Mathematics Courses

Runqing Zhuo

Course List (click to jump)

- Independent Reading *MATH 499*
- Pseudodifferential Operators *graduate level*
- Riemannian Geometry *graduate level*
- Homology Theory *graduate level*
- Foundations of Representation Theory *graduate level*

Course Details

Mathematical Analysis (I)

Instructor: Prof. Cheng, Chongqing Score: 93.1 *Fall 2022, Nanjing University*

Textbooks: Lecture notes (Chinese) by Prof. Cheng

Main Topics: Real number system (completeness), limits of sequences and series, differentiation (Mean Value Theorems, Taylor's expansion), and the Riemann integral (Fundamental Theorem of Calculus).

Higher Algebra (I)

Instructor: Prof. Ji, Qingzhong Score: 94.5 *Fall 2022, Nanjing University*

Textbooks: Liu, *Higher Algebra* (Chinese)

Main Topics: Polynomial ring theory, determinants, systems of linear equations, and matrix theory.

Analytic Geometry

Instructor: Prof. Zhang, Gaofei Score: 97.5 *Fall 2022, Nanjing University*

Introduction to Number Theory

Instructor: Prof. Qin, Hourong Score: 90 *Fall 2022, Nanjing University*

Mathematical Analysis (II)

Instructor: Prof. Cheng, Chongqing Score: 93 *Spring 2023, Nanjing University*

Textbooks: Lecture notes (Chinese) by Prof. Cheng

Main Topics: Uniform convergence, multivariable differentiation (Total derivative, Jacobian, Inverse/Implicit Function Theorems), multiple integrals, line and surface integrals (Green's, Gauss's, and Stokes' Theorems), and introduction to Fourier Analysis and Distributions.

Higher Algebra

Instructor: Prof. Ji, Qingzhong Score: 93 *Spring 2023, Nanjing University*

Textbooks: Liu, *Higher Algebra* (Chinese)

Main Topics: Vector spaces, linear maps, canonical forms (Jordan), inner product spaces (Euclidean/Unitary), quadratic forms, and symplectic spaces.

Mathematical Analysis (III)

Instructor: Prof. Cheng, Chongqing Score: 94 *Fall 2023, Nanjing University*

Textbooks: Stein & Shakarchi, *Fourier analysis*; Arnold, *Mathematical methods of classical mechanics*

Main Topics: Fourier series, L^2 theory, and the Fourier Transform on $\mathbb{R}^n$ (Plancherel's Theorem, Uncertainty Principle). Tempered Distributions and their applications to Partial Differential Equations (heat and wave equations). Introduction to mathematical methods of classical mechanics (Legendre Transform, Lagrangian, and Hamiltonian mechanics).

Ordinary Differential Equations

Instructor: Prof. Cheng, Wei Score: 86 *Fall 2023, Nanjing University*

Textbooks: Lecture notes (Chinese) by Prof. Cheng; Walter, *Ordinary Differential Equations*

Main Topics: Existence and uniqueness theory (Picard-Lindelöf, Peano), continuous dependence, stability theory, Gronwall's inequality, and the Method of Characteristics for first-order PDEs.

Abstract Algebra

Instructor: Prof. Chen, Ke Score: 91.6 *Fall 2023, Nanjing University*

Textbooks: Lecture notes (Chinese) by Prof. Chen

Main Topics: Group theory (Sylow Theorems), ring theory (ideals, PIDs, quotients, polynomial rings), field extensions, splitting fields, and Galois Theory.

Real Analysis

Instructor: Prof. Lv, Yong Score: 90 *Fall 2023, Nanjing University*

Textbooks: Lecture notes (Chinese) by Prof. Cheng; I read Stein & Shakarchi, *Real analysis* (whole book) myself

Main Topics: Lebesgue Measure Theory (construction, measurable functions, integration), convergence theorems (MCT, DCT), differentiation theory (BV/AC functions, Lebesgue Differentiation Theorem), and L^p spaces.

