

Improved Trajectory Planning for On-Road Self-Driving Vehicles via Combined Graph Search, Optimization & Topology Analysis

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Agenda

- Introduction / Related Work / Motivation
- Proposed Planning Framework
- Approach
 - Proposed Method I: Region Segmentation w/ Topology Analysis
 - Proposed Method II: Search-based Planning w/ Edge-augmented Graph
 - Adapted Method I: LQR Path/Trajectory Smoothing
 - Proposed Method III: Sampling-based Maneuver Pattern Analysis
 - Adapted Method II: Focused Trajectory Optimization w/ Iterative-LQR
- Results
- Conclusion/Contribution/Future Work

Introduction

- Self-driving / Autonomous-driving Vehicles

Boss



2007

Cadillac



2011

Google



2013

Uber



2016

- Social Benefits

Safety

Two million car crashes per year in the United States [1].

Good Autonomy Software!

Autonomous
Driving

Wasted hours in traffic jams.
Wasted parking space to city.

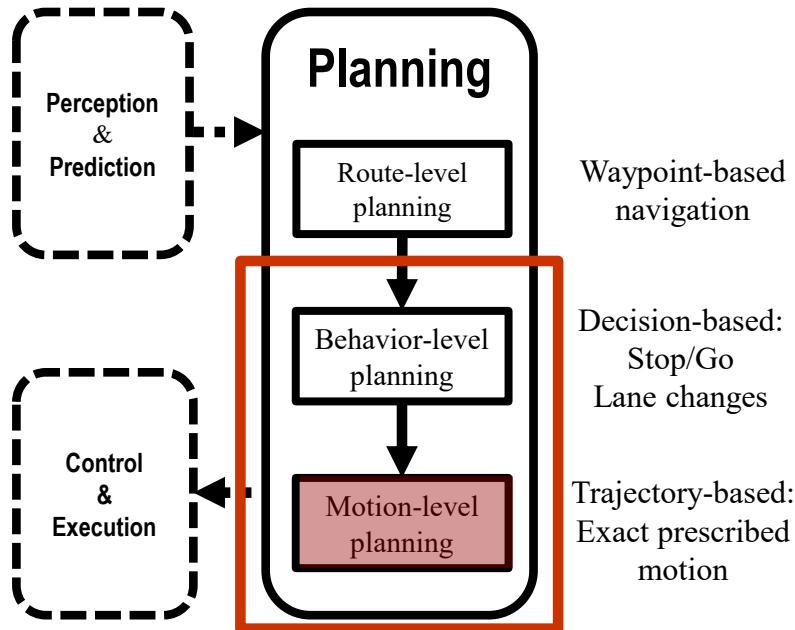
Efficiency

Freedom

Driving is exhausting.
Driving is difficult/impossible for some.

Introduction

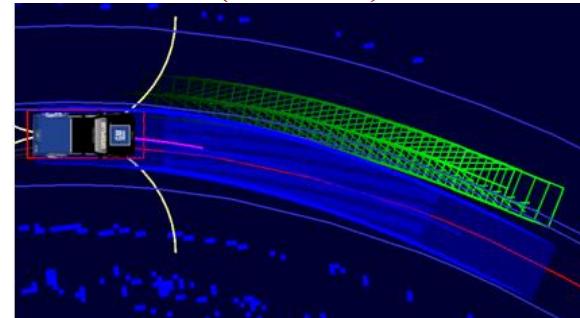
- Thesis Scope



Less constrained Environment [1]
(Off-road)



More constrained Environment [2]
(On-road)

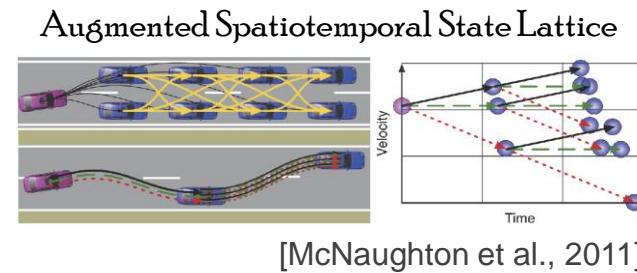
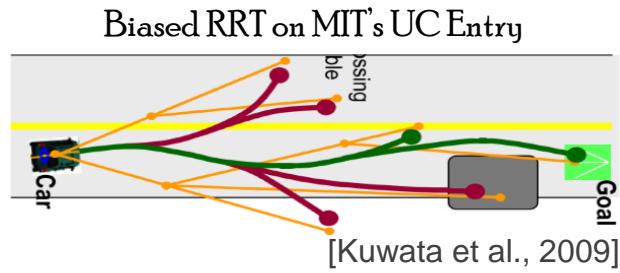
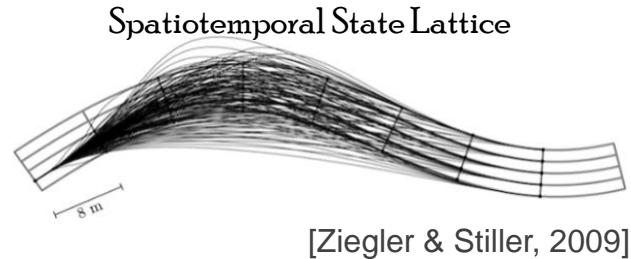
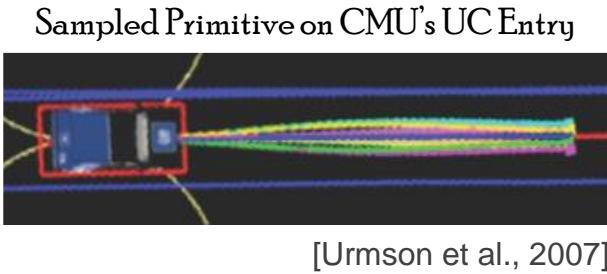


Related Work

- Sampling-based Planners
- Optimization-based Planners
- Hybrid Sampling/Optimization Planners
- Topology-aware Planners

Related Work

- Sampling-based Planners



1. Lightweight primitive sampling with model-based evaluation.
2. Deterministic runtime with termination guarantee.
3. Search space is comprehensive with respect to the continuous space of interest.

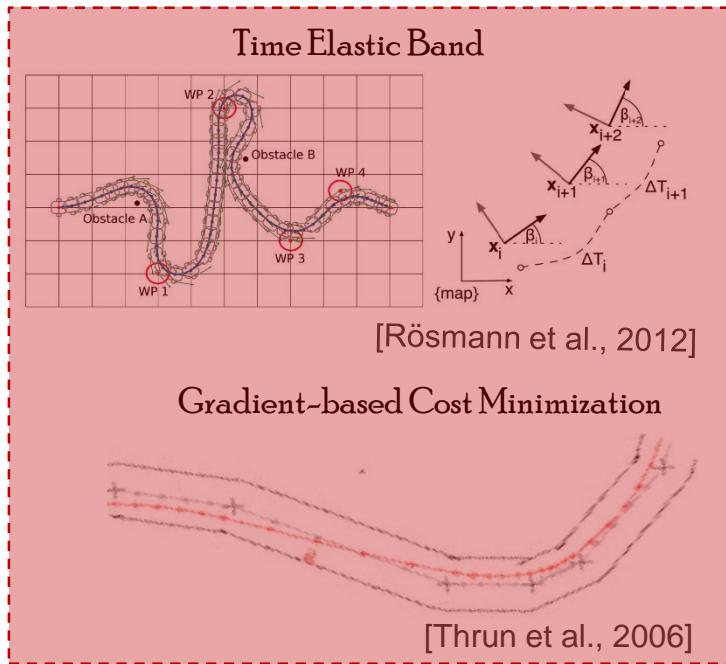
Pro

Con

1. Sampling sub-optimality, not converging to local minimum.
2. Search space blow-up due to the curse of dimensionality.

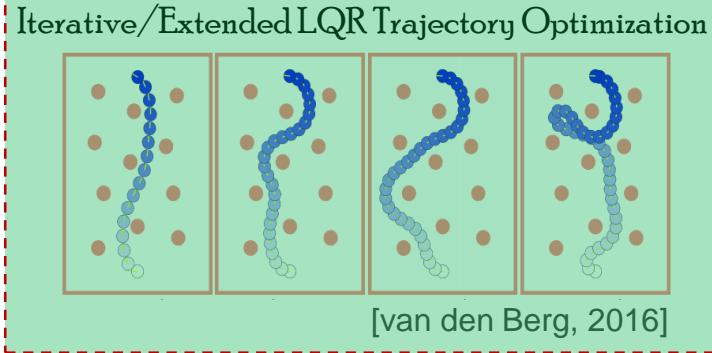
Related Work

- Optimization-based Planners



Direct Method

Indirect Method



- Search in the continuous space, alleviate sampling sub-optimality.
- Efficient if implemented properly, compared to the thousands, even millions of sampled trajectory evaluations.

Pro

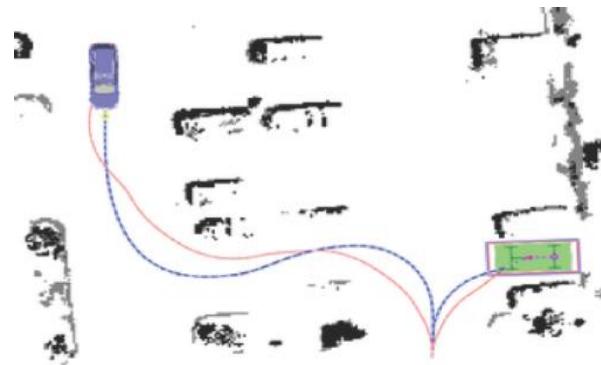
Con

- Lacks global awareness, can get stuck in the wrong local minimum.
- Non-deterministic runtime w/o termination guarantee
- Difficult to parallelize the computation.

Related Work

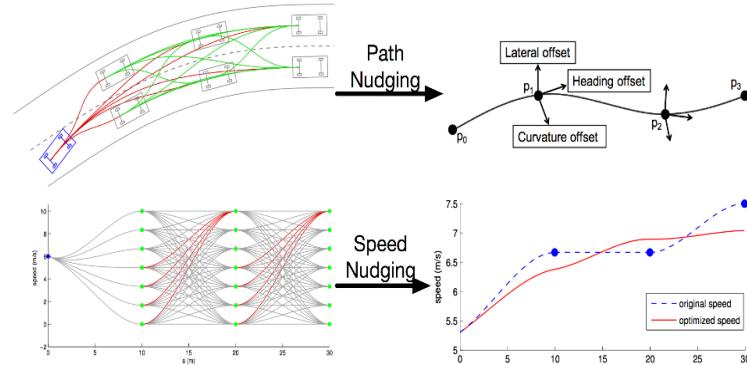
- Hybrid Sampling/Optimization Planners

Hybrid A* + Conjugate Gradient Optimization



[Dolgov et al., 2008]

Lattice Sampling + Simplex Optimization



[Xu et al., 2012]

- Achieve local minimum while having global awareness.
- Moderate computation overhead.

Pro

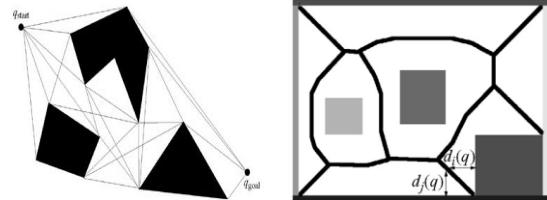
Con

- Pure path/speed planning, independent spatial/temporal planning.
- Direct trajectory optimization by manipulating the sampled configuration states. Difficult to guarantee plan feasibility.

Related Work

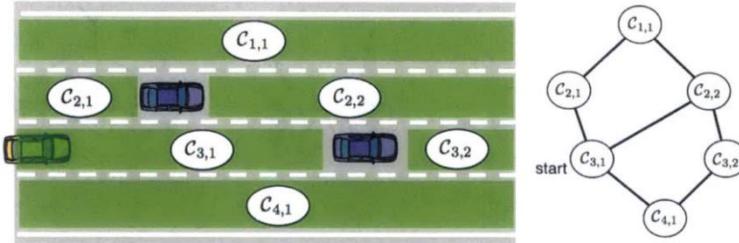
- Topology-aware Motion Planners

Visibility Graph & Voronoi Graph

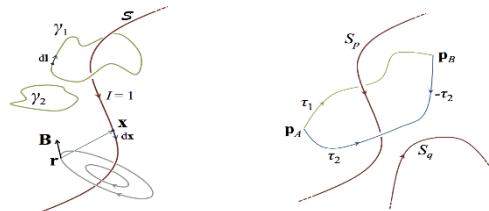


[Schmitzberger, 2002] & [Choset, 2005]

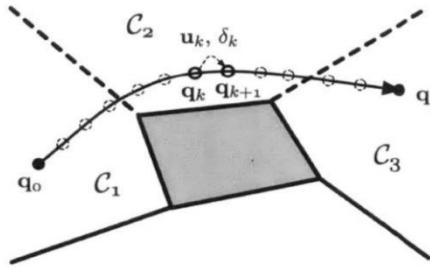
Cell Decomposition + Mixed-Integer-Quadratic Programming



Homology Marker Function



[Bhattacharya, 2012]



[Parker, 2016]

1. Explicit topological awareness.
2. Focus (narrow down) the search space.

Pro

1. Pure spatial topological analysis.

2. Co-terminal requirements for paths under analysis.

Motivation

- Thesis Statement:

By taking advantage of a combined sampling-n-search, optimization and topology analysis approach, we can avoid the pitfalls of standalone methods, and equip self-driving cars with improved high-level reasoning capabilities for on-road trajectory planning.

