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IMU and VO Loose Fusion based on ESKF

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July 19, 2021

IMU and VO
Fusion based
on ESKFHongchen
Gao

System State

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ThankYou

the nominal-state

$$\mathbf{x} = \begin{bmatrix} \mathbf{p} \\ \mathbf{v} \\ \mathbf{q} \\ \mathbf{b}_a \\ \mathbf{b}_g \end{bmatrix} \in \mathbb{R}^{16 \times 1}$$

the error-state

$$\delta \mathbf{x} = \begin{bmatrix} \delta \mathbf{p} \\ \delta \mathbf{v} \\ \delta \boldsymbol{\theta} \\ \delta \mathbf{b}_a \\ \delta \mathbf{b}_g \end{bmatrix} \in \mathbb{R}^{15 \times 1}$$

the true-state

$$\mathbf{x}_t = \mathbf{x} \oplus \delta \mathbf{x}$$

IMU and VO Fusion based on ESKF

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System State

State Prediction

IMU-driven system kinematics in discrete time

The error-state Jacobian and perturbation matrices

Measurement Update (Fusing IMU with VO data)

Measurement Models

Measurement Jacobian Matrix

Filter Correction

QA & Challenges

ThankYou

The nominal-state kinematics

$$\mathbf{p} \leftarrow \mathbf{p} + \mathbf{v}\Delta t + \frac{1}{2} (\mathbf{R} (\mathbf{a}_m - \mathbf{a}_b) + \mathbf{g}) \Delta t^2$$

$$\mathbf{v} \leftarrow \mathbf{v} + (\mathbf{R} (\mathbf{a}_m - \mathbf{a}_b) + \mathbf{g}) \Delta t$$

$$\mathbf{q} \leftarrow \mathbf{q} \otimes \mathbf{q} \{(\boldsymbol{\omega}_m - \boldsymbol{\omega}_b) \Delta t\}$$

$$\mathbf{a}_b \leftarrow \mathbf{a}_b$$

$$\boldsymbol{\omega}_b \leftarrow \boldsymbol{\omega}_b$$

The error-state kinematics

$$\delta \mathbf{p} \leftarrow \delta \mathbf{p} + \delta \mathbf{v} \Delta t$$

$$\delta \mathbf{v} \leftarrow \delta \mathbf{v} + (-\mathbf{R} [\mathbf{a}_m - \mathbf{a}_b]_{\times} \delta \boldsymbol{\theta} - \mathbf{R} \delta \mathbf{a}_b + \delta \mathbf{g}) \Delta t + \mathbf{v}_i$$

$$\delta \boldsymbol{\theta} \leftarrow \mathbf{R}^{\top} \{(\boldsymbol{\omega}_m - \boldsymbol{\omega}_b) \Delta t\} \delta \boldsymbol{\theta} - \delta \boldsymbol{\omega}_b \Delta t + \boldsymbol{\theta}_i$$

$$\delta \mathbf{a}_b \leftarrow \delta \mathbf{a}_b + \mathbf{a}_i$$

$$\delta \boldsymbol{\omega}_b \leftarrow \delta \boldsymbol{\omega}_b + \boldsymbol{\omega}_i$$

The error-state system is

$$\delta \mathbf{x} \leftarrow f(\mathbf{x}, \delta \mathbf{x}, \mathbf{i}) = \mathbf{F}_{\mathbf{x}}(\mathbf{x}) \cdot \delta \mathbf{x} + \mathbf{F}_{\mathbf{i}} \cdot \mathbf{i}$$

The ESKF prediction equations are written

$$\begin{aligned}\hat{\delta \mathbf{x}} &\leftarrow \mathbf{F}_{\mathbf{x}}(\mathbf{x}) \cdot \delta \mathbf{x} \\ \mathbf{P} &\leftarrow \mathbf{F}_{\mathbf{x}} \mathbf{P} \mathbf{F}_{\mathbf{x}}^{\top} + \mathbf{F}_{\mathbf{i}} \mathbf{Q}_{\mathbf{i}} \mathbf{F}_{\mathbf{i}}^{\top}\end{aligned}$$

where

$$\mathbf{F}_{\mathbf{x}} = \left. \frac{\partial f}{\partial \delta \mathbf{x}} \right|_{\mathbf{x}} = \begin{bmatrix} \mathbf{I} & \mathbf{I} \Delta t & 0 & 0 \\ 0 & \mathbf{I} & -\mathbf{R} [\mathbf{a}_m - \mathbf{a}_b]_{\times} \Delta t & -\mathbf{R} \Delta t \\ 0 & 0 & \mathbf{R}^{\top} \{(\boldsymbol{\omega}_m - \boldsymbol{\omega}_b) \Delta t\} & 0 \\ 0 & 0 & 0 & -\mathbf{I} \Delta t \\ 0 & 0 & 0 & \mathbf{I} \end{bmatrix}$$

$$\mathbf{F}_{\mathbf{i}} = \left. \frac{\partial f}{\partial \mathbf{i}} \right|_{\mathbf{x}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \mathbf{I} & 0 & 0 & 0 \\ 0 & \mathbf{I} & 0 & 0 \\ 0 & 0 & \mathbf{I} & 0 \\ 0 & 0 & 0 & \mathbf{I} \end{bmatrix}$$

