# 1 Problem Formulation

acados can handle the following optimization problem

with

- state vector  $x : \mathbb{R} \to \mathbb{R}^{n_x}$
- control vector  $u : \mathbb{R} \to \mathbb{R}^{n_{\mathrm{u}}}$
- algebraic state vector  $z : \mathbb{R} \to \mathbb{R}^{n_z}$

 $s_{11 \text{ h}}^{\text{e}}, s_{11 \text{ hx}}^{\text{e}}, s_{11 \text{ hu}}^{\text{e}}, s_{11 \text{ o}}^{\text{e}} > 0,$ 

- model parameters  $p \in \mathbb{R}^{n_p}$
- slacks for path constraints  $s_l(t) = (s_{l,bu}, s_{l,bx}, s_{l,g}, s_{l,h}) \in \mathbb{R}^{n_s}$  and  $s_u(t) = (s_{u,bu}, s_{u,bx}, s_{u,g}, s_{u,h}) \in \mathbb{R}^{n_s}$

(21)

• slacks for terminal constraints  $s_{l}^{e}(t) = (s_{l,bx}^{e}, s_{l,e}^{e}, s_{l,e}^{e}) \in \mathbb{R}^{n_{s}^{e}}$  and  $s_{u}^{e}(t) = (s_{u,bx}^{e}, s_{u,e}^{e}, s_{u,h}^{e}) \in \mathbb{R}^{n_{s}^{e}}$ 

Some of the following restrictions may apply to matrices in the formulation:

DIAG diagonal

SPUM horizontal slice of a permuted unit matrix SPUME like SPUM, but with empty rows intertwined

**Document Purpose** This document is only associated to the MATLAB interface of acados. Here, the focus is to give a mathematical overview of the problem formulation and possible options to model it within acados. The problem formulation and the possibilities of acados are similar in the PYTHON interface, however, some of the string identifiers are different. The documentation is not exhaustive and does not contain a full description for the MATLAB interface.

You can find examples int the directory <acados>/examples/acados\_matlab\_octave. The source code of the acados MATLAB interface is found in: <acados>/interfaces/acados\_matlab\_octave and should serve as a more extensive, complete and up-to-date documentation about the possibilities.

# 2 Dynamics

The system dynamics term is used to connect state trajectories from adjacent shooting nodes by means of equality constraints. The system dynamics equation (3) is replaced with a discrete-time dynamic system. The dynamics can be formulated in different ways in acados: As implicit equations in continuous time (22), or as explicit equations in continuous time (23) or directly as discrete-time dynamics (24). This section and table 1 summarizes the options.

## 2.1 Implicit Dynamics

The most general way to provide a continuous time ODE in acados is to define the function  $f_{\text{impl}}: \mathbb{R}^{n_x} \times \mathbb{R}^{n_z} \times \mathbb{R}^{n_z} \times \mathbb{R}^{n_z} \times \mathbb{R}^{n_p} \to \mathbb{R}^{n_x+n_z}$  which is fully implicit DAE formulation describing the system as:

$$f_{\text{impl}}(x, \dot{x}, u, z, p) = 0.$$
 (22)

acados can discretize  $f_{\text{impl}}$  with a classical implicit Runge-Kutta (irk) or a structure exploiting implicit Runge-Kutta method (irk\_gnsf). Both discretization methods are set using the 'sim\_method' identifier in a acados\_ocp\_opts class instance.

#### 2.2 Explicit Dynamics

Alternatively, acados offers an explicit Runge-Kutta integrator (erk), which can be used with explicit ODE models, i.e., models of the form

$$f_{\text{expl}}(x, u, p) = \dot{x}. \tag{23}$$

## 2.3 **Discrete Dynamics**

Another option is to provide a discrete function that maps state  $x_i$ , control  $u_i$  and parameters  $p_i$  from shooting node i to the state  $x_{i+1}$  of the next shooting node i + 1, i.e., a function

$$x_{i+1} = f_{\text{disc}}(x_i, u_i, p_i).$$
 (24)

Table 1: Dynamics definitions

Term	String identifier	Data type	Required
$f_{ m impl}$ respectively $f_{ m expl}$ $f_{ m disc}$ -	dyn_expr_f dyn_exp_phi dyn_type	CasADi expression CasADi expression string ('explicit', 'implicit' or 'discrete')	yes yes yes

## 3 Cost

There are different acados modules to model the cost functions in equation (1).

