Chapter 13 含磁耦合的电路

13.1 概述

13.2 耦合电感
Coupled inductors

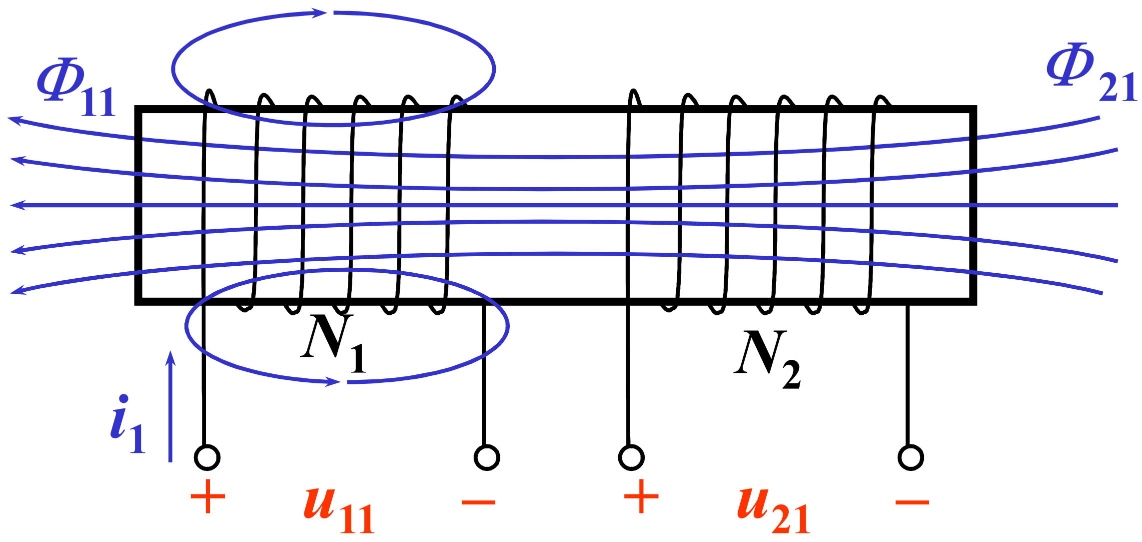
13.3 含耦合电感电路的分析
Analysis of coupled circuits

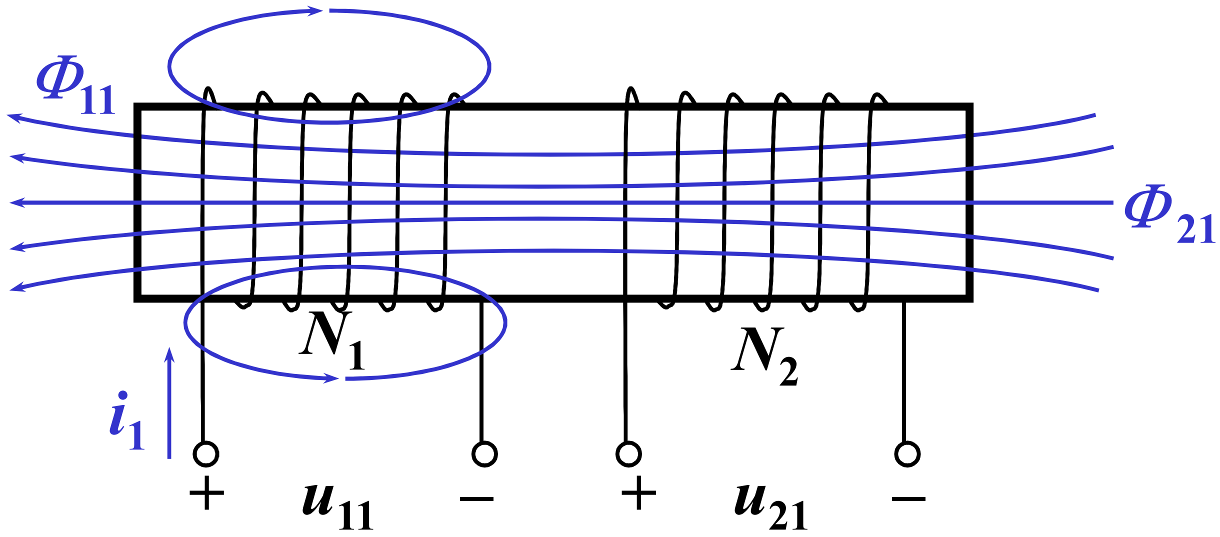
13.4 变压器
Transformers

13.2　耦合电感

1 自感、 互感现象和耦合电感元件

- 当线圈1中通入电流i_1时，在线圈1中产生磁通(*magnetic flux*)，
- 同时，有部分磁通穿过临近线圈2，
- 当i_1为时变电流时，磁通也将随时间变化，从而在线圈两端产生感应电压。u_{11}称为自感电压，u_{21}称为互感电压。





根据电磁感应定律和楞次定律（Ψ：磁链，$\psi=N\,\Phi$)

$$u_{11}=\frac{\mathrm{d}\Psi_{11}}{\mathrm{d}t} \qquad \boxed{u_{11}=L_1\frac{\mathrm{d}i_1}{\mathrm{d}t}}$$

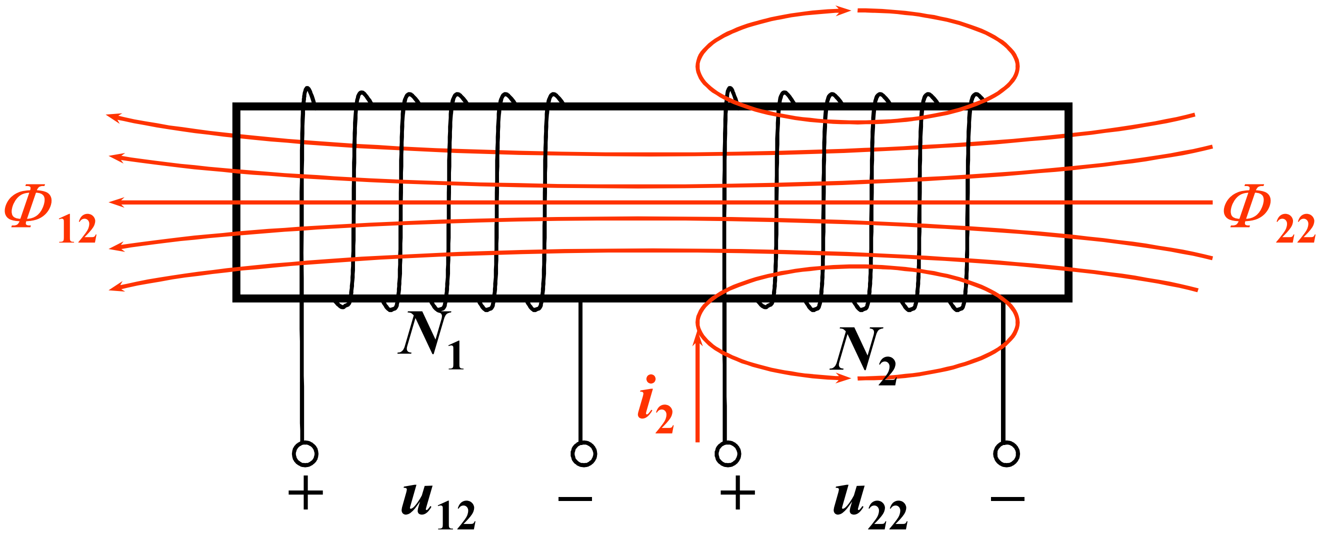
$$u_{21}=\frac{\mathrm{d}\Psi_{21}}{\mathrm{d}t} \qquad \boxed{u_{21}=M_{21}\frac{\mathrm{d}i_1}{\mathrm{d}t}}$$

当线圈周围的磁介质为线性时，Ψ_{11}、Ψ_{22}与i_1成线性关系，即：

$L_1=\dfrac{\psi_{11}}{i_1}$　L为线圈的电感　$M_{21}=\dfrac{\psi_{21}}{i_1}$　M_{21}为线圈1对2的互感

L、M_{21}与线圈的几何尺寸、匝数 、和周围的介质磁导率有关。
M_{21}还与两个线圈的相互位置有关。

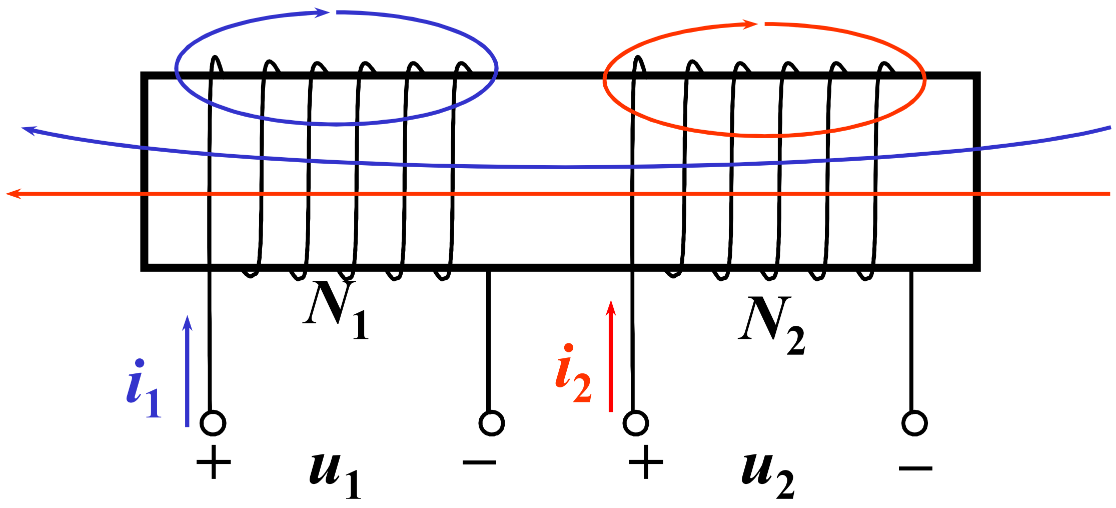
同理，当线圈2中通电流i_2时会产生磁通Φ_{22}，Φ_{12}。i_2为时变时，线圈2和线圈1两端分别产生感应电压u_{22}，u_{12}。



$$u_{12}=\frac{\mathrm{d}\Psi_{12}}{\mathrm{d}t}=M_{12}\frac{\mathrm{d}i_2}{\mathrm{d}t}\qquad(\Psi_{12}=N_1\Phi_{12}=M_{12}i_2)$$

$$u_{22}=\frac{\mathrm{d}\Psi_{22}}{\mathrm{d}t}=L_2\frac{\mathrm{d}i_2}{\mathrm{d}t}\qquad(\Psi_{22}=N_2\Phi_{22}=L_2i_2)$$

通过分析线圈储能（参见附录）可以证明：$M_{12}=M_{21}=M$。



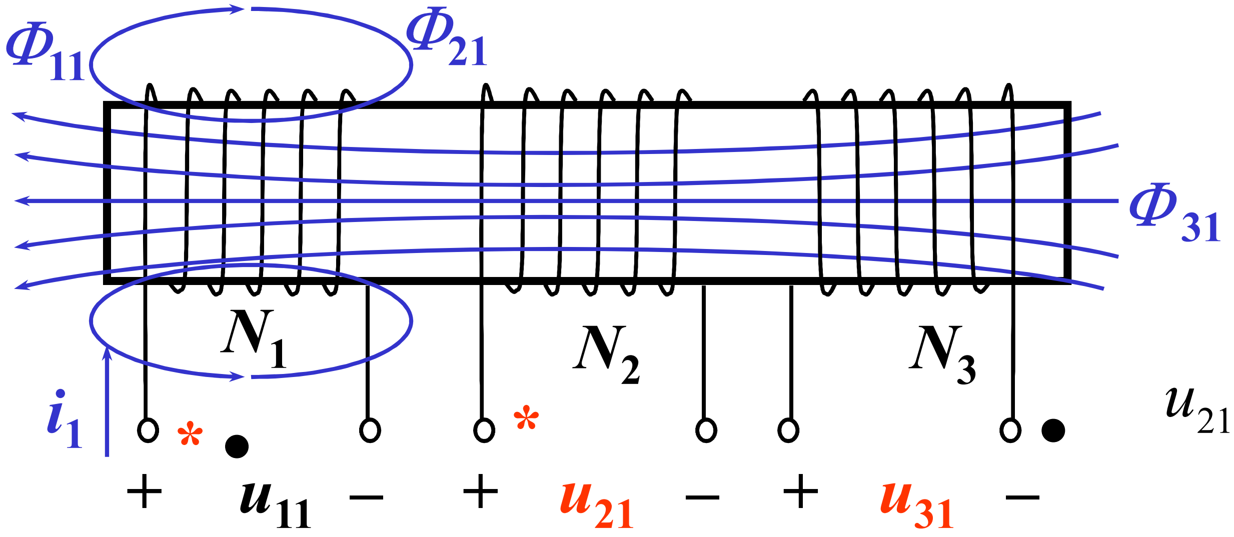
当两个线圈同时通以电流时，每个线圈两端的电压均包含自感电压和互感电压：

$$u_1 = L_1 \frac{\mathrm{d}i_1}{\mathrm{d}t} + M \frac{\mathrm{d}i_2}{\mathrm{d}t}$$

$$u_2 = M \frac{\mathrm{d}i_1}{\mathrm{d}t} + L_2 \frac{\mathrm{d}i_2}{\mathrm{d}t}$$

2 同名端

对互感电压，因产生该电压的的电流在另一线圈上，因此，要确定其符号，就必须知道两个线圈的绕向。这在电路分析中显得很不方便。



$$u_{21}=M_{21}\frac{\mathrm{d}i_1}{\mathrm{d}t}$$

$$u_{31}=-M_{31}\frac{\mathrm{d}i_1}{\mathrm{d}t}$$

引入同名端可以解决这个问题。

同名端：当两个电流分别从两个线圈的对应端子流入，其所产生的磁场相互加强时，则这两个对应端子称为同名端。

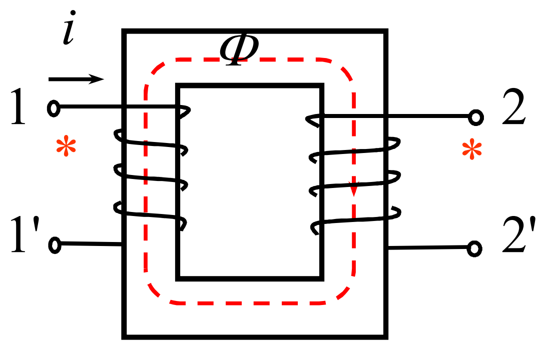
2 同名端

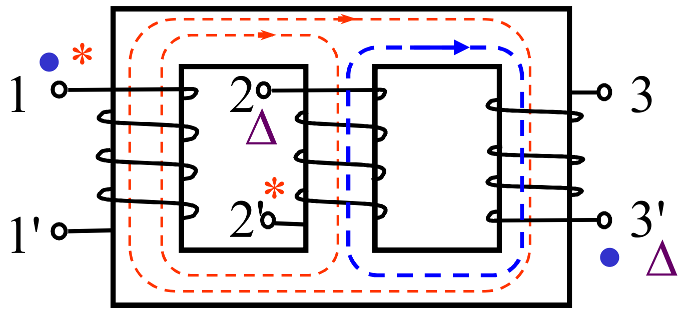
同名端表明了线圈的相互绕法关系。

确定同名端的方法:

(1) 当两个线圈中电流同时由同名端流入(或流出)时，两个电流产生的磁场相互增强。

【例】.确定图示电路的同名端



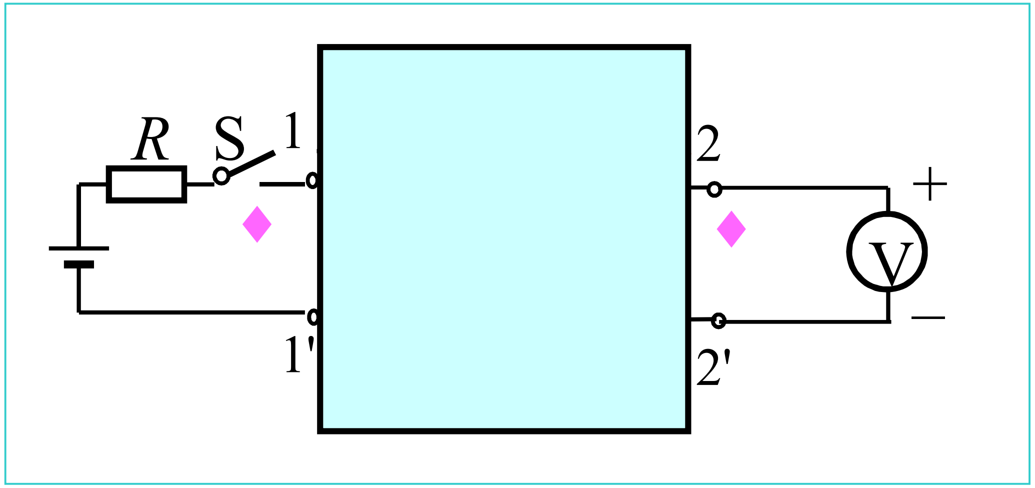


注意：线圈的同名端必须两两确定。

2 同名端

(2) 当随时间增大的时变电流从一线圈的一端流入时，将会引起另一线圈相应同名端的电位升高。

同名端的实验测定:



当闭合开关S时，i增加，

$$\frac{di}{dt} > 0, \qquad u_{22'} = M\frac{di}{dt} > 0$$ 电压表正偏。

当两组线圈装在黑盒里，只引出四个端线组，要确定其同名端，就可以利用上面的结论来加以判断。

3. 耦合系数 (coupling coefficient)k：

k 表示两个线圈磁耦合的紧密程度。 $k \overset{\text{def}}{=} \dfrac{M}{\sqrt{L_1 L_2}} \quad (0 \le k \le 1)$

设：全耦合: $\Phi_{s1} = \Phi_{s2} = 0$ 即 $\Phi_{11} = \Phi_{21}$， $\Phi_{22} = \Phi_{12}$

$$k = \frac{M}{\sqrt{L_1 L_2}} = \sqrt{\frac{M_{12} M_{21}}{L_1 L_2}} = \sqrt{\frac{\dfrac{\psi_{12}}{i_2} \cdot \dfrac{\psi_{21}}{i_1}}{\dfrac{\psi_{11}}{i_1} \cdot \dfrac{\psi_{22}}{i_2}}} = \sqrt{\frac{N_1 \phi_{12} \cdot N_2 \phi_{21}}{N_1 \phi_{11} \cdot N_2 \phi_{22}}} = \sqrt{\frac{\phi_{12} \cdot \phi_{21}}{\phi_{11} \cdot \phi_{22}}} \le 1$$

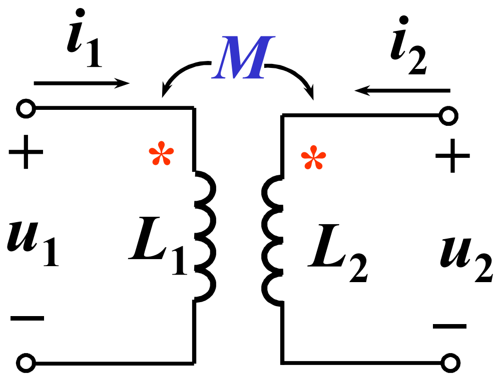
可以证明，$k \le 1$。全耦合时，$k=1$.

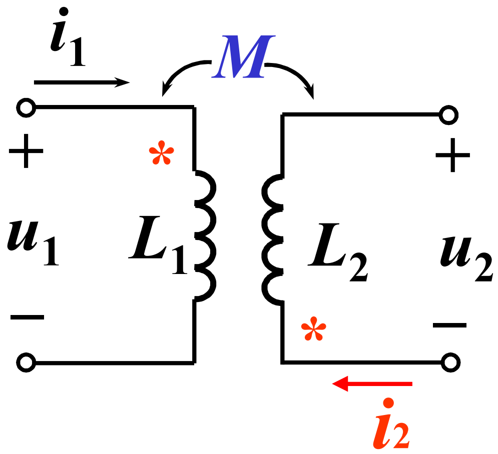
M能否表示两个线圈磁耦合的紧密程度？M 的上下限？

$$0 \le M \le \sqrt{L_1 L_2}$$

4 耦合电感元件的u-i关系

有了同名端，以后表示两个线圈相互作用，就不再考虑实际绕向，而只画出同名端及参考方向即可。(参考前图，标出同名端得到下面结论)。



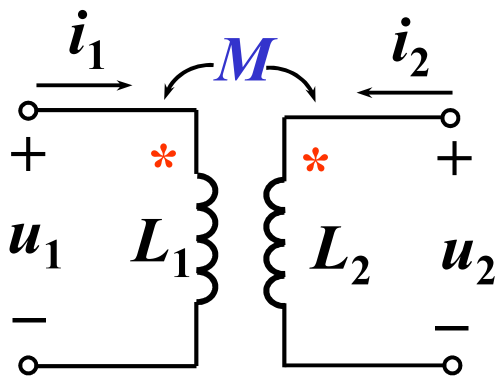


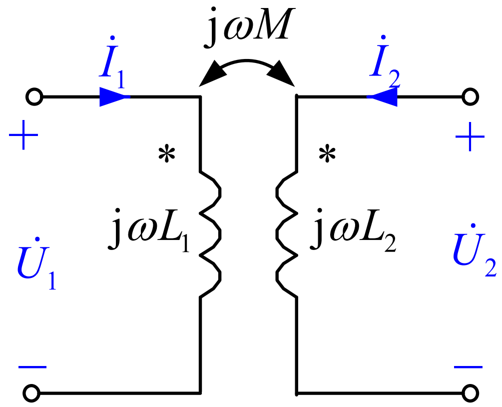
时域形式:

$$u_1 = L_1 \frac{\mathrm{d}i_1}{\mathrm{d}t} + M \frac{\mathrm{d}i_2}{\mathrm{d}t} \qquad u_1 = L_1 \frac{\mathrm{d}i_1}{\mathrm{d}t} \mp M \frac{\mathrm{d}i_2}{\mathrm{d}t}$$

$$u_2 = M \frac{\mathrm{d}i_1}{\mathrm{d}t} + L_2 \frac{\mathrm{d}i_2}{\mathrm{d}t} \qquad u_2 = -M \frac{\mathrm{d}i_1}{\mathrm{d}t} \pm L_2 \frac{\mathrm{d}i_2}{\mathrm{d}t}$$

4 耦合电感元件的u-i关系





时域形式：

$$u_1 = L_1\frac{\mathrm{d}i_1}{\mathrm{d}t} + M\frac{\mathrm{d}i_2}{\mathrm{d}t}$$

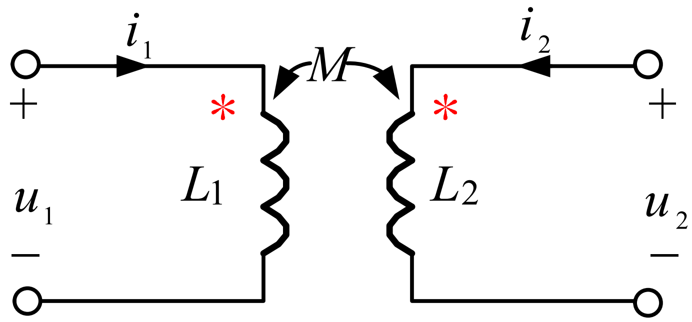
$$u_2 = M\frac{\mathrm{d}i_1}{\mathrm{d}t} + L_2\frac{\mathrm{d}i_2}{\mathrm{d}t}$$

相量形式：

$$\dot{U}_1 = \mathrm{j}\omega L_1\dot{I}_1 + \mathrm{j}\omega M\dot{I}_2$$

$$\dot{U}_2 = \mathrm{j}\omega M\dot{I}_1 + \mathrm{j}\omega L_2\dot{I}_2$$

5 耦合电感的储能



$$w=\int_{-\infty}^{t} p(t)\,\mathrm{d}\,t$$

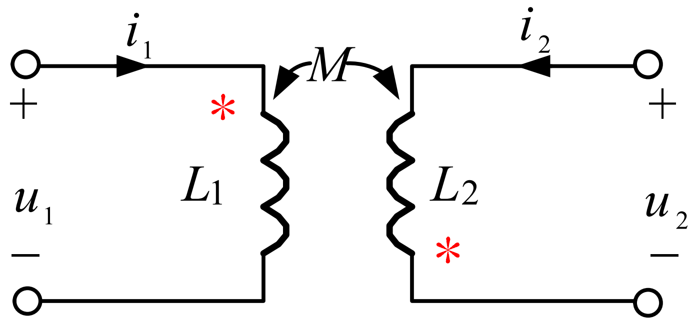
$$=\int_{-\infty}^{t}(u_1 i_1+u_2 i_2)\;\mathrm{d}\,t$$

$$=\int_{-\infty}^{t}[(L_1\frac{\mathrm{d}\,i_1}{\mathrm{d}\,t}+M\frac{\mathrm{d}\,i_2}{\mathrm{d}\,t})i_1+(+M\frac{\mathrm{d}\,i_1}{\mathrm{d}\,t}+L_2\frac{\mathrm{d}\,i_2}{\mathrm{d}\,t})i_2]\mathrm{d}\,t$$

$$=\int_{-\infty}^{i_1}L_1 i_1\mathrm{d}\,i_1+\int_{-\infty}^{i_2}L_2 i_2\mathrm{d}\,i_2+\int_{-\infty}^{i_1、i_2}+M(i_2\mathrm{d}\,i_1+i_1\mathrm{d}\,i_2)$$

$$=\frac{1}{2}L_1 i_1^2+\frac{1}{2}L_2 i_2^2+M i_1 i_2$$

5 耦合电感的储能



$$w=\int_{-\infty}^{t} p(t)\mathrm{d}\,t$$

$$=\int_{-\infty}^{t}(u_1 i_1+u_2 i_2)\ \mathrm{d}\,t$$

$$=\int_{-\infty}^{t}[(L_1\frac{\mathrm{d}\,i_1}{\mathrm{d}\,t}\pm M\frac{\mathrm{d}\,i_2}{\mathrm{d}\,t})i_1+(\pm M\frac{\mathrm{d}\,i_1}{\mathrm{d}\,t}+L_2\frac{\mathrm{d}\,i_2}{\mathrm{d}\,t})i_2]\mathrm{d}\,t$$

$$=\int_{-\infty}^{i_1} L_1 i_1 \mathrm{d}\,i_1+\int_{-\infty}^{i_2} L_2 i_2 \mathrm{d}\,i_2+\int_{-\infty}^{i_1、i_2} \pm M(i_2\mathrm{d}\,i_1+i_1\mathrm{d}\,i_2)$$

$$=\frac{1}{2}L_1 i_1^2+\frac{1}{2}L_2 i_2^2\pm M i_1 i_2$$

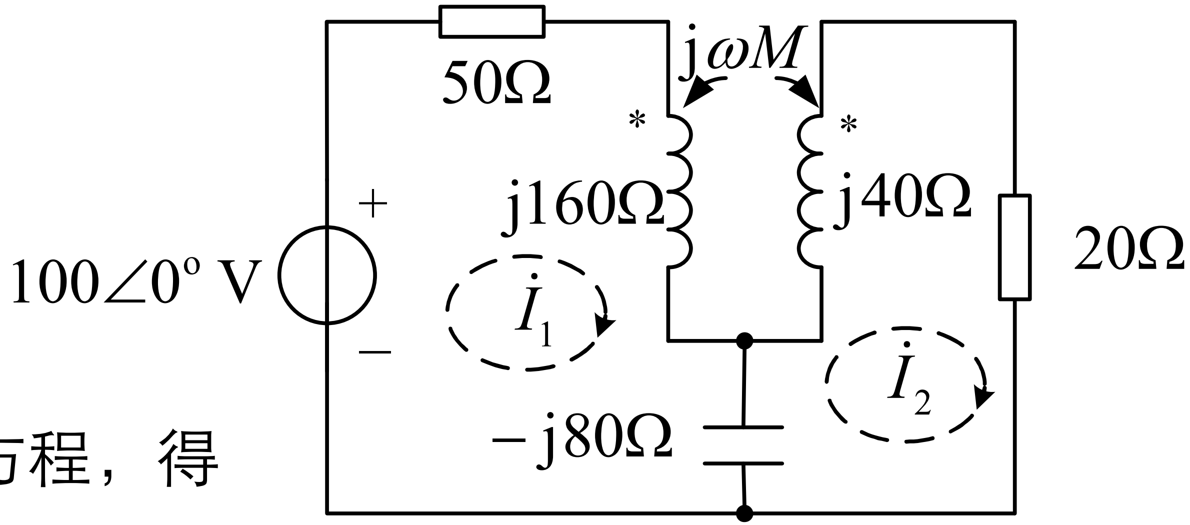
$\frac{1}{2}L_1 i_1^2$、$\frac{1}{2}L_2 i_2^2$ 线圈的自感储能

$\pm M i_1 i_2$ 线圈的互感储能

13.3　含有耦合电感元件的电路分析

13.3.1 网孔分析法的应用：耦合电感的VCR应用

【例1】：图示电路中，耦合电感的耦合系数K=0.5，求电路消耗的总功率。



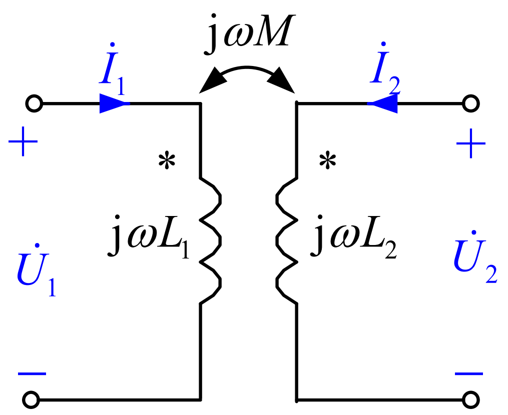
解:列写网孔电流方程，得

$$\begin{cases}(50+\mathrm{j}160-\mathrm{j}80)\dot{I}_1-(-\mathrm{j}80)\dot{I}_2-\mathrm{j}40\dot{I}_2=100\angle 0^\circ\\(20+\mathrm{j}40-\mathrm{j}80)\dot{I}_2-(-\mathrm{j}80)\dot{I}_1-\mathrm{j}40\dot{I}_1=0\end{cases}$$