Functions of one Complex Variable

Instructor: Prof. Liao, Liangwen Score: 90 *Spring 2024, Nanjing University*

Textbooks: I read Stein & Shakarchi, *Complex analysis* (Ch.1-5, 8) myself

Main Topics: Holomorphic and harmonic functions (Cauchy's Theorem/Formula, singularities, Maximum Modulus Principle), meromorphic functions, the Residue Theorem, and conformal mappings (Schwarz Lemma, Riemann Mapping Theorem).

Foundations of Probability Theory

Instructor: Prof. Dai, Xiongping Score: 94 *Spring 2024, Nanjing University*

Textbooks: Li, *Foundations of Probability Theory* (Chinese)

Main Topics: Axiomatic and geometric probability, conditional probability, and statistical independence (Bayes', Bernoulli trials). Introduces random variables, vectors, and their distributions. Focuses on numerical characteristics: expectation, variance, and characteristic functions. Concludes with convergence types, the strong and weak Laws of Large Numbers, and the Central Limit Theorem.

Numerical analysis

Instructor: Prof. Deng, Weibing Score: 90.5 *Spring 2024, Nanjing University*

Textbooks: Lin, *Numerical Computation Methods* (Chinese)

Main Topics: Error analysis and propagation theory. Function approximation (best uniform/square approximation, orthogonal polynomials, FFT) and interpolation methods (Lagrange, Newton, Hermite, Splines). Numerical integration techniques (Newton-Cotes, Romberg, Gauss quadrature). Numerical solutions for ODEs: single-step and linear multi-step methods, including analysis of consistency, convergence, and stability.

Selected Topics in Analysis

Instructor: Prof. Qiu, Hua Score: 95 *Spring 2024, Nanjing University*

Textbooks: Heinonen, *Lectures on Analysis on Metric Space* (Ch.1-4); Falconer, *Fractal Geometry*

Main Topics: Analysis on Metric Spaces: Hausdorff measure and dimension, covering lemmas, singular integrals, geometric measure theory, the Hardy-Littlewood maximal operator, and Poincaré inequalities.

Theoretical Mechanics

Instructor: Prof. Fan, Renhao Score: 100 *Spring 2024, Nanjing University*

Textbooks: I read Arnold, *Mathematical methods of classical mechanics* myself

Main Topics: Foundational principles of classical mechanics: Lagrangian mechanics (Euler-Lagrange equations) and Hamiltonian mechanics.

Mathematical Methods for Physics

Instructor: Prof. Wu, Shengjun Score: 93 *Spring 2024, Nanjing University*

Textbooks: Liang, *Mathematical Methods for Physics* (Chinese)

Main Topics: Application-focused mathematical techniques: Complex Analysis applications, variational methods, Green's functions, integral transforms, theory of linear ODEs, and series solutions for PDEs (Fourier series).

Partial Differential Equations

Instructor: Prof. Ruan, Zhuoping Score: 93 *Fall 2024, Nanjing University*

Textbooks: Jiang, *Lecture Notes on Equations of Mathematical Physics* (Chinese)

Main Topics: Derivation of PDEs from conservation laws (momentum, energy, mass) and variational principles. Solution methods for Wave, Heat, and Potential equations: characteristics, Fourier transform, separation of variables, Green's functions, and maximum principles. Includes well-posedness, energy estimates, and an introduction to linear PDE classification.

Topology

Instructor: Prof. Shi, Weixue Score: 93 *Fall 2024, Nanjing University*

Textbooks: You, *Lectures on Basic Topology* (Chinese)

Main Topics: Point-Set Topology (axioms, compactness, connectedness, Urysohn Lemma, Tietze Extension Theorem), metric spaces, and introduction to Algebraic Topology (homotopy, the fundamental group).

Galois Theory

Instructor: Prof. Qin, Hourong Score: 97 *Fall 2024, Nanjing University*

Textbooks: N/A

Main Topics: Advanced seminar covering the fundamental concepts of Galois Theory: field extensions, the Galois group, and the Fundamental Theorem. Explores applications like solvability by radicals and classical construction problems. Seminar topics include general equations, cyclotomic and Kummer extensions, infinite Galois theory, and group cohomology.

