

# Parallel Sudoku Solver

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# Introduction

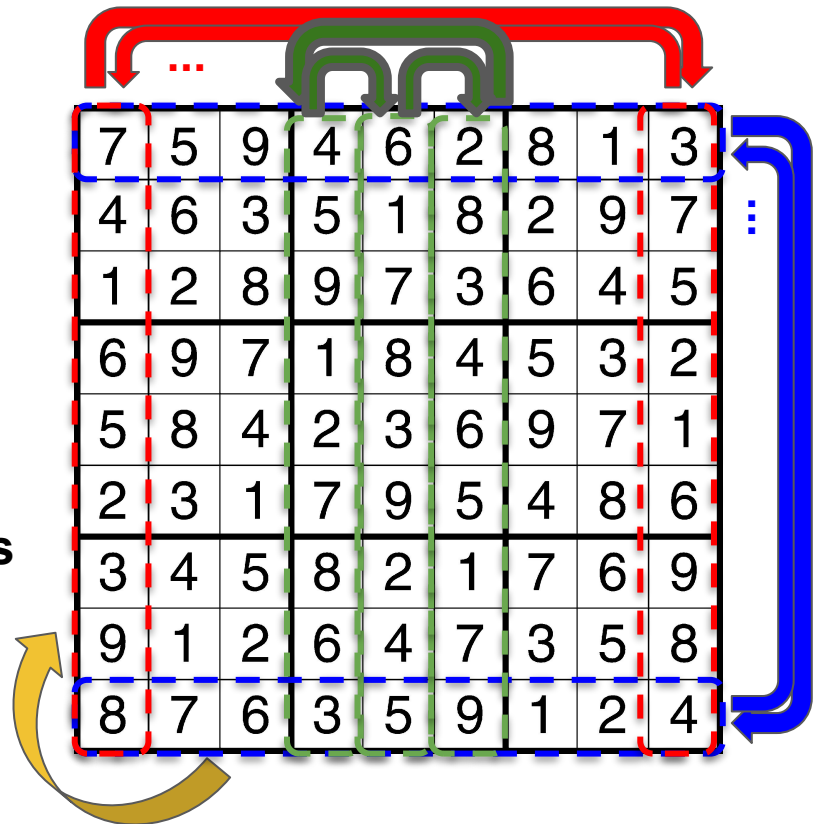
Each row, each column, and each 3 X 3 subgrid contains all digits 1 - 9.

7	5	9	4	6	2	8	1	3
4	6	3	5	1	8	2	9	7
1	2	8	9	7	3	6	4	5
6	9	7	1	8	4	5	3	2
5	8	4	2	3	6	9	7	1
2	3	1	7	9	5	4	8	6
3	4	5	8	2	1	7	6	9
9	1	2	6	4	7	3	5	8
8	7	6	3	5	9	1	2	4

# Sudoku Generator

Steps:

1. Copy a Sudoku Array from online resource
2. Flip / rotate for creating new random Sudoku Array
3. Random remove defined number of cells in the Sudoku Array



# Sudoku Generator

Steps:

3. Random remove defined number of cells in the Sudoku Array

7	5	9	4	6	2	8	1	3
4	6	3	5	1	8	2	9	7
1	2	8	9	7	3	6	4	5
6	9	7	1	8	4	5	3	2
5	8	4	2	3	6	9	7	1
2	3	1	7	9	5	4	8	6
3	4	5	8	2	1	7	6	9
9	1	2	6	4	7	3	5	8
8	7	6	3	5	9	1	2	4

RmNum = 35



	5		4	6			1	3
4	6			1	8	2		
1		8	9		3			5
		7	1			5	3	
5	8		2		6	9		1
2		1	7		5		8	
3		5	8	2			6	9
9	1				7	3	5	
	7	6		5	9		2	

# Sudoku Solver - Serial

We use a stack to tack the DFS tree

- Explore the solution space using depth first search

7	5	9	4	6	2	8	1	3
4	6	3	5	1	8	2	9	7
1	2	8	9	7	3	6	4	5
6	9	7	1	8	4	5	3	2
5	8	4	2	3	6	9	7	1
2	3	1	7	9	5	4	8	6
3	4	5	8	2	1	7	6	9
9	1	2	6	4	7	3	5	8
8	7	6	3	5	9			4

