2/13/09

Investigation of Long Memory in Quarterly Home Price Index Data

Vikas Garg, Yasin Khan, Shirley Lu, Timothy Roberts, Yifan Tang, Wesley Tillu

I. Introduction

This project seeks to understand the housing price process in a discrete time series setting. As key drivers of mortgage rates and valuation, addressing house price evolution is critical to investing in mortgage backed assets. Fundamental considerations relevant to our project scope will include, but will not be limited to, the effect of default and prepayment risk on mortgage pool valuation and the significance of geography in the housing market. Additionally, this project will investigate the presence of long memory in the housing price process, with a specific focus on long memory in housing price volatility.

In general, an understanding of the housing price process should lend itself to a model from which time series forecasts can be derived using appropriate methods and techniques.

II. Methods

Should describe data and their source(s)

A. Initial Analysis

1) Exploratory Data Analysis

The following steps were followed for the initial analysis of the data:

The simple return series was created from the HPI data and time series plots were generated for all the states. If the series is stationary we determine the AR/MA, ARMA order (using the PACF, ACF plots) and estimate the parameters of the model. If the series is not stationary, it's tested for unit root stationarity. We create the first differenced series, for series which show the presence of a unit root, and try fitting an appropriate ARIMA model to the series. Box-Ljung test is used on residuals and squared residuals to test the model adequacy and presence of ARCH effects. Given below are charts prepared for AK, which show a non stationary behavior in the simple return series, and a slowly decaying ACF plot. But the differenced series appears to be stationary.

define

Hard to read — needs larger fonts.

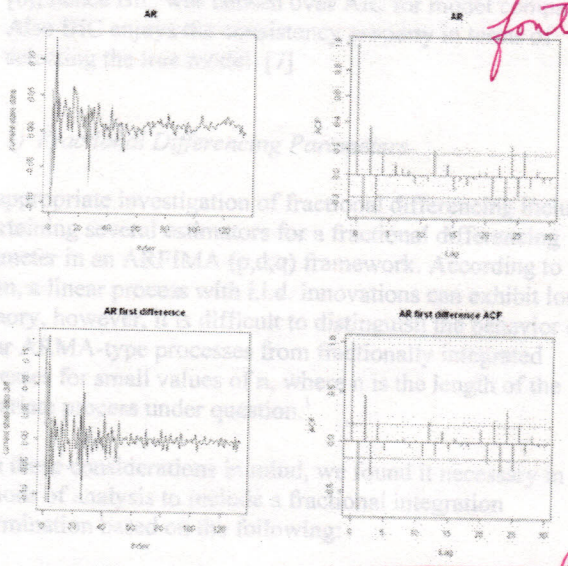

Figure 1.1.1: Arkansas OFHEO HPI simple returns, autocorrelation function of simple returns, simple returns first difference, and first-differenced simple returns autocorrelation function.

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2) Data Transformations

Many parameter estimators are based on the assumption of Gaussian distribution. In particular, the Whittle estimator for the fractional differencing parameter is one such estimator. The Whittle estimator is generally preferred over other fractional differencing parameters as it is consistent, unbiased, and asymptotically efficient.

The Box Cox transformation aims to apply a power transformation to ensure that the data follows an approximately normal distribution. The Box Cox transformation stabilizes variance and induces homoscedacity in the time series. It is defined as

$$y^{(\lambda)} = \begin{cases} \dfrac{y^{\lambda} - 1}{\lambda}, \lambda \neq 0 \\ \log(y), \lambda = 0 \end{cases}$$

Not a monotonic transformation

In practice, the y's must be strictly positive. The simple returns series of the state-level housing price data include negative data points and thus, the absolute value of the minimum observed data value plus a small epsilon was added to each data point.

Not the usual method to apply Box-Cox. Rather, Box-Cox is applied before differencing.

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If the return series failed prior tests of normality, the Box Cox test was used to estimate the appropriate power of λ along with its standard error. Then, the interval (λ +/- std. error) was evaluated to determine the power to apply. If the interval contained 0, a log transformation was used. Otherwise, the estimate or its closest fraction was used to transform the data.

The effectiveness of the data transformation was tested graphically with density plots and statistically using formal normality tests.

