三(2)衍射的MATLAB 计算

- 一、狭缝衍射的Fresnel积分计算(直接)
- 二、远场衍射计算(FT)
- 三、近场衍射计算(看做卷积，先求FT，再X，再IFFT)
- 四、直接卷积核(看做卷积，直接计算)
- 五、园域函数
- 六、FT平面位移与干涉

一、Fresnel 衍射

固有

$$u_2(x)=\boxed{\left[\frac{e^{jkz_{12}}}{j\lambda z_{12}}\right]^{\frac{1}{2}}}\int u_1(\alpha)q\left(\alpha-x;\frac{1}{\lambda z_{12}}\right)d\alpha \qquad q(x;a)=e^{j\pi ax^2}$$

$$u_2(x)=\left[\frac{e^{jkz_{12}}}{j\lambda z_{12}}\right]^{\frac{1}{2}}\boxed{q\left(x;\frac{1}{\lambda z_{12}}\right)}\int u_1(\alpha)q\left(\alpha;\frac{1}{\lambda z_{12}}\right)\exp\left(-j\frac{2\pi}{\lambda z_{12}}\alpha x\right)d\alpha$$

采用归一化的孔径坐标，则衍射公式可以写成:

$$u_2(x)=\left[\frac{e^{jkz_{12}}}{j}N_f\right]^{\frac{1}{2}}\underline{\int u_1(\alpha)q(\alpha-x;N_f)d\alpha} \qquad N_f=\frac{w^2}{\lambda z_{12}}$$

$$u_2(x)=\left[\frac{e^{jkz_{12}}}{j}N_f\right]^{\frac{1}{2}}q\left(\frac{x}{w};N_f\right)\int u_1(\alpha)q(\alpha;N_f)\exp\left(-j\frac{2\pi}{b}\alpha x\right)d\alpha$$

狭缝的Fresnel衍射

$$u_2(x)=\left[\frac{e^{jkz_{12}}}{j\lambda z_{12}}\right]^{\frac{1}{2}}\int_{-w}^{w} q\left(\alpha-x;\frac{1}{\lambda z_{12}}\right)d\alpha \qquad u_1(x)=\mathrm{rect}\left(\frac{x}{2w}\right)$$

1、通过直接积分计算得到

$$u_2(x)=\left[\frac{e^{jkz_{12}}}{j\lambda z_{12}}\right]^{\frac{1}{2}}\int_{-w}^{w} e^{j\frac{\pi}{\lambda z_{12}}(\alpha-x)^2}\,d\alpha \qquad t=\sqrt{\frac{2}{\lambda z_{12}}}(\alpha-x)$$

$$u_2(x)=\left[\frac{e^{jkz_{12}}}{2j}\right]^{\frac{1}{2}}\int_{-t_1}^{t_2} e^{j\frac{\pi}{2}t^2}\,dt \qquad t_1=\sqrt{\frac{2}{\lambda z_{12}}}(w+x) \qquad t_2=\sqrt{\frac{2}{\lambda z_{12}}}(w-x)$$

$$\mathrm{fresnel}(x)=\int_0^x e^{j\frac{\pi}{2}t^2}\,dt \qquad \int_{-t_1}^{0} e^{j\frac{\pi}{2}t^2}\,dt=-\int_{t_1}^{0} e^{j\frac{\pi}{2}t^2}\,dt=\int_0^{t_1} e^{j\frac{\pi}{2}t^2}\,dt$$

change variables　　change limits

$$u_2(x)=\left[\frac{e^{jkz_{12}}}{2j}\right]^{\frac{1}{2}}\left[\mathrm{fresnel}\left(\sqrt{\frac{2}{\lambda z_{12}}}(w-x)\right)+\mathrm{fresnel}\left(\sqrt{\frac{2}{\lambda z_{12}}}(w+x)\right)\right]$$

Fresnel 积分 见本书matlab 程序**fresnel**

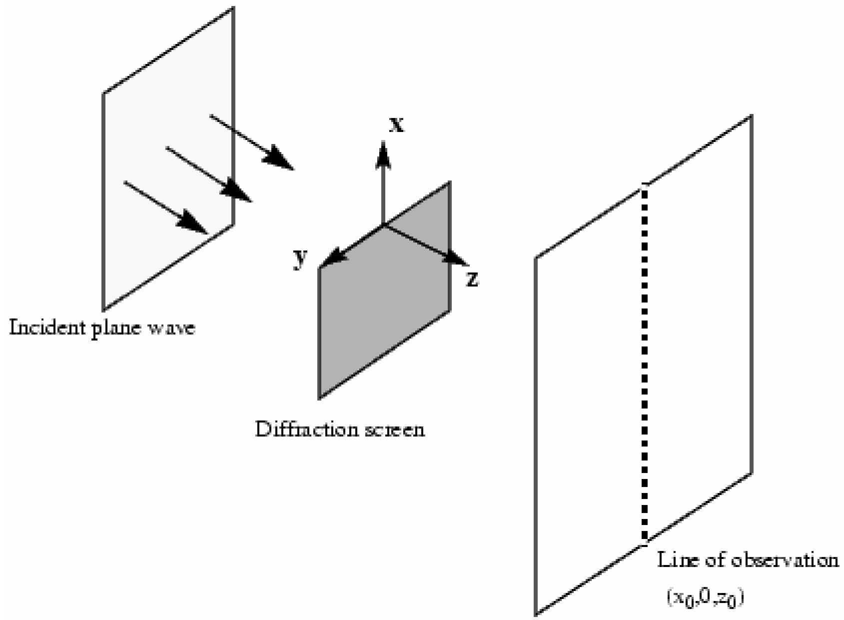
$$\mathrm{fresnel}(x)=\int_0^x e^{j\frac{\pi}{2}t^2}\,dt$$

$$C(x)=\int_0^x \cos\left(\tfrac{\pi}{2}t^2\right)dt$$

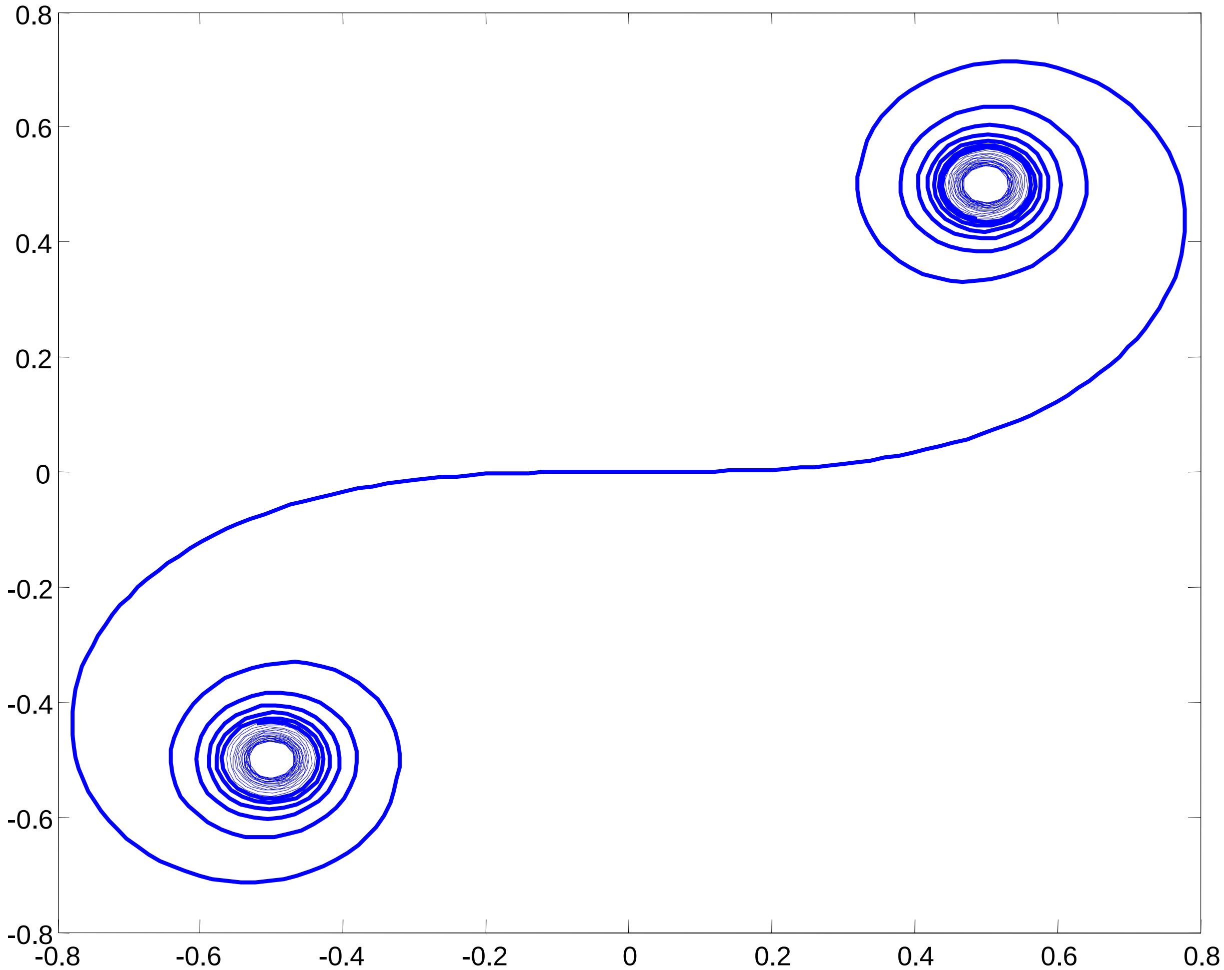
$$S(x)=\int_0^x \sin\left(\tfrac{\pi}{2}t^2\right)dt$$

Matlab 本身 FresnelC(x) ；FresnelS(x)

```
x = -50:50;
C = mfun('FresnelC',x);
S = mfun('FresnelS',x);
I0 = 1;
T = (C+1/2).^2 + (S+1/2).^2;
I = (I0/2)*T;
plot(x,I);
xlabel('x');
ylabel('I(x)');
title('Intensity of Diffracted Wave');
```



Cornu Spiral



狭缝衍射的归一化

$$fresnel(x) = \int_0^x e^{j\frac{\pi}{2}t^2}\,dt$$

$$t_1 = \sqrt{\frac{2}{\lambda z_{12}}}(w+x) \qquad t_2 = \sqrt{\frac{2}{\lambda z_{12}}}(w-x)$$

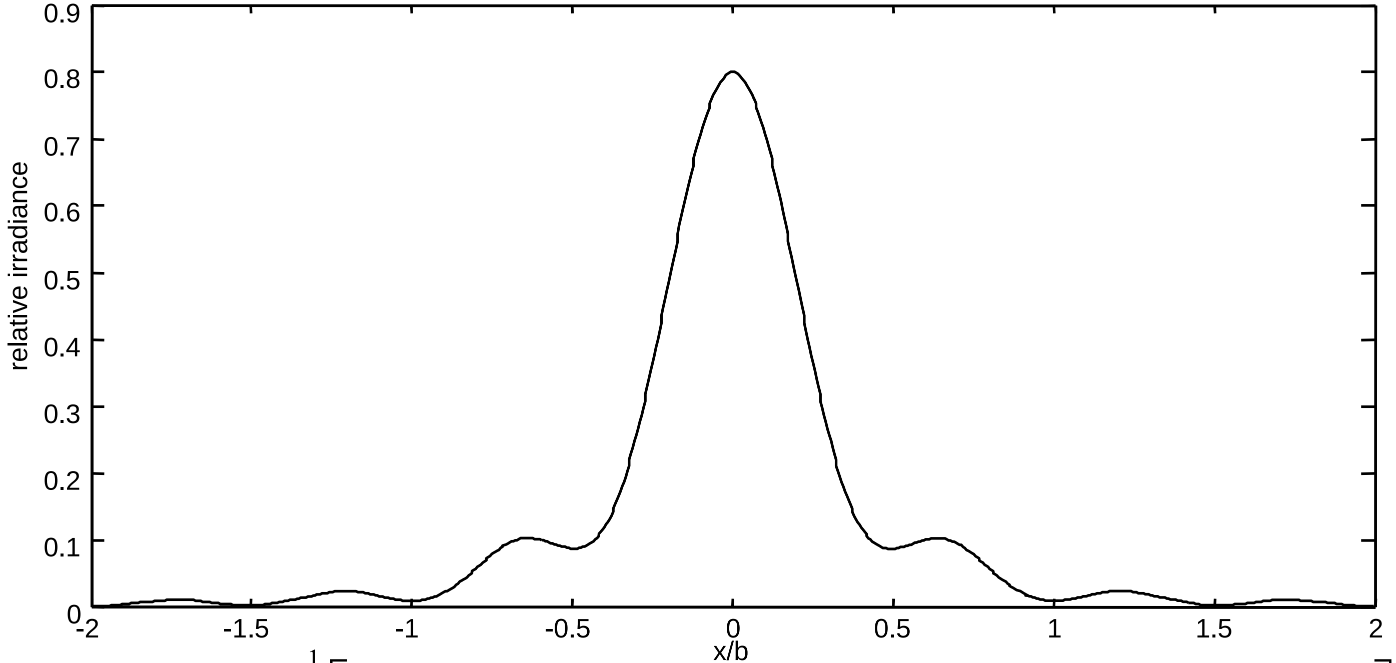
$$u_2(x) = \left[\frac{e^{jkz_{12}}}{2j}\right]^{\frac{1}{2}}\left[\text{fresnel}\left(\sqrt{\frac{2}{\lambda z_{12}}}(w-x)\right) + \text{fresnel}\left(\sqrt{\frac{2}{\lambda z_{12}}}(w+x)\right)\right]$$

$$N_f = \frac{w^2}{\lambda z_{12}} \qquad b = \frac{\lambda z_{12}}{w} = \frac{w}{N_f}$$

$$N_f \le 1 \qquad u_2(x) = \left[\frac{e^{jkz_{12}}}{2j}\right]^{\frac{1}{2}}\left[\text{fresnel}\left(\sqrt{\frac{2}{N_f}}\left(N_f - \frac{x}{b}\right)\right) + \text{fresnel}\left(\sqrt{\frac{2}{N_f}}\left(N_f + \frac{x}{b}\right)\right)\right]$$

$$N_f \ge 1 \qquad u_2(x) = \left[\frac{e^{jkz_{12}}}{2j}\right]^{\frac{1}{2}}\left[\text{fresnel}\left(\sqrt{2N_f}\left(1 - \frac{x}{w}\right)\right) + \text{fresnel}\left(\sqrt{2N_f}\left(1 + \frac{x}{w}\right)\right)\right]$$

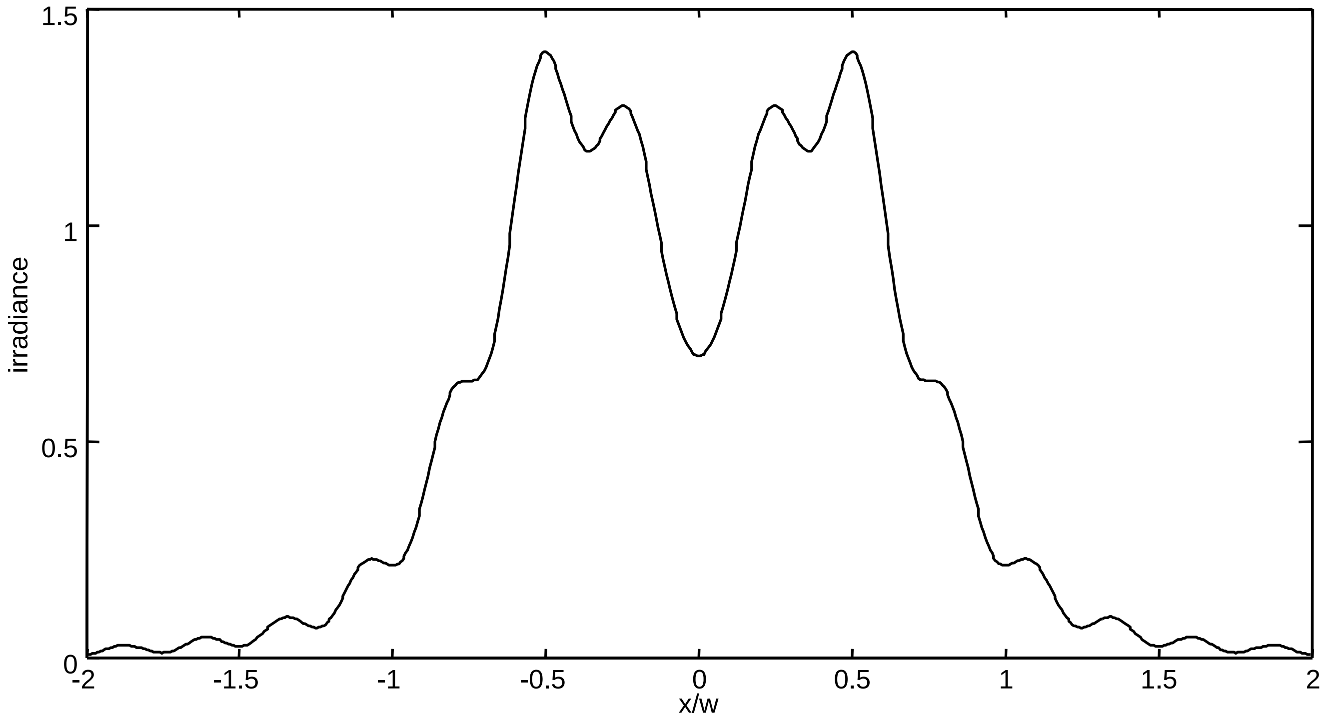
Fresnel Number = 0.5



$$u_2(x) = \left[\frac{1}{2j}\right]^{\frac{1}{2}} \left[\mathrm{fresnel}\left(\sqrt{\frac{2}{N_f}}\left(N_f - \frac{x}{b}\right)\right) + \mathrm{fresnel}\left(\sqrt{\frac{2}{N_f}}\left(N_f + \frac{x}{b}\right)\right)\right]$$

```
a=0.5;
x = linspace(-2,2,801);
[cc2,ss2] = fresnel(sqrt(2/a)*(a-x));
[cc1,ss1] = fresnel(sqrt(2/a)*(a+x));
z = sqrt(1/(2*j))*complex(cc2+cc1,ss2+ss1);
f = z.*conj(z);
f = (0.25/a)*f;
```

Fresnel Number = 2



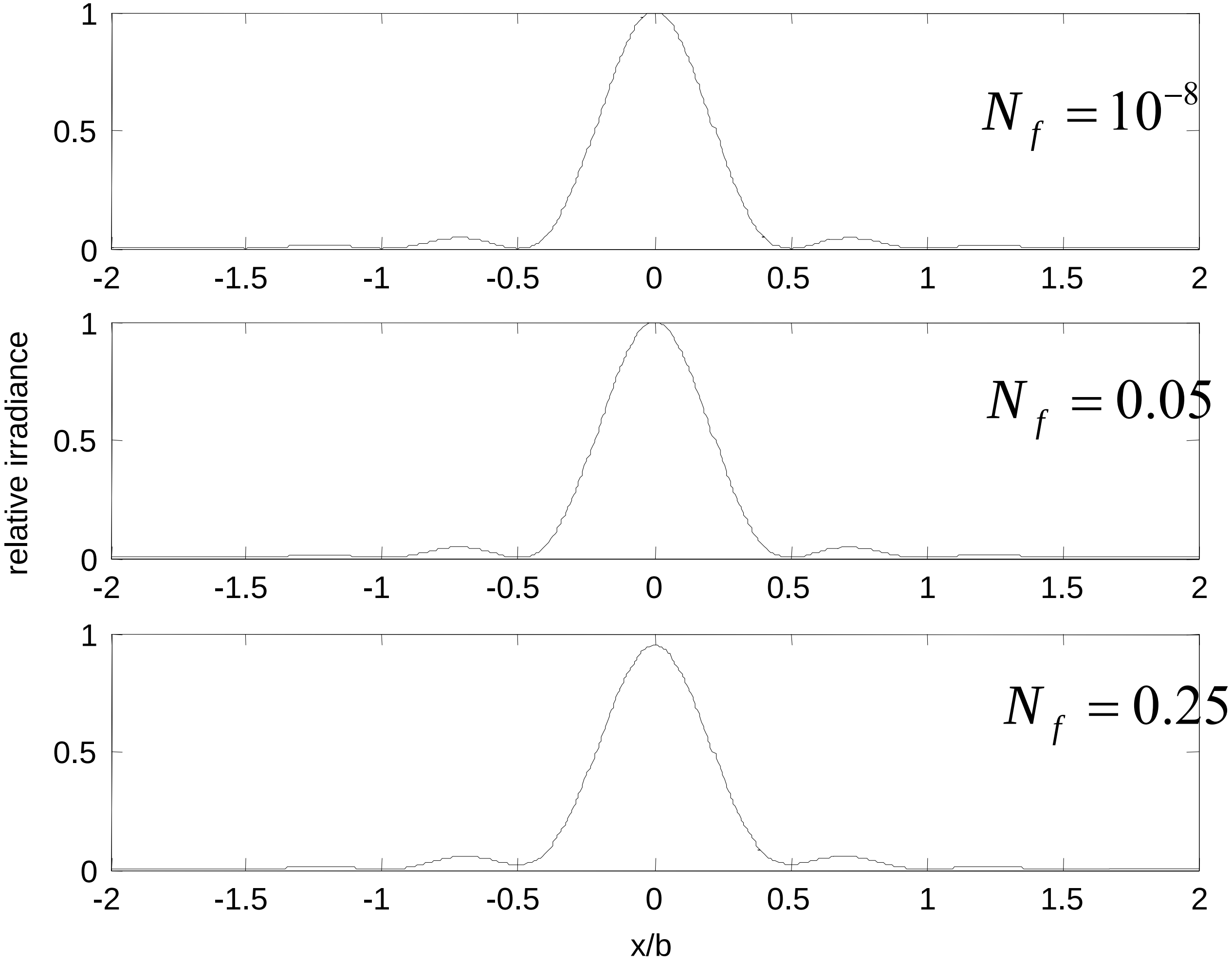
$$u_2(x) = \left[\frac{1}{2j}\right]^{\frac{1}{2}} \left[\text{fresnel}\left(\sqrt{2N_f}\left(1 - \frac{x}{w}\right)\right) + \text{fresnel}\left(\sqrt{2N_f}\left(1 + \frac{x}{w}\right)\right) \right]$$

```
a=2;
x = linspace(-2,2,801);
[cc2,ss2] = fresnel(sqrt(2*a)*(1.0-x));
[cc1,ss1] = fresnel(sqrt(2*a)*(1.0+x));
z = sqrt(1/(2*j))*complex(cc2+cc1,ss2+ss1);
f = z.*conj(z);
```