Step 0

Initial problem

7	5	9	4	6	2	8	1	3
4	6	3	5	1	8	2	9	7
1	2	8	9	7	3	6	4	5
6	9	7	1	8	4	5	3	2
5	8	4	2	3	6	9	7	1
2	3	1	7	9	5	4	8	6
3	4	5	8	2	1	7	6	9
9	1	2	6	4	7	3	5	8
8	7	6	3	5	9	1		4

Step 1

Try 1 at cell (8, 6)

7	5	9	4	6	2	8	1	3
4	6	3	5	1	8	2	9	7
1	2	8	9	7	3	6	4	5
6	9	7	1	8	4	5	3	2
5	8	4	2	3	6	9	7	1
2	3	1	7	9	5	4	8	6
3	4	5	8	2	1	7	6	9
9	1	2	6	4	7	3	5	8
8	7	6	3	5	9	1	1	4

Step 2

Try 1 at cell (8, 7)

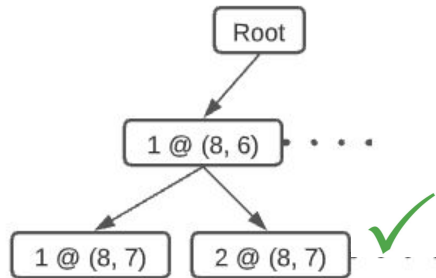
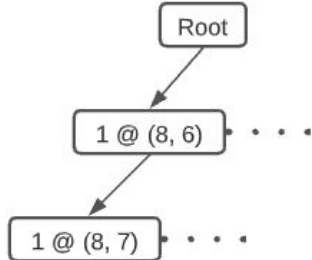
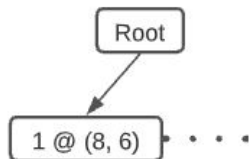
Fail and backtrack

7	5	9	4	6	2	8	1	3
4	6	3	5	1	8	2	9	7
1	2	8	9	7	3	6	4	5
6	9	7	1	8	4	5	3	2
5	8	4	2	3	6	9	7	1
2	3	1	7	9	5	4	8	6
3	4	5	8	2	1	7	6	9
9	1	2	6	4	7	3	5	8
8	7	6	3	5	9	1	2	4

Step 3

Try 2 at cell (8, 7)

Success



# Sudoku Solver - Serial

- Pruning

## Rule 1

Some possibilities are ruled out by its peers

	4	5		6	7
1					
2					
3					

## Rule 2

If all of its peers has a specific number ruled out, the cell itself must contain that number

				3			
			3				
	3						
		3					

# Sudoku Solver - Serial

Original parallel reference needs  
several seconds to solve a sudoku  
problem (TOO SLOW!!!)

## Optimization - Memory Pool

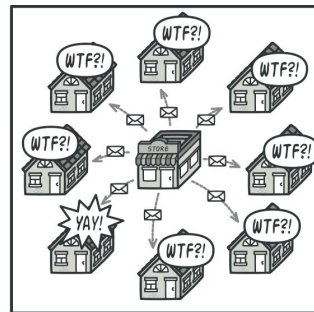
- **Heap allocation: 1000 new operations take around 1 ms, and can cause heap fragmentation**
- **STL containers use new operation by default**
- **Preallocating all the memory makes things faster**

```
MemPool <T> {  
    T* Apply();  
    void Delete(T*);  
}
```

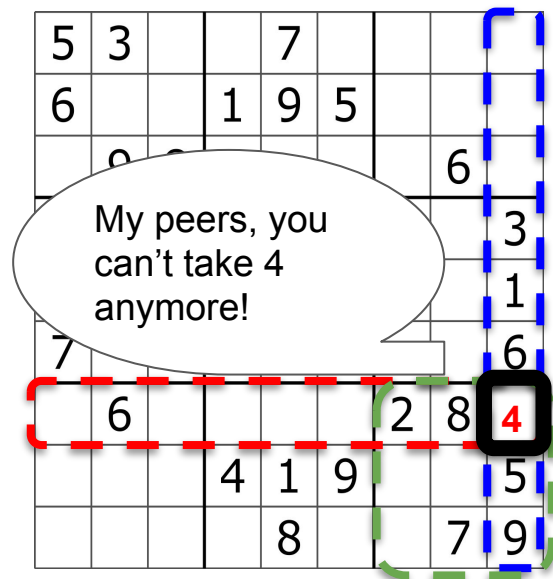


# Sudoku Solver - Serial

## Optimization - Observer Design Pattern



### Prune Rule Propagation



### CellState Class

Both subscriber and publisher

```
CellState {  
    Subscribe(CellState *);  
    Unsubscribe(CellState *);  
    NotifyConstraints();  
}
```

Elements with values filled should  
unsubscribe all it's publishers  
Faster to broadcast a message!