A weakness of the Box Cox transformation is that if the true distribution is far from Gaussian, there will not exist a sufficient λ that will make the data normal. However, even in cases where no power transformation could bring the distribution to exactly normal, the usual estimates of λ will lead to a distribution that satisfies certain restrictions on the first 4 moments, thus will be usually symmetric.

3) I(1) Testing

To test for unit root stationary, primarily, the Augmented Dickey-Fuller (ADF) test was used. The ADF test checks for the presence of unit root in the time series. The hypothesis tests are summarized as follows:

Ho: yt ~ I(1), series contains a unit root
Ha: yt ~ I(0), series is stationary
where $y_t \sim$ i.i.d

The ADF test was implemented using adf.test within R (using the package "tseries"). To choose the lag input, the order obtained through an AIC vs. BIC calculation was used; subsequently, the lowest lag order was chosen.

Additionally, the Phillips-Perron test was added, using pp.test within R, to substantiate test results from the ADF test. The hypotheses are the same as those from the ADF test.

4) Normality Testing

As stated before, testing the univariate data for normality was an essential part of this analysis, since Gaussian MLE's such as the Whittle estimate have parametric assumptions about the process under test. Graphical and statistical tests were employed in this study.

B. Parameter Estimation for Model Building

The current methodology estimates the model orders and the fractional differencing parameter of the model by a two step process:

In the first step we determine the ARMA orders to fit the mean equation by inspecting the PACF/ACF/EACF plots. Using these orders determine the value of "d" (the fractional differencing parameter) by the GPH methodology and create a fractionally differenced series. We estimate the model parameters by trial and error approach to determining an appropriate model that best fits the data.

Box-Ljung test is applied at each stage to check for model adequacy and to investigate the presence of ARCH effects. Model selection is based on BIC, the Bayesian Information Criterion.

$$\mathrm{BIC}(\ell) = \ln(\hat{\sigma}_\ell^2) + \frac{\ell \ln(T)}{T}$$

The model with the lowest BIC is accepted as the final model. BIC is found to give more superior results as compared to AIC for autoregressive model order selection [6]; hence BIC was chosen over AIC for model comparison. Also BIC enjoys the consistency property in terms of selecting the true model. [7]

1) Fractional Differencing Parameters

An appropriate investigation of fractional differencing includes entertaining several estimators for a fractional differencing parameter in an ARFIMA (p,d,q) framework. According to Beran, a linear process with i.i.d. innovations can exhibit long memory, however, it is difficult to distinguish the behavior of linear ARMA-type processes from fractionally integrated processes for small values of n, where n is the length of the univariate process under question.[1]

With these considerations in mind, we found it necessary in our methods of analysis to include a fractional integration determination based on the following:

- An efficiency study of different estimators for the fractional differencing d, including "removal of fractional differencing."
- A formal statistical hypothesis test for fractional differencing. Specifically, Dolando *et. al.*[2] has proposed a Fractional Dickey Fuller test, in which, in the following regression setup,

  $$\Delta y_t = \mu + \varphi \Delta^d y_{t-1} + A(L)\Delta y_t + \varepsilon_t$$

  the null hypothesis of unit-root non-stationarity ($d=d_0=1$) is tested against fractional integration ($d=d_1 \varepsilon (-1/2,1/2)$).

  H_0: $\varphi = 0$, y_t is $FI(d_0)$
  H_A: $\varphi < 0$, y_t is $FI(d_1)$

  Initial results of this test are promising in simulation, but further investigation has to be carried out to validate statistical accuracy with respect to MARS-derived critical values (see Sephton[3]).

- An objective comparison of ARFIMA(p,d,q) (long memory) with an alternative model, using forecast errors.

2) Joint Estimation of ARFIMA(p,d,q) Parameters

The current methodologies for estimating the model parameters follow a two step approach, the first step is to obtain the estimate of fractional differencing parameter "d" and an ARMA estimation technique is applied to the fractionally differenced time series in the second.

We are currently researching the possibility of implementing a one-step joint estimation framework of the ARIMA(p,d,q) model to the return series, ones the ARCH effects are confirmed.