狭缝衍射的Matlab 代码

```
function [f,x] = slit(a)  %a is N
N=800; D=4;
x = linspace(-1,1,N+1)*D/2;
if (a<1)
  [cc2,ss2] = fresnel(sqrt(2/a)*(a-x));
  [cc1,ss1] = fresnel(sqrt(2/a)*(a+x));
else
  [cc2,ss2] = fresnel(sqrt(2*a)*(1.0-x));
  [cc1,ss1] = fresnel(sqrt(2*a)*(1.0+x));
end
z = sqrt(1/(2*j))*complex(cc2+cc1,ss2+ss1);
f = z.*conj(z);
if (a<1)
    f = (0.25/a)*f;
end
plot(x,f,'k');
```

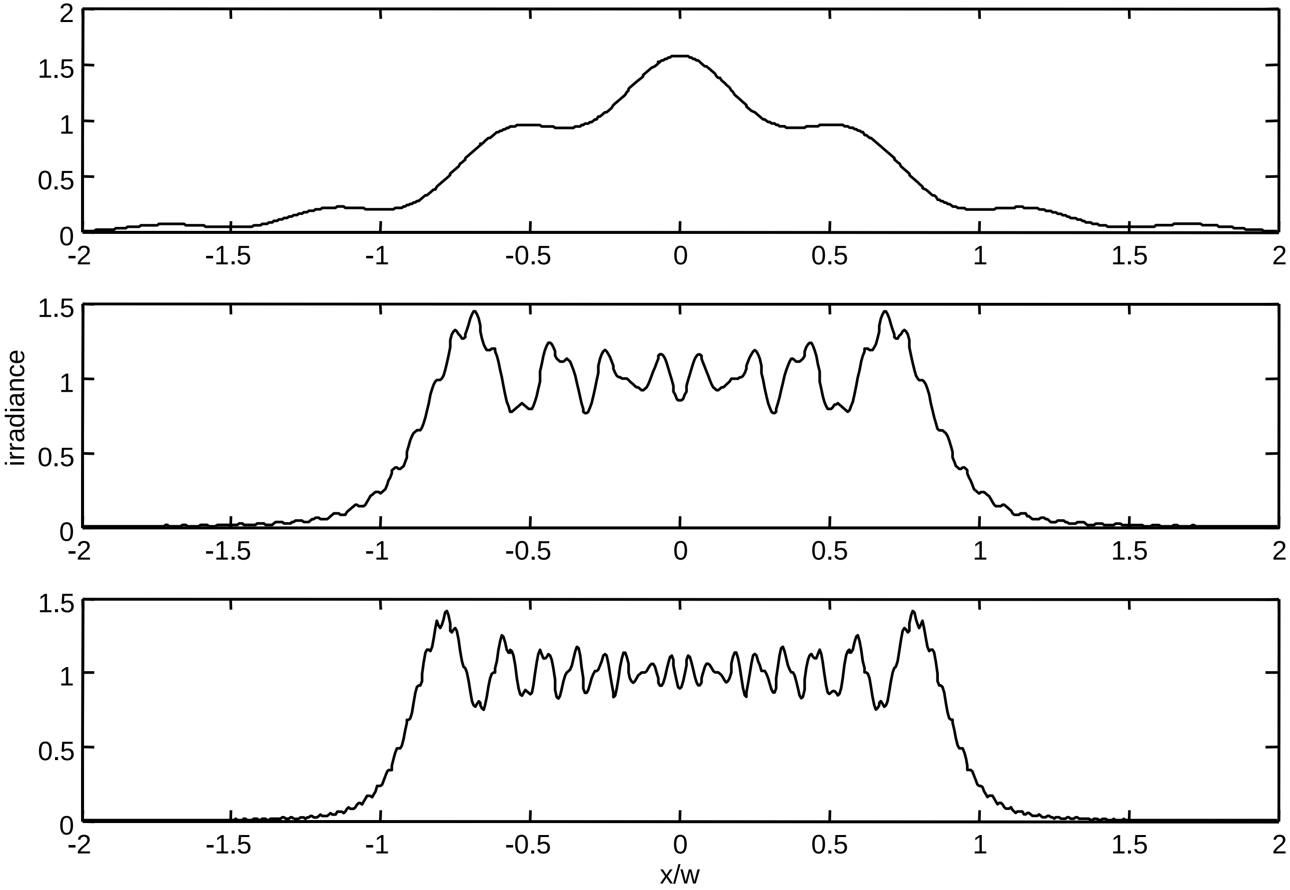
远场衍射




```
subplot(3,1,1);
slit(1e-8);
subplot(3,1,2);
slit(0.05);
ylabel('relative
irradiance');
subplot(3,1,3);
slit(0.25);
xlabel('x/b');
```

形状基本相同, 但大小同 $b = (\lambda z_{12})/ w$ 成正比；辐射同 (λz_{12}) 成反比

近场衍射



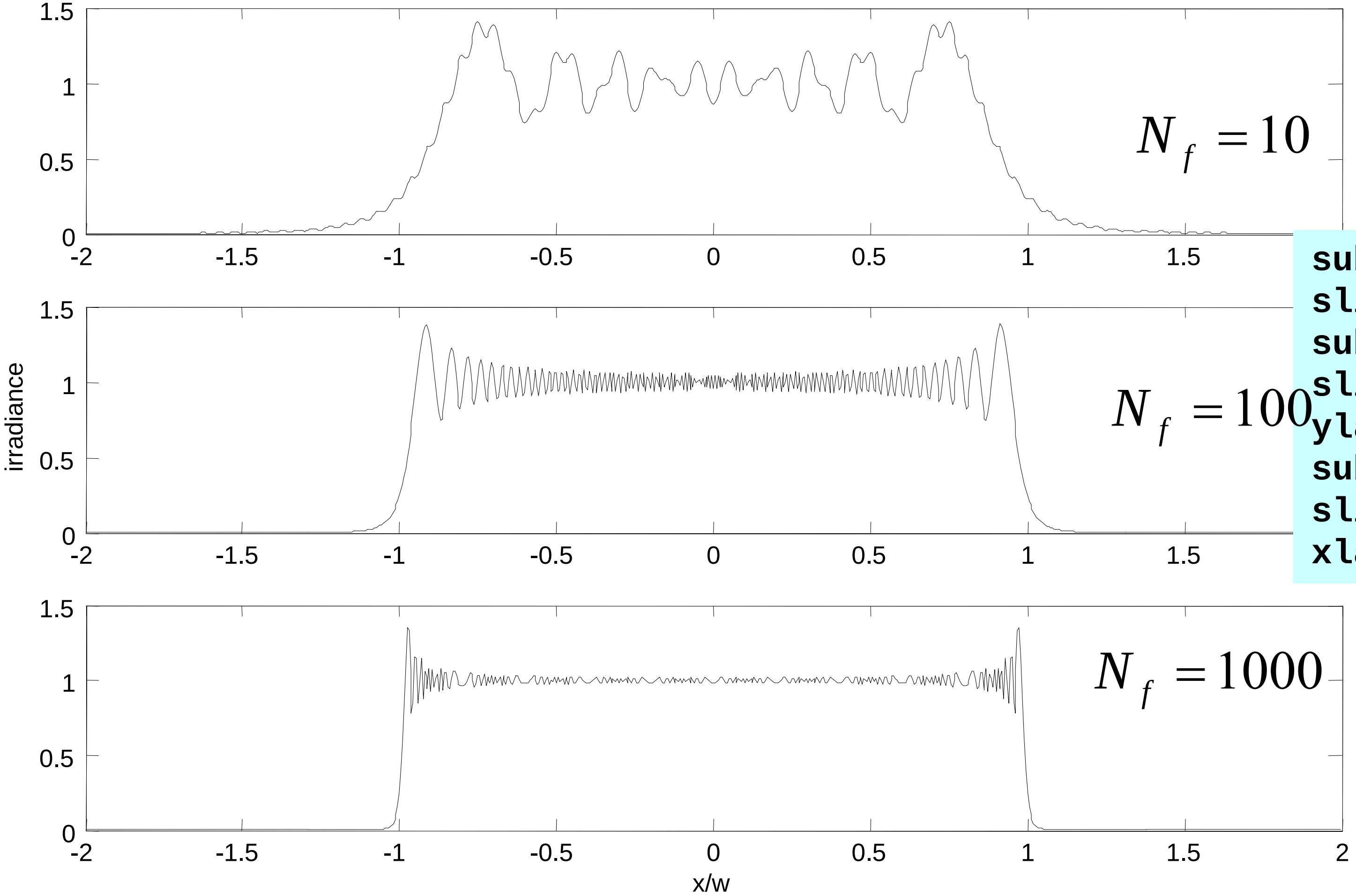
$N_f = 1$

$N_f = 8$

$N_f = 16$

```
subplot(3,1,1);
slit(1);
subplot(3,1,2);
slit(8);
ylabel('irradiance');
subplot(3,1,3);
slit(16);
xlabel('x/w');
```

极近场衍射



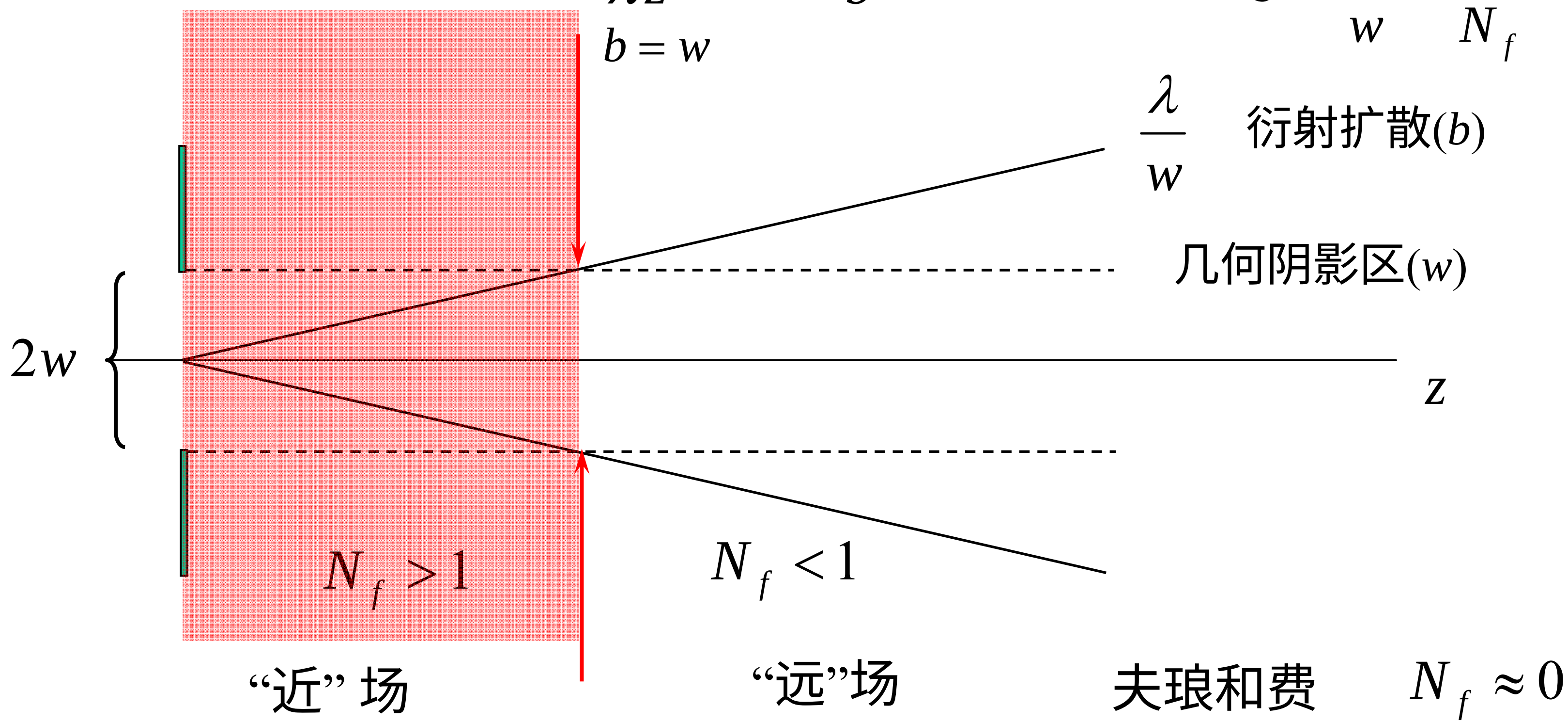

```
subplot(3,1,1);
slit(10);
subplot(3,1,2);
slit(100);
ylabel('irradiance')
subplot(3,1,3);
slit(1000);
xlabel('x/w');
```

衍射区域

Fresnel 数 $$N_f = \frac{w^2}{\lambda z}$$

$$N_f = 1 \Rightarrow N_f = \frac{w^2}{\lambda z} = 1; \Rightarrow \frac{w}{b} = 1$$

$$b = \frac{\lambda z}{w} = \frac{w}{N_f}$$



Fresnel 区域

$$\frac{2\pi}{\lambda}\frac{1}{8z_{12}^3}\left(x^2+y^2\right)^2 << 1$$ 展开中忽略的第一项

$$\frac{2\pi}{\lambda}\frac{1}{8z_{12}^3}w^4 << 1$$ 在 “近场” $N_f > 1$ $$N_f = \frac{w^2}{\lambda z_{12}}$$

$$\frac{2\pi}{\lambda}\frac{1}{8}\left(\frac{\lambda N_f}{w^2}\right)^3 w^4 << 1$$ $$\frac{\pi}{4} \approx 1$$

$$\boxed{N_f << \left(\frac{w}{\lambda}\right)^{\frac{2}{3}}}$$

令 $\lambda = 0.5$ μm 以及 $w = 1$ mm, 那么 $N_f << 160$

令 $l = 0.5$ mm,当$w = 10$ mm, 那么 $N_f << 740$

二、远场衍射的计算

$$u_2(x)=\left[\frac{e^{jkz_{12}}}{j}N_f\right]^{\frac{1}{2}}q\left(\frac{x}{w};N_f\right)\int u_1(\alpha)\boxed{q(\alpha;N_f)}\exp\left(-j\frac{2\pi}{b}\alpha x\right)d\alpha$$

$$N_f=\frac{w^2}{\lambda z_{12}}\qquad u_1(\alpha)=\mathrm{rect}\left(\frac{\alpha}{2}\right)$$

归一化坐标中狭缝宽度 $=2w$

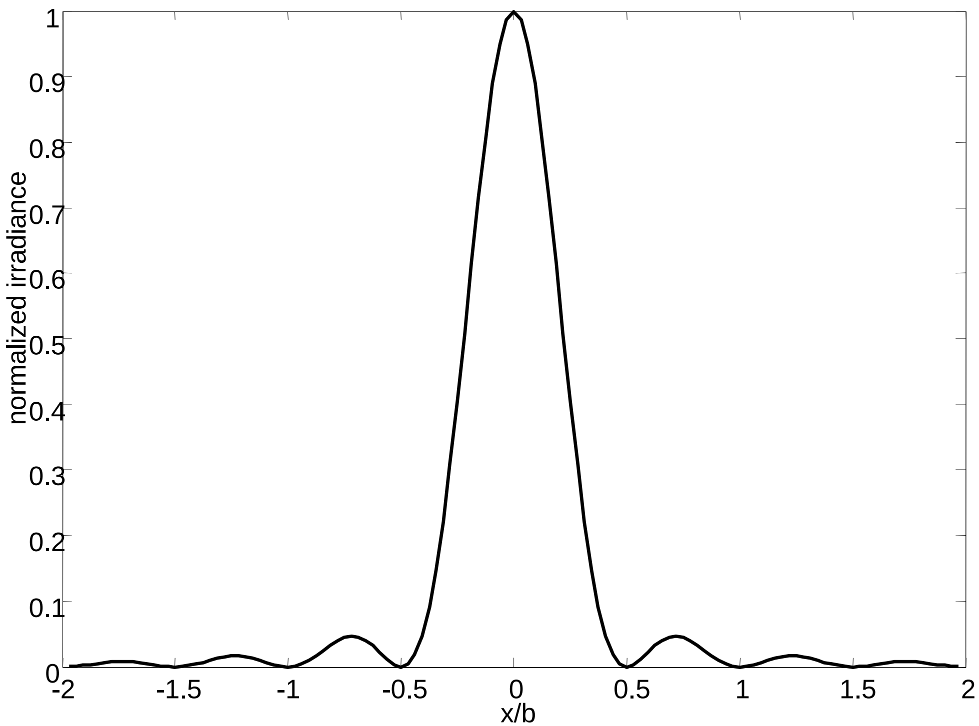
$$b=\frac{\lambda z_{12}}{w}\qquad U_1(\xi)=2\,\mathrm{sinc}(2\xi)\qquad \xi=\frac{x}{b}$$

$$|u_2|^2=4N_f\,\mathrm{sinc}^2\left(2\frac{x}{b}\right)$$

将峰值归一化: 将irradiance 乘以0.25， 并且不考虑 N_f.