Requirement 1: a deliberative trajectory planning system.

Requirement 2: spatiotemporal (trajectory) planning.

Requirement 3: tactical reasoning capability with topological awareness.

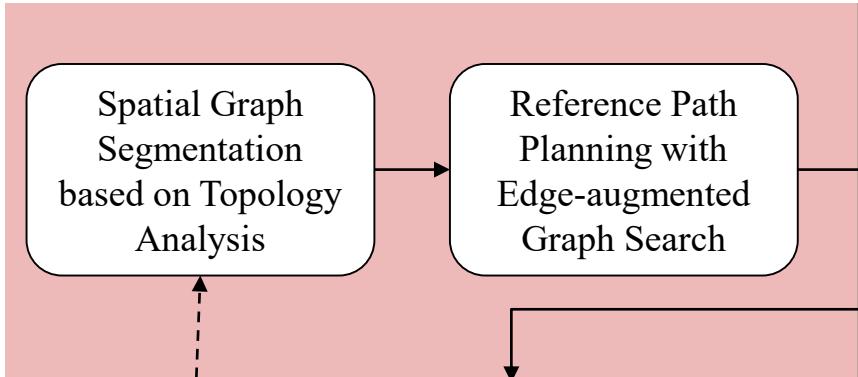
Requirement 4: global awareness with the ability to converge to a local optimum.

Requirement 5: apply to self-driving passenger vehicle on-road driving.

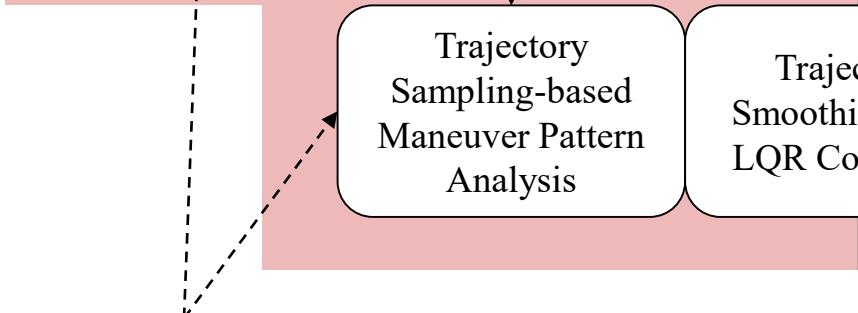
Proposed Planning Framework

- Technical Overview

Sampling&Search/Topology-based Planning



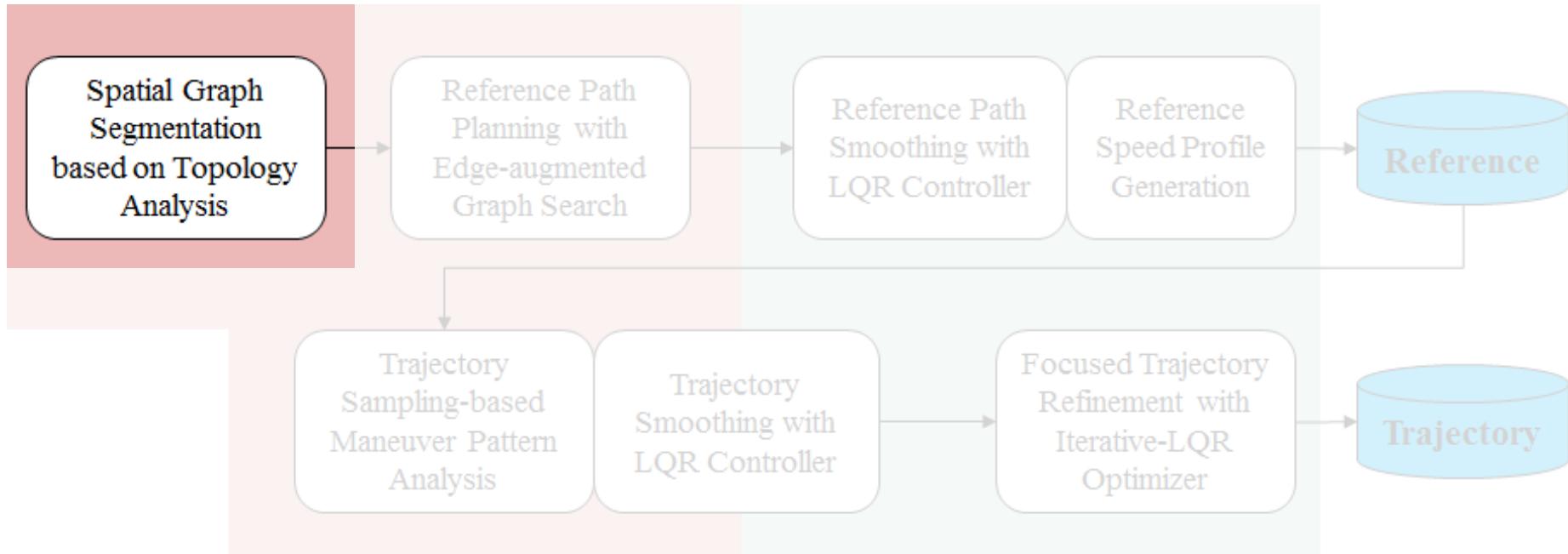
Optimization-based Planning



High-level Tactical Reasoning Capability

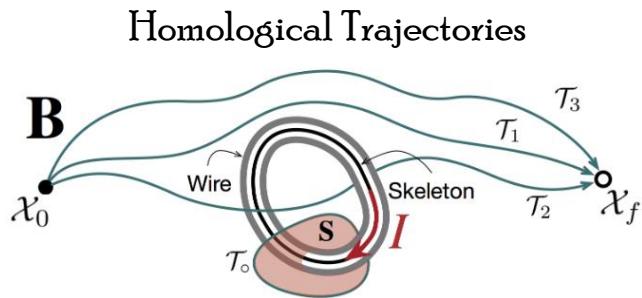
*By taking advantage of a **combined sampling-n-search, optimization and topology analysis approach**, we can avoid pitfalls of standalone methods, and equip self-driving cars with **improved high-level reasoning capabilities** for on-road trajectory planning.*

Proposed Method I: Region Segmentation w/ Topology Analysis



Proposed Method I: Region Segmentation w/ Topology Analysis

- Homology Marker Function-based Topological Analysis



Biot-Savart Law:

A steady current flowing through a wire generates a magnetic field \mathbf{B} .

$$\mathbf{B}(r) = \frac{\mu_0 \cdot I}{4\pi} \int_{\mathcal{W}} \frac{dl \times (l - r)}{\|l - r\|^3}$$

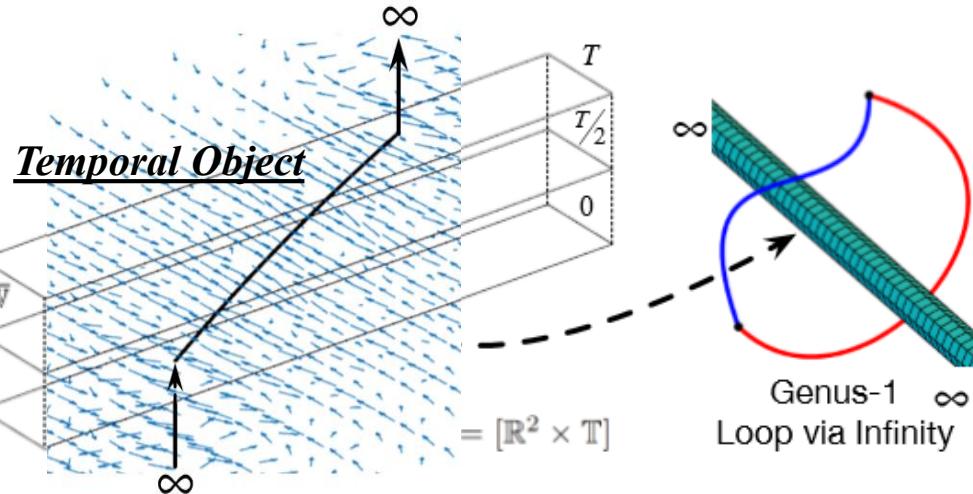
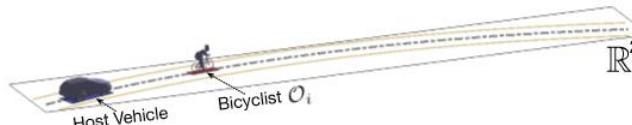
Homology Marker Function

$$\mathcal{H}(\mathcal{T}) = \int_{\mathcal{T}} \mathbf{B} \cdot dl$$

$$\mathcal{H}(\mathcal{T}_1) = \mathcal{H}(\mathcal{T}_3) \neq \mathcal{H}(\mathcal{T}_2)$$

→ \mathcal{T}_1 & \mathcal{T}_3 are homological.
 \mathcal{T}_1 & \mathcal{T}_2 are NOT homological.

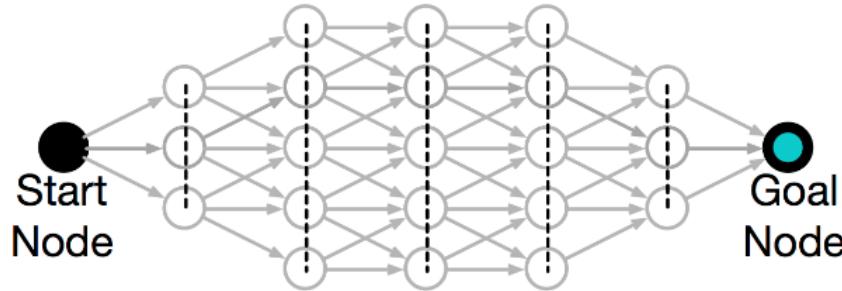
Analysis in Spatiotemporal Configuration Space



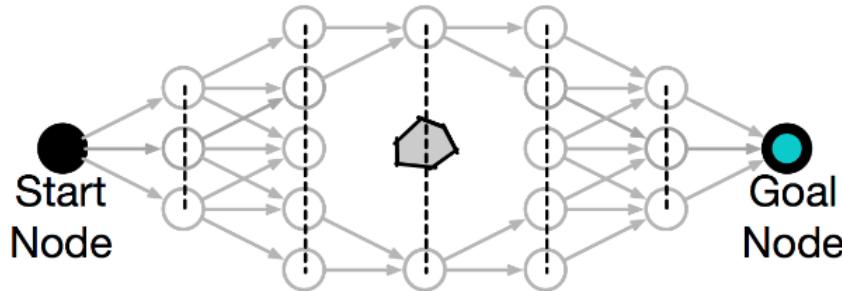
Magnetic field generated to distinguish trajectories.

