1. The following map $\mathbf{G} : D \subset R^3 \to R^3$ has a unique fixed point in D. Write a Matlab function to approximate the solution to within 10^{-5} in l_∞ norm using the fixed-point iteration.

$$\mathbf{G}(x_1, x_2, x_3) = \left(\frac{1}{3}\left(\cos(x_2 x_3) + 0.5\right), \frac{1}{25}\sqrt{x_1^2 + 0.3125} - 0.03, -\frac{1}{20}e^{-x_1 x_2} - \frac{1}{60}\left(10\pi - 3\right) \right)^T,$$

$$D = \left\{ (x_1, x_2, x_3)^T | -1 \le x_i \le 1,\ i = 1, 2, 3 \right\}.$$

2. The nonlinear system

$$5x_1^2 - x_2^2 = 0, \quad x_2 - 0.25(\sin x_1 + \cos x_2) = 0$$

has a unique solution near $(0.25, 0.25)^T$.

Write a Matlab function to approximate the solution to within 10^{-5} in l_∞ norm using the fixed-point iteration.

Algorithm (Fixed-Point iteration):

Input: initial approximation $x^{(0)}$; tolerance TOL; maximum number of iterations N.
Output: approximation solution x or a message that the number of iteration was exceeded.

Set $k = 1$.
While $k \le N$
    $x^{(k)} = G(x^{(k-1)})$
    if $\|x^{(k)} - x^{(k-1)}\| \le TOL$, then
        Output($x^{(k)}$); STOP
    endif
    set $k := k + 1$
End
Output("Max number of iteration exceeded").