- $l: \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_z} \to \mathbb{R}$  is the Lagrange objective term.
- $m: \mathbb{R}^{n_x} \times \mathbb{R}^{n_z} \to \mathbb{R}$  is the Mayer objective term.

to define which one is used set  $cost_type for l$ ,  $cost_type_e for m$ .

Setting the slack penalties in equation (1) is done in the same way for all cost modules, see table 2 for an overview. Moreover,

Table 2: Cost module slack variable options

Term	String id	Data type	Required
$Z_{\mathrm{l}}$	cost_Zl	double, <b>DIAG</b>	no
$Z_{ m u}$	cost_Zu	double, <b>DIAG</b>	no
$z_{ m l}$	cost_zl	double	no
$z_{\rm u}$	cost_zu	double	no
$Z_{ m l}^{ m e}$	cost_Zl_e	double, <b>DIAG</b>	no
$Z_{ m u}^{ m e} \ z_{ m l}^{ m e}$	cost_Zu_e	double, <b>DIAG</b>	no
$z_{\mathrm{l}}^{\mathrm{e}}$	cost_zl_e	double	no
$z_{\rm u}^{\rm e}$	cost_zu_e	double	no

you can specify  $cost_Z$ , to set  $Z_l$ ,  $Z_u$  to the same values, i.e., use a symmetric L2 slack penalty. Similarly,  $cost_Z_e$ ,  $cost_Z_e$ ,  $cost_Z_e$ ,  $cost_Z_e$  can be used to set symmetric slack L1 penalties, respectively penalties for the terminal slack variables.

Note, that the dimensions of the slack variables  $s_l(t)$ ,  $s_l^e(t)$ ,  $s_u(t)$  and  $s_u^e(t)$  are determined by acados from the associated matrices ( $Z_l$ ,  $Z_u$ ,  $J_{sh}$ ,  $J_{sg}$ ,  $J_{sbu}$ ,  $J_{sbx}$  etc.).

#### 3.1 Cost module: auto

Set cost\_type to auto (default). In this case acados detects if the cost function specified is a linear least squares term and transcribes it in the corresponding form. Otherwise, it is formulated using the external cost module. Note: slack penalties are optional and we plan to detected them from the expressions in future versions. Table 3 shows the available options.

Table 3: Cost module auto options

Term	String identifier	Data type	Required
1	cost_expr_ext_cost	CasADi expression	yes

## 3.2 Cost module: external

Set cost\_type to ext\_cost. See table 4 for the available options.

Table 4: Cost module external options

Term	String identifier	Data type	Required
1	cost_expr_ext_cost	CasADi expression	yes
m	cost_expr_ext_cost_e	CasADi expression	yes

#### 3.3 Cost module: linear least squares

In order to activate the linear least squares cost module, set cost\_type to linear\_ls. The Lagrange cost term has the form

$$l(x, u, z) = \frac{1}{2} \left\| \underbrace{V_x \, x + V_u \, u + V_z \, z}_{y} - y_{\text{ref}} \right\|_{w}^{2}$$
(25)

where matrices  $V_x \in \mathbb{R}^{n_y \times n_x}$ ,  $V_u \in \mathbb{R}^{n_y \times n_u}$  are  $V_z \in \mathbb{R}^{n_y \times n_z}$  map x, u and z onto y, respectively and  $W \in \mathbb{R}^{n_y \times n_y}$  is the weighing matrix. The vector  $y_{\text{ref}} \in \mathbb{R}^{n_y}$  is the reference.

Similarly, the Mayer cost term has the form

$$\underline{m(x,u,z)} = \frac{1}{2} \left\| \underbrace{V_x^{\text{e}} x - y_{\text{ref}}^{\text{e}}}_{y^{\text{e}}} \right\|_{W^{\text{e}}}^{2}$$
(26)

where matrix  $V_x^e \in \mathbb{R}^{n_ye \times n_x}$  maps x onto  $y^e$  and  $W^e \in \mathbb{R}^{n_ye \times n_ye}$  is the weighing matrix. The vector  $y_{\text{ref}}^e \in \mathbb{R}^{n_ye}$  is the reference.

See table 5 for the available options of this cost module.

Table 5: Cost module linear\_ls options

Term	String identifier	Data type	Required
$\overline{V_x}$	cost_Vx	double	yes
$V_u$	cost_Vu	double	yes
$V_z$	cost_Vz	double	yes
W	cost_W	double	yes
$y_{ m ref}$	cost_y_ref	double	yes
$V_x^{\mathrm{e}}$	cost_Vx_e	double	yes
$W^{e}$	cost_W_e	double	yes
$\mathcal{Y}_{\mathrm{ref}}^{\mathrm{e}}$	cost_y_ref_e	double	yes

#### 3.4 Cost module: nonlinear least squares

In order to activate the nonlinear least squares cost module, set cost\_type to nonlinear\_ls.