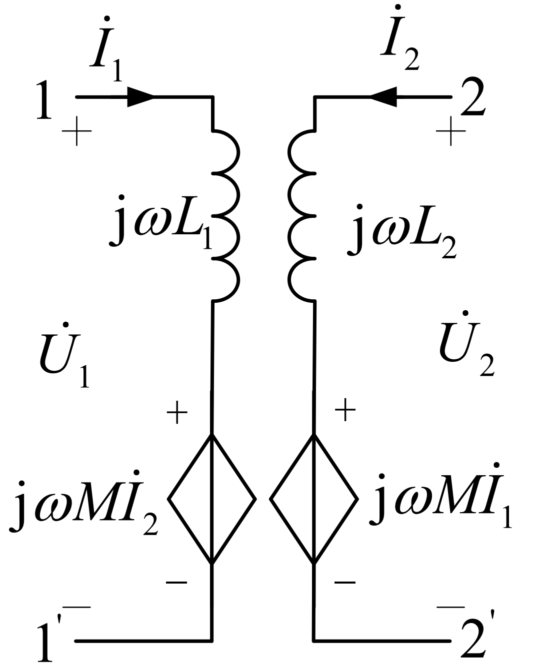
$$\dot{I}_1=0.77\angle-59^\circ\ \mathrm{A},\dot{I}_2=0.69\angle-85.6^\circ\ \mathrm{A}$$

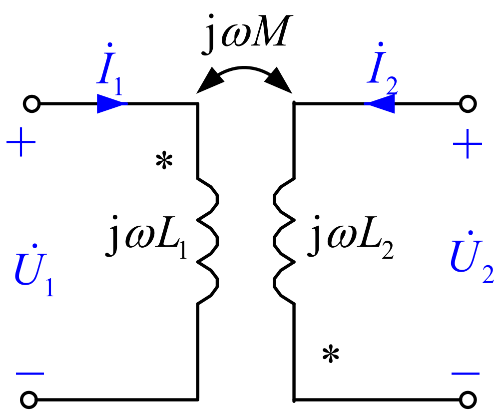
$$P=100\times 0.77\times\cos 59^\circ=39.7\ \mathrm{W}$$

用受控源表示互感电压

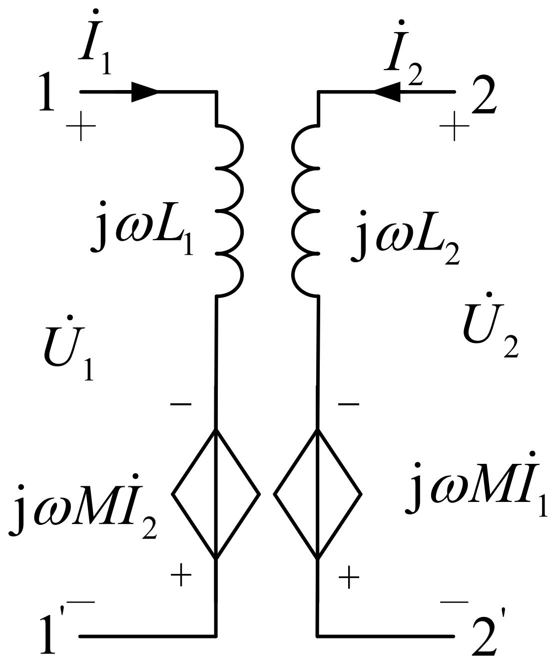


$$\dot{U}_1 = \mathrm{j}\omega L_1\dot{I}_1 + \mathrm{j}\omega M\dot{I}_2$$
$$\dot{U}_2 = \mathrm{j}\omega L_2\dot{I}_2 + \mathrm{j}\omega M\dot{I}_1$$

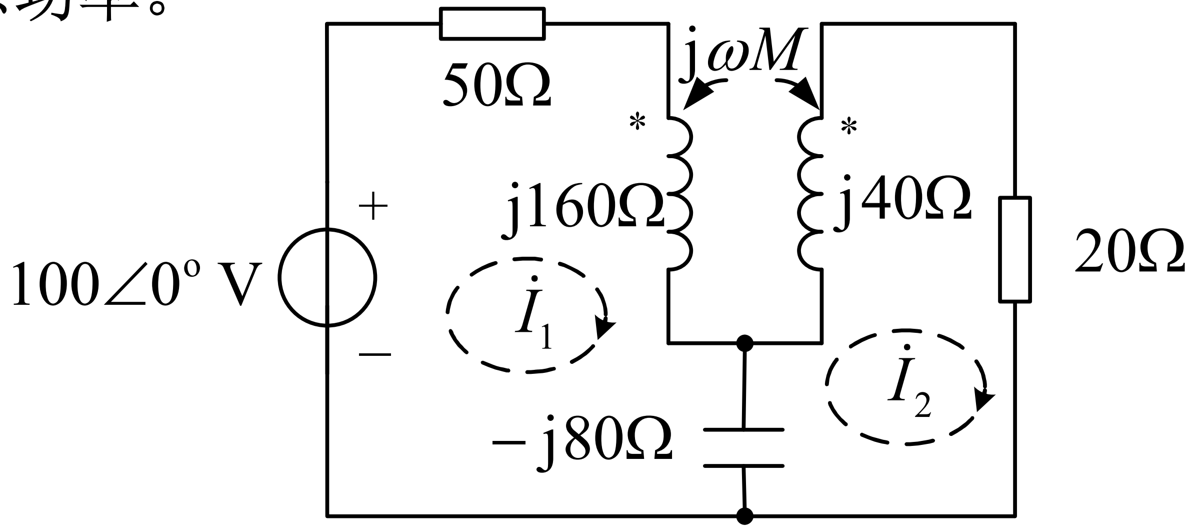


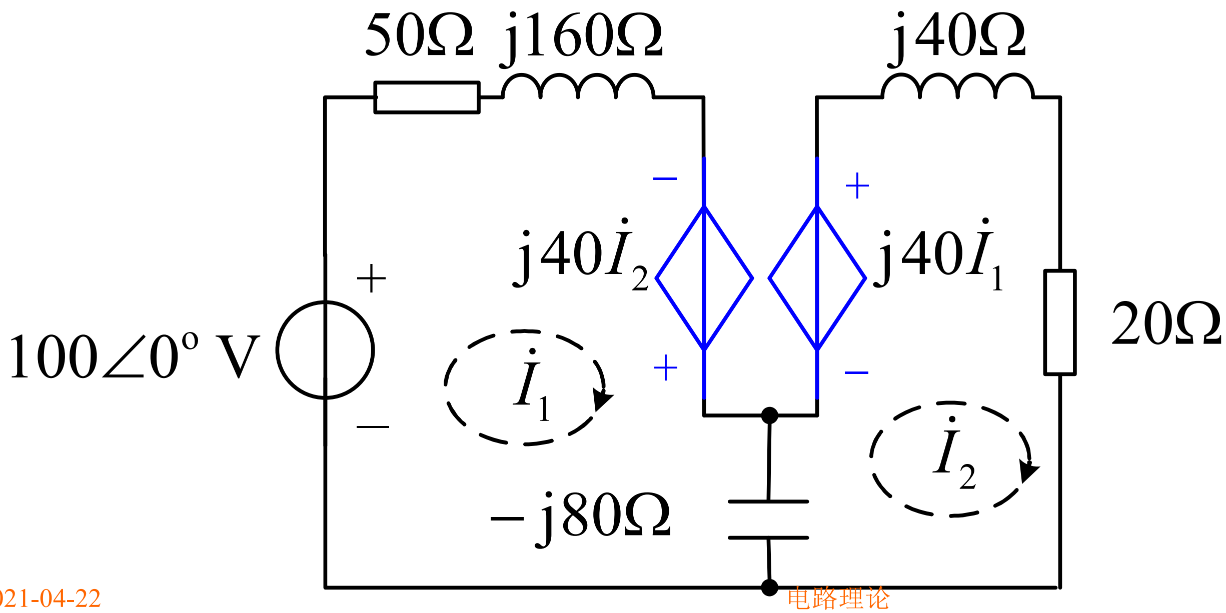


$$\dot{U}_1 = \mathrm{j}\omega L_1\dot{I}_1 - \mathrm{j}\omega M\dot{I}_2$$
$$\dot{U}_2 = \mathrm{j}\omega L_2\dot{I}_2 - \mathrm{j}\omega M\dot{I}_1$$



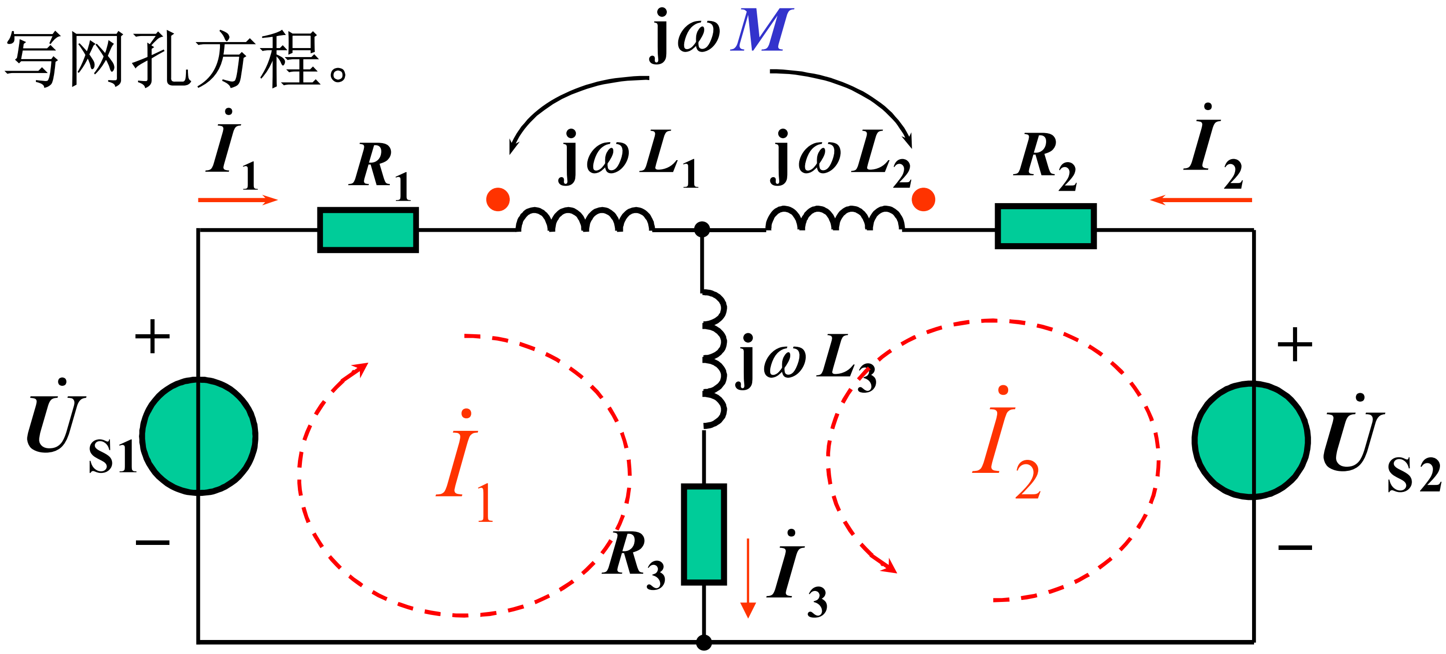
【例1】：图示电路中，耦合电感的耦合系数K=0.5，求电路消耗的总功率。





有互感的电路的计算仍属正弦稳态分析，前面介绍的相量分析的的方法均适用。只需注意互感线圈上的电压除自感电压外，还应包含互感电压。

【课下练习】列写网孔方程。



网孔方程：（考虑互感）

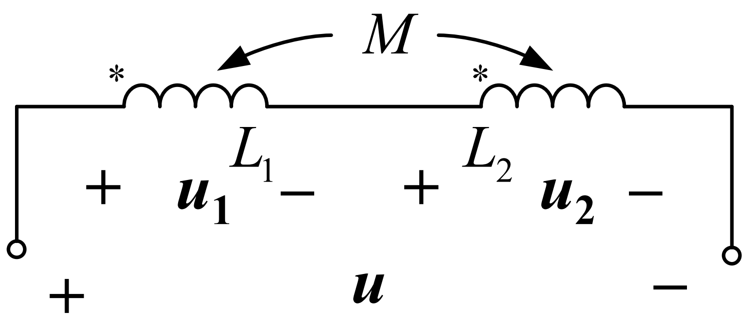
$$
\begin{cases}
(R_1 + j\omega L_1 + R_3 + j\omega L_3)\dot{I}_1 + (R_3 + j\omega L_3)\dot{I}_2 + j\omega M\dot{I}_2 = \dot{U}_{S1} \\
(R_2 + j\omega L_2 + R_3 + j\omega L_3)\dot{I}_2 + (R_3 + j\omega L_3)\dot{I}_1 + j\omega M\dot{I}_1 = \dot{U}_{S2}
\end{cases}
$$

注意：互感线圈的互感电压的的表示式及正负号。

13.3.2 去耦等效电路及应用

1 耦合电感线圈的串联

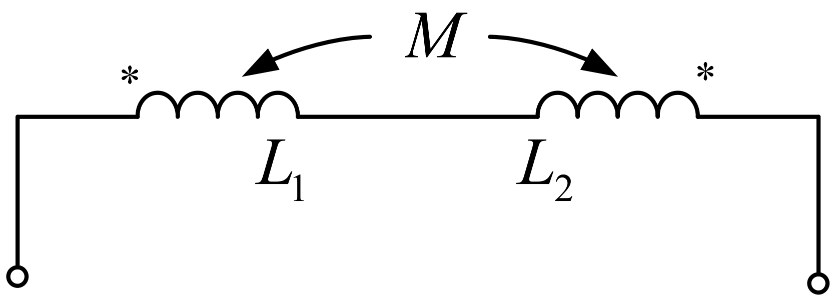
Series-aiding connection (顺向串联):



$$u = u_1 + u_2 = (L_1 \frac{di}{dt} + M \frac{di}{dt}) + (L_2 \frac{di}{dt} + M \frac{di}{dt}) = (L_1 + L_2 + 2M) \frac{di}{dt}$$

$$\therefore \quad L = L_1 + L_2 + 2M$$

Series-opposing connection (反向串联):



$$u = u_1 + u_2 = (L_1 \frac{di}{dt} - M \frac{di}{dt}) + (L_2 \frac{di}{dt} - M \frac{di}{dt}) = (L_1 + L_2 - 2M)\frac{di}{dt}$$

$$\therefore \quad L = L_1 + L_2 - 2M$$

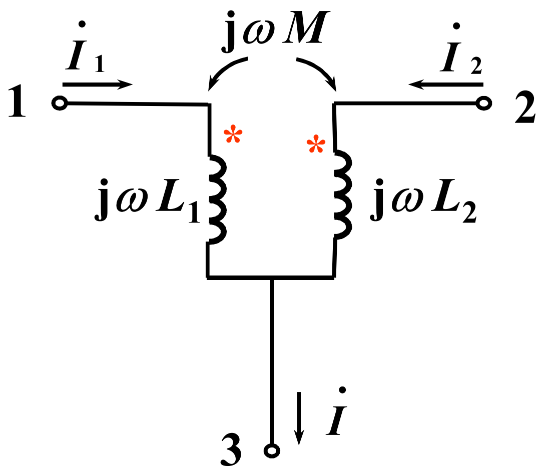
互感的测量方法:

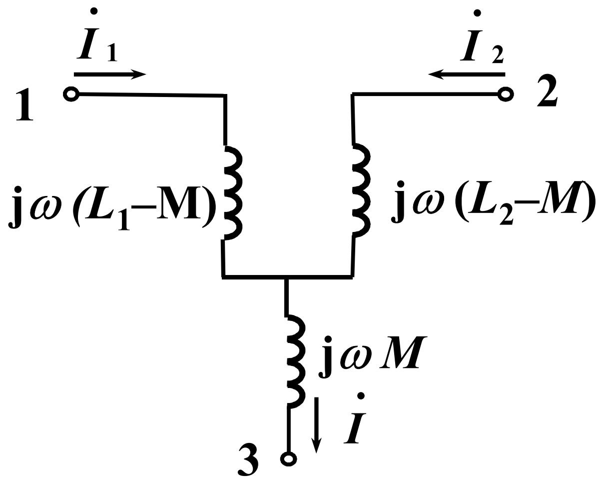
* 顺接一次，反接一次，就可以测出互感:

$$M = \frac{L_{顺} - L_{反}}{4}$$

2. T形连接的耦合电感的去耦等效电路

(a) 同名端同侧联接



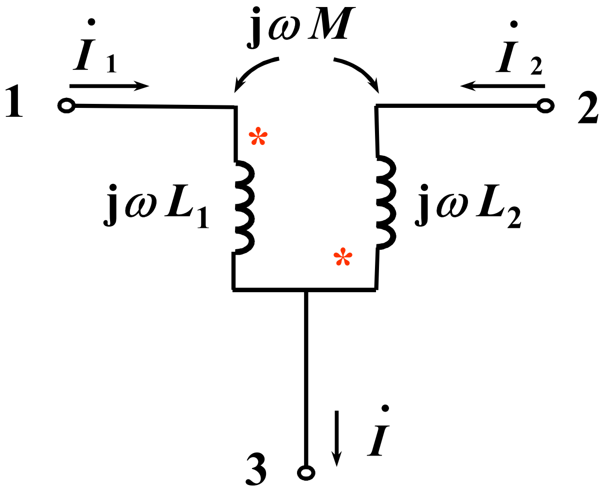


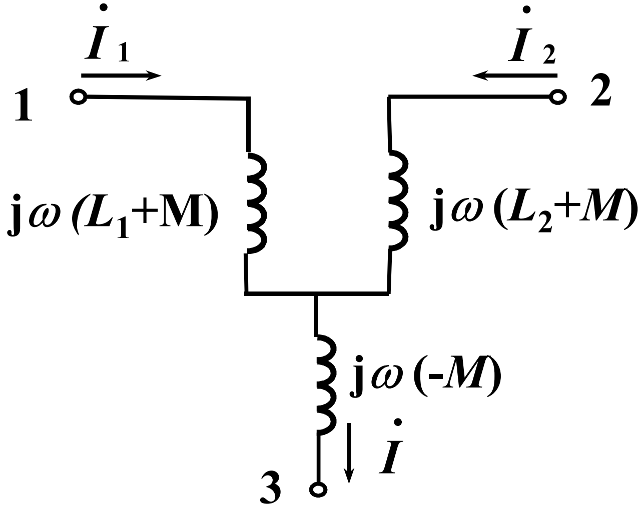
$$\begin{cases} \dot{U}_{13} = \mathrm{j}\omega L_1 \dot{I}_1 + \mathrm{j}\omega M \dot{I}_2 \\ \dot{U}_{23} = \mathrm{j}\omega L_2 \dot{I}_2 + \mathrm{j}\omega M \dot{I}_1 \\ \dot{I} = \dot{I}_1 + \dot{I}_2 \end{cases}$$

整理得

$$\begin{cases} \dot{U}_{13} = \mathrm{j}\omega (L_1 - M) \dot{I}_1 + \mathrm{j}\omega M \dot{I} \\ \dot{U}_{23} = \mathrm{j}\omega (L_2 - M) \dot{I}_2 + \mathrm{j}\omega M \dot{I} \\ \dot{I} = \dot{I}_1 + \dot{I}_2 \end{cases}$$

(b) 同名端异侧联接



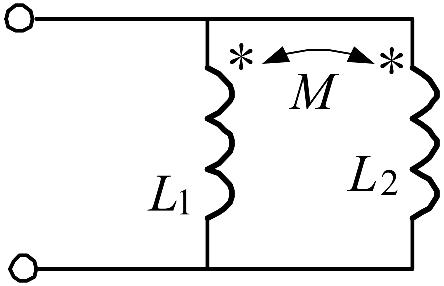


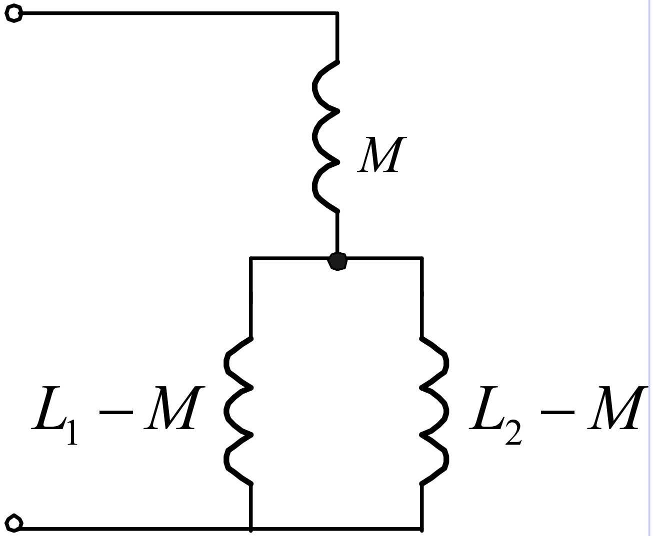
$$\begin{cases} \dot{U}_{13} = j\omega L_1\dot{I}_1 - j\omega M\dot{I}_2 \\ \dot{U}_{23} = j\omega L_2\dot{I}_2 - j\omega M\dot{I}_1 \\ \dot{I} = \dot{I}_1 + \dot{I}_2 \end{cases}$$

整理得

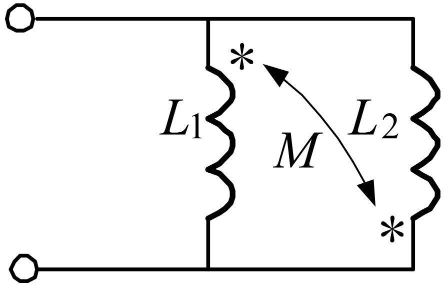
$$\begin{cases} \dot{U}_{13} = j\omega(L_1 + M)\dot{I}_1 - j\omega M\dot{I} \\ \dot{U}_{23} = j\omega(L_2 + M)\dot{I}_2 - j\omega M\dot{I} \\ \dot{I} = \dot{I}_1 + \dot{I}_2 \end{cases}$$

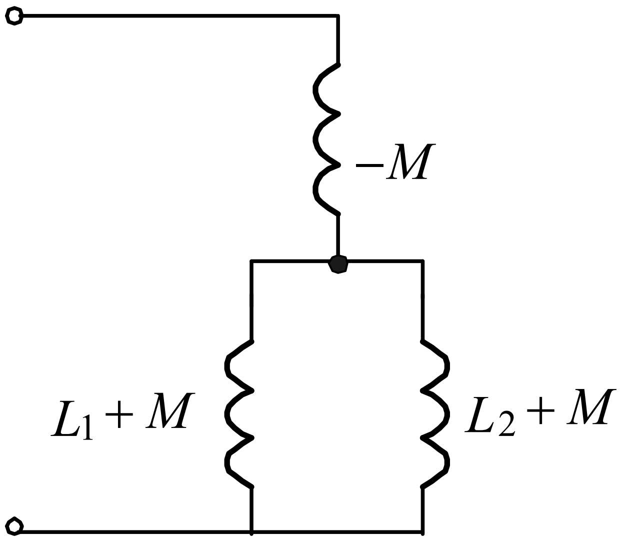
练习：等效电路（p528测13-6）



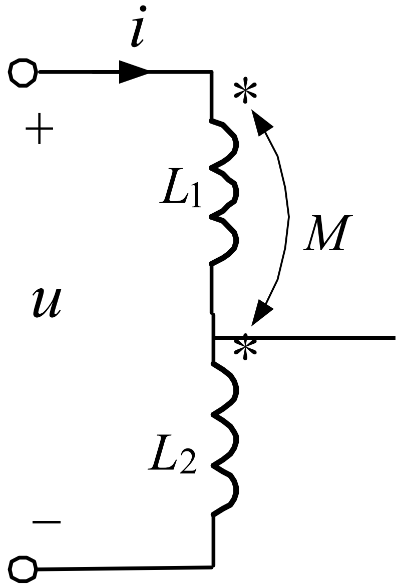


$$L_{eq}=\frac{(L_1L_2-M^2)}{L_1+L_2-2M}$$

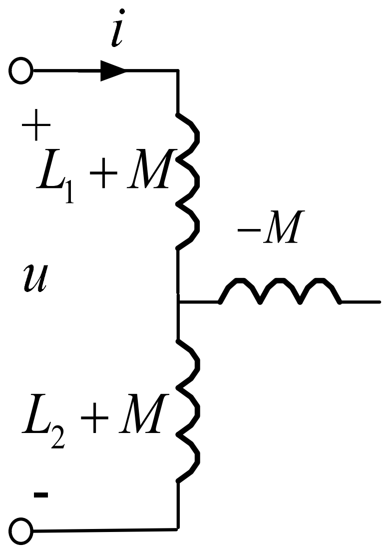




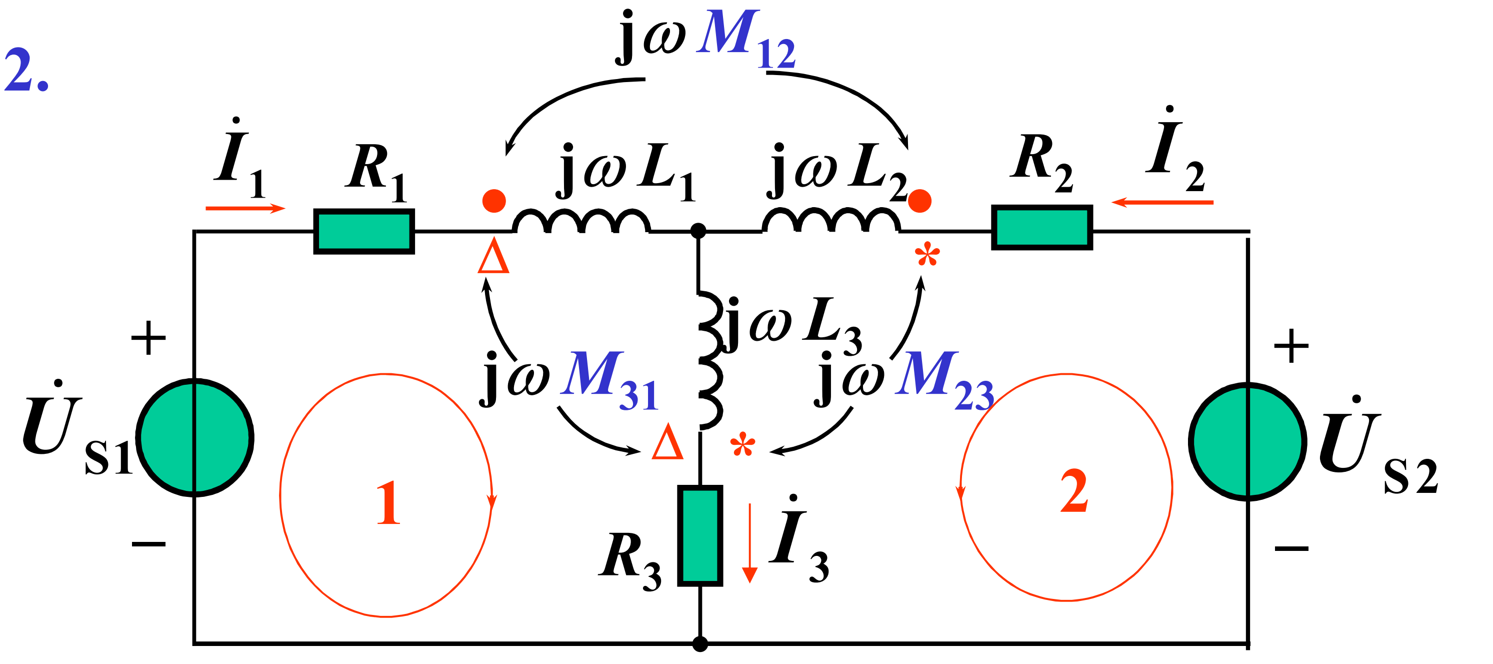
$$L_{eq}=\frac{(L_1L_2-M^2)}{L_1+L_2+2M}$$



$$L_{\mathrm{eq}}=L_1+L_2+2M$$



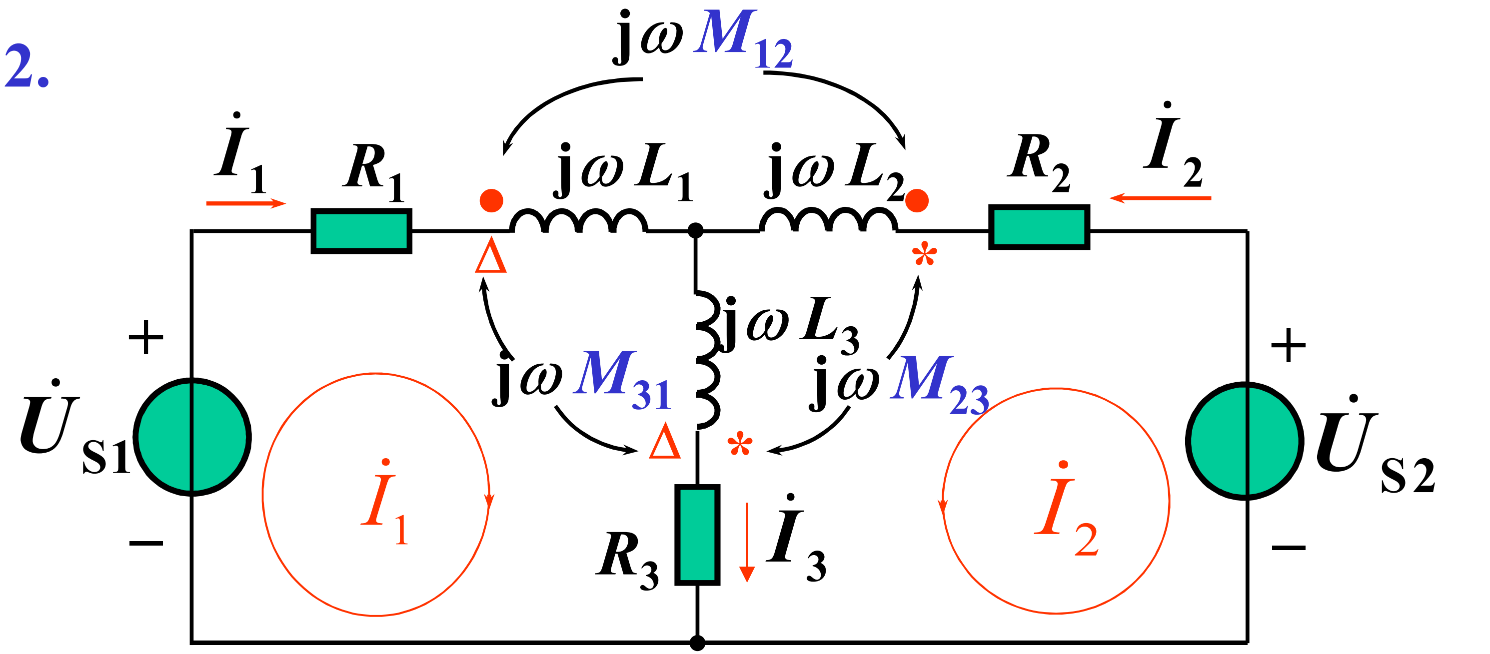
例 2.