远场例子

```
N=1024;
D=32;

k=-N/2:N/2-1;
x = (D/N)*k;
dx = x(2)-x(1);

f = rect(x/2);

z = fftshift(fft(fftshift(f)))*dx;
fx = (1/D)*k;
fy = z.*conj(z)/4;

frange=2;
idx = find(abs(fx)<frange);
plot(fx(idx),fy(idx),'k');
xlabel('x/b');
ylabel('normalized irradiance');
```

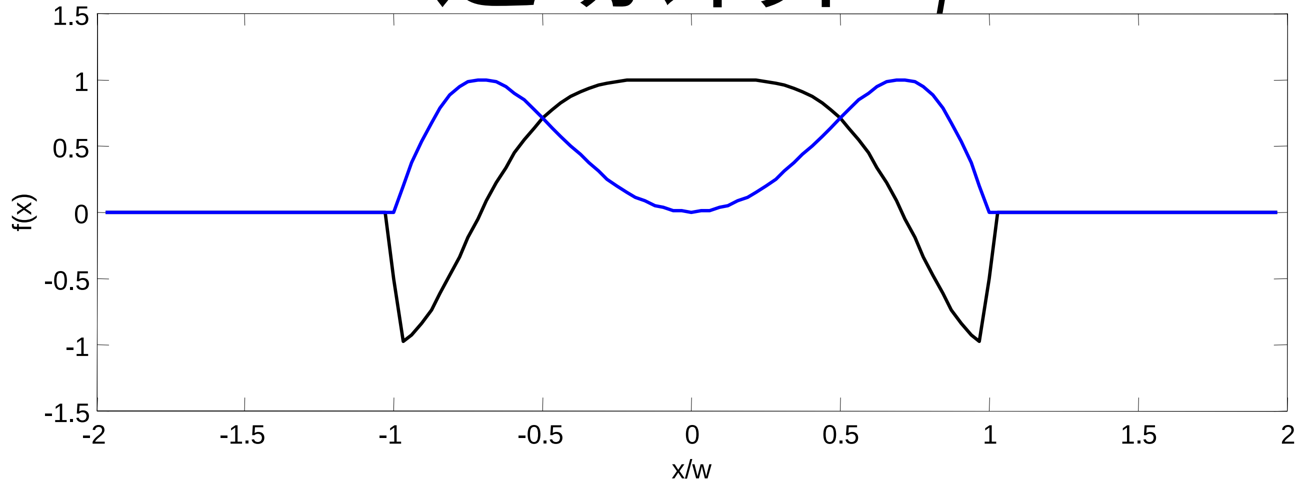


远场衍射的Matlab 代码

```
function [fy, fx] = fslit(a)

N=1024;
D=32;

k=-N/2:N/2-1;
x = (D/N)*k;
dx = x(2)-x(1);

f = qchirp(x,a).*rect(x/2);

xrange = 2;
idx=find(abs(x)<xrange);
subplot(2,1,1);
plot(x(idx),real(f(idx)),'k',x(idx),imag(f(idx)),'b');
axis([-xrange xrange -1.5 1.5]);
xlabel('x/w');
ylabel('f(x)');

z = fftshift(fft(fftshift(f)))*dx;
```

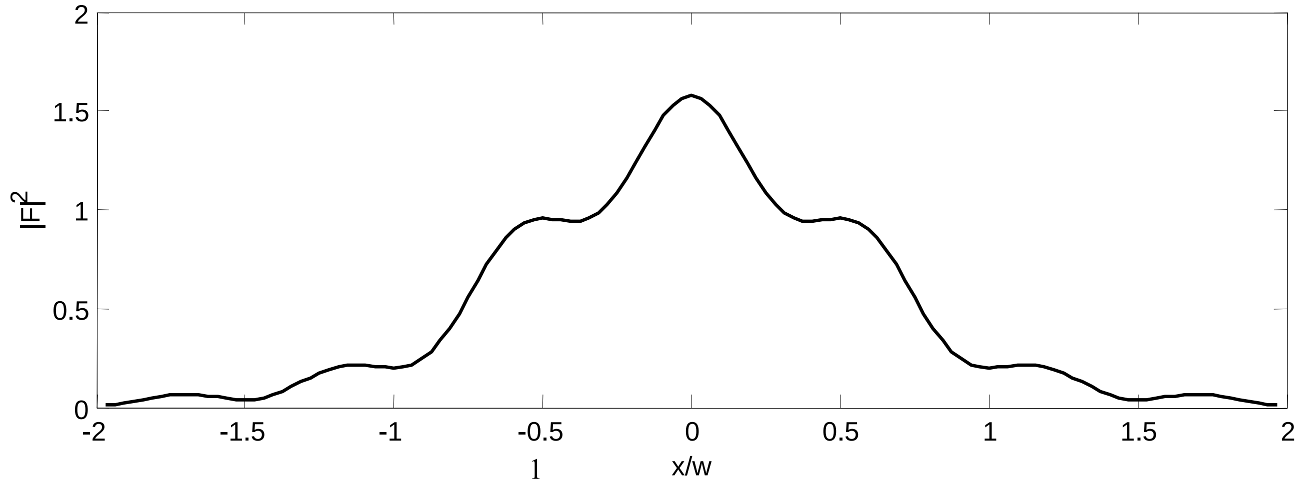
```
fx = (1/D)*k;
fy = z.*conj(z);
if (a<1)
    fy = 0.25*fy;
else
    fy = fy*a;
    fx = fx/a;
end

frange=2;
idx = find(abs(fx)<frange);
subplot(2,1,2);
plot(fx(idx),fy(idx),'k');
%axis([-frange frange 0 1.0]);
if (a<1)
    xlabel('x/b');
else
    xlabel('x/w');
end
ylabel('|F|^2');
```

远场计算$N_f = 1$



黑色为实数部分，
兰色为虚部部分t

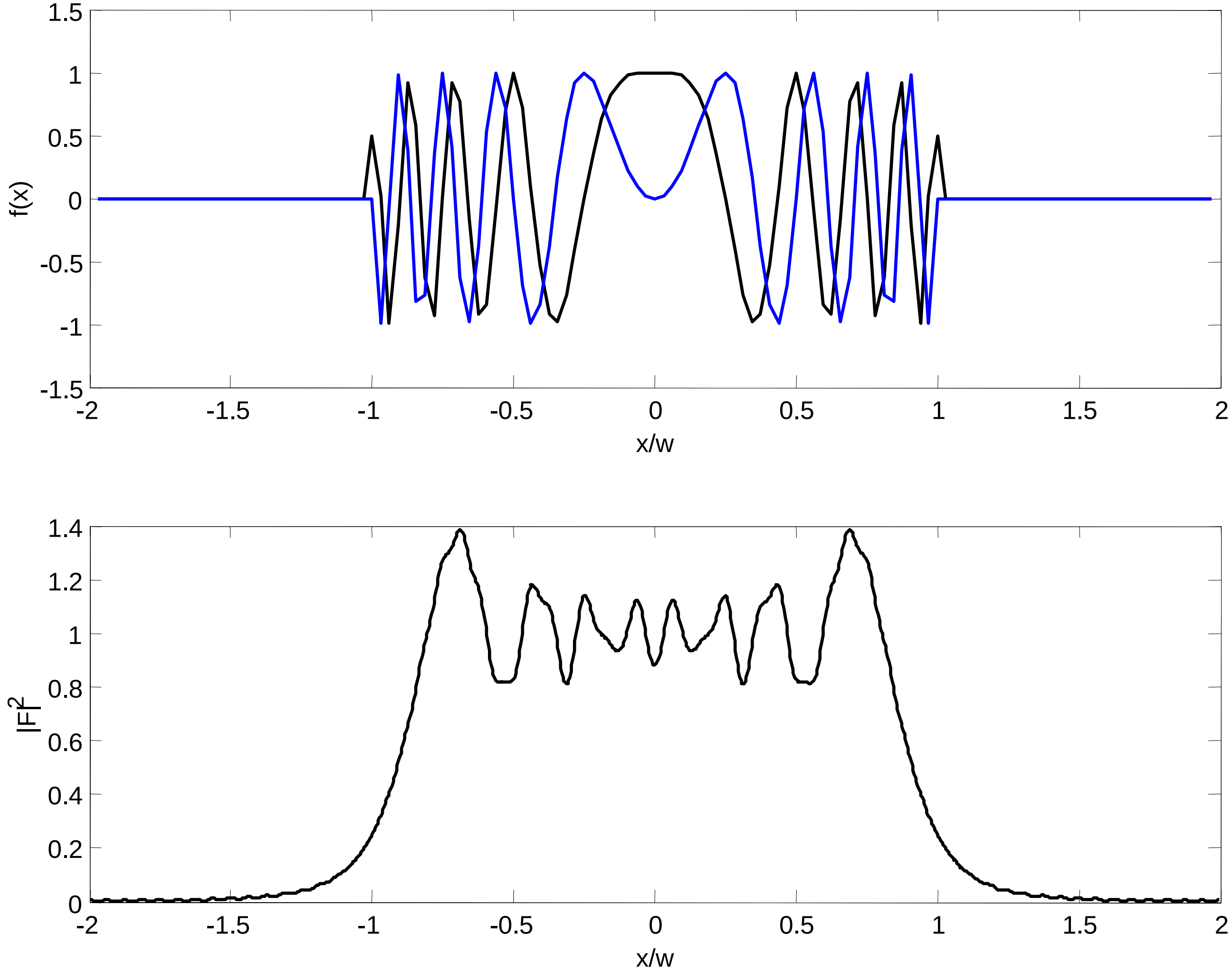
$$u_1(\alpha)q(\alpha;N_f)$$



$$u_2(x)=\left[\frac{e^{jkz_{12}}}{j}N_f\right]^{\frac{1}{2}}q\left(\frac{x}{w};N_f\right)\int u_1(\alpha)q(\alpha;N_f)\exp\left(-j\frac{2\pi}{b}\alpha x\right)d\alpha$$

```
fslit(1);
```

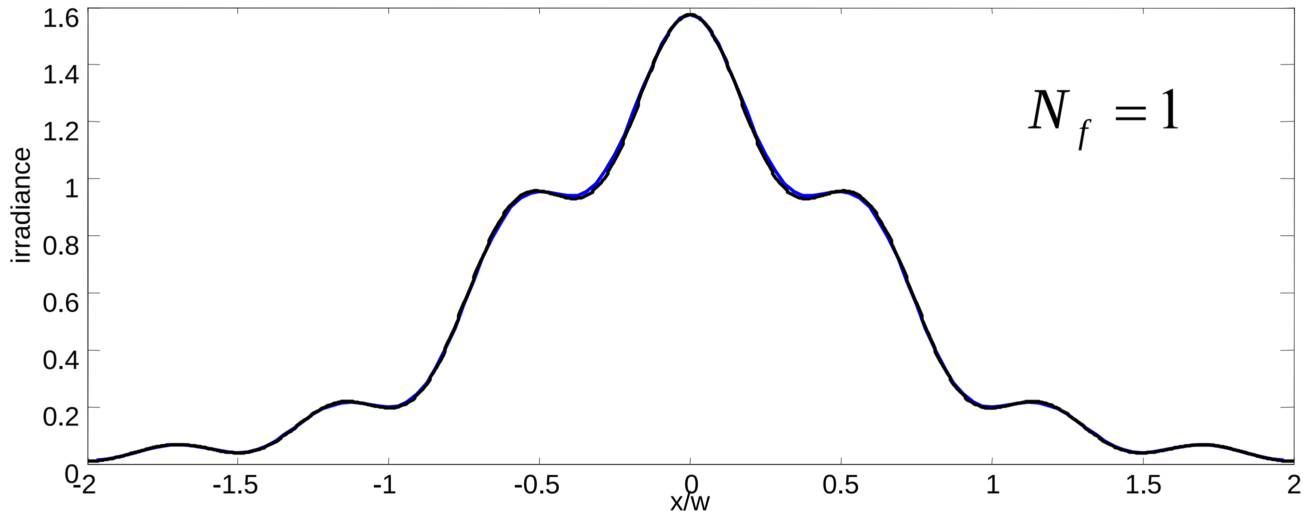
远场计算 $N_f = 8$



黑色为实数部分,
兰色为虚部部分t

`fslit(8);`

同直接计算的比较



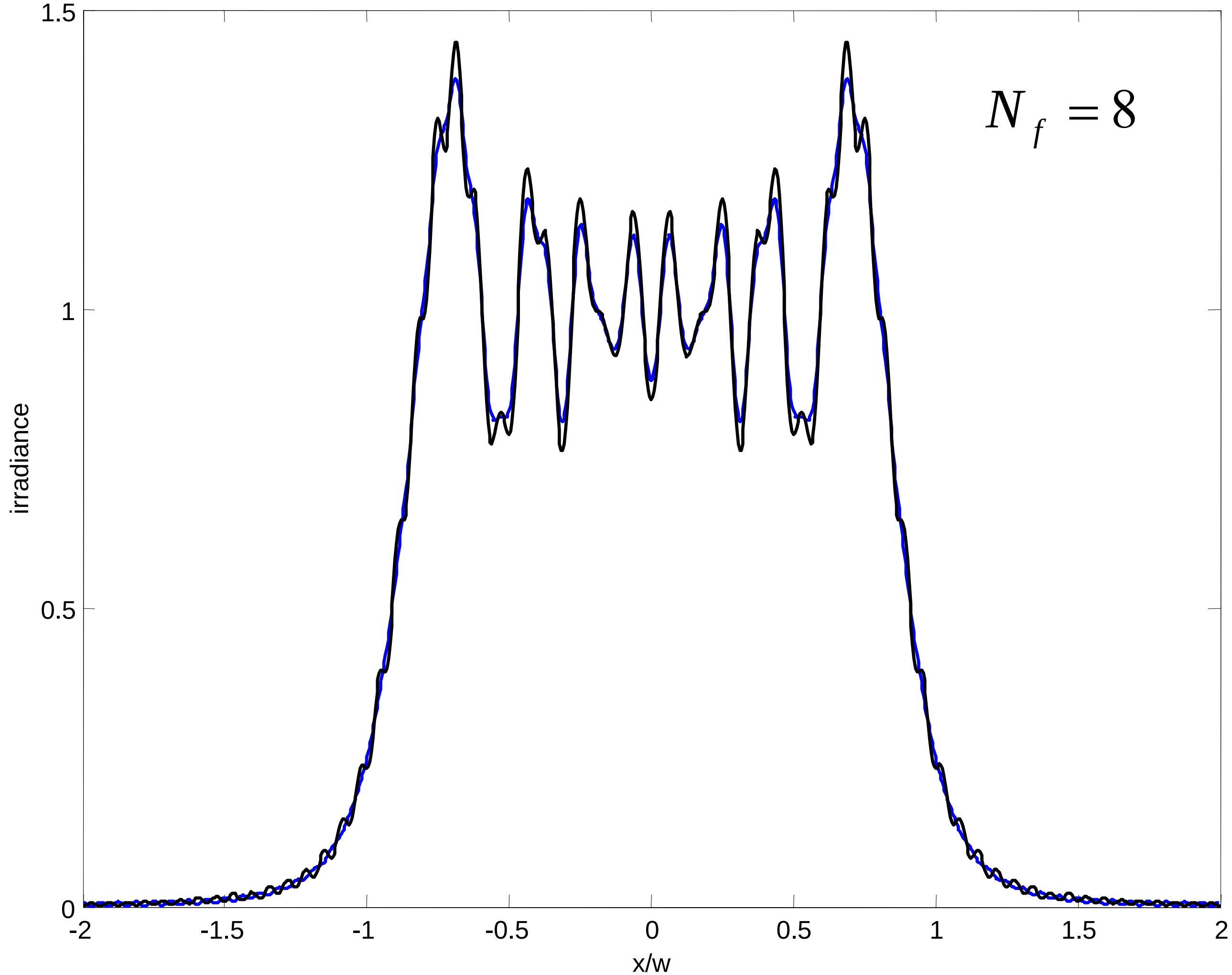
Blue：远场计算,

Black：直接计算.

```
function compare(a)

[fy fx] = fslit(a);
[y x] = slit(a);
idx = find(abs(fx)<2.0);
subplot(1,1,1);
plot(fx(idx),fy(idx),'b',x,y,'k');
if (a<1)
    xlabel('x/b');
    ylabel('normalized irradiance');
else
    xlabel('x/w');
    ylabel('irradiance');
end
```

```
compare(1);
```

远场比较$N_f = 8$

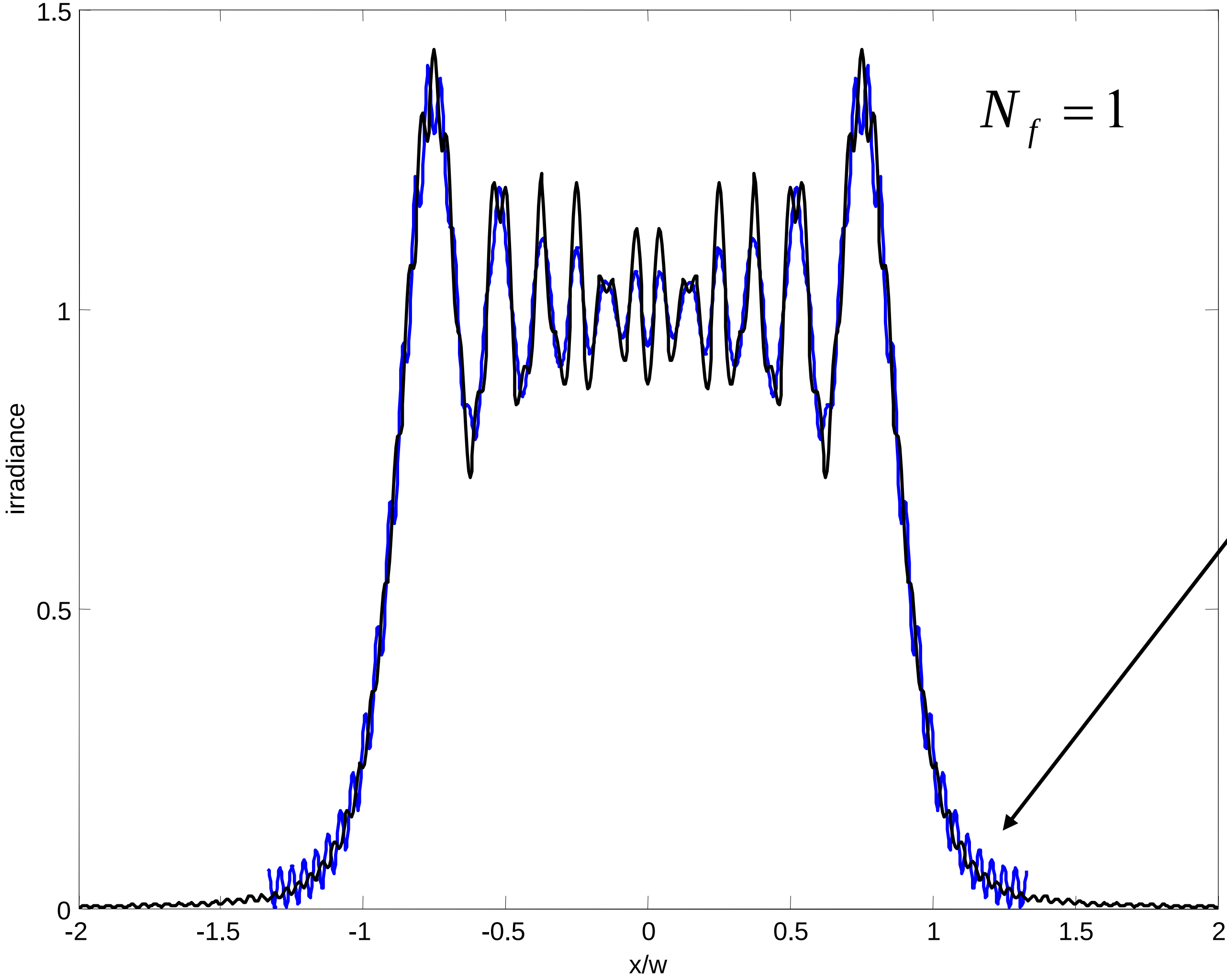


Blue：远场计算,

Black：直接计算.

`compare(8);`

远场计算比较 $N_f = 12$



FFT aliasing

$$\left(\frac{x}{w}\right)_{\max} = \frac{N}{2D}\frac{1}{N_f} = \frac{1024}{2\cdot 32}\frac{1}{N_f} = \frac{16}{N_{f22}} = \frac{4}{3}$$

`compare(12);`

Blue：远场计算,

Black：直接计算.

三、近场衍射计算

GASKILL

$$u_2(x)=\left[\frac{e^{jkz_{12}}}{j}N_f\right]^{\frac{1}{2}}\int u_1(\alpha)q(\alpha-x;N_f)d\alpha$$

$$N_f=\frac{w^2}{\lambda z_{12}}$$

$$u_2(x)=\left[\frac{e^{jkz_{12}}}{j}N_f\right]^{\frac{1}{2}}u_1(x)*q(x;N_f)$$

$$q(x;a)=e^{j\pi ax^2}$$

$$U_2(\xi)=\left[\frac{e^{jkz_{12}}}{j}N_f\right]^{\frac{1}{2}}U_1(\xi)Q(\xi;N_f)$$

$$Q(\xi;a)=\frac{e^{j\frac{\pi}{4}}}{\sqrt{a}}e^{-j\pi\frac{1}{a}\xi^2}=\sqrt{\frac{j}{a}}q\left(\xi;-\frac{1}{a}\right)$$

$$\boxed{U_2(\xi)=\left[e^{jkz_{12}}\right]^{\frac{1}{2}}U_1(\xi)q\left(\xi;-\frac{1}{N_f}\right)}$$

$$\boxed{u_2(x)=\left[e^{jkz_{12}}\right]^{\frac{1}{2}}\int U_1(\xi)q\left(\xi;-\frac{1}{N_f}\right)\exp(j2\pi\xi x)d\xi}$$

归一化坐标

$$x\leftarrow\frac{x}{w}$$

$$\xi\leftarrow w\xi$$

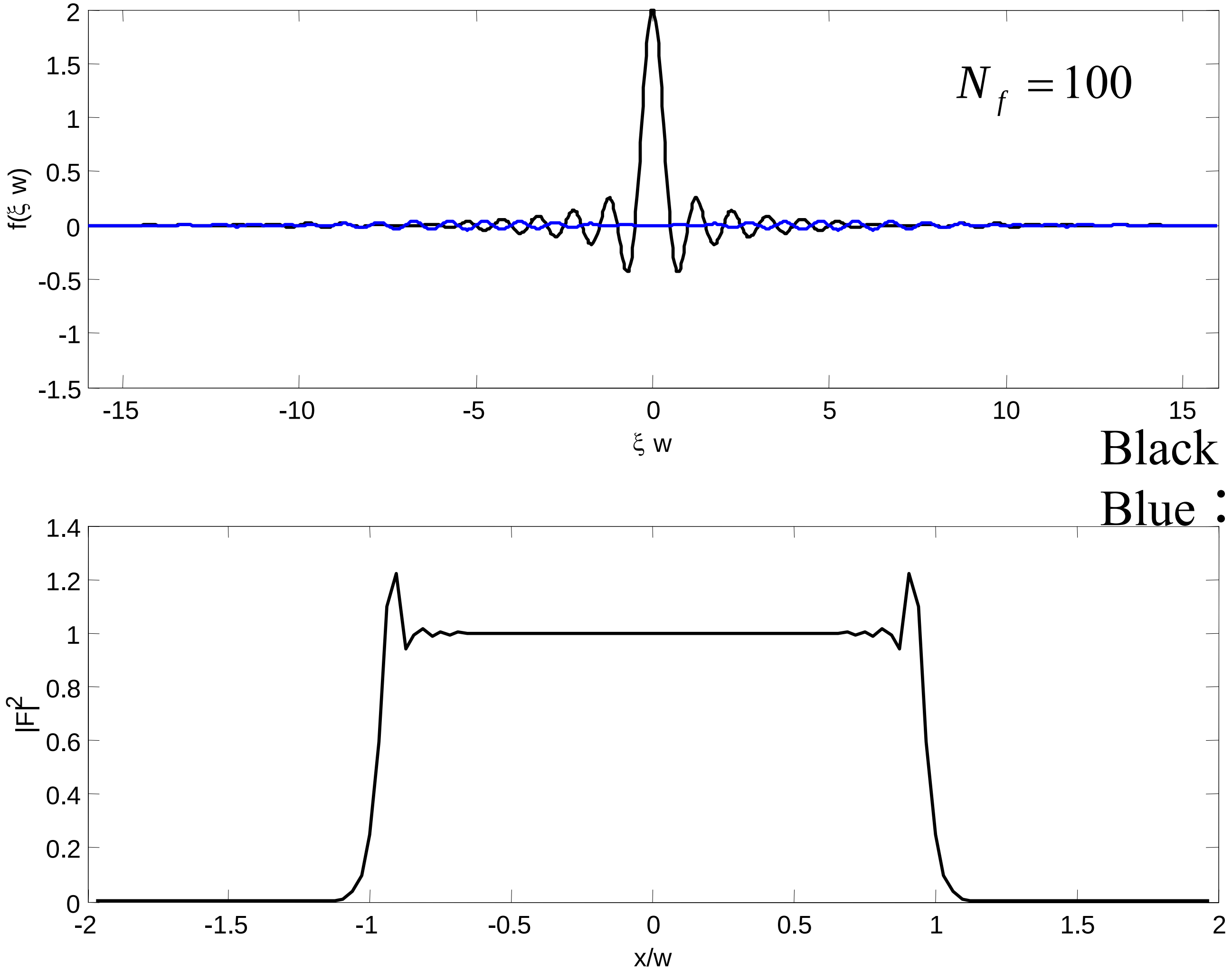
近场Matlab 代码

```
function [y, x] = nslit(a)%n->near

N=1024;
D=32;

k=-N/2:N/2-1;
x = (D/N)*k;
dx = x(2)-x(1);

g = rect(x/2);
gf = fftshift(fft(fftshift(g)))*dx;
%gf = 2*sinc(2*x);

fx = (1/D)*k;
f = gf.*qchirp(fx,-a);

frange =N/(2*D);
subplot(2,1,1);
plot(fx,real(f),'k',fx,imag(f),'b');
axis([-frange frange -1.5 2]);
xlabel('\xi w');
ylabel('f(\xi w)');

z = fftshift(ifft(fftshift(f)))*(N*dx);
```

```
y = z.*conj(z);
if (a>1)
    y = y*(a/4);
    x = x/a;
end


xrange=2;
idx = find(abs(x)<xrange);
subplot(2,1,2);
plot(x(idx),y(idx),'k');
if (a>1)
    xlabel('x/b');
else
    xlabel('x/w');
end
ylabel('|F|^2');
```

