section, in order to see the difference macro dynamics generated by two borrowing constraints in a detailed mechanism.

Figure 1 below plots the impulse response functions of macro variables to a positive permanent technology shock.[11] When productivity increases, firm would like to invest more, but can only increase its investment by the amount that its capital increases in value. In contrast, consumption increases by more in the economy than investment. This is because when productivity improves, the firm would like to borrow more (debt borrows increase in both cases) to increase its investment, but is unable to do so by much because of the borrowing constraint always binding. Because investment can't go up by much, consumption goes up by more. But consumption going up by more means that hours worked essentially decrease (there is a bigger offsetting wealth effect on labor supply function (1)). Firm's earnings as well as capital price (firm's market value of capital) also increase, then a higher wage goes into household sector, further induces a continuous increase in consumption. Here, alternative borrowing constraints also imply different debt dynamics but both a positive sign.

This is different for the investment shock, which leads to higher efficiency in the economy's investment margin. This induces investment and stronger economic activity accompanied by growing earnings. However, a reduction in capital price through the collateral constraint induces lower debt, while a growing earning through the earnings-

[11]To keep in line with vast literature in RBC studies, I also report the IRFs of consumption, investment, wage, stock price and risk-free rate as by-products.

Figure 1: Baseline model IRFs to a permanent technology shock



Note: The figure displays model IRFs of key variables to a positive technology factor productivity (TFP) shock, under two alternative calibrations in which only the earnings-based constraint (dotted blue line) or only the collateral constraint (red line) is present. The figure highlights that the responses of debt to shocks have a different dynamics while others are remaining the same under the alternative borrowing constraints.

based constraint induces higher debt. The responses to this shock thus imply sharply different debt dynamics depending on the relevant borrowing constraint.

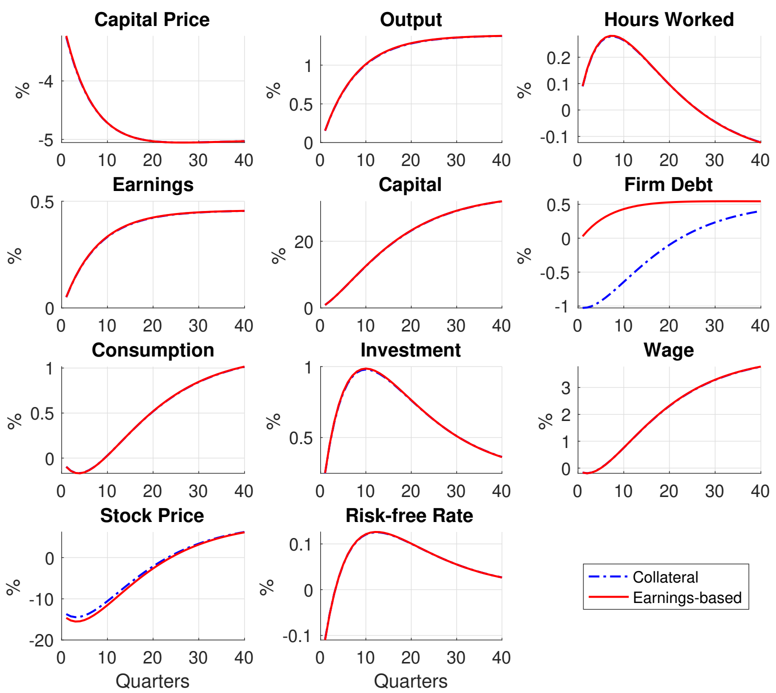
Specifically, in Figure 2, we observe that the investment shock also leads to a sizable increase in output, earnings, investments, capital, and hours worked, however, with reductions in consumption, capital price, and stock price. The intuition is as follows. A positive investment shock (an increase of $1/v_t$) raises the marginal efficiency of investment and thus the rate with which firm savings are transformed into productive capital. since turning investment into capital is more efficient, it makes sense to save more through capital, hence, consumption jumps down. By the mechanism in equation (1), an inward shift of the labor supply curve results in an increase in hours worked and a slightly reduction in wage. The increase in labor leads to an increase in output, which combined with the reduction in consumption means investment increases. As a supply type shock, a positive investment shock also yields a decline in the price of capital. In the presence of borrowing constraints, the counter-cyclical asset price movement tends to tighten the borrowing constraint and this mechanism reduces the contribution of investment shocks. Finally, through the channel of equation (3) and (2), risk-free rate and stock price decrease with a reduction in consumption immediately, and recover along with consumption.

The borrowing constraint acts as a link between firm investment and equity valuations. It is well known that in the data, stock prices co-move with investment, too. This could be simply because news about investment opportunities affect stock prices and predict investment at the same time.[12] However, We do not generate correct dynamics of consumption and hours worked in response to a positive investment shock. There are two main issues.

First, the counter-factual movement between consumption and hours worked is typical in a neoclassical setting, mainly because of the "wealth effect" of consumption in the standard labor supply in equation (1). To reconcile this anomaly, I later on extend the model following a class of literature which requires the complete absence of a wealth effect on labor supply. Second, the short-run negative response of consumption after a positive investment shock is a common failure among a class of DSGE models. This is because the substitution effect dominates over the weak income effect. Following a positive investment shock, investment is more profitable, so agents substitute consumption for investment, hence the response of consumption is negative on impact of a positive investment shock and remains negative for about five quarters. A recent work by Ascari et al. (2019) proposes a New-Keynesian model incorporating firm's roundabout production and trend output growth which could survive the "Barro-King" Curse without removing the wealth effect on labor supply.

[12]See Barro (1990), and Blanchard et al. (1993)

Figure 2: Baseline model IRFs to a permanent investment shock



Note: The figure displays model IRFs of key variables to a positive investment shock, under two alternative calibrations in which only the earnings-based constraint (dotted blue line) or only the collateral constraint (red line) is present. The figure highlights that the responses of debt to shocks have a different signs under the alternative borrowing constraints, which is consistent with the results in Drechsel (2019).

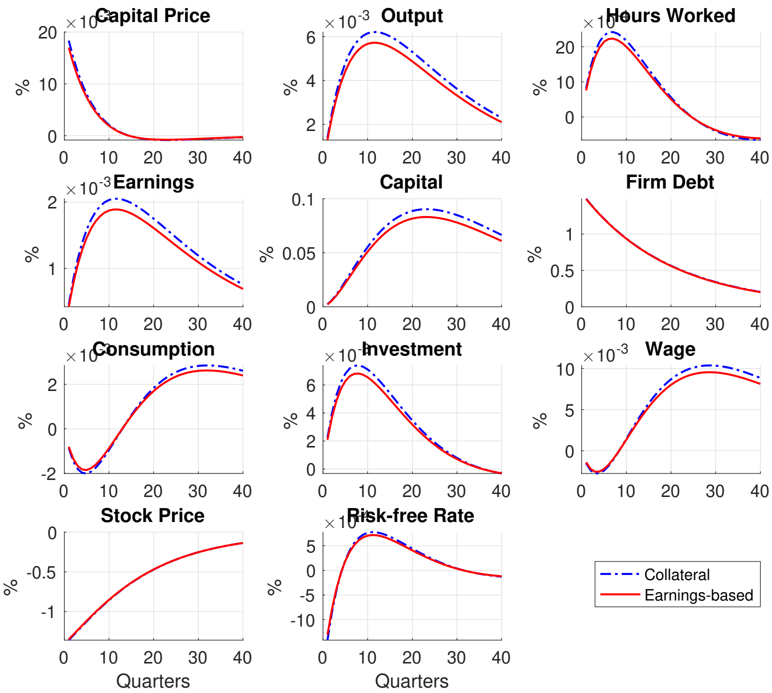
In addition, Figure 3 presents the impulse response functions to a positive financial shock which are consistent with literature that a negative financial shock can cause aggregate investment, employment and consumption to fall with output.[13] Despite this realistic comovement among macro quantities, a positive financial shock generates an equity price decrease as the shock looses firms' financing constraint. This counterfactual response of the equity price is robust to a wide range of variations in how financial frictions are modeled and whether financial shocks affect asset liquidity or firms' either collateral constraints or earnings-based constraints.

