

I.

Fig.S 1 compares the phase trajectory of the original system (red solid line) with its 2nd–7th order Taylor approximations (colored dash-dotted lines). The horizontal axis denotes the phase angle φ , and the vertical axis denotes the angular velocity ω . Higher-order expansions show closer agreement with the original trajectory, demonstrating improved accuracy in capturing the system's nonlinear dynamics.

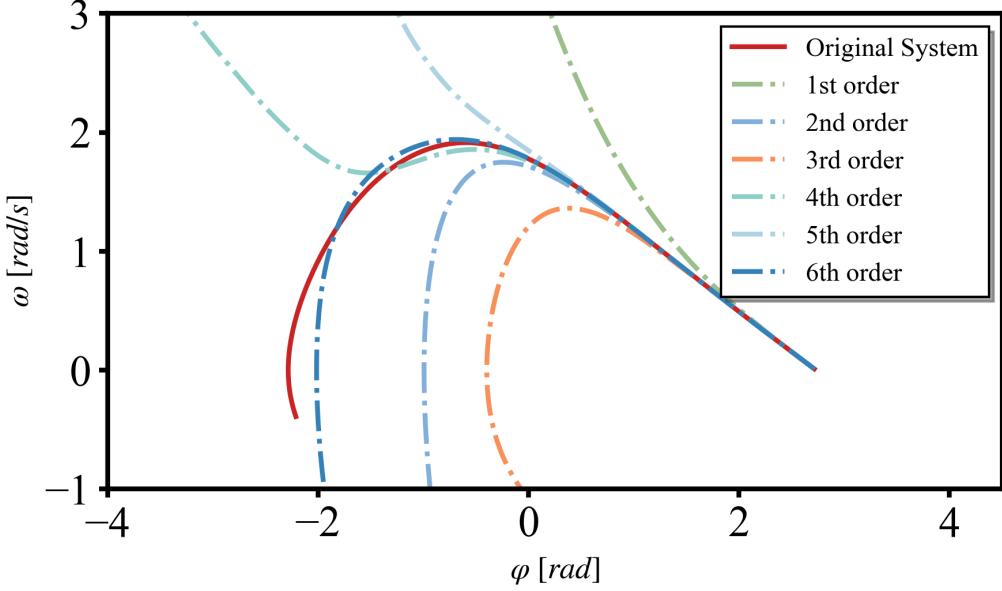


FIG. 1. Phase trajectories of the original system (red solid line) and the 2nd–7th order Taylor approximations (colored dash-dotted lines). The horizontal axis denotes φ , and the vertical axis denotes ω

II.

In this section, we apply the fifth-order RG method to the GSE. We fully describe the evolution from the original GSE to the renormalized align.

First, starting from the GSE, to solve for the manifold trajectory along the stable direction of the align starting from the saddle point, we need to replace t with $-t$, resulting in the following align:

$$\begin{cases} \dot{\varphi}(t) = -\omega(t) \\ \dot{\omega}(t) = -I + \sin(\varphi(t)) + (\alpha \cos(\varphi(t)) - D)\omega(t). \end{cases} \quad (1)$$

Perform a fifth-order Taylor expansion of the trigonometric functions in the align at the saddle point $[a_3, b_3] = [\pi - \arcsin(I), 0]$. Expanding $\sin(\varphi(t))$ at $\varphi(t) = a_3$:

$$\begin{aligned} \sin(\varphi(t)) &= \sin(a_3 + (\varphi(t) - a_3)) \\ &= \sin(\pi - \arcsin(I) + (\varphi(t) - a_3)) \\ &= I \cos(\varphi(t) - a_3) - \sqrt{1 - I^2} \sin(\varphi(t) - a_3). \end{aligned} \quad (2)$$

Expand the cosine and sine functions:

$$\begin{aligned} \cos(\varphi(t) - a_3) &= 1 - \frac{(\varphi(t) - a_3)^2}{2} + \frac{(\varphi(t) - a_3)^4}{24} + O((\varphi(t) - a_3)^6) \\ \sin(\varphi(t) - a_3) &= (\varphi(t) - a_3) - \frac{(\varphi(t) - a_3)^3}{6} + \frac{(\varphi(t) - a_3)^5}{120} + O((\varphi(t) - a_3)^7). \end{aligned} \quad (3)$$

Then,

$$\begin{aligned}\sin(\varphi(t)) &= I \left(1 - \frac{(\varphi(t) - a_3)^2}{2} + \frac{(\varphi(t) - a_3)^4}{24} \right) \\ &\quad - \sqrt{1 - I^2} \left((\varphi(t) - a_3) - \frac{(\varphi(t) - a_3)^3}{6} + \frac{(\varphi(t) - a_3)^5}{120} \right) + O((\varphi(t) - a_3)^6)\end{aligned}\quad (4)$$

$$\begin{aligned}\cos(\varphi(t)) &= -\sqrt{1 - I^2} \cos(\varphi(t) - a_3) - I \sin(\varphi(t) - a_3) \\ &= -\sqrt{1 - I^2} \left(1 - \frac{(\varphi(t) - a_3)^2}{2} + \frac{(\varphi(t) - a_3)^4}{24} \right) \\ &\quad - I \left((\varphi(t) - a_3) - \frac{(\varphi(t) - a_3)^3}{6} + \frac{(\varphi(t) - a_3)^5}{120} \right) + O((\varphi(t) - a_3)^6)\end{aligned}\quad (5)$$

Substitute the expansion into $d\omega(t)$:

$$\begin{aligned}\dot{\omega}(t) &= -I + \sin(\varphi(t)) + (\alpha \cos(\varphi(t)) - D)\omega(t) \\ &= (-\alpha \sqrt{1 - I^2} - D)\omega(t) \\ &\quad + (-\sqrt{1 - I^2} - \alpha I \omega(t))(\varphi(t) - a_3) \\ &\quad + \frac{1}{2}(-I + \alpha \sqrt{1 - I^2} \omega(t))(\varphi(t) - a_3)^2 \\ &\quad + \frac{1}{6}(\sqrt{1 - I^2} + \alpha I \omega(t))(\varphi(t) - a_3)^3 \\ &\quad + \frac{1}{24}(I - \alpha \sqrt{1 - I^2} \omega(t))(\varphi(t) - a_3)^4 \\ &\quad + \frac{1}{120}(-\sqrt{1 - I^2} - \alpha I \omega(t))(\varphi(t) - a_3)^5 + O((\varphi(t) - a_3)^6)\end{aligned}\quad (6)$$

The final result is:

$$\left\{ \begin{array}{l} \dot{\varphi}(t) = -\omega(t) \\ \dot{\omega}(t) = (-\alpha \sqrt{1 - I^2} - D)\omega(t) \\ \quad + (-\sqrt{1 - I^2} - \alpha I \omega(t))(\varphi(t) - a_3) \\ \quad + \frac{1}{2}(-I + \alpha \sqrt{1 - I^2} \omega(t))(\varphi(t) - a_3)^2 \\ \quad + \frac{1}{6}(\sqrt{1 - I^2} + \alpha I \omega(t))(\varphi(t) - a_3)^3 \\ \quad + \frac{1}{24}(I - \alpha \sqrt{1 - I^2} \omega(t))(\varphi(t) - a_3)^4 \\ \quad + \frac{1}{120}(-\sqrt{1 - I^2} - \alpha I \omega(t))(\varphi(t) - a_3)^5 \end{array} \right. \quad (7)$$

Perform coordinate transformation by substitution:

$$\begin{cases} \varphi(t) = a_1 z(t) + b_1 w(t) + a_3 \\ \omega(t) = a_2 z(t) + b_2 w(t) + b_3 \end{cases} \quad (8)$$

Among these, $[a_1, a_2]$ is the eigenvector representing the unstable direction at the saddle point, $[b_1, b_2]$ is the eigenvector representing the stable direction at the saddle point, and $[a_3, b_3]$ is the saddle point itself. Rearranging yields:

$$\begin{aligned}
\dot{w}(t) = & \frac{-1}{(-a_2 b_1 + a_1 b_2)} \left(-a_2(b_2 w(t) + a_2 z(t)) \right. \\
& + a_1 \left(- \left(-\alpha \sqrt{1 - I^2} - D \right) (b_2 w(t) + a_2 z(t)) \right. \\
& - (b_1 w(t) + a_1 z(t)) \left(-\sqrt{1 - I^2} - \alpha I(b_2 w(t) + a_2 z(t)) \right) \\
& - \frac{1}{120} (b_1 w(t) + a_1 z(t))^5 \left(-\sqrt{1 - I^2} - \alpha I(b_2 w(t) + a_2 z(t)) \right) \\
& - \frac{1}{6} (b_1 w(t) + a_1 z(t))^3 \left(\sqrt{1 - I^2} + \alpha I(b_2 w(t) + a_2 z(t)) \right) \\
& - \frac{1}{24} (b_1 w(t) + a_1 z(t))^4 \left(I - \alpha \sqrt{1 - I^2} (b_2 w(t) + a_2 z(t)) \right) \\
& \left. \left. - \frac{1}{2} (b_1 w(t) + a_1 z(t))^2 \left(-I + \alpha \sqrt{1 - I^2} (b_2 w(t) + a_2 z(t)) \right) \right) \right), \tag{9}
\end{aligned}$$