近场计算例子$N_f = 100$

$$U_1(\xi)q\left(\xi;-\frac{1}{N_f}\right)$$

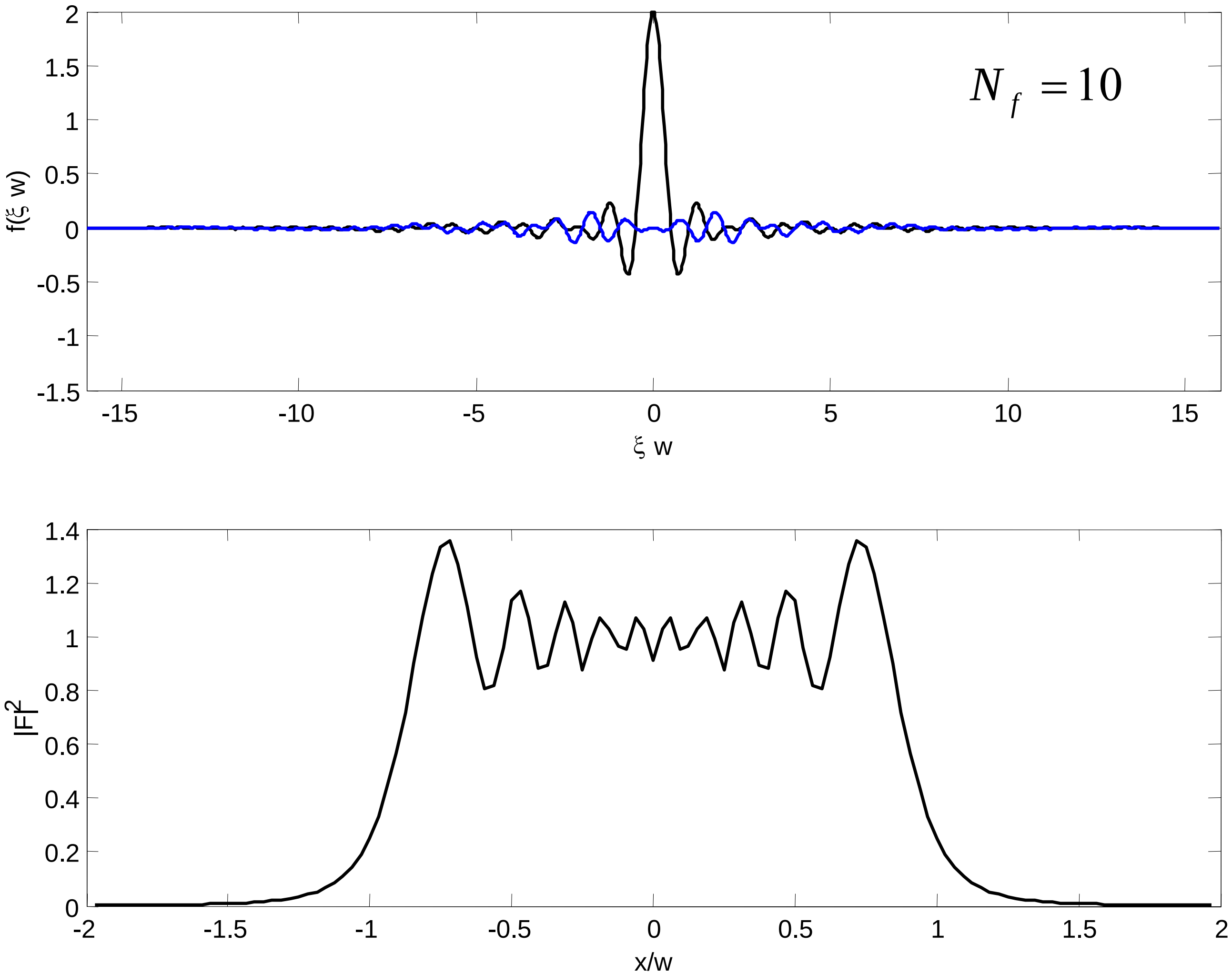


Black：谱实部，
Blue：谱虚部.

```
nslit(1/100);
```

近场计算例子$N_f = 10$

$$U_1(\xi)q\left(\xi;-\frac{1}{N_f}\right)$$

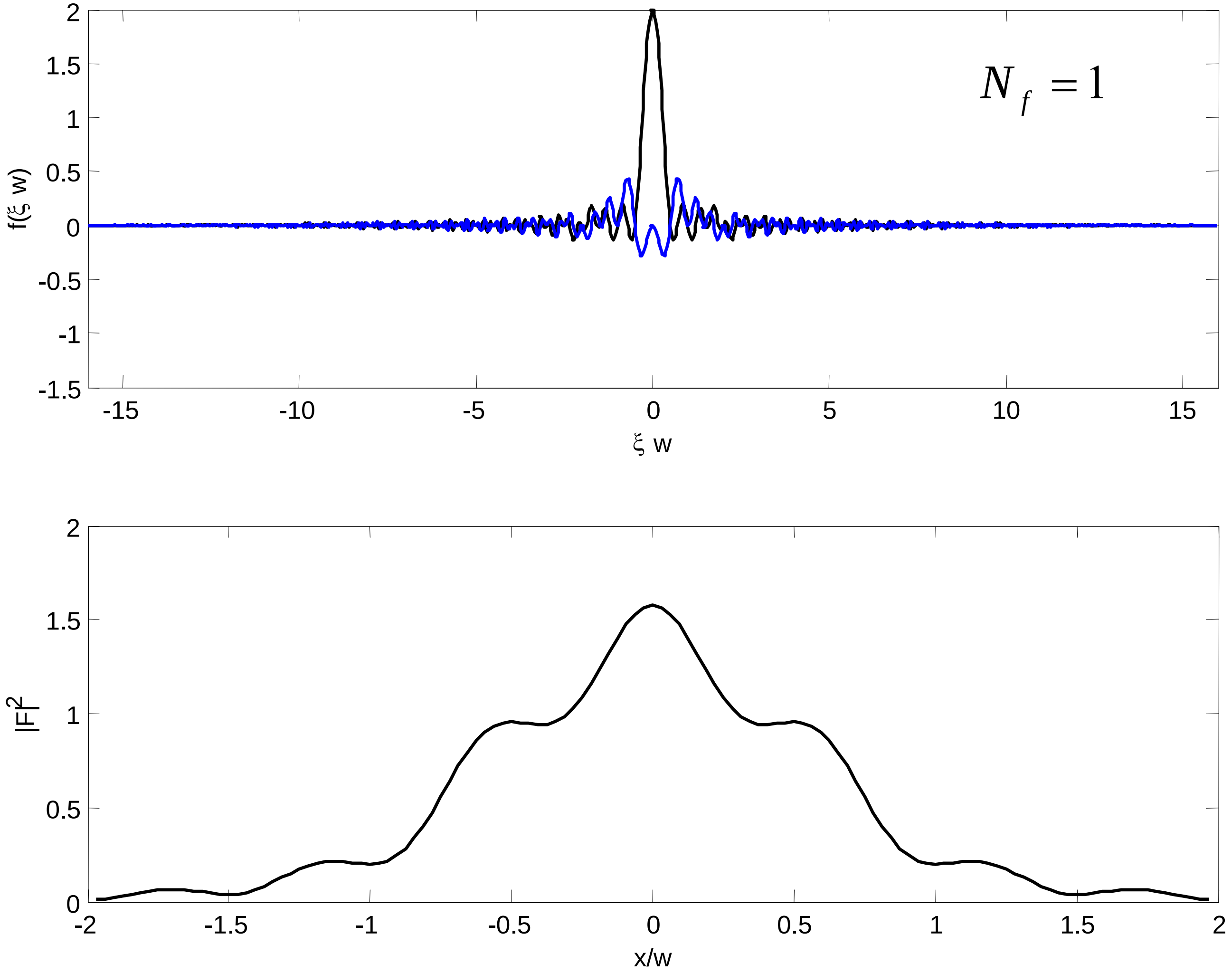


Black：谱实部，
Blue：谱虚部.

```
nslit(1/10);
```

近场计算例子 $N_f = 1$

$$U_1(\xi)q\left(\xi;-\frac{1}{N_f}\right)$$

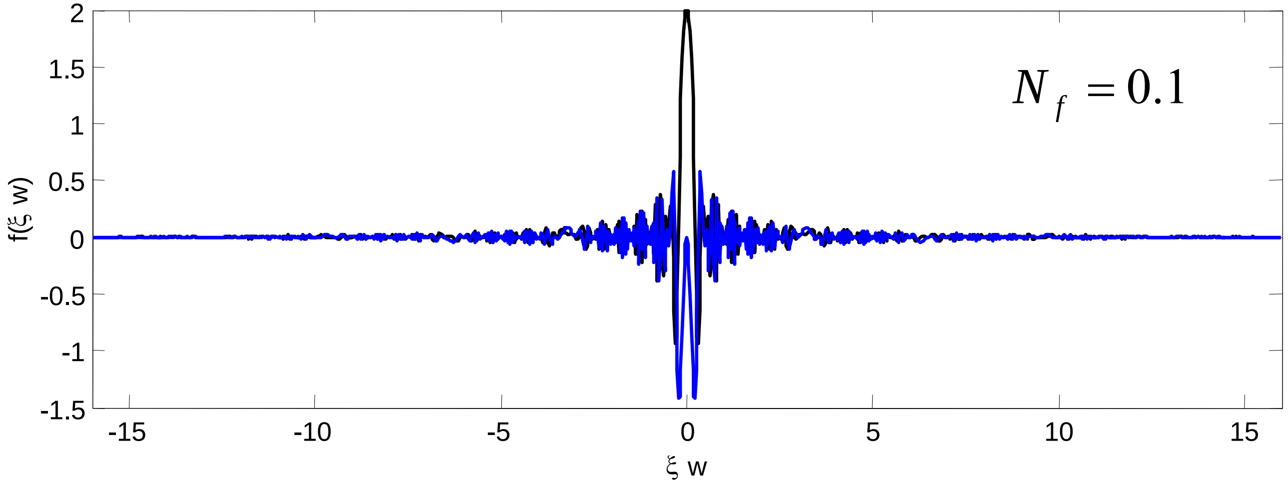


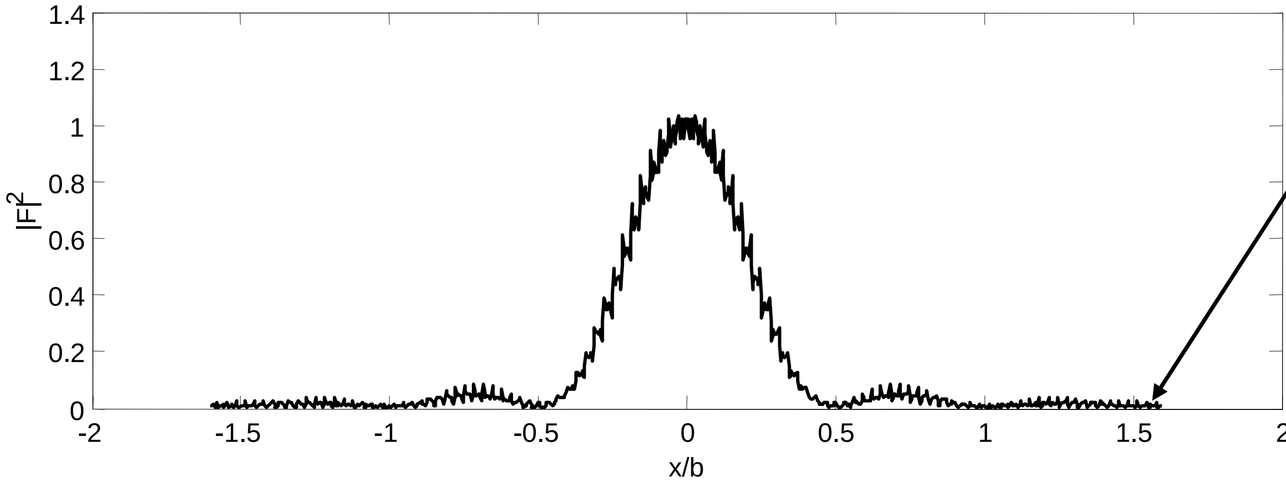
Black：谱实部，
Blue：谱虚部.

`nslit(1);`

近场计算例子 $N_f = 0.1$

$$U_1(\xi)q\left(\xi;-\frac{1}{N_f}\right)$$



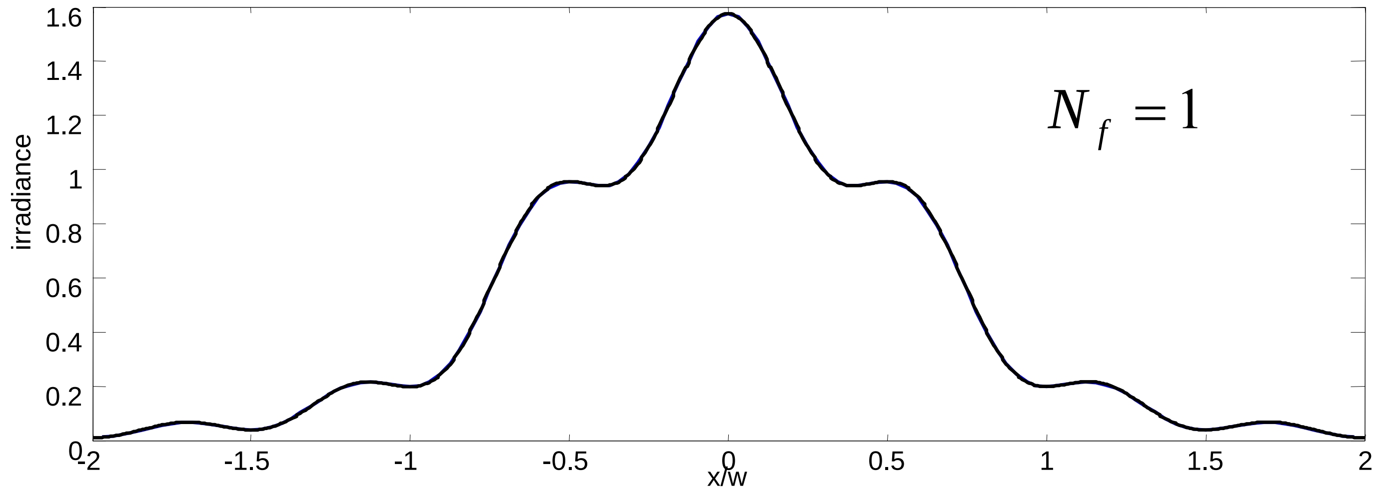


FFT aliasing

```
nslit(10);
```

Black：谱实部，
Blue：谱虚部.

近场计算与直接计算比较

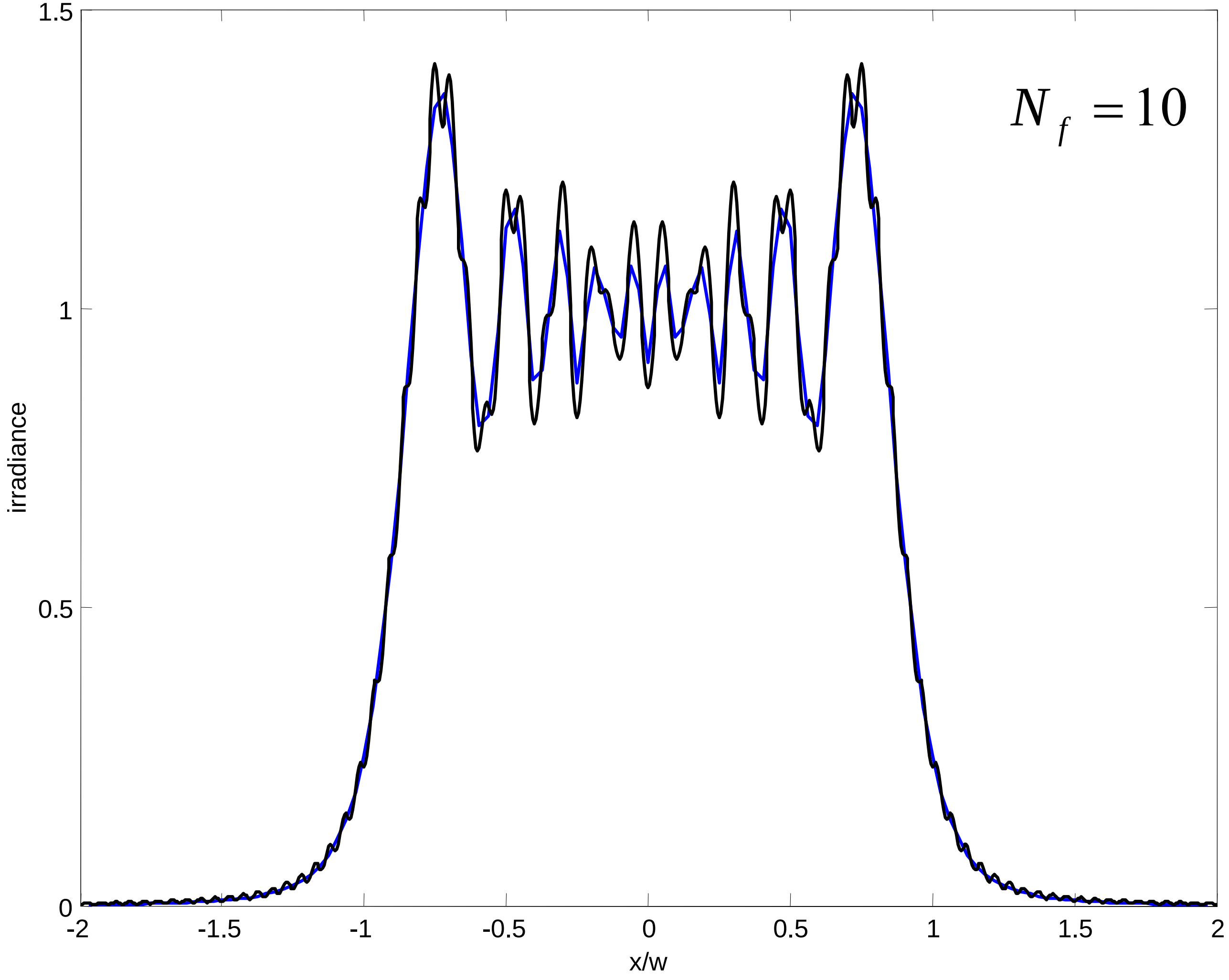


Black：近场计算，
Blue：远场计算

```
function ncompare(a)  %n->near

[fy fx] = nslit(1/a);
[y x] = slit(a);
idx = find(abs(fx)<2.0);
subplot(1,1,1);
plot(fx(idx),fy(idx),'b',x,y,'k');
if (a<1)
    xlabel('x/b');
    ylabel('normalized irradiance');
else
    xlabel('x/w');
    ylabel('irradiance');
end
```

ncompare(1);

近场比较 $N_f = 10$



`ncompare(10);`

Black：直接计算，
Blue：远场计算

四、直接卷积核

$$u_2(x) = \left[\frac{e^{jkz_{12}}}{j} N_f\right]^{\frac{1}{2}} u_1(x) * q(x; N_f) \qquad N_f = \frac{w^2}{\lambda z_{12}}$$

