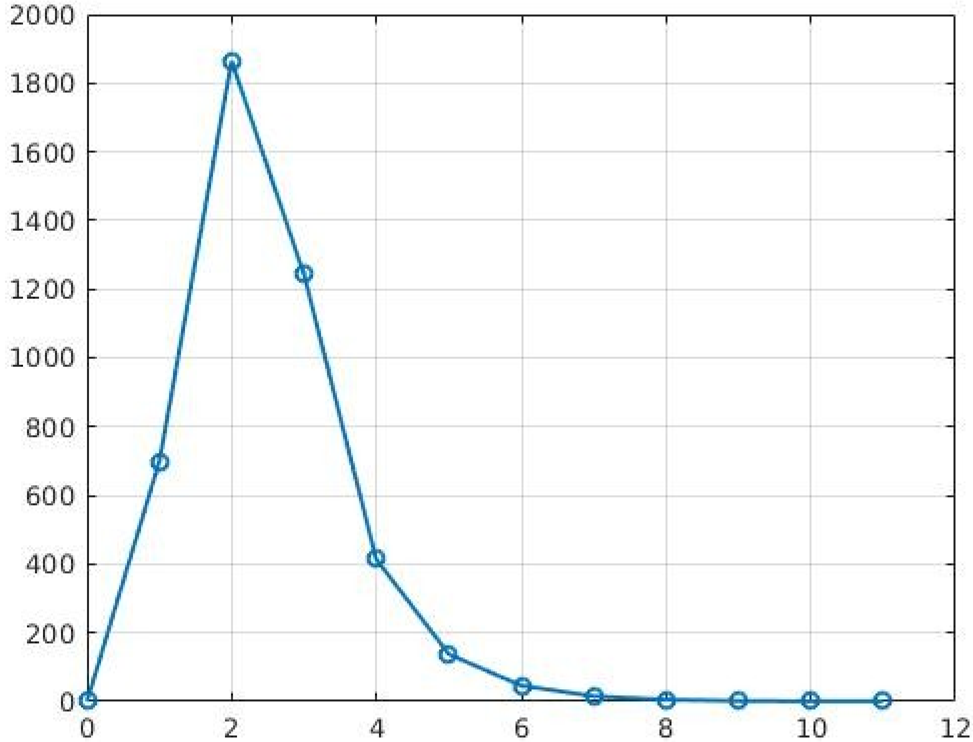
1.1 代码运行如下

```
zy@zy-ThinkPad-E490 ~/CurveFitting_LM/build/app $ ./testCurveFitting

Test CurveFitting start...
iter: 0 , chi= 36048.3 , Lambda= 0.001
iter: 1 , chi= 30015.5 , Lambda= 699.051
iter: 2 , chi= 13421.2 , Lambda= 1864.14
iter: 3 , chi= 7273.96 , Lambda= 1242.76
iter: 4 , chi= 269.255 , Lambda= 414.252
iter: 5 , chi= 105.473 , Lambda= 138.084
iter: 6 , chi= 100.845 , Lambda= 46.028
iter: 7 , chi= 95.9439 , Lambda= 15.3427
iter: 8 , chi= 92.3017 , Lambda= 5.11423
iter: 9 , chi= 91.442 , Lambda= 1.70474
iter: 10 , chi= 91.3963 , Lambda= 0.568247
iter: 11 , chi= 91.3959 , Lambda= 0.378832
problem solve cost: 0.572448 ms
   makeHessian cost: 0.332089 ms
-------After optimization, we got these parameters :
0.941939  2.09453 0.965586
-------ground truth:
1.0,  2.0,  1.0
```