支路法:

$$
\begin{cases}
\dot{I}_1 + \dot{I}_2 = \dot{I}_3 \\
R_1\dot{I}_1 \ +(\mathrm{j}\omega L_1\dot{I}_1 + \mathrm{j}\omega M_{12}\dot{I}_2 - \mathrm{j}\omega M_{31}\dot{I}_3) \\
\qquad +(\mathrm{j}\omega L_3\dot{I}_3 - \mathrm{j}\omega M_{31}\dot{I}_1 - \mathrm{j}\omega M_{23}\dot{I}_2) + R_3\dot{I}_3 = \dot{U}_{S1} \\
R_2\dot{I}_2 + (\mathrm{j}\omega L_2\dot{I}_2 + \mathrm{j}\omega M_{12}\dot{I}_1 - \mathrm{j}\omega M_{23}\dot{I}_3) + \\
\qquad (\mathrm{j}\omega L_3\dot{I}_3 - \mathrm{j}\omega M_{31}\dot{I}_1 - \mathrm{j}\omega M_{23}\dot{I}_2) + R_3\dot{I}_3 = \dot{U}_{S2}
\end{cases}
$$

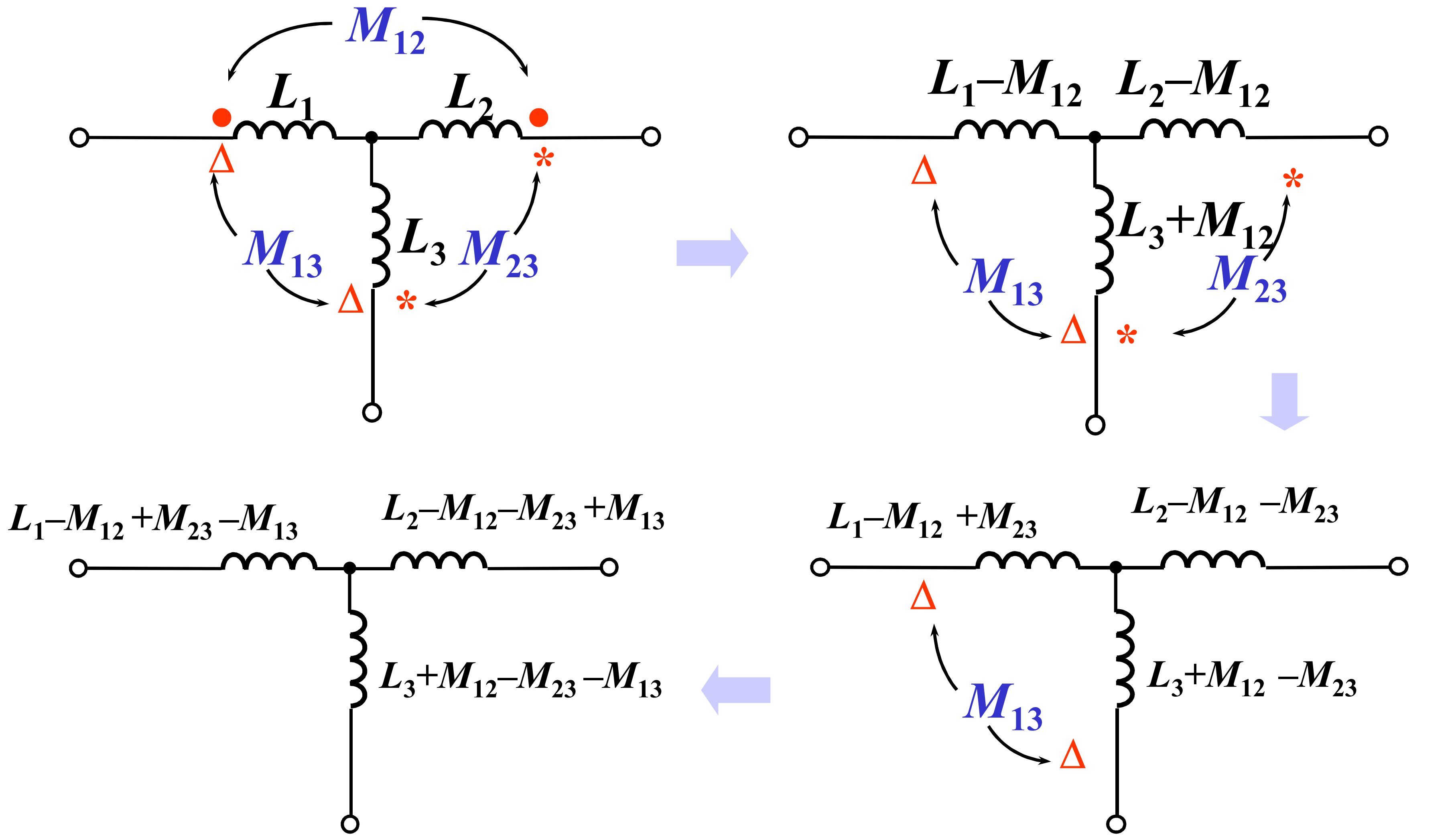
例 2.



网孔法：

$$
\begin{cases}
(R_1+j\omega L_1+j\omega L_3+R_3)\dot{I}_1+(R_3+j\omega L_3)\dot{I}_2 \\
+j\omega M_{12}\dot{I}_2-j\omega M_{31}(\dot{I}_1+\dot{I}_2)-j\omega M_{31}\dot{I}_1-j\omega M_{23}\dot{I}_2=\dot{U}_{S1} \\
(R_2+j\omega L_2+j\omega L_3+R_3)\dot{I}_2+(R_3+j\omega L_3)\dot{I}_1 \\
+j\omega M_{12}\dot{I}_1-j\omega M_{23}\dot{I}_1-j\omega M_{23}\dot{I}_2-j\omega M_{31}\dot{I}_1-j\omega M_{23}\dot{I}_2=\dot{U}_{S2}
\end{cases}
$$

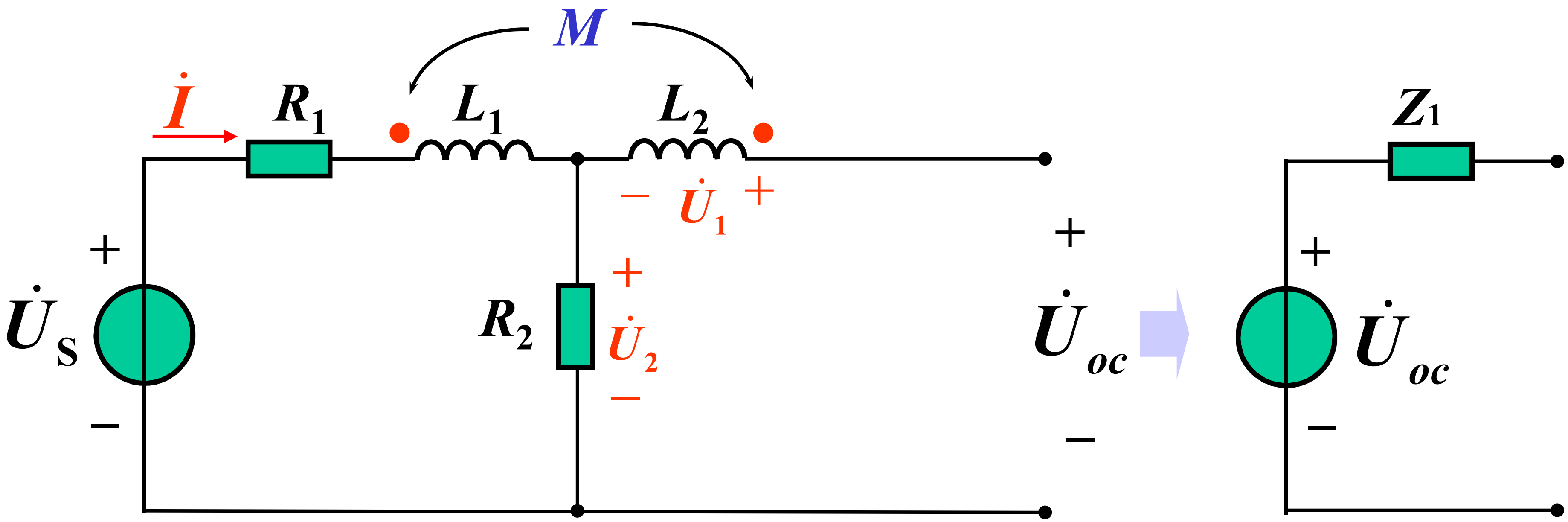
此题可先作出去耦等效电路，再列方程(一对一对消)：



若$L_1 = L_2 = L_3 = L$；$M_{12} = M_{23} = M_{31} = M$，则三个电感均为$L - M$。

【例3】 $\omega L_1 = \omega L_2 = 10\Omega\,,\omega M = 5\Omega\,,R_1 = R_2 = 6\Omega\,,U_S = 6V\,,$

求其戴维南等效电路。

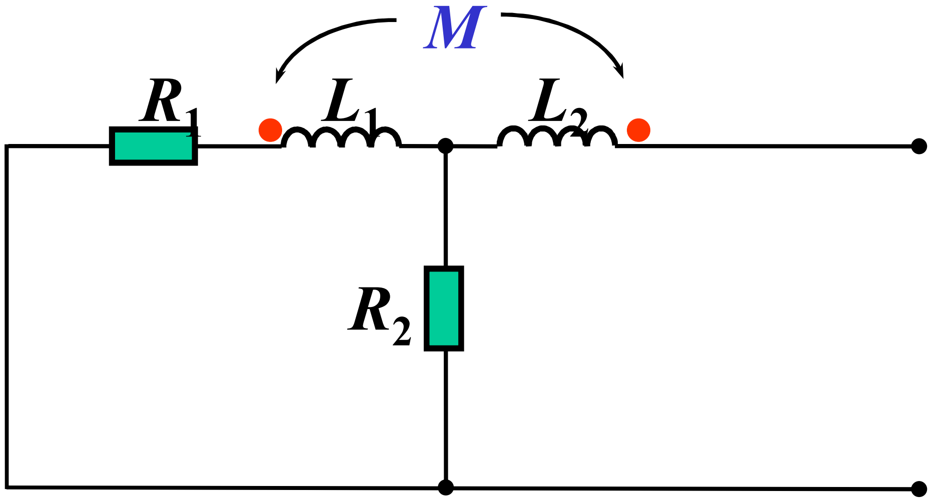


计算开路电压$\dot{U}_{OC}$ 。

$$\dot{U}_{OC} = \dot{U}_1 + \dot{U}_2 = j\omega M\,\dot{I} + R_2\dot{I} = (6 + j5)\times 0.384\angle -39.8^\circ = 3\angle 0^\circ\ V$$

$$\dot{I} = \frac{\dot{U}_S}{R_1 + j\omega L_1 + R_2} = \frac{6\angle 0^\circ}{12 + j10} = \frac{6\angle 0^\circ}{15.62\angle 39.8^\circ} = 0.384\angle -39.8^\circ\ A$$

【例3】 $\omega L_1=\omega L_2=10\Omega$，$\omega M=5\Omega$，$R_1=R_2=6\Omega$，$U_S=6V$，求其戴维南等效电路。

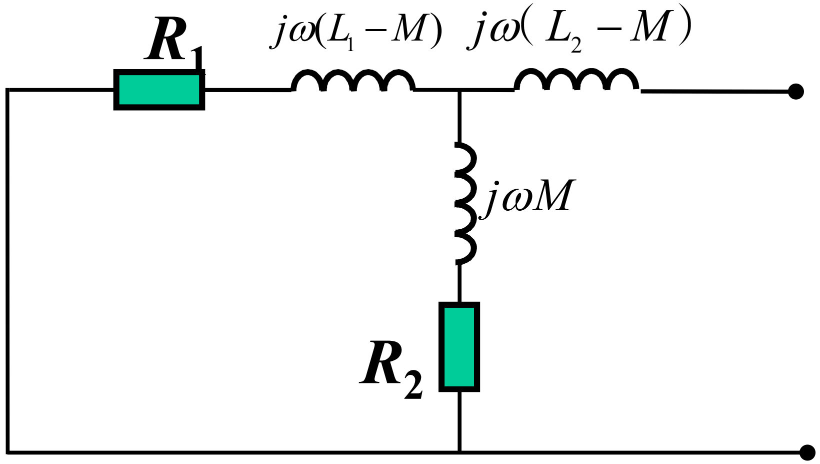


求内阻：Z_{eq}

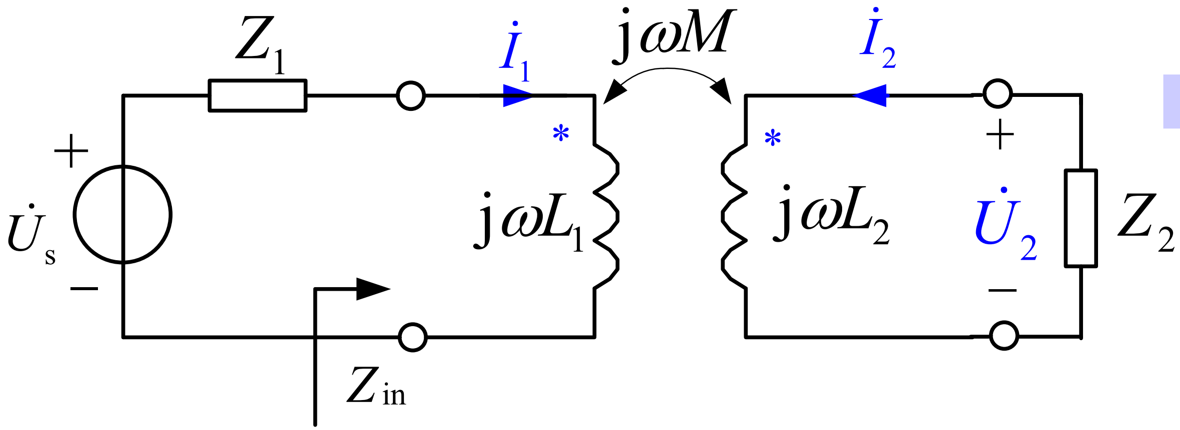
$$Z_{eq}=j\omega(L_2-M)+R_1+j\omega(L_1-M)//(R_2+j\omega M)$$

$$=j5+\frac{(6+j5)(6+j5)}{(6+j5)+(6+j5)}$$

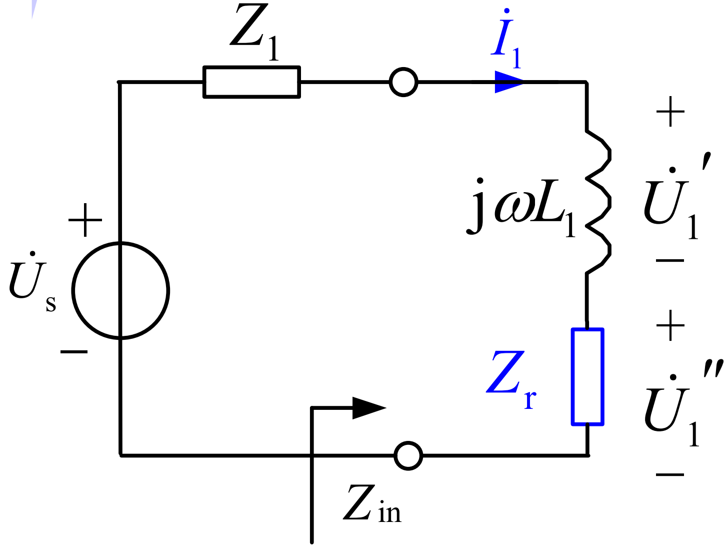
$$=8.08\angle 68.2^{\circ}\Omega$$



13.3.3. 映射阻抗及应用



电源回路等效电路:



$$\begin{cases}(Z_1 + \mathrm{j}\omega L)\dot{I}_1 + \mathrm{j}\omega M\dot{I}_2 = \dot{U}_S \\ \mathrm{j}\omega M\dot{I}_1 + (Z_2 + \mathrm{j}\omega L_2)\dot{I}_2 = 0\end{cases}$$