### Modified Subscriber List

O(1) Insertion

O(1) Deletion

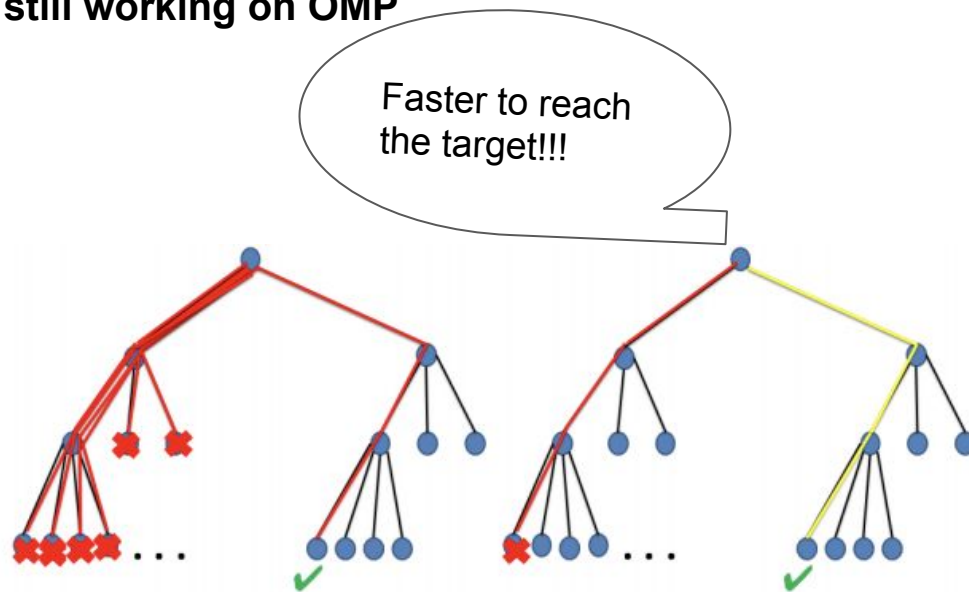
O(1) Lookup

**Serial solver speed:**  
**Simple sudoku: less than 1 ms**  
**Most can be solved within 5 ms**

# Sudoku Solver - Parallel

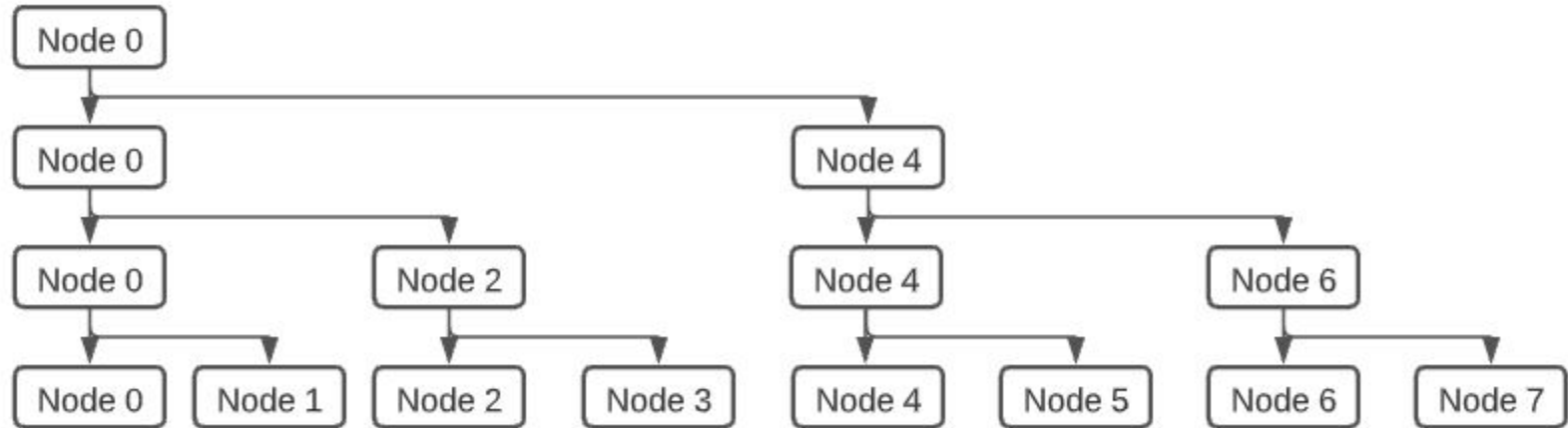
## Parallel searching

Finished MPI, still working on OMP



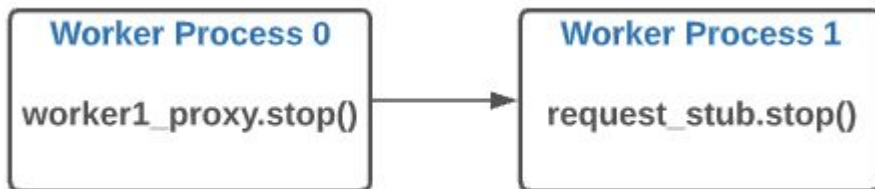
# Sudoku Solver - Parallel

Problem distribution  $O(\log(N))$



# Sudoku Solver - Parallel

A remote procedure call (RPC) style design like Charm++



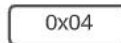
