1. Consider a circle inscribed in a unit square. Given that the circle and the square have a ratio of areas that is $\pi/4$, the value of π can be approximated using a Monte Carlo method.

Generate a number of points that are uniformly distributed on the square (this can be done in Matlab using RAND for the x and y coordinates respectively). Count the number of points inside the circle and the total number of points. The ratio of the two counts is an estimate of the ratio of the two areas, which is $\pi/4$. Investigate the rate of convergence by plotting the error against the number of samples (make a log-log plot).

2. Assume the underlying stock price is a geometric Brownian motion

$$S_t = S_0 \exp\left\{ \left(r - \frac{1}{2}\sigma^2 \right) t + \sigma W_t \right\},$$

where W_t is a standard Brownian motion.

Write a Matlab function to estimate the price of a binary put option with maturity T:

$$p = \mathrm{E}\left[e^{-rT} 1_{\{S_T \le K\}} \right],$$

where $1_{\{S_T \le K\}} = 1$ if $S_T \le K$, and 0 otherwise. The input parameters are S_0, r, σ, K, T, and the sample size n. The function should output the estimate of the price and its standard error. Report your results for

$$S_0 = 30, \ r = 0.05, \ \sigma = 0.2, \ K = 30, \ T = 0.5.$$

Try different sample size (e.g. $n = 10000$, 40000, etc.) and compare the error.