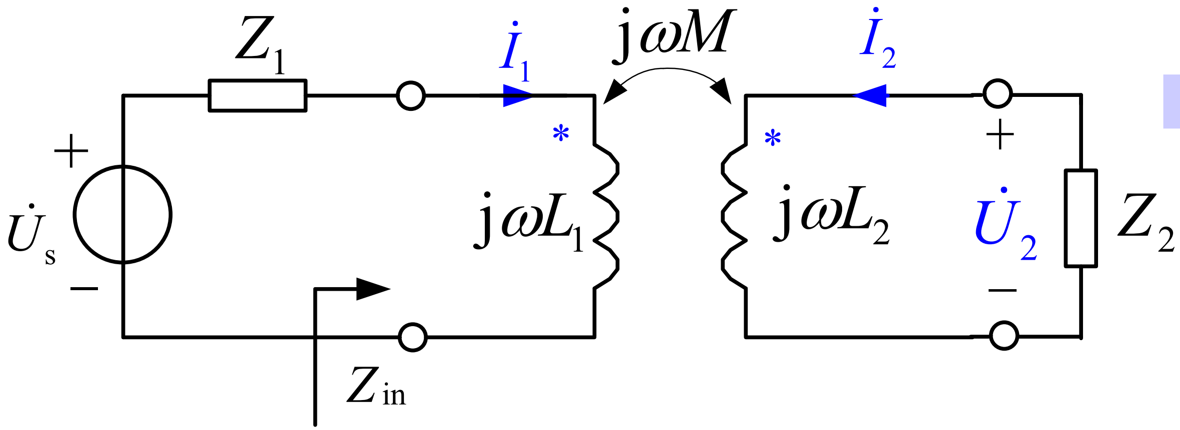
$$\dot{I}_1 = \frac{\dot{U}_S}{Z_1 + \mathrm{j}\omega L_1 + \dfrac{(\omega M)^2}{Z_2 + \mathrm{j}\omega L_2}}$$

$$\boxed{Z_r = \frac{(\omega M)^2}{Z_2 + \mathrm{j}\omega L_2}}$$

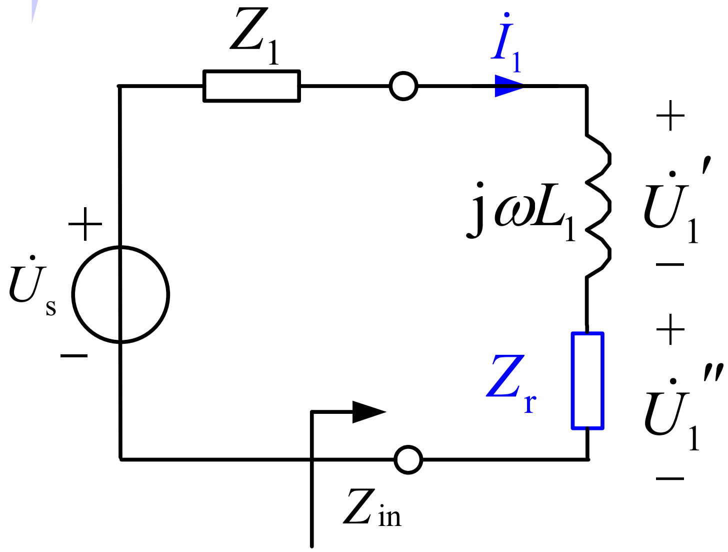
副方对原方的映射阻抗

$$Z_{\mathrm{in}} = \frac{\dot{U}_S}{\dot{I}_1} = Z_1 + \mathrm{j}\omega L_1 + \frac{(\omega M)^2}{Z_2 + \mathrm{j}\omega L_2}$$

13.3.3. 映射阻抗及应用

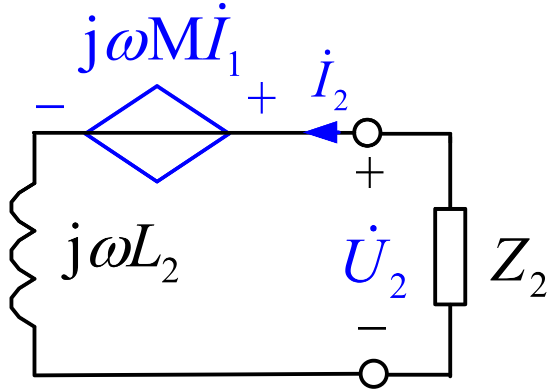


电源回路等效电路:



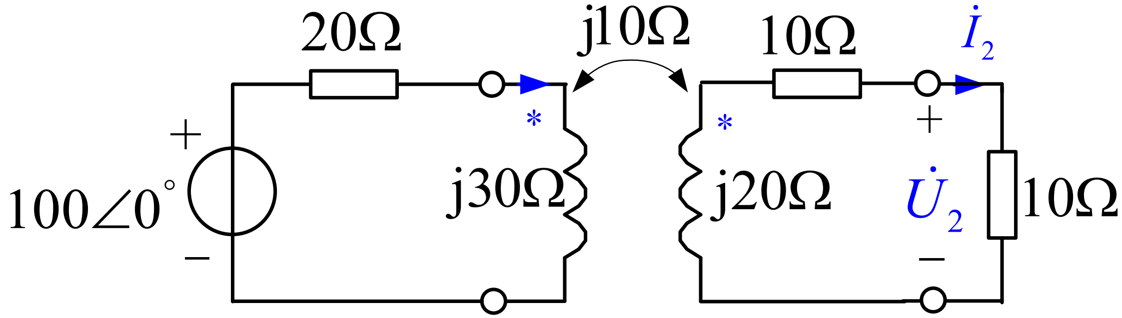
$$\begin{cases} (Z_1 + j\omega L)\dot{I}_1 + j\omega M\dot{I}_2 = \dot{U}_S \\ j\omega M\dot{I}_1 + (Z_2 + j\omega L_2)\dot{I}_2 = 0 \end{cases}$$

负载回路等效电路:



- 负载回路对电源回路的影响可以用映射阻抗来考虑。
- 虽然电源和负载回路没有电的联系，但由于互感作用使负载回路产生电流，反过来这个电流又影响电源回路。

【例1】求I_2 (P530例13-3-1)

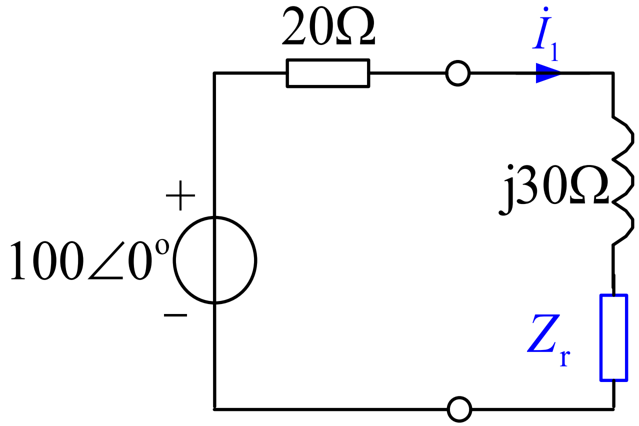


解：电源回路等效电路：

$$Z_r=\frac{(\omega M)^2}{10+10+\mathrm{j}20}=2.5-\mathrm{j}2.5\ \ \Omega$$

$$\dot{I}_1=\frac{100\angle 0^\circ}{[(20+\mathrm{j}30)+Z_r]}=2.814\angle -50.71^\circ\mathrm{A}$$

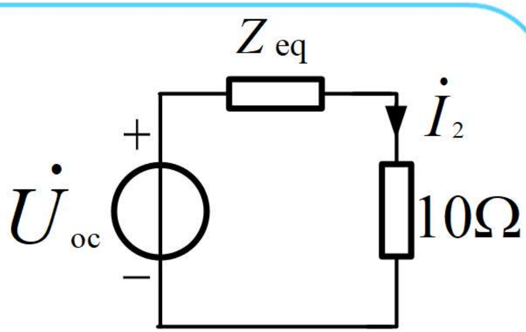
$$\dot{I}_2=\frac{\mathrm{j}10\dot{I}_1}{10+10+\mathrm{j}20}=0.995\angle -5.71^\circ\mathrm{A}$$



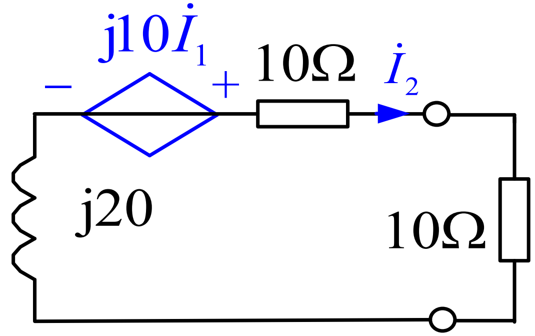
方法2：映射阻抗求I_2

$$\dot{U}_{oc}=j\omega M\dot{I}_1$$

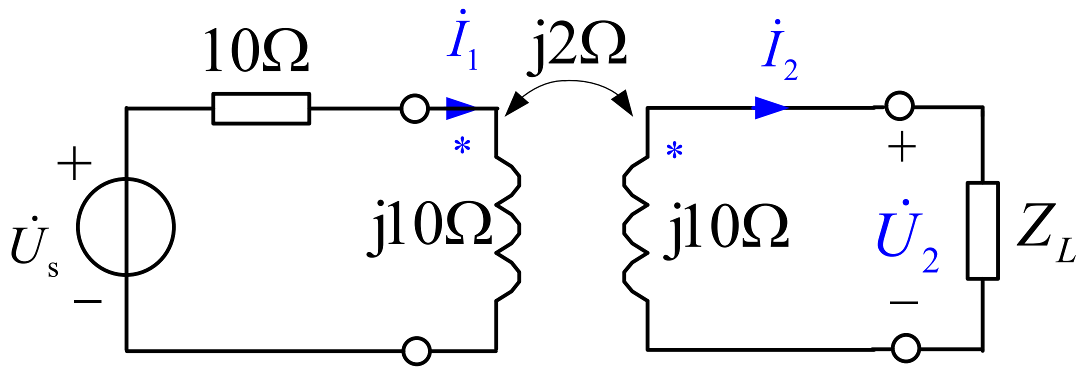
$$=j10\times\frac{100\angle 0^\circ}{20+j30}$$

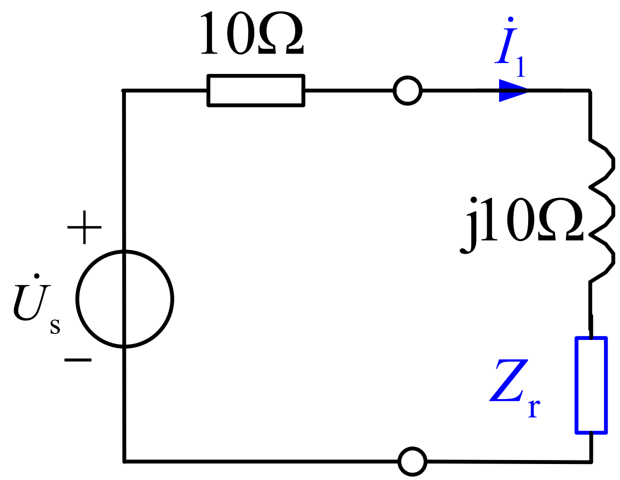


$$Z_{eq}=Z_{22}+Z_r=(10+j20)+\frac{10^2}{20+j30}$$



【例2】. 已知U_S=20 V ,映射阻抗 Z_r=10–j10Ω。求: Z_L 并求负载获得的有功功率。





解：电源回路等效电路：

$$Z_r = \frac{(\omega M)^2}{Z_L + \mathrm{j}10} = \frac{4}{Z_L + \mathrm{j}10} \qquad \therefore Z_L = (0.2 - \mathrm{j}9.8)\Omega$$

$$\dot{I}_1 = \frac{20}{10 + \mathrm{j}10 + 10 - \mathrm{j}10} = 1\,\mathrm{A}$$

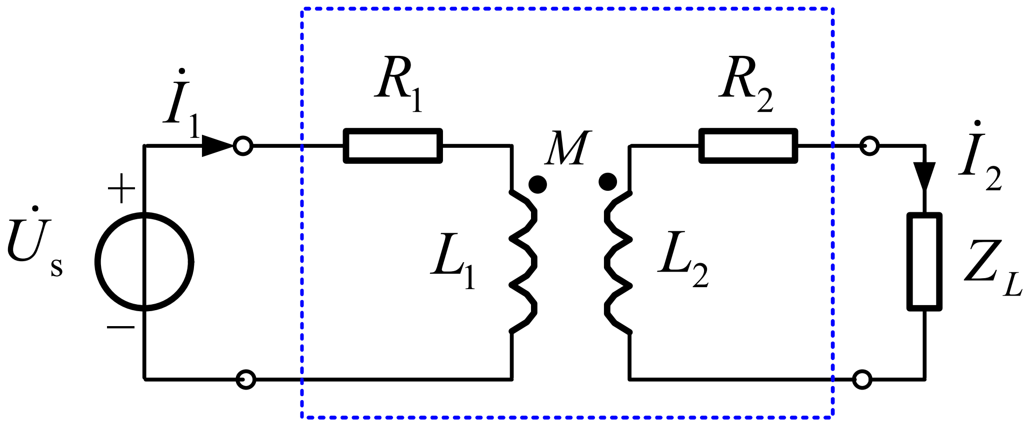
$$\dot{I}_2 = \frac{\mathrm{j}2\dot{I}_1}{\mathrm{j}10 + 0.2 - j9.8} = \frac{2\angle 90^\circ}{0.2\sqrt{2}\angle 45^\circ} = 5\sqrt{2}\angle 45^\circ\,\mathrm{A}$$

此时负载获得的功率： $P = (I_2)^2 R_L = (5\sqrt{2})^2 \times 0.2 = 10\,\mathrm{W}$

或者由功率守恒： $P = P_{R映射} = (\frac{20}{10+10})^2 R_r = 10\,\mathrm{W}$

13.4 变压器　Transformers

13.4.1. 线性变压器 Linear transformers（Air-core transformers)



$\mu = \text{constant}$

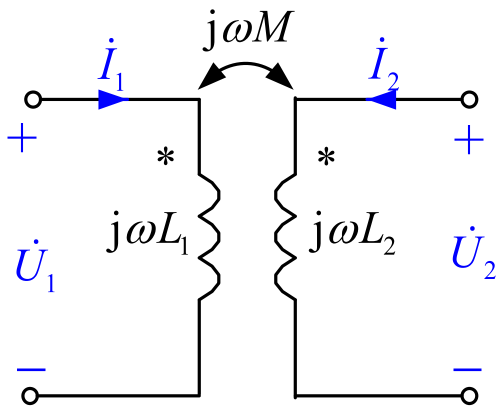
Primary winding　　secondary winding

初级(原方)　　次级(副方)

- 线性变压器，即空心变压器。自感不大，低频下自感抗低，因而线圈电流大。一般用在电子电路中。优点是没有铁心损耗。分析时用线性耦合电感为模型。
- 为了加大自感，采用铁心，即为铁心变压器，是非线性耦合系统。由于自感大，可以用于电压高、频率低的场合，存在铁心损耗。分析时近似为理想变压器。