```
WorkerProxy {  
    SetProblem(Solvable*);  
    PushProblem(Solvable*);  
    Stop();  
    AskForWork();  
    Kill();  
    SetConstraints(size_t x_idx, size_t y_idx, int val);  
}
```

## Example of messages

### PUSH\_PROBLEM



### KILL

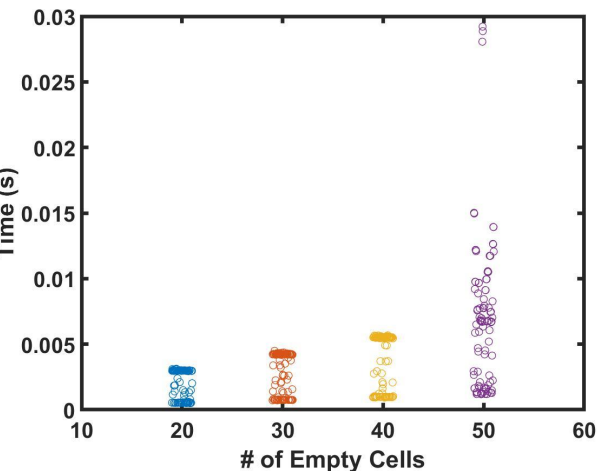


# Performance Tests

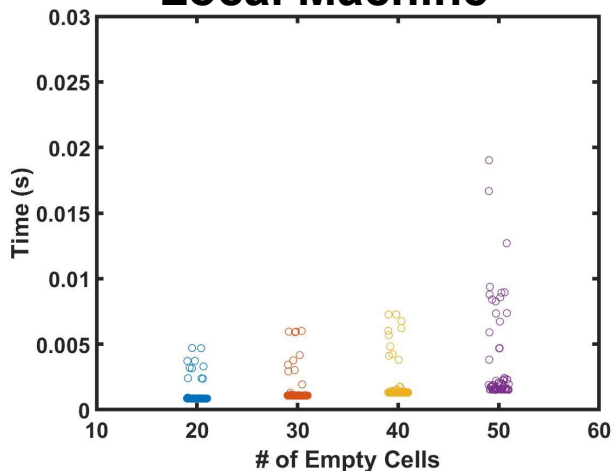
- The following performance tests are conducted on both **local machine** and **Deepphought2**
- Each set-up was experimented with 100 random Sudoku problems
  - # of Process: 1, 2, 4, 8
  - # of Empty Cells: 20, 30, 40, 50