Analysis

Instructor: Prof. Sun, Yongzhong Score: 88 *Fall 2024, Nanjing University*

Textbooks: Lecture notes (Chinese) by Prof. Sun; Folland, *Real Analysis* (Ch.1-7)

Main Topics: Measure Theory (Hahn-Kolmogorov, Fubini-Tonelli, Radon-Nikodym), foundations of Functional Analysis (Bounded Linear Operators, Dual Spaces), Riesz Representation Theorem, weak/weak* topologies, and the Banach-Alaoglu Theorem.

Harmonic Analysis

Instructor: Prof. Ruan, Zhuoping Score: 92 *Fall 2024, Nanjing University*

Textbooks: Grafakos, *GTM249* (Ch.1,2,5,6) & *GTM 250* (Ch.3)

Main Topics: Real-variable Harmonic Analysis: Hardy-Littlewood maximal operator, Calderón-Zygmund theory, boundedness of Singular Integral Operators (SIOs), Littlewood-Paley Theory, and H^1-BMO theory.

Functional Analysis

Instructor: Prof. Qin, Lizhen Score: 97 *Spring 2025, Nanjing University*

Textbooks: Lecture notes (Chinese) by Prof. Qin

Main Topics: Covers metric spaces (completeness, compactness, fixed point theorems), and core Banach and Hilbert space theory (bases, orthogonal systems). Focuses on bounded linear operators: fundamental theorems (Open Mapping, Closed Graph, Uniform Boundedness), dual spaces, adjoint operators, and spectral theory, including self-adjoint and compact operators.

Differential Geometry

Instructor: Prof. Chen, Xuezhang Score: 92 *Spring 2025, Nanjing University*

Textbooks: Do Carmo, *Differential Geometry of Curves and Surfaces*

Main Topics: Classical Differential Geometry: Frenet-Serret apparatus (curvature/torsion), surface theory via the first/second fundamental forms, Gauss map, and Gaussian/Mean curvature for surfaces in $\mathbb{R}^3$.

Analysis (II)

Instructor: Prof. Sun, Yongzhong Score: 90 *Spring 2025, Nanjing University*

Textbooks: Xia, *A Second Course in Functional Analysis* (Chinese)

Main Topics: Analysis in Banach Spaces: Geometric foundations (Hahn-Banach Theorem, separation of convex sets) and the calculus of vector-valued functions (Bochner integral).

Manifolds and Geometry

Instructor: Prof. Xu, Yiyan Score: 93 *Spring 2025, Nanjing University*

Textbooks: Mei, *Manifold and Geometry* (Chinese); I read Lee, *Introduction to Smooth Manifolds* (Ch.1-10) myself

Main Topics: Differentiable Manifolds: charts, atlases, smooth maps, partitions of unity, tangent and cotangent bundles. Vector fields, flows, Lie brackets, differentials, and the Regular Value Theorem (submanifolds).

From calculus to cohomology

Instructor: Prof. Shi, Yalong Score: 100 *Spring 2025, Nanjing University*

Textbooks: Lecture notes (Chinese) by Prof. Shi; Madsen, *From calculus to cohomology*; Bott & Tu, *Differential Forms in Algebraic Topology* (Ch. 1-5)

Main Topics: Differential forms, exterior algebra, chain complexes, and de Rham cohomology. Differentiable manifolds, differential forms on manifolds, integration, and computational methods for de Rham cohomology. Concepts of mapping degree and vector field index, leading to the Poincaré-Hopf and Poincaré Duality theorems. Advanced topics include vector bundles, connections, curvature, and characteristic classes (Euler and Chern classes).

Sino-Italian mathematical course in analysis

Instructor: Prof. Sorrentino, Alfonso; Prof. Cannarsa, Piermarco Score: 90Spring 2025, Nanjing University

Course 1: Geometric Hamiltonian Dynamics
Instructor: Prof. Sorrentino, Alfonso
Textbooks: Moser & Zehnder, *Notes on Dynamical Systems*
Main Topics: Symplectic manifolds, Hamiltonian flows, action-angle variables, Poisson brackets, canonical transformations, and the Arnold-Liouville theorem.

