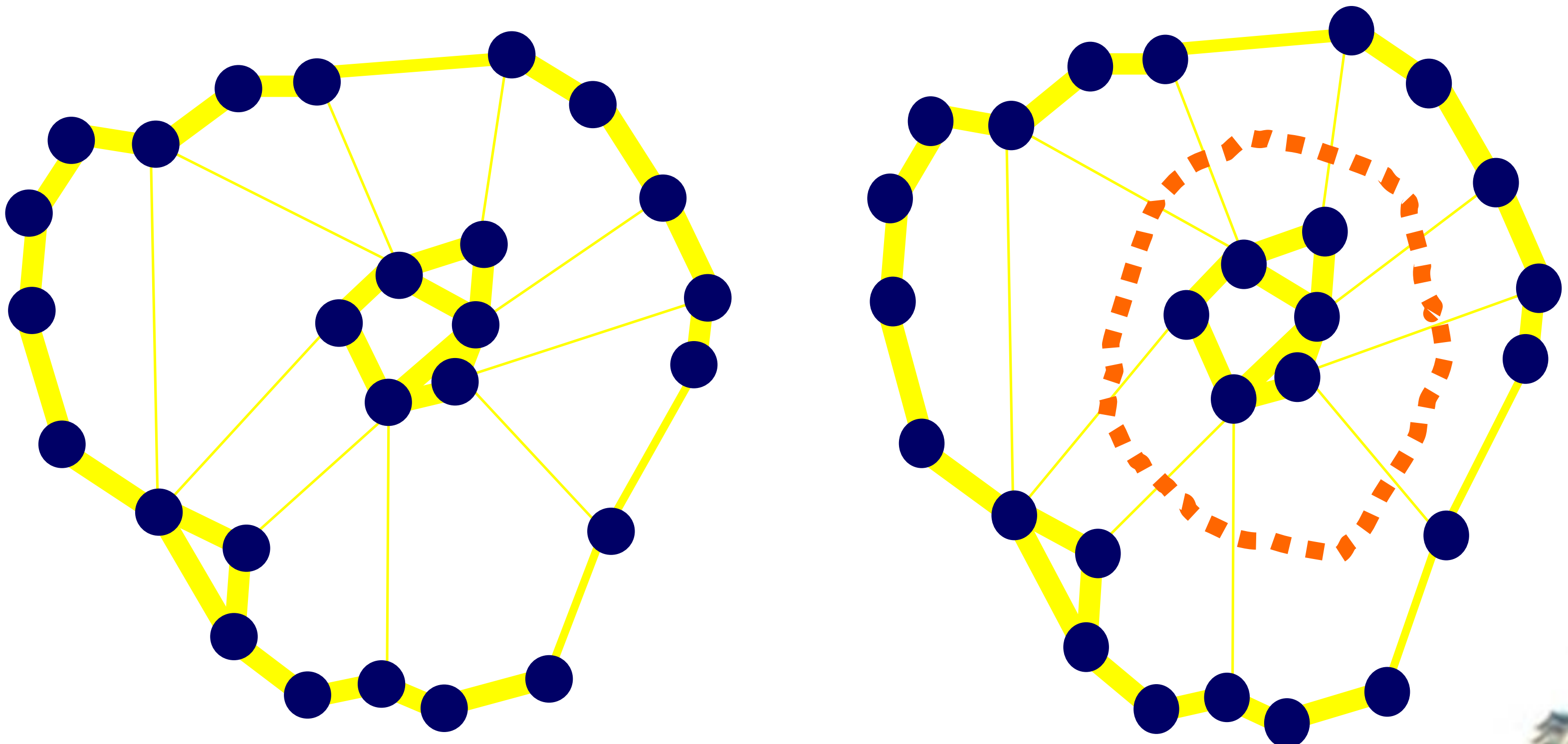
数字图像处理
Digital Image Processing

彩色图像分割
Color Image Segmentation

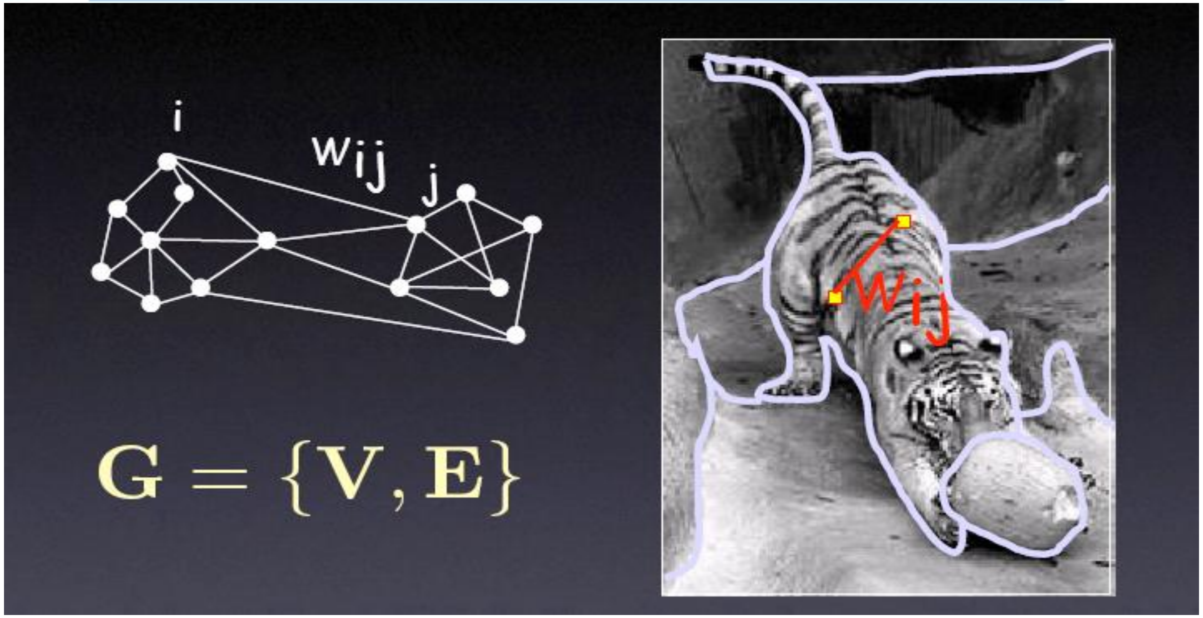
彩色图像分割及处理

1. 分水岭算法
2. Mean shift分割
3. Normalized cuts(Ncuts)分割
4. Ncuts分割改进算法

图的划分



- 将节点间的关系采用带权图来表达
- 将图划分成两个部分或多个部分



如果将图像中的每个象素看作一个节点，每对节点均用一条边连接起来，边的权值反映这两个象素之间的相似性，那么我们就可以构建一个带权的无向图G=(V,E) 。利用象素的灰度值以及它们的空间位置，可以定义图G中连接两个节点*u*和*v*的边的权值

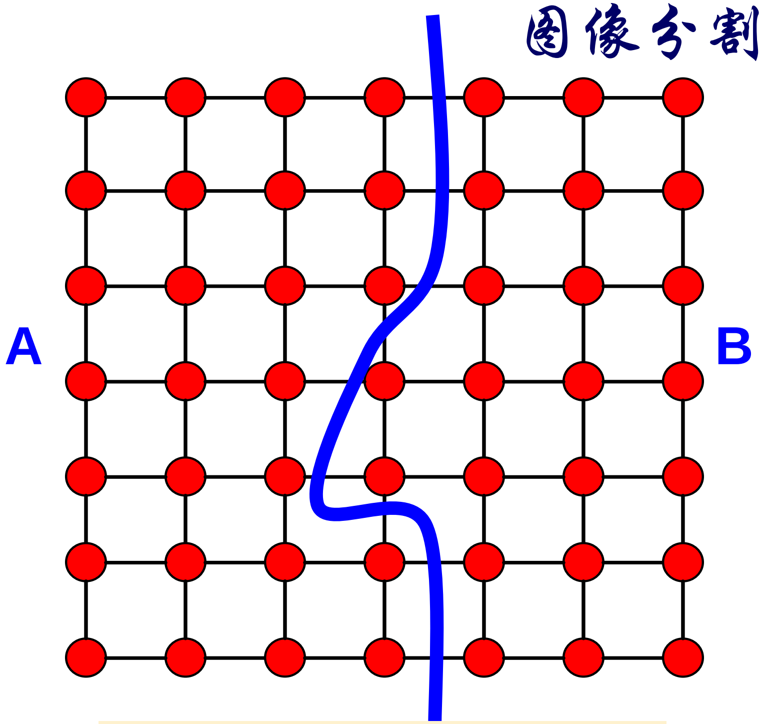
$$w(u,v)=\begin{cases} e^{-\left[\frac{\|F(u)-F(v)\|_2^2}{d_I}+\frac{\|X(u)-X(v)\|_2^2}{d_X}\right]} & \text{if } \|X(u)-X(v)\|_2 < r \\ 0 & \text{otherwise} \end{cases}$$

