Speed Profile Planning in Dynamic Environments via Temporal Optimization

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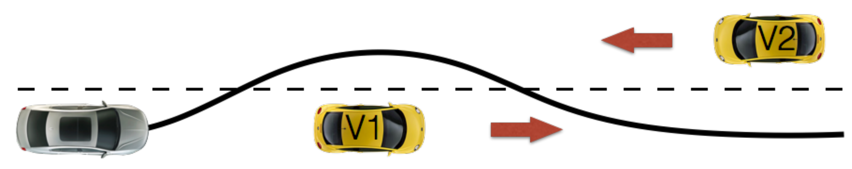
Speed Profile Planning in Dynamic Environments

What Is a Speed Profile?

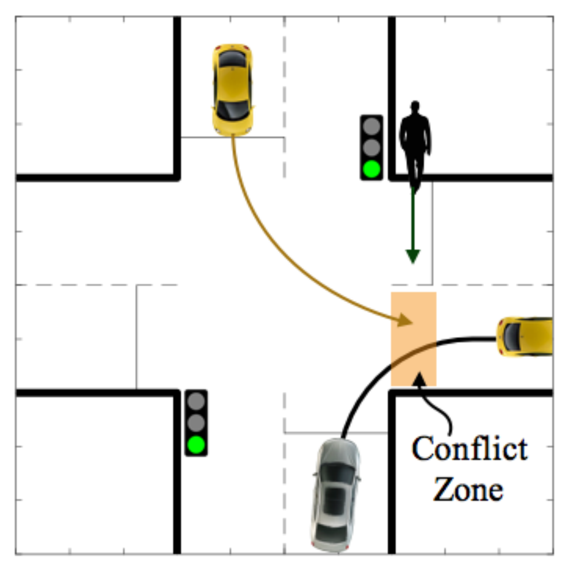
- A one-to-one mapping between the time domain T and the distance domain s of a path.

Why Speed Profile Planning?

- Layered approaches (path planning + speed profile planning) are usually more computationally efficient than integrated approaches for trajectory planning (case 1).
- In certain cases, a path is spatially fixed. The vehicle can only resort to temporal maneuvers to respond to other road participants (case 2).

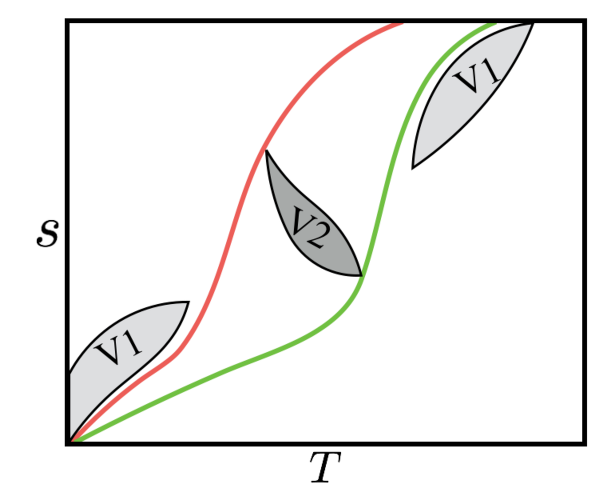


Case 1: Overtake Using the Opposite Lane



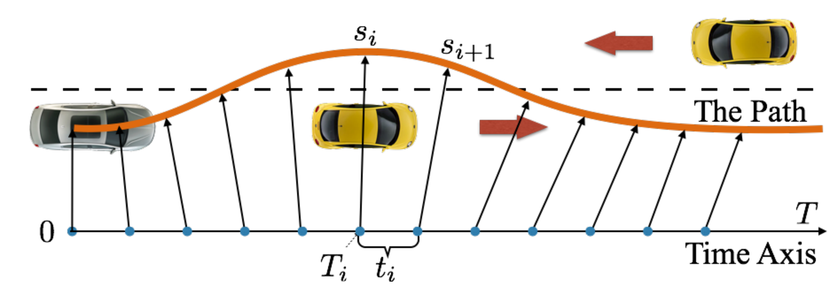
Case 2: Right Turn at Intersection.