The easing of the constraint also induces a substitution away from private consumption towards investment. Investment increases (since firm can issue more debt hence borrow more so as to invest more). The easing of the constraint also results in consumption immediate declines, means that you substitute away from consumption and into investment. The fall in consumption results in hours worked going up and thus the wage going down. As a result of the increase in hours, output goes up, and further affect the macro dynamics in long horizon. The binding borrowing constraint results in increase in the volatility of investment relative to output. This is fairly intuitive. One thing the firm can do to ease the constraint is to accumulate more capital. While this won't let the firm hire more labor immediately, it will once the capital comes on line. This means that the firm has an incentive to do more investment, which strengthens the model's internal propagation mechanism. Put another way, binding financial constraints often have the effect of increasing propagation and persistence, in a way somewhat similar to the inclusion of investment adjustment costs. This is an older point that is made in Carlstrom and Fuerst (1997) in the context of a costly state verification (CSV) setup.

To conclude, for IRFs to TFP and IST shocks, we see a similar behavior under alternative borrowing constraints except a different sign of firm's debt response. We see a significant negative co-movement between consumption and hours worked. In order to capture macro dynamics in a better fit, next section further studies the empirical relevance of earnings-based constraints. Then I leave such questions on model predictions to Section 5, where I extend the baseline model to capture the correct dynamics. As for financial shocks, there's a slightly more amplification in the collateral constraint. This result is even consistent with Ingholt (2019) which suggests, for the housing sector, a larger amplification mechanism from the Loan-to-Value(LTV) constraint rather than Debt-Service-to-Income(DTI) constraint. Hence, it suggests a more effective credit limit policy should be carried on via earnings-based constraint since a lower collateral limit could contrarily not have prevented the boom, since soaring asset prices slackened this constraint.

[13]See Jermann and Quadrini (2012), Nezafat and Slavik (2015), Jaccard (2017), Winkler (2019).

Figure 3: Baseline Model IRFs to a persistent financial shock



Note: The figure shows the IRFs of selected economic variables to a positive financial shock. The shock size and persistence to create the IRFs is the same across models.

4 Empirical Evidence: SVAR on aggregate US data

This section introduces the empirical model I employ to estimate the responses of selected macroeconomic variables to the identified structural shocks. Since earnings-based constraints and collateral constraints indicate sharply contrasting predictions of debt dynamics in the baseline model, the empirical analysis focuses on identifying the total factor productivity (TFP) shock and investment-specific technology (IST) shock. The attention to IST shock is because the convenience of data where literature commonly choose the inverse relative price of investment goods. I build on such previous work to see how macro variables response to TFP shocks and IST shocks.

4.1 Model Specification

I closely follow the model specification in Fisher (2006). Let y_t be an $n \times 1$ vector of variables that may be integrated of order 1 and possibly co-integrated. The structural form is given by:

$$B_0 y_t = B_1 y_{t-1} + \cdots + B_p y_{t-p} + u_t = BY_{t-1} + u_t, \tag{15}$$

where, $Y'_{t-1} \equiv (y'_{t-1}, \ldots, y'_{t-p})$, $B \equiv [B_1, \ldots, B_p]$, and u_t is an $n \times 1$ vector of structural shocks with variance-covariance matrix $\Omega_u = I_n$. By Wold decomposition, the corresponding reduced form is:

$$y_t = A_1 y_{t-1} + \cdots + A_p y_{t-p} + w_t = AY_{t-1} + w_t,$$

where $A \equiv [A_1, \ldots, A_p] = B_0^{-1} B$ and $w_t = B_0^{-1} u_t \sim (0, \Sigma_w = B_0^{-1} B_0^{-1\prime})$.

4.2 Identification

In order to identify both permanent neutral technology shocks and investment-specific technology shocks, the structural model is identified by the long-run restrictions following Fisher (2006). First, let $y_t = (d\log(p_{kt}) \quad d\log(y_t/n_t) \quad \log(n_t))'$, which is a stationary vector as $\log(p_{kt})$, the log real price of investment goods and $\log(y_t/n_t)$, the log of real labor productivity are treated as $I(1)$, whereas $\log(n_t)$, the log of per capita hours worked follows $I(0)$. The co-integrating rank of the model for y_t is $r = 1$ due to the inclusion of one $I(0)$ variable, so there's at least two shocks with permanent effects. In this case, identification involves imposing long-run exclusion restrictions on the elements of $n \times n$ cumulative structural impulse response matrix:

$$\Theta(1) = \sum_{i=0}^{\infty} \Theta_i = B(1)^{-1} = A(1)^{-1} B_0^{-1},$$

where $B(1)^{-1} = [B_0 - B_1 - \cdots - B_p]^{-1}$ denotes the long-run multiplier.

Thus, the matrix u_t contains three structural shocks: Total Factor Productivity (TFP) shocks, Investment Specific Technology (IST) shocks, and a transitory shocks. The transitory shock is identified by a zero column in the long-run effects matrix. There are two assumptions for identifying these two permanent shocks, which are explicitly derived from a real business cycle model based on Fisher (2006). First, only IST shocks have a long-run effect on the the relative price of investment. Second, both TFP shocks and IST shocks affect the economy's labor productivity in the long run. Since I only identify two permanent shocks, and keep the rest of $B(1)^{-1}$ unrestricted, I also includes additional variables that keep in line with the macro variables from theoretical model predictions:[14]

$$Y_t = [d \log(p_{kt}) \quad d \log(y_t/n_t) \quad \log(n_t) \quad d \log(\pi_t) \quad d \log(p_{kt}k_t) \quad d \log(b_t)]', \tag{16}$$

where p_{kt} is the relative price of investment, which corresponds to v_t^{-1} if v_t captures IST, π_t denotes the firm's earnings and b_t is firm's debt. As some of the variables display low frequency movements after log differencing, I detrend some of the series before estimating the VAR.