We have come across a couple of methodologies which are based on Bayesian inference for autoregressive fractionally integrated moving average (ARFIMA) models using Markov chain Monte Carlo methods [5] and a pseudo linear method for the estimation of Fractionally Integrated ARMA models [4], but a feasible implementation of e methodologies is yet to be confirmed in the context of our analysis.

delete — perhaps replace by "there"

III. RESULTS

A. Initial Analysis

1) Data Transformations

Analysis of the states demonstrated that the distribution of the returns deviates too far from Gaussian and thus, any Box Cox transformation attempt fails. Figure 3.1.1 shows graphical results from before and after the transformation for Arkansas and Table 1.2 shows numerical results.

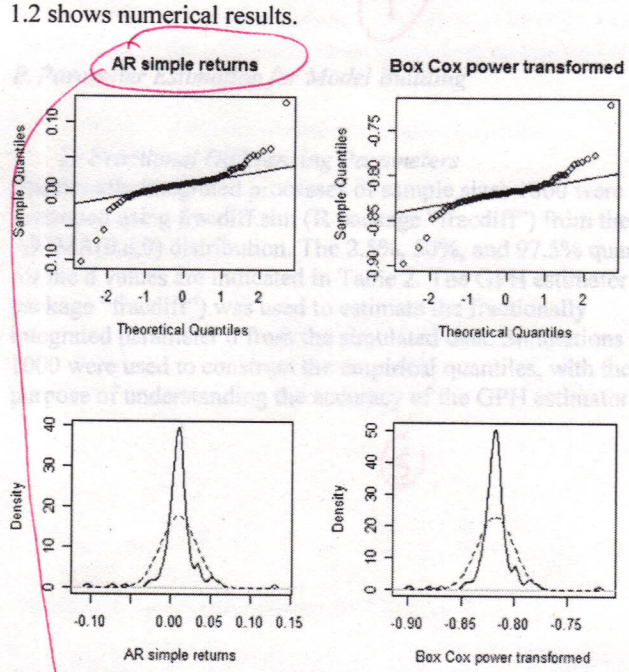


Figure 3.1.1: QQ plots and density plots of the Arkansas simple return series (left) and the Box Cox transformed series with $\hat{\lambda}$=1.11 (right)

What does "AR" mean here? Arkansas

p-value	AR	AR Box Cox transformed (λ = 1.11)
Shapiro	1.75E-12	3.07E-12
JB	2.20E-16	2.20E-16
AD	2.20E-16	2.20E-16
KS	2.20E-16	2.20E-16

Table 1.2: p-values from normality tests for the Arkansas simple return series and the Box Cox transformed series with $\hat{\lambda}$=1.11 (right)

The QQ plots and density plots show significant departures from normality. The p-values of various normality tests strongly suggest non-normal distributions. It is concluded that a Box Cox transformation is inadequate in making the return series Gaussian.

YES

2) I(1) Testing

Results of unit root tests are shown in Table 1. As shown, the p-values all show the rejection of the H_o for the presence of a unit root; thus, these states were determined to be stationary.

	Test Lag	ADF	PP
AK	0	0.01	0.01
AL	4	0.01	0.01
AR	4	0.03	0.01

Table 3.2.1: p-values obtained from applying the Augmented Dickey-Fuller and Phillips-Perron tests to a sample of states

3) Normality Testing

134 data points, 1000 simulations

p-value	$N(0,\sigma^2)$	σ(1)	σ(2)	σ(3)
Shapiro	2.5%	0.028	0.023	0.018
	50.0%	0.509	0.491	0.523
	97.5%	0.973	0.979	0.974
JB	2.5%	0.016	0.013	0.017
	50.0%	0.550	0.550	0.568
	97.5%	0.978	0.976	0.969
AD	2.5%	0.021	0.019	0.026
	50.0%	0.498	0.486	0.511
	97.5%	0.964	0.968	0.966
CVM	2.5%	0.018	0.020	0.021
	50.0%	0.486	0.496	0.526
	97.5%	0.968	0.977	0.975
KS	2.5%	0.029	0.021	0.019
	50.0%	0.510	0.503	0.486
	97.5%	0.977	0.979	0.971