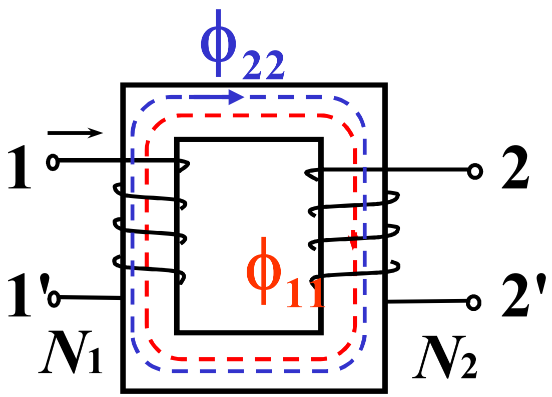
13.4 变压器 Transformers

13.4.2. 铁心变压器



$$\begin{cases} \dot{U}_1 = \mathrm{j}\omega L_1 \dot{I}_1 + \mathrm{j}\omega M \dot{I}_2 \\ \dot{U}_2 = \mathrm{j}\omega L_2 \dot{I}_2 + \mathrm{j}\omega M \dot{I}_1 \end{cases}$$

全耦合时 $M = \sqrt{L_1 L_2}$ ， $k = 1$



$$\dot{I}_1 = \frac{\dot{U}_2 - \mathrm{j}\omega L_2 \dot{I}_2}{\mathrm{j}\omega M}$$

$$\dot{U}_1 = \frac{L_1}{M}(\dot{U}_2 - \mathrm{j}\omega L_2 \dot{I}_2) + \mathrm{j}\omega M \dot{I}_2$$

$$= \frac{L_1}{M}\dot{U}_2$$

$$= n\dot{U}_2$$

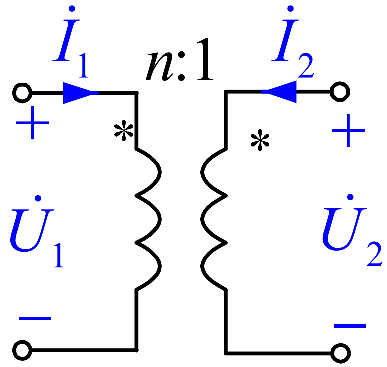
$$n = \frac{N_1}{N_2} = \frac{N_1 \Phi_{11}}{N_2 \Phi_{21}} = \frac{L_1 i_1}{M i_1} = \frac{L_1}{M} = \frac{M}{L_2} = \sqrt{\frac{L_1}{L_2}}$$

全耦合变压器的电压、电流关系：

$$\begin{cases} \dot{U}_1 = n\dot{U}_2 \\ \dot{I}_1 = \dfrac{\dot{U}_2 - j\omega L_2 \dot{I}_2}{j\omega M} = \dfrac{1}{j\omega Mn}\dot{U}_1 - \dfrac{j\omega L_2}{j\omega M}\dot{I}_2 = \dfrac{\dot{U}_1}{j\omega L_1} - \dfrac{1}{n}\dot{I}_2 \end{cases}$$

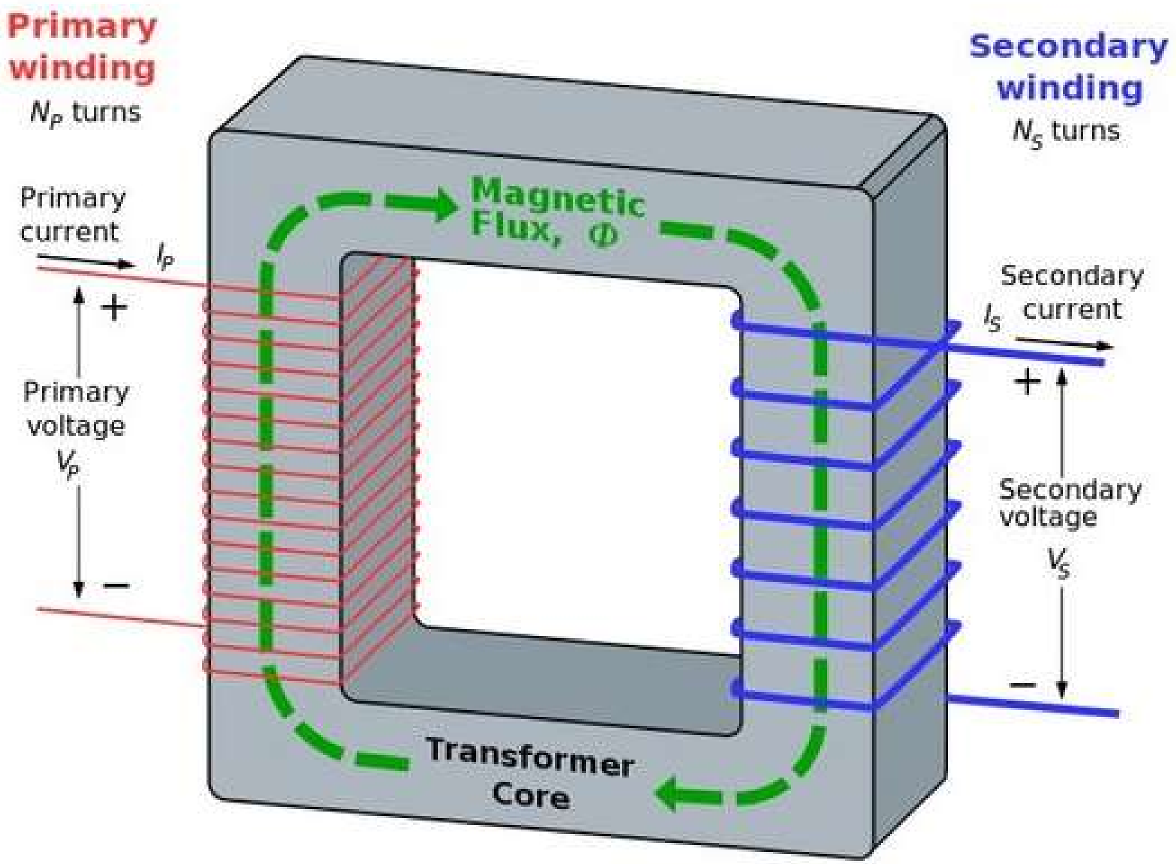
当L_1 ,M, $L_2 \to \infty$，L_1/L_2 比值不变 (磁导率$\mu \to \infty$) ，则有

$$\begin{cases} \dot{U}_1 = n\dot{U}_2 \\ \dot{I}_1 = -\dfrac{1}{n}\dot{I}_2 \end{cases}$$



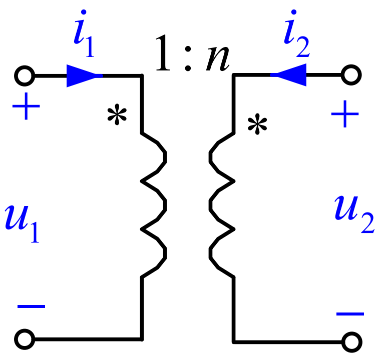
理想变压器的元件特性

理想变压器的电路模型

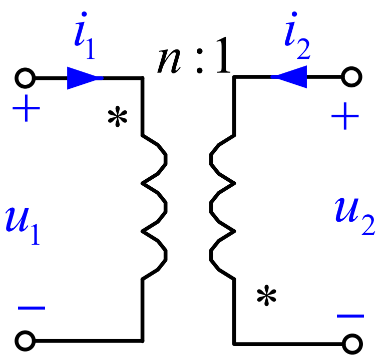


- 接电压源的线圈称为：一次绕组，接负载的线圈称为二次绕组。
- 一次二次绕组匝数比n
- 一次、二次绕组电压与电流。

13.5 理想变压器



$$\frac{u_1}{u_2}=\frac{1}{n} \qquad \frac{i_1}{i_2}=-\frac{n}{1}$$



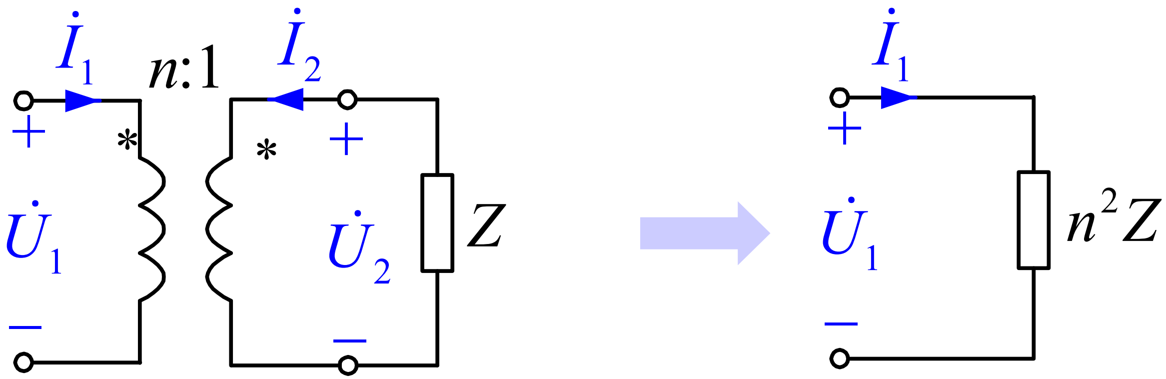
$$\frac{u_1}{u_2}=-n \qquad \frac{i_1}{i_2}=\frac{1}{n}$$

- 一次绕组电压和二次绕组电压的正极性均在同名端，则电压比等于匝数比。
- 一次绕组电流和二次绕组电流均为流入同名端时，电流比等于匝数比倒数的负值。

13.5 理想变压器

理想变压器的性质：

(a) 阻抗变换性质

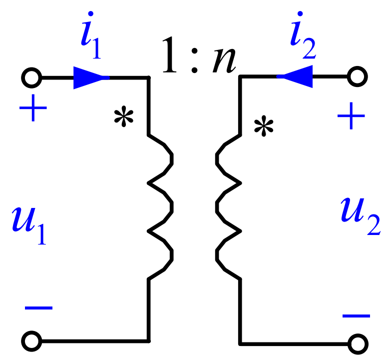


$$\frac{\dot{U}_1}{\dot{I}_1}=\frac{n\dot{U}_2}{-1/n\dot{I}_2}=n^2\left(-\frac{\dot{U}_2}{\dot{I}_2}\right)=n^2Z$$

(b) 功率性质:

理想变压器的特性方程为代数关系，因此无记忆作用。

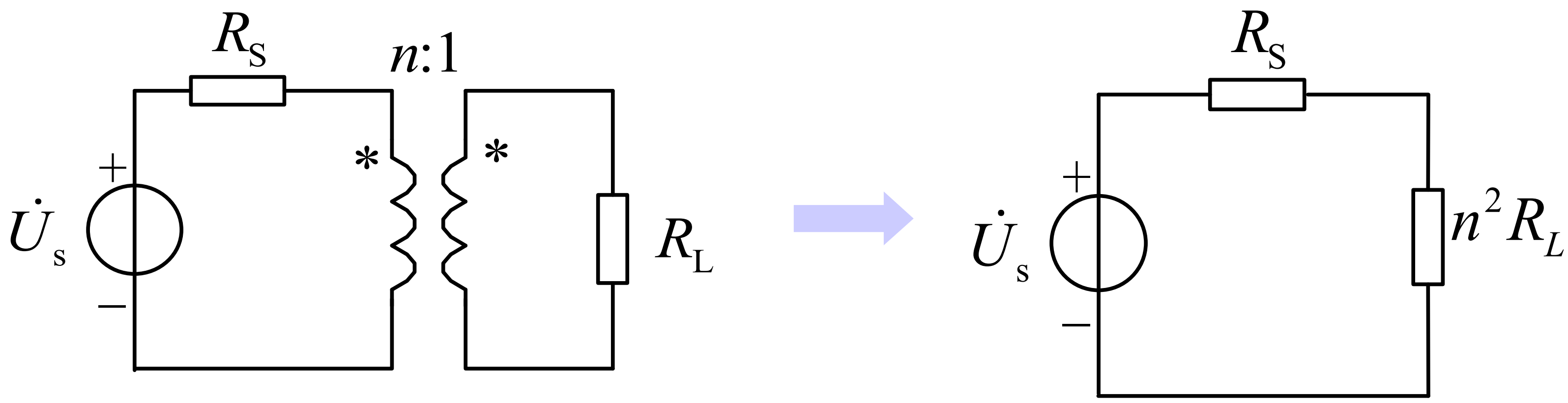
$$\begin{cases} u_1 = nu_2 \\ i_1 = -\dfrac{1}{n} i_2 \end{cases}$$



$$p = u_1 i_1 + u_2 i_2 = u_1 i_1 + \frac{1}{n} u_1 \times (-n i_1) = 0$$

由此可以看出，理想变压器既不储能，也不耗能，在电路中只起传递信号和能量的作用。

【例1】已知电源内阻R_S=1kΩ，负载电阻R_L=10Ω。为使R_L上获得最大功率，求理想变压器的变比n。

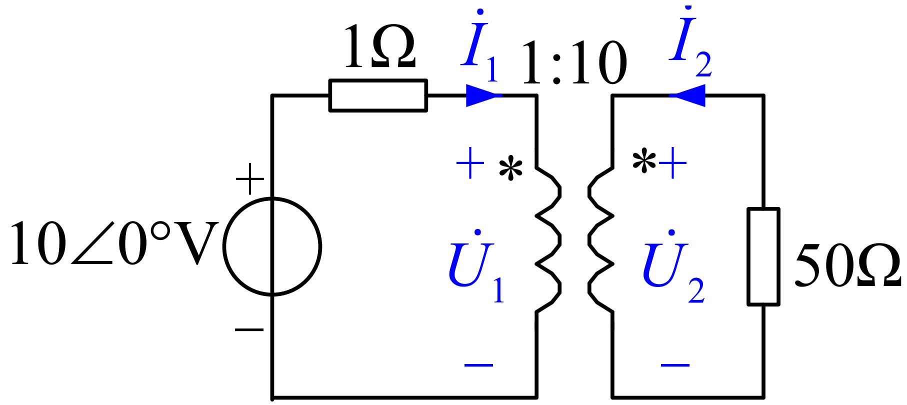


当 $n^2R_L=R_S$时匹配，即

$$10n^2=1000$$

$$\therefore\ n^2=100,\quad n=10\ .$$

【例2】.求$\dot{U}_2$.