对整个取样空间积分:

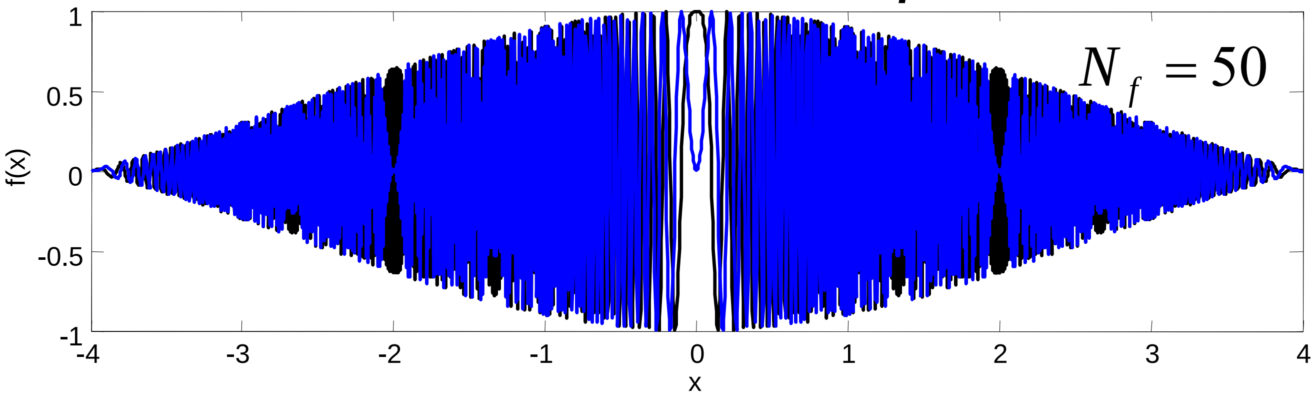
$$q(x;a) = e^{j\pi a x^2} \qquad a x^2 = \frac{1}{2} t^2 \qquad t = \sqrt{2a}\, x$$

$$q(x;a) * \frac{1}{dx} \mathrm{rect}\left(\frac{x}{dx}\right) \qquad \int_0^x e^{j\pi a x^2} dx = \frac{1}{\sqrt{2a}} \int_0^{\sqrt{2a}\,x} e^{j\frac{\pi}{2} t^2} dt$$

$$\frac{1}{dx} \int_{x-\frac{dx}{2}}^{x+\frac{dx}{2}} e^{j\pi a x^2} dx \qquad \int_0^x e^{j\pi a x^2} dx = \frac{1}{\sqrt{2a}} \mathrm{fresnel}\left(\sqrt{2a}\, x\right)$$

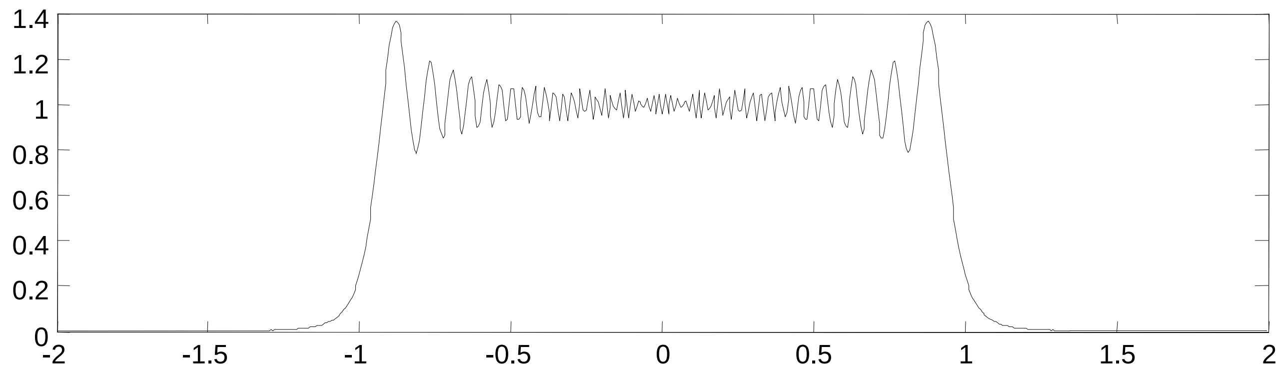
```
[cc2,ss2] = fresnel(sqrt(2*a)*(x+dx/2));
[cc1,ss1] = fresnel(sqrt(2*a)*(x-dx/2));

f = (1/sqrt(2*a))*complex(cc2-cc1,ss2-ss1)/dx;
```

直接卷积 $N_f = 50$



核

$$f = q(x; N_f)$$



卷积

$$g = \mathrm{rect}\left(\frac{x}{2}\right)$$

$$z = g * f$$

$$y = |z|^2$$

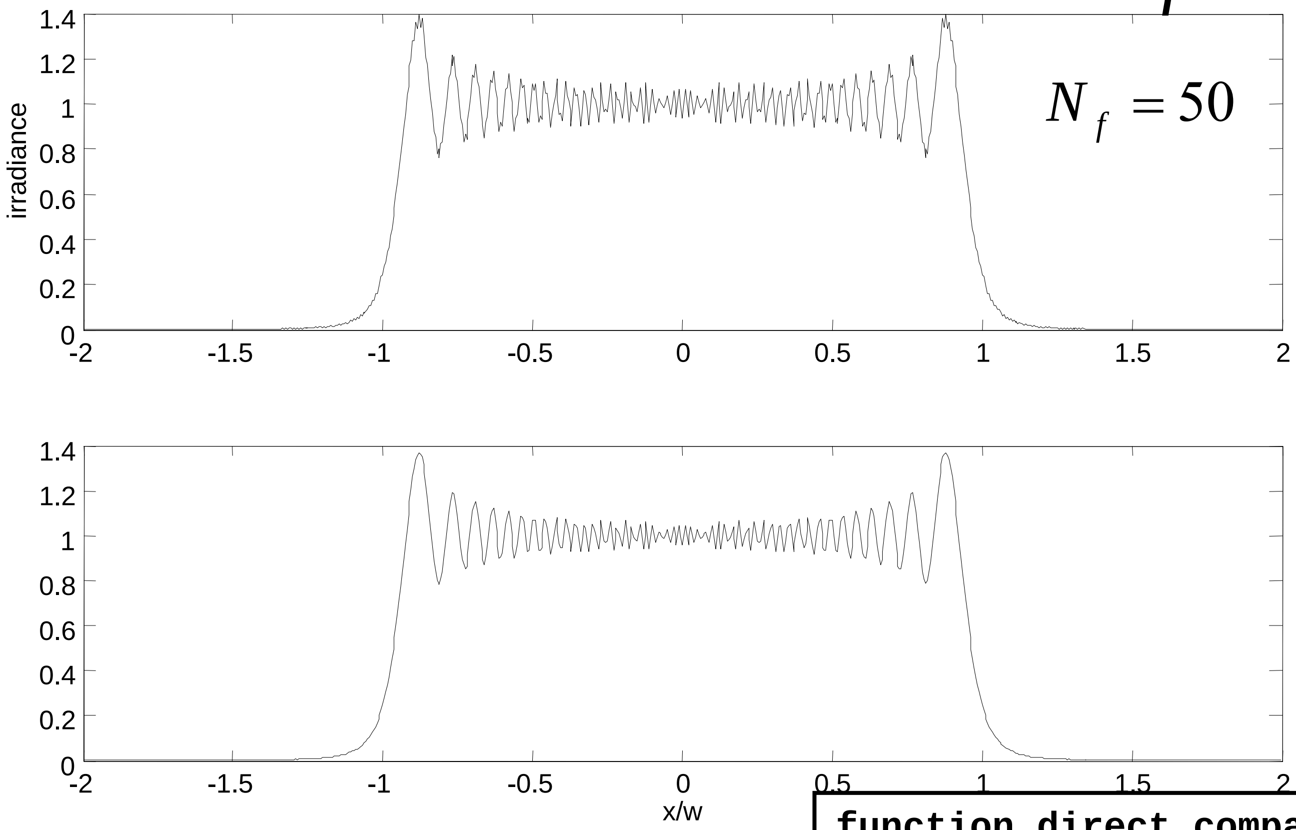
```
g = rect(x/2);
z = convn(g,f,'same')*(sqrt(a)*dx);
y = z.*conj(z);
```

卷积的Matlab Code

```
function [y, x] = prop_response(a)

N=1600;
D=4;

x=linspace(-1,1,N+1)*D;
dx = x(2)-x(1);

[cc2,ss2] = fresnel(sqrt(2*a)*(x+dx/2));
[cc1,ss1] = fresnel(sqrt(2*a)*(x-dx/2));


f = (1/sqrt(2*a))*complex(cc2-cc1,ss2-
ss1)/dx;
%f = qchirp(x,a);
```

```
xrange = D;
subplot(2,1,1);
plot(x,real(f),'k',x,imag(f),'b');
xlabel('x');
ylabel('f(x)');

g = rect(x/2);
z = convn(g,f,'same')*(sqrt(a)*dx);
y = z.*conj(z);

if (a<1)
    y = y/4;
end

subplot(2,1,2);
xrange = 2;
idx = find(abs(x)<xrange);
plot(x(idx),y(idx),'k');
```

直接与卷积计算比较$N_f = 50$



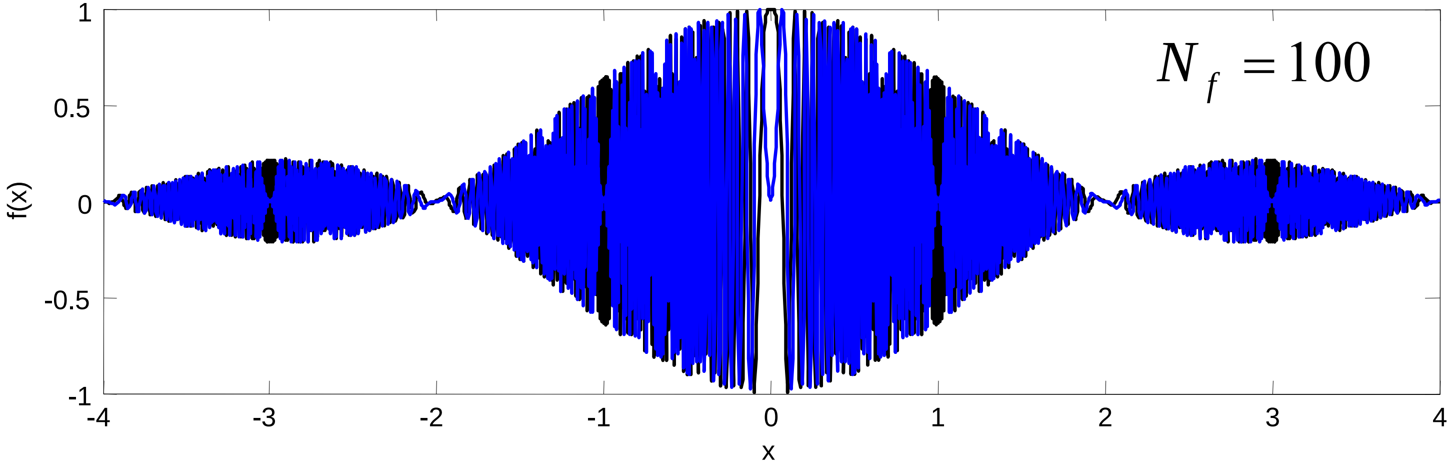
直接计算

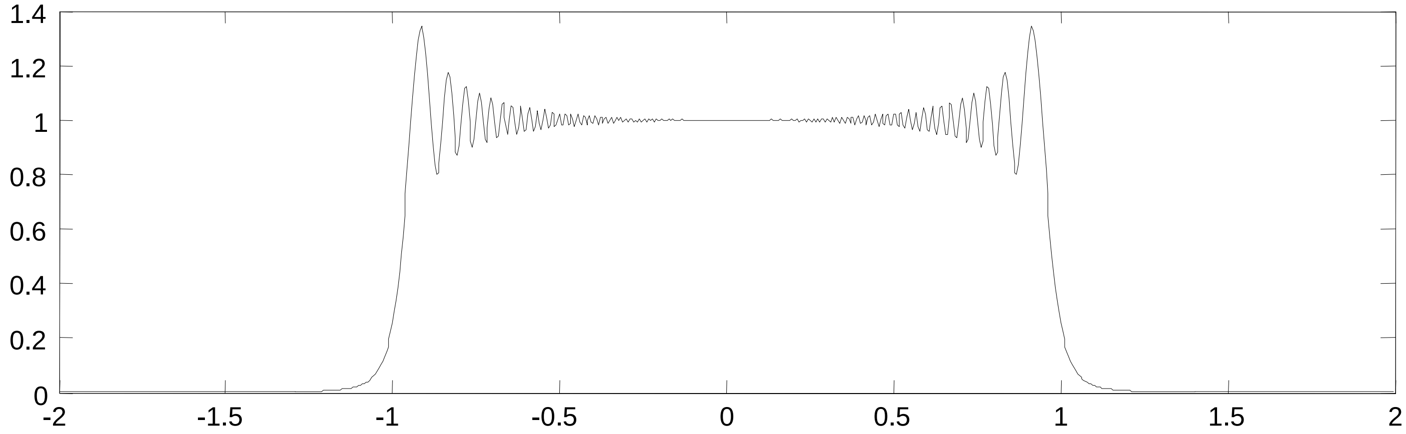
卷积

```
function direct_compare(a)

prop_response(a);
xlabel('x/w');
subplot(2,1,1);
slit(a);
ylabel('irradiance');
```

```
Direct_compare(50);
```