4.3 Data

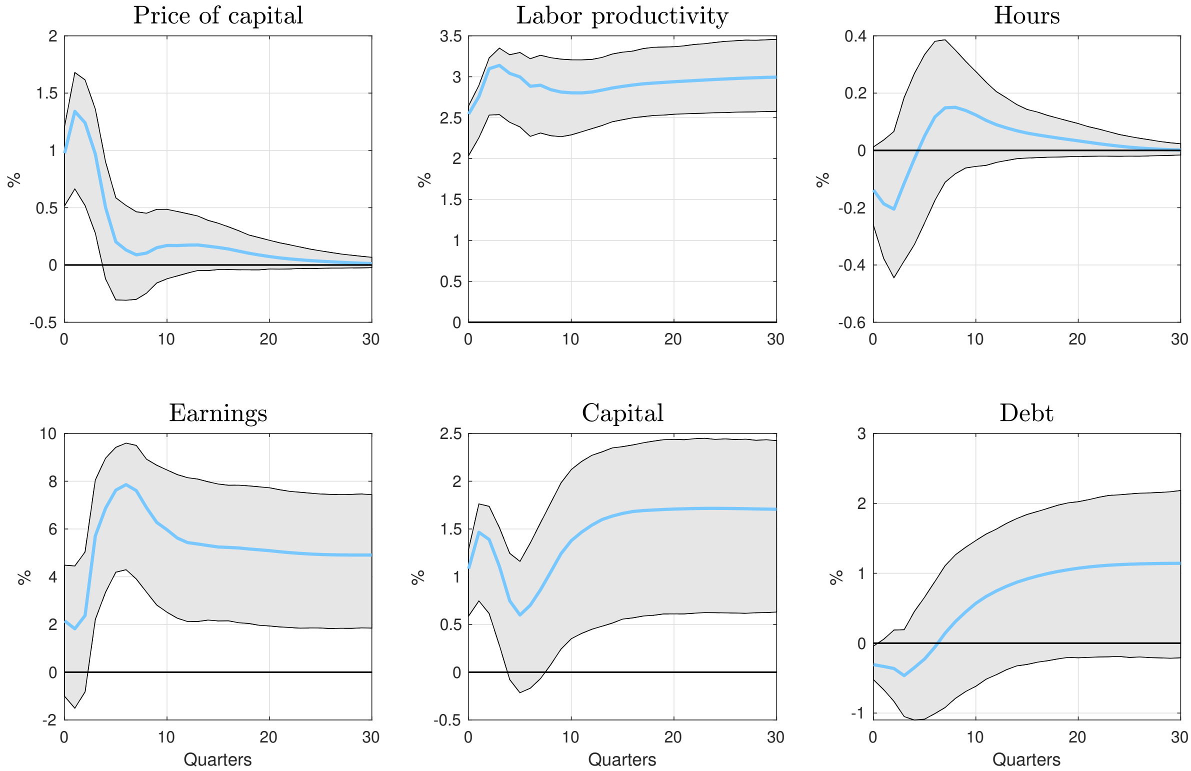
The macroeconomic time series variables are drawn from the FRED database and the US Financial Accounts (Flow of Funds) for the total non-financial business sector from the period 1952Q1-2016Q4. Details in Appendix A.2. To compute real variables I use nominal data deflated by the consumption deflater for nondurable goods and services. An important consideration lies in the choice of data for p_{kt}. Following the literature on IST shocks, I use the relative price of equipment investment. To make sure the IRFs are in line with Drechsel (2019) who studies firm's debt in response to different structural shocks, I also include firm's debt. I estimate the reduced form VAR using OLS, recover the IRFs from inverting the relevant matrices under the identifying restrictions, and compute 68% error bands using bootstrap techniques.

[14] a unit root test to confirm that both series are non-stationary in levels and stationary after first-differencing, as required by the identification scheme of the SVAR. Besides, as explained in Section 3, the impulse response of investment and consumption are common knowledge, here the selection of extra variables is in line with Drechsel (2019).

4.4 SVAR results

Figure 4 and 5 present the IRFs to a positive permanent TFP shock identified based on its long-run impact on the output productivity and a positive permanent IST shock identified based on its long-run impact on the relative price of capital. Regarding the effects of a permanent positive TFP shock, Figure 4 shows that a positive response of capital price, labor productivity, earnings and capital stock to TFP shock, but an overshot feature of firm's debt and hours worked, which is not captured by the baseline model prediction. Total hours worked instantaneously fall below its baseline level for about 5 quarters, then overshoot slightly and finally back to zero since it is not affected in the long run. Ramey (2016) points out that a positive TFP shock typically features an initial decline in hours worked before rising, consistent with different class of identification strategy among Justiniano et al. (2011), Fernald (2014) and Francis et al. (2014). A permanent TFP shock also shift firm's debt down for about 5 quarters, then firm's debt overshoots slightly and keep in a positive response. This represents a closely relationship between hours worked and firm's debt, section 5 successfully captures such dynamics by incorporating the working capital formation in borrowing constraints.

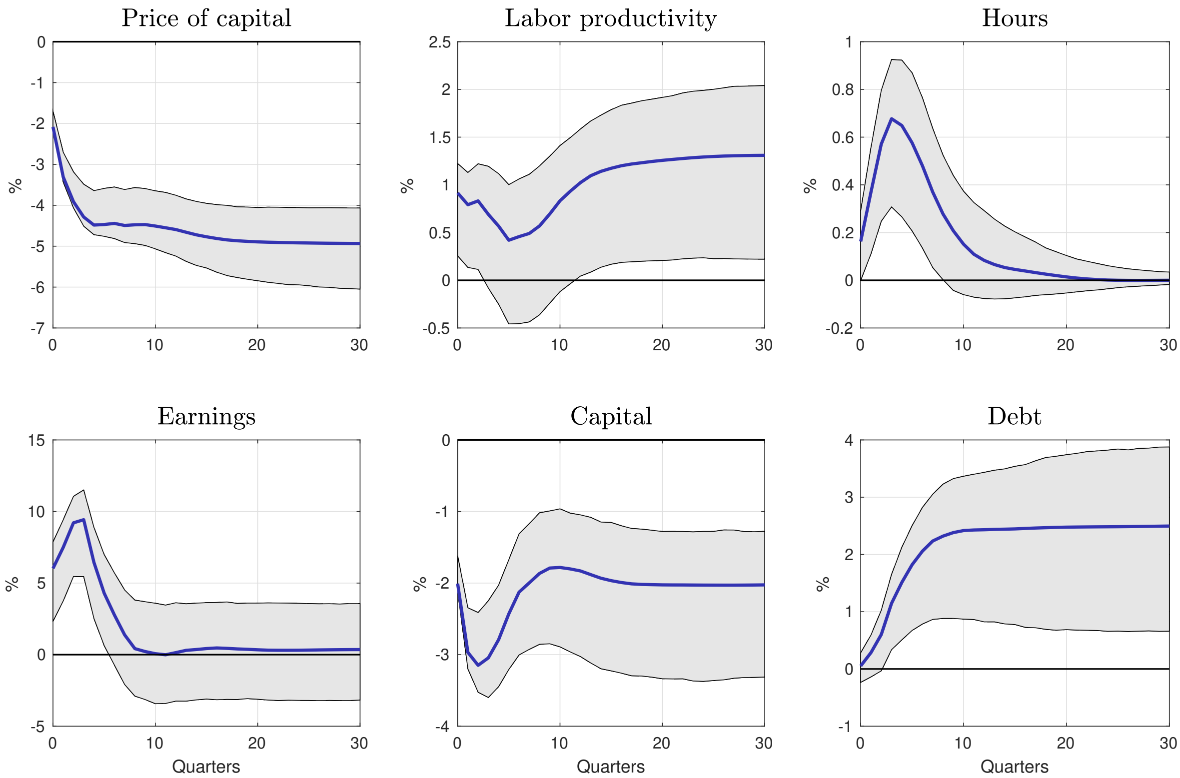
Figure 4: IRFs to a TFP shock identified from an estimated SVAR model using aggregate US data