Representing the Speed Profile — *s-T* graph

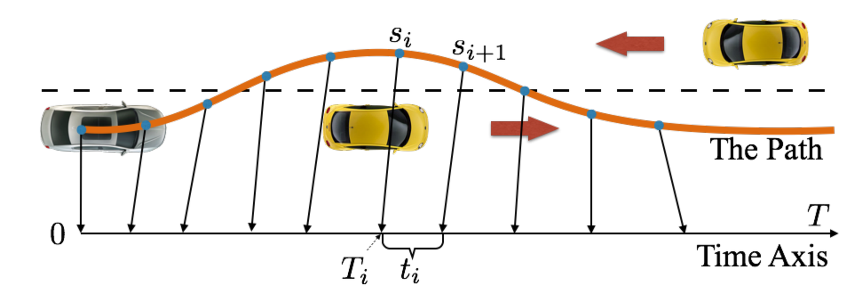


- Illustration under case 1.
- Red profile: Overtake before V2 passes.
- Green profile: Overtake after V2 passes.
- The homotopy class should be set as constraints before optimization.

Optimizing over station



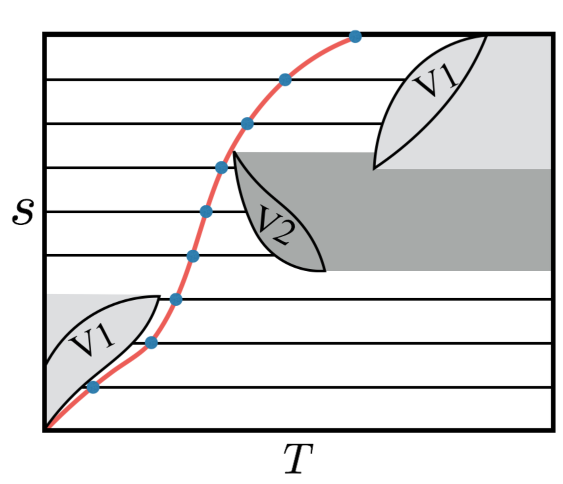
Optimizing over time



- Major advantage of temporal optimization: Do not require parametric representation of the path.

Temporal Optimization Using the Slack Convex Feasible Set (SCFS) Algorithm

Temporal Optimization



v	velocity $\in \mathbb{R}^2$;
a	acceleration $\in \mathbb{R}^2$;
j	jerk $\in \mathbb{R}^2$;
θ	vehicle heading $\in [0, 2\pi)$;
τ	longitudinal direction $\in \mathbb{R}^2$;
η	lateral direction $\in \mathbb{R}^2$;
t	relative time interval $\in \mathbb{R}$;
T	absolute time stamp $\in \mathbb{R}$;
h	horizon of the problem $\in \mathbb{N}$.

Problem 1: Temporal Optimization Problem

$$\min_{t_1,\cdots,t_h} w_1 \sum |a_i^\tau|^2 + w_2 \sum |a_i^\eta|^2 + w_3 \sum |j_i^\tau|^2 + w_4 \sum |j_i^\eta|^2 + w_5 \sum (v^r - v_i^\tau)^2$$

$$s.t.\ T_i \in [T_i^{min}, T_i^{max}]$$

$$|a^\tau| \leq \bar{a}, |a^\eta| \leq \bar{a}$$

Problem 1: Compact Form

$$\min_{\mathbf{t}} \mathbf{u}^T R \mathbf{u}$$

$$s.t.\ A\mathbf{t} \leq b, -\bar{a} \leq u_i^j \leq \bar{a}, \forall i, \forall j = 1, 2$$

$$f_i^j(\mathbf{t}) + h_i^j(\mathbf{t}) u_i^j = 0, \forall i, j,$$

$$\mathbf{t} := [t_1, \cdots, t_h]$$

$$\mathbf{u} := [u_1, \cdots, u_h]$$

$$u_i := [a_i^\tau, a_i^\eta, j_i^\tau, j_i^\eta, v^r - v_i^\tau]$$

The Slack Convex Feasible Set Algorithm

- Iteratively solve a convex subproblem
- The convex subproblem is constructed by
  1) relaxation of the nonlinear equality constraints;
  2) linearization of non-convex inequality constraints.

