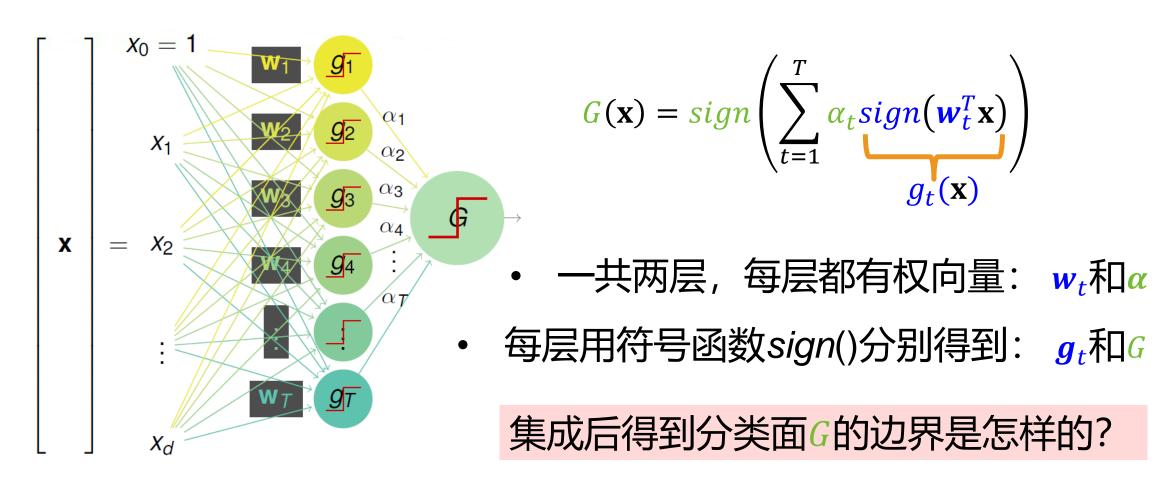
第十讲 神经网络到深度学习(From Neural Networks to Deep Learning)

- 10.1 神经网络动机 (Motivation of Neural Networks)
- 10.2 神经网络模型(Neural Network Hypothesis)
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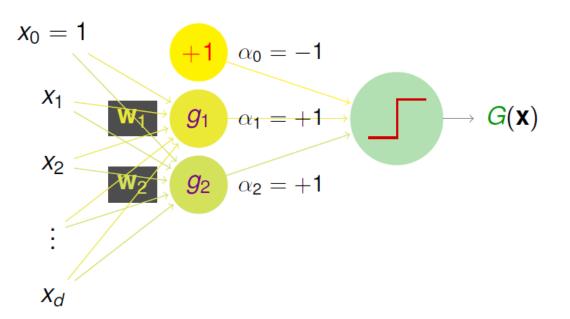
多个感知器的线性集成(Linear Aggregation of Perceptrons)

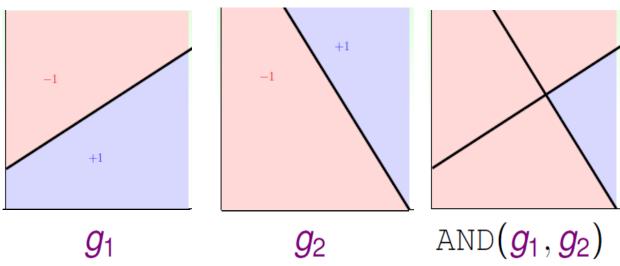


Source: NTU-LIN



用集成方式实现逻辑运算





$$G(\mathbf{x}) = sign(-1 + g_1(\mathbf{x}) + g_2(\mathbf{x}))$$

if
$$g_1(\mathbf{x}) = g_2(\mathbf{x}) = +1$$
: $G(\mathbf{x}) = +1$

otherwise:
$$G(\mathbf{x}) = -1$$

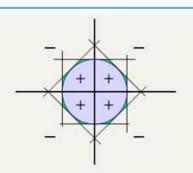
$$G(\mathbf{x}) \equiv AND(g_1(\mathbf{x}), g_2(\mathbf{x}))$$

集成也能实现逻辑运算OR、NOT

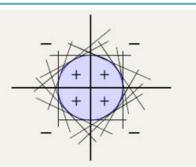


两层集成的强大能力:

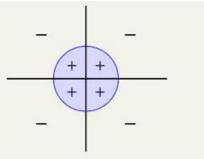
足够多的感知器可得到 光滑的非线性分类面



8 perceptrons



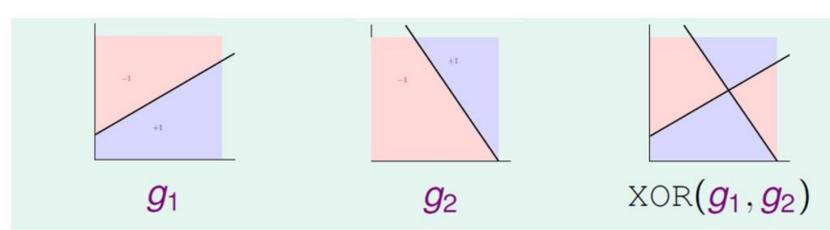
16 perceptrons



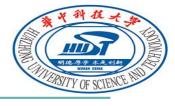
target boundary

两层集成的不足:

在两层结构下,做不到 "线性可分",无法实现 "异或(XOR)"逻辑



Source: NTU-LIN



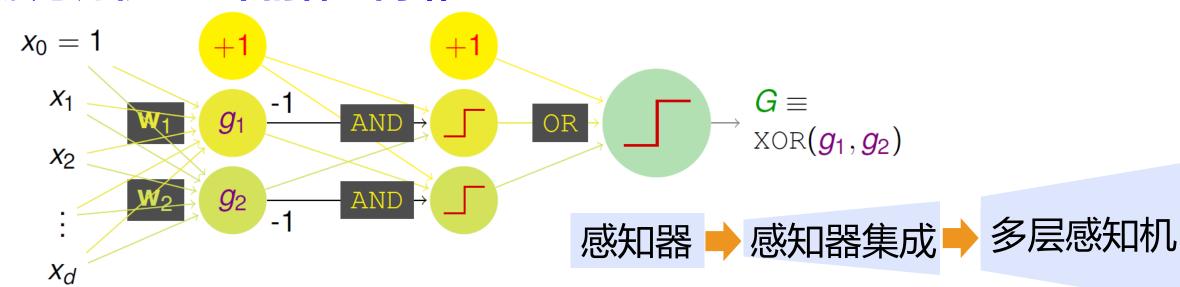
如何实现"异或"逻辑?

• 途径1: 非线性变换

• 途径2: 在两层实现简单逻辑基础上, 再增加层数

$$XOR(g_1, g_2) = OR(AND(-g_1, g_2) + AND(g_1, -g_2))$$

多层感知机—基本的神经网络

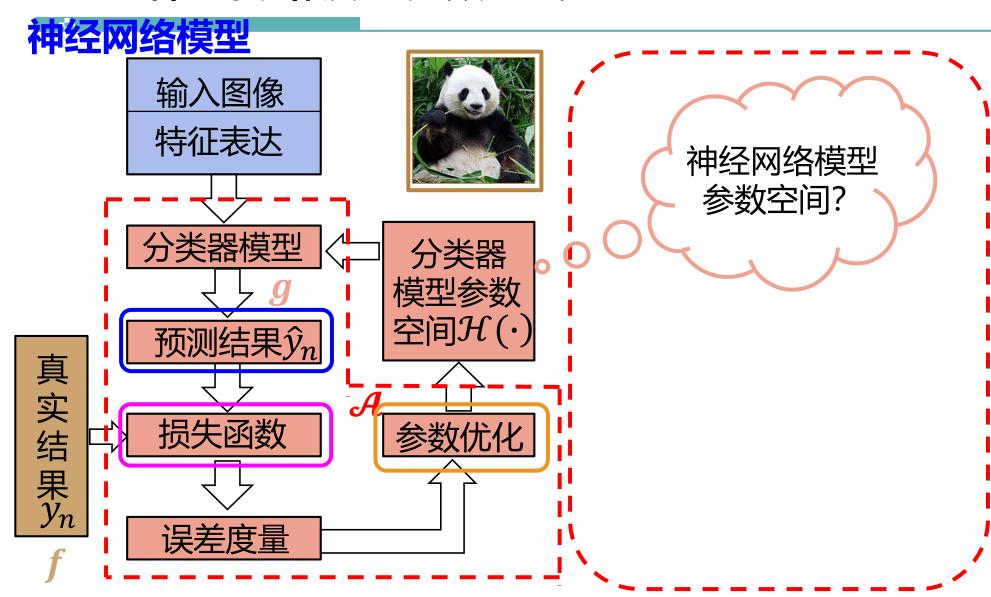


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第十讲 神经网络到深度学习(From Neural Networks to Deep Learning)