Note: The figure displays the IRFs to an investment-specific shock identified from an estimated SVAR model using US data. The identification scheme relies on long-run restrictions following Fisher (2006). The responses are shown for all six variables included in the system, in percent. The unit of the shock is one standard deviation. The results are based on quarterly US data from 1952Q2 to 2016Q4 for $p = 4$. 68% error bands are calculated using bootstrap techniques. The figure shows a positive response of debt to an investment shock, which is in line with the predictions arising from a earnings-based borrowing constraint in the theoretical macro model.

Figure 5 shows a positive response of firm's debt, which is in line with the model predictions under earnings-based constraint rather than collateral constraint. Besides, it also in line with the baseline model predictions in the sense that the increase of firm's debt is accompanied by increasing earnings and output, and a fall in capital price. However, investment specific shocks lead to a significant expansion in hours worked, as well as a fall of capital stock which baseline model fails to predict. The empirical results are in line with Drechsel (2019) that the existence of earnings-based constraint but not collateral constraint, the extended model in next section is able to predict most of the empirical evidence with supporting earnings-based constraint.

Figure 5: IRFs to a IST shock identified from an estimated SVAR model using aggregate US data



Note: The figure displays the IRFs to an investment-specific shock identified from an estimated SVAR model using US data. The identification scheme relies on long-run restrictions following Fisher (2006). The responses are shown for all six variables included in the system, in percent. The unit of the shock is one standard deviation. The results are based on quarterly US data from 1952Q2 to 2016Q4 for $p = 4$. 68% error bands are calculated using bootstrap techniques. The figure shows a positive response of debt to an investment shock, which is in line with the predictions arising from a earnings-based borrowing constraint in the theoretical macro model.

To conclude, empirical analysis further support the economy-wide relevance of earnings-based borrowing constraints, since increase of earnings induces borrowers increase their debt liabilities in response to a positive investment shock, while firms subject to collateral constraints do not.

5 The Extended Model

As can be see from section 3, baseline model predictions under earnings-based constraints are in line with empirical evidence on debt dynamics to investment shocks. However, in some extents, the baseline model predictions poorly match their empirical counterparts since we observe an overshoot feature of hours worked and firm's debt in response to TFP shocks from the data and a significant positive response of hour worked and firm's debt. This section 5 attempts to extend the benchmark model to better match the profiles of debt and hours worked dynamics. Solving the negative response of hours worked problem, as discussed in section 3, requires a weak wealth effects in labor supply and working capital loans to pre-finance firm's expenditures. Besides, Jermann and Quadrini (2012) argues that hours worked and debt financing are tied through a channel where the flexibility with which the firm can change its financial structure, that is, the composition of debt and equity (stock) financing. Thus, consider the rigidities affecting the substitution between equity and debt is a key mechanism to break down the anomaly between debt and hours worked.

This section extends the baseline model in Section 3 to incorporate features of non-separable preference of utility, working capital formulation of borrowing constraints, firm's equity payout cost as well as additional shocks.[15]

5.1 Households

Here I consider preferences of utility feature both intra-temporal and inter-temporal non-separability. As discussed previously, to weaken the wealth effects in labor supply curve, I incorporate intra-temporal non-separability following Greenwood, Hercowitz, and Huffman (1988) who propose a popular utility specification that features non-separability between consumption and labor. Such preferences eliminate the wealth effect on labor supply which will result in more amplification. For inter-temporal non-separability, the state-of-the-art paper on habits is Ravn, Schmitt-Grohé, and Uribe (2006). Household's expected lifetime utility is given by:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \gamma_t \beta^t \frac{1}{1-\sigma} \left[(C_t - hC_{t-1}) - \chi \frac{N_t^{1+\theta}}{1+\theta} \right]^{1-\sigma},$$

where $\beta \in (0,1)$ is the discount factor, $h \in (0,1)$ is the habit persistence parameter, χ measures the dis-utility of labor and θ denotes the inverse of Frisch elasticity, γ_t denotes the preference shock. C_t and N_t denote consumption and labor respectively.

[15] Modern DSGE models with habits and adjustment costs include Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007), firm's equity payout cost is abstracted from Jermann and Quadrini (2012).

σ and χ are risk aversion and relative dis utility of labor.

The representative household's budget constraint remains the same:

$$C_t + \frac{b_t}{1 + r_t} + P_t S_t = w_t N_t + b_{t-1} + S_{t-1}(P_t + D_t),$$

where $S_t, P_t, D_t, R_t, b_t, w_t$ denote share holdings, the aggregate stock price of all firms, the aggregate dividend, the bond interest rate, bond holdings, and the wage rate, respectively.

5.2 Firm

Analogous to Section 3, the representative firm has access to two means of financing its operations, equity and debt, firm's flow of funds constraints are considered as follows:

$$\Psi(D_t) = y_t - w_t N_t + \frac{b_t}{R_t} - b_{t-1} - i_t,$$

where D_t denotes equity payouts (dividend), and $\Psi(\cdot)$ is equity payout cost function, following Jermann and Quadrini (2012), now raising equity is costly and firm has motives to smooth dividend:

$$\Psi(D_t) = D_t + \kappa(D_t - \bar{D})^2,$$

where $\kappa \geq 0$, $\bar{D}$ is a coefficient whose value is equal to the long-run payout target (steady state level of dividend).

Borrowing constraints have advocated the working capital formulation as a way to introduce an interaction between the labor wedge and financial frictions as an important amplification mechanism that delivers quantitatively more elevated responses to shocks.[16] This leaves the representative firm problem features the same property with the baseline setup in Section 3, but the borrowing constraints change to:

$$\frac{b_t}{1 + r_t} + w_t N_t \leq \xi_t^\pi \theta^\pi f(\pi_{t-3}, \pi_{t-2}, \pi_{t-1}, \pi_t), \tag{17}$$

$$\frac{b_t}{1 + r_t} + w_t N_t \leq \xi_t^k \theta^k \mathbb{E}_t Q_{t+1}(1 - \delta) k_t \tag{18}$$

Here I allow a more general form of earnings-based constraint, where $f(\cdot)$ denotes a linear polynomial which gives the 4-quarter trailing averages.[17] Details on model's

[16] This is motivated by microeconomic data provided by Drechsel (2019). Among the most frequently stated "deal purpose" in corporate covenants, working capital has the second large share.