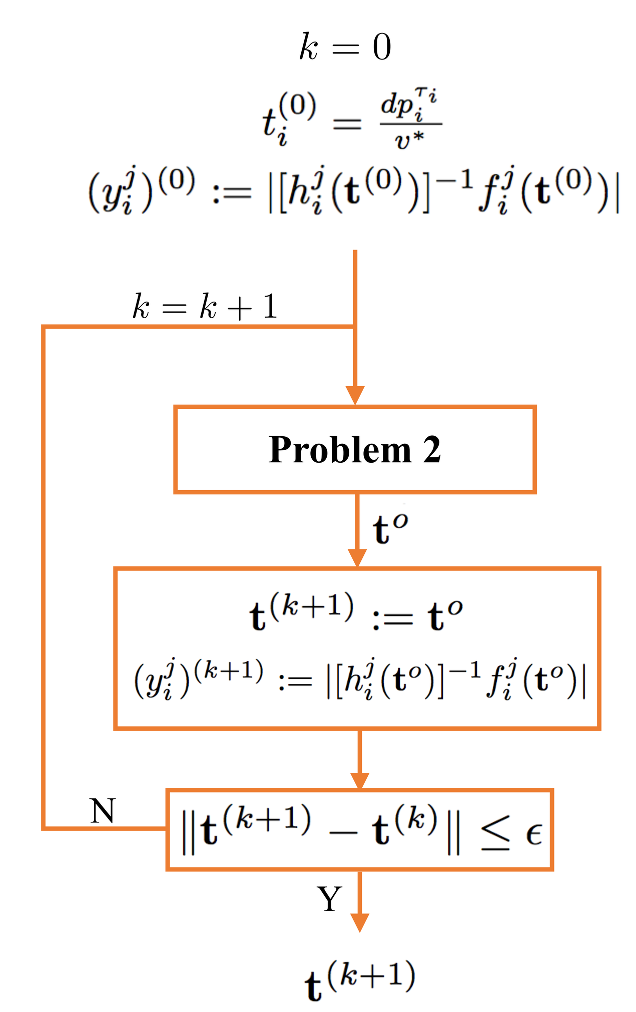
Problem 2: The Approximated Quadratic Program

$$\min_{\mathbf{t},\mathbf{y}} \mathbf{y}^T R \mathbf{y}$$

$$s.t.\ A\mathbf{t} \leq b, y_i^j \leq \bar{a}, \forall i, \forall j = 1, 2$$

$$\left[\nabla_{\mathbf{t}} f_i^j(\mathbf{t}^{(k)}) + \nabla_{\mathbf{t}} h_i^j(\mathbf{t}^{(k)})(y_i^j)^{(k)}\right](\mathbf{t} - \mathbf{t}^{(k)}) + h_i^j(\mathbf{t}^{(k)}) y_i^j + f_i^j(\mathbf{t}^{(k)}) \geq 0$$

