IMU and 6 DoF Odometry (Stereo Visual Odometry) Loosely-Coupled Fusion Localization based on ESKF

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IMU and VO Loose Fusion based on ESKF

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the nominal-state

$$\mathbf{x} = \begin{bmatrix} \mathbf{p} \\ \mathbf{v} \\ \mathbf{q} \\ \mathbf{b}_a \\ \mathbf{b}_g \end{bmatrix} \in \mathbb{R}^{16\times1}$$

the error-state

$$\delta\mathbf{x} = \begin{bmatrix} \delta\mathbf{p} \\ \delta\mathbf{v} \\ \delta\boldsymbol{\theta} \\ \delta\mathbf{b}_a \\ \delta\mathbf{b}_g \end{bmatrix} \in \mathbb{R}^{15\times1}$$

the true-state

$$\mathbf{x}_t = \mathbf{x} \oplus \delta\mathbf{x}$$

The nominal-state kinematics

$$
\begin{aligned}
&\mathbf{p} \leftarrow \mathbf{p} + \mathbf{v}\Delta t + \tfrac{1}{2}\left(\mathbf{R}\left(\mathbf{a}_m - \mathbf{a}_b\right) + \mathbf{g}\right)\Delta t^2 \\
&\mathbf{v} \leftarrow \mathbf{v} + \left(\mathbf{R}\left(\mathbf{a}_m - \mathbf{a}_b\right) + \mathbf{g}\right)\Delta t \\
&\mathbf{q} \leftarrow \mathbf{q} \otimes \mathbf{q}\left\{\left(\boldsymbol{\omega}_m - \boldsymbol{\omega}_b\right)\Delta t\right\} \\
&\mathbf{a}_b \leftarrow \mathbf{a}_b \\
&\boldsymbol{\omega}_b \leftarrow \boldsymbol{\omega}_b
\end{aligned}
$$

The error-state kinematics

$$
\begin{aligned}
\delta\mathbf{p} &\leftarrow \delta\mathbf{p} + \delta\mathbf{v}\Delta t \\
\delta\mathbf{v} &\leftarrow \delta\mathbf{v} + \left(-\mathbf{R}\left[\mathbf{a}_m - \mathbf{a}_b\right]_\times \delta\boldsymbol{\theta} - \mathbf{R}\delta\mathbf{a}_b + \delta\mathbf{g}\right)\Delta t + \mathbf{v_i} \\
\delta\boldsymbol{\theta} &\leftarrow \mathbf{R}^\top\left\{\left(\boldsymbol{\omega}_m - \boldsymbol{\omega}_b\right)\Delta t\right\}\delta\boldsymbol{\theta} - \delta\boldsymbol{\omega}_b\Delta t + \boldsymbol{\theta}_\mathbf{i} \\
\delta\mathbf{a}_b &\leftarrow \delta\mathbf{a}_b + \mathbf{a_i} \\
\delta\boldsymbol{\omega}_b &\leftarrow \delta\boldsymbol{\omega}_b + \boldsymbol{\omega}_\mathbf{i}
\end{aligned}
$$

The error-state system is

$$\delta \mathbf{x} \leftarrow f\left(\mathbf{x}, \delta \mathbf{x}, \mathbf{i}\right) = \mathbf{F}_{\mathbf{x}}\left(\mathbf{x}\right) \cdot \delta \mathbf{x} + \mathbf{F}_{\mathbf{i}} \cdot \mathbf{i}$$

The ESKF prediction equations are written

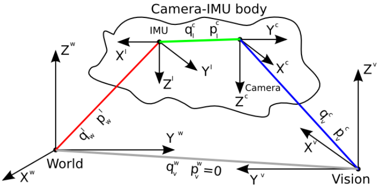
$$\begin{aligned} \hat{\delta \mathbf{x}} &\leftarrow \mathbf{F}_{\mathbf{x}}\left(\mathbf{x}\right) \cdot \hat{\delta \mathbf{x}} \\ \mathbf{P} &\leftarrow \mathbf{F}_{\mathbf{x}} \mathbf{P} \mathbf{F}_{\mathbf{x}}^{\top} + \mathbf{F}_{\mathbf{i}} \mathbf{Q}_{\mathbf{i}} \mathbf{F}_{\mathbf{i}}^{\top} \end{aligned}$$

where

$$\mathbf{F}_{\mathbf{x}} = \left.\frac{\partial f}{\partial \delta \mathbf{x}}\right|_{\mathbf{x}} = \begin{bmatrix} \mathbf{I} & \mathbf{I}\Delta t & 0 & 0 & 0 \\ 0 & \mathbf{I} & -\mathbf{R}\left[\mathbf{a}_m - \mathbf{a}_b\right]_{\times} \Delta t & -\mathbf{R}\Delta t & 0 \\ 0 & 0 & \mathbf{R}^{\top}\left\{\left(\boldsymbol{\omega}_m - \boldsymbol{\omega}_b\right)\Delta t\right\} & 0 & -\mathbf{I}\Delta t \\ 0 & 0 & 0 & \mathbf{I} & 0 \\ 0 & 0 & 0 & 0 & \mathbf{I} \end{bmatrix}$$

$$\mathbf{F}_{\mathbf{i}} = \left.\frac{\partial f}{\partial \mathbf{i}}\right|_{\mathbf{x}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \mathbf{I} & 0 & 0 & 0 \\ 0 & \mathbf{I} & 0 & 0 \\ 0 & 0 & \mathbf{I} & 0 \\ 0 & 0 & 0 & \mathbf{I} \end{bmatrix}$$