The nonlinear least squares cost function has the same basic form as eqns. (25 - 26) of the linear least squares cost module. The only difference is that y and  $y^e$  are defined by means of CasADi expressions, instead of via matrices  $V_x$ ,  $V_u$ ,  $V_z$  and  $V_x^e$ . See table 6 for the available options of this cost module.

Table 6: Cost module nonlinear\_ls options

Term	String identifier	Data type	Required
y W y <sub>ref</sub>	<pre>cost_expr_y    cost_W    cost_y_ref</pre>	CasADi expression double double	yes yes yes
$y^{ m e} \ W^{ m e} \ y^{ m e}_{ m ref}$	<pre>cost_expr_y_e    cost_W_e   cost_y_ref_e</pre>	CasADi expression double double	yes yes yes

#### 4 Constraints

This section is about how to define the constraints equations (2) and (4 - 21).

The Matlab interface supports the constraint module bgh, which is able to handle simple **b**ounds (on x and u), **g**eneral linear constraints and general nonlinear constraints. Meanwhile, the Python interface also supports the acados constraint module bgp, which can handle convex-over-nonlinear constraints in a dedicated fashion.

#### 4.1 Initial State

Note: An initial state is not required. For example for moving horizon estimation (MHE) problems it should not be set.

Two possibilities exist to define the initial states equation (2): a simple syntax and an extended syntax.

**Simple syntax** defines the full initial state  $x(0) = \bar{x}_0$ . The options are found in table 7.

Table 7: Simple syntax for setting the initial state

Term	String identifier	Data type	Required
$\bar{x}_0$	constr_x0	double	no

**Extended syntax** allows to define upper and lower bounds on a subset of states. The options for the extended syntax are found in table 8.

Table 8: Extended syntax for setting the initial state

Term	String identifier	Data type	Required
$\underline{x}_0$	constr_lbx_0	double	no
$\frac{x}{\bar{x}_0}$	constr_ubx_0	double	no
$J_{ m bx,0}$	constr_Jbx_0	double	no

#### 4.2 Path Constraints

Table 9 shows the options for defining the path constraints equations (4 - 13). Here, matrices

- $J_{sh}$ , maps lower slack vectors  $s_{l,h}(t)$  and upper slack vectors  $s_{u,h}(t)$  onto the non-linear constraint expressions h(x,u,p).
- $J_{\text{bx}}, J_{\text{bu}} \text{ map } x(t) \text{ and } u(t) \text{ onto its bounds vectors } \underline{x}, \bar{x} \text{ and } \underline{u}, \bar{u}, \text{ respectively.}$
- $J_{sx}$ ,  $J_{su}$  map lower slack vectors  $s_{l,bx}(t)$ ,  $s_{l,bu}(t)$  and upper slack vectors  $s_{u,bx}(t)$ ,  $s_{u,bu}(t)$  onto x(t) and u(t), respectively.
- $J_{sg}$  map lower slack vectors  $s_{l,g}(t)$  and upper slack vectors  $s_{u,g}(t)$  onto lower and upper equality bounds  $\underline{g}$ ,  $\bar{g}$ , respectively.
- C, D map x(t) and u(t) onto lower and upper inequality bounds g,  $\bar{g}$  (polytopic constraints)

#### 4.3 Terminal Constraints

Table 10 shows the options for defining the terminal constraints equations (14 - 21). Here, matrices

- $J_{sh}^e$ , maps lower slack vectors  $s_{l,h}^e(t)$  and upper slack vectors  $s_{u,h}^e(t)$  onto non-linear terminal constraint expressions  $h^e(x(T), p)$ .
- $J_{hv}^{e}$  maps x(T) onto its bounds vectors  $\underline{x}^{e}$  and  $\bar{x}^{e}$ .
- $J_{\text{sbx}}^{\text{e}}$  maps lower slack vectors  $s_{\text{l.bx}}^{\text{e}}$  and upper slack vectors  $s_{\text{u.bx}}^{\text{e}}$  onto x(T).
- $J_{sg}^{e}$  map lower slack vectors  $s_{l,g}^{e}(t)$  and upper slack vectors  $s_{u,g}^{e}(t)$  onto lower and upper equality bounds  $\underline{g}^{e}$ ,  $\bar{g}^{e}$ , respectively.

