SOUTH UNIVERSITY OF SCIENCE AND TECHNOLOGY OF CHINA

Methods of Applied Math: Assignment 3

Due Date: October 28, 2019

This assignment covers 6.1 to 6.6 in the textbook "Advanced Mathematical Methods for Scientists and Engineers".

✏ **(Exercise 1)** Use Taylor expansion of e^{-t^2} to find an asymptotic expansion of the integral:

$$I(x) = \int_0^{\frac{1}{x}} e^{-t^2} \mathrm{d}t, \quad x \to 0^+.$$

You are required to show that the asymptotic expansion of the integrand holds uniformly for t.

✏ **(Exercise 2)** Consider the integral:

$$I(x) = \int_0^{\infty} \frac{e^{-t}}{x^2 + t} \mathrm{d}t, \quad x \to +\infty.$$

1. Using integration by parts, show that

$$I(x) = \sum_{n=1}^{N} \frac{(-1)^{n-1}(n-1)!}{x^{2n}} + (-1)^N N! \int_0^{\infty} \frac{e^{-t}}{(x^2+t)^{N+1}} \mathrm{d}t.$$

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2. Using definition, show that

$$I(x) \sim \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(n-1)!}{x^{2n}}, \quad x \to +\infty.$$

✏ **(Exercise 3)** Consider the asymptotic behavior of the integral:

$$I(x) = \int_1^3 e^{-x(\frac{9}{t}+t)}\mathrm{d}t, \quad x \to +\infty.$$

1. Find the leading asymptotic behavior using Laplace's method.
2. Rewrite the integral using variable $u = \frac{9}{t} + t - 6$ and find the first two leading terms in the asymptotic expansion (using Watson's lemma).

✏ **(Exercise 4)** Consider the asymptotic behavior of the integral:

$$I(x) = \int_0^1 \cos(xt^4)\tan t\,\mathrm{d}t, \quad x \to +\infty.$$

Find the leading asymptotic behavior using the method of stationary phase (Remember to use the Generalized Riemann-Lebesgue Lemma).

✏ **(Exercise 5)** Using the method of steepest descent to find the full asymptotic behaviors of:

$$I(x) = \int_0^1 e^{ixt^3}\mathrm{d}t, \quad x \to +\infty.$$

✏ **(Exercise 6)**

1. Show that an integral representation of the Airy function $Ai(x)$ is given by

   $$Ai(x) = \frac{1}{2\pi i}\int_C e^{xt-t^3/3}\mathrm{d}t$$

   where C is a contour which originates at $\infty e^{-2\pi i/3}$ and terminates at $\infty e^{2\pi i/3}$
2. Use this integral representation to show that the Taylor series expansion of $Ai(x)$ about $x = 0$ is as given in our class notes (or (3.2.1) in the textbook).
3. Using the method of steepest descents, find the asymptotic behaviour of $Ai(x)$ as $x \to +\infty$.