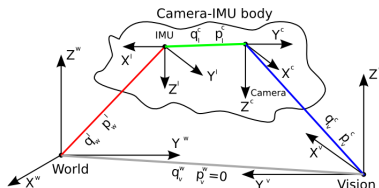
$$\mathbf{Q}_{\mathbf{i}} = \begin{bmatrix} \mathbf{V}_{\mathbf{i}} & 0 & 0 & 0 \\ 0 & \boldsymbol{\Theta}_{\mathbf{i}} & 0 & 0 \\ 0 & 0 & \mathbf{A}_{\mathbf{i}} & 0 \\ 0 & 0 & 0 & \boldsymbol{\Omega}_{\mathbf{i}} \end{bmatrix} \quad \text{with} \quad \begin{cases} \mathbf{V}_{\mathbf{i}} = \sigma_{\tilde{\mathbf{a}}_n}^2 \Delta t^2 \mathbf{I} \\ \boldsymbol{\Theta}_{\mathbf{i}} = \sigma_{\tilde{\boldsymbol{\omega}}_n}^2 \Delta t^2 \mathbf{I} \\ \mathbf{A}_{\mathbf{i}} = \sigma_{\tilde{\mathbf{a}}_w}^2 \Delta t \mathbf{I} \\ \boldsymbol{\Omega}_{\mathbf{i}} = \sigma_{\tilde{\boldsymbol{\omega}}_w}^2 \Delta t \mathbf{I} \end{cases} \begin{bmatrix} m^2/s^2 \\ rad^2 \\ m^2/s^4 \\ rad^2/s^2 \end{bmatrix}$$

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Measurement Function

$$\begin{aligned}
 h(\hat{x}) &\leftarrow \underbrace{T_{c_0 c_m} \cdot T_{cb} \cdot T_{b_0 b_m}^{-1}}_{T_{vw}} \cdot T_{b_0 b_n} \cdot T_{cb}^{-1} \\
 &= T_{vw} \cdot T \cdot T_{cb}^{-1} \\
 &= \begin{bmatrix} R_{vw} R R_{cb}^T & R_{vw}(t + R t_{bc}) + R_{c_0 c_m} t_{cb} + t_{c_0 c_m} \\ 0 & 1 \end{bmatrix}
 \end{aligned}$$

Measurement Residual

$$r = z \ominus h(\hat{x}) = \begin{bmatrix} t_z - \hat{t} \\ \theta_z \ominus \hat{\theta} \end{bmatrix} = \begin{bmatrix} t_z - \hat{t} \\ 2[\hat{q}^* \otimes q_z]_{vec} \end{bmatrix} \in \mathbb{R}^6$$

Measurement Jacobian Matrix w.r.t Error-State

$$H = \left. \frac{\partial h(x)}{\partial \delta x} \right|_{\mathbf{x}=\hat{\mathbf{x}}} = - \left. \frac{\partial \mathbf{r}}{\partial \delta x} \right|_{\mathbf{x}=\hat{\mathbf{x}}}$$

 h_t w.r.t δt

$$\frac{\partial h_t}{\partial \delta t} = R_{vw}$$

 h_t w.r.t $\delta \theta$

$$\begin{aligned} \frac{\partial h_t}{\partial \delta \theta} &= \frac{\partial R_{vw} R t_{bc}}{\partial \delta \theta} \\ &= R_{vw} \cdot \frac{\partial R t_{bc}}{\partial \delta \theta} \\ &= -R_{vw} \cdot R \cdot t_{bc}^{\wedge} \end{aligned}$$

 h_{θ} w.r.t $\delta \theta$

$$\begin{aligned} \frac{\partial h_{\theta}}{\partial \delta \theta} &= \frac{\partial \theta \{R_{vw} R R_{cb}^T\}}{\partial \delta \theta} \\ &= \frac{\partial^2 [q_{vw} \otimes q \otimes q_{cb}^T]_{vec}}{\partial \delta \theta} \\ &= 2 [0 \quad I] \cdot \frac{\partial q_{vw} q q_{cb}^T}{\partial \delta \theta} \\ &= 2 [0 \quad I] \cdot \frac{\partial q_{vw} \otimes (q \otimes \delta q) \otimes q_{cb}^T}{\partial \delta q} \cdot \frac{\partial \delta q}{\partial \delta \theta} \\ &= 2 [0 \quad I] \cdot \frac{\partial L(q_{vw} \otimes q) \cdot R(q_{cb}^T) \cdot \delta q}{\partial \delta q} \cdot \frac{\partial \begin{bmatrix} 1 \\ \frac{1}{2} \delta \theta \end{bmatrix}}{\partial \delta \theta} \\ &= [0 \quad I]_{3 \times 4} \cdot L(q_{vw} \otimes q) \cdot R(q_{cb}^T) \cdot \begin{bmatrix} 0 \\ I \end{bmatrix}_{4 \times 3} \end{aligned}$$

the Kalman Gain

$$K = PH^T(HPH^T + R)^{-1}$$

the error-state

$$\delta x \leftarrow Kr$$

the covariance matrix

$$P \leftarrow (I - KH)P$$

the best true-state estimation

$$p = \hat{p} + \delta p$$

$$v = \hat{v} + \delta v$$

$$x_t = x \oplus \delta x \longrightarrow R = \hat{R} \cdot \Delta R\{\delta\theta\}$$

$$b_a = \hat{b}_a + \delta b_a$$

$$b_g = \hat{b}_g + \delta b_g$$

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- 1 Measurement Function
- 2 Measurement Jacobian Matrix of ESKF
- 3 Rotation Perturbation in the Formula of Residual and Correction
- 4 EKF Tuning: R , Q , P

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