Proposed Method I: Region Segmentation w/ Topology Analysis

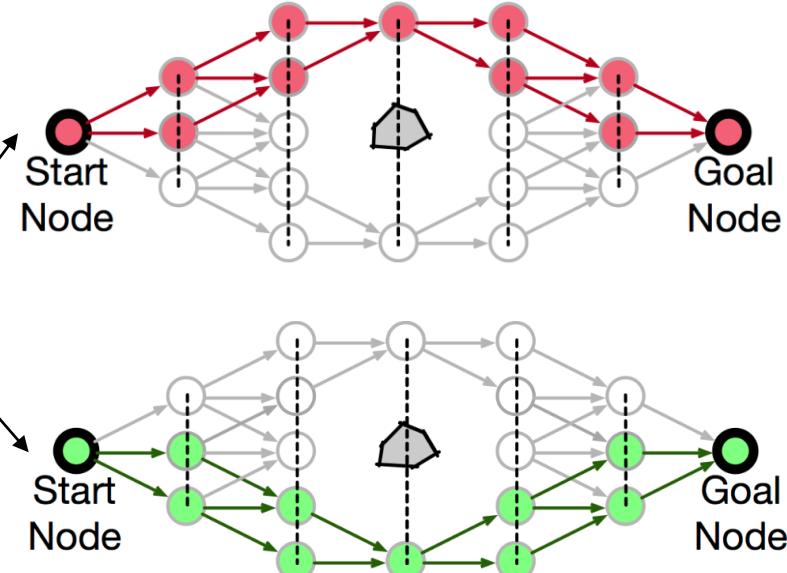
- Segment directed acyclic graph (DAG) into several sub-graphs



No obstacle → No segmentation



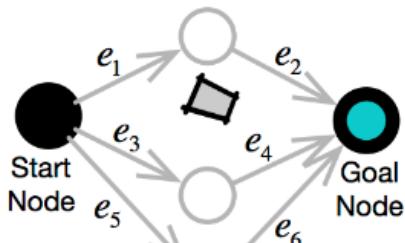
Obstacle → Two Sub-Graphs



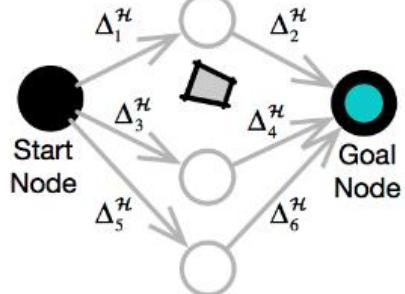
Proposed Method I: Region Segmentation w/ Topology Analysis

- Dynamic Programming-Inspired Backward Topology Induction

Step 1: Construct Topology Graph



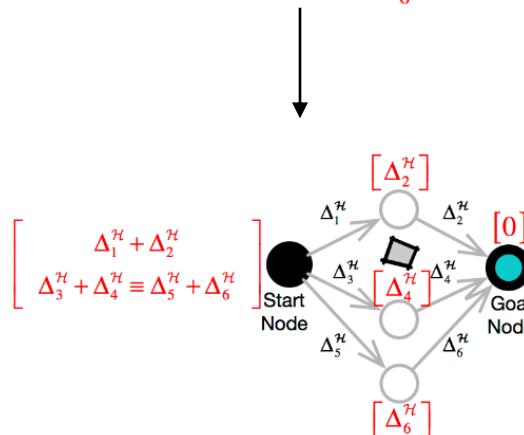
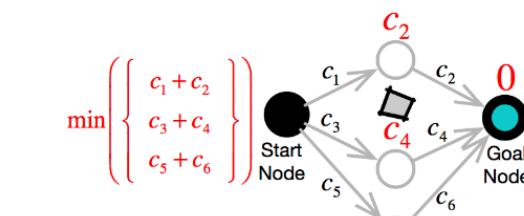
$$\Delta_i^H = H(e_i) = \int_{e_i} B \cdot dl$$



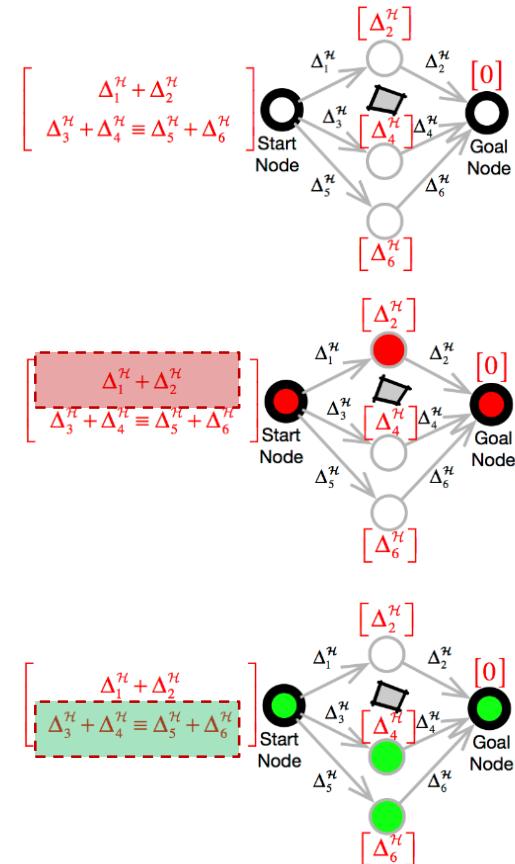
Step 2: Backward Topology Induction

Dynamic Programming

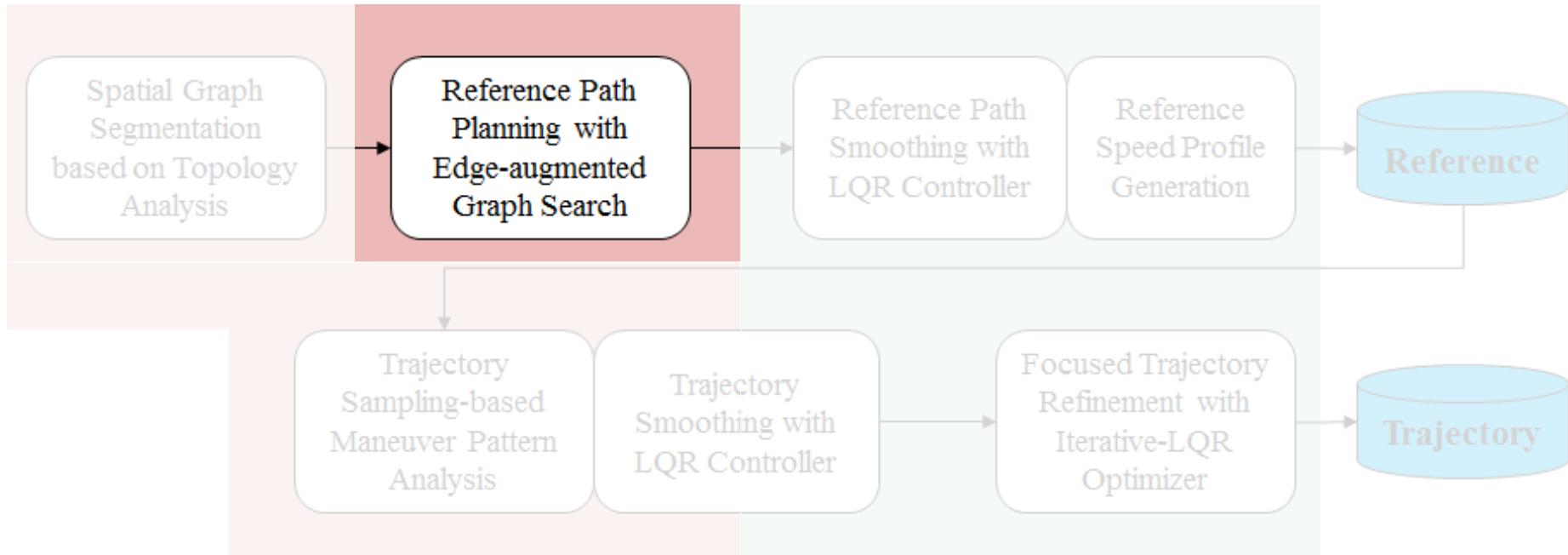
$$\min \left(\begin{Bmatrix} c_1 + c_2 \\ c_3 + c_4 \\ c_5 + c_6 \end{Bmatrix} \right)$$



Step 3: Forward Region Marking

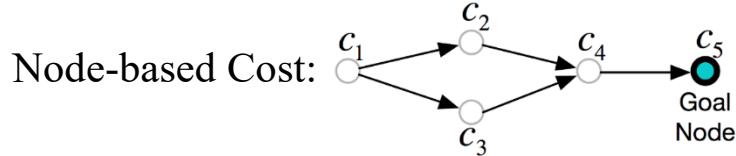
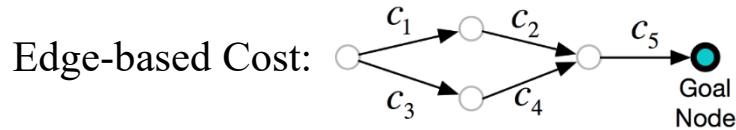
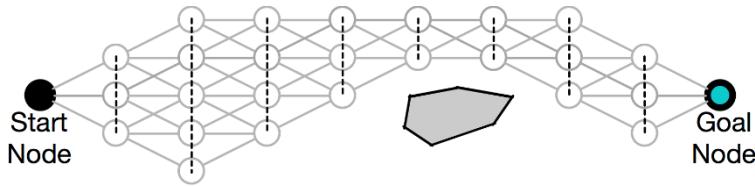


Proposed Method II: Search-based Planning w/ Edge-augmented Graph

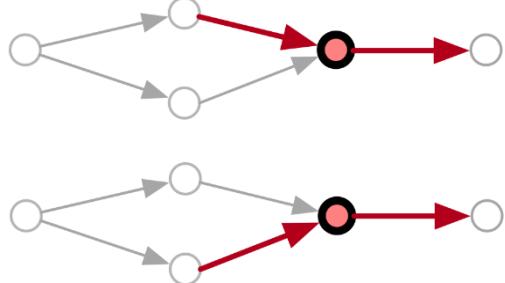


Proposed Method II: Search-based Planning w/ Edge-augmented Graph

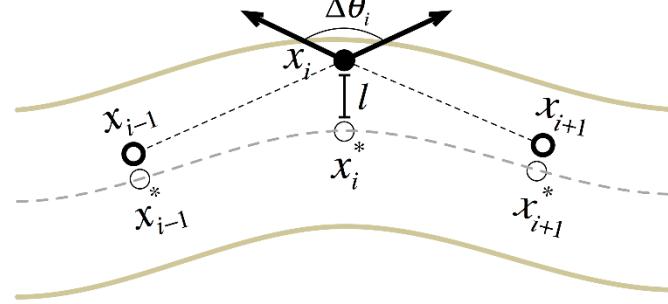
- Motivation: why do we need edge-augmented graph?



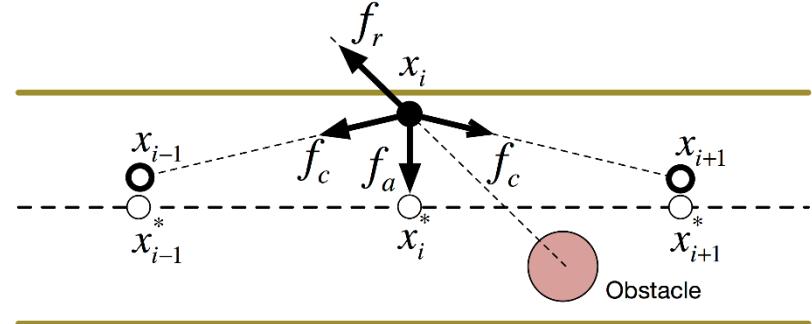
What if the cost is associated with two neighboring edges?



Smoothing Cost



Contractive Force in Elastic-Band



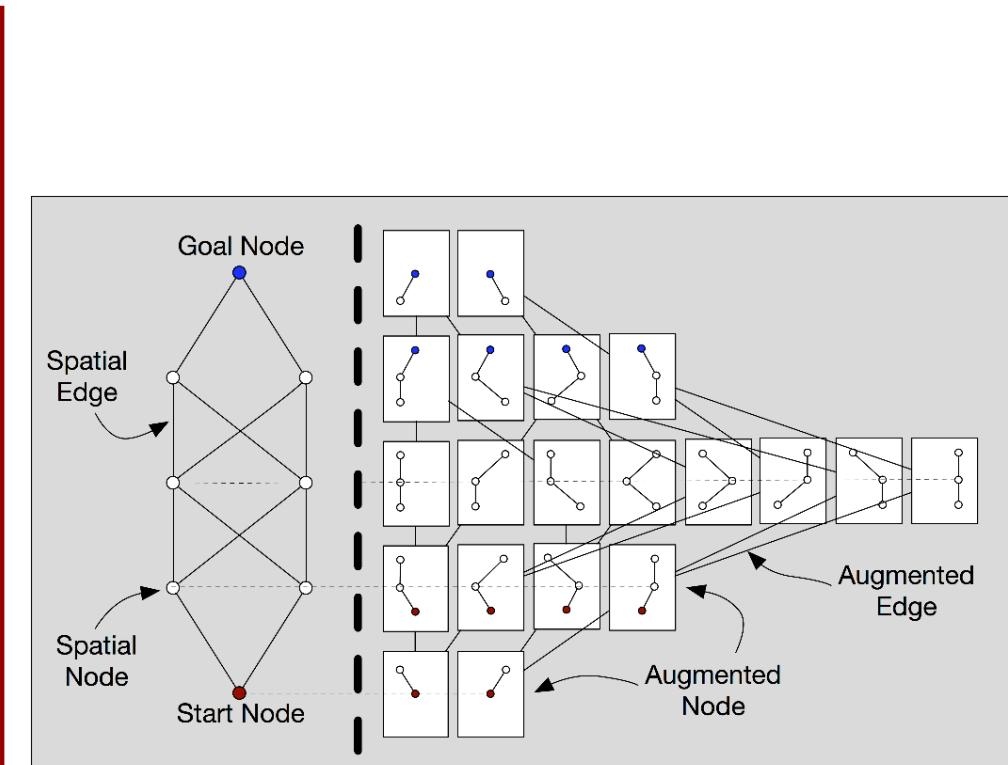
Proposed Method II: Search-based Planning w/ Edge-augmented Graph