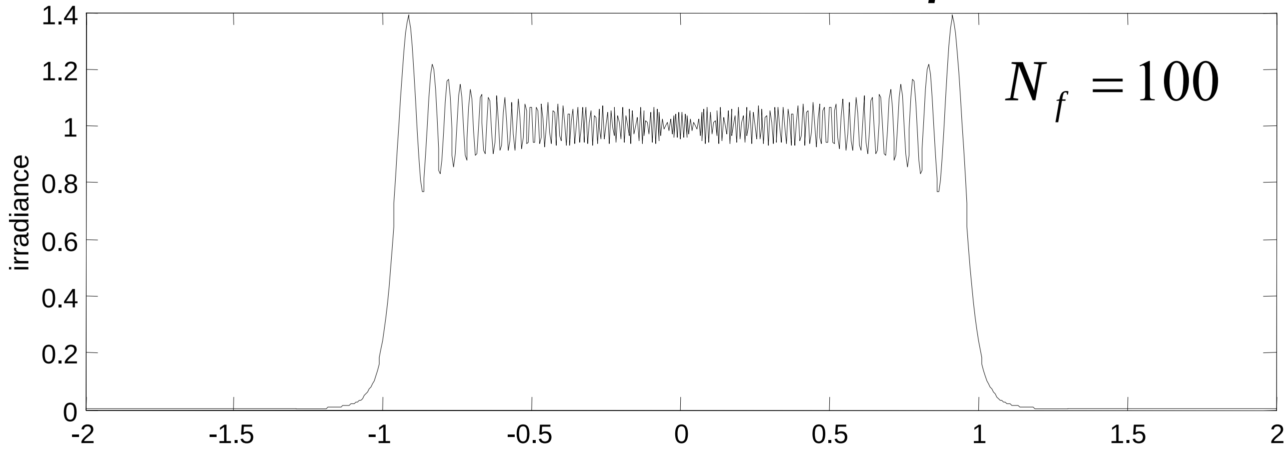
直接卷积比较$N_f = 100$



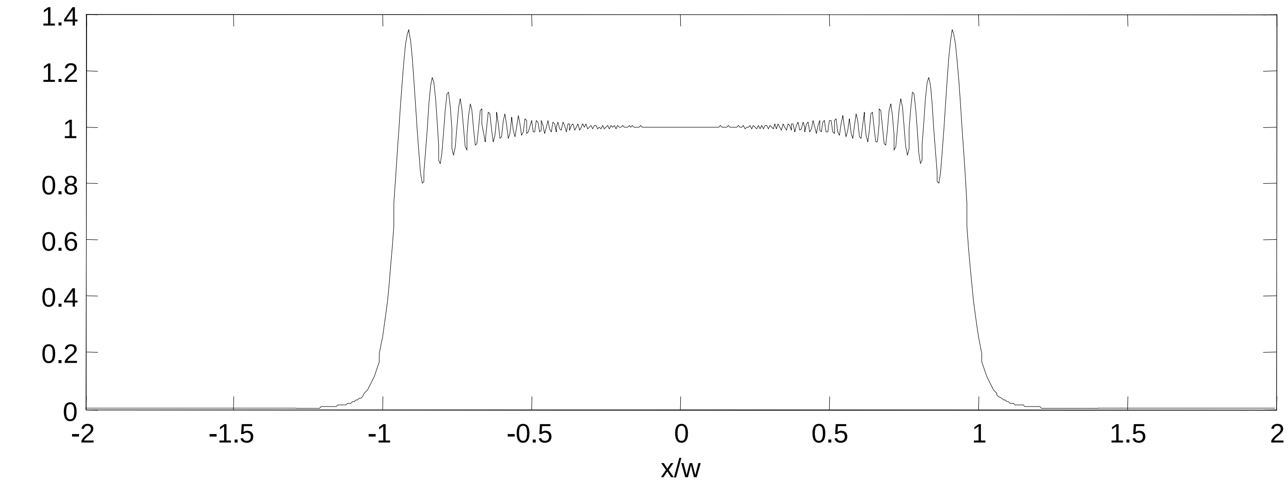



```
prop_response(100);
```

卷积比较 $N_f = 100$



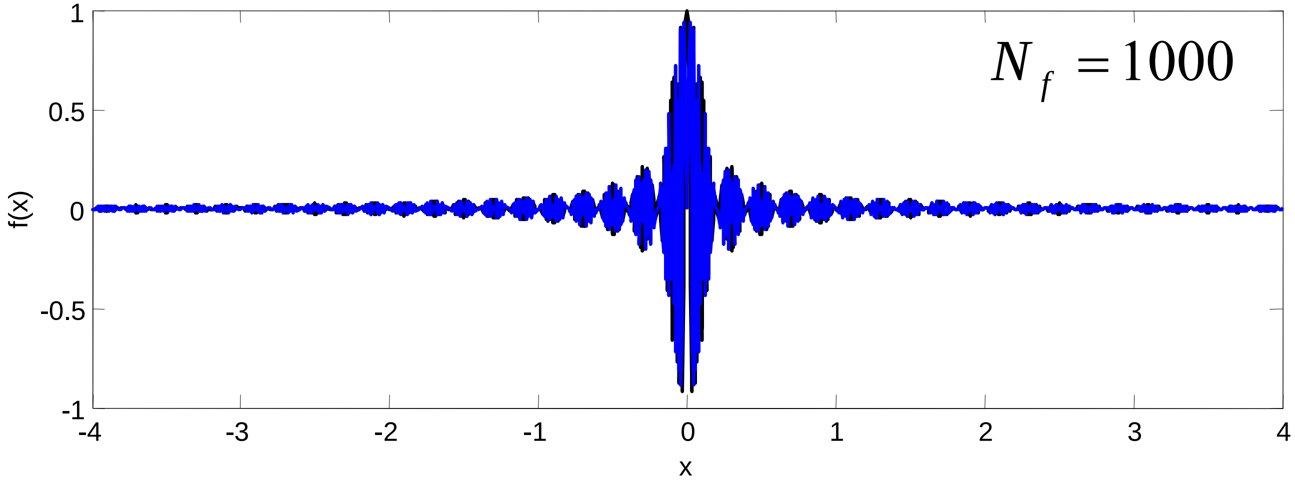
直接计算

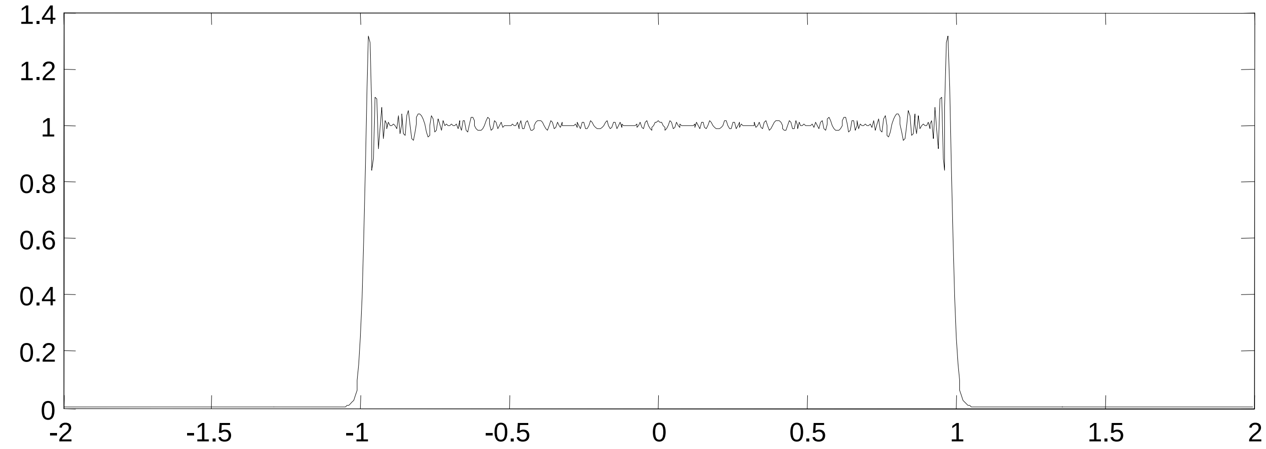


卷积

```
Direct_compare(100);
```

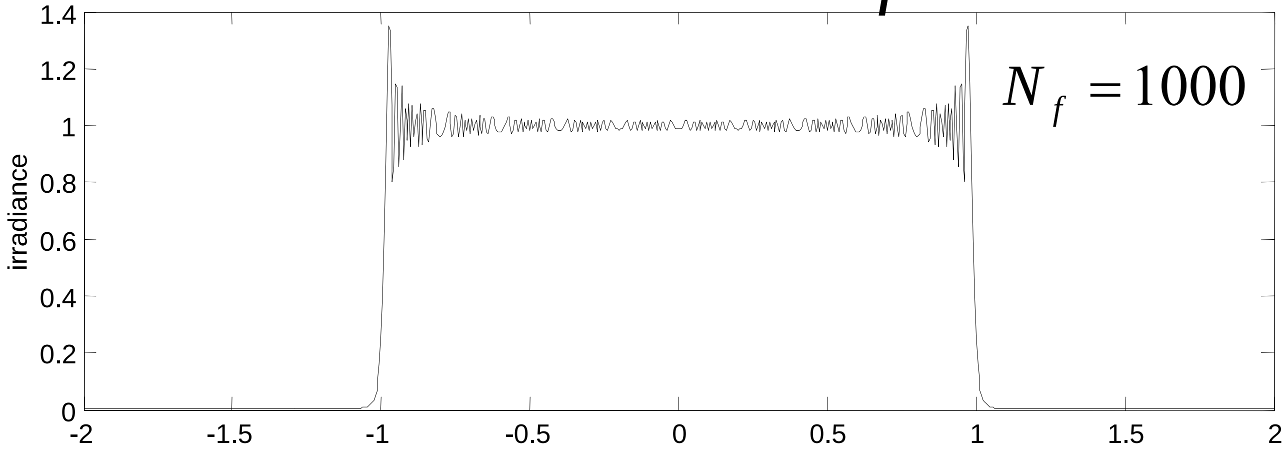
卷积比较$N_f = 1000$



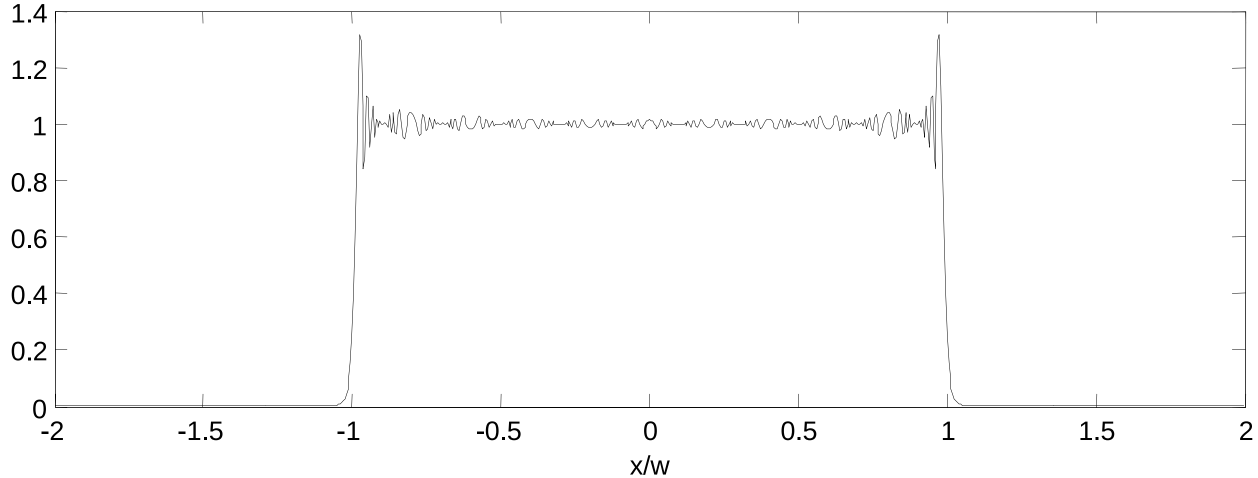



```
prop_response(1000);
```

卷积比较$N_f = 1000$



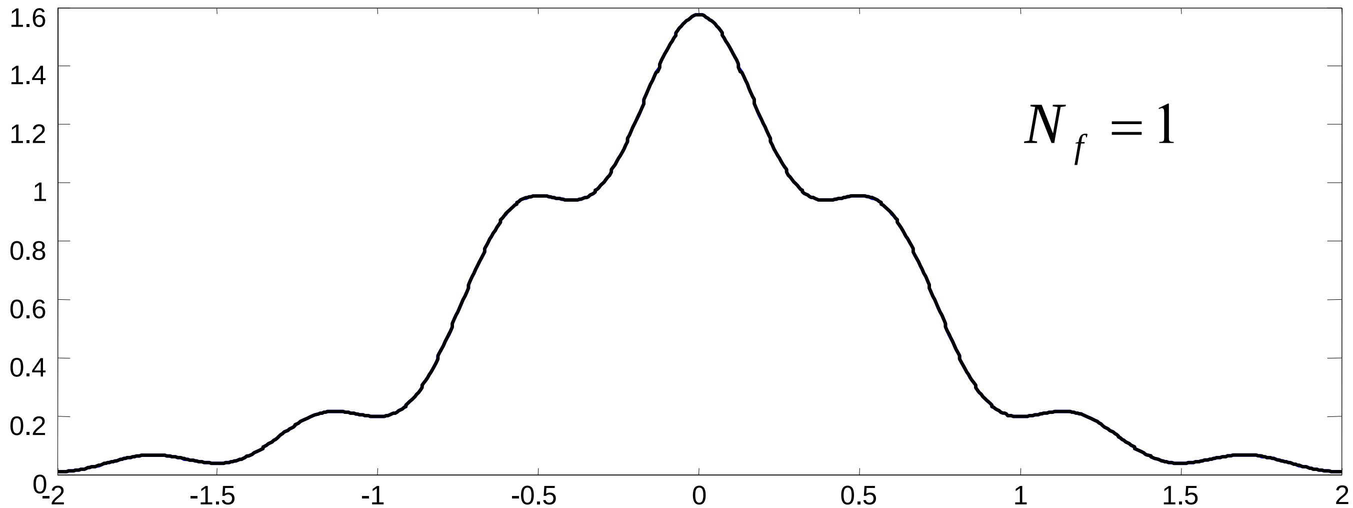
直接计算



卷积

```
Direct_compare(1000);
```

卷积与远场计算的比较

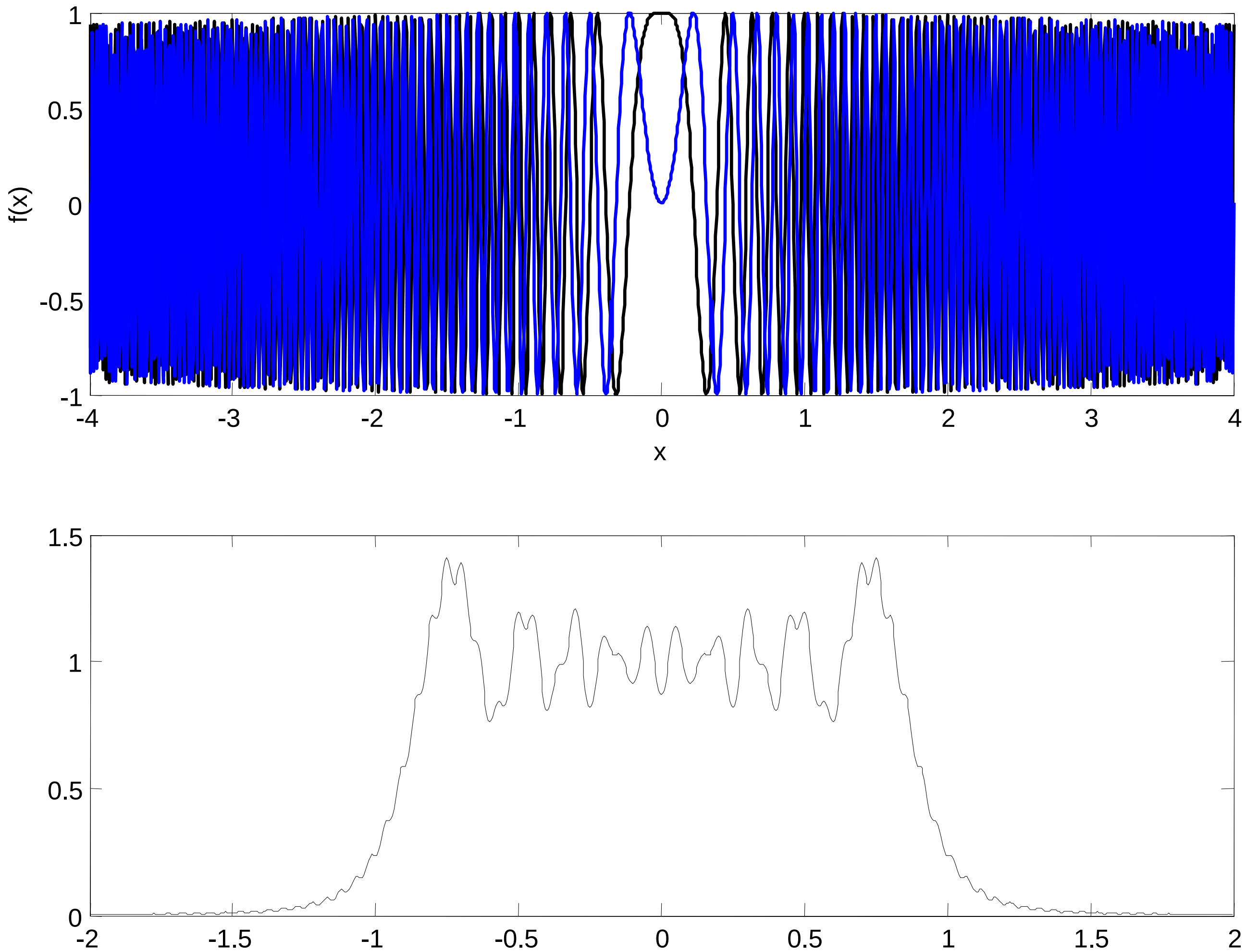



```
function pcompare(a)

[fy fx] = prop_response(a);
[y x] = slit(a);
idx = find(abs(fx)<2.0);
subplot(1,1,1);
plot(fx(idx),fy(idx),'b',x,y,'k');
if (a<1)
    xlabel('x/b');
    ylabel('normalized irradiance');
else
    xlabel('x/w');
    ylabel('irradiance');
end
```

Blue：远场计算,
Black：直接计算.

pcompare(1);

直接卷积 $N_f = 10$

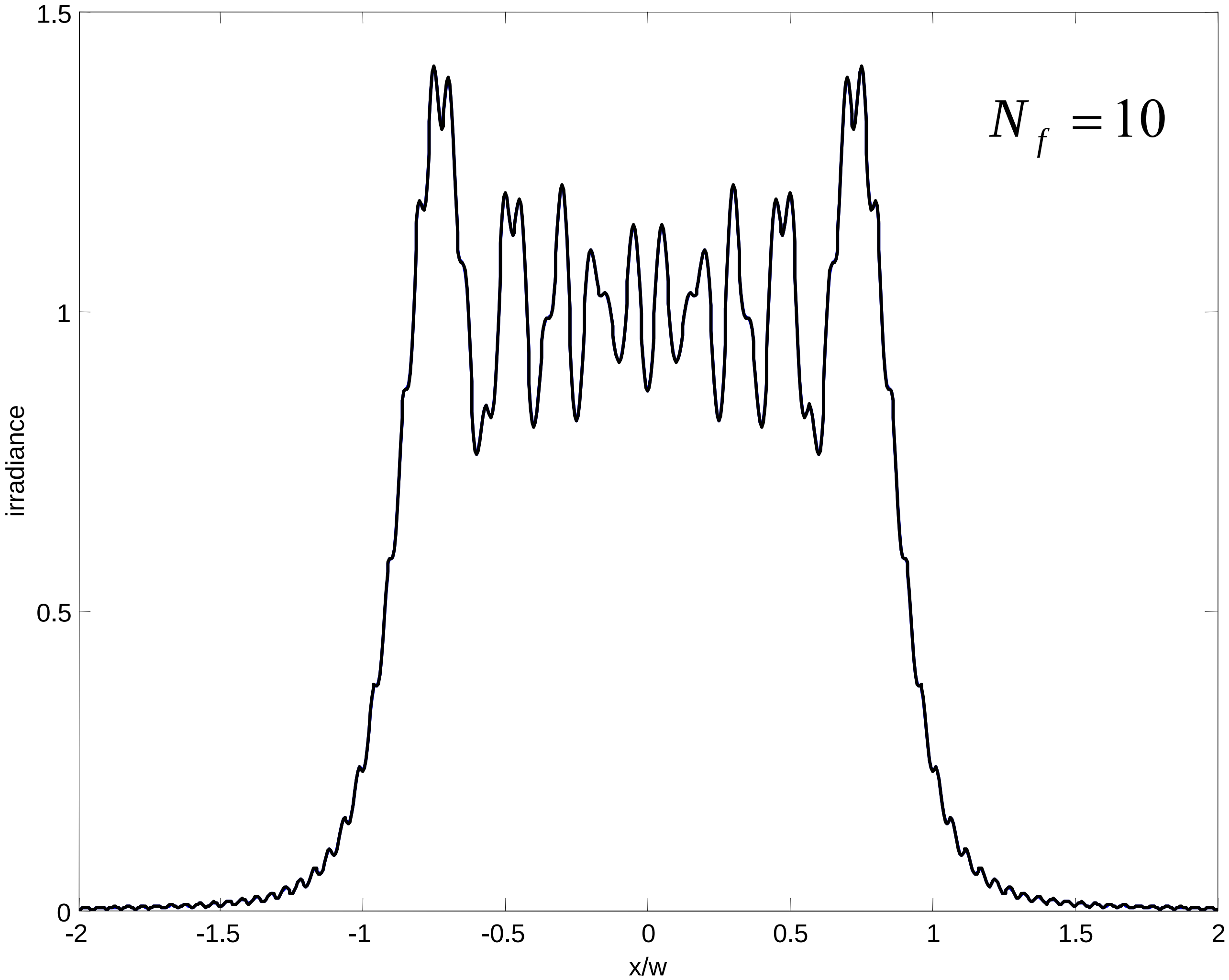


$$N_f = 10$$

$$f = q(x; N_f)$$

```
prop_response(10);
```

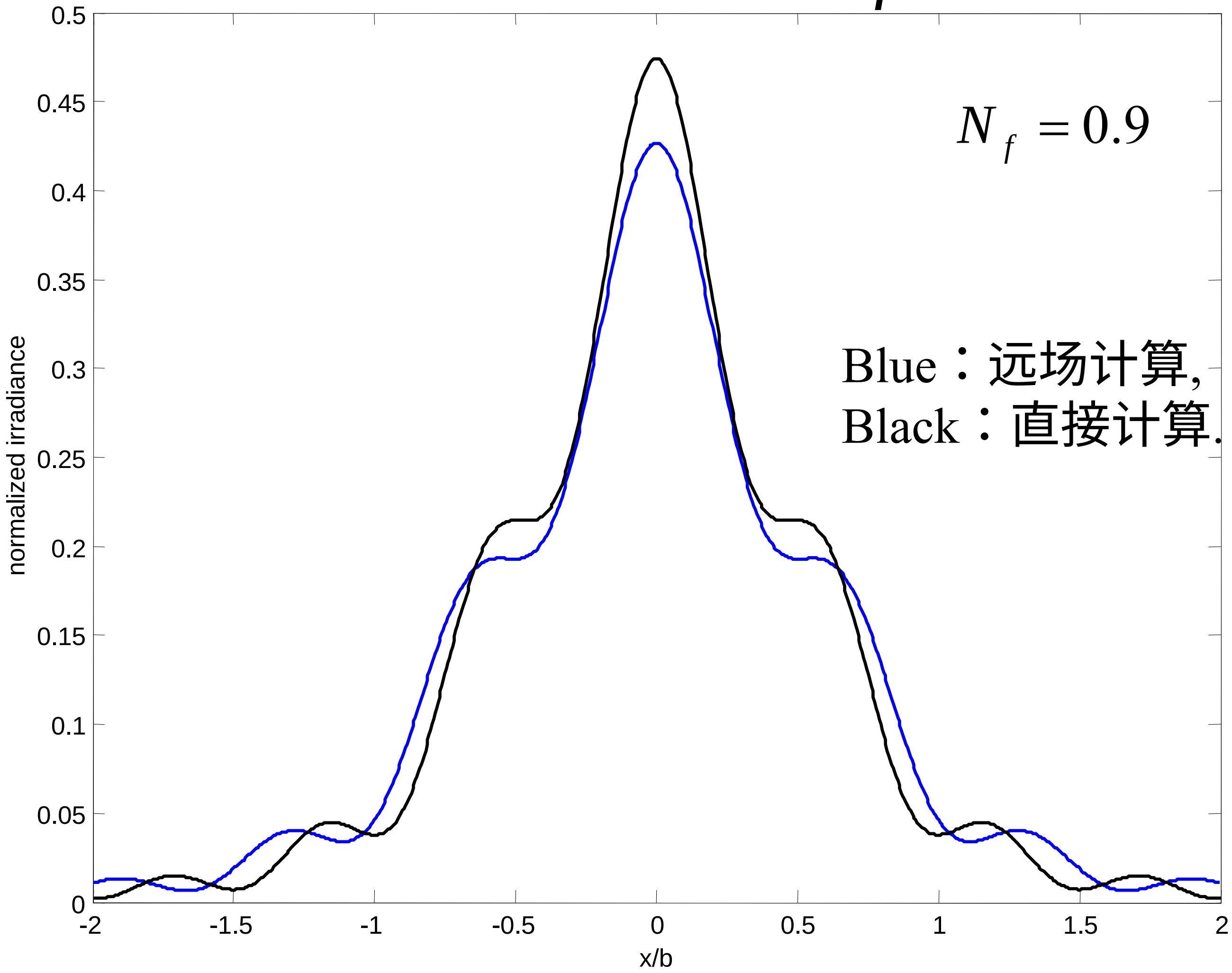
卷积计算与直接计算比较$N_f = 10$



Blue：远场计算，
Black：直接计算.