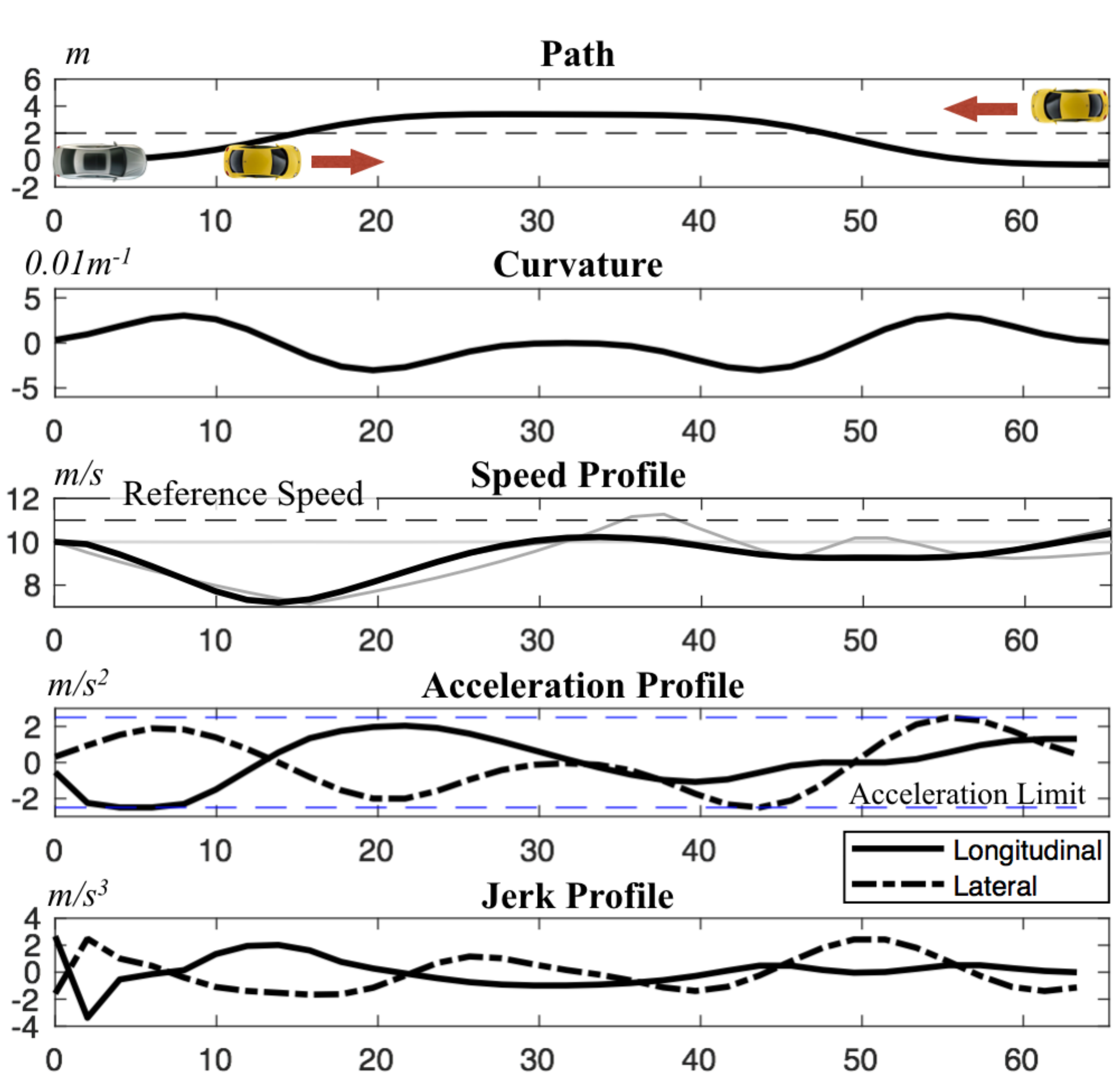
$$\left[\nabla_{\mathbf{t}} f_i^j(\mathbf{t}^{(k)}) - \nabla_{\mathbf{t}} h_i^j(\mathbf{t}^{(k)})(y_i^j)^{(k)}\right](\mathbf{t} - \mathbf{t}^{(k)}) - h_i^j(\mathbf{t}^{(k)}) y_i^j + f_i^j(\mathbf{t}^{(k)}) \leq 0.$$