- Construct & Search over Edge-augmented Graph

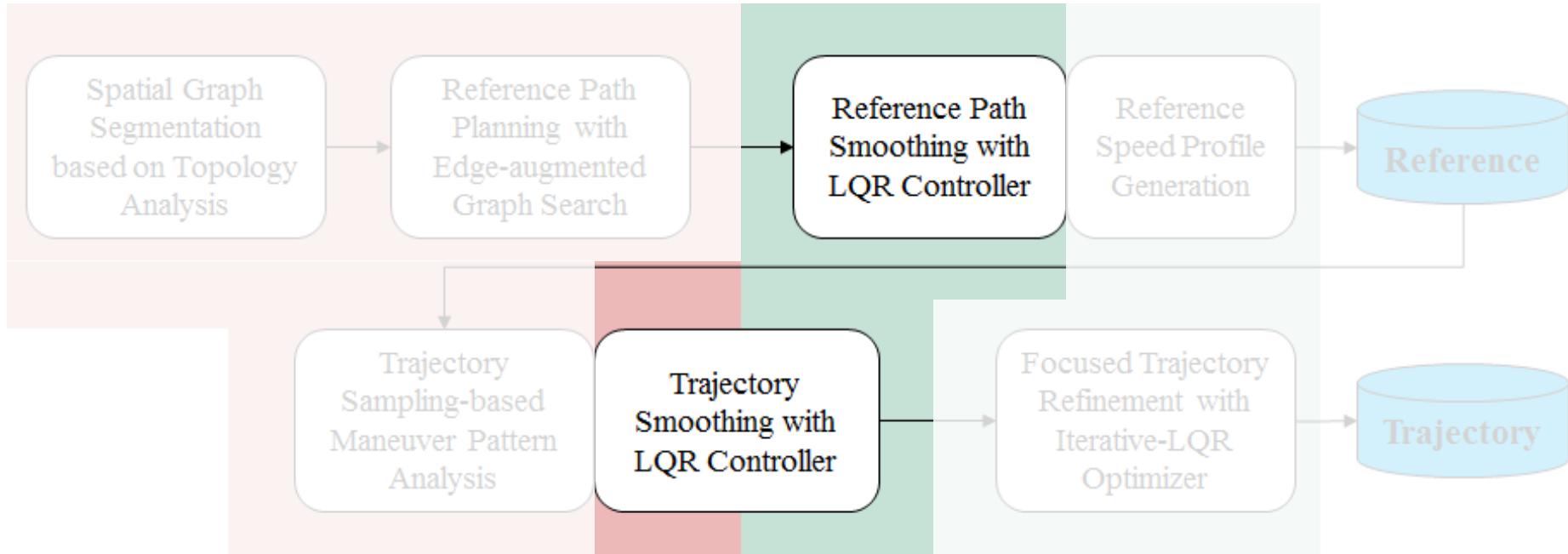
1. Construct regular DAG
2. Evaluate graph node/edge
3. Construct edge-augmented node
4. Evaluate edge-augmented node
5. Build edge-augmented graph

Edge-augmented graph is still DAG!

6. Search DAG with dynamic programming or topological search.

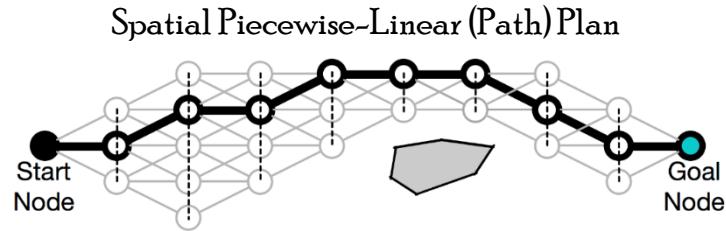


Adapted Method I: LQR-based Path/Trajectory Smoothing

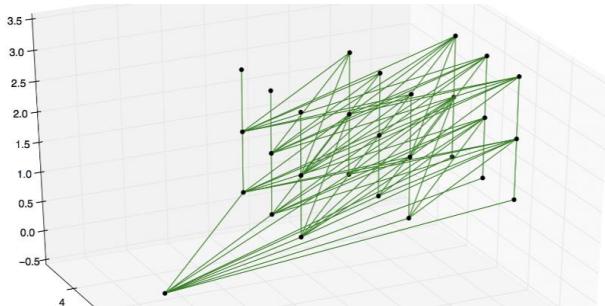


Adapted Method I: LQR-based Path/Trajectory Smoothing

- Motivation: How to convert coarse graph plan to smooth path/trajectory?



Spatiotemporal Piecewise-Linear (Trajectory) Plan



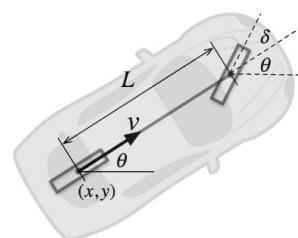
Roughly captures the gist of maneuver.

Not model-feasible, non-smooth.

Solution:

- Treat piecewise linear plan as a coarse reference.
- Use a realistic vehicle model and a trajectory tracking controller to “follow” the reference.
- Keep the trace of the model states as the smoothed trajectory.

What vehicle model to use?



$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\delta} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & \cos(\theta) \\ 0 & 0 & 0 & 0 & \sin(\theta) \\ 0 & 0 & 0 & 0 & \frac{\tan(\delta)}{L} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ \theta \\ \delta \\ v \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \gamma \\ a \end{bmatrix}$$

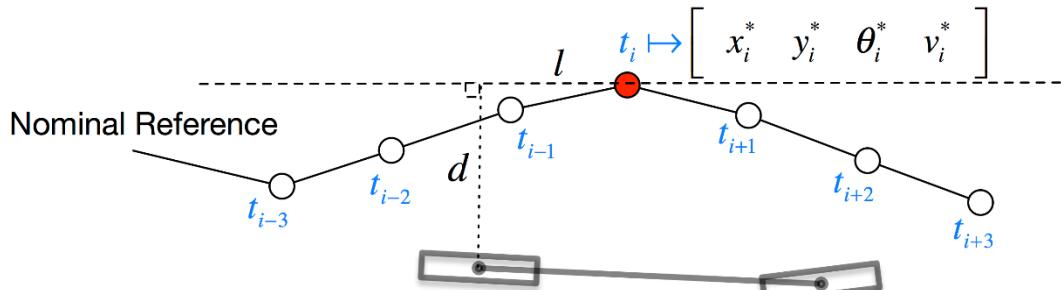
Non-linear, non-holonomic!

What tracking controller to use?

- Geometric tracker, e.g., pure pursuit.
- Optimal trackers, e.g., LQR-based tracker.

Adapted Method I: LQR-based Path/Trajectory Smoothing

- Path/Trajectory Smoothing



Lateral Tracking Control

State Transformation:

$$\begin{aligned} x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} &= \begin{bmatrix} d \\ \theta^* - \theta \\ \delta \end{bmatrix} \\ \dot{x}_1 &= \begin{bmatrix} 0 & v & 0 \\ 0 & 0 & -\frac{v}{L} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \dot{\delta} \end{aligned}$$

Dynamics Linearization:

Linear Quadratic Regulator

Minimize: $J = \int_0^\infty (x^T Q x + u^T R u) \cdot dt$

Control: $u = -K \cdot x$

$$K = R^{-1} B^T P$$

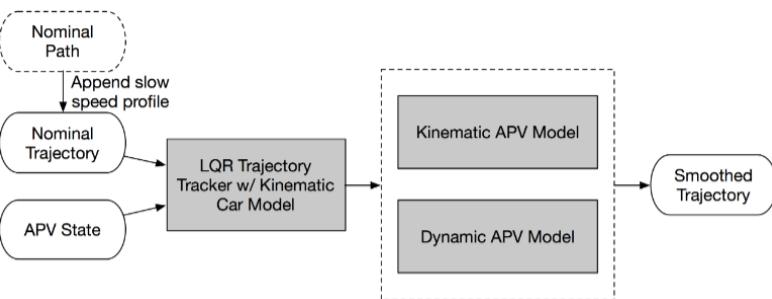
$$A^T P + P A - P B R^{-1} B^T P + Q = 0$$

Longitudinal Tracking Control

State Transformation:

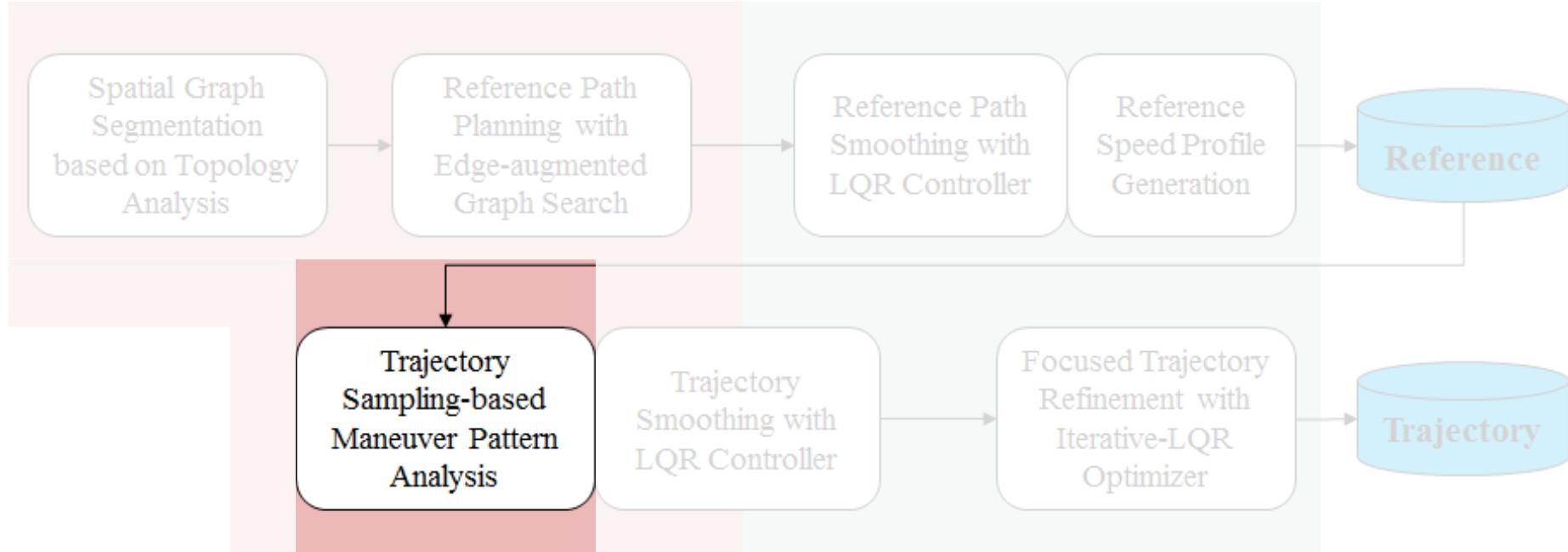
$$\begin{aligned} x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= \begin{bmatrix} l \\ v^* - v \end{bmatrix} \\ \dot{x}_1 &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} a \end{aligned}$$

Dynamics Linearization:



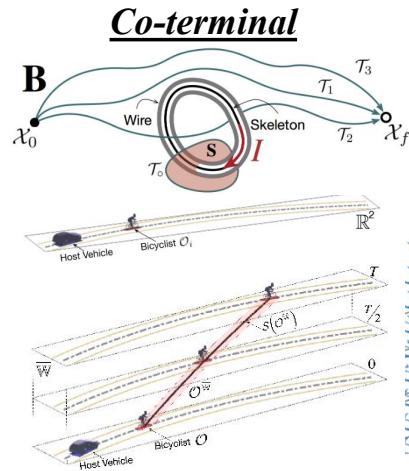
Smoothed model-feasible path/trajectory

Proposed Method III: Sampling-based Maneuver Pattern Analysis

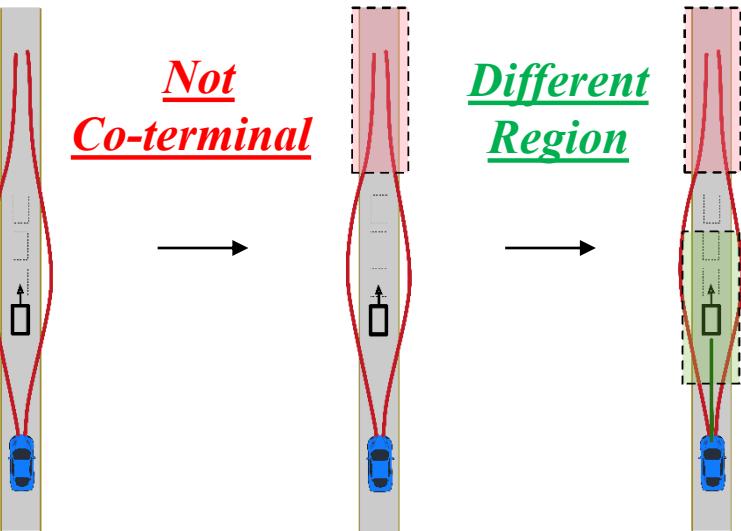
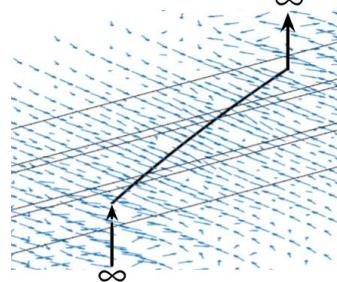


Proposed Method III: Sampling-based Maneuver Pattern Analysis

- Motivation: factors other than topology that matter for pattern distinction of spatiotemporal trajectory?