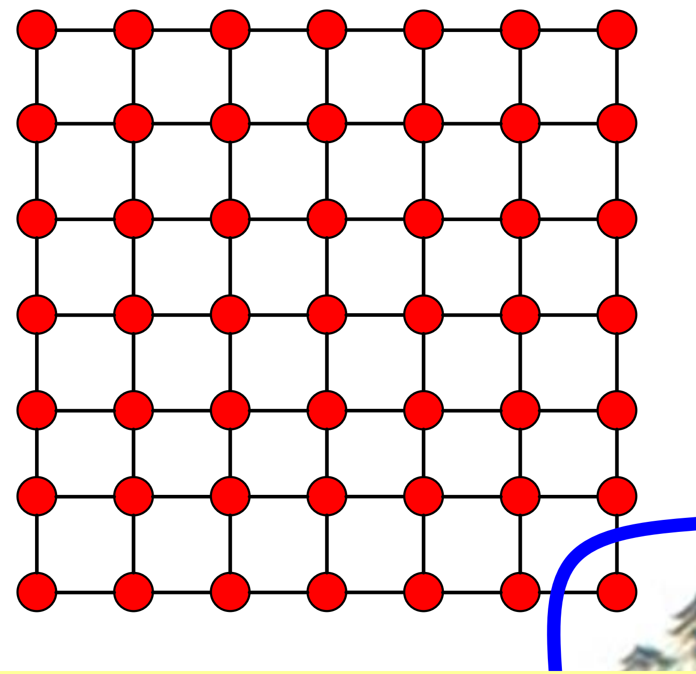
Segmentation=Graph Partition

图割(Graph Cuts)优化算法

图像 — 图

图像分割 — 图划分





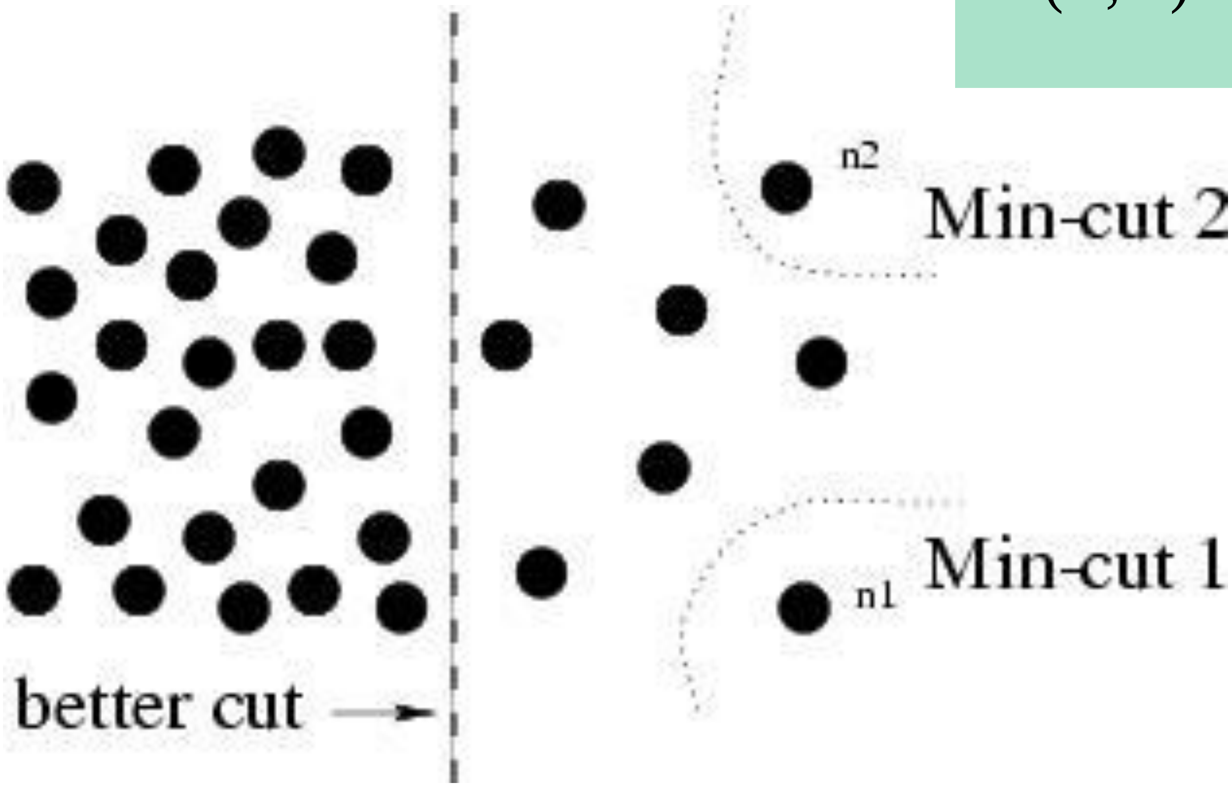
$$c(A,B)=\sum_{u\in A}\sum_{v\in B}c(u,v)$$

Minimum Cut 容易产生孤立点

- 图的割集是与切割的边的数量及权值相关的
- 一个割集 **cut(A, B)** 将图分为独立的两个部分

Disassociation Measures

$$cut(A,B)=\sum_{u\in A, v\in B} w(u,v)$$



- Minimizing the cut will give a partition with the maximum disassociation.
- However, this measure favors cutting to small sets of isolated nodes.

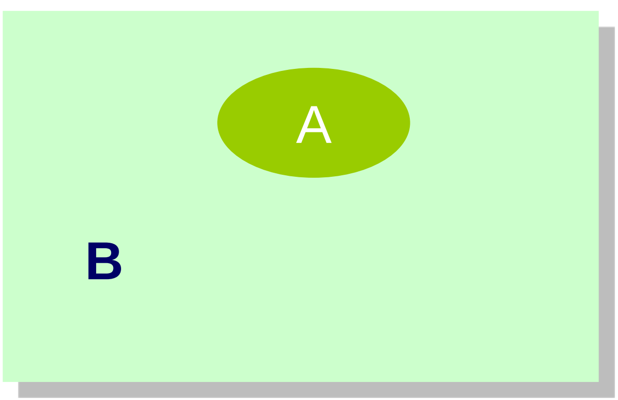
Disassociation Measures

- Normalized cut Ncut(A,B) measures similarity between two groups, normalized by the “volume” they occupy in the whole graph [Shi and Malik, 2000].

- It is more appropriate to measure the disassociation between groups A and B.

minimize

$$Ncut(A,B) = \frac{cut(A,B)}{asso(A,V)} + \frac{cut(A,B)}{asso(B,V)}$$

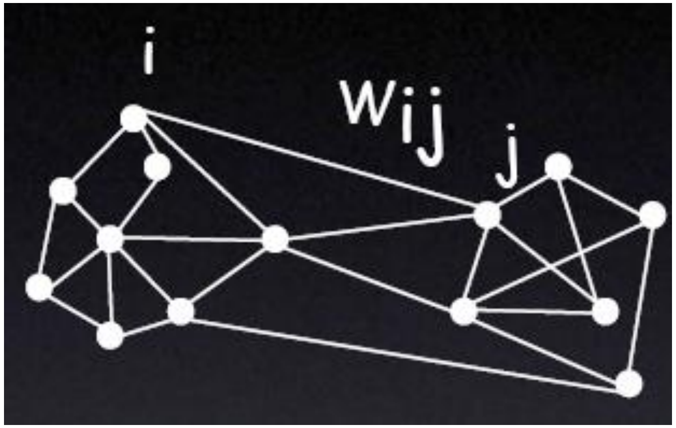


A + B = V

$$asso(A,V) = asso(A,A) + cut(A,B)$$

$$asso(B,V) = asso(B,B) + cut(A,B)$$

Graph-based Image Segmentation



G = {V,E}

V: graph nodes
E: edges connection nodes

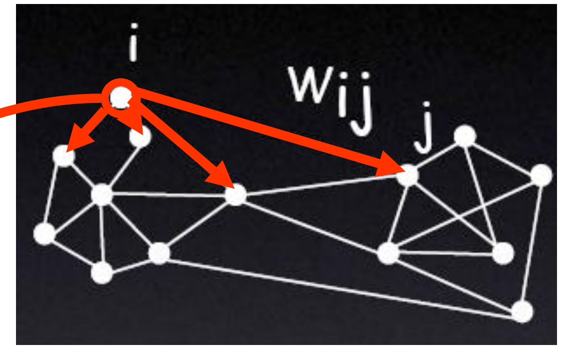
Pixels
Pixel similarity

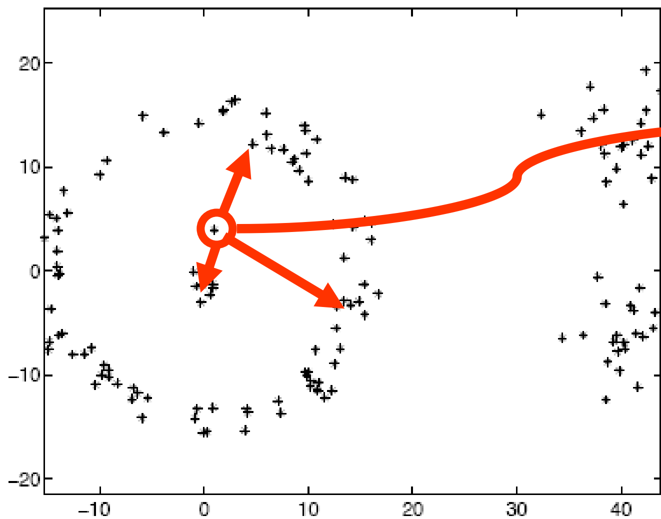
Slides from Jianbo Shi

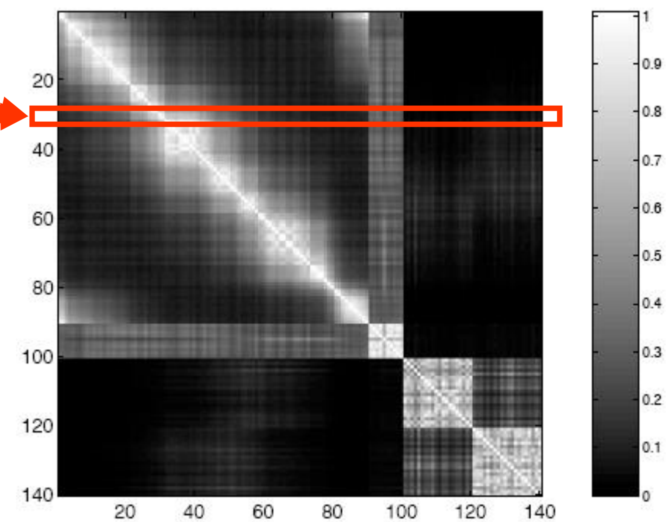
Graph terminology

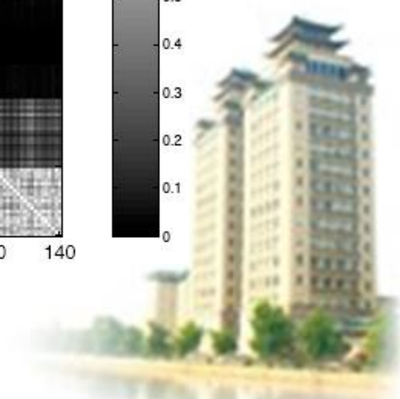
- Similarity matrix: $W = \left[w_{i,j} \right]$