Course 2: Evolution Equations
Instructor: Prof. Cannarsa, Piermarco
Textbooks: Lecture notes by Prof. Cannarsa
Main Topics: Generation of strongly continuous semigroups of bounded linear operators, solvability of inhomogeneous Cauchy problems, and controllability properties (approximate, null, and exact) of abstract linear control systems.

MATH 656, Introduction to Partial Differential Equations

Instructor: Prof. Sijue Wu Score: Pending *Fall 2025, University of Michigan*

Textbooks: F. John (Ch. 1-2, 3.6, 4-5, 7); Evans (Ch. 1-3, 5)

Main Topics: Nonlinear first-order equations; linear elliptic, hyperbolic, and parabolic equations. Method of characteristics, energy methods, maximum principles, Fourier transform, distributions, and Sobolev spaces.

MATH 399, Independent Reading

Instructor: Prof. Tian Jing Score: Pending *Fall 2025, University of Michigan*

Textbooks: Journé, *Calderon-Zygmund Operators, Pseudo-Differential Operators and the Cauchy Integral of Calderon* (Ch. 0-3)

Main Topics: Preliminaries on $L^p_B(\mathbb{R}^n, d\mu)$, the Hardy-Littlewood maximal operator, A_p weights, and BMO$(\mathbb{R}^n, dx)$. (Weekly presentations).

MATH 499, Independent Reading

Instructor: Prof. Xiao Ma Score: Pending *Fall 2025, University of Michigan*

Textbooks: Evans (Ch. 6); Reed & Simon, *Methods of Modern Mathematical Physics I*; Chen, *Pseudo-differential Operators*; Tao's blog

Main Topics: Elliptic PDEs, functional analysis, foundations of harmonic analysis, pseudo-differential operators, Fourier restriction theory, and modern harmonic analysis.

Pseudodifferential Operators

Instructor: Prof. Albert Ai Score: N/A *Spring 2026, BICMR*

Textbooks: Bahouri et al., *Fourier Analysis and Nonlinear PDEs*; Taylor, *Pseudodifferential Operators and Nonlinear PDE*; Métivier, *Para-differential calculus*

Main Topics: Pseudodifferential and paradifferential operators with applications to nonlinear PDEs. Sobolev and Besov spaces, Littlewood-Paley decomposition, paraproducts, product estimates, symbol classes, symbolic calculus, and nonlinear hyperbolic systems. (Prerequisites: Real and Fourier analysis.)

Riemannian Geometry

Instructor: Prof. Ge, Huabin *Score: N/A* *Spring 2026, BICMR*

Textbooks: Do Carmo, *Riemannian geometry*; Petersen, *Riemannian geometry*; Wu et al., *Introduction to Riemannian Geometry* (Chinese)

Main Topics: Fundamentals of Riemannian Geometry: metrics, connections, covariant derivatives, geodesics, curvature, Jacobi fields, and variation formulas. Comparison theorems, the Morse Index Theorem, and the Sphere Theorem. Introduction to advanced topics: Gromov-Hausdorff convergence, Ricci flow, and the Calabi-Yau Theorem.

Homology Theory

Instructor: Prof. Xie, Yi *Score: N/A* *Spring 2026, BICMR*

Textbooks: Jiang, *Homology Theory* (Chinese); Bott & Tu, *Differential forms in algebraic topology*; Hatcher, *Algebraic topology*; Milnor, *Morse theory*

Main Topics: Homology and Cohomology theory: cup products, cap products, Poincaré Duality, and an introduction to Morse Theory. (Prerequisites: Basic topology, abstract algebra.)

Foundations of Representation Theory

Instructor: Prof. Yu, Jun *Score: N/A* *Spring 2026, BICMR*

Textbooks: Brion, *Introduction to actions of algebraic groups*; Dolgachev, *Lectures on invariant theory*; Howe, *Perspectives on invariant theory*; Knapp, *Lie groups*

Main Topics: Fundamental topics in Representation Theory: Lie algebras, Lie groups, algebraic invariant theory, and geometric invariant theory. (Prerequisites: Abstract algebra, commutative algebra, differentiable manifolds.)