$$\begin{aligned}
\dot{z}(t) = & \frac{-1}{120(-a_2 b_1 + a_1 b_2)} \left(120 b_2^2 w(t) - 120 b_1^2 \sqrt{1 - I^2} w(t) \right. \\
& - 120 b_1 b_2 \alpha \sqrt{1 - I^2} w(t) - 120 b_1 b_2 D w(t) - 60 b_1^3 I w(t)^2 \\
& - 120 b_1^2 b_2 \alpha I w(t)^2 + 20 b_1^4 \sqrt{1 - I^2} w(t)^3 \\
& + 60 b_1^3 b_2 \alpha \sqrt{1 - I^2} w(t)^3 + 5 b_1^5 I w(t)^4 + 20 b_1^4 b_2 \alpha I w(t)^4 \\
& - b_1^6 \sqrt{1 - I^2} w(t)^5 - 5 b_1^5 b_2 \alpha \sqrt{1 - I^2} w(t)^5 \\
& - b_1^6 b_2 \alpha I w(t)^6 + 120 a_2 b_2 z[t] - 120 a_1 b_1 \sqrt{1 - I^2} z(t) \\
& - 120 a_2 b_1 \alpha \sqrt{1 - I^2} z(t) - 120 a_2 b_1 D z(t) - 120 a_1 b_1^2 I w(t) z(t) \\
& - 120 a_2 b_1^2 \alpha I w(t) z[t] - 120 a_1 b_1 b_2 \alpha I w[t] z[t] + 60 a_1 b_1^3 \sqrt{1 - I^2} w[t]^2 z[t] \\
& + 60 a_2 b_1^3 \alpha \sqrt{1 - I^2} w[t]^2 z[t] + 120 a_1 b_1^2 b_2 \alpha \sqrt{1 - I^2} w[t]^2 z[t] \\
& + 20 a_1 b_1^4 I w[t]^3 z[t] + 20 a_2 b_1^4 \alpha I w[t]^3 z[t] + 60 a_1 b_1^3 b_2 \alpha I w[t]^3 z[t] \\
& - 5 a_1 b_1^5 \sqrt{1 - I^2} w(t)^4 z(t) - 5 a_2 b_1^5 \alpha \sqrt{1 - I^2} w(t)^4 z(t) \\
& - 20 a_1 b_1^4 b_2 \alpha \sqrt{1 - I^2} w[t]^4 z[t] - a_2 b_1^6 \alpha I w(t)^5 z(t) \\
& - 5 a_1 b_1^5 b_2 \alpha I w(t)^5 z(t) - 60 a_1^2 b_1 I z[t]^2 - 120 a_1 a_2 b_1 \alpha I z(t)^2 \\
& + 60 a_1^2 b_1^2 \sqrt{1 - I^2} w(t) z(t)^2 + 120 a_1 a_2 b_1^2 \alpha \sqrt{1 - I^2} w(t) z(t)^2 \\
& + 60 a_1^2 b_1 b_2 \alpha \sqrt{1 - I^2} w(t) z(t)^2 + 30 a_1^2 b_1^3 I w(t)^2 z(t)^2 \\
& + 60 a_1 a_2 b_1^3 \alpha I w(t)^2 z(t)^2 + 60 a_1^2 b_1^2 b_2 \alpha I w(t)^2 z(t)^2 - 10 a_1^2 b_1^4 \sqrt{1 - I^2} w(t)^3 z(t)^2 \\
& - 20 a_1 a_2 b_1^4 \alpha \sqrt{1 - I^2} w[t]^3 z(t)^2 - 30 a_1^2 b_1^3 b_2 \alpha \sqrt{1 - I^2} w(t)^3 z(t)^2 \\
& - 5 a_1 a_2 b_1^5 \alpha I w[t]^4 z[t]^2 - 10 a_1^2 b_1^4 b_2 \alpha I w(t)^4 z(t)^2 \\
& + 20 a_1^3 b_1 I \sqrt{1 - I^2} z(t)^3 + 60 a_1^2 a_2 b_1 \alpha \sqrt{1 - I^2} z(t)^3 \\
& + 20 a_1^3 b_1^2 I w(t) z(t)^3 + 60 a_1^2 a_2 b_1^2 \alpha I w(t) z(t)^3 \\
& + 20 a_1^3 b_1 b_2 \alpha I w(t) z(t)^3 - 10 a_1^3 b_1^3 \sqrt{1 - I^2} w(t)^2 z(t)^3 \\
& - 30 a_1^2 a_2 b_1^3 \alpha \sqrt{1 - I^2} w(t)^2 z(t)^3 - 20 a_1^3 b_1^2 b_2 \alpha \sqrt{1 - I^2} w(t)^2 z(t)^3 \\
& - 10 a_1^2 a_2 b_1^4 \alpha I w(t)^3 z(t)^3 - 10 a_1^3 b_1^3 b_2 \alpha I w(t)^3 z(t)^3 + 5 a_1^4 b_1 I \\
& \beta z[t]^4 + 20 a_1^3 a_2 b_1 \alpha I z(t)^4 - 5 a_1^4 b_1^2 \sqrt{1 - I^2} w[t] z(t)^4 \\
& - 20 a_1^3 a_2 b_1^2 \alpha \sqrt{1 - I^2} w(t) z(t)^4 - 5 a_1^4 b_1 b_2 \alpha \sqrt{1 - I^2} w(t) z(t)^4 \\
& - 10 a_1^3 a_2 b_1^3 \alpha I w(t)^2 z(t)^4 - 5 a_1^4 b_1^2 b_2 \alpha I w(t)^2 z(t)^4 - a_1^5 b_1 \\
& \sqrt{1 - I^2} z(t)^5 - 5 a_1^4 a_2 b_1 \alpha \sqrt{1 - I^2} z(t)^5 - 5 a_1^4 a_2 b_1^2 \alpha I w(t) z(t)^5 \\
& - a_1^5 b_1 b_2 \alpha I w[t] z(t)^5 - a_1^5 a_2 b_1 \alpha I z[t]^6 \Big) \tag{10}
\end{aligned}$$

Now we proceed with the perturbation expansion.

$$z(t) = \varepsilon z_1(t) + \varepsilon^2 z_2(t) + \varepsilon^3 z_3(t) + \varepsilon^4 z_4(t) + \varepsilon^5 z_5(t), \quad (11)$$

$$w(t) = \varepsilon w_1(t) + \varepsilon^2 w_2(t) + \varepsilon^3 w_3(t) + \varepsilon^4 w_4(t) + \varepsilon^5 w_5(t), \quad (12)$$

The derivation of the first-order equation is as follows:

$$\begin{aligned} \dot{z}_1(t) &= \left(-\frac{b_2^2}{-a_2 b_1 + a_1 b_2} + \frac{b_1^2 \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} + \frac{b_1 b_2 \alpha \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} + \frac{b_1 b_2 D}{-a_2 b_1 + a_1 b_2} \right) w_1(t) \\ &\quad + \left(-\frac{a_2 b_2}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1 \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1 \alpha \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1 D}{-a_2 b_1 + a_1 b_2} \right) z_1(t) \end{aligned} \quad (13)$$

$$\begin{aligned} \dot{w}_1(t) &= \left(-\frac{a_2 b_2}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_1 \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_2 \alpha \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_2 D}{-a_2 b_1 + a_1 b_2} \right) w_1(t) \\ &\quad + \left(-\frac{a_2^2}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} - \frac{a_1 a_2 \alpha \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} - \frac{a_1 a_2 D}{-a_2 b_1 + a_1 b_2} \right) z_1(t) \end{aligned} \quad (14)$$

It can be simply stated as

$$\begin{cases} \dot{z}_1(t) = A \cdot z_1(t) + B \cdot w_1(t) \\ \dot{w}_1(t) = C \cdot z_1(t) + D \cdot w_1(t) \end{cases}$$

In this case, regardless of the values of the original three parameters, B and C are both zero. Therefore, the first-order equations are two decoupled equations. Consequently, they can be simplified to:

$$\dot{z}_1(t) = \left(-\frac{a_2 b_2}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1 \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1 \alpha \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1 D}{-a_2 b_1 + a_1 b_2} \right) z_1(t) \quad (15)$$

$$\dot{w}_1(t) = \left(-\frac{a_2 b_2}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_1 \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_2 \alpha \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_2 D}{-a_2 b_1 + a_1 b_2} \right) w_1(t) \quad (16)$$

The result is directly calculated as:

$$w_1(t) = b(t_0) e^{-1.37037t + 1.37037t_0} \quad (17)$$

$$z_1(t) = a(t_0) e^{0.668809t - 0.668809t_0} \quad (18)$$

The derivation of the second-order formula is as follows:

$$\begin{aligned}
\dot{z}_2(t) &= \frac{b_1^3 I w_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} + \frac{b_1^2 b_2 \alpha I w_1(t)^2}{-a_2 b_1 + a_1 b_2} \\
&\quad + \left(-\frac{b_2^2}{-a_2 b_1 + a_1 b_2} + \frac{b_1^2 \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} + \frac{b_1 b_2 \alpha \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} + \frac{b_1 b_2 D}{-a_2 b_1 + a_1 b_2} \right) w_2(t) \\
&\quad + \frac{a_1 b_1^2 I w_1(t) z_1(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1^2 \alpha I w_1(t) z_1(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1 b_2 \alpha I w_1(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
&\quad + \frac{a_1^2 b_1 I z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} + \frac{a_1 a_2 b_1 \alpha I z_1(t)^2}{-a_2 b_1 + a_1 b_2} \\
&\quad + \left(-\frac{a_2 b_2}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1 \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1 \alpha \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1 D}{-a_2 b_1 + a_1 b_2} \right) z_2(t) \\
\dot{w}_2(t) &= -\frac{a_1 b_1^2 I w_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_1 b_1 b_2 \alpha I w_1(t)^2}{-a_2 b_1 + a_1 b_2} \\
&\quad + \left(\frac{a_2 b_2}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_1 \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_2 \alpha \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_2 D}{-a_2 b_1 + a_1 b_2} \right) w_2(t) \\
&\quad - \frac{a_1^2 b_1 I w_1(t) z_1(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1 a_2 b_1 \alpha I w_1(t) z_1(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_2 \alpha I w_1(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
&\quad - \frac{a_1^3 I z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_1^2 a_2 \alpha I z_1(t)^2}{-a_2 b_1 + a_1 b_2} \\
&\quad + \left(\frac{a_2^2}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} - \frac{a_1 a_2 \alpha \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} - \frac{a_1 a_2 D}{-a_2 b_1 + a_1 b_2} \right) z_2(t)
\end{aligned} \tag{19}$$

In particular, the coefficients preceding mutually coupled terms remain zero regardless of parameter variations. Simplification yields:

$$\begin{aligned}
\dot{z}_2(t) &= \frac{b_1^3 I w_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} + \frac{b_1^2 b_2 \alpha I w_1(t)^2}{-a_2 b_1 + a_1 b_2} \\
&\quad + \frac{a_1 b_1^2 I w_1(t) z_1(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1^2 \alpha I w_1(t) z_1(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1 b_2 \alpha I w_1(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
&\quad + \frac{a_1^2 b_1 I z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} + \frac{a_1 a_2 b_1 \alpha I z_1(t)^2}{-a_2 b_1 + a_1 b_2} \\
&\quad + \left(-\frac{a_2 b_2}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1 \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1 \alpha \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1 D}{-a_2 b_1 + a_1 b_2} \right) z_2(t) \\
\dot{w}_2(t) &= -\frac{a_1 b_1^2 I w_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_1 b_1 b_2 \alpha I w_1(t)^2}{-a_2 b_1 + a_1 b_2} \\
&\quad + \left(\frac{a_2 b_2}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_1 \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_2 \alpha \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_2 D}{-a_2 b_1 + a_1 b_2} \right) w_2(t) \\
&\quad - \frac{a_1^2 b_1 I w_1(t) z_1(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1 a_2 b_1 \alpha I w_1(t) z_1(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_2 \alpha I w_1(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
&\quad - \frac{a_1^3 I z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_1^2 a_2 \alpha I z_1(t)^2}{-a_2 b_1 + a_1 b_2}
\end{aligned} \tag{20}$$

Solving the above equation yields:

$$\begin{aligned}
z2 &= -0.013960138773144507 a^2 e^{(1.3376176263148816 t - 1.3376176263148816 t_0)} \\
&\quad + 0.013960138773144507 a^2 e^{(0.6688088131574408 t - 0.6688088131574408 t_0)}
\end{aligned} \tag{21}$$

$$w2 = -0.00706449 a^2 e^{1.33762 t - 1.33762 t_0}$$

The derivation of the third-order term is as follows:

$$\begin{aligned}
\dot{z}_3(t) = & -\frac{b_1^4 \sqrt{1 - I^2} w_1(t)^3}{6(-a_2 b_1 + a_1 b_2)} - \frac{b_1^3 b_2 \alpha \sqrt{1 - I^2} w_1(t)^3}{2(-a_2 b_1 + a_1 b_2)} + \frac{b_1^3 I w_1(t) w_2(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{2b_1^2 b_2 \alpha I w_1(t) w_2(t)}{-a_2 b_1 + a_1 b_2} + \left(-\frac{b_2^2}{-a_2 b_1 + a_1 b_2} + \frac{b_1^2 \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} \right. \\
& \left. + \frac{b_1 b_2 \alpha \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} + \frac{b_1 b_2 D}{-a_2 b_1 + a_1 b_2} \right) w_3(t) \\
& - \frac{a_1 b_1^3 \sqrt{1 - I^2} w_1(t)^2 z_1(t)}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_2 b_1^3 \alpha \sqrt{1 - I^2} w_1(t)^2 z_1(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 b_1^2 b_2 \alpha \sqrt{1 - I^2} w_1(t)^2 z_1(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1^2 I w_2(t) z_1(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1^2 \alpha I w_2(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1 b_1 b_2 \alpha I w_2(t) z_1(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_1^2 \sqrt{1 - I^2} w_1(t) z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 a_2 b_1^2 \alpha \sqrt{1 - I^2} w_1(t) z_1(t)^2}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_1 b_2 \alpha \sqrt{1 - I^2} w_1(t) z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1^3 b_1 \sqrt{1 - I^2} z_1(t)^3}{6(-a_2 b_1 + a_1 b_2)} - \frac{a_1^2 a_2 b_1 \alpha \sqrt{1 - I^2} z_1(t)^3}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1 b_1^2 I w_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1^2 \alpha I w_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1 b_2 \alpha I w_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1^2 b_1 I z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} + \frac{2a_1 a_2 b_1 \alpha I z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& + \left(-\frac{a_2 b_2}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1 \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1 \alpha \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1 D}{-a_2 b_1 + a_1 b_2} \right) z_3(t)
\end{aligned} \tag{22}$$