$$w_{i,j} = e^{\frac{-\left\| X_{(i)} - X_{(j)} \right\|_2^2}{\sigma_X^2}}$$

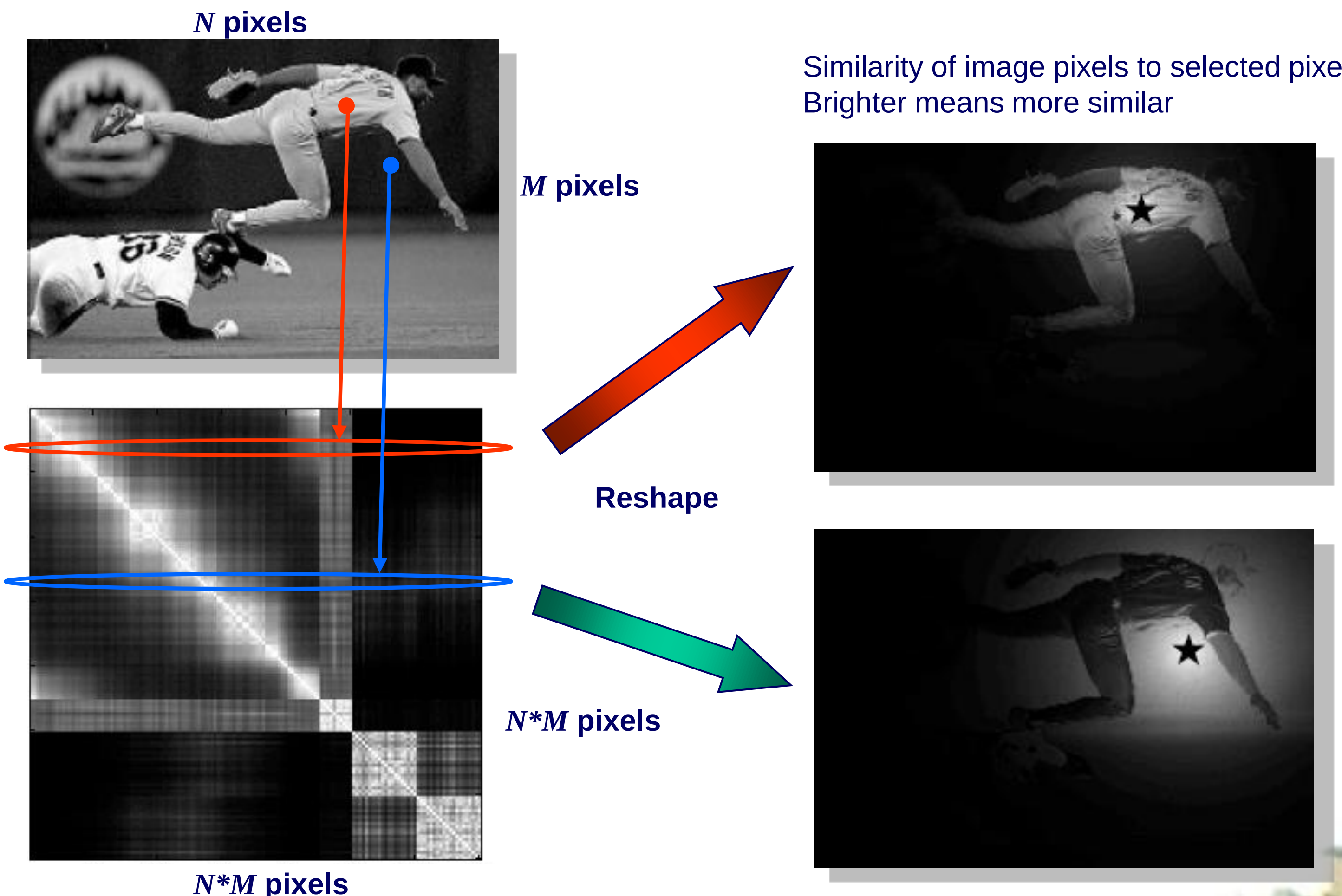








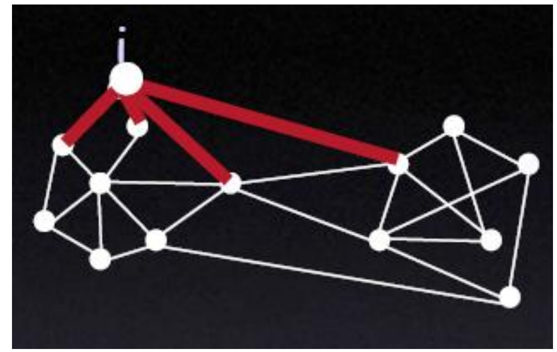
Affinity matrix

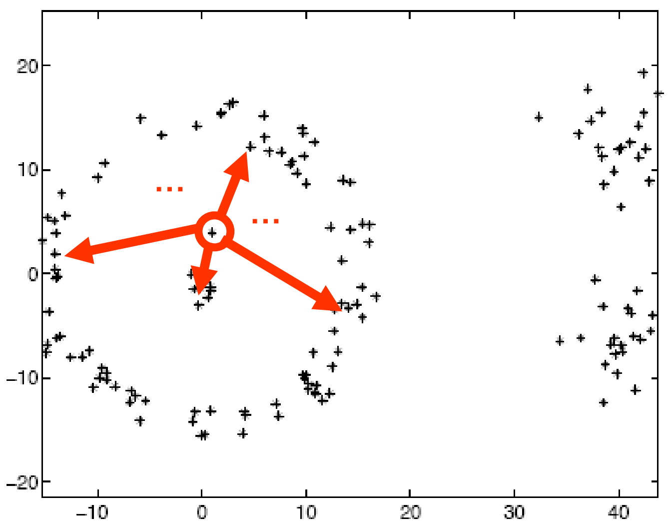


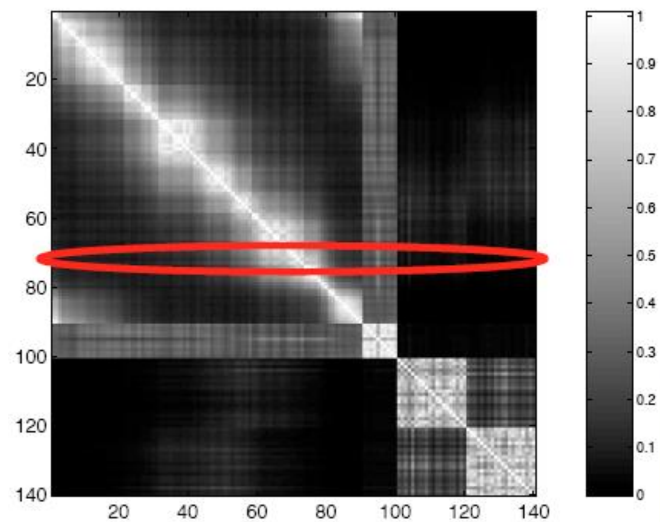
Graph terminology

- Degree of node:

$$d_i = \sum_j w_{i,j}$$



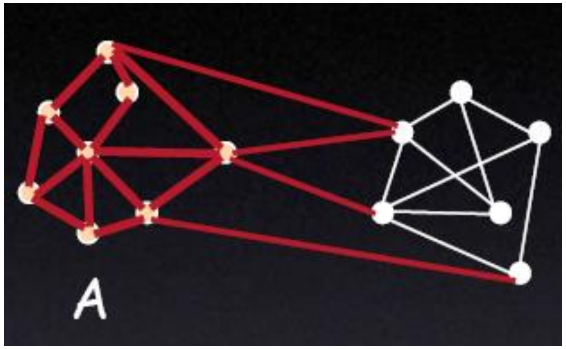


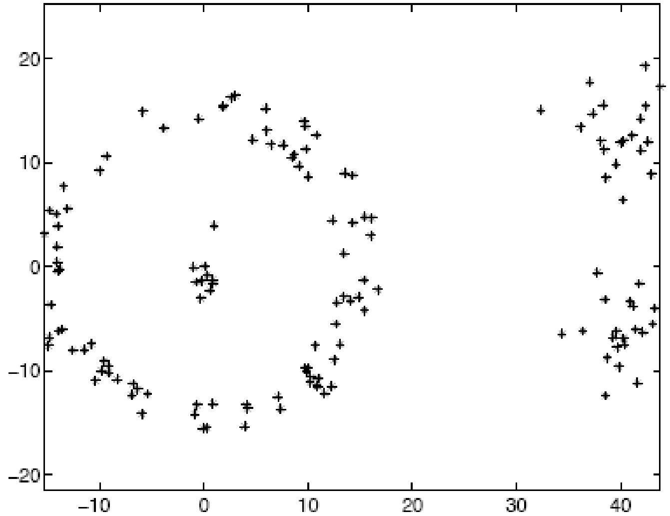


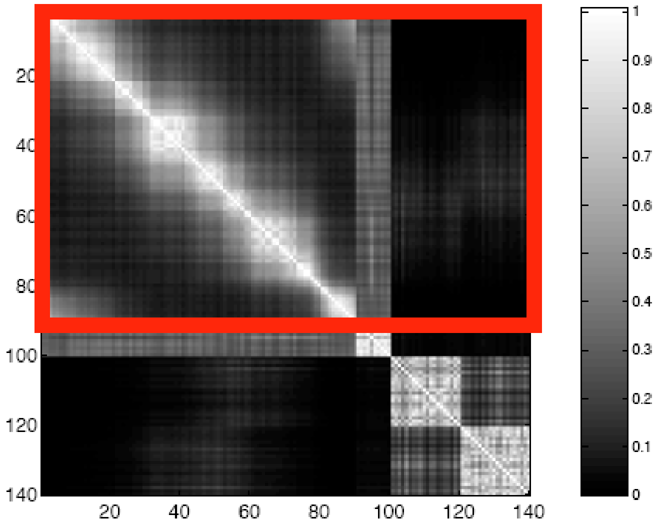
Graph terminology

- Volume of set:

$$vol(A) = \sum_{i \in A} d_i, A \subseteq V$$





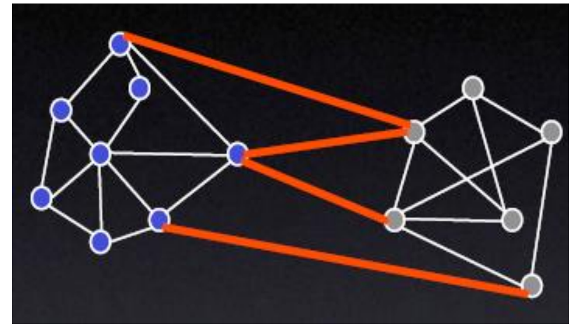


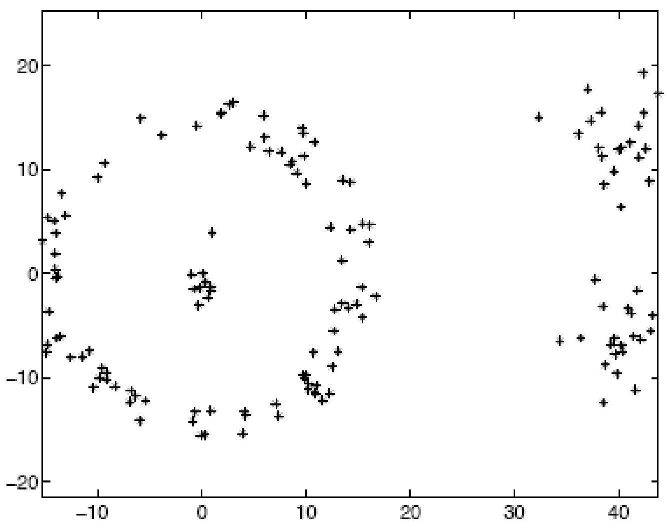
总计140个点，假定1-95为集合A，96-140为集合B

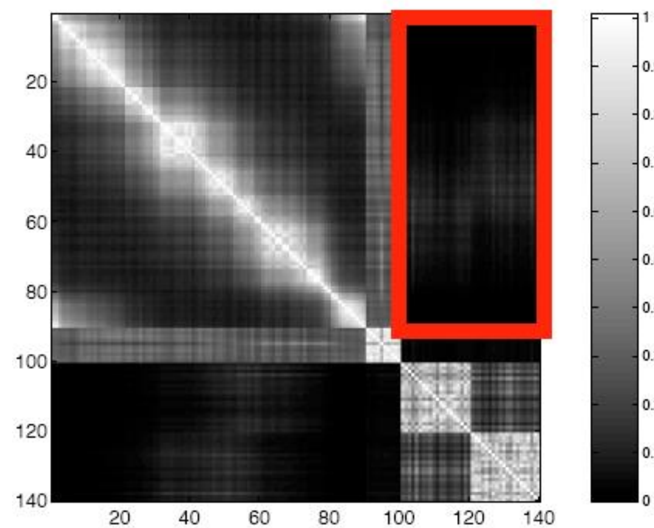
Graph terminology

- Cuts in a graph:

$$cut(A,\overline{A}) = \sum_{i \in A,\, j \in \overline{A}} w_{i,j}$$

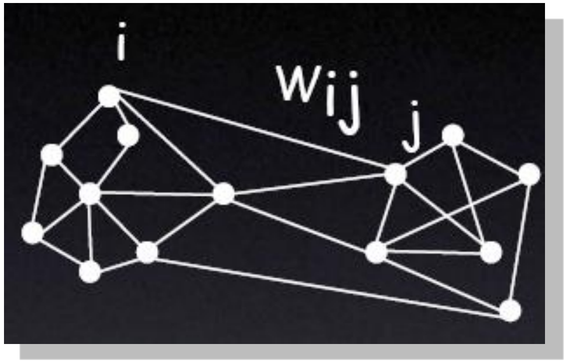




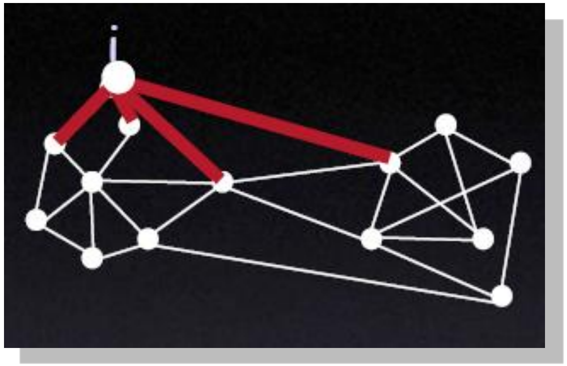


Graph terminology

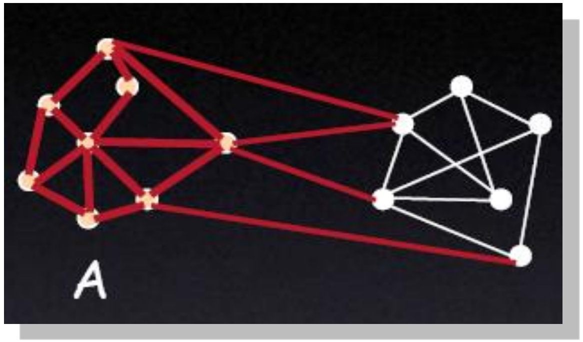
Similarity matrix: $W = \left[w_{i,j} \right]$