Table 3.3.1: Normality test p-values for simulated data, 134 data points

by simply with t-error

662 data points, 1000 simulations

p-value	N(0,σ²)	σ(1)	σ(2)	σ(3)
Shapiro	2.5%	0.027	0.022	0.031
	50.0%	0.494	0.483	0.492
	97.5%	0.972	0.964	0.972
JB	2.5%	0.018	0.015	0.026
	50.0%	0.522	0.515	0.529
	97.5%	0.968	0.974	0.972
AD	2.5%	0.031	0.028	0.023
	50.0%	0.506	0.516	0.494
	97.5%	0.968	0.962	0.976
CVM	2.5%	0.022	0.034	0.023
	50.0%	0.513	0.513	0.500
	97.5%	0.981	0.978	0.982
KS	2.5%	0.024	0.027	0.023
	50.0%	0.499	0.499	0.498
	97.5%	0.965	0.965	0.975

Table 3.3.2: Normality test p-values for simulated data, 662 data points

Data was simulated from a normal distribution, of both samples sizes 134 and 662, in Table 2 and 3, respectively. Normality tests were applied to the simulated data and the p-values were recorded. The simulation was repeated 1,000 times and the empirical quantiles of the p-values were recorded.

The purpose of this exercise is to determine the accuracy of the normality tests. The results were expected, with an approximately uniform distribution of p-values. This will be investigated further before conclusions are drawn.

4

B. Parameter Estimation for Model Building

1) Fractional Differencing Parameters

Fractionally integrated processes of sample sizes 1000 were generated using fracdiff.sim (**R** package "fracdiff") from the ARIMA(0,d,0) distribution. The 2.5%, 50%, and 97.5% quantiles for the d values are indicated in Table 2. The GPH estimator (**R** package "fracdiff") was used to estimate the fractionally integrated parameter d from the simulated data. Simulations of 1000 were used to construct the empirical quantiles, with the purpose of understanding the accuracy of the GPH estimator.

5

d = 0.4	2.5%	0.321
	50.0%	0.420
	97.5%	0.659
d = 0.3	2.5%	0.227
	50.0%	0.318
	97.5%	0.555
d=0.1	2.5%	0.016
	50.0%	0.111
	97.5%	0.357
d=-0.2	2.5%	-0.283
	50.0%	-0.185
	97.5%	0.098
d=-0.6	2.5%	-0.632
	50.0%	-0.526
	97.5%	-0.233

Table 4.1: Quantiles of d-parameter of simulated fractionally integrated processes, determined though the GPH estimator (n=1,000; Burn-In = 1,000; Simulations = 1,000; ARIMA(0,d,0))

Selected Density Plots:

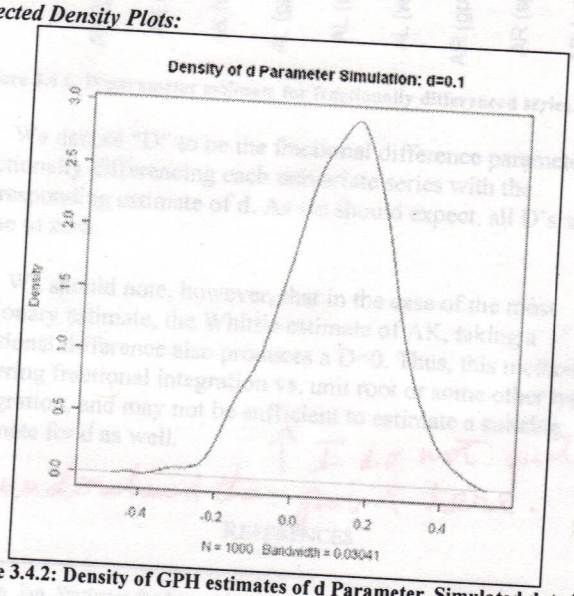
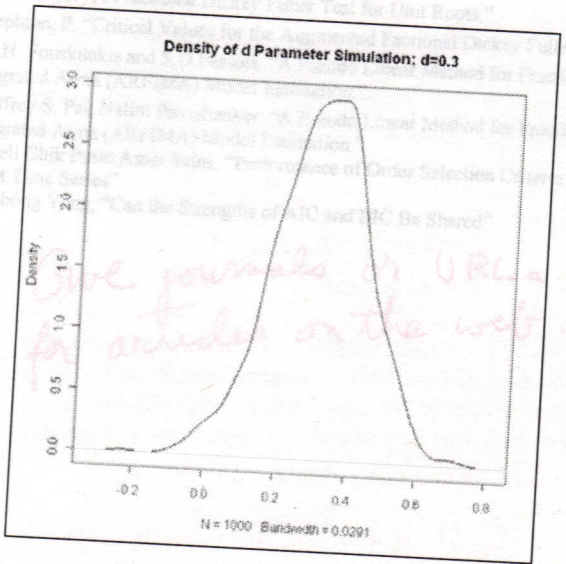


