ASSIGNMENT 3 - due 1 March, 2016

1. **(a.)** Show that $g(x) = \pi + \frac{1}{2}\sin(x/2)$ has a unique fixed point in $[0, 2\pi]$.

(b.) Use fixed-point iteration to find an approximation to the fixed point of $g(x)$. Carry out 4 steps of the iteration using $x = 0$ as the initial approximation. In the computation, write the decimal numbers in the form of $m \times 10^n$ where $1 \leq |m| < 10$ and n is an integer, and keep 5 digits after the decimal point in m.

(c.) Use the Newton's method to find the root of $g(x) - x = 0$. Carry out 4 steps of iteration using $x = 0$ as the initial approximation. Compare the residue $|g(x) - x|$ for the approximate solutions found in (b) and (c).

2. The nonlinear system

$$\begin{aligned} x_1^2 - 10x_1 + x_2^2 + 8 &= 0, \\ x_1 x_2^2 + x_1 - 10x_2 + 8 &= 0 \end{aligned}$$

can be transformed into the fixed-point problem

$$\begin{aligned} x_1 = g_1(x_1, x_2) &= \frac{1}{10}(x_1^2 + x_2^2 + 8), \\ x_2 = g_2(x_1, x_2) &= \frac{1}{10}(x_1 x_2^2 + x_1 + 8). \end{aligned}$$

The map $\mathbf{G} = (g_1, g_2)^t$ has a unique fixed point in

$$D = \left\{(x_1, x_2)^t | 0 \leq x_1, x_2 \leq 1.5\right\}.$$

Use fixed-point iteration with $\mathbf{x}^{(0)} = (0, 0)^t$ to approximate the solution within 10^{-3} (i.e. $\|\mathbf{x}^{(n)} - \mathbf{x}^{(n-1)}\|_\infty < 10^{-3}$).