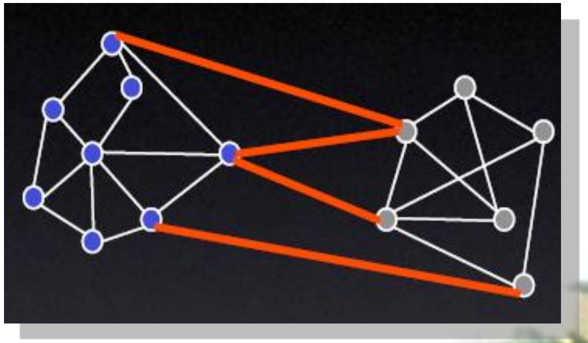
Degree of node: $d_i = \sum w_{i,j}$



Volume of set:



Graph cuts:

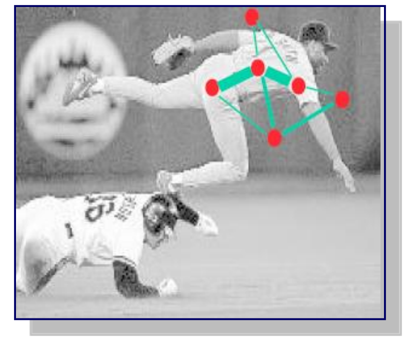


Representation

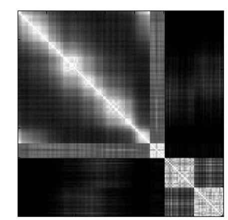
Partition matrix *X*:

$$X = [X_1, \ldots, X_K]$$

$$X = \begin{array}{cc} \text{segments} & \\ \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \text{pixels} \end{array}$$



Pair-wise similarity matrix *W*: $W(i,j) = aff(i,j)$



Degree matrix *D*: $D(i,i) = \sum_j w_{i,j}$

Laplacian matrix *L*: $L = D - W$

Pixel similarity functions

Intensity
$$W(i,j)=e^{\frac{-\left\|I_{(i)}-I_{(j)}\right\|_2^2}{\sigma_I^2}}$$

Distance
$$W(i,j)=e^{\frac{-\left\|X_{(i)}-X_{(j)}\right\|_2^2}{\sigma_X^2}}$$

Texture
$$W(i,j)=e^{\frac{-\left\|c_{(i)}-c_{(j)}\right\|_2^2}{\sigma_c^2}}$$

$$w(u,v)=\begin{cases} e^{-\left[\frac{\left\|F(u)-F(v)\right\|_2^2}{d_I}+\frac{\left\|X(u)-X(v)\right\|_2^2}{d_X}\right]} & \text{if } \left\|X(u)-X(v)\right\|_2<r \\ 0 & \text{otherwise} \end{cases}$$

Disassociation Measures

- The solution to the problem

minimize $$Ncut(A,B) = \frac{cut(A,B)}{asso(A,V)} + \frac{cut(A,B)}{asso(B,V)}$$

is given by the following eigen-system

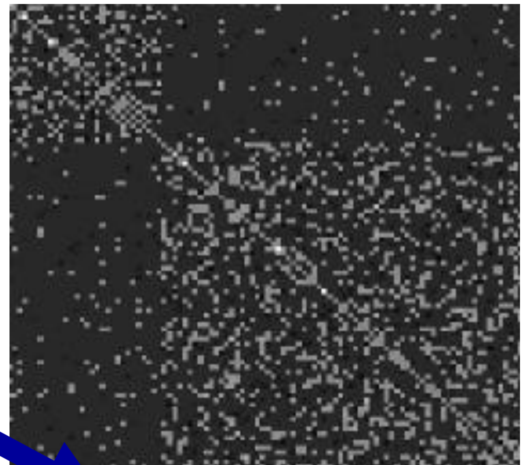
$$\mathbf{D}^{-\frac{1}{2}}(\mathbf{D}-\mathbf{W})\mathbf{D}^{-\frac{1}{2}}z = \lambda z$$

$$W_{ij} = W(x_i, x_j) \qquad D_{ii} = \sum_j W_{ij}$$

- For an image with N pixels, the matrix size is NxN.
- Computational cost increases dramatically as the image size increases!

Graph-based Image Segmentation

Image (I)

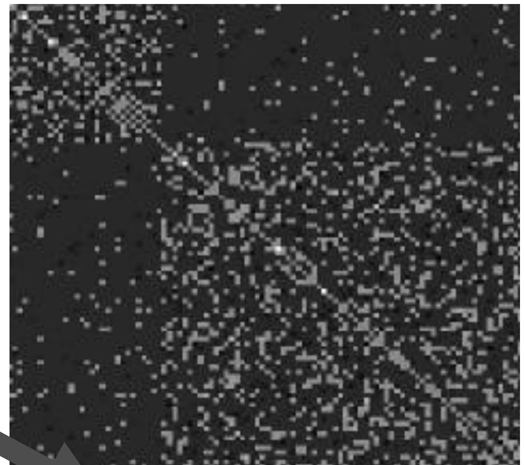


Graph Affinities (W)

Intensity
Color
Edges
Texture

Graph-based Image Segmentation

Image (I)



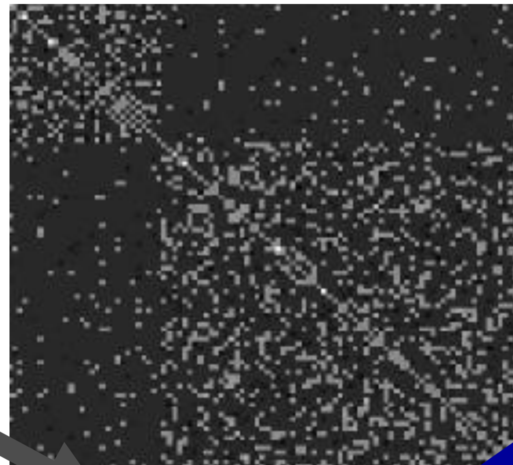
Graph Affinities
(W)

Intensity
Color
Edges
Texture

$$Ncut(A,B) = cut(A,B)\left(\frac{1}{vol(A)} + \frac{1}{vol(B)}\right)$$

Graph-based Image Segmentation

Image (I)



Graph Affinities
(W)

Eigenvector
X(W)

Intensity
Color
Edges
Texture

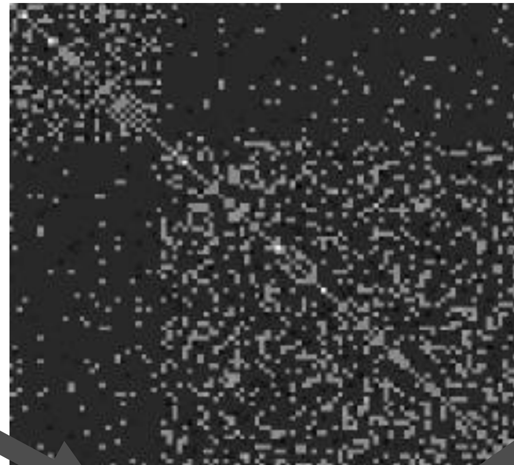
$$Ncut(A,B) = cut(A,B)\left(\frac{1}{vol(A)} + \frac{1}{vol(B)}\right)$$

$$(D-W)X = \lambda DX$$

$$X_A(i) = \begin{cases} 1 & if \quad i \in A \\ 0 & if \quad i \notin A \end{cases}$$

Graph-based Image Segmentation

Image (I)



Graph Affinities
(W)

Intensity
Color
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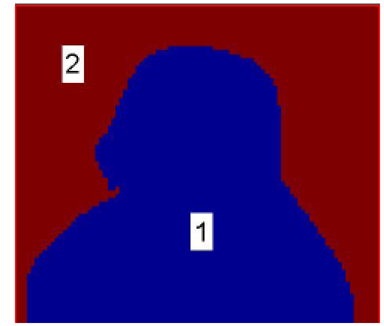
Eigenvector
X(W)



Discretization

$$Ncut(A,B) = cut(A,B)\left(\frac{1}{vol(A)} + \frac{1}{vol(B)}\right)$$

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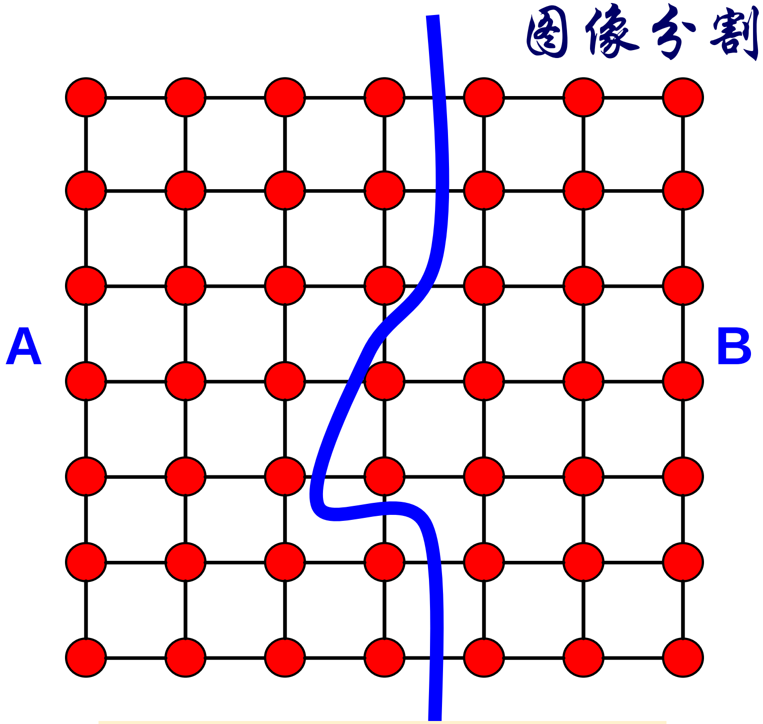
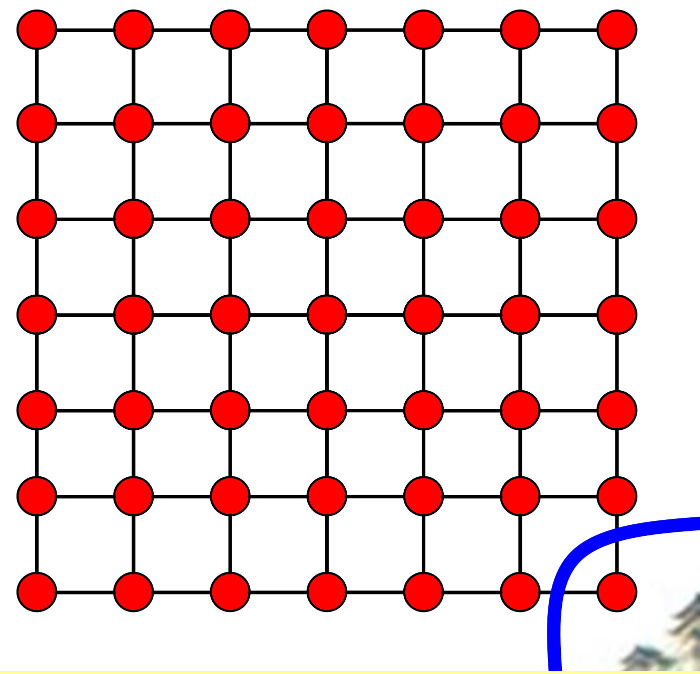
改进1——基于Ncuts的阈值分割

Wenbing Tao, et.al, "Image Thresholding Using Graph Cuts", *IEEE Transactions on Systems Man and Cybernetics Part A-Systems and Humans*. 2008.