[17] See Chodorow-Reich and Falato (2017).

optimality problems can be found in Appendix A.1.[18]

5.3 Government

The government's budget constraint now changes to:

$$T_t = \frac{b_t}{R_t} - \frac{b_t}{1+r_t} + G_t,$$

where T_t is the lump sum taxes levied on households, $\frac{b_t}{R_t} - \frac{b_t}{1+r_t}$ is the tax subsidy given to firms, and $G_t = \hat{g} \cdot g_t$ is a real government spending with $\hat{g}$ is exogenously choose to match the government purchases-to-output ratio, and g_t a government spending shock. Thus, the resource constraint should hold in the following manner:

$$y_t = C_t + i_t + G_t$$

5.4 Stochastic process

The model features six exogenous shocks, capturing shocks to TFP, investment, adjustment costs, preferences, financial constraints, and fiscal policy are the following:

$$\begin{aligned}
\log(z_t) &= (1-\rho_z)\log(\bar{z}) + \rho_z \log(z_{t-1}) + \varepsilon_t^z, \\
\log(v_t) &= (1-\rho_v)\log(\bar{v}) + \rho_v \log(v_{t-1}) + \varepsilon_t^v, \\
\log(\phi_t) &= (1-\rho_\phi)\log(\bar{\phi}) + \rho_\phi \log(\phi_{t-1}) + \varepsilon_t^\phi, \\
\log(\gamma_t) &= (1-\rho_\gamma)\log(\bar{\gamma}) + \rho_\gamma \log(\gamma_{t-1}) + \varepsilon_t^\gamma, \\
\log(\xi_t) &= (1-\rho_\xi)\log(\bar{\xi}) + \rho_\xi \log(\xi_{t-1}) + \varepsilon_t^\xi, \\
\log(g_t) &= (1-\rho_g)\log(\bar{g}) + \rho_g \log(g_{t-1}) + \varepsilon_t^g.
\end{aligned}$$

5.5 Model Parameterization

The specification of investment adjustment costs gives a steady state market value of capital 1. In steady state, $\Phi''(1) = \bar{\phi}$. Most parameter values are standard in business cycle paper for the U.S. Table 2 reports the additional and re-calibrated parameter values.

[18] Since debt repayment is the third share of stated deal purpose in corporate covenants from microeconomic data provided by ??. I also explore the model to include the flow debt borrowing constraints which does not change the dynamics in a significant way.

Table 2: Model parameterizations and calibration targets

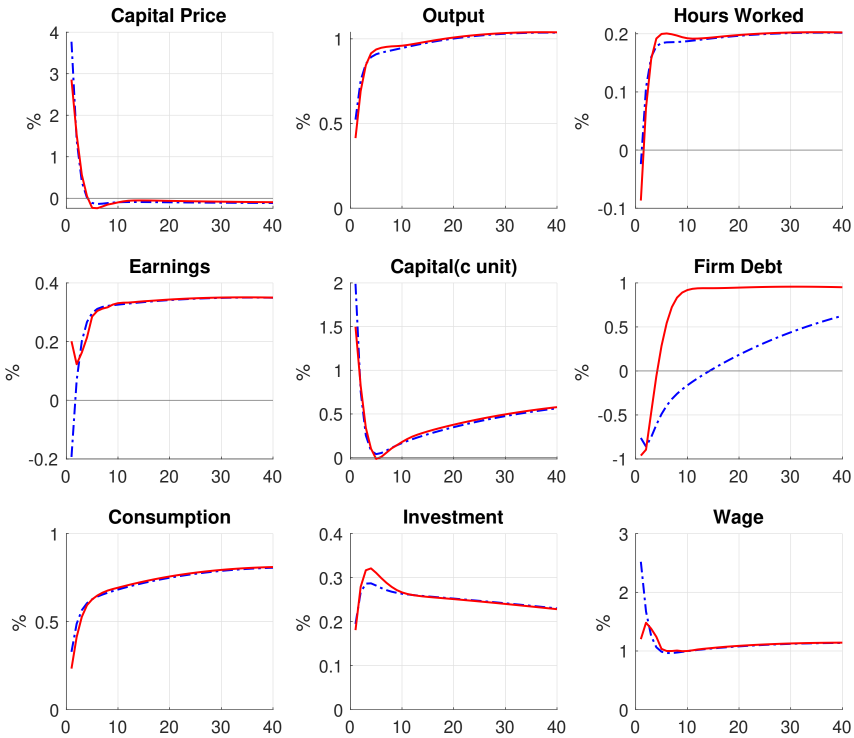
Parameter	Value	Source/target
χ	2.33	Target $n = 0.3$ in steady state
σ	1.5	Exogenously chosen
h	0.6	Exogenously chosen
θ^{π}	4.6/4	Exogenously chosen as in Section 3, Table 1
θ^{k}	0.2320	Same calibration procedure as in Section 3, Table 1
κ	0.46	Following Jermann and Quadrini (2012)
$\sigma_z, \sigma_v, \sigma_\phi, \sigma_\xi, \sigma_\psi, \sigma_g$	0.05	Following Drechsel (2019)
$\rho_z, \rho_v, \rho_\phi, \rho_\xi, \rho_g, \rho_\psi$	0.9	Perform persistent shocks

5.6 Macro dynamics implied by extended model

To compare with the empirical results and baseline model predictions, here I focus on the IRFs of macro variables to TFP and IST shocks, the IRFs to additional shocks are viewed as by-product of this analysis shown in Appendix A.4. Figure 6 below plots the impulse response functions of macro variables to a positive persistent technology shock. Since now the borrowing constraints incorporate working capital pre-finance version, thus increase of labor payment ($w_t N_t$) will induce a decrease of firm's debt in both cases. Differently than before, investment increases substantially less than output given that investment now must be financed via working capital. The smaller increase in investment means that we get less propagation of the productivity shock. Hence, the output impulse response is not as hump-shaped.

Unlike the baseline model, we are able to capture an overshoot feature of firm debt and hours worked in response to a positive TFP shock in the sense that firm debt and hours worked decrease first and recover later. Besides, it's also worth to notice that earnings are significant positive under earnings-based constraint which is supported by the empirical evidence. The mechanism behind is the following. When the economy encounters a positive persistent TFP shock, firm's output immediately increases, thus firm would like to increase investment to accumulate more capital, since investment increases substantially less than output, household consumption increases. However, since there exists a habit formation, thus wealth effects of consumption play a role in hours worked, further decrease the hours worked. Thus wage should increase more to compensate an increase of labor payment. However, wage increase more in the model with collateral constraint. By a working capital formation of borrowing constraint, firm has to decrease their debt in the beginning. Since firm's earnings defined by output minus the labor payment, earnings under earnings-based constraint positively response to TFP shocks while they do not under collateral constraint because of a higher increase of labor payment.

Figure 6: Extended model IRFs to a persistent technology shock



Note: The figure displays model IRFs of key variables to a positive technology factor productivity (TFP) shock, under two alternative calibrations in which only the earnings-based constraint (dotted blue line) or only the collateral constraint (red line) is present. The figure highlights that the responses of debt to TFP shocks have a significant overshot under earnings-based constraints while others are remaining the same under the alternative borrowing constraints.