$$\begin{aligned}
\dot{w}_3(t) = & \frac{a_1 b_1^3 \sqrt{1 - I^2} w_1(t)^3}{6(-a_2 b_1 + a_1 b_2)} + \frac{a_1 b_1^2 b_2 \alpha \sqrt{1 - I^2} w_1(t)^3}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_1 b_1^2 I w_1(t) w_2(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{2a_1 b_1 b_2 \alpha I w_1(t) w_2(t)}{-a_2 b_1 + a_1 b_2} + \left(\frac{a_2 b_2}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_1 \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} \right. \\
& \left. - \frac{a_1 b_2 \alpha \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_2 D}{-a_2 b_1 + a_1 b_2} \right) w_3(t) \\
& + \frac{a_1^2 b_1^2 \sqrt{1 - I^2} w_1(t)^2 z_1(t)}{2(-a_2 b_1 + a_1 b_2)} + \frac{a_1 a_2 b_1^2 \alpha \sqrt{1 - I^2} w_1(t)^2 z_1(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 b_1 b_2 \alpha \sqrt{1 - I^2} w_1(t)^2 z_1(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_1 I w_2(t) z_1(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1 a_2 b_1 \alpha I w_2(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^2 b_2 \alpha I w_2(t) z_1(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1^3 b_1 \sqrt{1 - I^2} w_1(t) z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 a_2 b_1 \alpha \sqrt{1 - I^2} w_1(t) z_1(t)^2}{-a_2 b_1 + a_1 b_2} + \frac{a_1^3 b_2 \alpha \sqrt{1 - I^2} w_1(t) z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^4 \sqrt{1 - I^2} z_1(t)^3}{6(-a_2 b_1 + a_1 b_2)} + \frac{a_1^3 a_2 \alpha \sqrt{1 - I^2} z_1(t)^3}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1^2 b_1 I w_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1 a_2 b_1 \alpha I w_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_2 \alpha I w_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^3 I z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} - \frac{2a_1^2 a_2 \alpha I z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& + \left(\frac{a_2^2}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} - \frac{a_1 a_2 \alpha \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} - \frac{a_1 a_2 D}{-a_2 b_1 + a_1 b_2} \right) z_3(t)
\end{aligned} \tag{23}$$

Similarly, simplifying the coupling terms yields:

$$\begin{aligned}
\dot{z}_3(t) = & -\frac{b_1^4 \sqrt{1-I^2} w_1(t)^3}{6(-a_2 b_1 + a_1 b_2)} - \frac{b_1^3 b_2 \alpha \sqrt{1-I^2} w_1(t)^3}{2(-a_2 b_1 + a_1 b_2)} + \frac{b_1^3 I w_1(t) w_2(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{2 b_1^2 b_2 \alpha I w_1(t) w_2(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1 b_1^3 \sqrt{1-I^2} w_1(t)^2 z_1(t)}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_2 b_1^3 \alpha \sqrt{1-I^2} w_1(t)^2 z_1(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 b_1^2 b_2 \alpha \sqrt{1-I^2} w_1(t)^2 z_1(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1^2 I w_2(t) z_1(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1^2 \alpha I w_2(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1 b_1 b_2 \alpha I w_2(t) z_1(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_1^2 \sqrt{1-I^2} w_1(t) z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 a_2 b_1^2 \alpha \sqrt{1-I^2} w_1(t) z_1(t)^2}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_1 b_2 \alpha \sqrt{1-I^2} w_1(t) z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1^3 b_1 \sqrt{1-I^2} z_1(t)^3}{6(-a_2 b_1 + a_1 b_2)} - \frac{a_1^2 a_2 b_1 \alpha \sqrt{1-I^2} z_1(t)^3}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1 b_1^2 I w_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1^2 \alpha I w_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1 b_2 \alpha I w_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1^2 b_1 I z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} + \frac{2 a_1 a_2 b_1 \alpha I z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& + \left(-\frac{a_2 b_2}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1 \sqrt{1-I^2}}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1 \alpha \sqrt{1-I^2}}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1 D}{-a_2 b_1 + a_1 b_2} \right) z_3(t)
\end{aligned} \tag{24}$$

$$\begin{aligned}
\dot{w}_3(t) = & \frac{a_1 b_1^3 \sqrt{1-I^2} w_1(t)^3}{6(-a_2 b_1 + a_1 b_2)} + \frac{a_1 b_1^2 b_2 \alpha \sqrt{1-I^2} w_1(t)^3}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_1 b_1^2 I w_1(t) w_2(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{2 a_1 b_1 b_2 \alpha I w_1(t) w_2(t)}{-a_2 b_1 + a_1 b_2} + \left(\frac{a_2 b_2}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_1 \sqrt{1-I^2}}{-a_2 b_1 + a_1 b_2} \right. \\
& \left. - \frac{a_1 b_2 \alpha \sqrt{1-I^2}}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_2 D}{-a_2 b_1 + a_1 b_2} \right) w_3(t) \\
& + \frac{a_1^2 b_1^2 \sqrt{1-I^2} w_1(t)^2 z_1(t)}{2(-a_2 b_1 + a_1 b_2)} + \frac{a_1 a_2 b_1^2 \alpha \sqrt{1-I^2} w_1(t)^2 z_1(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 b_1 b_2 \alpha \sqrt{1-I^2} w_1(t)^2 z_1(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_1 I w_2(t) z_1(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1 a_2 b_1 \alpha I w_2(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^2 b_2 \alpha I w_2(t) z_1(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1^3 b_1 \sqrt{1-I^2} w_1(t) z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 a_2 b_1 \alpha \sqrt{1-I^2} w_1(t) z_1(t)^2}{-a_2 b_1 + a_1 b_2} + \frac{a_1^3 b_2 \alpha \sqrt{1-I^2} w_1(t) z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^4 \sqrt{1-I^2} z_1(t)^3}{6(-a_2 b_1 + a_1 b_2)} + \frac{a_1^3 a_2 \alpha \sqrt{1-I^2} z_1(t)^3}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1^2 b_1 I w_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1 a_2 b_1 \alpha I w_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_2 \alpha I w_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^3 I z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} - \frac{2 a_1^2 a_2 \alpha I z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2}
\end{aligned} \tag{25}$$

Solving the above equation yields:

$$\begin{aligned}
z_3 = & 0.04971 a^3 e^{2.00643t - 2.00643t_0} \\
& - 0.000389771 a^3 e^{1.33762t - 1.33762t_0} \\
& - 0.0493203 a^3 e^{0.668809t - 0.668809t_0}
\end{aligned} \tag{26}$$

$$\begin{aligned} w_3 = & 0.0403466a^3 e^{2.00643t - 2.00643t_0} \\ & - 0.000197243a^3 e^{1.33762t - 1.33762t_0} \end{aligned} \quad (27)$$

The derivation of the fourth-order term is as follows:

$$\begin{aligned}
\dot{z}_4(t) = & -\frac{b_1^5 I w_1(t)^4}{24(-a_2 b_1 + a_1 b_2)} - \frac{b_1^4 b_2 \alpha I w_1(t)^4}{6(-a_2 b_1 + a_1 b_2)} \\
& - \frac{b_1^4 \sqrt{1 - I^2} w_1(t)^2 w_2(t)}{2(-a_2 b_1 + a_1 b_2)} - \frac{3b_1^3 b_2 \alpha \sqrt{1 - I^2} w_1(t)^2 w_2(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{b_1^3 I w_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} + \frac{b_1^2 b_2 \alpha I w_2(t)^2}{-a_2 b_1 + a_1 b_2} \\
& + \frac{b_1^3 I w_1(t) w_3(t)}{-a_2 b_1 + a_1 b_2} + \frac{2b_1^2 b_2 \alpha I w_1(t) w_3(t)}{-a_2 b_1 + a_1 b_2} \\
& + \left(-\frac{b_2^2}{-a_2 b_1 + a_1 b_2} + \frac{b_1^2 \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} + \frac{b_1 b_2 \alpha \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} + \frac{b_1 b_2 D}{-a_2 b_1 + a_1 b_2} \right) w_4(t) \\
& - \frac{a_1 b_1^4 I w_1(t)^3 z_1(t)}{6(-a_2 b_1 + a_1 b_2)} - \frac{a_2 b_1^4 \alpha I w_1(t)^3 z_1(t)}{6(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 b_1^3 b_2 \alpha I w_1(t)^3 z_1(t)}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_1 b_1^3 \sqrt{1 - I^2} w_1(t) w_2(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_2 b_1^3 \alpha \sqrt{1 - I^2} w_1(t) w_2(t) z_1(t)}{-a_2 b_1 + a_1 b_2} - \frac{2a_1 b_1^2 b_2 \alpha \sqrt{1 - I^2} w_1(t) w_2(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1 b_1^2 I w_3(t) z_1(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1^2 \alpha I w_3(t) z_1(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1 b_2 \alpha I w_3(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^2 b_1^3 I w_1(t)^2 z_1(t)^2}{4(-a_2 b_1 + a_1 b_2)} - \frac{a_1 a_2 b_1^3 \alpha I w_1(t)^2 z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1^2 b_1^2 b_2 \alpha I w_1(t)^2 z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_1^2 b_1^2 \sqrt{1 - I^2} w_2(t) z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 a_2 b_1^2 \alpha \sqrt{1 - I^2} w_2(t) z_1(t)^2}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_1 b_2 \alpha \sqrt{1 - I^2} w_2(t) z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1^3 b_1^2 I w_1(t) z_1(t)^3}{6(-a_2 b_1 + a_1 b_2)} - \frac{a_1^2 a_2 b_1^2 \alpha I w_1(t) z_1(t)^3}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1^3 b_1 b_2 \alpha I w_1(t) z_1(t)^3}{6(-a_2 b_1 + a_1 b_2)} - \frac{a_1^4 b_1 I z_1(t)^4}{24(-a_2 b_1 + a_1 b_2)} - \frac{a_1^3 a_2 b_1 \alpha I z_1(t)^4}{6(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 b_1^3 \sqrt{1 - I^2} w_1(t)^2 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_2 b_1^3 \alpha \sqrt{1 - I^2} w_1(t)^2 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 b_1^2 b_2 \alpha \sqrt{1 - I^2} w_1(t)^2 z_2(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1^2 I w_2(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_2 b_1^2 \alpha I w_2(t) z_2(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1 b_2 \alpha I w_2(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^2 b_1^2 \sqrt{1 - I^2} w_1(t) z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} - \frac{2a_1 a_2 b_1^2 \alpha \sqrt{1 - I^2} w_1(t) z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^2 b_1 b_2 \alpha \sqrt{1 - I^2} w_1(t) z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^3 b_1 \sqrt{1 - I^2} z_1(t)^2 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{3a_1^2 a_2 b_1 \alpha \sqrt{1 - I^2} z_1(t)^2 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} + \frac{a_1^2 b_1 I z_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1 a_2 b_1 \alpha I z_2(t)^2}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1^2 I w_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_2 b_1^2 \alpha I w_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1 b_2 \alpha I w_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1^2 b_1 I z_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} + \frac{2a_1 a_2 b_1 \alpha I z_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} \\
& + \left(-\frac{a_2 b_2}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1 \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1 \alpha \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1 D}{-a_2 b_1 + a_1 b_2} \right) z_4(t)
\end{aligned} \tag{28}$$