图像 — 图

图像分割 — 图划分





$$c(A,B)=\sum_{u\in A}\sum_{v\in B}c(u,v)$$

Minimum Cut 容易产生孤立点

- 图的割集是与切割的边的数量及权值相关的
- 一个割集 **cut(A, B)** 将图分为独立的两个部分

Disassociation Measures

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$$W_{ij}=W(x_i,x_j)$$

$$D_{ii}=\sum_j W_{ij}$$

- For an image with N pixels, the matrix size is NxN.
- Computational cost increases dramatically as the image size increases!

■正确的划分函数
■有效的优化算法

$$\boxed{Ncut(A,B)} = \frac{cut(A,B)}{asso(A,V)} + \frac{cut(A,B)}{asso(B,V)}$$

Jianbo Shi

最小化

NP难问题

$$\mathbf{D}^{-\frac{1}{2}}(\mathbf{D}-\mathbf{W})\mathbf{D}^{-\frac{1}{2}}z = \lambda z$$

计算 Laplacian 矩阵 D-W的特征矢量

矩阵D、W和D-W的维数为图像中象素的个数

问题：维数太高，效率低下？

基于图划分的阈值法基本原理

$$Ncut(A,B)=\frac{cut(A,B)}{asso(A,V)}+\frac{cut(A,B)}{asso(B,V)}$$

对每一个设定阈值T(0≦T≦255)计算*Ncut(A, B)*
最小的*Ncut*对应的阈值T为最佳阈值

计算高维权值矩阵耗时
权值矩阵太大无法存储

影响阈值方法的效率和实现

Proposed Approach

- Consider V_k, k = 0, …, 255 corresponds to the gray scale levels.

$$V_k = \{(x,y): f(x,y) = k, (x,y) \in V\}, k \in L$$

$$cut(V_i, V_j) = \sum_{u \in V_i, v \in V_j} w(u,v)$$

$$A = \bigcup_{k=0}^{t} V_k \qquad B = \bigcup_{k=t+1}^{255} V_k$$

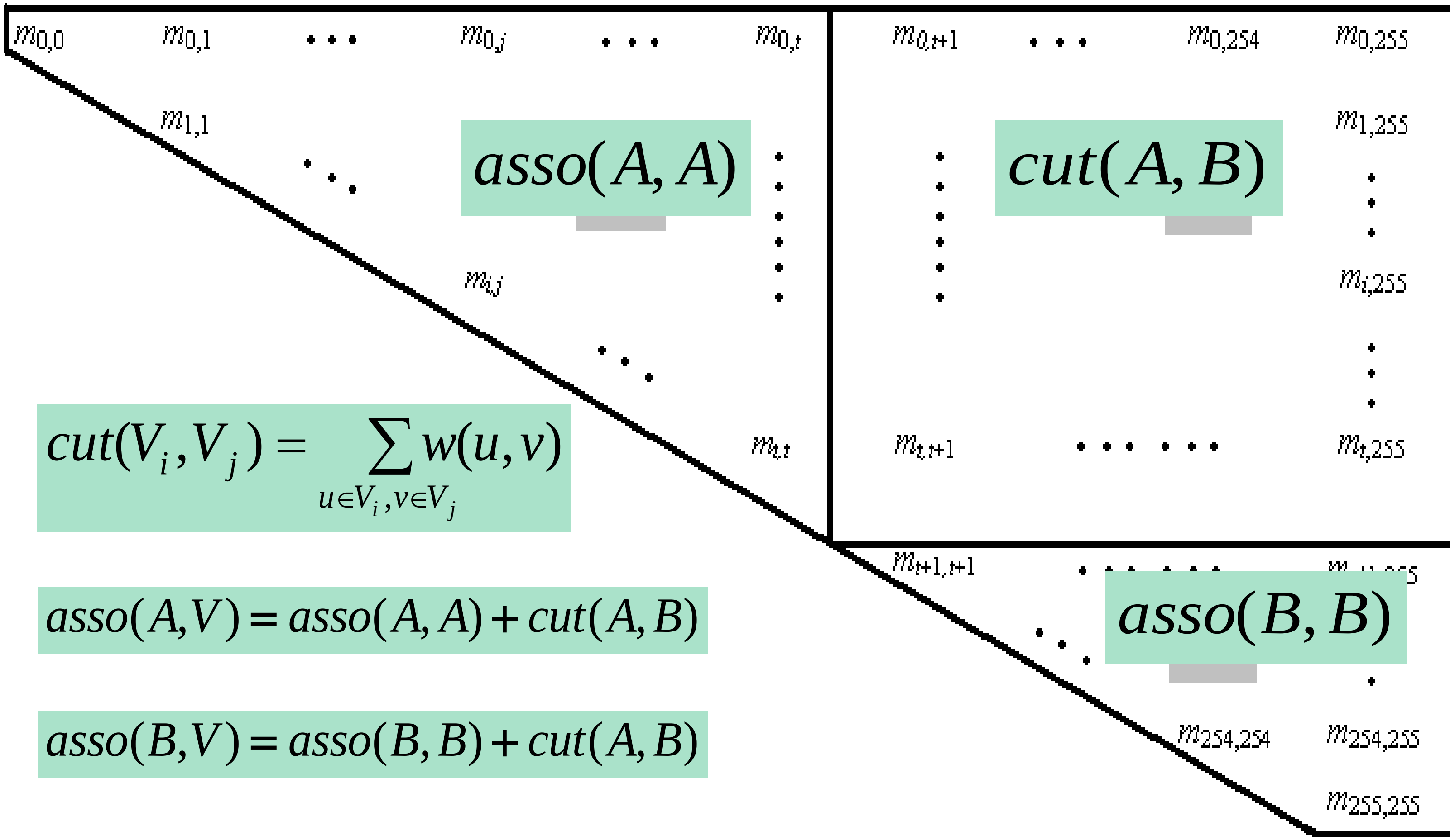
$$cut(A,B) = \sum_{u \in A, v \in B} w(u,v) = \sum_{u \in A}\Big[\sum_{v \in B} w(u,v)\Big]$$

$$= \sum_{i=0}^{t}\sum_{u \in V_i}\Big[\sum_{j=t+1}^{255}\sum_{v \in V_j} w(u,v)\Big] = \sum_{i=0}^{t}\sum_{j=t+1}^{255}\Big[\sum_{u \in V_i, v \in V_j} w(u,v)\Big] = \sum_{i=0}^{t}\sum_{j=t+1}^{255}[cut(V_i,V_j)]$$

$$asso(A,A) = \sum_{u \in A, v \in A} w(u,v) = \sum_{i=0}^{t}\sum_{j=i}^{t}\Big[\sum_{u \in V_i, v \in V_j} w(u,v)\Big] = \sum_{i=0}^{t}\sum_{j=i}^{t}[cut(V_i,V_j)]$$

$$asso(B,B) = \sum_{u \in B, v \in B} w(u,v) = \sum_{i=t+1}^{255}\sum_{j=i}^{255}\Big[\sum_{u \in V_i, v \in V_j} w(u,v)\Big] = \sum_{i=t+1}^{255}\sum_{j=i}^{255}[cut(V_i,V_j)]$$

Proposed Approach



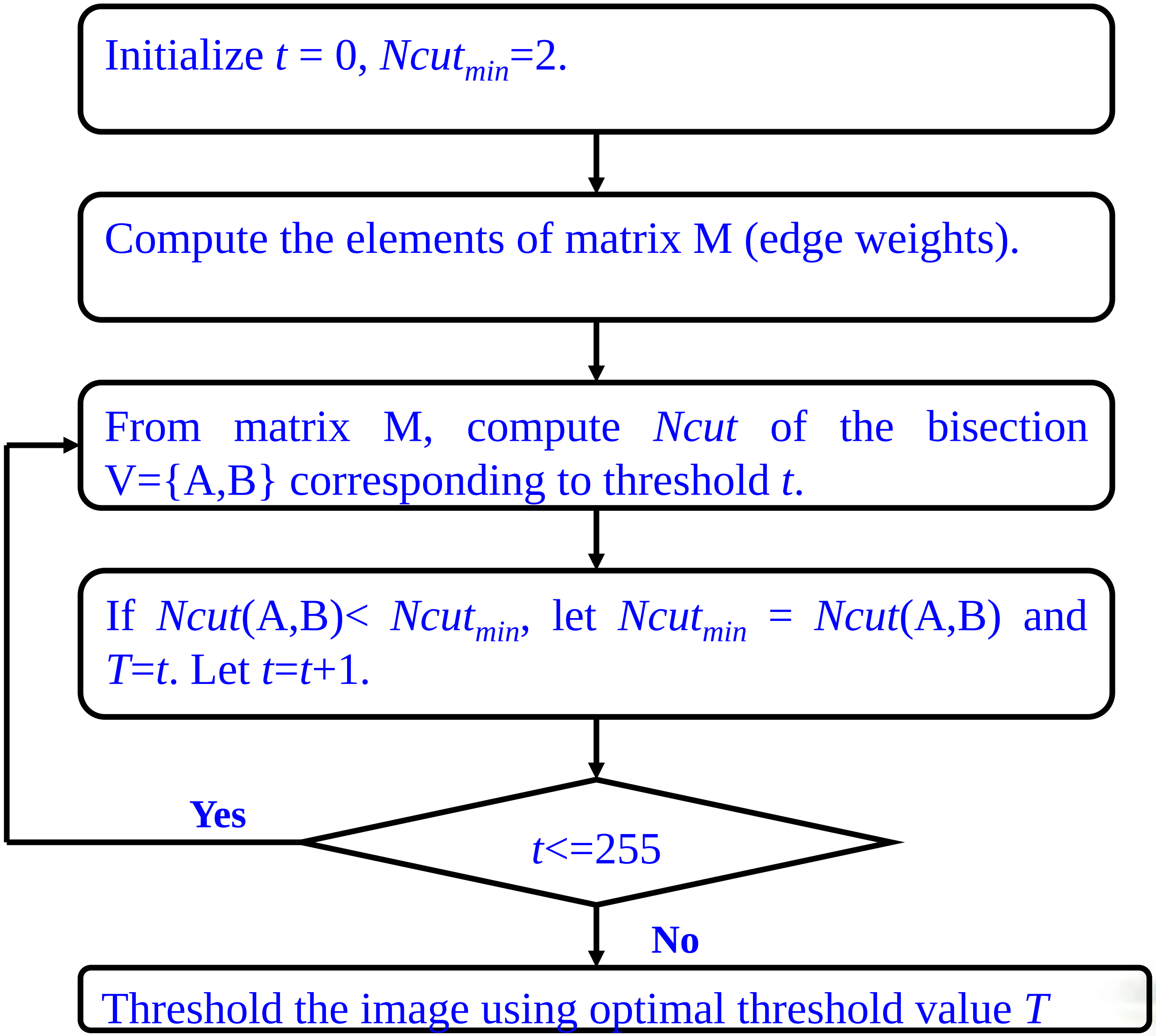
$$cut(V_i,V_j)=\sum_{u\in V_i, v\in V_j} w(u,v)$$

$$asso(A,V)=asso(A,A)+cut(A,B)$$

$$asso(B,V)=asso(B,B)+cut(A,B)$$

$$Ncut(A,B)=\frac{cut(A,B)}{asso(A,A)+cut(A,B)}+\frac{cut(A,B)}{asso(B,B)+cut(A,B)}$$