As for investment shock shown in Figure 7, a larger volatility of investment relative to output, and larger volatility in hours worked are generated due to the extended features in utility preferences which result in a weaker wealth effects. By extending the model to a broader dimension, we still obtain a different sign of debt dynamics under alternative borrowing constraints, and the one under earnings-based constraints is supported by the empirical results. There is a delayed and hump-shaped response under this version of the earnings-based constraint because of the weighted average of 4-quarters earnings show up in the borrowing limit, but the signs of the responses remain unchanged. Besides, hours worked are in significant positive response under earnings-based constraint. The debt borrowing constraints in always binding case deliver slightly different propagation channels for other macro variables due to the equity payout regidities.

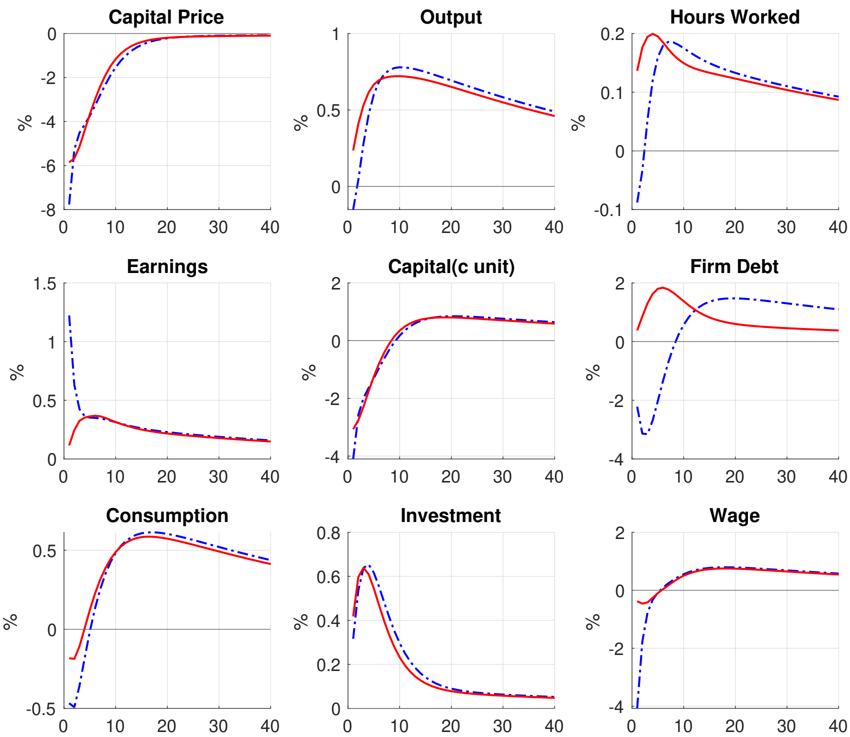
Different sign of debt dynamics under alternative borrowing constraints is explained in Section 3. However, with firm pre-financed its working capital, firm has to decrease not only debt but only labor payment under collateral constraint but this is not the case with earnings-based constraint. Since a large decrease of labor payment under collateral constraint, there's a higher increase of earnings. Other macro dynamics are similar with section 3. Here, investment shocks cannot be a major source of business cycle fluctuations as it does not generate the correct co-movement between consumption and other macro variables. This is a typical failure in the neoclassical model(See Appendix A.5) called "Barro-King Curse", recently, Ascari et al. (2019) consider a New Keynesian model with the realistic additions of roundabout production and real per capita output growth, which could reconcile such anomaly of consumption dynamics in reponse to investment shocks.

In addition, Figure 8 presents the impulse response functions to a positive financial shock. The capital price increases and the collateral constraint binds, the concurrent capital price appreciation amplifies the leveraging process, leading to a further increase in aggregate output. By contrast, when the capital price is high and the earnings-based constraint binds, amplification through asset price fluctuation does not translate through borrowing constraint so much, hence a relatively weaker amplification effect than model under collateral constraints.

6 Conclusion

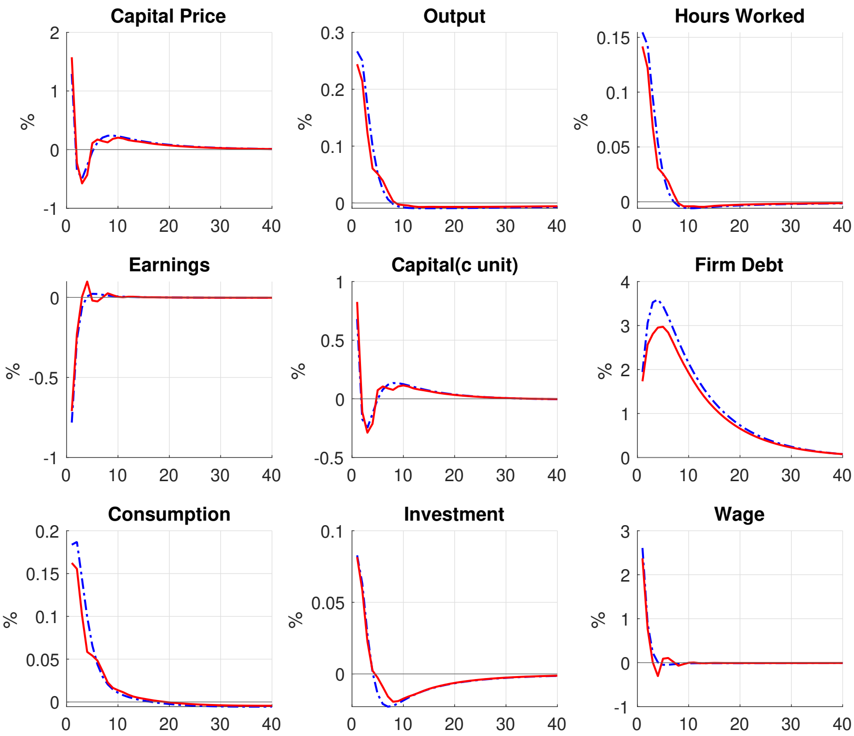
Corporate debt borrowing constraints imply sharply different firm's debt dynamics in response to investment shocks, but with other macro dynamics unchanged for alternative borrowing constraints under a benchmark real business cycle model. In this sense, empirical studies further supports the earnings-based constraints; a positive

Figure 7: Extended model IRFs to a persistent investment shock



Note: The figure displays model IRFs of key variables to a positive investment shock, under two alternative calibrations in which only the earnings-based constraint (dotted blue line) or only the collateral constraint (red line) is present. The figure highlights that the responses of debt to shocks still have a different signs under the alternative borrowing constraints, even in the working capital version of borrowing constraints.

Figure 8: Extended Model IRFs to a persistent financial shock



Note: The figure shows the IRFs of selected economic variables to a positive financial shock. The shock size and persistence to create the IRFs is the same across models.

response of debt to investment shocks. Empirical analysis reveals an overshoot feature of debt and hours worked in response to TFP shocks, and a significant positive response to IST shocks. To capture these evidence, I extend the model by featuring non-separable utility preferences in household, and firm subject to equity payout cost and constrained by working capital formulation borrowing constraints. These results suggest that the earnings-based constraint plays a key role in drawing quantitative conclusions about the transmission of shocks in the economy.

Further research can be done by examining how changes of debt-to-earnings ratio in models under occasional binding constraints affect the boom-bust cycles and its effectiveness as a macroprudential tool. Moreover, it's also worth to incorporate heterogenous agents and firm networking to study correlations between key variables and profiles of cross-correlations in a business cycle research.