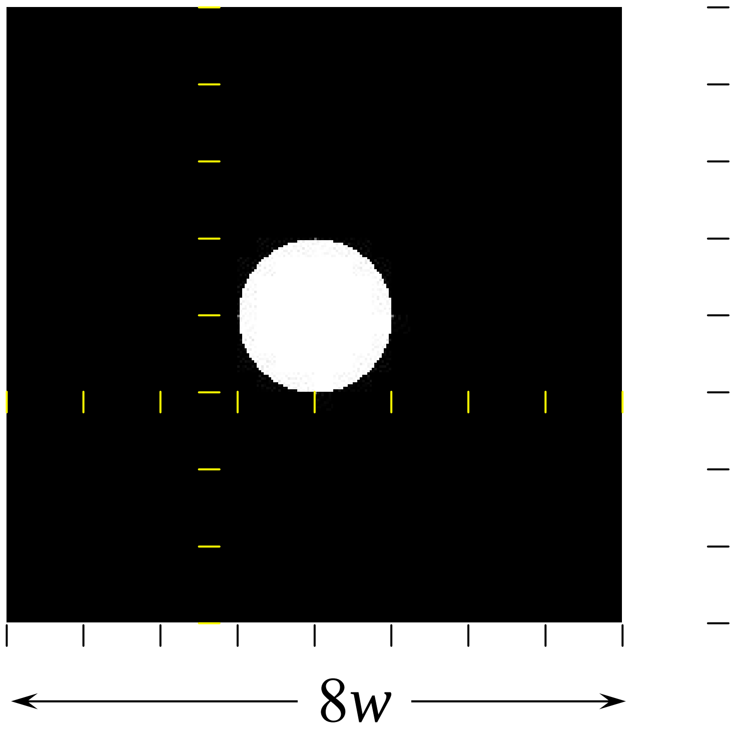
```
pcompare(10);
```

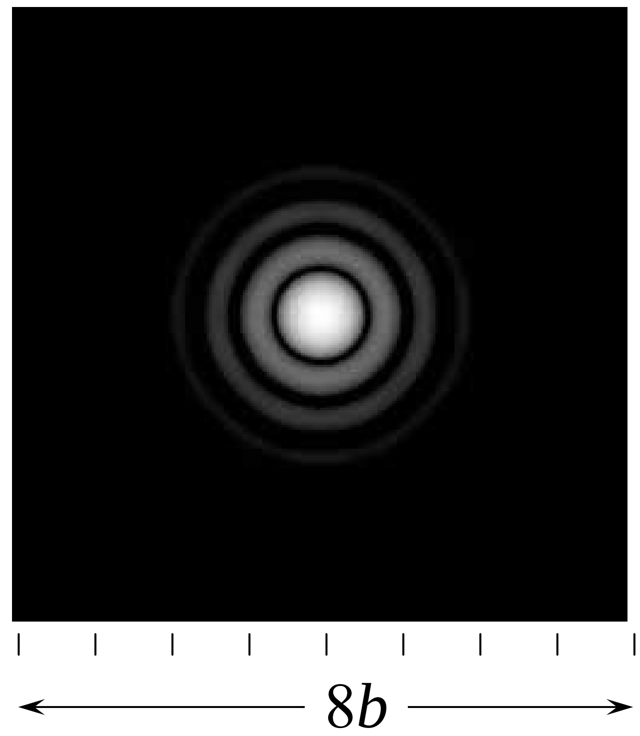
卷积计算比较 $N_f = 0.9$




```
pcompare(0.9);
```

五、园域函数, 直径为 $= 2w$



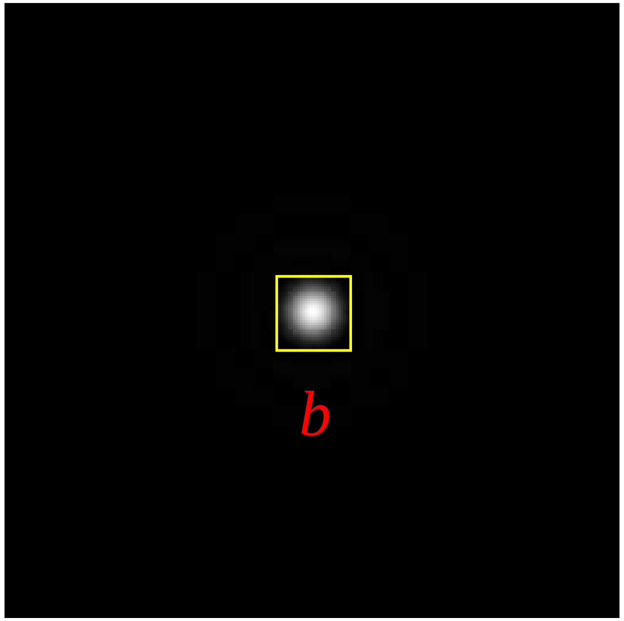


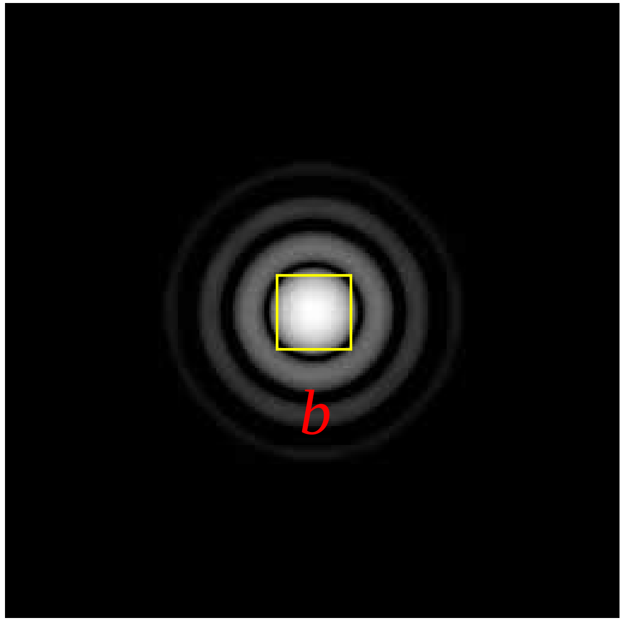

```
function f = func1(x,y)

f = cyl(x,y,2);
```

```
[out in] = fourier(@func1);
```

$$b = \frac{\lambda z}{w}$$

图像辐照度的缩放





$$b = \frac{\lambda z}{w}$$

$$\mathrm{cyl}(x, y; 2w)$$

单位辐照度孔径

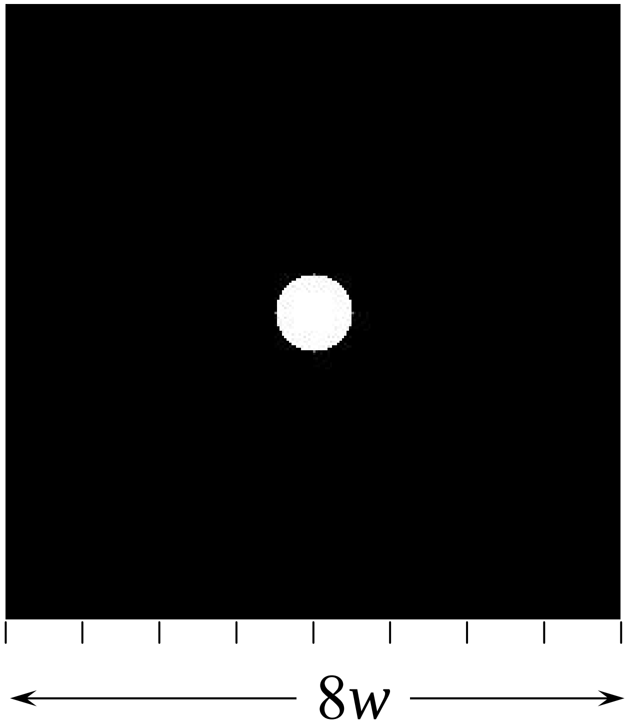
```
imshow(logim(out,3));
```

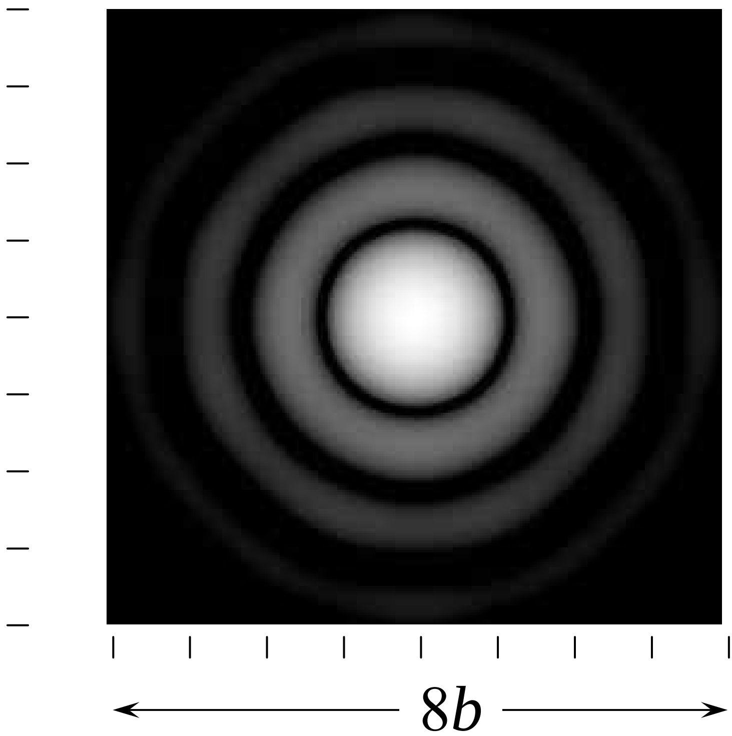
图像辐照度经过LOG变换
Image irradiance scaled logarithmically to 3 decades.

远场衍射的Matlab 代码

```
function [outp, inp] = fourier(fcn)
    ,
N=1024;
D=sqrt(N);
k = -N/2:N/2-1;
[xs,ys] = meshgrid(k*D/N);
dx = D/N;;
dy = D/N;

f = feval(fcn,xs,ys);
A = sum(sum(f))*dx*dy;

subplot(1,2,1);
range = 4;
xr = k/D;
idx = find( (-range <= xr) & (xr<range) );
inp = f(idx,idx);
imshow(inp);
x = xs(idx,idx);
y = ys(idx,idx);

z = fftshift(fft2(fftshift(f)))*(dx*dy/A);
fy = z.*conj(z);

subplot(1,2,2);
frange = 4;
idx = find( (-frange <= xr) & (xr<frange) );
outp = fy(idx,idx);
logim(outp,3);
```

园域函数, 直径 $= w$



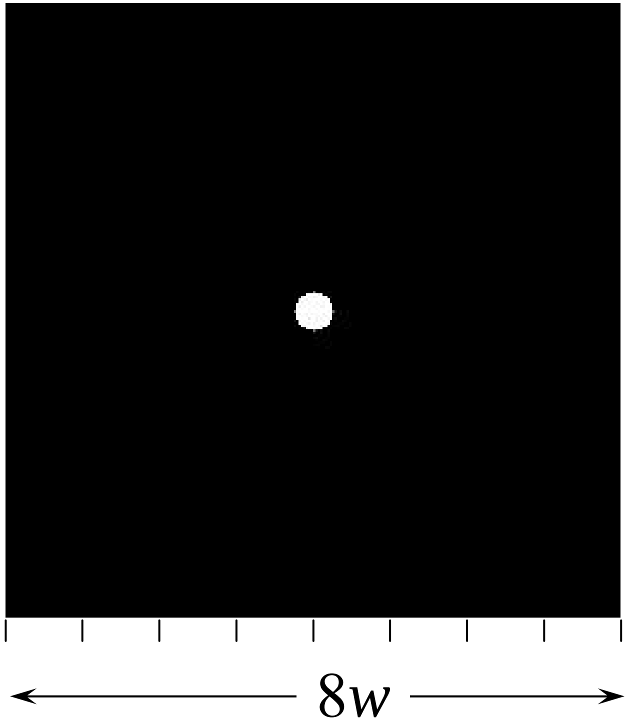


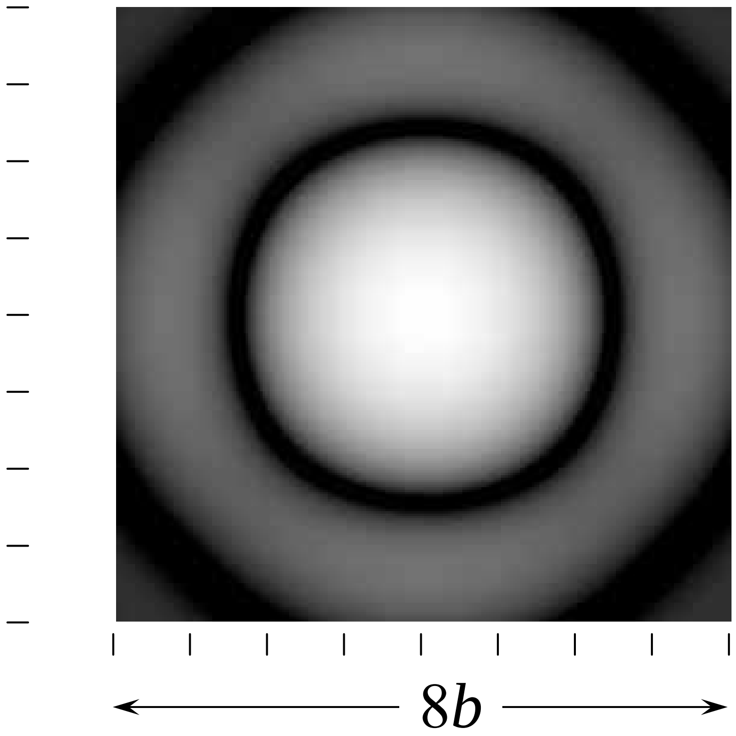

```
function f = func1(x,y)

f = cyl(x,y);
```

```
[out in] = fourier(@func1);
```

$$b = \frac{\lambda z}{w}$$

园域函数, 直径 = ½ w



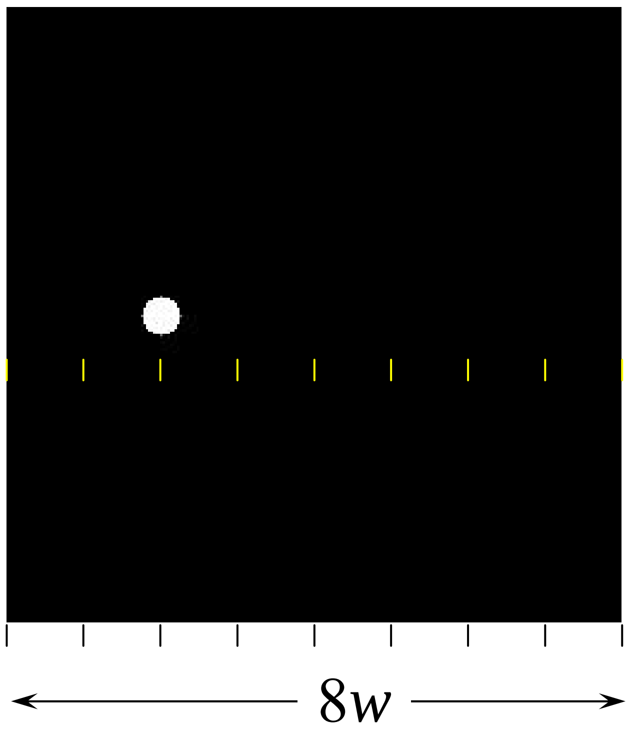


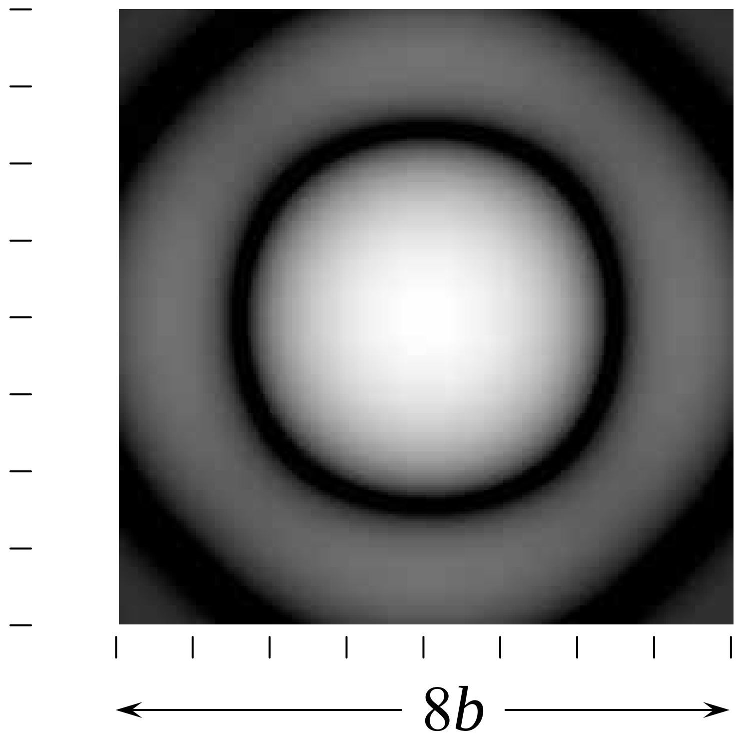

```
function f = func1(x,y)

f = cyl(x,y,0.5);
```

```
[out in] = fourier(@func1);
```

$$b = \frac{\lambda z}{w}$$

平移的园域函数



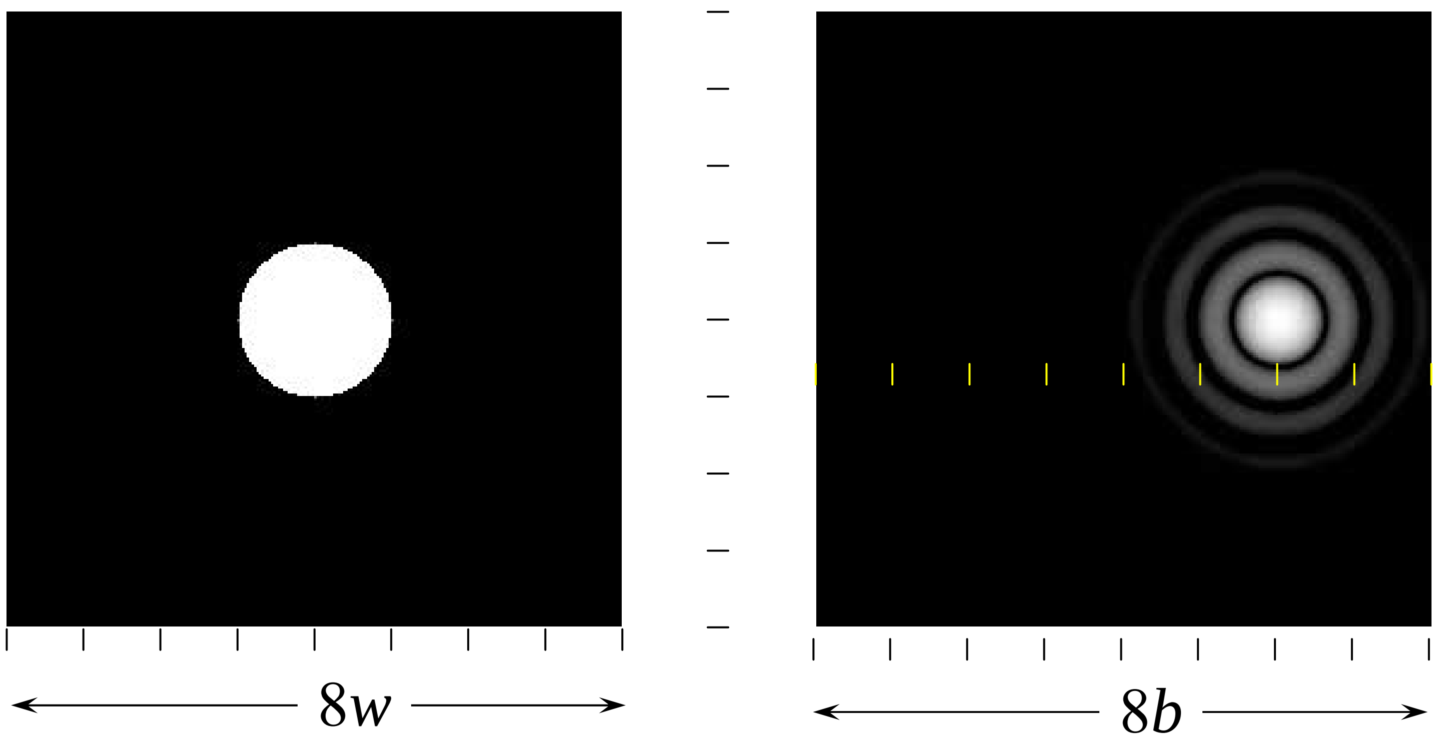



```
function f = func1(x,y)

f = cyl(x+2,y,0.5);
```

```
[out in] = fourier(@func1);
```

$$b = \frac{\lambda z}{w}$$

相移的园域函数




```
function f = func1(x,y)

xi = 2;
f = cyl(x,y,2).*exp(j*2*pi*xi*x);
```

```
[out in] = fourier(@func1);
```

$$b = \frac{\lambda z}{w} \qquad x_0 = \xi_0 b = 2b$$

六、FT平面的位移与干涉

$$e^{j2\pi\xi_0 x} \Leftrightarrow \delta(\xi - \xi_0)$$

$$f(x)e^{j2\pi\xi_0 x} \Leftrightarrow F(\xi) * \delta(\xi - \xi_0) = F(\xi - \xi_0)$$

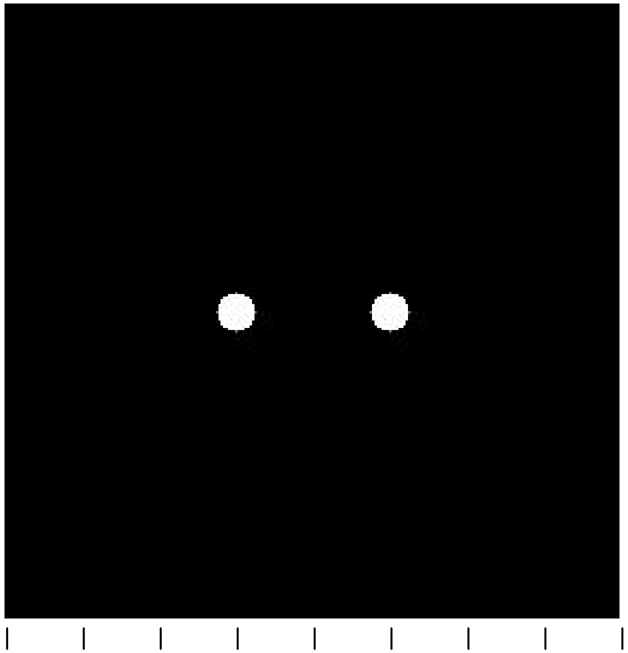
$$\xi = \frac{x}{b}$$

当 $\xi_0 = 2$

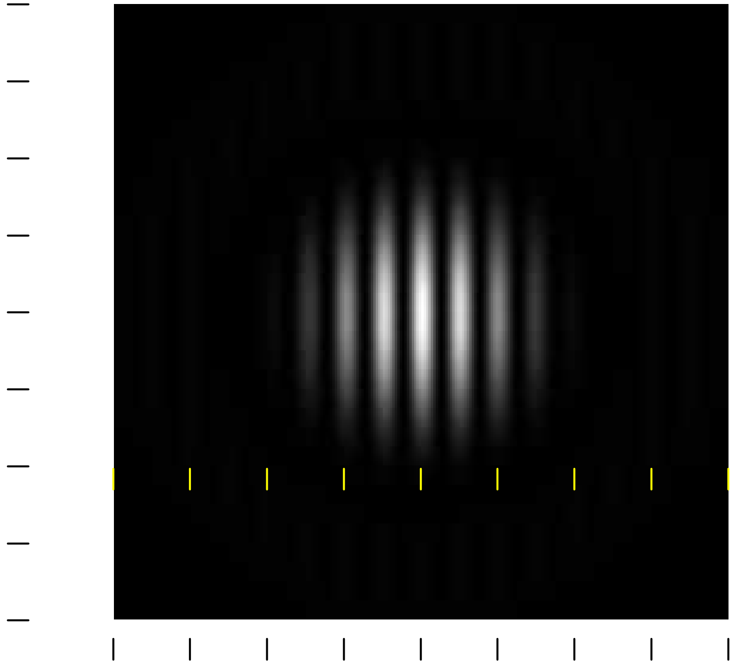
$$F(\xi - \xi_0) \rightarrow F\left(\frac{x}{b} - 2\right)$$

$$x_0 = 2b$$

两孔径的干涉



$\longleftarrow 8w \longrightarrow$



$\longleftarrow 8b \longrightarrow$

```
function f = func1(x,y)

f = cyl(x+1,y,0.5)+cyl(x-1,y,0.5);
```

```
[out in] = fourier(@func1);
```