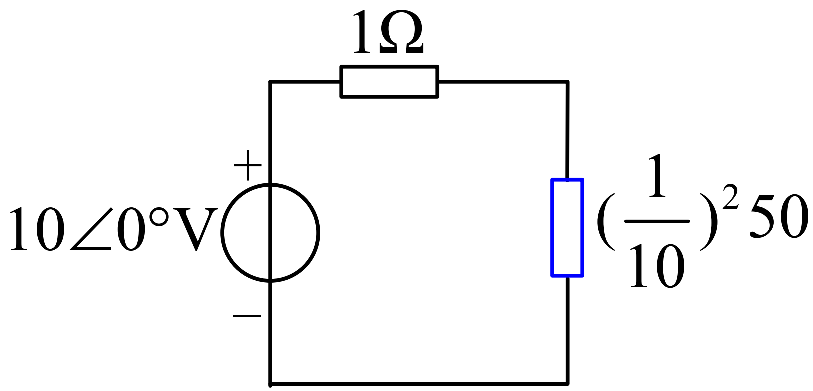


方法1：列方程

$$\begin{cases} 1\times \dot{I}_1+\dot{U}_1=10\angle 0^\circ \\ 50\dot{I}_2+\dot{U}_2=0 \\ \dot{U}_1=\dfrac{1}{10}\dot{U}_2 \\ \dot{I}_1=-10\dot{I}_2 \end{cases}$$

$$\dot{U}_2=33.33\angle 0^\circ\ \mathrm{V}$$

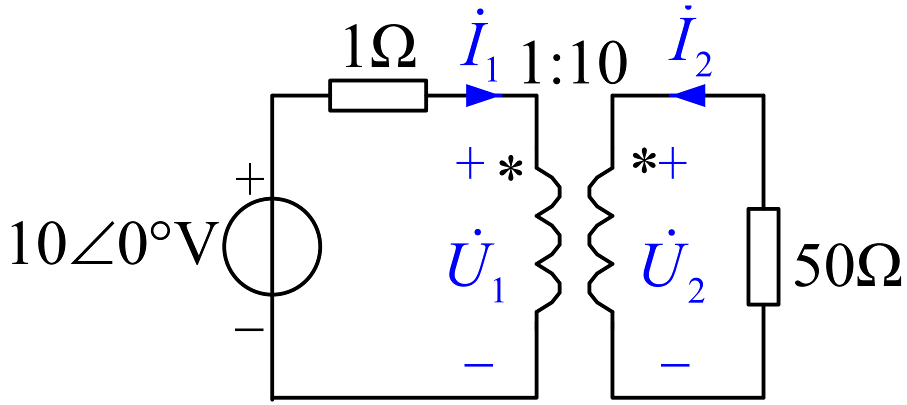
方法2：阻抗变换



$$\dot{U}_1=\frac{10\angle 0^\circ}{1+1/2}\times\frac{1}{2}=\frac{10}{3}\angle 0^\circ\ \mathrm{V}$$

$$\dot{U}_2=n\dot{U}_1=10\dot{U}_1$$
$$=33.33\angle 0^\circ\ \mathrm{V}$$

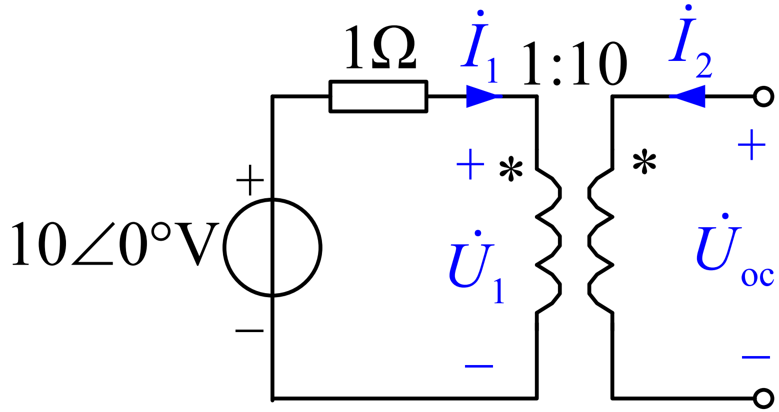
【例2】.求$\dot{U}_2$.



方法3：戴维南等效

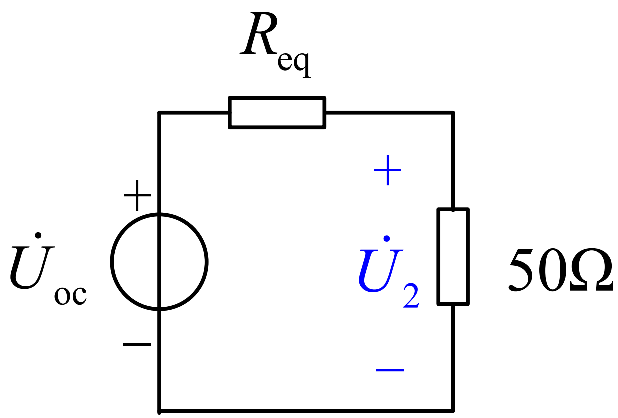
求$\dot{U}_{oc}$：

$$\dot{U}_{oc} = \dot{U}_2 = 10\dot{U}_1 = 100\angle 0^{\circ}\text{V}$$



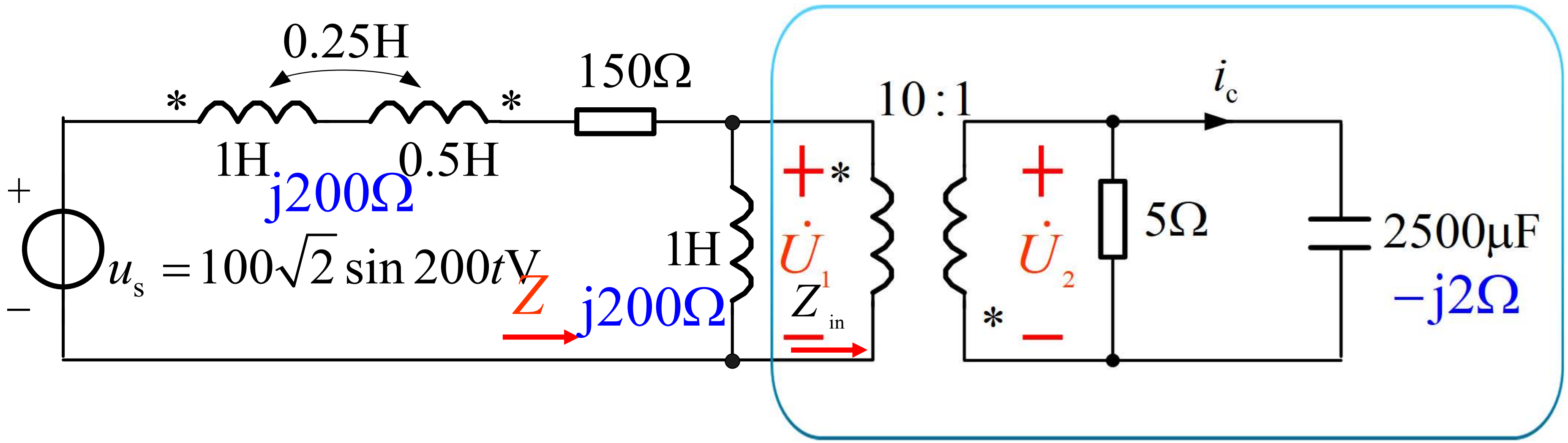
求R_{eq}：

$$R_{eq}=10^2\times 1=100\Omega$$



$$\dot{U}_2 = \frac{100\angle 0^{\circ}}{100+50}\times 50 = 33.33\angle 0^{\circ}\ \text{V}$$

【例】：计算i_C.



$$Z_{in} = n^2 Z_L = 10^2(5 // -j2)$$

$$Z = j200 // Z_{in} = j200 // 10^2(5 // -j2) = 500\Omega$$

$$\dot{U}_1 = \frac{Z}{150 + j200 + Z} \times 100\angle 0°$$

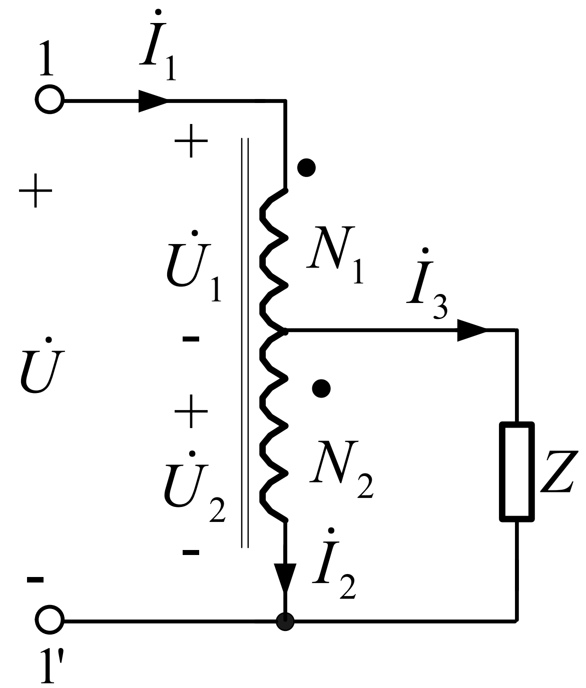
$$\dot{U}_2 = -\frac{\dot{U}_1}{10} \qquad \dot{I}_C = \frac{\dot{U}_2}{-j2}$$

（自学）13.5 理想变压器

Ideal autotransformers　自耦变压器

$$\frac{\dot U_1}{\dot U_2}=\frac{N_1}{N_2} \quad\Rightarrow\quad \frac{\dot U}{\dot U_2}=\frac{N_1+N_2}{N_2}$$

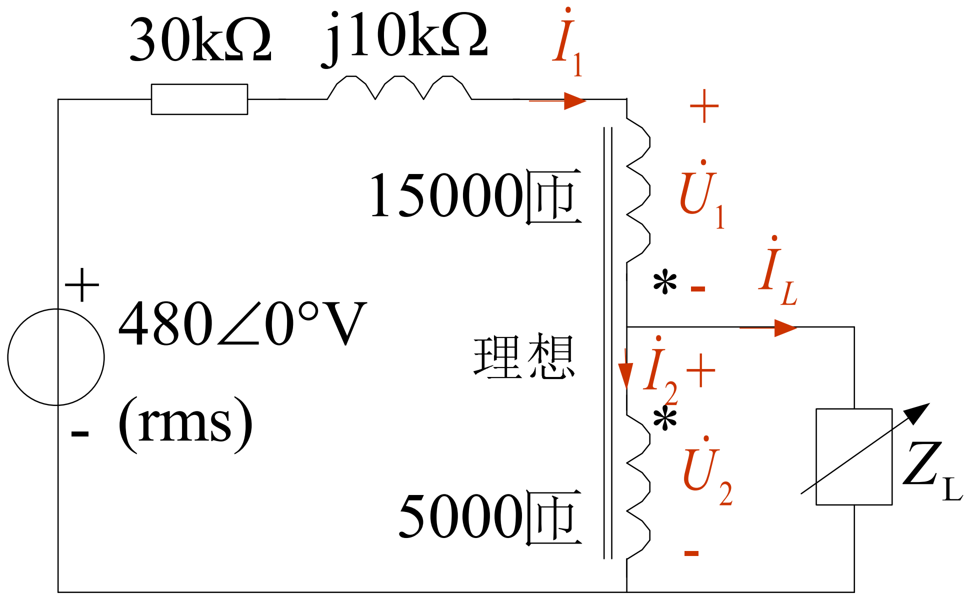
$$\frac{\dot I_1}{\dot I_2}=-\frac{N_2}{N_1} \quad\Rightarrow\quad \frac{\dot I_1}{\dot I_3}=\frac{\dot I_1}{\dot I_1-\dot I_2}=\frac{N_2}{N_1+N_2}$$



- 自耦变压器 :闭合铁心上只有一个线圈，中间接出一个抽头。或者两个线圈，串联起来作为一次或二次绕组。
- 广泛应用于电力系统，体积小，重量轻，但是不能电气隔离电源回路和负载回路。
- 自耦变压器的电压比、电流比与理想单相变压器一样。

【例3】计算负载的最大功率。

变压器方程：$\dfrac{\dot{U}_1}{\dot{U}_2} = -3 \quad \dfrac{\dot{I}_1}{\dot{I}_2} = \dfrac{1}{3}$



负载阻抗断开时：

$$\frac{\dot{I}_1}{\dot{I}_2} = \frac{1}{3} \Rightarrow \dot{I}_1 = \dot{I}_2 = 0$$

$$\dot{U}_1 + \dot{U}_2 = -3\dot{U}_2 + \dot{U}_2 = 480$$

$$\dot{U}_{2oc} = \dot{U}_2 = -240\angle 0^\circ$$

负载阻抗短接时：

$$\frac{\dot{U}_1}{\dot{U}_2} = -3 \Rightarrow \dot{U}_1 = \dot{U}_2 = 0$$

$$\dot{I}_1 = \frac{480}{30 + j10} = 14.4 - j4.8$$

$$\dot{I}_2 = 3\dot{I}_1$$

$$\dot{I}_{Lsc} = \dot{I}_1 - \dot{I}_2 = -2\dot{I}_1 = -28.8 + j9.6$$

$$Z_{eq} = \frac{\dot{U}_{2oc}}{\dot{I}_{Lsc}} = (7.5 + j2.5)\text{k}\Omega$$

$$Z_L = (7.5 - j2.5)\text{k}\Omega$$

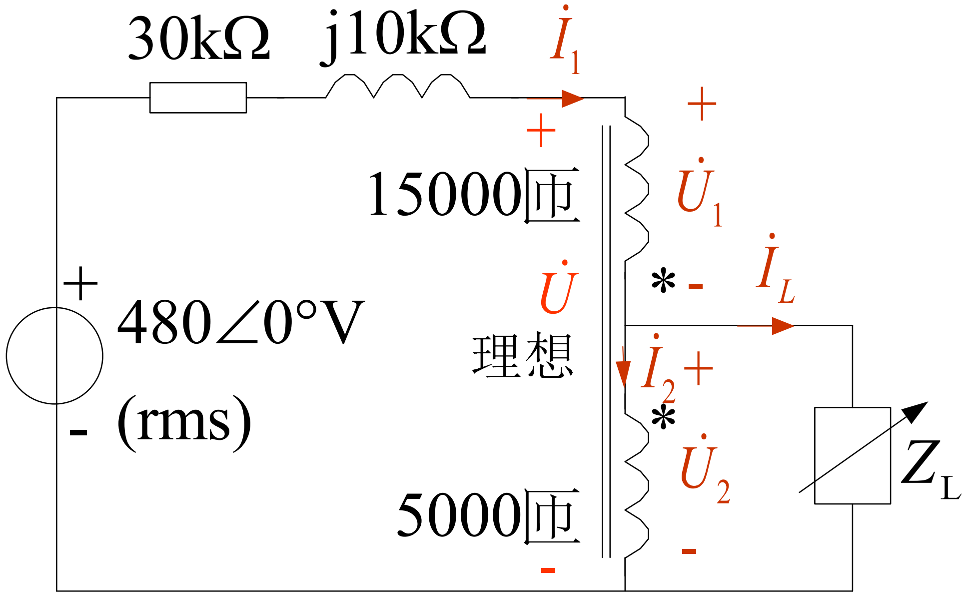
$$P_{L\max} = \frac{240^2}{4 \times 7.5} = 3.072\text{W}$$

【练习】计算负载的最大功率。

变压器方程：$\dfrac{\dot U_1}{\dot U_2}=-3 \quad \dfrac{\dot I_1}{\dot I_2}=\dfrac{1}{3}$

$$\frac{\dot U}{\dot U_2}=\frac{\dot U_1+\dot U_2}{\dot U_2}=-2$$

$$\frac{\dot I_1}{\dot I_L}=\frac{\dot I_1}{\dot I_1-\dot I_2}=-\frac{1}{2}$$



负载阻抗断开时：

$$\dot U_{2oc}=\dot U_2=\frac{1}{2}\dot U_1=-240\angle 0^\circ$$

负载阻抗短接时：

$$\dot I_1=\frac{480}{30+j10}=14.4-j4.8$$

$$\dot I_{Lsc}=-2\dot I_1=-28.8+j9.6$$

$$Z_{eq}=\frac{\dot U_{2oc}}{\dot I_{Lsc}}=\left(7.5+j2.5\right)\text{k}\Omega$$

$$Z_L=\left(7.5-j2.5\right)\text{k}\Omega$$

$$P_{L\max}=\frac{240^2}{4\times 7.5}=3.072\text{W}$$

计划学时：4学时；课后学习12学时

作业：
13-6，13-9 耦合电感
13-15 线性变压器
13-20 理想变压器
13-29综合（用到了14章知识，提示谐振的概念）