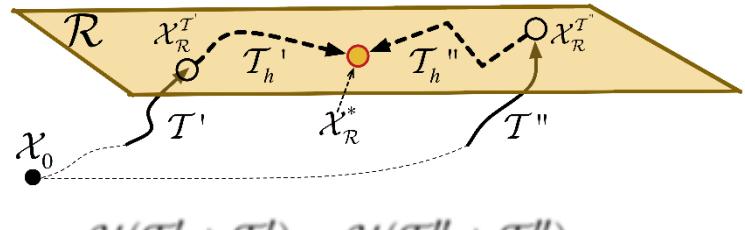


Homology
Marker Function
 $\mathcal{H}(\mathcal{T}) = \int_{\mathcal{T}} \mathbf{B} \cdot d\mathbf{l}$



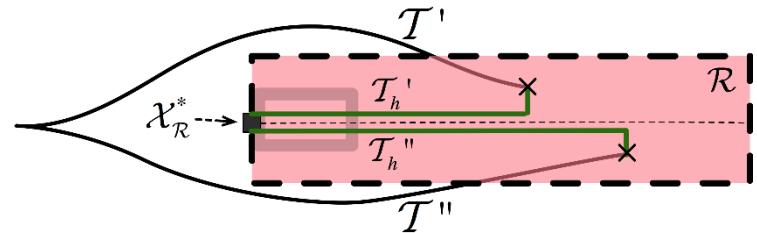
Region-based Distinction

Pseudo-Homology:



$$\mathcal{H}(\mathcal{T}' + \mathcal{T}_h') = \mathcal{H}(\mathcal{T}'' + \mathcal{T}_h'')$$

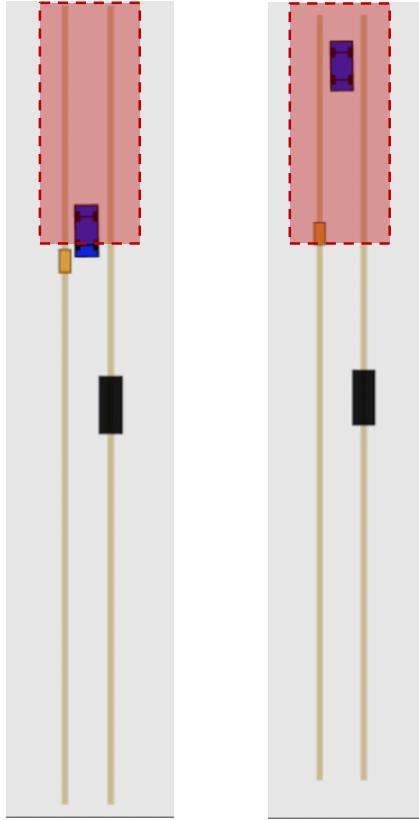
Helper trajectory for corridor-like region:



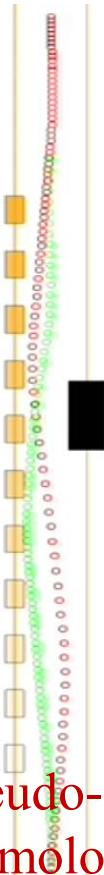
Proposed Method III: Sampling-based Maneuver Pattern Analysis

- Motivation: factors other than topology that matter for pattern distinction of spatiotemporal trajectory?

Conservative Aggressive



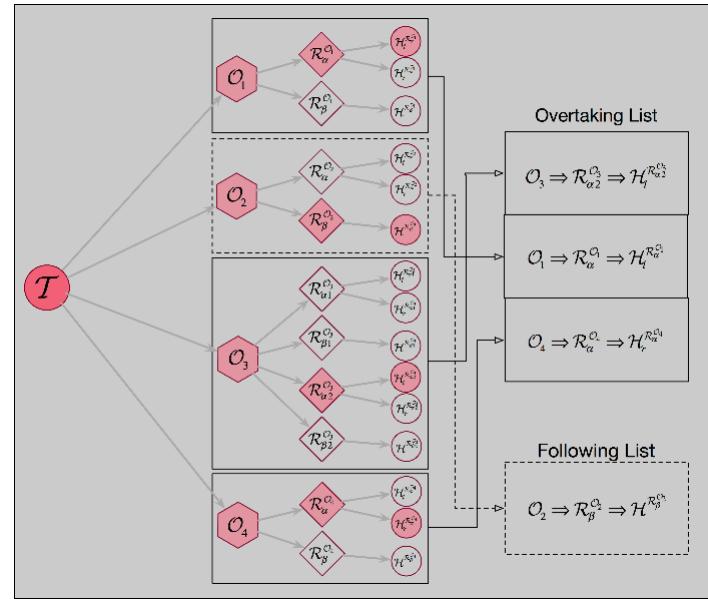
Pseudo-Homological



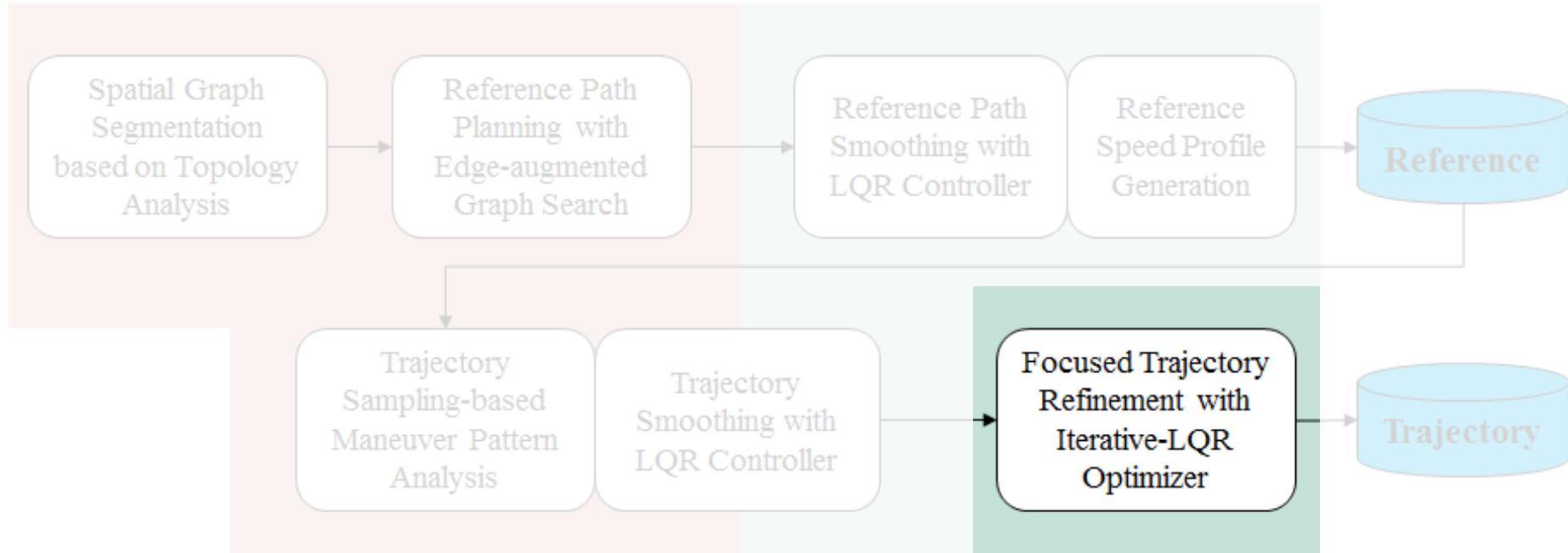
Overtaking Sequence-based Distinction

- Forward simulate APV & objects.
- Keep track of the overtaking timestamp.
- Sort obstacles (identifier) based on the overtaking timestamp.

Maneuver Pattern Distinction Tree

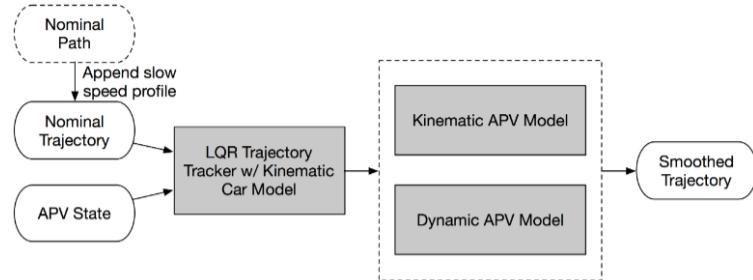


Adapted Method II: Focused Trajectory Optimization w/ Iterative-LQR



Adapted Method II: Focused Trajectory Optimization w/ Iterative-LQR

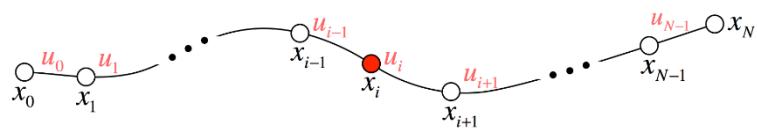
- Iterative LQR Backgrounds



Smoothed model-feasible path/trajecotry

- Does not consider any costs other than tracking errors, e.g., obstacles.
- Smoothed trajectory is only model-feasible, but not execution-feasible.

Trajectory Representation & Cost

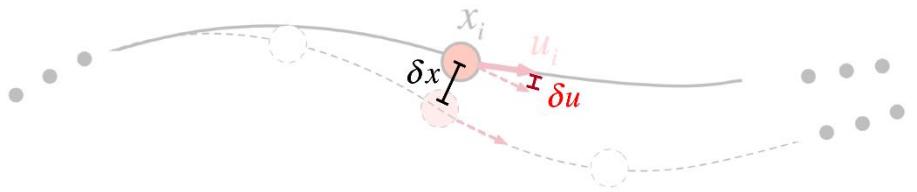


$$x_{i+1} = f_d(x_i, u_i)$$

$$X^{(k)} \doteq \{x_0, x_1, \dots, x_i, \dots, x_{N-1}, x_N\}$$

$$U^{(k)} \doteq \{u_0, u_1, \dots, u_i, \dots, u_{N-1}\}$$

**Iterative Linear Quadratic Regulator
(1st order Differential Dynamic Programming)**



Perturbation: δx incurred at (i-1)th timestamp.
 δu to be determined at (i)th timestamp.

What is the optimal δu given the δx ?

$$\begin{aligned} \delta u^* &= \underset{\delta u}{\operatorname{argmin}} \tilde{Q}(\delta x, \delta u) \quad \text{Singular Value Decomposition} \\ &= k + K \cdot \delta x \end{aligned}$$

$$\begin{aligned} Q_{uu} &= P \Sigma Q^T \\ Q_{uu}^{-1} &= Q \Sigma^{-1} P^T \end{aligned}$$

$$\begin{aligned} k &= -Q_{uu}^{-1} Q_u \\ K &= -Q_{uu}^{-1} Q_{ux} \end{aligned} \quad \Sigma = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \ddots & \ddots \\ 0 & \ddots & \sigma_n \end{bmatrix} \quad \Sigma^{-1} = \begin{bmatrix} \frac{1}{\sigma_1 + \lambda} & 0 & 0 \\ 0 & \ddots & \ddots \\ 0 & \ddots & \frac{1}{\sigma_n + \lambda} \end{bmatrix} + \lambda$$

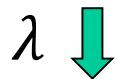
Adapted Method II: Focused Trajectory Optimization w/ Iterative-LQR

- Focused Iterative-LQR & Cost Function

Determine λ

Levenberg-Marquardt Heuristics

iLQR Optimization making progress:



iLQR Optimization not making progress:



Focused Line-Search Heuristics

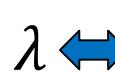
Optimization violating maneuver pattern constraint:



iLQR Optimization making progress:



Line-Search Optimization making progress:

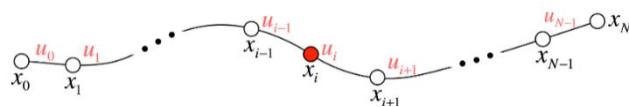


iLQR Optimization not making progress:



Cost Function

Cost penalizes certain undesirable aspect of the plan/trajectory.



$$\text{Total Cost: } J(X, U) = g_N(x_N) + \sum_{i=0}^{N-1} g(x_i, u_i)$$

$$\begin{aligned} g(x_i, u_i) &= \sum_{k=1}^M \omega_k \cdot c_k(x_i, u_i) \\ g_N(x_N) &= \sum_{k=1}^M \omega_k \cdot c_k(x_N, 0) \end{aligned}$$

Weights Cost Terms

Better be Continuous?



Edge-Augmented Graph Search



Topological Region/Pattern Selection



Iterative-LQR Trajectory Optimization

Adapted Method II: Focused Trajectory Optimization w/ Iterative-LQR

- Cost Function Design

iLQR is underlying Newton's Method:

- Convex Optimization
- 2nd-order Continuous

Key Results in Convex Optimization

2nd -order continuous
monotonically non-
decreasing convex function

2nd -order continuous
differentiable convex

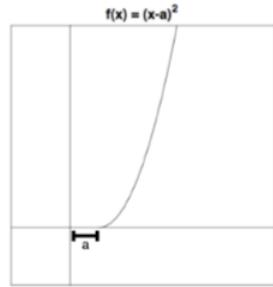
$$h(y)$$

$$g(x)$$

$$f = (h \circ g)(x) = h(g(x))$$

Still Convex!