Proposed Approach



Advantages of the Proposed Approach

- **Low computational cost**
- **Suited for real-time vision processing**
- **Provide superior and robust image thresholding performance**

Experimental Results

Compared methods

- **Pikaz**
- **Kittler**
- **Kapur**
- **Yanowitz**
- **Ramesh**
- **Pal**

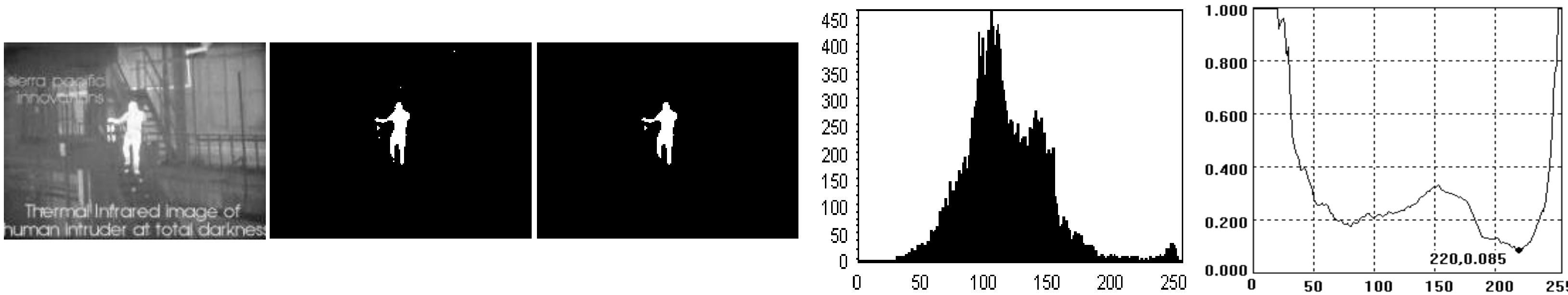
Test images

- **Infrared object images**
- **Standard test images**

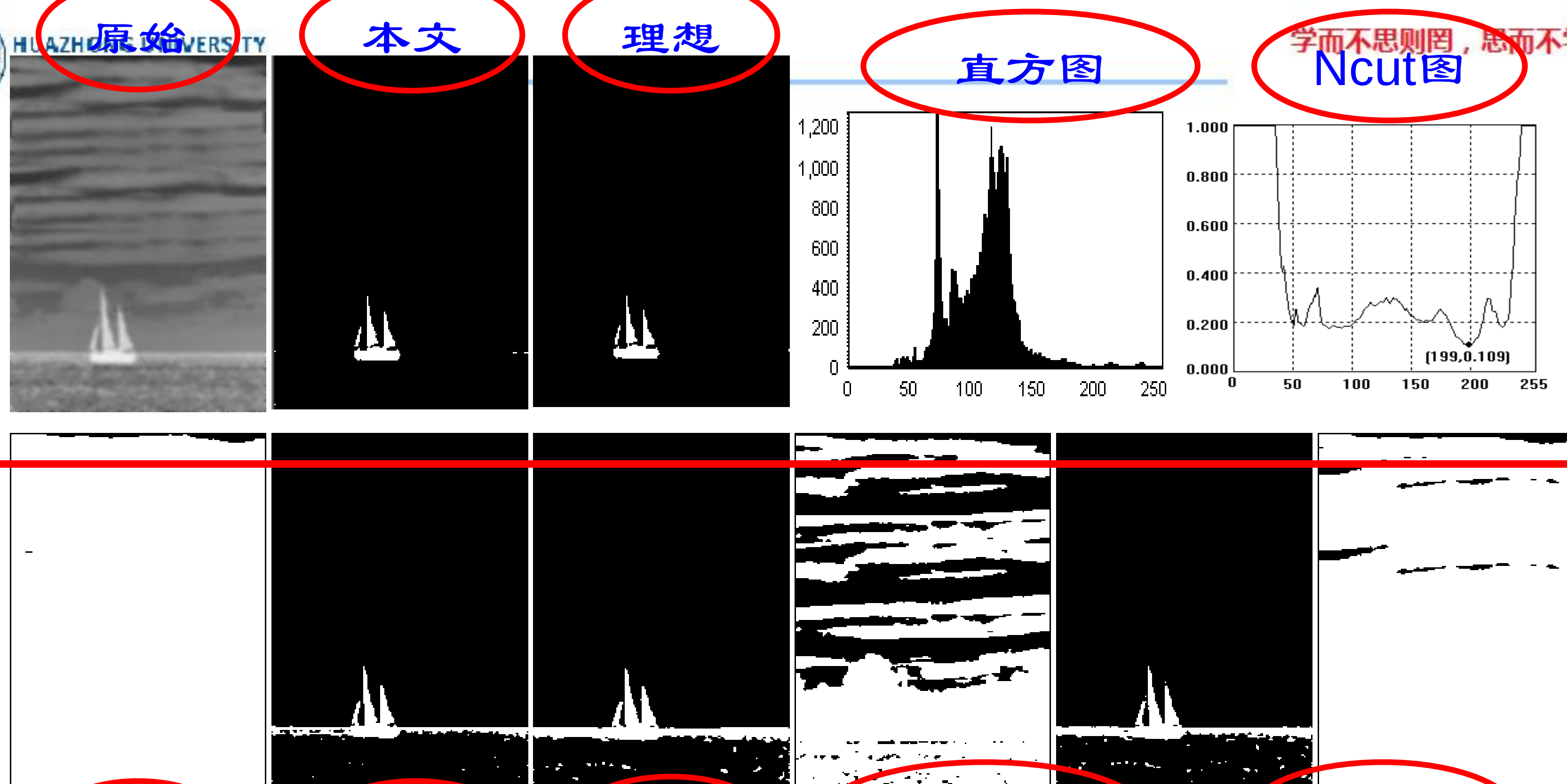
Experimental Results

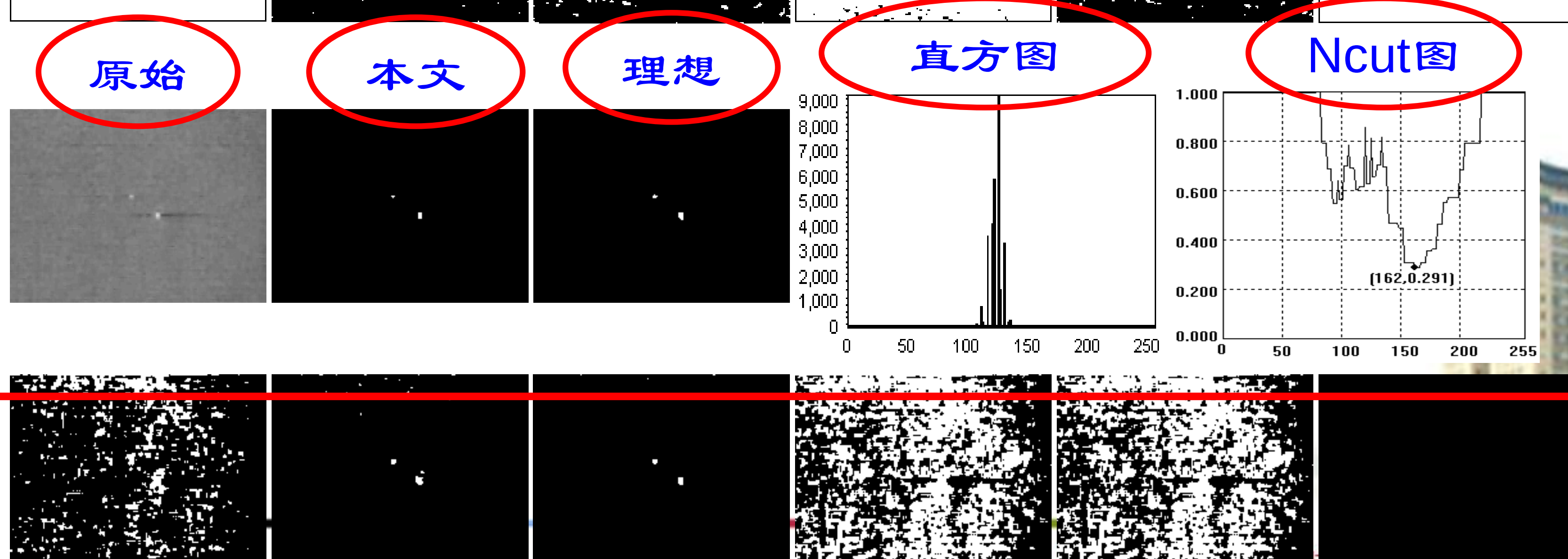
- **Intruder – infrared image: 185 x 141**

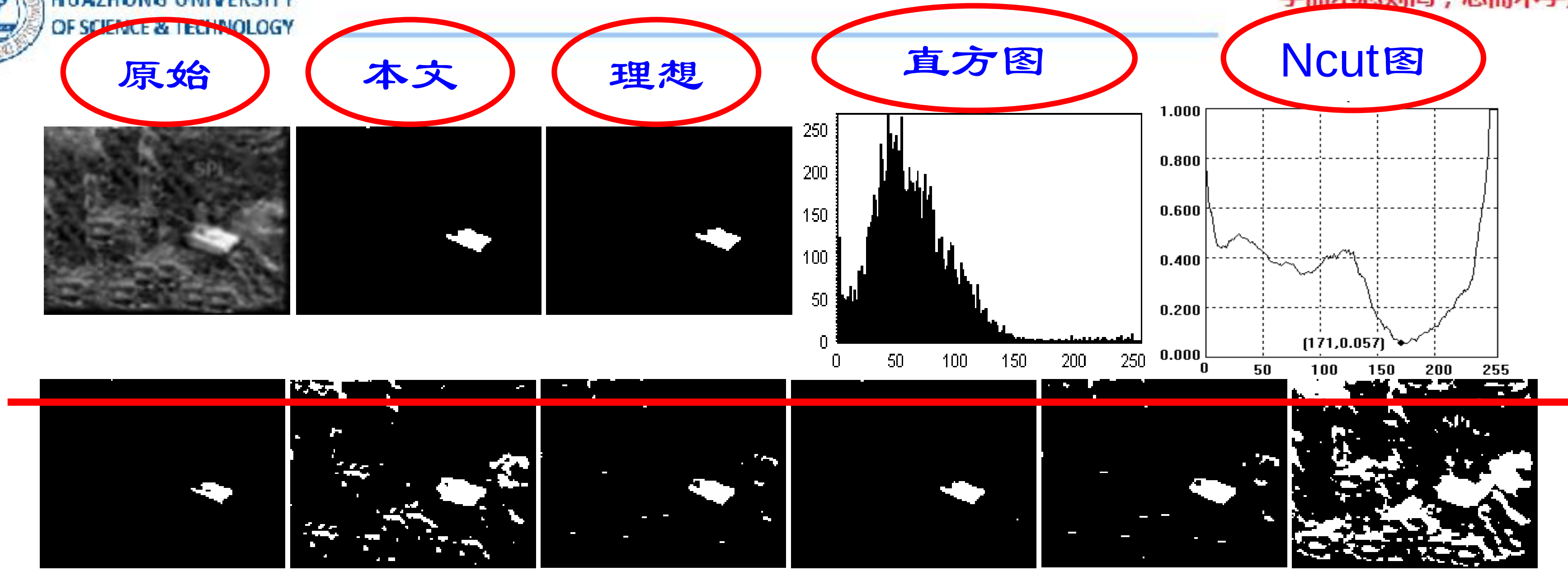
original | proposed | manual | histogram | Ncut