曲线图如下

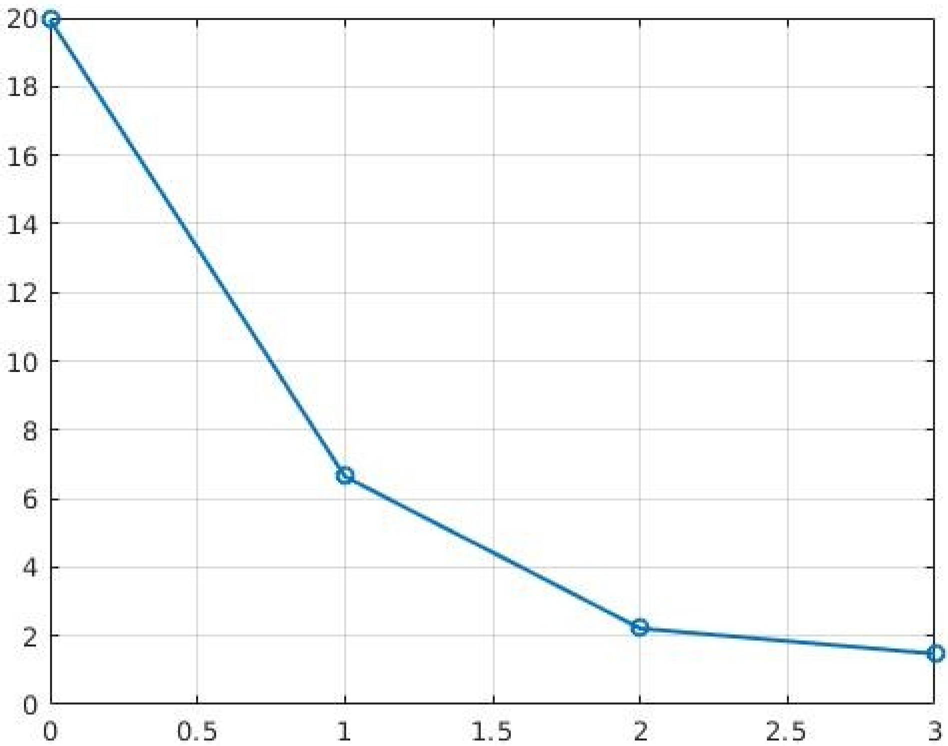


1.2 代码运行如下，代码见附件

```
zy@zy-ThinkPad-E490 ~/CurveFitting_LM/build $ ./app/testCurveFitting

Test CurveFitting start...
iter: 0 , chi= 3.21386e+06 , Lambda= 19.95
iter: 1 , chi= 974.658 , Lambda= 6.65001
iter: 2 , chi= 973.881 , Lambda= 2.21667
iter: 3 , chi= 973.88 , Lambda= 1.47778
problem solve cost: 1.21235 ms
   makeHessian cost: 0.986622 ms
-------After optimization, we got these parameters :
0.999588   2.0063 0.968786
-------ground truth:
1.0,  2.0,  1.0
```

曲线图如下



2 公式推导如下

$$f_{15} = \frac{\partial \partial_{b_i b_{k+1}}}{\partial \delta b_k^g} = \frac{\partial \frac{1}{2} a \delta t^2}{\partial \delta b_k^g} = \frac{1}{4}\delta t^2 \frac{\partial \left[q_{b_i b_k} \otimes \left[\frac{1}{\frac{1}{2} \omega' \delta t} \right] (a^{b_{k+1}} - b_k^a) \right]}{\partial \delta b_k^g}$$

$$= \frac{1}{4}\delta t^2 \frac{\partial \left[R_{b_i b_k} \exp([\omega \delta t]_\times)(a^{b_{k+1}} - b_k^a) \right]}{\partial \delta b_k^g}$$

$$= \frac{1}{4}\delta t^2 \lim_{\delta b_k^g \to 0} \frac{R_{b_i b_k} \exp([\omega - \delta b_k^g]_\times \delta t)(a^{b_{k+1}} - b_k^a) - R_{b_i b_k} \exp([\omega]_\times \delta t)(a^{b_{k+1}} - b_k^a)}{\delta b_k^g}$$