Figure 3.4.2: Density of GPH estimates of d Parameter, Simulated data has d=0.1



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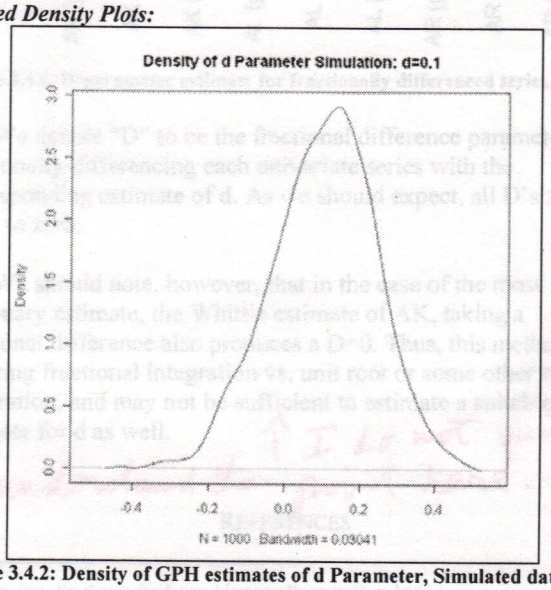

Figure 3.4.2: Density of GPH estimates of d Parameter, Simulated data has d=0.1

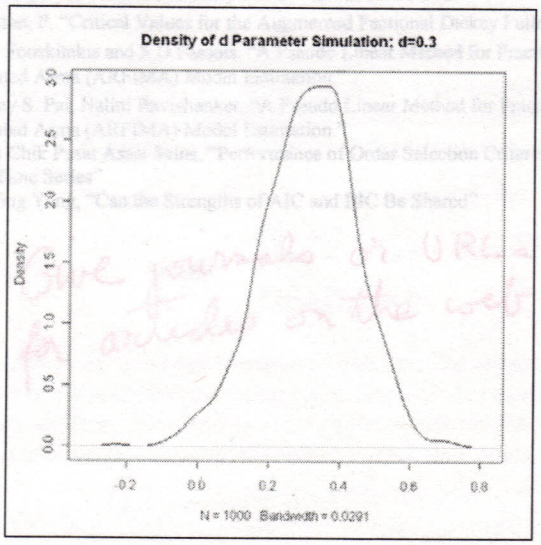

Figure 3.4.3: Density of GPH estimates of d Parameter, Simulated data has d=0.3

The density plot indicates that the GPH estimator appears to be a reasonable estimate of the fractionally integrated parameter. This is verified by the relatively light tails of the plots. The accuracy of the GPH estimator will be investigated further by comparing fractionally integrated parameter estimates to the results from other estimators.

Reference

Not the same numbers

	GPH	Sperio
AK	0.276	0.121
AL	0.215	0.209
AR	0.219	0.201

Table 3.4.4: d-parameter estimates using the GPH and Sperio estimates for a select sample of states

Using the GPH estimator as described above, d-parameters were estimated for the selected states. The results are shown above in Table 3. Additionally, d-parameters were estimated using the Sperio estimator, to gauge the accuracy of GPH. Results using the Sperio estimator confirmed the d estimates for Alabama and Arkansas. However, the results for Alaska verified the need for further investigation.

To supplement these findings, we've included a Monte Carlo simulation, where a random ARFIMA (0,d,0) process is simulated 1000 tines, for a series length of 134, where d is the point estimate derived from the corresponding estimation method. The following plot includes point estimates, in **black**, for the parameter itself, as well as standard errors derived from the asymptotic distributional assumptions of the estimation method. The results of the simulation study are superimposed and shown in **blue**.

