Write a Matlab function for the conjugate gradient method (algorithm attached).

Use your Matlab function to solve the following problem to within $10^{-5}$ in the $l_\infty$ norm.

**1.** Solve the linear system $A\mathbf{x} = \mathbf{b}$

$$a_{i,j} = \begin{cases} 2i, & \text{when } j = i \text{ and } i = 1, 2, \cdots, n \\ -1, & \text{when } j = i + 1 \text{ and } i = 1, 2, \cdots, n-1, \\ -1, & \text{when } j = i - 1 \text{ and } i = 2, 3, \cdots, n, \\ 0, & \text{otherwise,} \end{cases}$$

where $n = 40$. The entries of the right-hand side vector $\mathbf{b}$ are $b_i = 1.5i - 6$, for each $i = 1, 2, \cdots, n$.

Compare the performance of the conjugate gradient method with the SOR method with $\omega = 1$ (Gauss-Seidel), $\omega = 1.1$ and $\omega = 1.2$.

# The Conjugate Gradient Method

## CG algorithm

$\mathbf{x} := \mathbf{0}$, $\mathbf{r} := \mathbf{b}$, $\rho_0 := \|\mathbf{r}\|^2$, $\mathbf{p} := \mathbf{r}$

for $k = 1, 2, \cdots, N_{\max}$

quit if $\sqrt{\rho_{k-1}} \leq \varepsilon \|\mathbf{b}\|$

$\mathbf{w} := A\mathbf{p}$

$\alpha := \rho_{k-1} / \mathbf{p}^t \mathbf{w}$

$\mathbf{x} := \mathbf{x} + \alpha \mathbf{p}$

$\mathbf{r} := \mathbf{r} - \alpha \mathbf{w}$

$\rho_k := \|\mathbf{r}\|^2$

$\mathbf{p} := \mathbf{r} + (\rho_k / \rho_{k-1}) \mathbf{p}$