$$= \frac{1}{4}\delta t^2 \lim_{\delta b_k^g \to 0} \frac{R_{b_i b_k} \exp([\omega \delta t]_\times) \exp([J_r(\omega \delta t) \cdot (-\delta b_k^g \delta t)]_\times) - R_{b_i b_k} \exp([\omega \delta t]_\times)(a^{b_{k+1}} - b_k^a)}{\delta b_k^g}$$

$$= \frac{1}{4}\delta t^2 \lim_{\delta b_k^g \to 0} \frac{R_{b_i b_k} \exp([\omega \delta t]_\times)(I + [-J_r(\omega \delta t)\delta b_k^g \delta t]_\times) - R_{b_i b_k} \exp([\omega \delta t]_\times))(a^{b_{k+1}} - b_k^a)}{\delta b_k^g}$$

$$= \frac{1}{4}\delta t^2 \lim_{\delta b_k^g \to 0} \frac{R_{b_i b_k} \exp([\omega \delta t]_\times)[-J_r(\omega \delta t)\delta b_k^g \delta t]_\times (a^{b_{k+1}} - b_k^a)}{\delta b_k^g}$$

$$= \frac{1}{4}\delta t^2 \lim_{\delta b_k^g \to 0} \frac{-R_{b_i b_k} \exp([\omega \delta t]_\times)[a^{b_{k+1}} - b_k^a]_\times (-J_r(\omega \delta t)\delta b_k^g \delta t)}{\delta b_k^g}$$

$$= \frac{1}{4} R_{b_i b_k} \exp([\omega \delta t]_\times)[a^{b_{k+1}} - b_k^a]_\times J_r(\omega \delta t)\delta t^3$$

$$= \frac{1}{4} R_{b_i b_{k+1}} [a^{b_{k+1}} - b_k^a]_\times J_r(\omega \delta t)\delta t^3$$

$$\approx \frac{1}{4} R_{b_i b_{k+1}} [a^{b_{k+1}} - b_k^a]_\times \delta t^3$$

$$g_{12} = \frac{\partial \alpha_{b_i b_{k+1}}}{\partial \delta n_k^g} = \frac{\partial \frac{1}{2} a \delta t^2}{\partial \delta n_k^g} = \frac{1}{4}\delta t^2 \frac{\partial [q_{b_i b_k} \otimes \begin{bmatrix} 1 \\ \frac{1}{2}\omega \delta t \end{bmatrix} (a^{b_{k+1}} - b_k^a)]}{\partial \delta n_k^g}$$

$$= \frac{1}{4}\delta t^2 \frac{\partial [R_{b_i b_k} \exp([\omega \delta t]_\times)(a^{b_{k+1}} - b_k^a)]}{\partial \delta n_k^g}$$

$$= \frac{1}{4}\delta t^2 \lim_{\delta n_k^g \to 0} \frac{R_{b_i b_k} \exp([\omega + \frac{1}{2}\delta n_k^g]_\times \delta t)(a^{b_{k+1}} - b_k^a) - R_{b_i b_k} \exp([\omega]_\times \delta t)(a^{b_{k+1}} - b_k^a)}{\delta n_k^g}$$

$$= \frac{1}{4}\delta t^2 \lim_{\delta n_k^g \to 0} \frac{R_{b_i b_k} \exp([\omega \delta t]_\times) \exp([J_r(\omega \delta t) \frac{1}{2}\delta n_k^g \delta t]_\times) - R_{b_i b_k} \exp([\omega \delta t]_\times)(a^{b_{k+1}} - b_k^a)}{\delta n_k^g}$$

$$= \frac{1}{4}\delta t^2 \lim_{\delta n_k^g \to 0} \frac{R_{b_i b_k} \exp([\omega \delta t]_\times)(I + [\frac{1}{2} J_r(\omega \delta t) \delta n_k^g \delta t]_\times) - R_{b_i b_k} \exp([\omega \delta t]_\times))(a^{b_{k+1}} - b_k^a)}{\delta n_k^g}$$