# Performance Results - # of Empty Cells

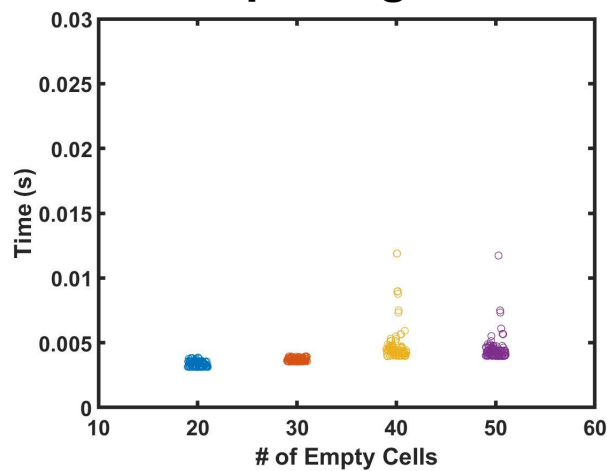
## Serial Solver



## 4 Process on Local Machine



## 4 Process on Deepthought2



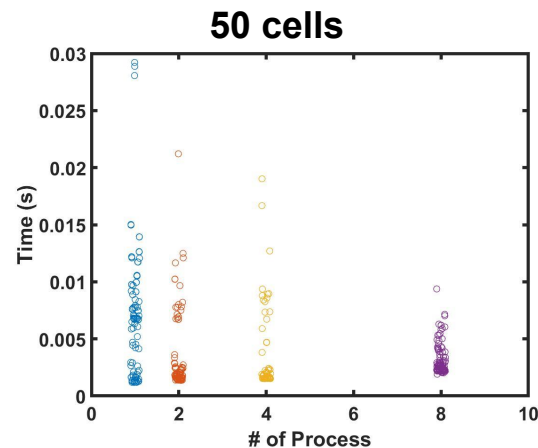
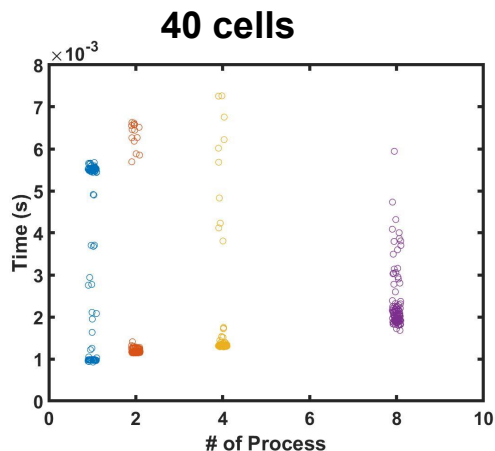
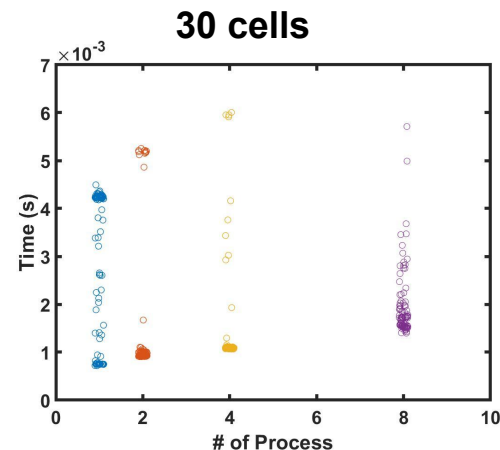
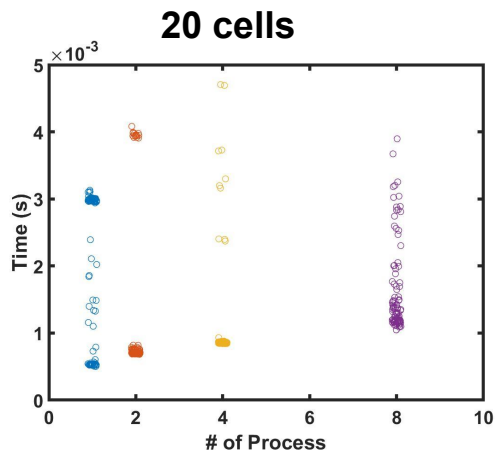
- More empty cells to solve lead to more solving time
- Lower performance variance with MPI parallel solver

# Performance Results - # of Process

- MPI solver performance is more stable

In most cases, MPI model:

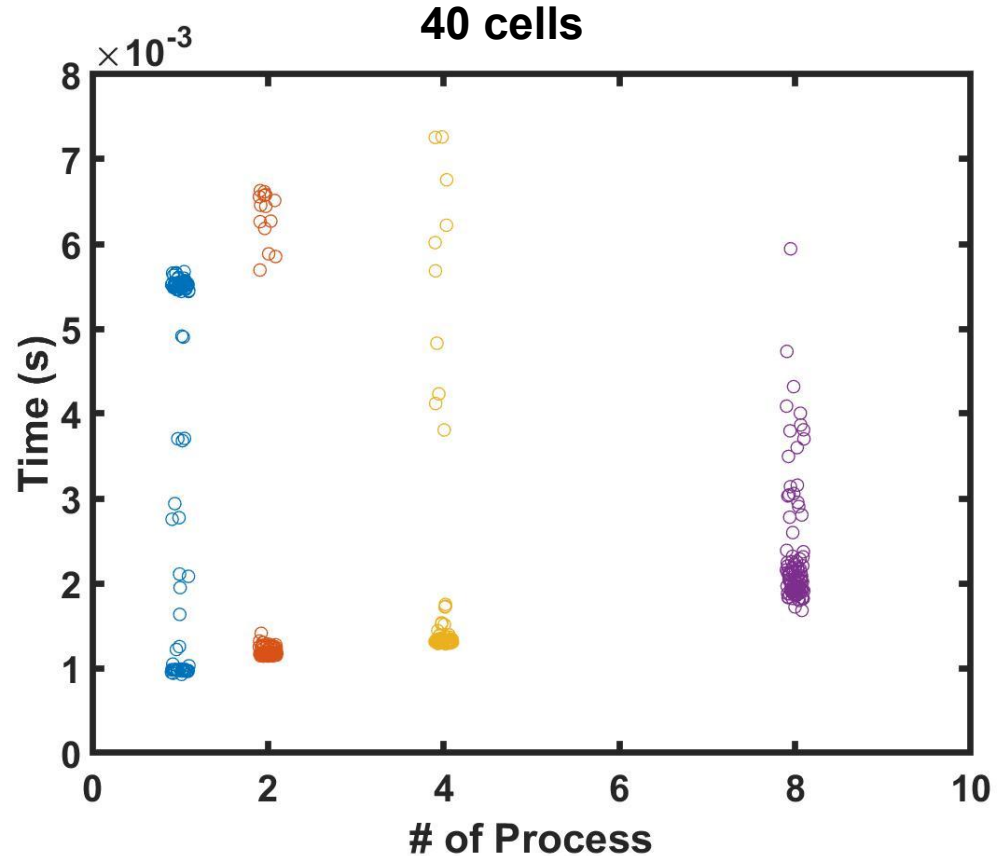
- worse than the best performance of serial solver
- But better than bad performance of serial solver



# Performance Results - # of Process

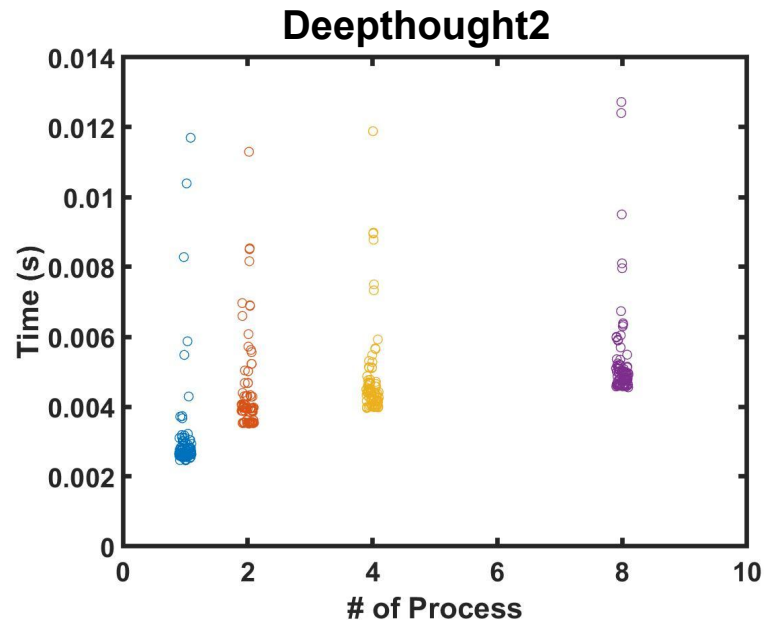
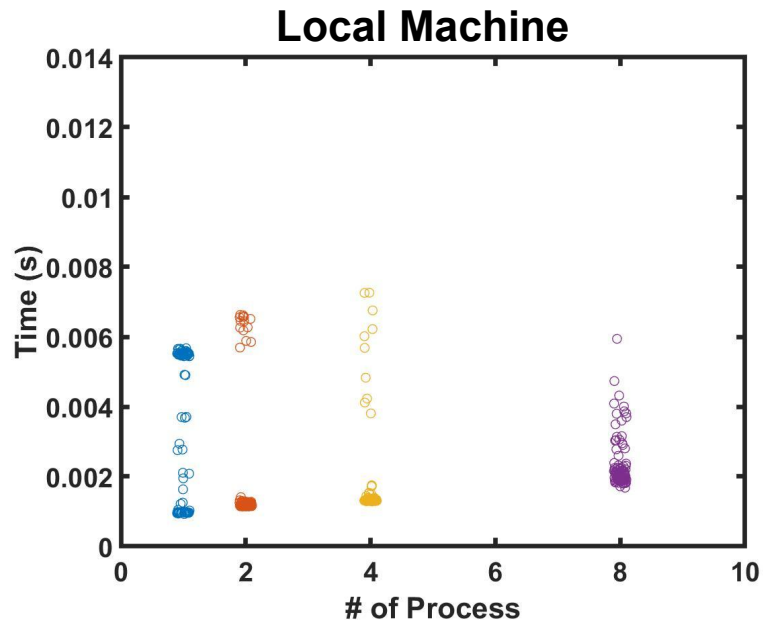
Why?