Table 9: Path constraints options

Term	String identifier	Data type	Required
$J_{ m bx}$	constr_Jbx	double, SPUM	no
$\frac{x}{\bar{x}}$	constr_lbx	double	no
$\bar{x}$	constr_ubx	double	no
$J_{ m bu}$	constr_Jbu	double, <b>SPUM</b>	no
<u>u</u>	constr_lbu	double	no
<u>и</u> ū	constr_ubu	double	no
С	constr_C	double	no
D	constr_D	double	no
g	constr_lg	double	no
$\frac{g}{\bar{g}}$	constr_ug	double	no
h	constr_expr_h	CasADi expression	no
$rac{h}{ar{h}}$	constr_lh	double	no
ħ	constr_uh	double	no
$J_{ m sbx}$	constr_Jsbx	double, <b>SPUME</b>	no
$J_{ m sbu}$	constr_Jsbu	double, SPUME	no
$J_{ m sg}$	constr_Jsg	double, SPUME	no
$J_{ m sbx}$	constr_Jsh	double, <b>SPUME</b>	no

•  $C^e$  maps x(T) onto lower and upper inequality bounds  $\underline{g}^e$ ,  $\bar{g}^e$  (polytopic constraints)

Table 10: Terminal constraints options

	14010 101 10111111	ar constraints options	
Term	String identifier	Data type	Required
$J_{ m bx}^{ m e} \ rac{x^{ m e}}{ar{x}^{ m e}}$	constr_Jbx_e	double, SPUM	no
$\underline{x}^{\mathrm{e}}$	constr_lbx_e	double	no
$\bar{x}^{\mathrm{e}}$	constr_ubx_e	double	no
$C^{\mathrm{e}}$	constr_C_e	double	no
ge	constr_lg	double	no
$\frac{g^{\rm e}}{\bar{g}^{\rm e}}$	constr_ug	double	no
$h^{\mathrm{e}}$	constr symm b s	CoolDi orranoccion	<b>n</b> o
	constr_expr_h_e	CasADi expression	no
$rac{\underline{h}^{\mathrm{e}}}{ar{h}^{\mathrm{e}}}$	constr_lh_e	double	no
$ar{h}^{\mathrm{e}}$	constr_uh_e	double	no
$J_{-1}^{\mathrm{e}}$	constr_Jsbx	double, <b>SPUME</b>	no
$J_{ m sbx}^{ m e} \ J_{ m sg}^{ m e}$	constr_Jsg_e	double, SPUME	no
$J_{ m sbx}^{ m sg}$	constr_Jsh_e	double, <b>SPUME</b>	no

# 5 External links

A table sheet with additional info is found here:

# 6 Model

A model instance is created using ocp\_model = acados\_ocp\_model(). It contains all model definitions for simulation and for usage in the OCP solver. See table 11 for the available options. Furthermore, see ocp\_model.model\_struct to see what

Table 11: Model set(id, data) options

String id	Data type	Description	Required
name	string	model name, used for code generation, default: 'ocp_model'	no
T	double	end time	yes
sym_x	CasADi expr.	state vector $x$ in problem formulation in sec. 1	yes
sym_u	CasADi expr.	control vector $u$ in problem formulation in sec. 1	only in OCP
sym_xdot	CasADi expr.	derivative of the state $\dot{x}$ in implicit dynamics eq. (3)	if IRK is used
sym_z	CasADi expr.	algebraic state $z$ in implicit dynamics eq. (3)	no, only with IRK
sym_p	CasADi expr.	parameters $p$ of the problem formulation in sec. 1	no
	Addition	: ally, options from tables 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10, apply her	e.
		•	

other fields can be set via direct access.

# 7 Solver & Options

An instance of the solver options class is created by using: ocp\_opts = acados\_ocp\_opts(). Together with the model these options are used when instancing the solver interface class: ocp = acados\_ocp(ocp\_model, ocp\_opts).

Tables 12, 13 and 14 show (almost) all available options. These options are set in MATLAB via ocp\_opts.set(<stringid>,

<value>). Furthermore, the struct ocp\_opts.opts\_struct can be used as a reference for what other fields are available.Note that some options of the solver can be modified after creation using the routine: set(<stringid>, <value>). Some options can only be set before the solver is created, especially options that influence the memory requirements of OCP solver, such as the modules used in the formulation, the QP solver, etc.