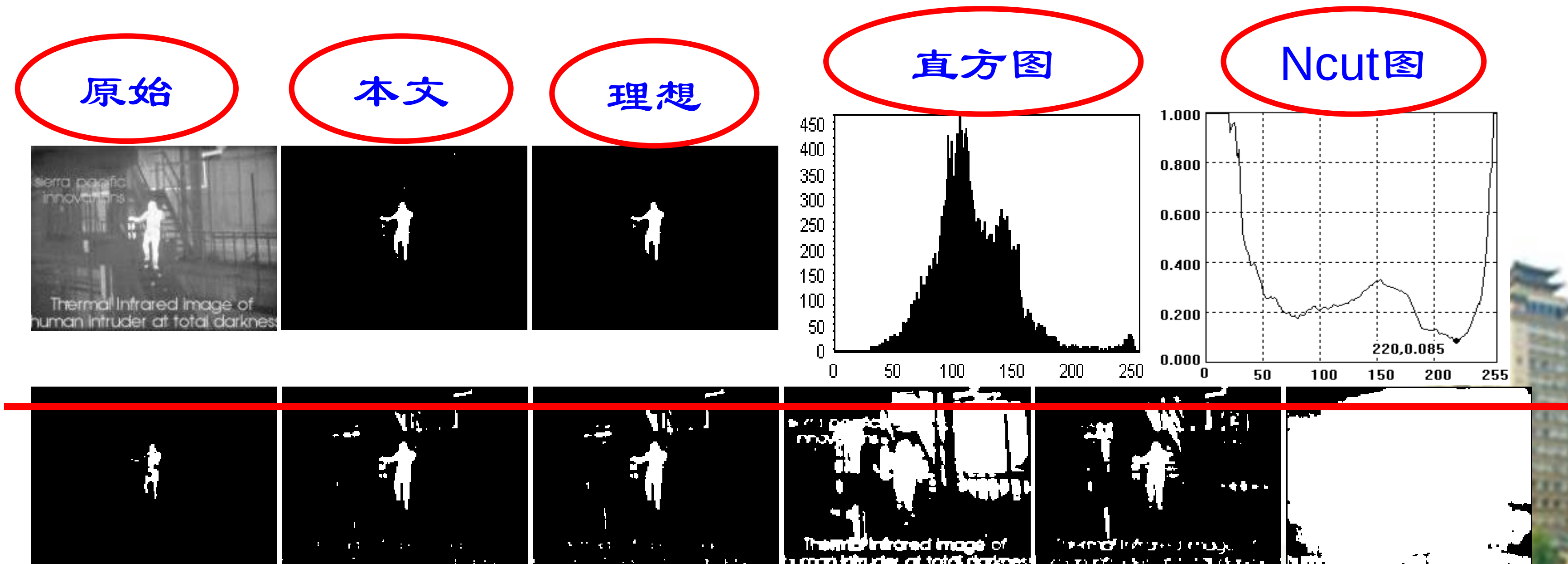


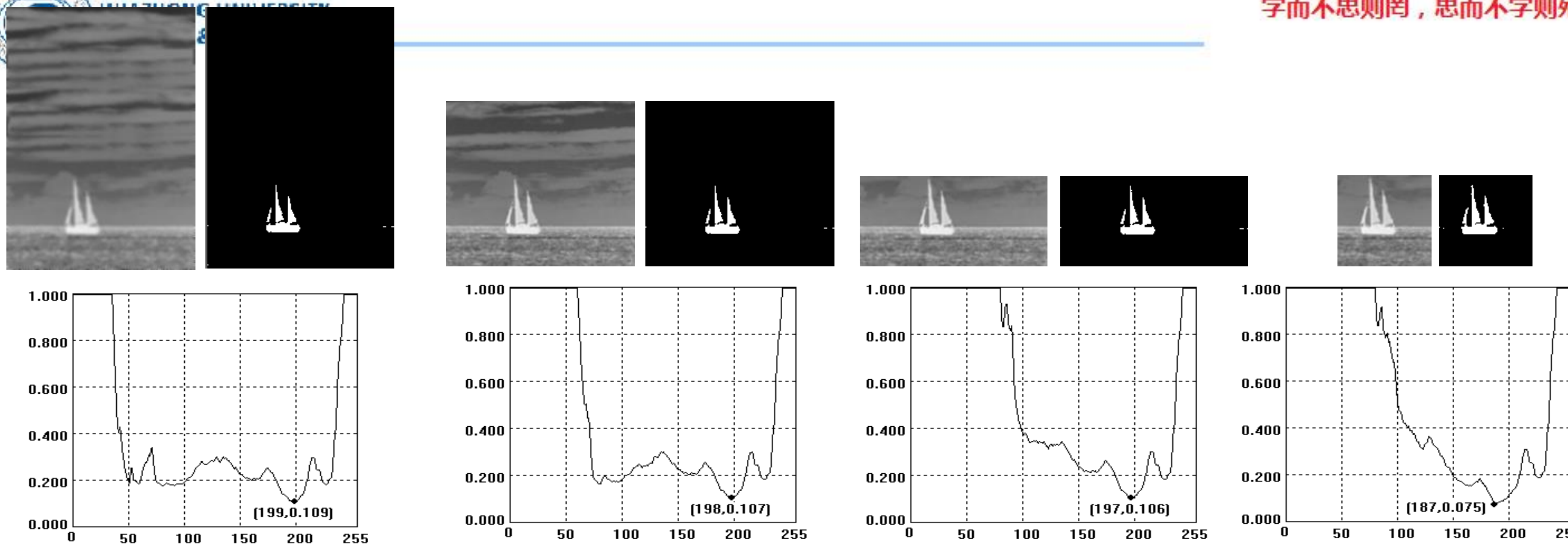
Other methods

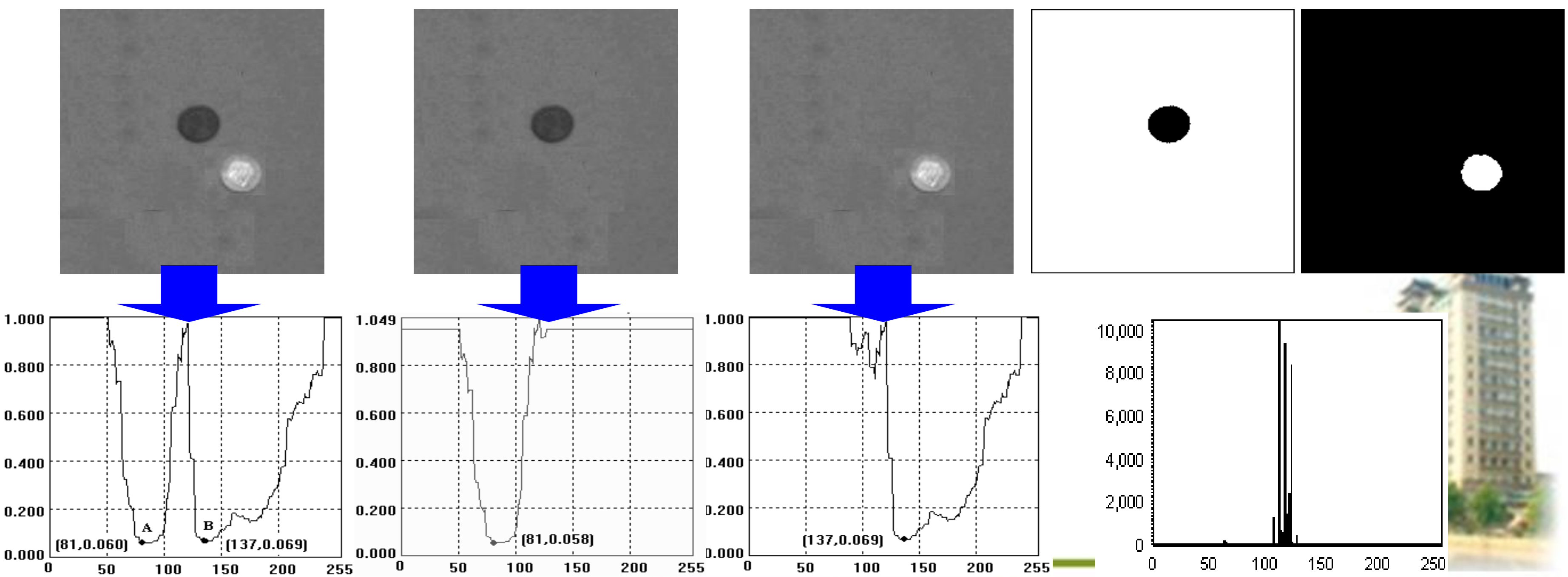

原始
本文
理想
直方图
Ncut图
(199,0.109)


原始
本文
理想
直方图
Ncut图
(162,0.291)


原始
本文
理想
直方图
Ncut图
250
200
150
100
50
0
0 50 100 150 200 250
1.000
0.800
0.600
0.400
0.200
0.000
0 50 100 150 200 255
(171,0.057)


原始
本文
理想
直方图
Ncut图
sierra pacific innovations
Thermal Infrared image of human intruder at total darkness
450
400
350
300
250
200
150
100
50
0
0 50 100 150 200 250
1.000
0.800
0.600
0.400
0.200
0.000
0 50 100 150 200 255
220,0.085


(199,0.109)
(198,0.107)
(197,0.106)
(187,0.075)


A
B
(81,0.060)
(137,0.069)
(81,0.058)
(137,0.069)

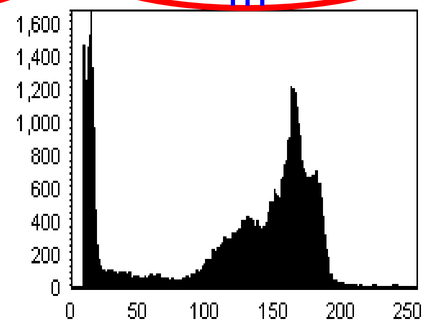
Experimental Results

- **Good results for standard test images**

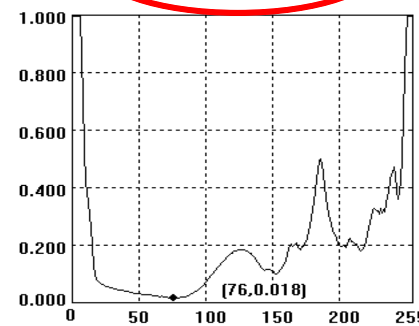
original

proposed

histogram



Ncut



Other methods

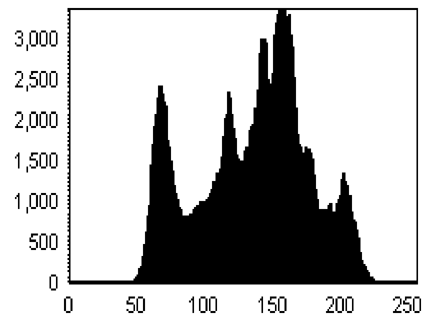
Experimental Results

- **Good results for standard test images**

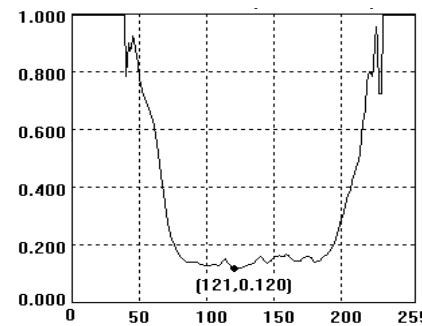
original

proposed

histogram



Ncut



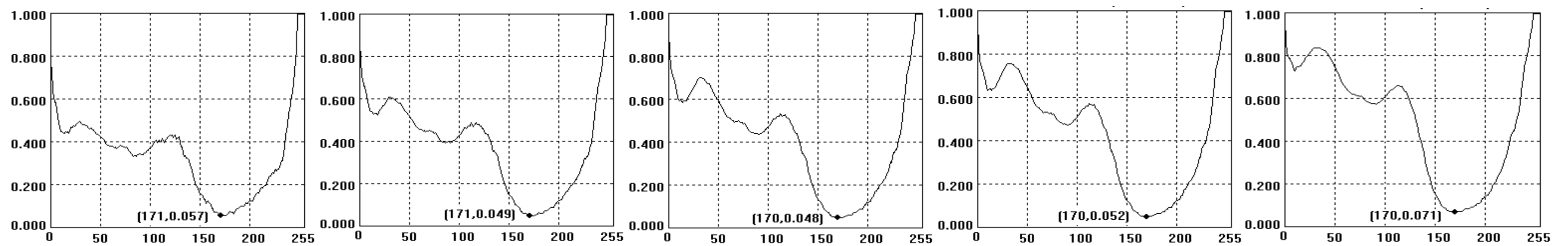
Other methods

Experimental Results

$$w(u,v)=\begin{cases} e^{-\left[\frac{\|F(u)-F(v)\|_2^2}{d_I}+\frac{\|X(u)-X(v)\|_2^2}{d_X}\right]} & \text{if } \|X(u)-X(v)\|_2<r \\ 0 & \text{otherwise} \end{cases}$$