- Difficulty of Sudoku problems has huge variance
- Some problems are easy to solver by filling the cell with only one possible value one by one
- Parallel solver only works well for problem with lots of possible values





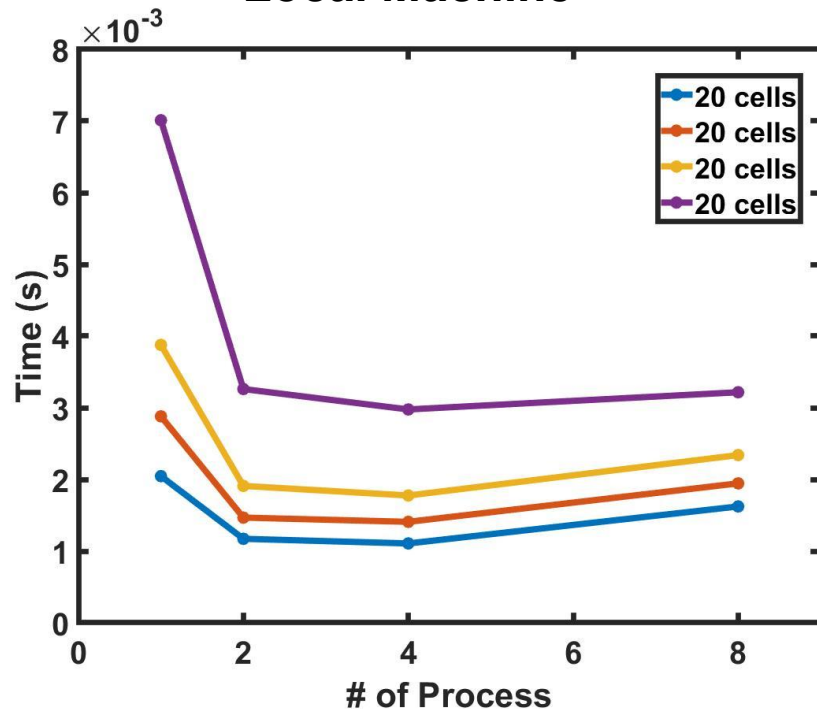
# Performance Results - Local Machine vs. Deepthought 2