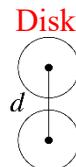
Quadratic



State/Control Variable

E.g., steering, speed, swirl, acceleration, etc.

Distance Function

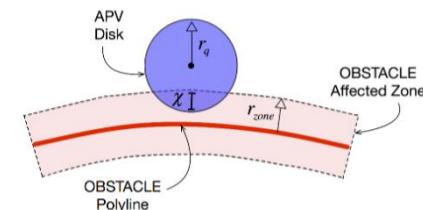
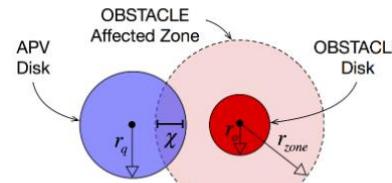


Convex
Modulation
Function

Convex
Feature
Function

Non-convex Features

Penetrated Distance:



Distance

Penetrated
Distance

Non-Convex!

Local cost quadratization \rightarrow Topological Structure

Total Cost Function

Behavioral Feature

$$g = c_{obstacle}^B(\omega_{obstacle}) + c_{ref}^B(\omega_{ref}) + c_{speed}^B(\omega_{speed}) + c_{lat_acc}^B(\omega_{lat_acc}) + c_{swirl}^B(\omega_{swirl}) + c_{lon_acc}^B(\omega_{acc}, \omega_{dec}) \\ + c_{obstacle}^C(\Omega) + c_{lane}^C(\Omega) + c_{speed}^C(\Omega) + c_{lat_acc}^C(\Omega) + c_{swirl}^C(\Omega) + c_{lon_acc}^C(\Omega)$$

Constraint Feature:

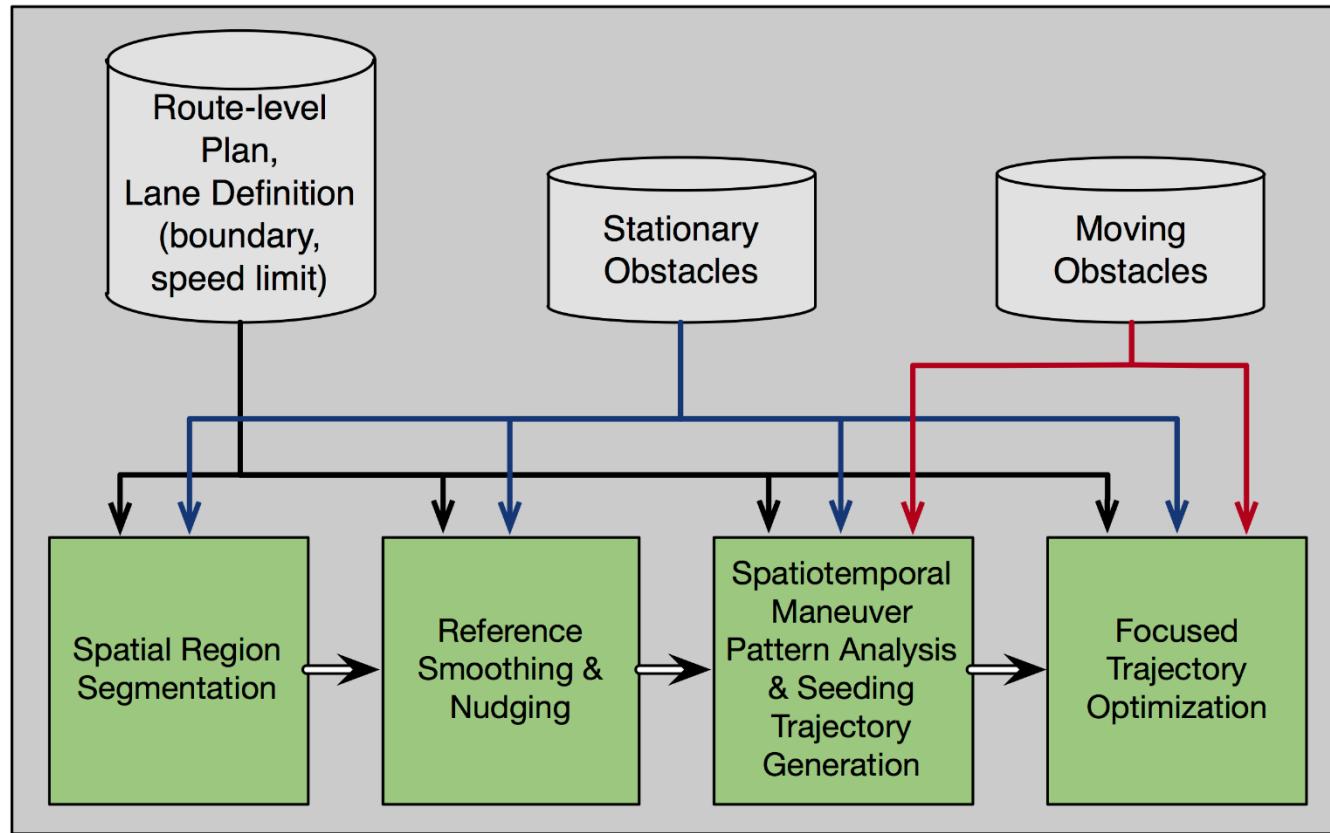
Penalty
Method

Constrained
Iterative-LQR
Optimization

Execution
Feasible
Trajectory

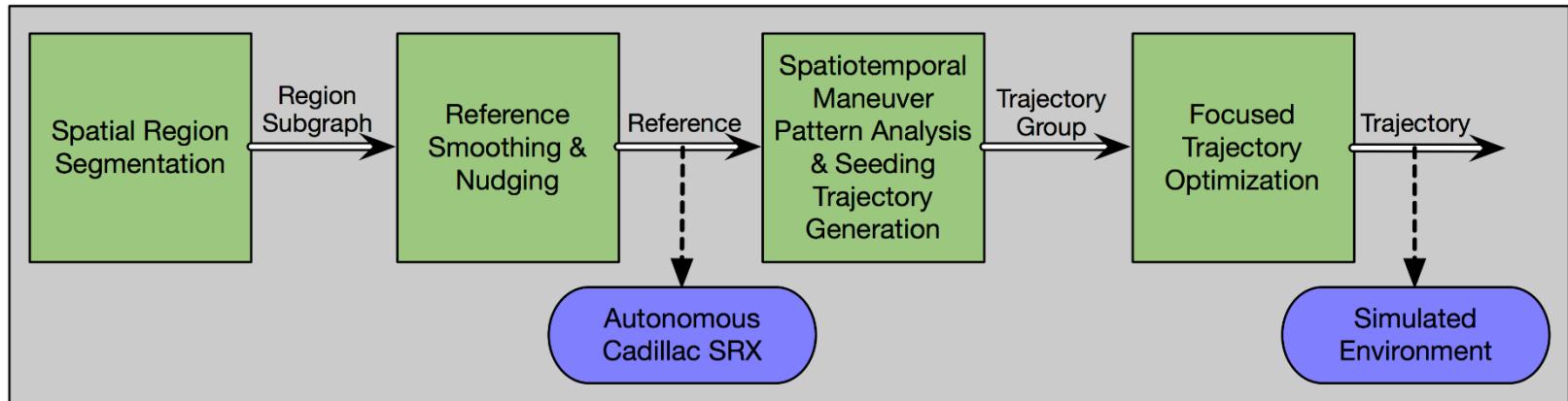
Results

- Planning Framework / Algorithmic Flow

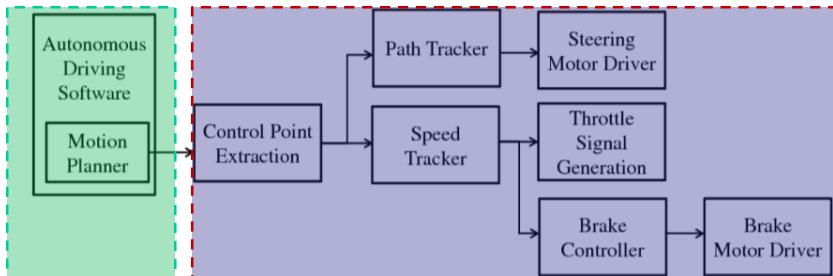


Results

- Experiment Setup



Cadillac Setup



Reference Generation

Tracking Control

Simulation Setup

Scenario:

A particular setup of the environment elements including lanes, obstacles and the APV.

Snapshot:

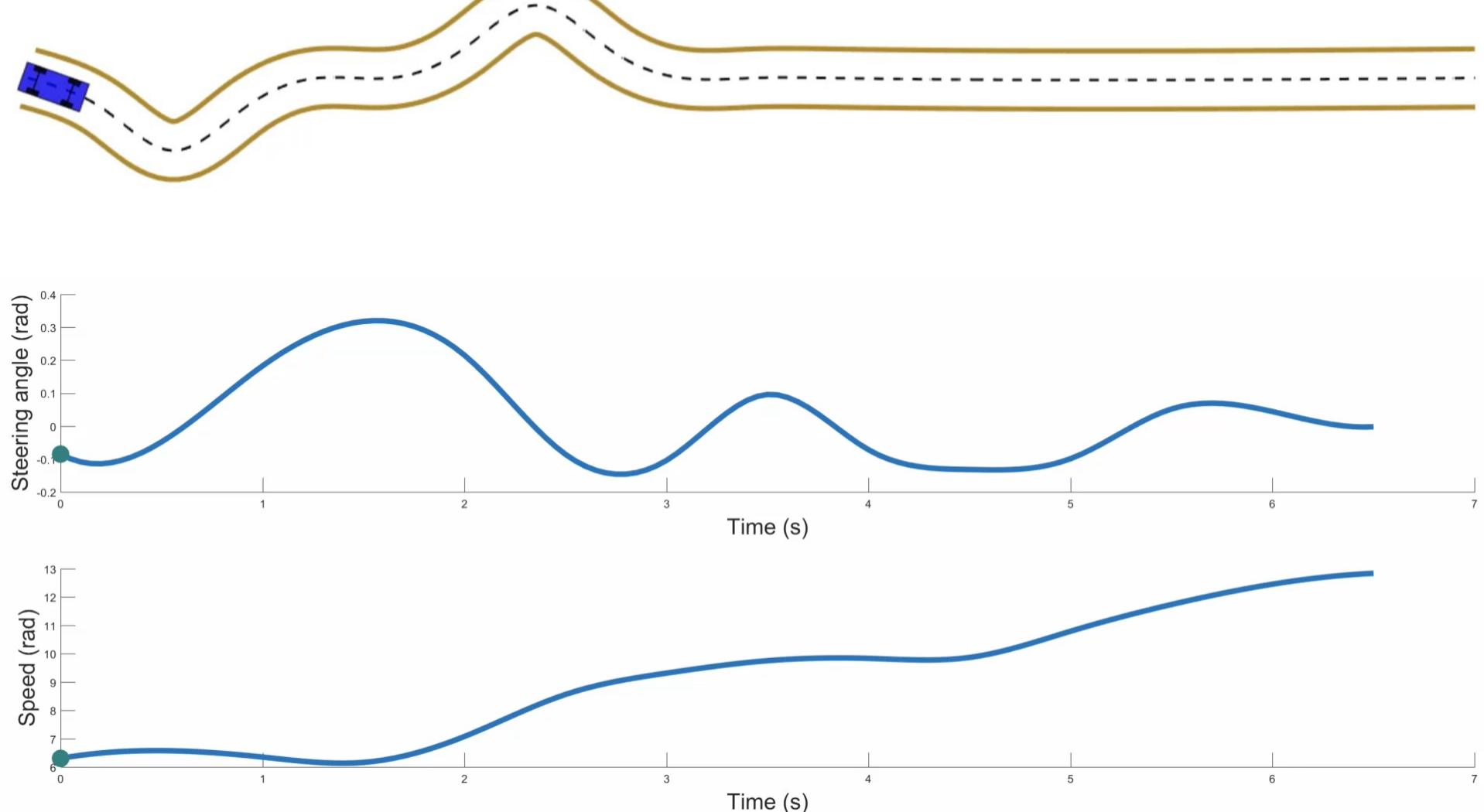
Plots of the environment elements' states and the planning outcome of each planning module at a given time-stamp.

Overlay:

A plot of the overlaid states of the environment elements' states over a time period that the APV demonstrates a maneuver.