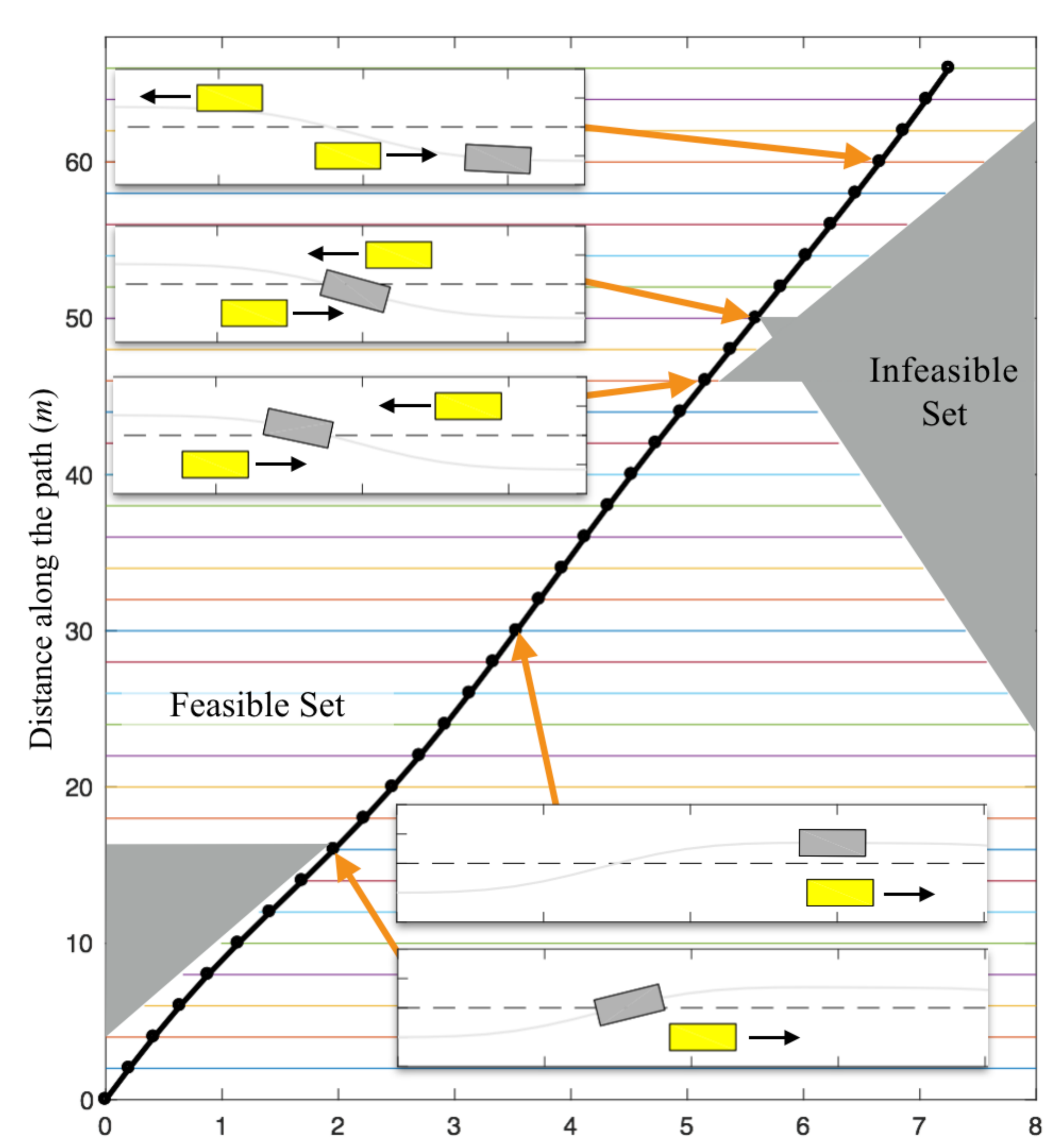


*C. Liu and M. Tomizuka, "Geometric considerations on real time trajectory optimization for nonlinear systems," submitted to *System & Control Letters*, 2016.

