SOUTH UNIVERSITY OF SCIENCE AND TECHNOLOGY OF CHINA

Methods of Applied Math: Assignment 1

Due Date: September 23, 2019

✏ **(Exercise 1)** Find by using dimensional analysis the self-similar solution $u(\mathbf{x}, t)$ to the following initial value problem

$$\begin{cases} \frac{\partial u}{\partial t} = \nu \Delta u, \qquad \text{in} \quad \mathbb{R}^d \\ u(\mathbf{x}, 0) = E\delta(\mathbf{x}) \end{cases}$$

(Hint: you need to assume the rotation invariance in the solution)

✏ **(Exercise 2)** Prove, using dimensional analysis, Pythagoras' theorem

$$c^2 = a^2 + b^2,$$

where c is the hypotenuse of the right-angled triangle, a and b are the other two sides.

✏ **(Exercise 3)** Find the rescalings for the roots of

$$\epsilon^2 x^3 + x^2 + 2x + \epsilon = 0,$$

and find two terms in the approximation for each root.

✏ **(Exercise 4)** Find the first two terms of $x(\epsilon)$ the solution near 0 of

$$\sqrt{2}\sin(x + \frac{\pi}{4}) - 1 - x + \frac{1}{2}x^2 = -\frac{1}{6}\epsilon.$$

✏ **(Exercise 5)** Find the first 5 terms in an approximation for the solution of

$$\frac{e^{-x^2}}{x} = \epsilon.$$

✏ **(Exercise 6)** Find the second order perturbations of the eigenvalues of the matrix

$$\begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} + \begin{pmatrix} 0 & \omega \\ -\omega & 0 \end{pmatrix}$$

for small ω and for large ω.