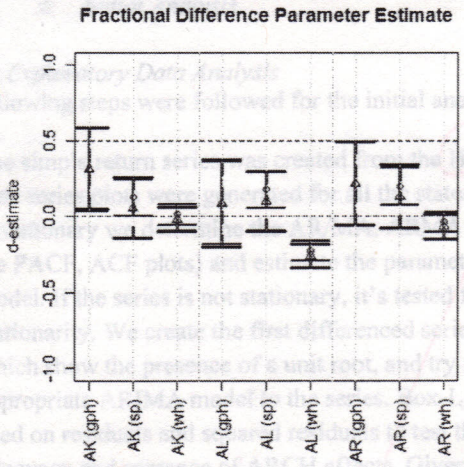

Figure 3.4.5. d-parameter estimates for AK, AL, AR.

Additionally, we included point estimates for the fractionally differenced series, anticipating that any degree of fractional integration should be "filtered-out" by taking a fractional difference, using the previous d parameter estimates.

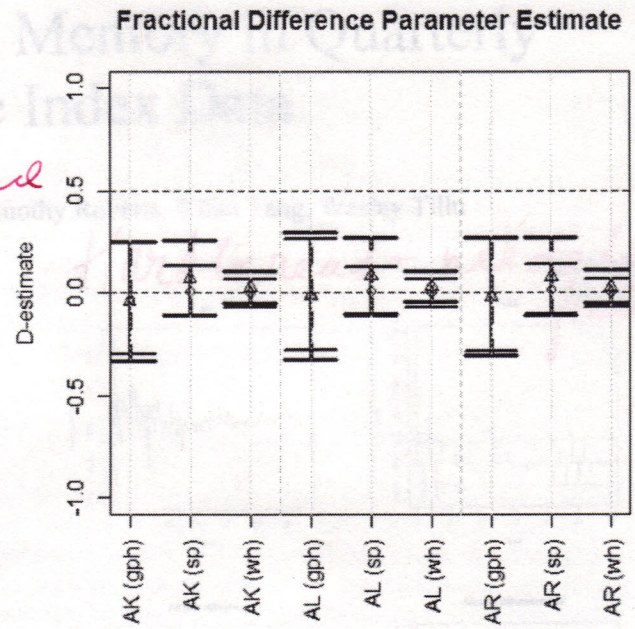

Figure 3.4.6. D-parameter estimate for fractionally differenced series.

We denote "D" to be the fractional difference parameter after fractionally differencing each univariate series with the corresponding estimate of d. As we should expect, all D's are close to zero.

We should note, however, that in the case of the most stationary estimate, the Whittle estimate of AK, taking a fractional difference also produces a D=0. Thus, this method of inferring fractional integration vs. unit root or some other type of integration, and may not be sufficient to estimate a suitable estimate for d as well.

I do not quite understand the point here.

REFERENCES

[1] Beran, Jan. *Statistics for Long Memory Processes.* p.144.

[2] Dolando, J. "A Fractional Dickey Fuller Test for Unit Roots."

[3] Sephton, P. "Critical Values for the Augmented Factional Dickey Fuller Test".

[4] G.N. Fouskitakis and S.D.Fassois. "A Pseudo Linear Method for Fractionally Integrated Arma (ARFIMA) Model Estimation."

[5] Jeffrey S. Pai, Nalini Ravishanker. "A Pseudo Linear Method for Fractionally Integrated Arma (ARFIMA) Model Estimation."

6 Zazli Chik Pusat Asasi Sains, "Performance of Order Selection Criteria for Short Time Series"

7 Yuhong Yang, "Can the Strengths of AIC and BIC Be Shared"

Give journals or URLs for articles on the web.

① What does AIC <u>versus</u> BIC mean?

② How did you use Box-Ljung to check for ARCH effects?

③ The problem is that the data are symmetric and heavy-tailed. Box-Cox works best for asymmetric data.

④ Some of the non-normality may be the result of ARCH effects — this can be removed by ~~an~~ GARCH model. Probably, there will still be heavy tails. A t-distribution could be used instead.

⑤ It would be very useful to run some simulations with t-errors to see the effects of non-normality. Perhaps ~~g~~ normality is not important.

⑥ Light tails are good but a small variance is more important.