$$= \frac{1}{4}\delta t^2 \lim_{\delta n_k^g \to 0} \frac{R_{b_i b_k} \exp([\omega \delta t]_\times)[\frac{1}{2} J_r(\omega \delta t) \delta n_k^g \delta t]_\times (a^{b_{k+1}} - b_k^a)}{\delta n_k^g}$$

$$= \frac{1}{4}\delta t^2 \lim_{\delta n_k^g \to 0} \frac{-R_{b_i b_k} \exp([\omega \delta t]_\times)[a^{b_{k+1}} - b_k^a]_\times \frac{1}{2} J_r(\omega \delta t) \delta t \delta n_k^g}{\delta n_k^g}$$

$$= -\frac{1}{8} R_{b_i b_k} R_{b_i b_k} \exp([\omega \delta t]_\times)[a^{b_{k+1}} - b_k^a]_\times J_r(\omega \delta t) \delta t^3$$

$$= -\frac{1}{8} R_{b_i b_{k+1}} [a^{b_{k+1}} - b_k^a]_\times J_r(\omega \delta t) \delta t^3$$

$$\approx -\frac{1}{8} R_{b_i b_{k+1}} [a^{b_{k+1}} - b_k^a]_\times \delta t^3$$

3 证明如下

$$\Delta x = \arg\min_{\Delta x} F(x+\Delta x) = \arg\min_{\Delta x} \frac{1}{2}\|f(x+\Delta x)\|^2 = \arg\min_{\Delta x} \frac{1}{2}\|f(x+J\Delta x)\|^2$$

$$= \arg\min_{\Delta x} \left\{ \frac{1}{2} f^T(x) f(x) + \Delta x^T J^T f(x) + \frac{1}{2}\Delta x^T J^T J \Delta x \right\}$$

令 $\frac{\partial F(x+\Delta x)}{\partial \Delta x} = 0 \Rightarrow (J^T J + \mu I)\Delta x = -J^T f(x)$

对 $J^T J$ 进行分解: $J^T J = V \Lambda V^T = [v_1\ v_2 \cdots v_m] \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \ddots \\ & & & \lambda_m \end{bmatrix} \begin{bmatrix} v_1^T \\ v_2^T \\ \vdots \\ v_m^T \end{bmatrix}$

$$= v_1 \lambda_1 v_1^T + v_2 \lambda_2 v_2^T + \cdots v_m \lambda_m v_m^T$$

则 $J^T J + \mu I = V \Lambda V^T + V V^T \mu I V V^T$

$$= V \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_m \end{bmatrix} V^T + V \begin{bmatrix} u & & \\ & \ddots & \\ & & u \end{bmatrix} V^T$$

$$= [v_1\ v_2 \cdots v_m] \begin{bmatrix} \lambda_1 + u & & \\ & \ddots & \\ & & \lambda_m + \mu \end{bmatrix} \begin{bmatrix} v_1^T \\ v_2^T \\ v_m^T \end{bmatrix} = V \Lambda' V^T$$

将 $\Lambda' = V^T(J^T J + \mu I)V$, $F'(x) = (J^T f(x))^T$ 代入 $(J^T J + \mu I)\Delta x = -J^T f(x)$

$$V^T (J^T J + \mu I) V \cdot V^T \Delta x = -V^T F'(x)^T$$

$$\Lambda' (V^T \Delta x) = -V^T F'(x)^T$$

$$V^T \Delta x = -\Lambda'^{-1} V^T F'(x)^T$$

$$\Delta x = -V\Lambda'^{-1} V^T F'(x)^T = -[v_1\ v_2 \cdots v_m] \begin{bmatrix} \lambda_1 + u & & \\ & \ddots & \\ & & \lambda_m + u \end{bmatrix}^{-1} \begin{bmatrix} v_1^T \\ \vdots \\ v_m^T \end{bmatrix} F'(x)^T$$

$$= -\left\{ \sum_{j=1}^{m} \frac{v_j v_j^T}{\lambda_j + u} \right\} F'(x)^T$$

$$= -\sum_{j=1}^{m} \frac{v_j v_j^T F'(x)^T}{\lambda_j + \mu}$$