输出未经对数缩放

$$f(x,y) = p(x,y) * \left(\delta(x - x_0) + \delta(x + x_0)\right)$$

$$x_0 = 1$$

$$\text{period} = \frac{b}{2x_0} = \frac{b}{2}$$

两个孔径的FT

```
function f = func1(x,y)

f = cyl(x+1,y,0.5)+cyl(x-1,y,0.5);
```

$$p(x,y)=\mathrm{cyl}(x,y;0.5)$$

$$FT\{f(x-x_0)\}=e^{-j2\pi\xi x_0}F(\xi)$$

$$f(x,y)=p(x,y)*(\delta(x-x_0)+\delta(x+x_0))$$

$$F(\xi,\eta)=P(\xi,\eta)\left(e^{-j2\pi x_0\xi}+e^{j2\pi x_0\xi}\right)=2P(\xi,\eta)\cos(2\pi x_0\xi)$$

$$F\left(\frac{x}{b},\frac{y}{b}\right)=2P\left(\frac{x}{b},\frac{y}{b}\right)\cos\left(2\pi x_0\frac{x}{b}\right)$$

$$E\left(\frac{x}{b},\frac{y}{b}\right)=2\left|P\left(\frac{x}{b},\frac{y}{b}\right)\right|^2\cos^2\left(2\pi x_0\frac{x}{b}\right)$$

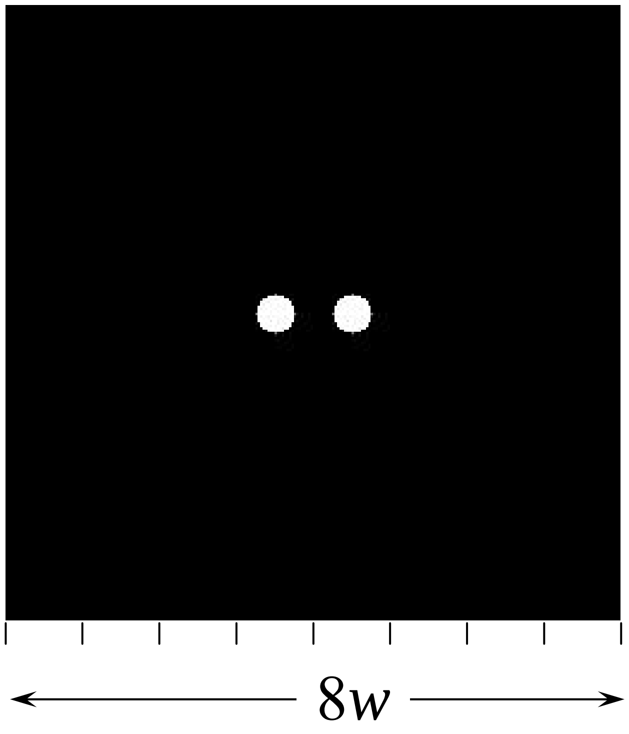
$$E\left(\frac{x}{b},\frac{y}{b}\right)=2\left|P\left(\frac{x}{b},\frac{y}{b}\right)\right|^2\frac{1+\cos\left(4\pi x_0\frac{x}{b}\right)}{2}$$

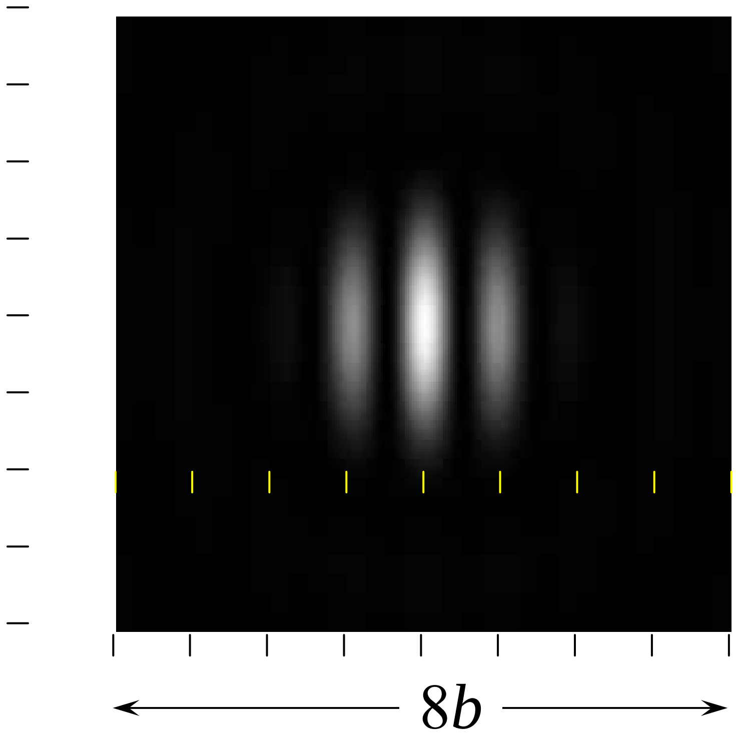
周期由下式决定

$$4\pi x_0\frac{x}{b}=2\pi$$

$$周期=\frac{b}{2x_0}$$

两孔径移近






```
function f = func1(x,y)

x0=0.5;
f = cyl(x+x0,y,0.5)+cyl(x-x0,y,0.5);
```

```
[out in] = fourier(@func1);
```

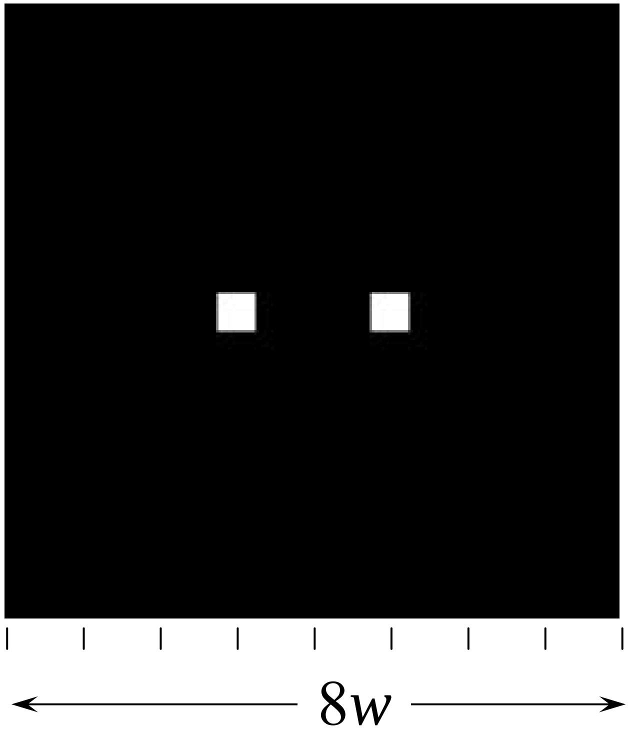
输出未经对数缩放

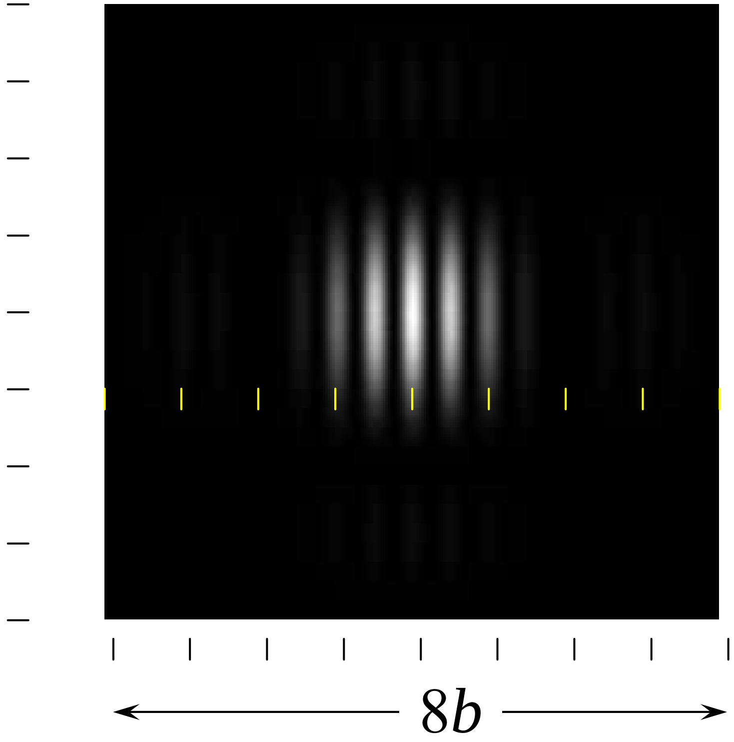
$$f(x,y) = p(x,y) * \left(\delta\left(x - x_0\right) + \delta\left(x + x_0\right)\right)$$

$$\text{period} = \frac{b}{2x_0} = b$$

$$x_0 = \tfrac{1}{2}$$

两方形孔径的干涉






```
function f = func1(x,y)
d=1;
f = (rect(x-d,0.5)+rect(x+d,0.5)).*rect(y,0.5);
```

```
[out in] = fourier(@func1);
```

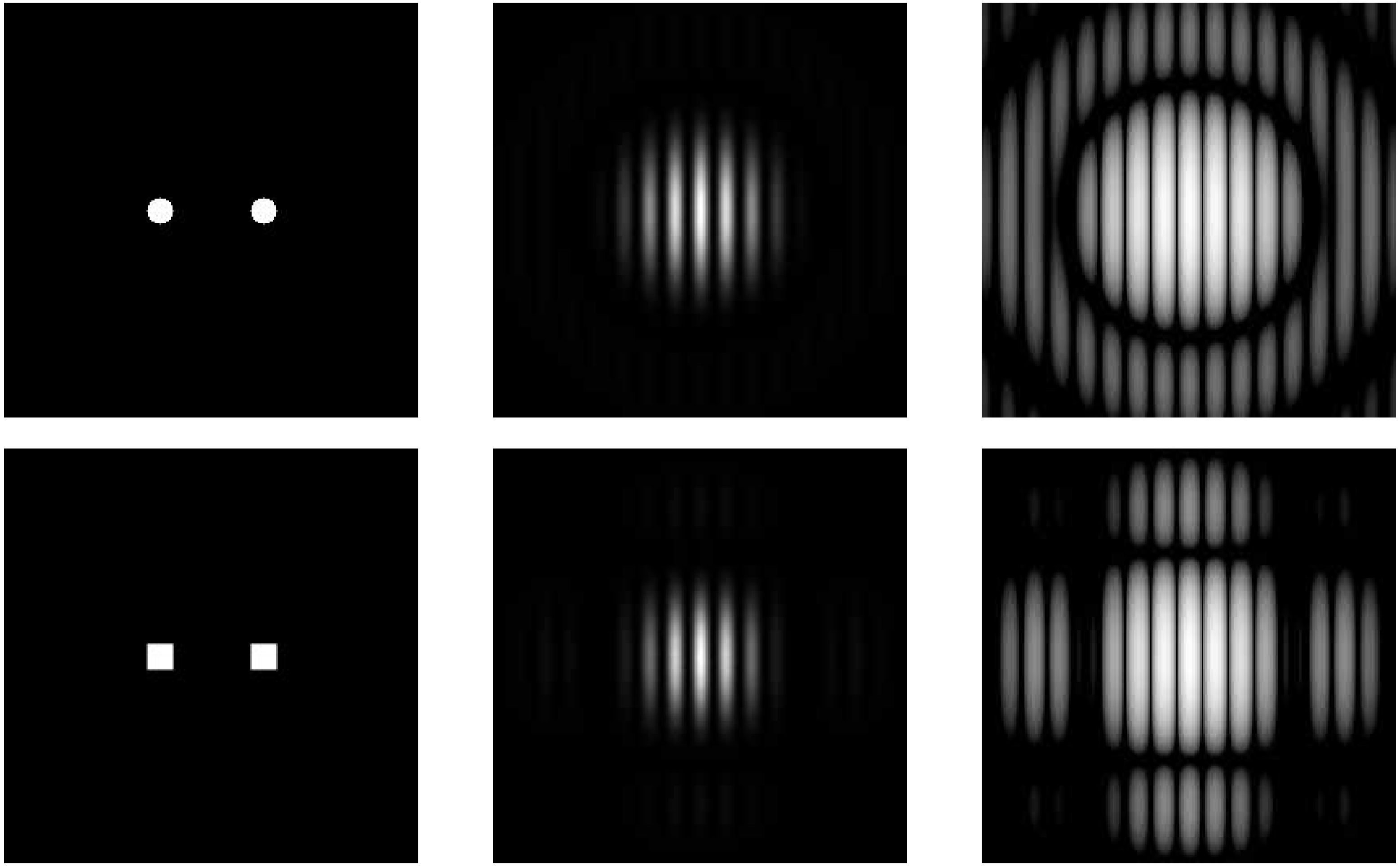
$$f(x,y) = p(x,y) * \left(\delta\left(x-d\right)+\delta\left(x+d\right)\right)$$

$$d = 1$$

输出未经对数缩放

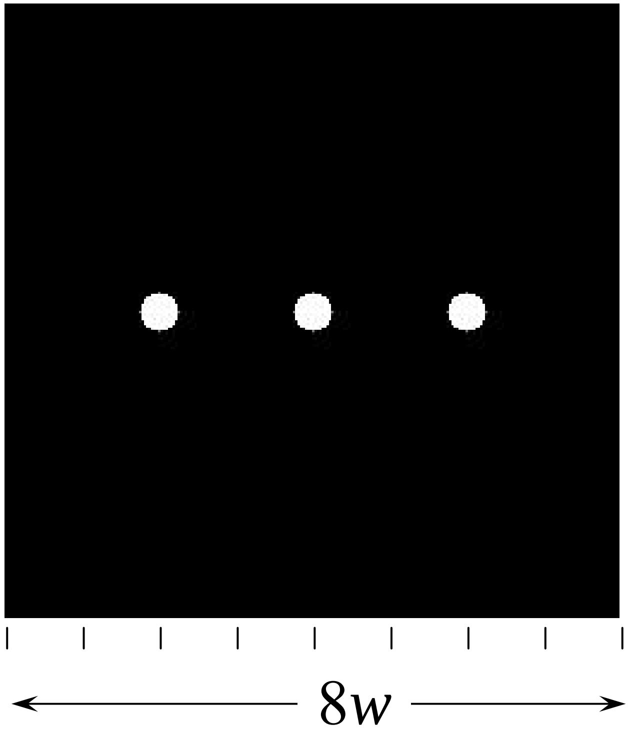
$$\text{period} = \frac{b}{2d} = \frac{b}{2}$$

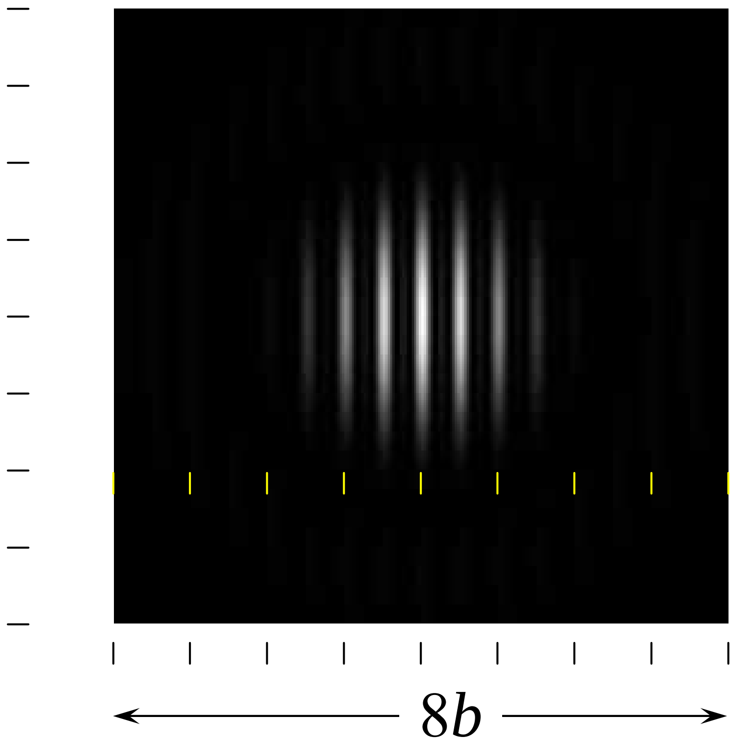
孔径形状的影响



输入　　输出　　对数缩放后的图像输出

三个孔径的干涉



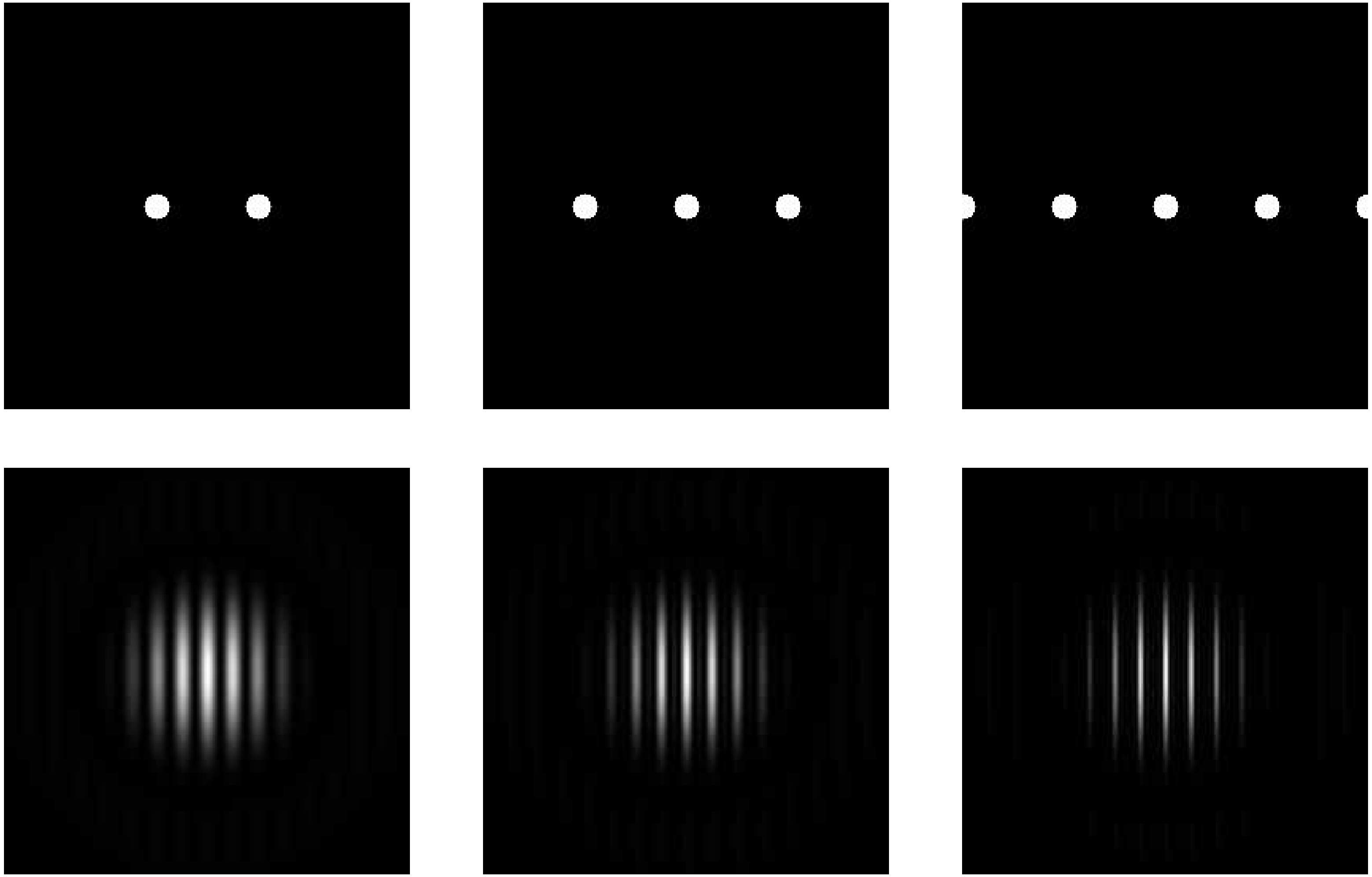



```
function f = func3(x,y)

d=2;
n=1;
f = zeros(size(x));
for i=-n:n
    f = f + cyl(x-i*d,y,0.5);
end
```

```
[out in] = fourier(@func3);
```

输出未经对数缩放

多孔径效应



2 3 5

作业

3、编写一个通用的普通函数的衍射函数，实现输入函数的菲涅尔衍射。注意各参数、代码的通用性。要求文档齐全。