

Write a Matlab function for Newton's method. Use your Matlab function to solve the following problem to  $\|\mathbf{x}^{(k)} - \mathbf{x}^{(k-1)}\|_\infty < 10^{-5}$ .

$$\begin{aligned}4x_1 - x_2 + x_3 &= x_1x_4, \\-x_1 + 3x_2 - 2x_3 &= x_2x_4, \\x_1 - 2x_2 + 3x_3 &= x_3x_4, \\x_1^2 + x_2^2 + x_3^2 &= 1.\end{aligned}$$

# Newton's Method for System of Nonlinear Equations

Given an initial approximation  $\mathbf{x}$ .

## Algorithm

Step 1. Set  $k = 1$ .

Step 2. While  $k \leq N$  do Steps 3-7.

Step 3. Compute  $\mathbf{F}(\mathbf{x})$  and  $J(\mathbf{x})$ .

Step 4. Solve the  $n \times n$  linear systems  $J(\mathbf{x})\mathbf{y} = -\mathbf{F}(\mathbf{x})$ .

Step 5. Set  $\mathbf{x} := \mathbf{x} + \mathbf{y}$ .

Step 6. If  $\|\mathbf{y}\| < \varepsilon$  then output  $\mathbf{x}$ ; Stop.

Step 7. Set  $k = k + 1$ .

Step 8. Output("Maximum number of iterations exceeded"); Stop.