$$\begin{aligned}
\dot{w}_4(t) = & \frac{a_1 b_1^4 I w_1(t)^4}{24(-a_2 b_1 + a_1 b_2)} + \frac{a_1 b_1^3 b_2 \alpha I w_1(t)^4}{6(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1 b_1^3 \sqrt{1 - I^2} w_1(t)^2 w_2(t)}{2(-a_2 b_1 + a_1 b_2)} + \frac{3 a_1 b_1^2 b_2 \alpha \sqrt{1 - I^2} w_1(t)^2 w_2(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 b_1^2 I w_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_1 b_1 b_2 \alpha I w_2(t)^2}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1 b_1^2 I w_1(t) w_3(t)}{-a_2 b_1 + a_1 b_2} - \frac{2 a_1 b_1 b_2 \alpha I w_1(t) w_3(t)}{-a_2 b_1 + a_1 b_2} \\
& + \left(\frac{a_2 b_2}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_1 \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_2 \alpha \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_2 D}{-a_2 b_1 + a_1 b_2} \right) w_4(t) \\
& + \frac{a_1^2 b_1^3 I w_1(t)^3 z_1(t)}{6(-a_2 b_1 + a_1 b_2)} + \frac{a_1 a_2 b_1^3 \alpha I w_1(t)^3 z_1(t)}{6(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 b_1^2 b_2 \alpha I w_1(t)^3 z_1(t)}{2(-a_2 b_1 + a_1 b_2)} + \frac{a_1^2 b_1^2 \sqrt{1 - I^2} w_1(t) w_2(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1 a_2 b_1^2 \alpha \sqrt{1 - I^2} w_1(t) w_2(t) z_1(t)}{-a_2 b_1 + a_1 b_2} + \frac{2 a_1^2 b_1 b_2 \alpha \sqrt{1 - I^2} w_1(t) w_2(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^2 b_1 I w_3(t) z_1(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1 a_2 b_1 \alpha I w_3(t) z_1(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_2 \alpha I w_3(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1^3 b_1^2 I w_1(t)^2 z_1(t)^2}{4(-a_2 b_1 + a_1 b_2)} + \frac{a_1^2 a_2 b_1^2 \alpha I w_1(t)^2 z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^3 b_1 b_2 \alpha I w_1(t)^2 z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} + \frac{a_1^3 b_1 \sqrt{1 - I^2} w_2(t) z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 a_2 b_1 \alpha \sqrt{1 - I^2} w_2(t) z_1(t)^2}{-a_2 b_1 + a_1 b_2} + \frac{a_1^3 b_2 \alpha \sqrt{1 - I^2} w_2(t) z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^4 b_1 I w_1(t) z_1(t)^3}{6(-a_2 b_1 + a_1 b_2)} + \frac{a_1^3 a_2 b_1 \alpha I w_1(t) z_1(t)^3}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^4 b_2 \alpha I w_1(t) z_1(t)^3}{6(-a_2 b_1 + a_1 b_2)} + \frac{a_1^5 I z_1(t)^4}{24(-a_2 b_1 + a_1 b_2)} + \frac{a_1^4 a_2 \alpha I z_1(t)^4}{6(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 b_1^2 \sqrt{1 - I^2} w_1(t)^2 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} + \frac{a_1 a_2 b_1^2 \alpha \sqrt{1 - I^2} w_1(t)^2 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 b_1 b_2 \alpha \sqrt{1 - I^2} w_1(t)^2 z_2(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_1 I w_2(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1 a_2 b_1 \alpha I w_2(t) z_2(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_2 \alpha I w_2(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1^3 b_1 \sqrt{1 - I^2} w_1(t) z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} + \frac{2 a_1^2 a_2 b_1 \alpha \sqrt{1 - I^2} w_1(t) z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1^3 b_2 \alpha \sqrt{1 - I^2} w_1(t) z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1^4 \sqrt{1 - I^2} z_1(t)^2 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{3 a_1^3 a_2 \alpha \sqrt{1 - I^2} z_1(t)^2 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_1^3 I z_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1^2 a_2 \alpha I z_2(t)^2}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_1 I w_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1 a_2 b_1 \alpha I w_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_2 \alpha I w_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^3 I z_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} - \frac{2 a_1^2 a_2 \alpha I z_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} \\
& + \left(\frac{a_2^2}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} - \frac{a_1 a_2 \alpha \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} - \frac{a_1 a_2 D}{-a_2 b_1 + a_1 b_2} \right) z_4(t)
\end{aligned} \tag{29}$$

Processing the coupling terms yields the simplified expression:

$$\begin{aligned}
\dot{z}_4(t) = & -\frac{b_1^5 I w_1(t)^4}{24(-a_2 b_1 + a_1 b_2)} - \frac{b_1^4 b_2 \alpha I w_1(t)^4}{6(-a_2 b_1 + a_1 b_2)} \\
& - \frac{b_1^4 \sqrt{1 - I^2} w_1(t)^2 w_2(t)}{2(-a_2 b_1 + a_1 b_2)} - \frac{3b_1^3 b_2 \alpha \sqrt{1 - I^2} w_1(t)^2 w_2(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{b_1^3 I w_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} + \frac{b_1^2 b_2 \alpha I w_2(t)^2}{-a_2 b_1 + a_1 b_2} \\
& + \frac{b_1^3 I w_1(t) w_3(t)}{-a_2 b_1 + a_1 b_2} + \frac{2b_1^2 b_2 \alpha I w_1(t) w_3(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1 b_1^4 I w_1(t)^3 z_1(t)}{6(-a_2 b_1 + a_1 b_2)} - \frac{a_2 b_1^4 \alpha I w_1(t)^3 z_1(t)}{6(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 b_1^3 b_2 \alpha I w_1(t)^3 z_1(t)}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_1 b_1^3 \sqrt{1 - I^2} w_1(t) w_2(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_2 b_1^3 \alpha \sqrt{1 - I^2} w_1(t) w_2(t) z_1(t)}{-a_2 b_1 + a_1 b_2} - \frac{2a_1 b_1^2 b_2 \alpha \sqrt{1 - I^2} w_1(t) w_2(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1 b_1^2 I w_3(t) z_1(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1^2 \alpha I w_3(t) z_1(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1 b_2 \alpha I w_3(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^2 b_1^3 I w_1(t)^2 z_1(t)^2}{4(-a_2 b_1 + a_1 b_2)} - \frac{a_1 a_2 b_1^3 \alpha I w_1(t)^2 z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1^2 b_1^2 b_2 \alpha I w_1(t)^2 z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_1^2 b_1^2 \sqrt{1 - I^2} w_2(t) z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 a_2 b_1^2 \alpha \sqrt{1 - I^2} w_2(t) z_1(t)^2}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_1 b_2 \alpha \sqrt{1 - I^2} w_2(t) z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1^3 b_1^2 I w_1(t) z_1(t)^3}{6(-a_2 b_1 + a_1 b_2)} - \frac{a_1^2 a_2 b_1^2 \alpha I w_1(t) z_1(t)^3}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1^3 b_1 b_2 \alpha I w_1(t) z_1(t)^3}{6(-a_2 b_1 + a_1 b_2)} - \frac{a_1^4 b_1 I z_1(t)^4}{24(-a_2 b_1 + a_1 b_2)} - \frac{a_1^3 a_2 b_1 \alpha I z_1(t)^4}{6(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 b_1^3 \sqrt{1 - I^2} w_1(t)^2 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_2 b_1^3 \alpha \sqrt{1 - I^2} w_1(t)^2 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 b_1^2 b_2 \alpha \sqrt{1 - I^2} w_1(t)^2 z_2(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1^2 I w_2(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_2 b_1^2 \alpha I w_2(t) z_2(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1 b_2 \alpha I w_2(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^2 b_1^2 \sqrt{1 - I^2} w_1(t) z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} - \frac{2a_1 a_2 b_1^2 \alpha \sqrt{1 - I^2} w_1(t) z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^2 b_1 b_2 \alpha \sqrt{1 - I^2} w_1(t) z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^3 b_1 \sqrt{1 - I^2} z_1(t)^2 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{3a_1^2 a_2 b_1 \alpha \sqrt{1 - I^2} z_1(t)^2 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} + \frac{a_1^2 b_1 I z_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1 a_2 b_1 \alpha I z_2(t)^2}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1^2 I w_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_2 b_1^2 \alpha I w_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1 b_2 \alpha I w_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1^2 b_1 I z_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} + \frac{2a_1 a_2 b_1 \alpha I z_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} \\
& + \left(-\frac{a_2 b_2}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1 \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1 \alpha \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1 D}{-a_2 b_1 + a_1 b_2} \right) z_4(t)
\end{aligned} \tag{30}$$

$$\begin{aligned}
\dot{w}_4(t) = & \frac{a_1 b_1^4 I w_1(t)^4}{24(-a_2 b_1 + a_1 b_2)} + \frac{a_1 b_1^3 b_2 \alpha I w_1(t)^4}{6(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1 b_1^3 \sqrt{1 - I^2} w_1(t)^2 w_2(t)}{2(-a_2 b_1 + a_1 b_2)} + \frac{3 a_1 b_1^2 b_2 \alpha \sqrt{1 - I^2} w_1(t)^2 w_2(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 b_1^2 I w_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_1 b_1 b_2 \alpha I w_2(t)^2}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1 b_1^2 I w_1(t) w_3(t)}{-a_2 b_1 + a_1 b_2} - \frac{2 a_1 b_1 b_2 \alpha I w_1(t) w_3(t)}{-a_2 b_1 + a_1 b_2} \\
& + \left(\frac{a_2 b_2}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_1 \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_2 \alpha \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_2 D}{-a_2 b_1 + a_1 b_2} \right) w_4(t) \\
& + \frac{a_1^2 b_1^3 I w_1(t)^3 z_1(t)}{6(-a_2 b_1 + a_1 b_2)} + \frac{a_1 a_2 b_1^3 \alpha I w_1(t)^3 z_1(t)}{6(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 b_1^2 b_2 \alpha I w_1(t)^3 z_1(t)}{2(-a_2 b_1 + a_1 b_2)} + \frac{a_1^2 b_1^2 \sqrt{1 - I^2} w_1(t) w_2(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1 a_2 b_1^2 \alpha \sqrt{1 - I^2} w_1(t) w_2(t) z_1(t)}{-a_2 b_1 + a_1 b_2} + \frac{2 a_1^2 b_1 b_2 \alpha \sqrt{1 - I^2} w_1(t) w_2(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^2 b_1 I w_3(t) z_1(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1 a_2 b_1 \alpha I w_3(t) z_1(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_2 \alpha I w_3(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1^3 b_1^2 I w_1(t)^2 z_1(t)^2}{4(-a_2 b_1 + a_1 b_2)} + \frac{a_1^2 a_2 b_1^2 \alpha I w_1(t)^2 z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^3 b_1 b_2 \alpha I w_1(t)^2 z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} + \frac{a_1^3 b_1 \sqrt{1 - I^2} w_2(t) z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 a_2 b_1 \alpha \sqrt{1 - I^2} w_2(t) z_1(t)^2}{-a_2 b_1 + a_1 b_2} + \frac{a_1^3 b_2 \alpha \sqrt{1 - I^2} w_2(t) z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^4 b_1 I w_1(t) z_1(t)^3}{6(-a_2 b_1 + a_1 b_2)} + \frac{a_1^3 a_2 b_1 \alpha I w_1(t) z_1(t)^3}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^4 b_2 \alpha I w_1(t) z_1(t)^3}{6(-a_2 b_1 + a_1 b_2)} + \frac{a_1^5 I z_1(t)^4}{24(-a_2 b_1 + a_1 b_2)} + \frac{a_1^4 a_2 \alpha I z_1(t)^4}{6(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 b_1^2 \sqrt{1 - I^2} w_1(t)^2 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} + \frac{a_1 a_2 b_1^2 \alpha \sqrt{1 - I^2} w_1(t)^2 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 b_1 b_2 \alpha \sqrt{1 - I^2} w_1(t)^2 z_2(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_1 I w_2(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1 a_2 b_1 \alpha I w_2(t) z_2(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_2 \alpha I w_2(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1^3 b_1 \sqrt{1 - I^2} w_1(t) z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} + \frac{2 a_1^2 a_2 b_1 \alpha \sqrt{1 - I^2} w_1(t) z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1^3 b_2 \alpha \sqrt{1 - I^2} w_1(t) z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1^4 \sqrt{1 - I^2} z_1(t)^2 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{3 a_1^3 a_2 \alpha \sqrt{1 - I^2} z_1(t)^2 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_1^3 I z_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1^2 a_2 \alpha I z_2(t)^2}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_1 I w_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1 a_2 b_1 \alpha I w_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_2 \alpha I w_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^3 I z_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} - \frac{2 a_1^2 a_2 \alpha I z_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2}
\end{aligned} \tag{31}$$