Table 12: Solver options

			Table 12: Solver options
String identifier	Туре	Default	Description
Code generation			
compile_interface	string	'auto'	<pre>in ('auto', 'true', 'false')</pre>
codgen_model	string	'true'	in ('true', 'false')
compile_model	string	'true'	in ('true', 'false')
output_dir	string		codegen output directory
			, , , , , , , , , , , , , , , , , , ,
Shooting nodes	1	10	
param_scheme_N	int > 1	10	uniform grid: number of shooting nodes; acts together with end time T from model.
	Jb1	F7	
shooting_nodes or param_	-doubles	[]	nonuniform grid option 1: direct definition of the shooting node times
scheme_shooting_nodes	Jb1	F7	if
time_steps	doubles	[]	nonuniform grid option 2: definition of deltas between shooting nodes
Integrator			
sim_method	string	'irk'	'erk','irk','irk_gnsf'
sim_method_num_stages	int	4	Runge-Kutta int. stages: (1) RK1, (2) RK2, (4) RK4
sim_method_num_steps	int	1	
sim_method_newton_iter	int	3	
<pre>gnsf_detect_struct</pre>	string	'true'	
NLP solver			
nlp_solver	string	'sqp'	<pre>in ('sqp', 'sqp_rti')</pre>
nlp_solver_max_iter	int > 1	100	maximum number of NLP iterations
nlp_solver_tol_stat	double	$10^{-6}$	stopping criterion
nlp_solver_tol_eq	double	$10^{-6}$	stopping criterion
nlp_solver_tol_ineq	double	$10^{-6}$	stopping criterion
nlp_solver_tol_comp	double	$10^{-6}$	stopping criterion
nlp_solver_ext_qp_res	int	0	compute QP residuals at each NLP iteration
nlp_solver_step_length	double	1.0	fixed step length in SQP algorithm
rti_phase	int	0	RTI phase: (1) preparation, (2) feedback, (0) both
QP solver			
qp_solver	string	<b>→</b>	Defines the quadratic programming solver and condensing strategy. See ta-
	:	F0	ble 13
qp_solver_iter_max	int	50	maximum number of iterations per QP solver call
<pre>qp_solver_cond_ric_alg</pre>	int	0	factorize hessian in the condensing: (0) no, (1) yes
<pre>qp_solver_ric_alg</pre>	int	0	HPIPM specific
qp_solver_warm_start	int	0	(0) cold start, (1) warm start primal variables, (2) warm start and dual variables
start 6: ast an	:	0	
warm_start_first_qp	int	0	warm start even in first SQP iteration: (0) no, (1) yes
globalization			
globalization	string '	fixed_step	'globalization strategy in ('fixed_step', 'merit_backtracking'), note
			merit_backtracking is a preliminary implementation.
alpha_min	double	0.05	minimum step-size, relevant for globalization
alpha_reduction	double	0.7	step-size reduction factor, relevant for globalization
Hessian approximation			
nlp_solver_exact_hessian	string	'false'	use exact hessian calculation: (")in ('true', 'false'), use exact
regularize_method	string	<i>→</i>	Defines the hessian regularization method. See table 14
levenberg_marquardt	double	0.0	in case of a singular hessian, setting this > 0 can help convergence
exact_hess_dyn	int	1	in (0, 1), compute and use hessian in dynamics, only if 'nlp_solver
,			exact_hessian' = 'true'
exact_hess_cost	int	1	in (0, 1), only if 'nlp_solver_exact_hessian' = 'true'
exact_hess_constr	int	1	in (0, 1), only if 'nlp_solver_exact_hessian' = 'true'
	·		
Other	: ^	0	and a six of the column (O) silver (S O) and a Cont OD and I leave 1 1 1 2
print_level	$int \ge 0$	0	verbosity of the solver: (0) silent, (> 0) print first QP problems and solution
			during SQP

Table 13: Solver set('qp\_solver', <stringid>) options. The availability depends on for which solver interfaces acados was linked to.

Solver lib	Condensing	String identifier
HPIPM	partial full	partial_condensing_hpipm* full_condensing_hpipm
HPMPC	partial	partial_condensing_hpmpc
OOQP	partial full	<pre>partial_condensing_ooqp full_condensing_ooqp</pre>
OSQP QORE qpDUNES qpOASES	partial full partial full	<pre>partial_condensing_osqp   full_condensing_qore partial_condensing_qpdunes   full_condensing_qpoases</pre>

<sup>\*</sup> default

Table 14: Solver set('regularize\_method', <stringid>) options

	` •	<u> </u>
	String identifier	Description
	no_regularize*	don't regularize
	mirror	see Verschueren2017
	project	see Verschueren2017
	<pre>project_reduc_hess</pre>	preliminary
convexify		see Verschueren2017

<sup>\*</sup> default