流程工业综合自动化国家重点实验室
STATE KEY LABORATORY OF SYNTHETICAL AUTOMATION FOR PROCESS INDUSTRIES

Spiking Neuron Model and Python Implement

Yan Zhou
Northeastern University (zhouy@mail.neu.edu.cn)

Seminar, Shenyang, 22nd October 2016

Outline

Neuronal Coding

Introduction

- Traditionally it has been thought that most, if not all, of the relevant information was contained in the mean firing rate of the neuron. Neuronal coding is a way of convert the pulses to analog values.
- The relation between the measured firing frequency v and the applied input current I_0 is sometimes called the frequency-current curve of the neuron.
- In models, we formalize the relation between firing frequency (rate) and input current and write $v = g(I_0)$. We refer to g as the neuronal gain function or transfer function[2].
- For instance, the quantity of electric charge $\Delta t I_0 = \int_t^{t+\Delta t} \sum_i e(\tau - t_i)\, d\tau$, and the firing rate satisfy $\Delta t v = \int_t^{t+\Delta t} \sum_i \delta(\tau - t_i)\, d\tau$.

Neuronal Coding

Rate Codes

- Average over time.
- Average over several repetitions of the experiment.
- Average over a population of neurons.

Rate as a Spike Count (Average over Time)

$$\nu = \frac{n_{sp}(T)}{T} = \frac{\int_0^T \sum_{i=1}^{n_{sp}} \delta(\tau - t_i) d\tau}{T} \tag{1}$$

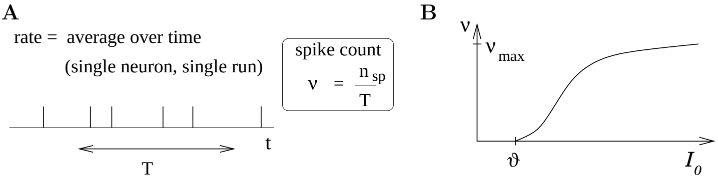

Figure: A. Definition of the mean firing rate via a temporal average. B. Gain function, schematic. The output rate ν is given as a function of the total input I_0.

Rate as a Spike Density (Average over Several Runs)

$$v = \frac{1}{\Delta t} \frac{n_k(t; t + \Delta t)}{K} \tag{2}$$

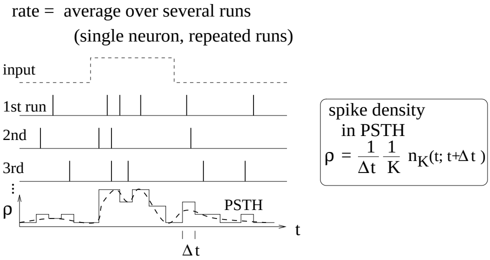

Figure: Definition of the spike density in the Peri-Stimulus-Time Histogram(PSTH) as an average over several runs of the experiment.

Rate as a Population Activity (Average over Several Neurons)

$$v = \frac{1}{\Delta t}\frac{n_{act}(t; t+\Delta t)}{N} = \frac{1}{\Delta t}\frac{\int_t^{t+\Delta t}\sum_j\sum_f \delta(\tau - t_j^{(f)})d\tau}{N} \qquad (3)$$

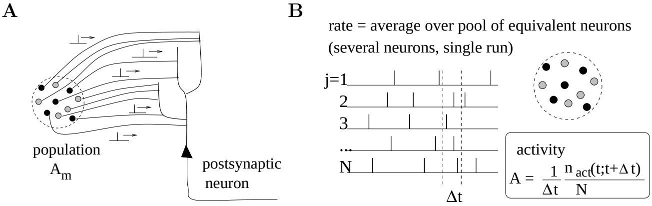

Figure: A. A postsynaptic neuron receives spike input from the population m with activity A_m. B. The population activity is defined as the fraction of neurons that are active in a short interval $[t; t+\Delta t]$ divided by Δt.