Solving the above equation yields:

$$\begin{aligned}
w_4 = & -0.0105108a^4 e^{2.67524t - 2.67524t_0} \\
& + 0.00168973a^4 e^{2.00643t - 2.00643t_0} \\
& + 0.000695468a^4 e^{1.33762t - 1.33762t_0}
\end{aligned} \tag{32}$$

$$\begin{aligned}
z_4 = & -0.0103433a^4 e^{2.67524t - 2.67524t_0} \\
& + 0.00208188a^4 e^{2.00643t - 2.00643t_0} \\
& + 0.00137432a^4 e^{1.33762t - 1.33762t_0} \\
& + 0.00688712a^4 e^{0.668809t - 0.668809t_0}
\end{aligned} \tag{33}$$

The derivation of the fifth-order term is as follows:

$$\begin{aligned}
\dot{z}_5(t) = & \frac{b_1^6 \sqrt{1 - I^2} w_1(t)^5}{120(-a_2 b_1 + a_1 b_2)} + \frac{b_1^5 b_2 \alpha \sqrt{1 - I^2} w_1(t)^5}{24(-a_2 b_1 + a_1 b_2)} \\
& - \frac{b_1^5 I w_1(t)^3 w_2(t)}{6(-a_2 b_1 + a_1 b_2)} - \frac{2b_1^4 b_2 \alpha I w_1(t)^3 w_2(t)}{3(-a_2 b_1 + a_1 b_2)} \\
& - \frac{b_1^4 \sqrt{1 - I^2} w_1(t) w_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} - \frac{3b_1^3 b_2 \alpha \sqrt{1 - I^2} w_1(t) w_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{b_1^4 \sqrt{1 - I^2} w_1(t)^2 w_3(t)}{2(-a_2 b_1 + a_1 b_2)} - \frac{3b_1^3 b_2 \alpha \sqrt{1 - I^2} w_1(t)^2 w_3(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{b_1^3 I w_2(t) w_3(t)}{-a_2 b_1 + a_1 b_2} + \frac{2b_1^2 b_2 \alpha I w_2(t) w_3(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{b_1^3 I w_1(t) w_4(t)}{-a_2 b_1 + a_1 b_2} + \frac{2b_1^2 b_2 \alpha I w_1(t) w_4(t)}{-a_2 b_1 + a_1 b_2} \\
& + \left(-\frac{b_2^2}{-a_2 b_1 + a_1 b_2} + \frac{b_1^2 \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} + \frac{b_1 b_2 \alpha \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} + \frac{b_1 b_2 D}{-a_2 b_1 + a_1 b_2} \right) w_5(t) \\
& + \frac{a_1 b_1^5 \sqrt{1 - I^2} w_1(t)^4 z_1(t)}{24(-a_2 b_1 + a_1 b_2)} + \frac{a_2 b_1^5 \alpha \sqrt{1 - I^2} w_1(t)^4 z_1(t)}{24(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1 b_1^4 b_2 \alpha \sqrt{1 - I^2} w_1(t)^4 z_1(t)}{6(-a_2 b_1 + a_1 b_2)} - \frac{a_1 b_1^4 I w_1(t)^2 w_2(t) z_1(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_2 b_1^4 \alpha I w_1(t)^2 w_2(t) z_1(t)}{2(-a_2 b_1 + a_1 b_2)} - \frac{3a_1 b_1^3 b_2 \alpha I w_1(t)^2 w_2(t) z_1(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 b_1^3 \sqrt{1 - I^2} w_2(t)^2 z_1(t)}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_2 b_1^3 \alpha \sqrt{1 - I^2} w_2(t)^2 z_1(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 b_1^2 b_2 \alpha \sqrt{1 - I^2} w_2(t)^2 z_1(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_1^3 \sqrt{1 - I^2} w_1(t) w_3(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_2 b_1^3 \alpha \sqrt{1 - I^2} w_1(t) w_3(t) z_1(t)}{-a_2 b_1 + a_1 b_2} - \frac{2a_1 b_1^2 b_2 \alpha \sqrt{1 - I^2} w_1(t) w_3(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1 b_1^2 I w_4(t) z_1(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1^2 \alpha I w_4(t) z_1(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1 b_2 \alpha I w_4(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1^2 b_1^4 \sqrt{1 - I^2} w_1(t)^3 z_1(t)^2}{12(-a_2 b_1 + a_1 b_2)} + \frac{a_1 a_2 b_1^4 \alpha \sqrt{1 - I^2} w_1(t)^3 z_1(t)^2}{6(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 b_1^3 b_2 \alpha \sqrt{1 - I^2} w_1(t)^3 z_1(t)^2}{4(-a_2 b_1 + a_1 b_2)} - \frac{a_1^2 b_1^3 I w_1(t) w_2(t) z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 a_2 b_1^3 \alpha I w_1(t) w_2(t) z_1(t)^2}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_1^2 b_2 \alpha I w_1(t) w_2(t) z_1(t)^2}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^2 b_1^2 \sqrt{1 - I^2} w_3(t) z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_1 a_2 b_1^2 \alpha \sqrt{1 - I^2} w_3(t) z_1(t)^2}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^2 b_1 b_2 \alpha \sqrt{1 - I^2} w_3(t) z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} + \frac{a_1^3 b_1^3 \sqrt{1 - I^2} w_1(t)^2 z_1(t)^3}{12(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 a_2 b_1^3 \alpha \sqrt{1 - I^2} w_1(t)^2 z_1(t)^3}{4(-a_2 b_1 + a_1 b_2)} + \frac{a_1^3 b_1^2 b_2 \alpha \sqrt{1 - I^2} w_1(t)^2 z_1(t)^3}{6(-a_2 b_1 + a_1 b_2)}
\end{aligned}$$

$$\begin{aligned}
& - \frac{a_1^3 b_1^2 I w_2(t) z_1(t)^3}{6(-a_2 b_1 + a_1 b_2)} - \frac{a_1^2 a_2 b_1^2 \alpha I w_2(t) z_1(t)^3}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1^3 b_1 b_2 \alpha I w_2(t) z_1(t)^3}{6(-a_2 b_1 + a_1 b_2)} + \frac{a_1^4 b_1^2 \sqrt{1 - I^2} w_1(t) z_1(t)^4}{24(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^3 a_2 b_1^2 \alpha \sqrt{1 - I^2} w_1(t) z_1(t)^4}{6(-a_2 b_1 + a_1 b_2)} + \frac{a_1^4 b_1 b_2 \alpha \sqrt{1 - I^2} w_1(t) z_1(t)^4}{24(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^5 b_1 \sqrt{1 - I^2} z_1(t)^5}{120(-a_2 b_1 + a_1 b_2)} + \frac{a_1^4 a_2 b_1 \alpha \sqrt{1 - I^2} z_1(t)^5}{24(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 b_1^4 I w_1(t)^3 z_2(t)}{6(-a_2 b_1 + a_1 b_2)} - \frac{a_2 b_1^4 \alpha I w_1(t)^3 z_2(t)}{6(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 b_1^3 b_2 \alpha I w_1(t)^3 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_1 b_1^3 \sqrt{1 - I^2} w_1(t) w_2(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_2 b_1^3 \alpha \sqrt{1 - I^2} w_1(t) w_2(t) z_2(t)}{-a_2 b_1 + a_1 b_2} - \frac{2 a_1 b_1^2 b_2 \alpha \sqrt{1 - I^2} w_1(t) w_2(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1 b_1^2 I w_3(t) z_2(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1^2 \alpha I w_3(t) z_2(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1 b_2 \alpha I w_3(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^2 b_1^3 I w_1(t)^2 z_1(t) z_2(t)}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_1 a_2 b_1^3 \alpha I w_1(t)^2 z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^2 b_1^2 b_2 \alpha I w_1(t)^2 z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_1^2 \sqrt{1 - I^2} w_2(t) z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{2 a_1 a_2 b_1^2 \alpha \sqrt{1 - I^2} w_2(t) z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_1 b_2 \alpha \sqrt{1 - I^2} w_2(t) z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^3 b_1^2 I w_1(t) z_1(t)^2 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} - \frac{3 a_1^2 a_2 b_1^2 \alpha I w_1(t) z_1(t)^2 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1^3 b_1 b_2 \alpha I w_1(t) z_1(t)^2 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_1^4 b_1 I z_1(t)^3 z_2(t)}{6(-a_2 b_1 + a_1 b_2)} \\
& - \frac{2 a_1^3 a_2 b_1 \alpha I z_1(t)^3 z_2(t)}{3(-a_2 b_1 + a_1 b_2)} - \frac{a_1^2 b_1^2 \sqrt{1 - I^2} w_1(t) z_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 a_2 b_1^2 \alpha \sqrt{1 - I^2} w_1(t) z_2(t)^2}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_1 b_2 \alpha \sqrt{1 - I^2} w_1(t) z_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1^3 b_1 \sqrt{1 - I^2} z_1(t) z_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} - \frac{3 a_1^2 a_2 b_1 \alpha \sqrt{1 - I^2} z_1(t) z_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 b_1^3 \sqrt{1 - I^2} w_1(t)^2 z_3(t)}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_2 b_1^3 \alpha \sqrt{1 - I^2} w_1(t)^2 z_3(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 b_1^2 b_2 \alpha \sqrt{1 - I^2} w_1(t)^2 z_3(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1^2 I w_2(t) z_3(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_2 b_1^2 \alpha I w_2(t) z_3(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1 b_2 \alpha I w_2(t) z_3(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^2 b_1^2 \sqrt{1 - I^2} w_1(t) z_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} - \frac{2 a_1 a_2 b_1^2 \alpha \sqrt{1 - I^2} w_1(t) z_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^2 b_1 b_2 \alpha \sqrt{1 - I^2} w_1(t) z_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^3 b_1 \sqrt{1 - I^2} z_1(t)^2 z_3(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{3 a_1^2 a_2 b_1 \alpha \sqrt{1 - I^2} z_1(t)^2 z_3(t)}{2(-a_2 b_1 + a_1 b_2)} + \frac{a_1^2 b_1 I z_2(t) z_3(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{2 a_1 a_2 b_1 \alpha I z_2(t) z_3(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1^2 I w_1(t) z_4(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_2 b_1^2 \alpha I w_1(t) z_4(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1 b_2 \alpha I w_1(t) z_4(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1^2 b_1 I z_1(t) z_4(t)}{-a_2 b_1 + a_1 b_2} + \frac{2 a_1 a_2 b_1 \alpha I z_1(t) z_4(t)}{-a_2 b_1 + a_1 b_2} \\
& + \left(-\frac{a_2 b_2}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1 \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1 \alpha \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1 D}{-a_2 b_1 + a_1 b_2} \right) z_5(t) \tag{34}
\end{aligned}$$

