Theory of 2D-Rockets basically

Zhao H.Q.

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1 Assumption

1.2Dimensional motion: Our rockets only consider 2Dimension,x-z flat (The z-axis points upward, and the x-axis is horizontal.)
2.rigid body: The rocket is treated as a rigid body, meaning it does not deform during flight.
3.Translation and rotation: The rocket's motion includes both translation (movement through space) and rotation (spinning around its center of mass).
4.Aerodynamic force only acts as drag.
5.Gravity acts downward along the z-axis with a constant acceleration of $g = 9.81\,\mathrm{m/s^2}$.
6.constant air density of $\rho = 1.225\,\mathrm{kg/m^3}$ (sea level standard conditions).

2 Coordinate system and state variables

2.1 1.Definition of system coordinate

1.Inertial frame of reference: origin at launch point
2.Body axis system: origin at the center of gravity (CG), $\hat{x}_b$forward along the rocket's longitudinal axis, $\hat{y}_b$laterally (right-hand rule), and $\hat{z}_b$upward.

2.2 2.State variables

We can use 6 variables to describe the state of the rocket at any time t: $s(t) = \begin{bmatrix} x(t) \\ z(t) \\ v_x(t) \\ v_z(t) \\ \theta(t) \\ \omega(t) \end{bmatrix}$

where:
$x(t)$: Horizontal position of the rocket's center of mass (CG) at time t.
$z(t)$: Vertical position of the rocket's center of mass (CG) at time t.
$v_x(t)$: Horizontal velocity component of the rocket at time t.
$v_z(t)$: Vertical velocity component of the rocket at time t.
$\theta(t)$: Pitch angle of the rocket relative to the horizontal axis at time t.
$\omega(t)$: Angular velocity (rate of change of pitch angle) of the rocket at time t.

$\omega = \frac{d\theta}{dt}$

3 Force Analysis

3.1 Thrust Force $\vec{T}$

- Direction: Along the rocket's longitudinal axis (body x-axis).
- Magnitude: Given by the rocket motor's thrust profile $T(t)$.