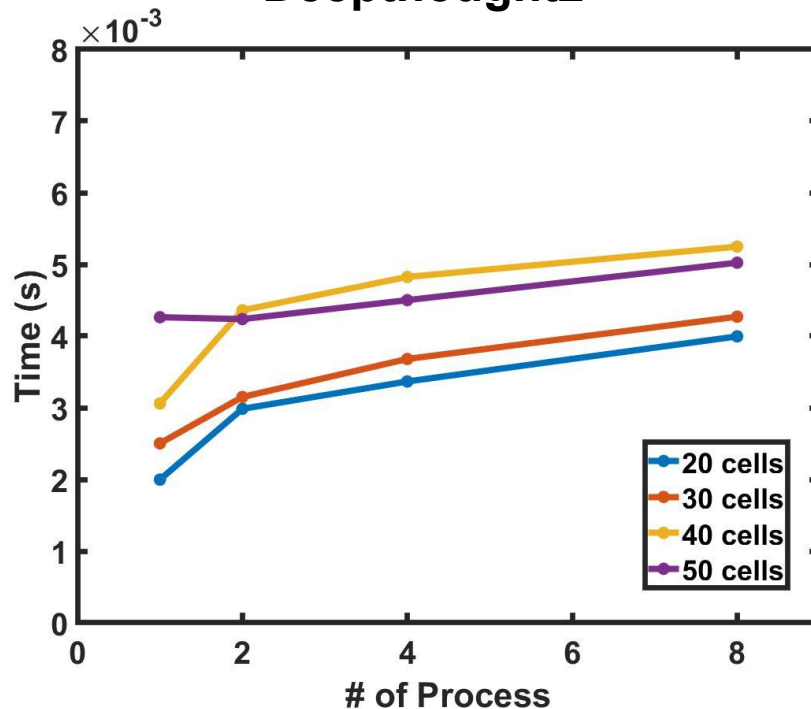
- Local machine has better performance (averaged)
- Deepthought2 has lower performance variance

# Performance Results - Scaling Performance

## Local Machine

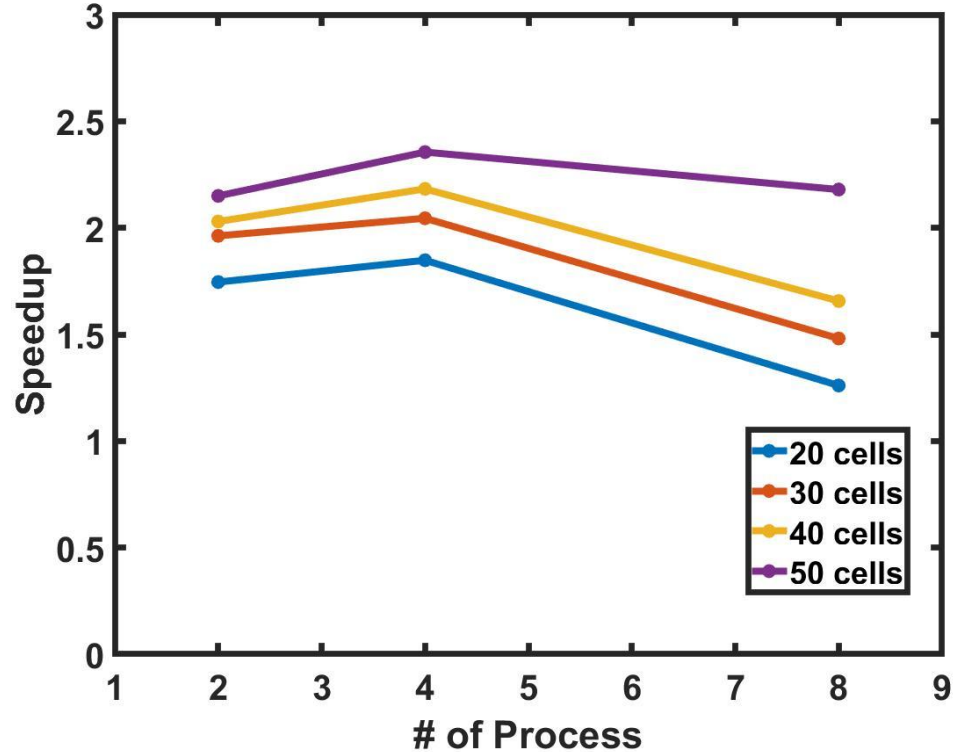


## Deepthought2



# Performance Results - Speedup

- 4-process model works the best
- The more cells in the problem to solve, parallel solver works the better



# Conclusions

- **Serial solver works better on some easy Sudoku problems**
- **Even though more processes may handle trial tasks simultaneously, the communication takes a lot of time**
- **Communication on Deepthought2 costs more time than on our local machine**
  - **We ran a simple MPI\_Send and receive test, local machine took  $4e-6$  s while dp2 took  $7e-5$  s**

# Question Time

