Download the Matlab functions *FD_linearBVP.m* and *FD_nonlinearBVP.m* from the course website.

1. Use the finite difference method (*FD_linearBVP.m*) to solve the linear BVP:

$$y'' = -\frac{4}{x}y' - \frac{2}{x^2}y + \frac{2\ln x}{x^2}, \quad 1 \leq x \leq 2$$

$$y(1) = 1/2, \quad y(2) = \ln 2.$$

Compare the numerical solution with the exact solution

$$y(x) = \frac{4}{x} - \frac{2}{x^2} + \ln x - \frac{3}{2}.$$

Try different step size (e.g. $N = 20$, 40, etc), and plot the error versus the step size h.

2. Use the finite difference method with $TOL = 10^{-6}$ (*FD_nonlinearBVP.m*) to solve the nonlinear BVP:

$$y'' = y' \cos x - y \ln y, \quad 0 \leq x \leq \pi/2$$

$$y(0) = 1, \quad y\left(\frac{\pi}{2}\right) = e.$$

Compare the numerical solution with the exact solution $y(x) = e^{\sin x}$. Try different step size (e.g. $N = 20$, 40, etc), and plot the error versus the step size h.