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1  function K = MTGP_covSEconU(hyp, x, z, i)
2
3  % Squared Exponential covariance function with different distance
4  % measure for each task.
5  %
6  % Based on the covSEisoU.m function of the GPML Toolbox -
7  %   with the following changes:
8  %   - only elements of x(:,1:end-1)/z(:,1:end-1) will be analyzed,
9  %   - x(:,end)/z(:,end) contains the label information
10 %   - task specific input-scaling hyperparameter based on convolution
11 %   - output-scaling hyperparameter is fixed to 1
12 %
13 % The covariance function is parameterized for a two inputs x_1 and x_2 from
14 % task l=1 and l=2 as:
15 %
16 %  $k(x_1, x_2) = \sqrt{2\ell_1\ell_2/(\ell_1^2\ell_2^2)} \exp(-(x_1 -$ 
17 %  $x_2)^2/(\ell_1^2+\ell_2^2));$ 
18 %
19 % The hyperparameters are:
20 %
21 % hyp = [    log(ell_1)
22 %          ...
23 %          log(ell_nL)]
24 %
25 %
26 % by Robert Duerichen
27 % 01/21/2014
28
29
30 if nargin<2, K = 'nL'; return; end           % report number of
parameters                                     %
31                                              % here nL is number of
                                              % tasks
32 if nargin<3, z = []; end                     % make sure, z exists
33 xeqz = numel(z)==0; dg = strcmp(z,'diag') && numel(z)>0; % determine mode
34
35 nL = max(x(:,2));                             % determine dimension
36
37 ell = exp(hyp(1:nL));                         % characteristic length
scale
38
39
40 if dg                                          % vector kxx
41     K = zeros(size(x(:,1),1),1,1);           % initialise correlation
vector
42     r = zeros(size(x(:,1),1),1,1);           % initialise squared
distance vector
43     hyp_mat = ones(size(x(:,1),1),1,4);       % initialise 4D matrix
to store hyperparameter results
44 else
45     if xeqz                                  % symmetric matrix Kxx
46
47         r = sq_dist(x(:,1)');                % compute squared distance
48
49         len_x = length(x(:,1));
50         hyp_mat = zeros(len_x,len_x,4);       % initialise 4D matrix
to store hyperparameter results
51         hyp_mat(:,:,1) = repmat(ell(x(:,2))',[1,len_x]); % repeat hyperparameter
in dim 2 of matrix
52         hyp_mat(:,:,2) = repmat(ell(x(:,2))',[len_x,1]); % repeat hyperparameter
in dim 1 of matrix
53
54     else                                     % cross covariances Kxz
55         r = sq_dist(x(:,1)',z(:,1)');        % compute squared distance
56
57         len_x = length(x(:,1));
58         len_z = length(z(:,1));
59         hyp_mat = zeros(len_x,len_z,4);       % initialise 4D matrix
to store hyperparameter results
60         hyp_mat(:,:,1) = repmat(ell(x(:,2))',[1,len_z]); % repeat hyperparameter
in dim 2 of matrix

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61     hyp_mat(:, :, 2) = repmat(ell(z(:, 2)), [len_x, 1]); % repeat hyperparameter
        in dim 1 of matrix
62 end
63
64     hyp_mat(:, :, 3) = (hyp_mat(:, :, 1).^2 + hyp_mat(:, :, 2).^2); % comp. squared sum of
        hyperpara.
65     hyp_mat(:, :, 4) = sqrt(2 * hyp_mat(:, :, 1) .* hyp_mat(:, :, 2) ./ (hyp_mat(:, :, 3)));
66
67     K = r ./ (hyp_mat(:, :, 3)); % comp. correlation matrix
68 end
69
70 if nargin < 4 % covariances
71     K = hyp_mat(:, :, 4) .* exp(-K);
72 else % derivatives
73     if i <= nL
74         % case hyper ell(i) does not belong to x_1 and x_2
75         K(x(:, 2) ~= i, x(:, 2) ~= i) = 0;
76
77         % case x_1 and x_2 belong to do not belong to the same task (l_1 ~= l_2 -->
        ell_1 ~= ell_2)
78         K(x(:, 2) ~= i, x(:, 2) == i) = hyp_mat(x(:, 2) ~= i, x(:, 2) == i, 1)
            .* hyp_mat(x(:, 2) ~= i, x(:, 2) == i, 2) .* exp(-K(x(:, 2) ~= i, x(:, 2) ==
            i)) .* (4 * r(x(:, 2) ~= i, x(:, 2) == i) .* hyp_mat(x(:, 2) ~= i, x(:, 2) ==
            i, 1).^2 - hyp_mat(x(:, 2) ~= i, x(:, 2) == i, 1).^4 + hyp_mat(x(:, 2) ~= i, x(:, 2) ==
            i, 2).^4) ./ ...
79         (hyp_mat(x(:, 2) ~= i, x(:, 2) == i, 4) .* hyp_mat(x(:, 2) ~= i, x(:, 2) ==
            i, 3).^3);
80
81         K(x(:, 2) == i, x(:, 2) ~= i) = K(x(:, 2) ~= i, x(:, 2) == i)';
82
83         % case x_1 and x_2 belong to same task (l_1 = l_2 --> ell_1 = ell_2)
84         K(x(:, 2) == i, x(:, 2) == i) = hyp_mat(x(:, 2) == i, x(:, 2) == i, 1) .*
            exp(-K(x(:, 2) == i, x(:, 2) == i)) .* r(x(:, 2) == i, x(:, 2) == i) ./ ...
            (hyp_mat(x(:, 2) == i, x(:, 2) == i, 1).^3);
85     else
86         error('Unknown hyperparameter')
87     end
88 end
89 end

```