

1.1-1.2 如下

1.1 绘制信息矩阵  $\Lambda$

	$\epsilon_1$	$\epsilon_2$	$\epsilon_3$	$L_1$	$L_2$	$L_3$
$\epsilon_1$	shaded	shaded	white	shaded	shaded	white
$\epsilon_2$	shaded	shaded	shaded	shaded	shaded	shaded
$\epsilon_3$	white	shaded	shaded	white	shaded	shaded
$L_1$	shaded	shaded	white	shaded	white	white
$L_2$	shaded	shaded	shaded	white	shaded	white
$L_3$	white	shaded	shaded	white	white	shaded

1.2.  $\epsilon_1$  被 marg 后的信息矩阵  $\Lambda'$

$$\begin{array}{c}
 \begin{array}{c|ccccc}
 & \epsilon_2 & \epsilon_3 & L_1 & L_2 & L_3 \\
 \hline
 \epsilon_2 & \text{shaded} & \text{shaded} & \text{shaded} & \text{shaded} & \text{shaded} \\
 \epsilon_3 & \text{shaded} & \text{shaded} & \text{white} & \text{shaded} & \text{shaded} \\
 L_1 & \text{shaded} & \text{white} & \text{shaded} & \text{white} & \text{white} \\
 L_2 & \text{shaded} & \text{shaded} & \text{white} & \text{shaded} & \text{white} \\
 L_3 & \text{shaded} & \text{shaded} & \text{white} & \text{white} & \text{shaded}
 \end{array} \\
 \Lambda_{22}
 \end{array}
 -
 \begin{array}{c}
 \begin{array}{ccccc}
 \text{shaded} & \text{white} & \text{shaded} & \text{white} & \text{white} \\
 \text{white} & \text{white} & \text{white} & \text{white} & \text{white} \\
 \text{shaded} & \text{white} & \text{shaded} & \text{shaded} & \text{white} \\
 \text{shaded} & \text{white} & \text{shaded} & \text{shaded} & \text{white} \\
 \text{white} & \text{white} & \text{white} & \text{white} & \text{white}
 \end{array} \\
 \Lambda_{22} \Lambda_{\epsilon_1 \epsilon_1}^{-1} \Lambda_{\epsilon_1 \epsilon_2}
 \end{array}
 =
 \begin{array}{c}
 \begin{array}{c|ccccc}
 & \epsilon_2 & \epsilon_3 & L_1 & L_2 & L_3 \\
 \hline
 \epsilon_2 & \text{shaded} & \text{shaded} & \text{shaded} & \text{shaded} & \text{shaded} \\
 \epsilon_3 & \text{shaded} & \text{shaded} & \text{white} & \text{shaded} & \text{shaded} \\
 L_1 & \text{shaded} & \text{white} & \text{shaded} & \text{white} & \text{white} \\
 L_2 & \text{shaded} & \text{shaded} & \text{white} & \text{shaded} & \text{white} \\
 L_3 & \text{shaded} & \text{shaded} & \text{white} & \text{white} & \text{shaded}
 \end{array} \\
 \Lambda'
 \end{array}$$

2 证明如下

2. 证明如下：

$$p(\theta) = (2\pi)^{-\frac{N\theta}{2}} |\Sigma_\theta|^{-\frac{1}{2}} \exp \left[ -\frac{1}{2}(\theta - \theta^*)^T \Sigma_\theta^{-1} (\theta - \theta^*) \right]$$

$$J(\theta) = -\ln p(\theta) = \frac{N\theta}{2} \ln 2\pi + \frac{1}{2} \ln |\Sigma_\theta| + \frac{1}{2}(\theta - \theta^*)^T \Sigma_\theta^{-1} (\theta - \theta^*)$$

$$H^{(l,l')}(\theta^*) = \frac{\partial^2 J(\theta)}{\partial \theta_l \partial \theta_{l'}} \Big|_{\theta=\theta^*} = \left[ \frac{\partial}{\partial \theta_l} \left( \frac{\partial J(\theta)}{\partial \theta_{l'}} \right) \right]_{\theta=\theta^*} = \left[ \frac{\partial J'(\theta)}{\partial \theta_l} \right]_{\theta=\theta^*}$$

对  $J(\theta)$  求一次偏导，得

$$dJ = \left( \frac{1}{2} d\theta \right)^T (\Sigma_\theta^{-1} \theta - \Sigma_\theta^{-1} \theta^*) + \frac{1}{2} (\theta - \theta^*)^T (\Sigma_\theta^{-1} d\theta)$$

$$= (\Sigma_\theta^{-1} \theta - \Sigma_\theta^{-1} \theta^*)^T \left( \frac{1}{2} d\theta \right) + \frac{1}{2} (\theta - \theta^*)^T (\Sigma_\theta^{-1} d\theta)$$

$$= (\Sigma_\theta^{-1} (\theta - \theta^*))^T \left( \frac{1}{2} d\theta \right) + \frac{1}{2} (\theta - \theta^*)^T (\Sigma_\theta^{-1} d\theta)$$

$$= (\theta - \theta^*)^T (\Sigma_\theta^{-1})^T \left( \frac{1}{2} d\theta \right) + \frac{1}{2} (\theta - \theta^*)^T (\Sigma_\theta^{-1} d\theta)$$

$$= \frac{1}{2} (\theta - \theta^*)^T (\Sigma_\theta^{-1})^T (d\theta) + \frac{1}{2} (\theta - \theta^*)^T \Sigma_\theta^{-1} (d\theta)$$

$$= \frac{1}{2} (\theta - \theta^*)^T \Sigma_\theta^{-1} (d\theta) + \frac{1}{2} (\theta - \theta^*)^T \Sigma_\theta^{-1} (d\theta)$$

$$= (\theta - \theta^*)^T \Sigma_\theta^{-1} (d\theta)$$

$$\frac{\partial J}{\partial \theta} = (\Sigma_\theta^{-1})^T (\theta - \theta^*) = J'(\theta)$$

$$2.1) \frac{\partial J'}{\partial \theta} = \Sigma_\theta^{-1}$$

3 代码和结果如下

```
H.block(i*6,i*6,6,6) += jacobian_Ti.transpose() * jacobian_Ti;
H.block(i*6, poseNums*6+j*3, 6, 3) += jacobian_Ti.transpose() * jacobian_Pj;
H.block(poseNums*6+j*3, i*6, 3, 6) += jacobian_Pj.transpose() * jacobian_Ti;
H.block(poseNums*6+j*3, poseNums*6+j*3, 3, 3) += jacobian_Pj.transpose() * jacobian_Pj;
```

3.21708e-17

2.06732e-17

1.43188e-17

7.66992e-18

6.08423e-18

6.05715e-18

3.94363e-18