$$\begin{aligned}
\dot{w}_5(t) = & -\frac{a_1 b_1^5 \sqrt{1 - I^2} w_1(t)^5}{120(-a_2 b_1 + a_1 b_2)} - \frac{a_1 b_1^4 b_2 \alpha \sqrt{1 - I^2} w_1(t)^5}{24(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1 b_1^4 I w_1(t)^3 w_2(t)}{6(-a_2 b_1 + a_1 b_2)} + \frac{2 a_1 b_1^3 b_2 \alpha I w_1(t)^3 w_2(t)}{3(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1 b_1^3 \sqrt{1 - I^2} w_1(t) w_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} + \frac{3 a_1 b_1^2 b_2 \alpha \sqrt{1 - I^2} w_1(t) w_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1 b_1^3 \sqrt{1 - I^2} w_1(t)^2 w_3(t)}{2(-a_2 b_1 + a_1 b_2)} + \frac{3 a_1 b_1^2 b_2 \alpha \sqrt{1 - I^2} w_1(t)^2 w_3(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 b_1^2 I w_2(t) w_3(t)}{-a_2 b_1 + a_1 b_2} - \frac{2 a_1 b_1 b_2 \alpha I w_2(t) w_3(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1 b_1^2 I w_1(t) w_4(t)}{-a_2 b_1 + a_1 b_2} - \frac{2 a_1 b_1 b_2 \alpha I w_1(t) w_4(t)}{-a_2 b_1 + a_1 b_2} \\
& + \left(\frac{a_2 b_2}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_1 \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_2 \alpha \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_2 D}{-a_2 b_1 + a_1 b_2} \right) w_5(t) \\
& - \frac{a_1^2 b_1^4 \sqrt{1 - I^2} w_1(t)^4 z_1(t)}{24(-a_2 b_1 + a_1 b_2)} - \frac{a_1 a_2 b_1^4 \alpha \sqrt{1 - I^2} w_1(t)^4 z_1(t)}{24(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1^2 b_1^3 b_2 \alpha \sqrt{1 - I^2} w_1(t)^4 z_1(t)}{6(-a_2 b_1 + a_1 b_2)} + \frac{a_1^2 b_1^3 I w_1(t)^2 w_2(t) z_1(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1 a_2 b_1^3 \alpha I w_1(t)^2 w_2(t) z_1(t)}{2(-a_2 b_1 + a_1 b_2)} + \frac{3 a_1^2 b_1^2 b_2 \alpha I w_1(t)^2 w_2(t) z_1(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 b_1^2 \sqrt{1 - I^2} w_2(t)^2 z_1(t)}{2(-a_2 b_1 + a_1 b_2)} + \frac{a_1 a_2 b_1^2 \alpha \sqrt{1 - I^2} w_2(t)^2 z_1(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 b_1 b_2 \alpha \sqrt{1 - I^2} w_2(t)^2 z_1(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1^2 b_1^2 \sqrt{1 - I^2} w_1(t) w_3(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1 a_2 b_1^2 \alpha \sqrt{1 - I^2} w_1(t) w_3(t) z_1(t)}{-a_2 b_1 + a_1 b_2} + \frac{2 a_1^2 b_1 b_2 \alpha \sqrt{1 - I^2} w_1(t) w_3(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^2 b_1 I w_4(t) z_1(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1 a_2 b_1 \alpha I w_4(t) z_1(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_2 \alpha I w_4(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^3 b_1^3 \sqrt{1 - I^2} w_1(t)^3 z_1(t)^2}{12(-a_2 b_1 + a_1 b_2)} - \frac{a_1^2 a_2 b_1^3 \alpha \sqrt{1 - I^2} w_1(t)^3 z_1(t)^2}{6(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1^3 b_1^2 b_2 \alpha \sqrt{1 - I^2} w_1(t)^3 z_1(t)^2}{4(-a_2 b_1 + a_1 b_2)} + \frac{a_1^3 b_1^2 I w_1(t) w_2(t) z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 a_2 b_1^2 \alpha I w_1(t) w_2(t) z_1(t)^2}{-a_2 b_1 + a_1 b_2} + \frac{a_1^3 b_1 b_2 \alpha I w_1(t) w_2(t) z_1(t)^2}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1^3 b_1 \sqrt{1 - I^2} w_3(t) z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} + \frac{a_1^2 a_2 b_1 \alpha \sqrt{1 - I^2} w_3(t) z_1(t)^2}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1^3 b_2 \alpha \sqrt{1 - I^2} w_3(t) z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_1^4 b_1^2 \sqrt{1 - I^2} w_1(t)^2 z_1(t)^3}{12(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1^3 a_2 b_1^2 \alpha \sqrt{1 - I^2} w_1(t)^2 z_1(t)^3}{4(-a_2 b_1 + a_1 b_2)} - \frac{a_1^4 b_1 b_2 \alpha \sqrt{1 - I^2} w_1(t)^2 z_1(t)^3}{6(-a_2 b_1 + a_1 b_2)}
\end{aligned}$$

$$\begin{aligned}
& + \frac{a_1^4 b_1 I w_2(t) z_1(t)^3}{6(-a_2 b_1 + a_1 b_2)} + \frac{a_1^3 a_2 b_1 \alpha I w_2(t) z_1(t)^3}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^4 b_2 \alpha I w_2(t) z_1(t)^3}{6(-a_2 b_1 + a_1 b_2)} - \frac{a_1^5 b_1 \sqrt{1 - I^2} w_1(t) z_1(t)^4}{24(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1^4 a_2 b_1 \alpha \sqrt{1 - I^2} w_1(t) z_1(t)^4}{6(-a_2 b_1 + a_1 b_2)} - \frac{a_1^5 b_2 \alpha \sqrt{1 - I^2} w_1(t) z_1(t)^4}{24(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1^6 \sqrt{1 - I^2} z_1(t)^5}{120(-a_2 b_1 + a_1 b_2)} - \frac{a_1^5 a_2 \alpha \sqrt{1 - I^2} z_1(t)^5}{24(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 b_1^3 I w_1(t)^3 z_2(t)}{6(-a_2 b_1 + a_1 b_2)} + \frac{a_1 a_2 b_1^3 \alpha I w_1(t)^3 z_2(t)}{6(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 b_1^2 b_2 \alpha I w_1(t)^3 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} + \frac{a_1^2 b_1^2 \sqrt{1 - I^2} w_1(t) w_2(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1 a_2 b_1^2 \alpha \sqrt{1 - I^2} w_1(t) w_2(t) z_2(t)}{-a_2 b_1 + a_1 b_2} + \frac{2 a_1^2 b_1 b_2 \alpha \sqrt{1 - I^2} w_1(t) w_2(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^2 b_1 I w_3(t) z_2(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1 a_2 b_1 \alpha I w_3(t) z_2(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_2 \alpha I w_3(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1^3 b_1^2 I w_1(t)^2 z_1(t) z_2(t)}{2(-a_2 b_1 + a_1 b_2)} + \frac{a_1^2 a_2 b_1^2 \alpha I w_1(t)^2 z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1^3 b_1 b_2 \alpha I w_1(t)^2 z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1^3 b_1 \sqrt{1 - I^2} w_2(t) z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{2 a_1^2 a_2 b_1 \alpha \sqrt{1 - I^2} w_2(t) z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1^3 b_2 \alpha \sqrt{1 - I^2} w_2(t) z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1^4 b_1 I w_1(t) z_1(t)^2 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} + \frac{3 a_1^3 a_2 b_1 \alpha I w_1(t) z_1(t)^2 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^4 b_2 \alpha I w_1(t) z_1(t)^2 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} + \frac{a_1^5 I z_1(t)^3 z_2(t)}{6(-a_2 b_1 + a_1 b_2)} \\
& + \frac{2 a_1^4 a_2 \alpha I z_1(t)^3 z_2(t)}{3(-a_2 b_1 + a_1 b_2)} + \frac{a_1^3 b_1 \sqrt{1 - I^2} w_1(t) z_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 a_2 b_1 \alpha \sqrt{1 - I^2} w_1(t) z_2(t)^2}{-a_2 b_1 + a_1 b_2} + \frac{a_1^3 b_2 \alpha \sqrt{1 - I^2} w_1(t) z_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^4 \sqrt{1 - I^2} z_1(t) z_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} + \frac{3 a_1^3 a_2 \alpha \sqrt{1 - I^2} z_1(t) z_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 b_1^2 \sqrt{1 - I^2} w_1(t)^2 z_3(t)}{2(-a_2 b_1 + a_1 b_2)} + \frac{a_1 a_2 b_1^2 \alpha \sqrt{1 - I^2} w_1(t)^2 z_3(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 b_1 b_2 \alpha \sqrt{1 - I^2} w_1(t)^2 z_3(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_1 I w_2(t) z_3(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1 a_2 b_1 \alpha I w_2(t) z_3(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_2 \alpha I w_2(t) z_3(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1^3 b_1 \sqrt{1 - I^2} w_1(t) z_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} + \frac{2 a_1^2 a_2 b_1 \alpha \sqrt{1 - I^2} w_1(t) z_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1^3 b_2 \alpha \sqrt{1 - I^2} w_1(t) z_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1^4 \sqrt{1 - I^2} z_1(t)^2 z_3(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{3 a_1^3 a_2 \alpha \sqrt{1 - I^2} z_1(t)^2 z_3(t)}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_1^3 I z_2(t) z_3(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{2 a_1^2 a_2 \alpha I z_2(t) z_3(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_1 I w_1(t) z_4(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1 a_2 b_1 \alpha I w_1(t) z_4(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_2 \alpha I w_1(t) z_4(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^3 I z_1(t) z_4(t)}{-a_2 b_1 + a_1 b_2} - \frac{2 a_1^2 a_2 \alpha I z_1(t) z_4(t)}{-a_2 b_1 + a_1 b_2} \\
& + \left(\frac{a_2^2}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} - \frac{a_1 a_2 \alpha \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} - \frac{a_1 a_2 D}{-a_2 b_1 + a_1 b_2} \right) z_5(t) \tag{35}
\end{aligned}$$