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A Appendix

A.1 Model details

Here I provide the optimality problems of the extended model in Section 5, solutions to the baseline model are therefore straightforward.

Household's decision problem

I define the marginal utility of consumption as Λ_t. The household's first-order conditions are[19]:

$$\Lambda_t = U_{c,t} - \frac{\gamma_{t+1}}{\gamma_t}\beta h \mathbb{E}_t U_{c,t+1}, \tag{19}$$

$$\Lambda_t w_t = \chi N_t^{\theta} U_{c,t}, \tag{20}$$

$$\Lambda_t = \beta(1+r_t)\mathbb{E}_t \Lambda_{t+1}, \tag{21}$$

$$P_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t}(P_{t+1} + D_{t+1}), \tag{22}$$

where $U_{c,t} = \left[(C_t - hC_{t-1}) - \chi \frac{N_t^{1+\theta}}{1+\theta}\right]^{-\sigma}$. Equation (20) says to equate the marginal rate of substitution between leisure and consumption to the relative price of leisure (the real wage), and equation (21) says to equate the marginal rate of substitution between consumption today and tomorrow to the relative price of consumption today (the real interest rate).

Firm's decision problem

The firm discounts future dividends by the stochastic discount factor of the household, $\mathbb{E}_k \frac{\beta^k \Lambda_{t+k}}{\Lambda_t}$. Let Q_t be the multiplier on the capital accumulation equation and also the level of capital price.

$$\max \mathbb{E}_k \sum_{k=0}^{\infty} \frac{\beta^k \Lambda_{t+k}}{\Lambda_t} D_{t+k},$$

subject to (4), (5), (6), (8) and either of the borrowing constraints (17) or (18).

The firm's Lagrangian problem is presented by the following:

[19] Household problems are same under alternative borrowing constraints.

$$
\begin{aligned}
\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \Big\{ & D_t + \lambda_t (y_t - w_t N_t + \frac{b_t}{1+r_t} - i_t - b_{t-1} - \Psi(D_t)) \cdots \\
& + Q_t \left[v_t i_t \left(1 - \Phi\left(\frac{i_t}{i_{t-1}}\right)\right) + (1-\delta) k_{t-1} - k_t \right] \cdots \\
& + \mu_t^{\pi} [\xi_t \theta^{\pi} f(\pi_{t-3}, \pi_{t-2}, \pi_{t-1}, \pi_t) - w_t N_t - \frac{b_t}{1+r_t}] \cdots \\
& + \mu_t^{k} [\xi_t \theta^{k} \mathbb{E}_t Q_{t+1} (1-\delta) k_t - w_t N_t - \frac{b_t}{1+r_t}] \Big\}
\end{aligned}
$$

The first order conditions with respect to N_t, b_t, i_t, k_t are:[20]

$$
\lambda_t = \frac{1}{2\phi(D_t - \bar{D}) + 1} \tag{23}
$$

$$
\left[\Lambda_t \lambda_t + \mathbb{E}_t \sum_{j=0}^{3} \frac{\beta^j \Lambda_{t+j} \mu_{t+j}^{\pi} \xi_{t+j} \theta^{\pi}}{\bar{X}_j} \right] (F_{n,t} - w_t) = \Lambda_t (\mu_t^{\pi} + \mu_t^{k}) w_t \tag{24}
$$

$$
R_t \mathbb{E}_t \left\{ \frac{\beta \Lambda_{t+1} \lambda_{t+1}}{\Lambda_t} \right\} + \mathbb{1}_{\mu_t^k \neq 0} (\mu_t^k \frac{R_t}{1+r_t}) + \mathbb{1}_{\mu_t^{\pi} \neq 0} (\mu_t^{\pi} \frac{R_t}{1+r_t}) = \lambda_t \tag{25}
$$

$$
Q_t v_t (1 - \Phi_t' i_t - \Phi_t) + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} Q_{t+1} v_{t+1} i_{t+1} \Phi_{t+1}' = \lambda_t \tag{26}
$$

$$
\begin{aligned}
Q_t = \mu_t^k \xi_t \theta^k (1-\delta) \mathbb{E}_t Q_{t+1} + \mathbb{E}_t \left[\sum_{j=0}^{3} \beta^{j+1} \frac{\Lambda_{t+j+1}}{\Lambda_t} \frac{\mu_{t+j+1}^{\pi} \xi_{t+j+1} \theta^{\pi}}{\lambda_j} \right] F_{k,t+1} \\
+ \mathbb{E}_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} (\lambda_{t+1} F_{k,t+1} + (1-\delta) Q_{t+1})
\end{aligned} \tag{27}
$$

where $F_{n,t} = (1-\alpha)\frac{y_t}{n_t}$ and $F_{k,t+1} = \alpha \frac{y_{t+1}}{k_t}$ denote the marginal products of labor and capital, respectively. Q_t denotes the Lagrange multiplier on the capital accumulation (5), and since firm is either constrained by earnings-based constraint or collateral constraint, thus either $\mu^k = 0$ or $\mu^{\pi} = 0$.[21]

Non-Stochastic Steady States

This section documents the derivation of the steady-state solution of the model. An exact numerical solution can be reached by combining the resulting relations as it is done in the steady-state code. Here I present the algorithm of solving steady states for model with borrowing constraints.[22] Because I want to solve the model using

[20] Notice that for earnings-based constraint only depend on current earnings, Equation (24) can be rewritten as:
$w_t(1 + \theta^{\pi}\mu^{\pi} + \mu^{\pi}) = (1-\alpha)\frac{y_t}{n_t}(1 + \theta^{\pi}\mu^{\pi})$
and Equation (27) changes to $Q_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t}((1+\mu_t^{\pi}\xi_t^{\pi}\theta^{\pi})\alpha\frac{y_{t+1}}{k_t} + Q_{t+1}(1-\delta)) + \mathbb{E}_t \mu_t^k \xi_t^k \theta^k (1-\delta) Q_{t+1}$.
[21] For the purpose of simplifying the notation, here I assume $\lambda_i = 1/4$, for $i = 0, 1, 2, 3$.
[22] For ease of notation, I let earnings-based constraint only based on current earnings, it's straight-

perturbation techniques, I went to approximate about a point where the constraint binds, and then implicitly ignore any possibility of the constraint perhaps not binding at some point in the future. I therefore assume extract the steady states debt level from equation (17) and (18) for alternative borrowing constraints, respectively, which means that in the steady state both constraints bind.

