

```

1  function [K] = MTGP_covQPMisoUU_shift_fix(d,hyp, x, z, i)
2
3  % Stationary covariance function for a quasi-periodic function based on a
4  % multiplication of a Matern and Periodic function
5  %
6  %     - only elements of x(:,1:end-1)/z(:,1:end-1) will be analyzed,
7  %     - x(:,end)/z(:,end) will be ignored, as it contains only the label
  information
8  %     - independent of the label all x values will have the same hyp
9  %     - feature scaling hyperparameter is fixed to 1
10 %     - output scaling hyperparameter is fixed to 1
11 %
12 % k(x,y) = exp(-(x-theta_s) - ((y-theta_s)))'*inv(P)*((x-theta_s) -
  (y-theta_s)^q)/2) * ...
13 %         exp( -2*sin^2( pi*|(x-theta_s)-(y-theta_s)|/p ) )
14 %
15 % where the P matrix is ell^2 times the unit matrix.
16 % The hyperparameters are:
17 %
18 % hyp = [ log(ell)
19 %         log(p);
20 %         theta_s(1)
21 %         ...
22 %         theta_s(nL-1)]
23 %
24 % modified by Robert Duerichen
25 % 10/04/2014
26
27 if nargin<3, K = 'nL+1'; return; end % report number of
  parameters
28 if nargin<4, z = []; end % make sure, z exists
29 xeqz = numel(z)==0; dg = strcmp(z,'diag') && numel(z)>0; % determine mode
30
31
32 nL = max(x(:,2)); % get number of labels
33 ell = exp(hyp(1)); % characteristic length scale
34 p = exp(hyp(2)); % period
35 shift = (hyp(3:end)); % time shift hyp
36
37 %% define Matern function
38 if all(d~= [1,3,5]), error('only 1, 3 and 5 allowed for d'), end % degree
39
40 switch d
41     case 1, f = @(t) 1; df = @(t) 1;
42     case 3, f = @(t) 1 + t; df = @(t) t;
43     case 5, f = @(t) 1 + t.*(1+t/3); df = @(t) t.*(1+t)/3;
44 end
45 m = @(t,f) f(t).*exp(-t); dm = @(t,f) df(t).*exp(-t);
46
47
48
49 %% perform shift
50 for ii = 2:nL
51     x(x(:,2)== ii,1) = x(x(:,2)== ii,1)+shift(ii-1);
52     if ~isempty(z)
53         z(z(:,2)== ii,1) = z(z(:,2)== ii,1)+shift(ii-1);
54     end
55 end
56
57 % precompute distances
58 if dg % vector kxx
59     K_p = zeros(size(x(:,1)),1,1);
60     K_m = zeros(size(x(:,1)),1,1);
61 else
62     if xeqz % symmetric matrix Kxx
63         K_m = sqrt( sq_dist(sqrt(d)*x(:,1:end-1)'/ell) );
64         K_p = sqrt(sq_dist(x(:,1)'));
65     else % cross covariances Kxz
66         K_m = sqrt( sq_dist(sqrt(d)*x(:,1:end-1)'/ell,sqrt(d)*z(:,1:end-1)'/ell) );
67         K_p = sqrt(sq_dist(x(:,1)',z(:,1)'));
68     end

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69 end
70
71 K_p = pi*K_p/p;
72 if nargin<5 % covariances
73     K_p = sin(K_p); K_p = K_p.*K_p; K_p = exp(-2*K_p);
74     K_m = m(K_m,f);
75     K = K_p.*K_m;
76 else % derivatives
77     if i<=nL+1
78         if i==1 % derivatives of the se hyperparameter
79             K_p = sin(K_p); K_p = K_p.*K_p; K_p = exp(-2*K_p);
80             K = K_p.*K_m.*dm(K_m,f);
81         elseif i==2 % derivatives of the periodic hyperparameter
82             K_m = m(K_m,f);
83             R = sin(K_p); K = K_m.* 4.*exp(-2*R.*R).*R.*cos(K_p).*K_p;
84         elseif i > 2 && i <= nL+1 % derivatives of the shift hyperparameters
85             ind_i = (x(:,2) ==i-1);
86             ind_ni = (x(:,2) ~=i-1);
87             B = zeros(length(x));
88             B(ind_ni,ind_i) = ones(sum(ind_ni),sum(ind_i));
89             B(ind_i,ind_ni) = -ones(sum(ind_i),sum(ind_ni));
90             A = repmat(x(:,1) ,[1 length(x)]);
91
92             switch d
93                 case 1
94                     dK_m = B.*dm(K_m,f) ./ (ell) .*sign(A-A');
95                 case 3
96                     dK_m = B.*sqrt(d) .*dm(K_m,f) ./ (ell) .*sign(A-A');
97                 case 5
98                     dK_m = sqrt(d) .* (K_m.^2 +K_m) ./ (3*ell) .*exp(-K_m) .*B.*sign(A-A');
99             end
100
101             R = sin(K_p);
102             dK_p = B.*4.*exp(-2*R.*R).*R.*cos(K_p).*pi./p.*sign(A-A');
103
104             K_p = sin(K_p); K_p = K_p.*K_p; K_p = exp(-2*K_p);
105
106             K_m = m(K_m,f);
107
108             K = dK_m.*K_p + K_m.*dK_p;
109         end
110     else
111         error('Unknown hyperparameter')
112     end
113 end
114 end

```