After simplifying the coupling terms, we obtain:

$$\begin{aligned}
\dot{z}_5(t) = & \frac{b_1^6 \sqrt{1 - I^2} w_1(t)^5}{120(-a_2 b_1 + a_1 b_2)} + \frac{b_1^5 b_2 \alpha \sqrt{1 - I^2} w_1(t)^5}{24(-a_2 b_1 + a_1 b_2)} \\
& - \frac{b_1^5 I w_1(t)^3 w_2(t)}{6(-a_2 b_1 + a_1 b_2)} - \frac{2b_1^4 b_2 \alpha I w_1(t)^3 w_2(t)}{3(-a_2 b_1 + a_1 b_2)} \\
& - \frac{b_1^4 \sqrt{1 - I^2} w_1(t) w_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} - \frac{3b_1^3 b_2 \alpha \sqrt{1 - I^2} w_1(t) w_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{b_1^4 \sqrt{1 - I^2} w_1(t)^2 w_3(t)}{2(-a_2 b_1 + a_1 b_2)} - \frac{3b_1^3 b_2 \alpha \sqrt{1 - I^2} w_1(t)^2 w_3(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{b_1^3 I w_2(t) w_3(t)}{-a_2 b_1 + a_1 b_2} + \frac{2b_1^2 b_2 \alpha I w_2(t) w_3(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{b_1^3 I w_1(t) w_4(t)}{-a_2 b_1 + a_1 b_2} + \frac{2b_1^2 b_2 \alpha I w_1(t) w_4(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1 b_1^5 \sqrt{1 - I^2} w_1(t)^4 z_1(t)}{24(-a_2 b_1 + a_1 b_2)} + \frac{a_2 b_1^5 \alpha \sqrt{1 - I^2} w_1(t)^4 z_1(t)}{24(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1 b_1^4 b_2 \alpha \sqrt{1 - I^2} w_1(t)^4 z_1(t)}{6(-a_2 b_1 + a_1 b_2)} - \frac{a_1 b_1^4 I w_1(t)^2 w_2(t) z_1(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_2 b_1^4 \alpha I w_1(t)^2 w_2(t) z_1(t)}{2(-a_2 b_1 + a_1 b_2)} - \frac{3a_1 b_1^3 b_2 \alpha I w_1(t)^2 w_2(t) z_1(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 b_1^3 \sqrt{1 - I^2} w_2(t)^2 z_1(t)}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_2 b_1^3 \alpha \sqrt{1 - I^2} w_2(t)^2 z_1(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 b_1^2 b_2 \alpha \sqrt{1 - I^2} w_2(t)^2 z_1(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_1^3 \sqrt{1 - I^2} w_1(t) w_3(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_2 b_1^3 \alpha \sqrt{1 - I^2} w_1(t) w_3(t) z_1(t)}{-a_2 b_1 + a_1 b_2} - \frac{2a_1 b_1^2 b_2 \alpha \sqrt{1 - I^2} w_1(t) w_3(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1 b_1^2 I w_4(t) z_1(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1^2 \alpha I w_4(t) z_1(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1 b_2 \alpha I w_4(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1^2 b_1^4 \sqrt{1 - I^2} w_1(t)^3 z_1(t)^2}{12(-a_2 b_1 + a_1 b_2)} + \frac{a_1 a_2 b_1^4 \alpha \sqrt{1 - I^2} w_1(t)^3 z_1(t)^2}{6(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 b_1^3 b_2 \alpha \sqrt{1 - I^2} w_1(t)^3 z_1(t)^2}{4(-a_2 b_1 + a_1 b_2)} - \frac{a_1^2 b_1^3 I w_1(t) w_2(t) z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 a_2 b_1^3 \alpha I w_1(t) w_2(t) z_1(t)^2}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_1^2 b_2 \alpha I w_1(t) w_2(t) z_1(t)^2}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^2 b_1^2 \sqrt{1 - I^2} w_3(t) z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_1 a_2 b_1^2 \alpha \sqrt{1 - I^2} w_3(t) z_1(t)^2}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^2 b_1 b_2 \alpha \sqrt{1 - I^2} w_3(t) z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} + \frac{a_1^3 b_1^3 \sqrt{1 - I^2} w_1(t)^2 z_1(t)^3}{12(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 a_2 b_1^3 \alpha \sqrt{1 - I^2} w_1(t)^2 z_1(t)^3}{4(-a_2 b_1 + a_1 b_2)} + \frac{a_1^3 b_1^2 b_2 \alpha \sqrt{1 - I^2} w_1(t)^2 z_1(t)^3}{6(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1^3 b_1^2 I w_2(t) z_1(t)^3}{6(-a_2 b_1 + a_1 b_2)} - \frac{a_1^2 a_2 b_1^2 \alpha I w_2(t) z_1(t)^3}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1^3 b_1 b_2 \alpha I w_2(t) z_1(t)^3}{6(-a_2 b_1 + a_1 b_2)} + \frac{a_1^4 b_1^2 \sqrt{1 - I^2} w_1(t) z_1(t)^4}{24(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^3 a_2 b_1^2 \alpha \sqrt{1 - I^2} w_1(t) z_1(t)^4}{6(-a_2 b_1 + a_1 b_2)} + \frac{a_1^4 b_1 b_2 \alpha \sqrt{1 - I^2} w_1(t) z_1(t)^4}{24(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^5 b_1 \sqrt{1 - I^2} z_1(t)^5}{120(-a_2 b_1 + a_1 b_2)} + \frac{a_1^4 a_2 b_1 \alpha \sqrt{1 - I^2} z_1(t)^5}{24(-a_2 b_1 + a_1 b_2)}
\end{aligned}$$

$$\begin{aligned}
& - \frac{a_1 b_1^4 I w_1(t)^3 z_2(t)}{6(-a_2 b_1 + a_1 b_2)} - \frac{a_2 b_1^4 \alpha I w_1(t)^3 z_2(t)}{6(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 b_1^3 b_2 \alpha I w_1(t)^3 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_1 b_1^3 \sqrt{1 - I^2} w_1(t) w_2(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_2 b_1^3 \alpha \sqrt{1 - I^2} w_1(t) w_2(t) z_2(t)}{-a_2 b_1 + a_1 b_2} - \frac{2 a_1 b_1^2 b_2 \alpha \sqrt{1 - I^2} w_1(t) w_2(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1 b_1^2 I w_3(t) z_2(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1^2 \alpha I w_3(t) z_2(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1 b_2 \alpha I w_3(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^2 b_1^3 I w_1(t)^2 z_1(t) z_2(t)}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_1 a_2 b_1^3 \alpha I w_1(t)^2 z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^2 b_1^2 b_2 \alpha I w_1(t)^2 z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_1^2 \sqrt{1 - I^2} w_2(t) z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{2 a_1 a_2 b_1^2 \alpha \sqrt{1 - I^2} w_2(t) z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_1 b_2 \alpha \sqrt{1 - I^2} w_2(t) z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^3 b_1^2 I w_1(t) z_1(t)^2 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} - \frac{3 a_1^2 a_2 b_1^2 \alpha I w_1(t) z_1(t)^2 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1^3 b_1 b_2 \alpha I w_1(t) z_1(t)^2 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_1^4 b_1 I z_1(t)^3 z_2(t)}{6(-a_2 b_1 + a_1 b_2)} \\
& - \frac{2 a_1^3 a_2 b_1 \alpha I z_1(t)^3 z_2(t)}{3(-a_2 b_1 + a_1 b_2)} - \frac{a_1^2 b_1^2 \sqrt{1 - I^2} w_1(t) z_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 a_2 b_1^2 \alpha \sqrt{1 - I^2} w_1(t) z_2(t)^2}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_1 b_2 \alpha \sqrt{1 - I^2} w_1(t) z_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1^3 b_1 \sqrt{1 - I^2} z_1(t) z_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} - \frac{3 a_1^2 a_2 b_1 \alpha \sqrt{1 - I^2} z_1(t) z_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 b_1^3 \sqrt{1 - I^2} w_1(t)^2 z_3(t)}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_2 b_1^3 \alpha \sqrt{1 - I^2} w_1(t)^2 z_3(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 b_1^2 b_2 \alpha \sqrt{1 - I^2} w_1(t)^2 z_3(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1^2 I w_2(t) z_3(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_2 b_1^2 \alpha I w_2(t) z_3(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1 b_2 \alpha I w_2(t) z_3(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^2 b_1^2 \sqrt{1 - I^2} w_1(t) z_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} - \frac{2 a_1 a_2 b_1^2 \alpha \sqrt{1 - I^2} w_1(t) z_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^2 b_1 b_2 \alpha \sqrt{1 - I^2} w_1(t) z_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^3 b_1 \sqrt{1 - I^2} z_1(t)^2 z_3(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{3 a_1^2 a_2 b_1 \alpha \sqrt{1 - I^2} z_1(t)^2 z_3(t)}{2(-a_2 b_1 + a_1 b_2)} + \frac{a_1^2 b_1 I z_2(t) z_3(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{2 a_1 a_2 b_1 \alpha I z_2(t) z_3(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1^2 I w_1(t) z_4(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_2 b_1^2 \alpha I w_1(t) z_4(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1 b_2 \alpha I w_1(t) z_4(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1^2 b_1 I z_1(t) z_4(t)}{-a_2 b_1 + a_1 b_2} + \frac{2 a_1 a_2 b_1 \alpha I z_1(t) z_4(t)}{-a_2 b_1 + a_1 b_2} \\
& + \left(-\frac{a_2 b_2}{-a_2 b_1 + a_1 b_2} + \frac{a_1 b_1 \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1 \alpha \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} + \frac{a_2 b_1 D}{-a_2 b_1 + a_1 b_2} \right) z_5(t)
\end{aligned} \tag{36}$$