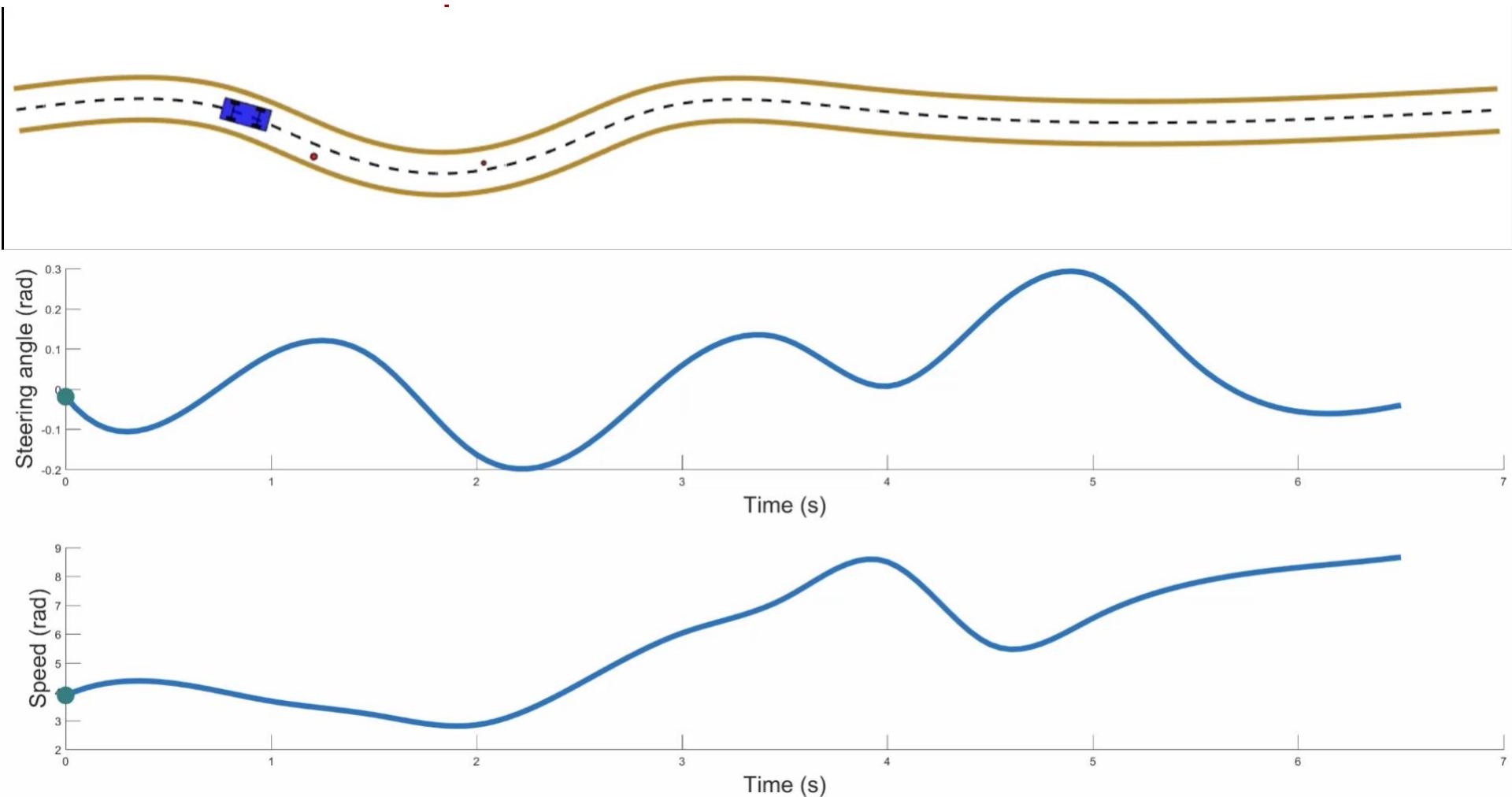
Results

- Simulation Scenario 1



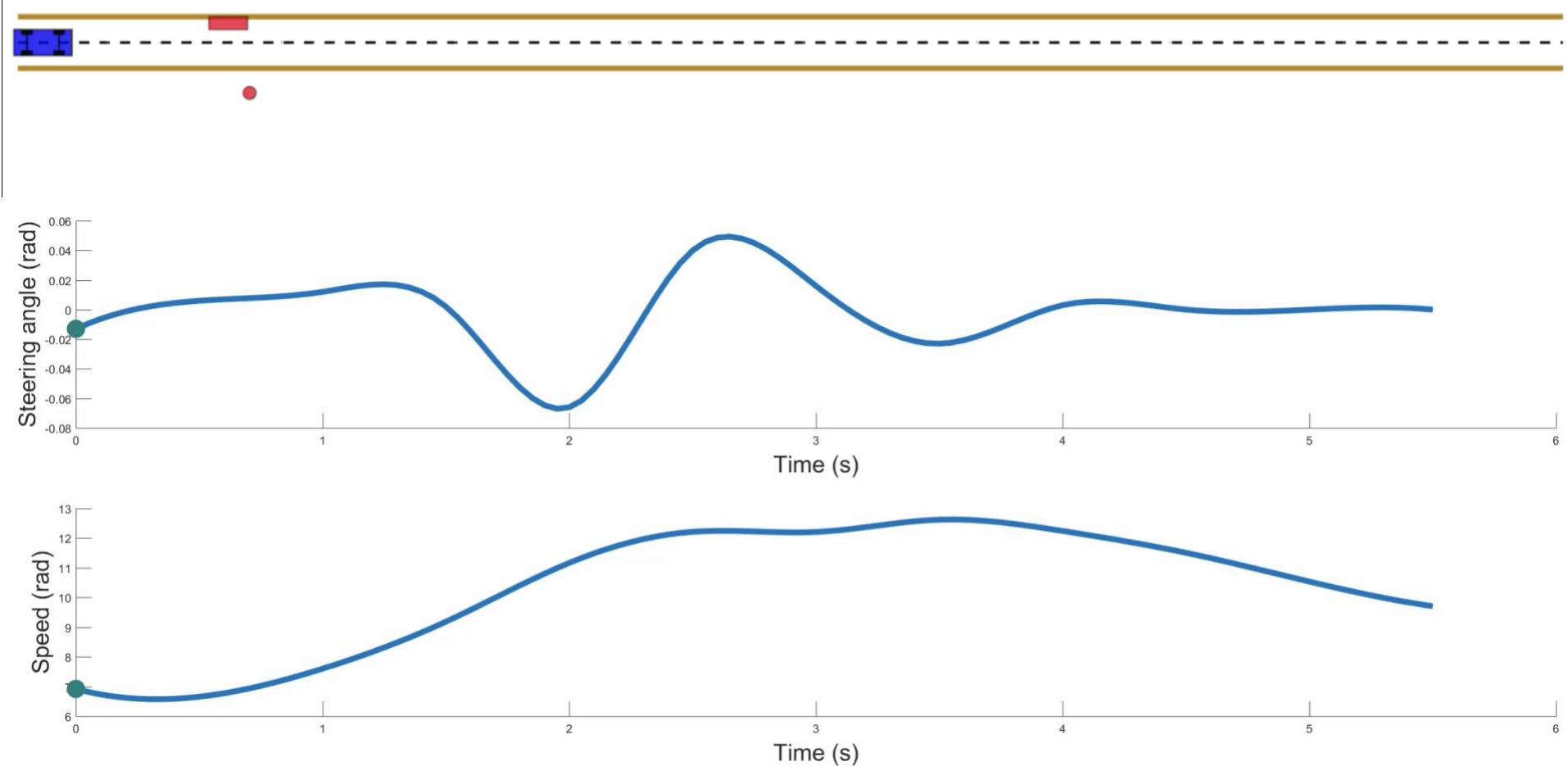
Results

- Simulation Scenario 2



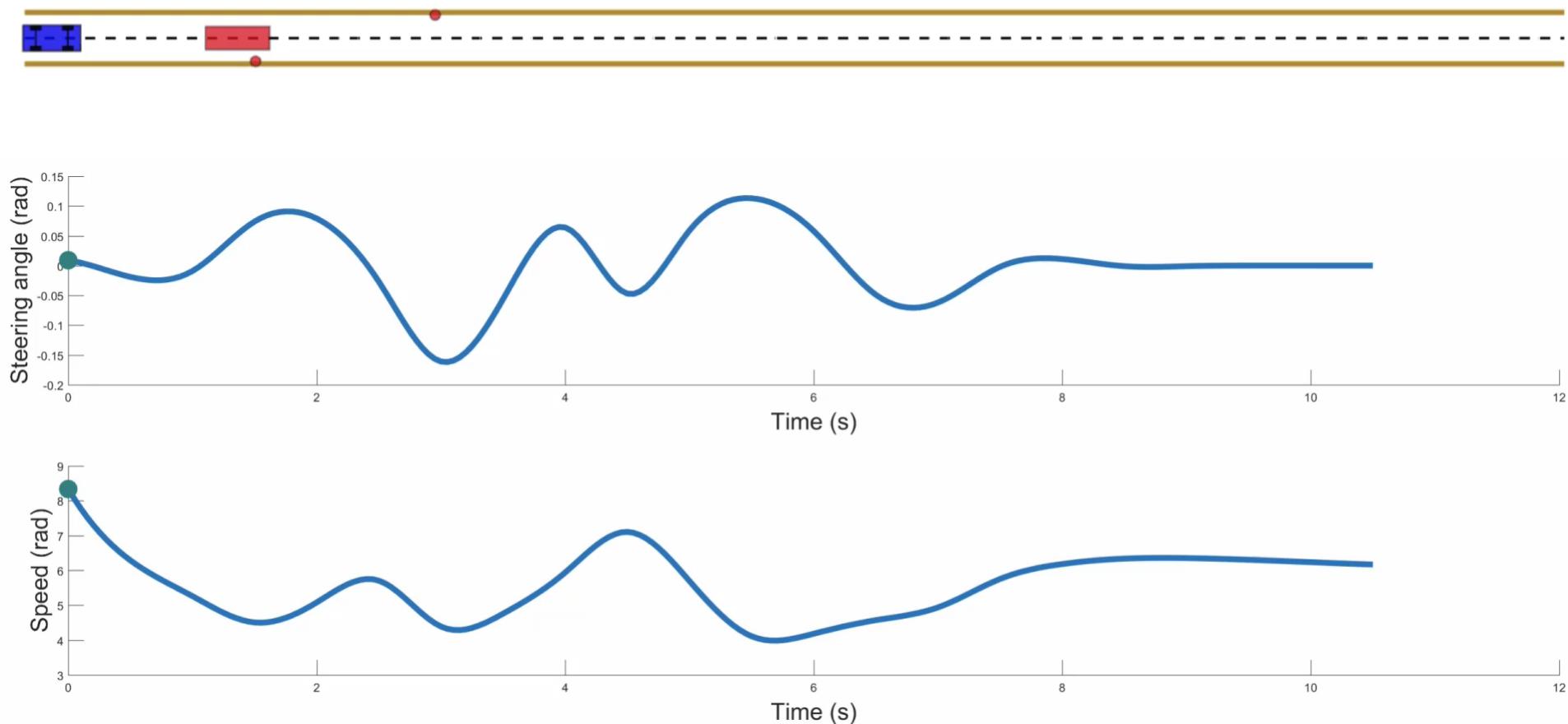
Results

- Simulation Scenario 3



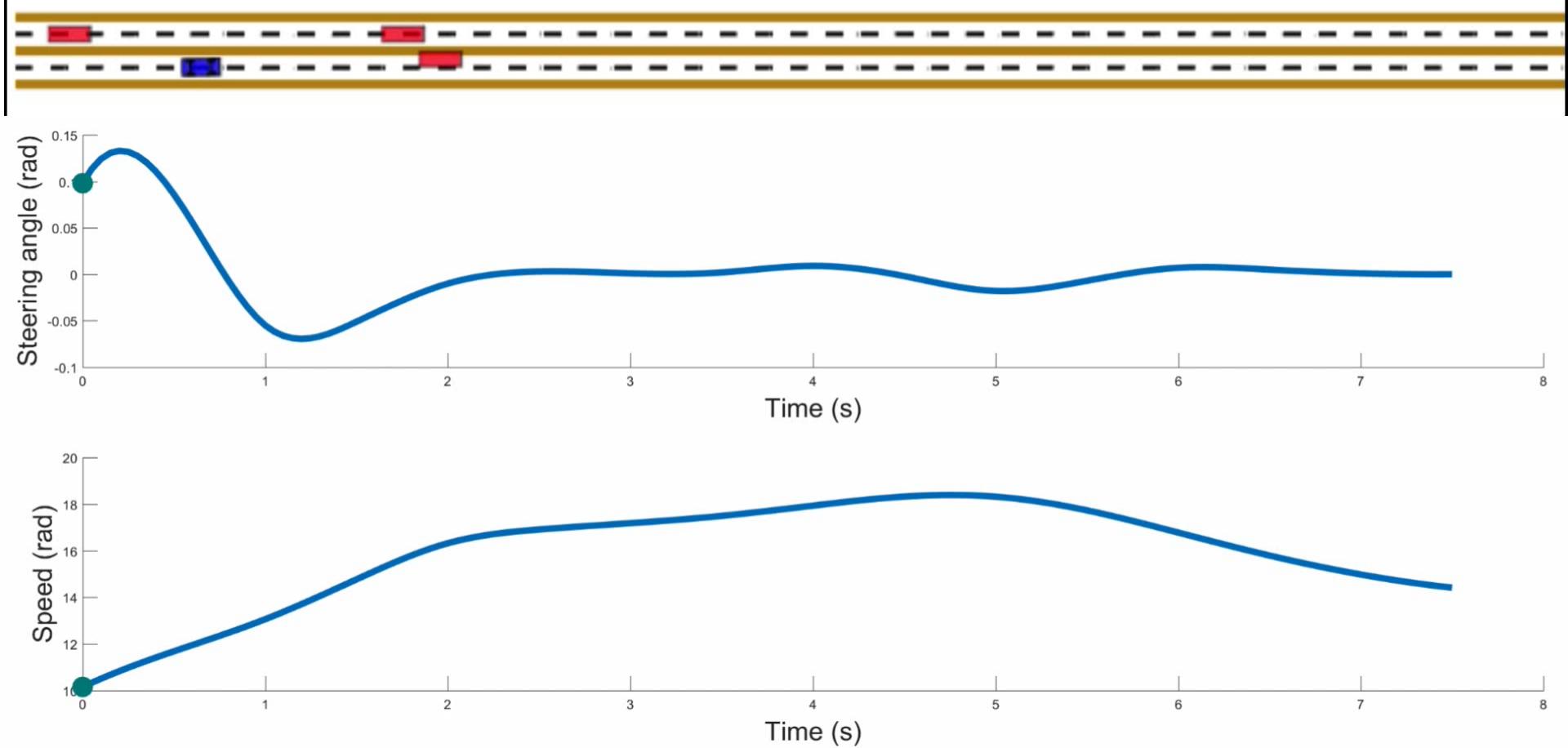
Results

- Simulation Scenario 4



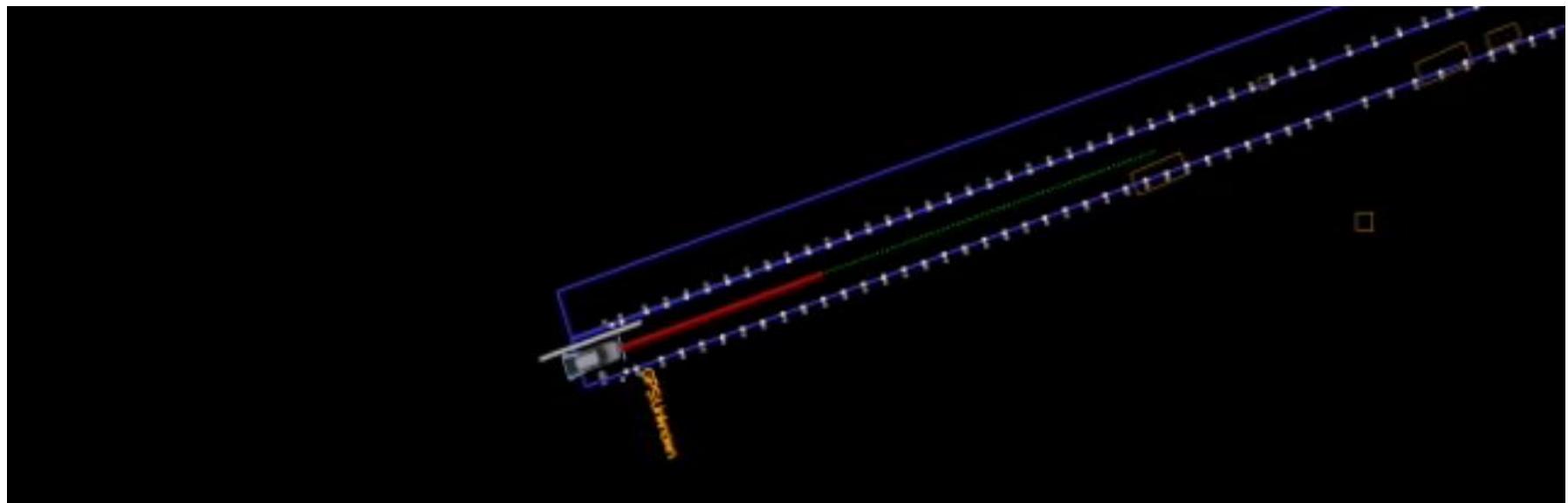
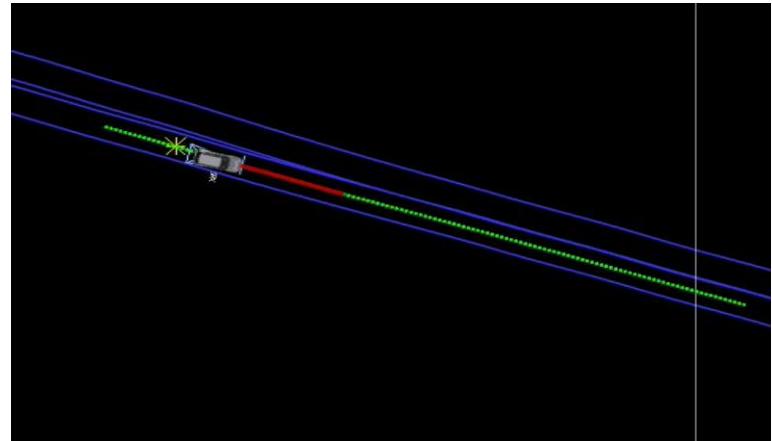
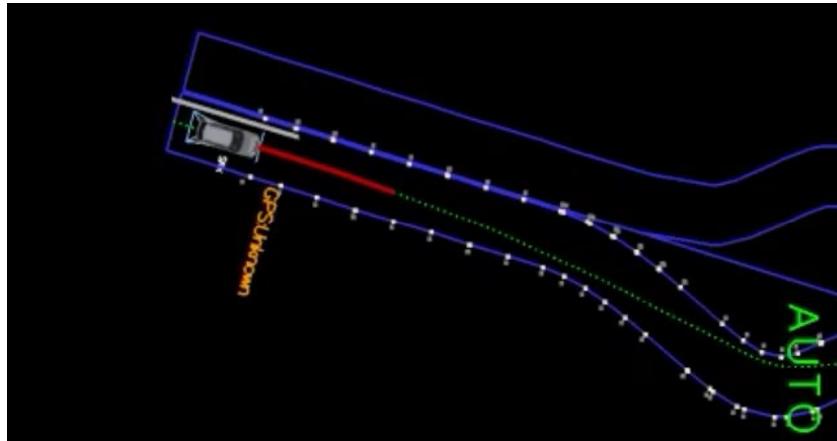
Results

- Simulation Scenario 5



Results

- On-Vehicle (Simulation)



Results

- On-Vehicle (Closed Course)

***Autonomous Driving Test
(On-Vehicle)***

Avoid Tightly Spaced Obstacles

Results

- On-Vehicle (Closed Course)



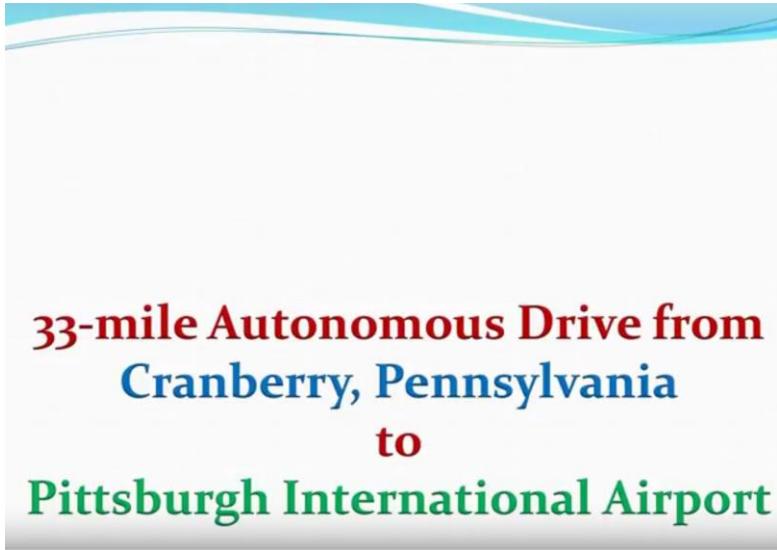
Results

- On-Vehicle (Schenley Park)



Results

- Past demo video footages.



[Link](#)

**Highway Driving
Near Capitol Hill, Washington DC
I-395 South
Multiple lane merges & changes**

[Link](#)

Conclusions

- Comparison

No #	Methods	\mathcal{F}_{1a}	\mathcal{F}_{1b}	\mathcal{F}_2	\mathcal{F}_3	\mathcal{F}_4	\mathcal{F}_5
M1	Boss's Local Planner [Ferguson et al., 2008]	✓	✓	✗	✗	✗	✗
M2	Junior's RNDF Follower [Montemerlo et al., 2008]	✓	✓	✓	✗	✗	✗
M3	Nonholonomic Potential field [Huang et al., 2006]	✓	✗	✗	✗	✗	✗
M4	Biased RRT [Kuwata et al., 2008]	✓	✓	✓	✓	✗	✗
M5	Spatiotemporal lattice [Ziegler and Stiller, 2009, McNaughton, 2011]	✓	✓	✓	✓	✗	✗
M6	Timed Elastic Band [Roesmann et al., 2012]	✓	✗	✓	✓	✗	✗
M7	Hybrid-A* + Conjugate Gradient Descent Smoothing [Dolgov et al., 2008]	✓	✗	✓	✗	✗	✓
M8	iLQR/DDP [Li and Todorov, 2004b, van den Berg, 2016, Tassa et al., 2014]	✓	✗	✓	✓	✗	✗
M9	Cell decomposition + Mixed-Integer Programming [Park et al., 2016]	✓	✗	✓	✗	✗	✓
M10	Homotopy marker function + Timed Elastic Band [Rösmann et al., 2015]	✓	✗	✓	✓	✗	✗
P	Proposed	✓	✓	✓	✓	✓	✓

Conclusions

- Summary

Requirement 1: a deliberative trajectory planning system.

Reference and Trajectory Planning have long spatial (100m) or temporal horizon (6s).

Requirement 2: spatiotemporal (trajectory) planning.

Maneuver pattern analysis and focused trajectory optimization explicitly plan spatiotemporally.

Requirement 3: tactical reasoning capability with topological awareness.

Region segmentation and maneuver pattern analysis provides tactical reasoning capability.

Requirement 4: global awareness with the ability to converge to a local optimum.

Reference planning and maneuver pattern analysis both perform global sampling.

Requirement 5: apply to self-driving passenger vehicle on-road driving.

Extensively experimented on a real self-driving vehicle.

Conclusions