Variables without time subscripts denote steady state values. We will consider two regions, one in which the collateral constraints binds and the other in which it does not. If the constraint j does not bind, then $\mu^j = 0$ and the steady state is the same as in the standard model. From the first order condition for investment as well as the capital accumulation equation, it is clear that $Q = 1$. Then, we can solve for the steady state capital-labor ratio from the dynamic capital supply Euler equation (27):

$$(\frac{k}{N})_{ss} = \left(\frac{1-(\beta+\mu^k\theta^k)(1-\delta)}{\beta\alpha(1+\mu^\pi\theta^\pi)}\right)^{\frac{1}{\alpha-1}}$$

Such expression holds regardless of whether the constraint binds or not but vary slightly between two alternative constraints. If the constraint does not bind, then this is the usual expression for the capital-labor ratio. Then, from the first order condition for labor demand (24), we obtain the steady states wage:

$$w_{ss} = \frac{1+\mu^\pi\theta^\pi}{1+\mu^\pi\theta^\pi+\mu^\pi+\mu^k}(1-\alpha)(\frac{k}{N})^\alpha_{ss}$$

Given steady state capital-labor ratio and wage, suppose that the borrowing constraint binds. Then we can solve wage and μ given the constraint evaluated in steady state:

$$w_{ss} = \xi(\frac{k}{N})_{ss}$$

Next, combine with household's optimality conditions and resource constraint, we solve equations to determine labor and consumption together by targeting steady state labor $n = 0.300$, with a standard minimization routine, we reach a target value of labor disutility parameter $\chi = 2.33$.

To allow for a small deviation of the adjustment cost in steady states, I specify $\iota = 0.999$:

$$\Phi_t\left(\frac{i_t}{i_{t-1}}\right) = \frac{\phi_t}{2}\left(\frac{i_t}{i_{t-1}} - \iota\right)^2. \tag{28}$$

Once we solve steady states level of consumption and labor, the rest of the steady state variables are straightforward, for example:

forward for the weighted sum of 4 quarters earnings-based constraints.

$$k_{ss} = (\frac{k}{N})_{ss} n_{ss} \tag{29}$$

$$y_{ss} = (\frac{k}{N})^{\alpha}_{ss} n_s s \tag{30}$$

$$i_{ss} = \delta k_{ss}/(1 - \Phi_{ss}) \tag{31}$$

$$g_{ss} = \Gamma y_{ss} \tag{32}$$

A.2 Data used in estimation of SVAR

This section describes how the data is constructed, and attached the related data code in parenthesis. Data are mainly downloaded from Federal Reserve Economic Data (FRED), and FED Financial Accounts F.102 Non-financial Business (USFA).

The consumption deflater corresponds to nondurable and service consumption, derived directly from the National Income and Product Accounts(NIPA).

For relative price of investment (log-diff), I follow Greenwood, Hercowitz, and Huffman (1988) which uses implicit price deflater of nonresidential fixed equipment investment (FRED: Y033RD3Q086SBEA), deflated with implicit price deflater of personal consumption expenditures of nondurable goods and services (FRED: CONSDEF).

Level of the capital stock (log-diff), following Drechsel (2019), is constructed from capital expenditures in the non-financial business sector (USFA: FA145050005.Q) minus depreciation (consumption of fixed capital in the non-financial business sector, USFA: FA106300083.Q) valued at the relative price of investment.

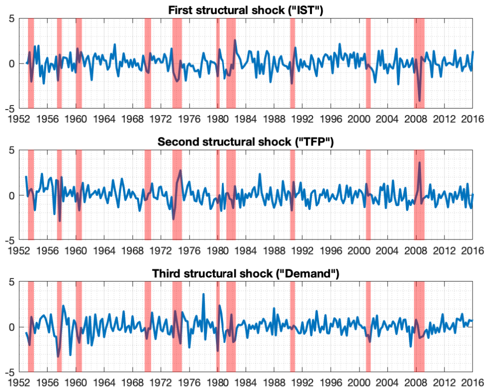
Hours worked is directly taken from Hours of all persons in the non-farm business sector (FRED: HOANBS).

Business sector earnings (log diff) is obtained from income before taxes from non-financial business sector (FA146110005.Q), deflated with consumption deflater (see above)

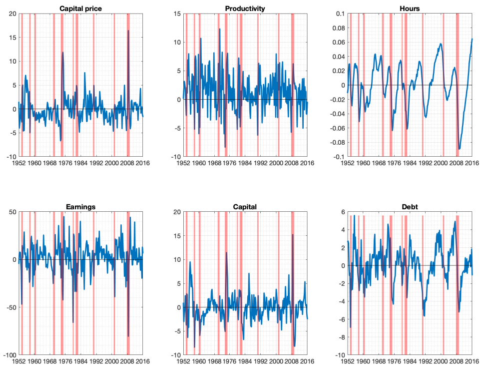
Output is computed by Nominal GDP (FRED: GDP), divided by population (FRED: B230RC0Q173SBEA), deflated with consumption deflater (see above).

A.3 Additional results for SVAR

Figure 9: Additional figures for SVAR



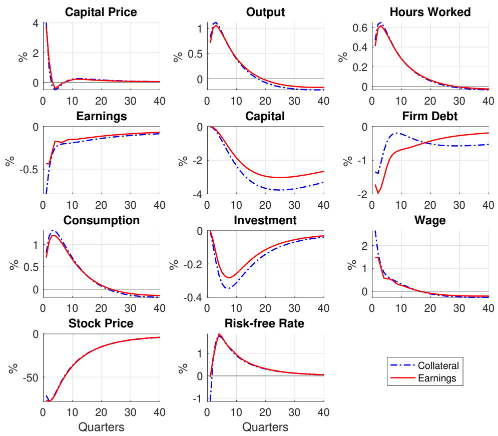
(a) Structural shocks identified from an estimated SVAR model using aggregate US data



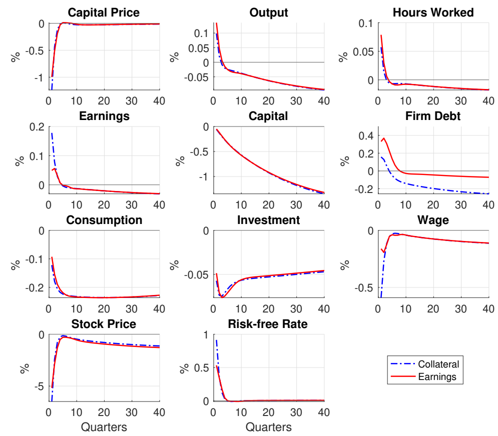
(b) Time series macroeconomic variables used in SVAR

A.4 Model IRFs of additional shocks

Figure 10: Extended model IRFs to additional shocks



(a) a persistent preference shock



(b) a persistent government spending shock

A.5 Model Variance Decompositions under alternative borrowing constraints

Table 3: Variance decomposition of model with borrowing constraints

	Extended Model				Baseline Model			
	Earnings-based Constraint		Collateral Constraint		Earnings-based Constraint		Collateral Constraint	
	TFP	IST	TFP	IST	TFP	IST	TFP	IST
Output	43.93	24.34	43.15	23.90	94.91	5.09	94.95	5.05
Earnings	25.75	27.00	22.47	23.78	94.91	5.09	94.95	5.05
Consumption	27.54	10.94	26.22	10.28	99.47	0.53	99.47	0.53
Investment	22.59	61.37	20.89	60.11	62.33	37.67	62.43	37.57
Capital	5.55	91.31	5.27	90.78	31.89	68.11	31.83	68.17
Hours Worked	11.82	11.55	11.27	10.76	63.79	36.21	64.38	35.62
Wage	45.28	7.69	40.40	6.62	99.22	0.78	99.22	0.78
Capital Price	9.64	76.94	9.44	76.87	5.12	94.88	5.01	94.99