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1  function K = MTGP_covMaternisoU_shift_mask(mask,d, hyp, x, z, i)
2
3  % Matern covariance function with nu = d/2 and isotropic distance measure. For
4  % d=1 the function is also known as the exponential covariance function or the
5  % Ornstein-Uhlenbeck covariance in 1d.
6  %
7  % Based on the covMaternisoU.m function of the GPML Toolbox -
8  %   with the following changes:
9  %       - only elements of x(:,1:end-1)/z(:,1:end-1) will be analyzed,
10 %       - x(:,end)/z(:,end) will be ignored, as it contains only the label information
11 %       - independent of the label all x values will have the same hyp
12 %       - output scaling hyperparameter is fixed to 1
13 %       - function considers additional hyperparameter theta_s for features shift
14 %         (function is limited to 1D features)
15 %       - mask parameter is a vector of size hyp and if mask(i) == 0, the
16 %         derivative of hyp(i) will be 0
17 %
18 % The covariance function is:
19 %
20 %    $k(x^p, x^q) = f(\sqrt{d} \cdot r) \cdot \exp(-\sqrt{d} \cdot r)$ 
21 %
22 % with  $f(t)=1$  for  $d=1$ ,  $f(t)=1+t$  for  $d=3$  and  $f(t)=1+t+t^2/3$  for  $d=5$ .
23 % Here  $r$  is the distance  $\sqrt{((x-\theta_s)^p - (x-\theta_s)^q)' \cdot \text{inv}(P) \cdot ((x-\theta_s)^p - (x-\theta_s)^q)}$ ,  $P$  is ell times the unit matrix.
24 % The hyperparameters are:
25 %
26 %   hyp = [ log(ell) ]
27 %           theta_s(1)
28 %           ...
29 %           theta_s(nL-1)]
30 %
31 % by Robert Duerichen
32 % 04/02/2014
33
34
35 if nargin<4, K = 'nL'; return; end % report number of parameters
36 if nargin<5, z = []; end % make sure, z exists
37 xeqz = numel(z)==0; dg = strcmp(z,'diag') && numel(z)>0; % determine mode
38
39 % check if size of mask is correct
40 if size(mask) ~= size(hyp)
41     error('Size of mask vector is not equivalent to hyperparameter vector');
42 end
43
44 % check if derivate shall be computed or not
45 if exist('i','var')
46     if mask(i) == 0
47         if xeqz % symmetric matrix Kxx
48             K = zeros(length(x));
49         else % cross covariances Kxz
50             K = zeros(length(x),length(z));
51         end
52         return; % terminate function
53     end
54 end
55
56
57 nL = max(x(:,end));
58 ell = exp(hyp(1));
59 shift = (hyp(2:end));
60 if all(d~= [1,3,5]), error('only 1, 3 and 5 allowed for d'), end % degree
61
62 switch d
63     case 1, f = @(t) 1; df = @(t) 1;
64     case 3, f = @(t) 1 + t; df = @(t) t;
65     case 5, f = @(t) 1 + t.*(1+t/3); df = @(t) t.*(1+t)/3;
66 end
67 m = @(t,f) f(t).*exp(-t); dm = @(t,f) df(t).*exp(-t);
68
69 %% perform shift
70 for ii = 2:nL
71     x(x(:,end)== ii,1) = x(x(:,end)== ii,1)+shift(ii-1);

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72     if ~isempty(z)
73         z(z(:,2)== ii,1) = z(z(:,2)== ii,1)+shift(ii-1);
74     end
75 end
76
77 % precompute distances
78 if dg % vector kxx
79     K = zeros(size(x(:,1:end-1),1),1);
80 else
81     if xeqz % symmetric matrix Kxx
82         K = sqrt( sq_dist(sqrt(d)*x(:,1)'/ell) );
83     else % cross covariances Kxz
84         K = sqrt( sq_dist(sqrt(d)*x(:,1)'/ell,sqrt(d)*z(:,1)'/ell) );
85     end
86 end
87
88 if nargin<6 % covariances
89     K = m(K,f);
90 else % derivatives
91     if i==1
92         K = K.*dm(K,f);
93     elseif i > 1 && i <= nL
94         dim = mod(i,2)+1;
95         ind_i = (x(:,2) ==i);
96         ind_ni = (x(:,2) ~=i);
97         B = zeros(length(x));
98         B(ind_ni,ind_i) = ones(sum(ind_ni),sum(ind_i));
99         B(ind_i,ind_ni) = -ones(sum(ind_i),sum(ind_ni));
100        A = repmat(x(:,dim) ,[1 length(x)]);
101        switch d
102            case 1
103                K = B.*exp(-K) ./ (ell) .*sign(A-A');
104            case 3
105                K = B.*sqrt(d) .*dm(K,f) ./ (ell) .*sign(A-A');
106            case 5
107                K = sqrt(d) .* (K.^2 +K) ./ (3*ell) .*exp(-K) .*B.*sign(A-A');
108        end
109    end
110 else
111     error('Unknown hyperparameter')
112 end
113 end

```