- Contributions
 1. A Hybrid Trajectory Planning Framework
 2. Search over Edge-augmented Graph for Reference Path Planning
 3. Topological Backward Propagation Algorithm for Region (Sub-Graph) Segmentation
 4. Sampling-based Maneuver Pattern Analysis/Seeding Trajectory Generation for On-Road Self-Driving
 5. Adapted Linear Quadratic Regulator (LQR) and Iterative-LQR for Trajectory Smoothing/Optimization
 6. Identification of Useful Cost Function Generation Principles

Future Work

- TBC: the fusion of sampling-based and optimization-based method.
- TBC: The inclusion in topological analysis in a trajectory planning system.
- Automated parameter tuning through machine learning techniques.
- Unstructured learning of driving skill, a.k.a, neural network, etc.
- Misc:
 - More complex/realistic vehicle model.
 - Planning with better shape representations.
 - Faster collision checking (distance function evaluation).
 - Faster homology information calculation.

Publications

- Gu, T., et al. (2016). Automated Motion-based Tactical Maneuver Discovery, Reasoning and Trajectory Planning for Autonomous Driving (IROS), 2016
- Gu, T., et al. (2016). Runtime-Bounded Tunable Motion Planning for Autonomous Driving (IV), 2016
- Gu, T., et al. (2016). Human-like Planning of Swerve Maneuvers for Autonomous Vehicles (IV), 2016
- Gu, T. and J. Dolan (2015). A Lightweight Simulator for Autonomous Driving Motion Planning Development. the Fourth International Conference on Intelligent Systems and Applications (INTELLI), 2015.
- Gu, T., et al. (2015). Tunable and Stable Real-Time Trajectory Planning for Urban Autonomous Driving. International Conference on Intelligent Robots and Systems (IROS), 2015. **Best student paper finalist**
- Wei, J., et al. (2014). A behavioral planning framework for autonomous driving. Intelligent Vehicles Symposium (IV), 2014, IEEE.
- Gu, T., et al. (2014). On-Road Trajectory Planning for General Autonomous Driving with Enhanced Tunability. Intelligent Autonomous Systems (IAS-13), 2014, Springer International Publishing Switzerland.
- Gu, T. and J. Dolan (2014). Toward human-like motion planning in urban environments. Intelligent Vehicles Symposium (IV), 2014, IEEE.
- Gu, T., et al. (2013). Focused trajectory planning for autonomous on-road driving. Intelligent Vehicles Symposium (IV), 2013, IEEE.
- Gu, T. and J. Dolan (2012). On-road motion planning for autonomous vehicles. International Conference on Intelligent Robotics and Applications (ICIRA), 2012, Springer.

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