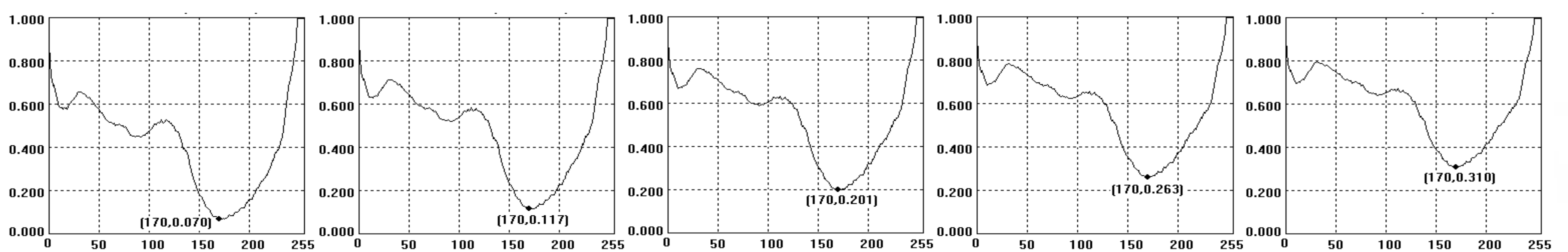
Tank image

- As r increases, the Ncut curve becomes smoother.



(a) r=2, d_I=625, d_X=4 (b) r=4, d_I=625, d_X=8 (c) r=8, d_I=625, d_X=16 (d) r=16, d_I=625, d_X=32 (e) r=32, d_I=625, d_X=250

- As d_I increases, Ncut values increases.

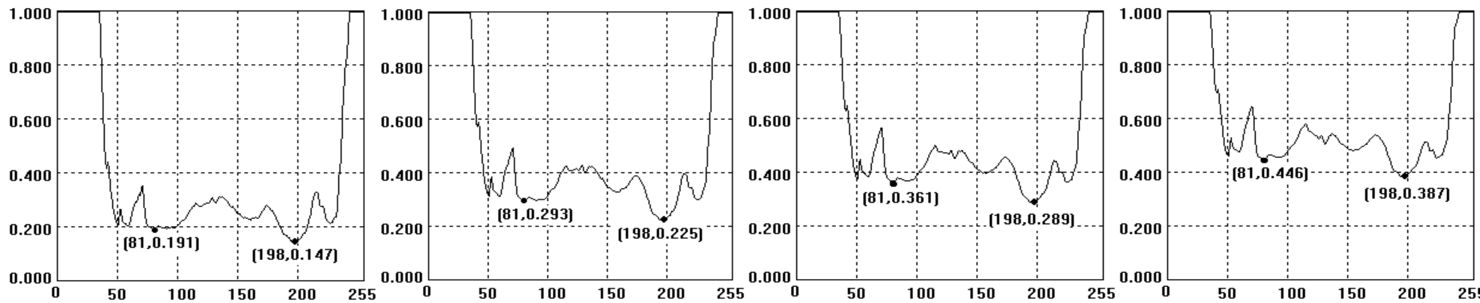


(f) r=4, d_I=1000, d_X=8 (g) r=4, d_I=2000, d_X=8 (h) r=4, d_I=5000, d_X=8 (i) r=4, d_I=10000, d_X=8 (j) r=4, d_I=20000, d_X=8

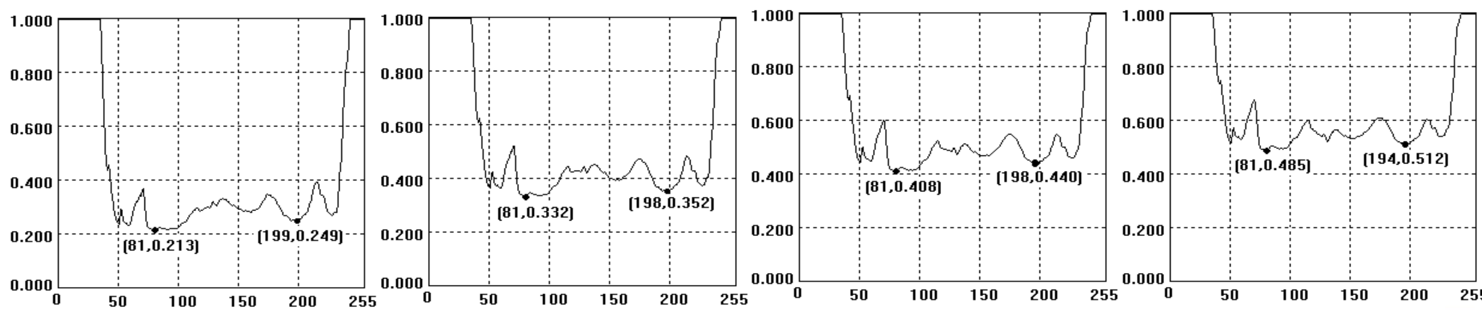
- In general, Ncut is insensitive to these parameters.

Experimental Results

Ship image



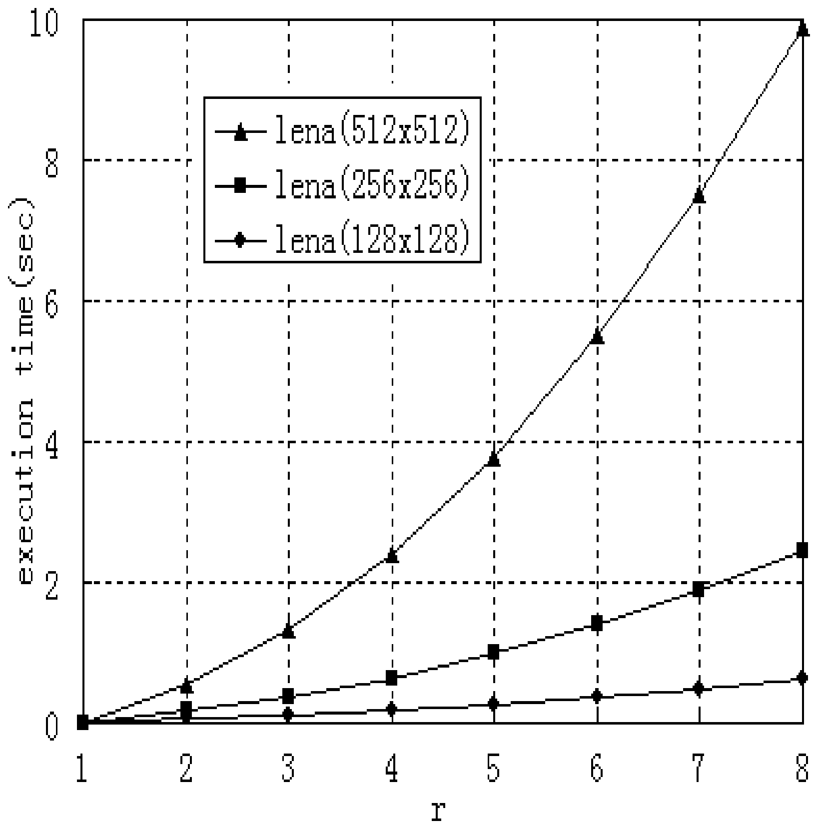
r=2, d_I=1000, d_X=4 | r=4, d_I=2000, d_X=8 | r=6, d_I=3000, d_X=10 | r=8, d_I=5000, d_X=16

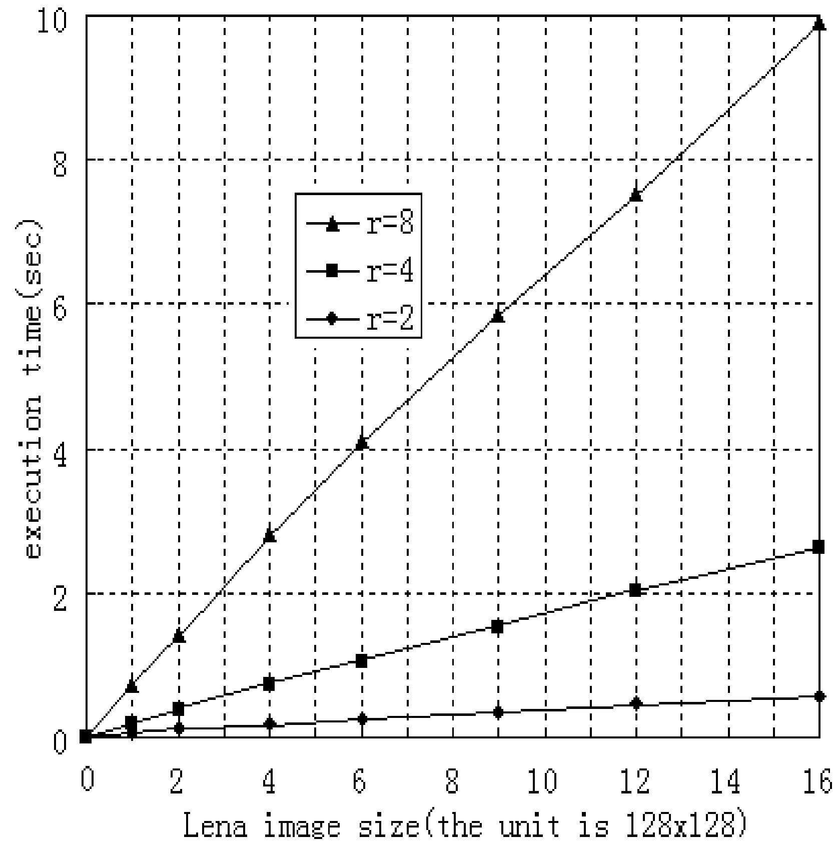


r=2, d_I=4000, d_X=4 | r=4, d_I=7000, d_X=8 | r=6, d_I=13000, d_X=10 | r=8, d_I=18000, d_X=16

- **As d_I increases to an extremely large value, the optimum value of T shifts from right to left.**
- **This does not usually happen for typical values of d_I (400 – 1000).**

Computational Complexity





Execution time vs. *r*.

Execution time vs. image size in the multiple of 128x128.