$$\begin{aligned}
\dot{w}_5(t) = & -\frac{a_1 b_1^5 \sqrt{1 - I^2} w_1(t)^5}{120(-a_2 b_1 + a_1 b_2)} - \frac{a_1 b_1^4 b_2 \alpha \sqrt{1 - I^2} w_1(t)^5}{24(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1 b_1^4 I w_1(t)^3 w_2(t)}{6(-a_2 b_1 + a_1 b_2)} + \frac{2 a_1 b_1^3 b_2 \alpha I w_1(t)^3 w_2(t)}{3(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1 b_1^3 \sqrt{1 - I^2} w_1(t) w_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} + \frac{3 a_1 b_1^2 b_2 \alpha \sqrt{1 - I^2} w_1(t) w_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1 b_1^3 \sqrt{1 - I^2} w_1(t)^2 w_3(t)}{2(-a_2 b_1 + a_1 b_2)} + \frac{3 a_1 b_1^2 b_2 \alpha \sqrt{1 - I^2} w_1(t)^2 w_3(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1 b_1^2 I w_2(t) w_3(t)}{-a_2 b_1 + a_1 b_2} - \frac{2 a_1 b_1 b_2 \alpha I w_2(t) w_3(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1 b_1^2 I w_1(t) w_4(t)}{-a_2 b_1 + a_1 b_2} - \frac{2 a_1 b_1 b_2 \alpha I w_1(t) w_4(t)}{-a_2 b_1 + a_1 b_2} \\
& + \left(\frac{a_2 b_2}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_1 \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_2 \alpha \sqrt{1 - I^2}}{-a_2 b_1 + a_1 b_2} - \frac{a_1 b_2 D}{-a_2 b_1 + a_1 b_2} \right) w_5(t) \\
& - \frac{a_1^2 b_1^4 \sqrt{1 - I^2} w_1(t)^4 z_1(t)}{24(-a_2 b_1 + a_1 b_2)} - \frac{a_1 a_2 b_1^4 \alpha \sqrt{1 - I^2} w_1(t)^4 z_1(t)}{24(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1^2 b_1^3 b_2 \alpha \sqrt{1 - I^2} w_1(t)^4 z_1(t)}{6(-a_2 b_1 + a_1 b_2)} + \frac{a_1^2 b_1^3 I w_1(t)^2 w_2(t) z_1(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1 a_2 b_1^3 \alpha I w_1(t)^2 w_2(t) z_1(t)}{2(-a_2 b_1 + a_1 b_2)} + \frac{3 a_1^2 b_1^2 b_2 \alpha I w_1(t)^2 w_2(t) z_1(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 b_1^2 \sqrt{1 - I^2} w_2(t)^2 z_1(t)}{2(-a_2 b_1 + a_1 b_2)} + \frac{a_1 a_2 b_1^2 \alpha \sqrt{1 - I^2} w_2(t)^2 z_1(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 b_1 b_2 \alpha \sqrt{1 - I^2} w_2(t)^2 z_1(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1^2 b_1^2 \sqrt{1 - I^2} w_1(t) w_3(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1 a_2 b_1^2 \alpha \sqrt{1 - I^2} w_1(t) w_3(t) z_1(t)}{-a_2 b_1 + a_1 b_2} + \frac{2 a_1^2 b_1 b_2 \alpha \sqrt{1 - I^2} w_1(t) w_3(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^2 b_1 I w_4(t) z_1(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1 a_2 b_1 \alpha I w_4(t) z_1(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_2 \alpha I w_4(t) z_1(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^3 b_1^3 \sqrt{1 - I^2} w_1(t)^3 z_1(t)^2}{12(-a_2 b_1 + a_1 b_2)} - \frac{a_1^2 a_2 b_1^3 \alpha \sqrt{1 - I^2} w_1(t)^3 z_1(t)^2}{6(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1^3 b_1^2 b_2 \alpha \sqrt{1 - I^2} w_1(t)^3 z_1(t)^2}{4(-a_2 b_1 + a_1 b_2)} + \frac{a_1^3 b_1^2 I w_1(t) w_2(t) z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 a_2 b_1^2 \alpha I w_1(t) w_2(t) z_1(t)^2}{-a_2 b_1 + a_1 b_2} + \frac{a_1^3 b_1 b_2 \alpha I w_1(t) w_2(t) z_1(t)^2}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1^3 b_1 \sqrt{1 - I^2} w_3(t) z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} + \frac{a_1^2 a_2 b_1 \alpha \sqrt{1 - I^2} w_3(t) z_1(t)^2}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1^3 b_2 \alpha \sqrt{1 - I^2} w_3(t) z_1(t)^2}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_1^4 b_1^2 \sqrt{1 - I^2} w_1(t)^2 z_1(t)^3}{12(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1^3 a_2 b_1^2 \alpha \sqrt{1 - I^2} w_1(t)^2 z_1(t)^3}{4(-a_2 b_1 + a_1 b_2)} - \frac{a_1^4 b_1 b_2 \alpha \sqrt{1 - I^2} w_1(t)^2 z_1(t)^3}{6(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^4 b_1 I w_2(t) z_1(t)^3}{6(-a_2 b_1 + a_1 b_2)} + \frac{a_1^3 a_2 b_1 \alpha I w_2(t) z_1(t)^3}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^4 b_2 \alpha I w_2(t) z_1(t)^3}{6(-a_2 b_1 + a_1 b_2)} - \frac{a_1^5 b_1 \sqrt{1 - I^2} w_1(t) z_1(t)^4}{24(-a_2 b_1 + a_1 b_2)} \\
& - \frac{a_1^4 a_2 b_1 \alpha \sqrt{1 - I^2} w_1(t) z_1(t)^4}{6(-a_2 b_1 + a_1 b_2)} - \frac{a_1^5 b_2 \alpha \sqrt{1 - I^2} w_1(t) z_1(t)^4}{24(-a_2 b_1 + a_1 b_2)}
\end{aligned}$$

$$\begin{aligned}
& -\frac{a_1^6 \sqrt{1-I^2} z_1(t)^5}{120(-a_2 b_1 + a_1 b_2)} - \frac{a_1^5 a_2 \alpha \sqrt{1-I^2} z_1(t)^5}{24(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 b_1^3 I w_1(t)^3 z_2(t)}{6(-a_2 b_1 + a_1 b_2)} + \frac{a_1 a_2 b_1^3 \alpha I w_1(t)^3 z_2(t)}{6(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 b_1^2 b_2 \alpha I w_1(t)^3 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} + \frac{a_1^2 b_1^2 \sqrt{1-I^2} w_1(t) w_2(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1 a_2 b_1^2 \alpha \sqrt{1-I^2} w_1(t) w_2(t) z_2(t)}{-a_2 b_1 + a_1 b_2} + \frac{2 a_1^2 b_1 b_2 \alpha \sqrt{1-I^2} w_1(t) w_2(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^2 b_1 I w_3(t) z_2(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1 a_2 b_1 \alpha I w_3(t) z_2(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_2 \alpha I w_3(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1^3 b_1^2 I w_1(t)^2 z_1(t) z_2(t)}{2(-a_2 b_1 + a_1 b_2)} + \frac{a_1^2 a_2 b_1^2 \alpha I w_1(t)^2 z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1^3 b_1 b_2 \alpha I w_1(t)^2 z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1^3 b_1 \sqrt{1-I^2} w_2(t) z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{2 a_1^2 a_2 b_1 \alpha \sqrt{1-I^2} w_2(t) z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1^3 b_2 \alpha \sqrt{1-I^2} w_2(t) z_1(t) z_2(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1^4 b_1 I w_1(t) z_1(t)^2 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} + \frac{3 a_1^3 a_2 b_1 \alpha I w_1(t) z_1(t)^2 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^4 b_2 \alpha I w_1(t) z_1(t)^2 z_2(t)}{2(-a_2 b_1 + a_1 b_2)} + \frac{a_1^5 I z_1(t)^3 z_2(t)}{6(-a_2 b_1 + a_1 b_2)} \\
& + \frac{2 a_1^4 a_2 \alpha I z_1(t)^3 z_2(t)}{3(-a_2 b_1 + a_1 b_2)} + \frac{a_1^3 b_1 \sqrt{1-I^2} w_1(t) z_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 a_2 b_1 \alpha \sqrt{1-I^2} w_1(t) z_2(t)^2}{-a_2 b_1 + a_1 b_2} + \frac{a_1^3 b_2 \alpha \sqrt{1-I^2} w_1(t) z_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^4 \sqrt{1-I^2} z_1(t) z_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} + \frac{3 a_1^3 a_2 \alpha \sqrt{1-I^2} z_1(t) z_2(t)^2}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 b_1^2 \sqrt{1-I^2} w_1(t)^2 z_3(t)}{2(-a_2 b_1 + a_1 b_2)} + \frac{a_1 a_2 b_1^2 \alpha \sqrt{1-I^2} w_1(t)^2 z_3(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{a_1^2 b_1 b_2 \alpha \sqrt{1-I^2} w_1(t)^2 z_3(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_1 I w_2(t) z_3(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1 a_2 b_1 \alpha I w_2(t) z_3(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_2 \alpha I w_2(t) z_3(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1^3 b_1 \sqrt{1-I^2} w_1(t) z_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} + \frac{2 a_1^2 a_2 b_1 \alpha \sqrt{1-I^2} w_1(t) z_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} \\
& + \frac{a_1^3 b_2 \alpha \sqrt{1-I^2} w_1(t) z_1(t) z_3(t)}{-a_2 b_1 + a_1 b_2} + \frac{a_1^4 \sqrt{1-I^2} z_1(t)^2 z_3(t)}{2(-a_2 b_1 + a_1 b_2)} \\
& + \frac{3 a_1^3 a_2 \alpha \sqrt{1-I^2} z_1(t)^2 z_3(t)}{2(-a_2 b_1 + a_1 b_2)} - \frac{a_1^3 I z_2(t) z_3(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{2 a_1^2 a_2 \alpha I z_2(t) z_3(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_1 I w_1(t) z_4(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1 a_2 b_1 \alpha I w_1(t) z_4(t)}{-a_2 b_1 + a_1 b_2} - \frac{a_1^2 b_2 \alpha I w_1(t) z_4(t)}{-a_2 b_1 + a_1 b_2} \\
& - \frac{a_1^3 I z_1(t) z_4(t)}{-a_2 b_1 + a_1 b_2} - \frac{2 a_1^2 a_2 \alpha I z_1(t) z_4(t)}{-a_2 b_1 + a_1 b_2}
\end{aligned} \tag{37}$$

Solve the following equations:

$$\begin{aligned}
z_5 = & -0.00203256a^5 e^{3.34405t - 3.34405t_0} \\
& - 0.000577577a^5 e^{2.67524t - 2.67524t_0} \\
& - 0.00732608a^5 e^{2.00643t - 2.00643t_0} \\
& - 0.000173067a^5 e^{1.33762t - 1.33762t_0} \\
& + 0.0101093a^5 e^{0.668809t - 0.668809t_0}
\end{aligned} \tag{38}$$

$$\begin{aligned}
w_5 = & -0.00236327a^5 e^{3.34405t - 3.34405t_0} \\
& - 0.000586928a^5 e^{2.67524t - 2.67524t_0} \\
& - 0.00594613a^5 e^{2.00643t - 2.00643t_0} \\
& - 0.0000875799a^5 e^{1.33762t - 1.33762t_0}
\end{aligned} \tag{39}$$

By integrating all solutions sets of z and w , we obtain the higher-order approximation:

$$\begin{aligned}
z(t) = & ae^{0.668809t - 0.668809t_0} \epsilon \\
& + (-0.0139601a^2 e^{1.33762t - 1.33762t_0} + 0.0139601a^2 e^{0.668809t - 0.668809t_0}) \epsilon^2 \\
& + (0.04971a^3 e^{2.00643t - 2.00643t_0} - 0.000389771a^3 e^{1.33762t - 1.33762t_0} \\
& \quad - 0.0493203a^3 e^{0.668809t - 0.668809t_0}) \epsilon^3 \\
& + (-0.0103433a^4 e^{2.67524t - 2.67524t_0} + 0.00208188a^4 e^{2.00643t - 2.00643t_0} \\
& \quad + 0.00137432a^4 e^{1.33762t - 1.33762t_0} + 0.00688712a^4 e^{0.668809t - 0.668809t_0}) \epsilon^4 \\
& + (-0.00203256a^5 e^{3.34405t - 3.34405t_0} - 0.000577577a^5 e^{2.67524t - 2.67524t_0} \\
& \quad - 0.00732608a^5 e^{2.00643t - 2.00643t_0} - 0.000173067a^5 e^{1.33762t - 1.33762t_0} \\
& \quad + 0.0101093a^5 e^{0.668809t - 0.668809t_0}) \epsilon^5
\end{aligned} \tag{40}$$

$$\begin{aligned}
w(t) = & -0.00706449a^2 e^{1.33762t - 1.33762t_0} \epsilon^2 \\
& + (0.0403466a^3 e^{2.00643t - 2.00643t_0} - 0.000197243a^3 e^{1.33762t - 1.33762t_0}) \epsilon^3 \\
& + (-0.0105108a^4 e^{2.67524t - 2.67524t_0} + 0.00168973a^4 e^{2.00643t - 2.00643t_0} \\
& \quad + 0.000695468a^4 e^{1.33762t - 1.33762t_0}) \epsilon^4 \\
& + (-0.00236327a^5 e^{3.34405t - 3.34405t_0} - 0.000586928a^5 e^{2.67524t - 2.67524t_0} \\
& \quad - 0.00594613a^5 e^{2.00643t - 2.00643t_0} - 0.0000875799a^5 e^{1.33762t - 1.33762t_0}) \epsilon^5
\end{aligned} \tag{41}$$

For $z(t)$, as $t \rightarrow t_0$, this yields $da(t_0)/dt_0$:

$$\dot{a}(t_0) = 0.668809\epsilon a(t_0) - 0.00933666\epsilon^2 a(t_0)^2 + 0.0662324\epsilon^3 a(t_0)^3 - 0.0170492\epsilon^4 a(t_0)^4 - 0.0165117\epsilon^5 a(t_0)^5 \tag{42}$$