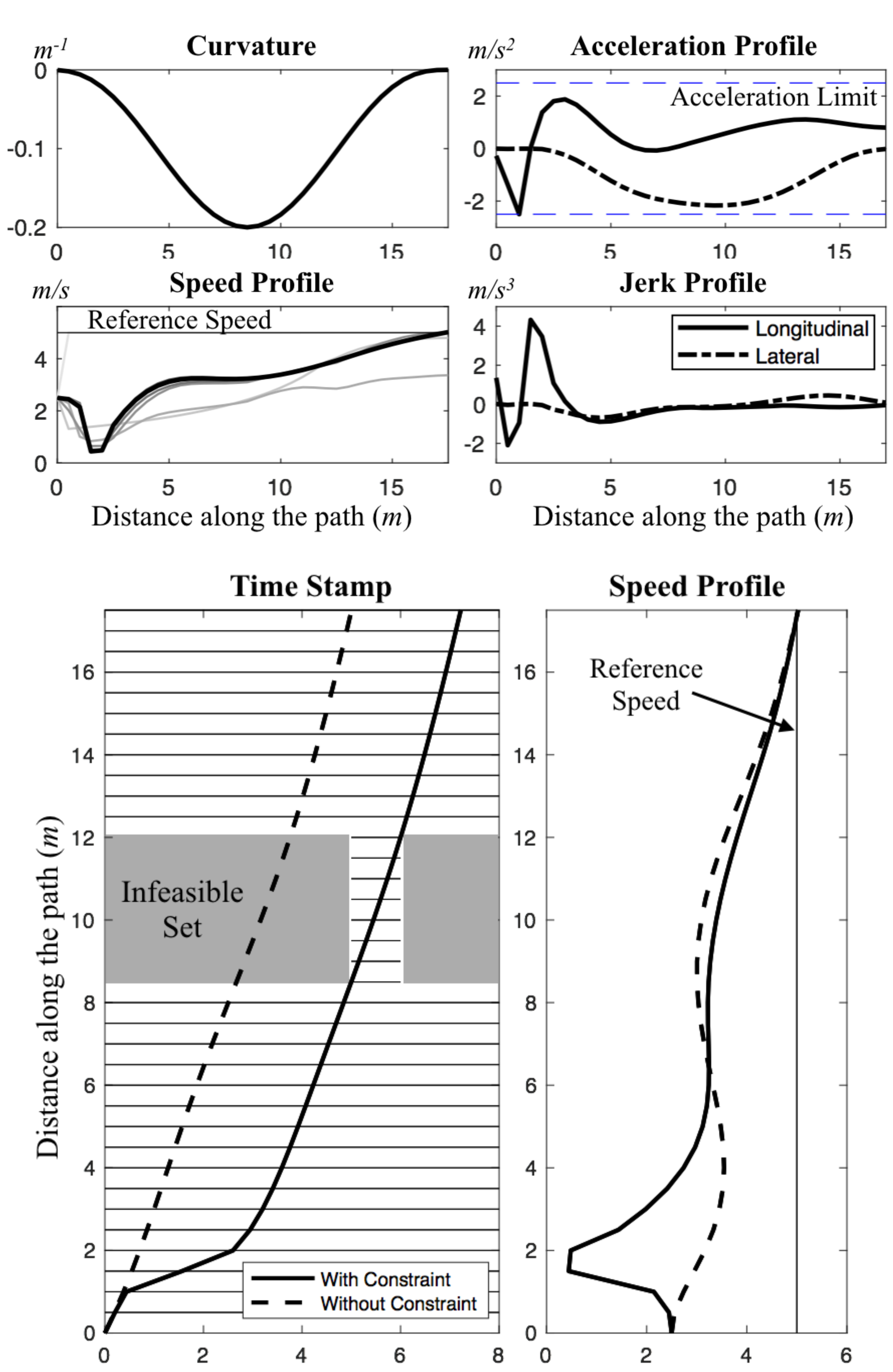
Case Studies

Case 1





Case 2



- Platform: Matlab on a Macbook of 2.3 GHz using Intel Core i7.
- In the SCFS algorithm, Problem 2 for each iteration was solved using the quadprog function.
- For comparison, Problem 1 was also solved using the SQP in the fmincon function.

	SCFS		SQP	
	Number of Iterations	Computation Time	Number of Iterations	Computation Time
Case 1	5	0.308s	48	28.513s
Case 2	8	0.522s	>100	-