$$\mathbf{Q}_{\mathbf{i}} = \begin{bmatrix} \mathbf{V}_{\mathbf{i}} & 0 & 0 & 0 \\ 0 & \boldsymbol{\Theta}_{\mathbf{i}} & 0 & 0 \\ 0 & 0 & \mathbf{A}_{\mathbf{i}} & 0 \\ 0 & 0 & 0 & \Omega_{\mathbf{i}} \end{bmatrix} \quad \text{with} \quad \begin{cases} \mathbf{V}_{\mathbf{i}} = \sigma_{\tilde{\mathbf{a}}_n}^2 \Delta t^2 \mathbf{I} & \left[m^2/s^2\right] \\ \Theta_{\mathbf{i}} = \sigma_{\tilde{\boldsymbol{\omega}}_n}^2 \Delta t^2 \mathbf{I} & \left[rad^2\right] \\ \mathbf{A}_{\mathbf{i}} = \sigma_{\mathbf{a}_w}^2 \Delta t \mathbf{I} & \left[m^2/s^4\right] \\ \boldsymbol{\Omega}_{\mathbf{i}} = \sigma_{\boldsymbol{\omega}_w}^2 \Delta t \mathbf{I} & \left[rad^2/s^2\right] \end{cases}$$



Measurement Function

$$h(\hat{x}) \longleftarrow \underbrace{T_{c_0 c_m} \cdot T_{cb} \cdot T_{b_0 b_m}^{-1}}_{T_{vw}} \cdot T_{b_0 b_n} \cdot T_{cb}^{-1}$$

$$= T_{vw} \cdot T \cdot T_{cb}^{-1}$$

$$= \begin{bmatrix} R_{vw} R R_{cb}^T & R_{vw}(t + R t_{bc}) + R_{c_0 c_m} t_{cb} + t_{c_0 c_m} \\ 0 & 1 \end{bmatrix}$$

Measurement Residual

$$r = z \ominus h(\hat{x}) = \begin{bmatrix} t_z - \hat{t} \\ \theta_z \ominus \hat{\theta} \end{bmatrix} = \begin{bmatrix} t_z - \hat{t} \\ 2\,[\hat{q}^* \otimes q_z]_{vec} \end{bmatrix} \in \mathbb{R}^6$$

Measurement Jacobian Matrix w.r.t Error-State

$$H = \left.\frac{\partial h(x)}{\partial \delta x}\right|_{\mathbf{x}=\hat{\mathbf{x}}} = -\left.\frac{\partial r}{\partial \delta x}\right|_{\mathbf{x}=\hat{\mathbf{x}}}$$

h_t w.r.t δt

$$\frac{\partial h_t}{\partial \delta t} = R_{vw}$$

h_t w.r.t $\delta\theta$

$$\begin{aligned}\frac{\partial h_t}{\partial \delta\theta} &= \frac{\partial R_{vw} R t_{bc}}{\partial \delta\theta} \\ &= R_{vw} \cdot \frac{\partial R t_{bc}}{\partial \delta\theta} \\ &= -R_{vw} \cdot R \cdot t_{bc}^{\wedge}\end{aligned}$$

h_θ w.r.t $\delta\theta$

$$\begin{aligned}\frac{\partial h_\theta}{\partial \delta\theta} &= \frac{\partial \theta\{R_{vw} R R_{cb}^T\}}{\partial \delta\theta} \\ &= \frac{\partial 2\left[q_{vw} \otimes q \otimes q_{cb}^T\right]_{vec}}{\partial \delta\theta} \\ &= 2\,[0 \quad I] \cdot \frac{\partial q_{vw} q q_{cb}^T}{\partial \delta\theta} \\ &= 2\,[0 \quad I] \cdot \frac{\partial q_{vw} \otimes (q \otimes \delta q) \otimes q_{cb}^T}{\partial \delta q} \cdot \frac{\partial \delta q}{\partial \delta\theta} \\ &= 2\,[0 \quad I] \cdot \frac{\partial L(q_{vw} \otimes q) \cdot R(q_{cb}^T) \cdot \delta q}{\partial \delta q} \cdot \frac{\partial \begin{bmatrix} 1 \\ \frac{1}{2}\delta\theta \end{bmatrix}}{\partial \delta\theta} \\ &= [0 \quad I]_{3\times 4} \cdot L(q_{vw} \otimes q) \cdot R(q_{cb}^T) \cdot \begin{bmatrix} 0 \\ I \end{bmatrix}_{4\times 3}\end{aligned}$$

the Kalman Gain

$$K = PH^T(HPH^T + R)^{-1}$$

the error-state

$$\delta x \longleftarrow Kr$$

the covariance matrix

$$P \longleftarrow (I - KH)P$$

the best true-state estimation

$$x_t = x \oplus \delta x \longrightarrow \begin{aligned} p &= \hat{p} + \delta p \\ v &= \hat{v} + \delta v \\ R &= \hat{R} \cdot \Delta R\{\delta\theta\} \\ b_a &= \hat{b_a} + \delta b_a \\ b_g &= \hat{b_g} + \delta b_g \end{aligned}$$

1. Measurement Function
2. Measurement Jacobian Matrix of ESKF
3. Rotation Perturbation in the Formula of Residual and Correction
4. EKF Tuning: R, Q, P

Thank You!