More Detailed Methods of Neuronal Coding

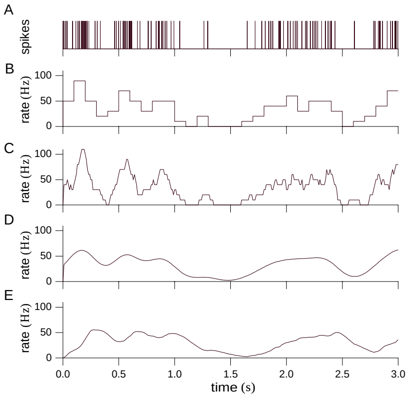

Figure: Firing rates approximated by different procedures[1].

Counting & Convolution

- **(A)** A spike train from a neuron.
- **(B)** Discrete time firing rate obtained by counting spikes
- **(C)** Approximate firing rate determined by sliding a rectangular window function along the spike train.
- **(D)** Approximate firing rate computed using a Gaussian window function.
- **(E)** Approximate firing rate for an α function window.

More Detailed Methods of Neuronal Coding

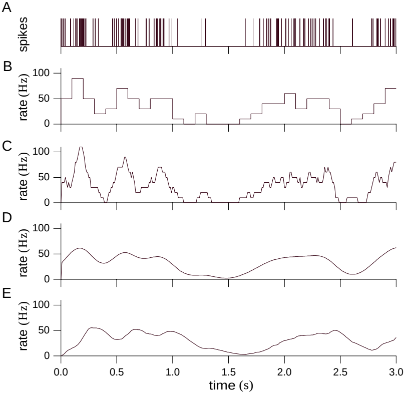

Figure: Firing rates approximated by different procedures[1].

Counting & Convolution

- $v_{approx}(t) = \sum_{i=1}^{n} w(t - t_i) = \int_{-\infty}^{+\infty} \sum_{i=1}^{n} d(\tau) w(\tau) \delta(t - \tau)$
- $w(\tau) = \begin{cases} 1/\Delta t & if -\Delta t/2 \geqslant t \geq \Delta t/2 \\ 0 & otherwise. \end{cases}$
- $w(\tau) = \frac{1}{\sqrt{2\pi\sigma}} exp(-\frac{\tau^2}{2\sigma^2})$.

Formal Spiking Neuron Model

Formal threshold models of neuronal firing

Spikes are generated whenever the membrane potential u crosses some threshold ϑ from below. The moment of threshold crossing defines the firing time $t(f)$.

$$t^{(f)}: \quad u(t^{(f)}) = \vartheta \quad and \quad \frac{du(t)}{dt}\Big|_{t=t(f)} > 0 \tag{4}$$

Formal Spiking Neuron Model

Formal threshold models of neuronal firing

- Integrate-and-fire model[2]
    - Leaky Integrate-and-Fire Model
    - nonlinear Integrate-and-Fire Model
- Izhikevich Model[3]

Leaky Integrate-and-Fire Model

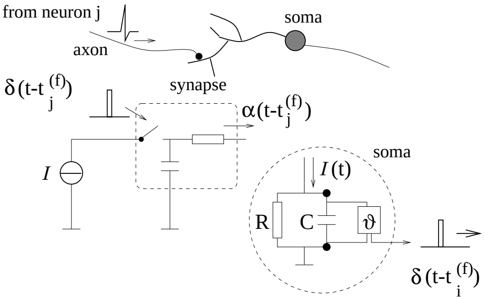

Figure:

Bibliography

Peter Dayan and Laurence F Abbott.
Theoretical neuroscience, volume 10.
Cambridge, MA: MIT Press, 2001.

Wulfram Gerstner and Werner M Kistler.
Spiking neuron models: Single neurons, populations, plasticity.
Cambridge university press, 2002.

Eugene M Izhikevich.
Simple model of spiking neurons.
IEEE Transactions on neural networks, 14(6):1569–1572, 2003.