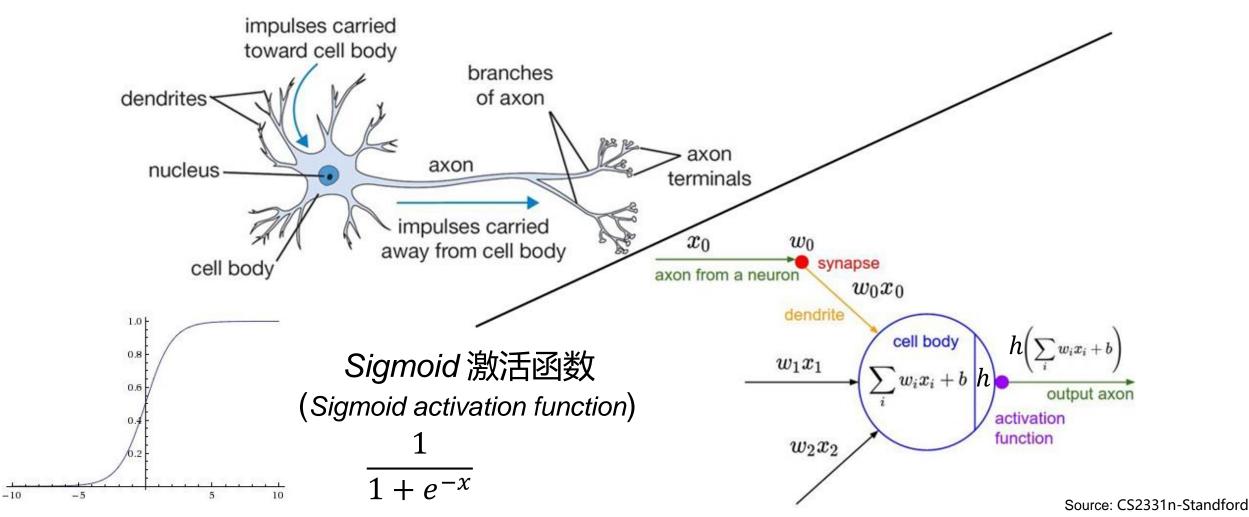
- 10.1 神经网络动机 (Motivation of Neural Networks)
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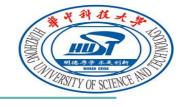




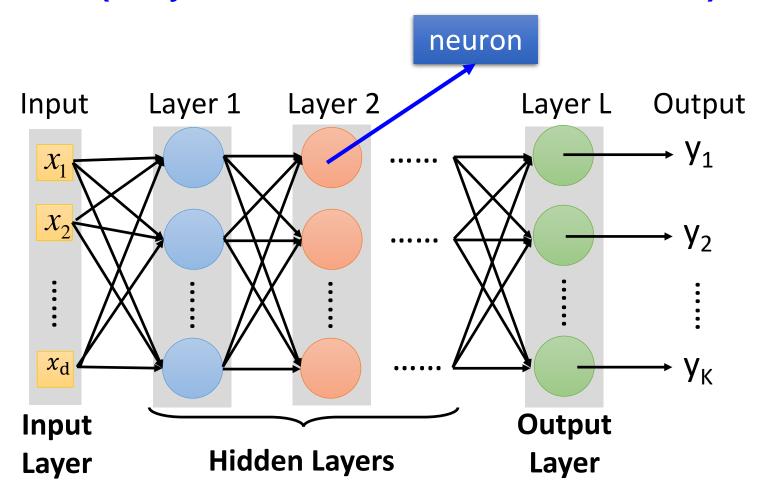


神经网络(Neural Networks):





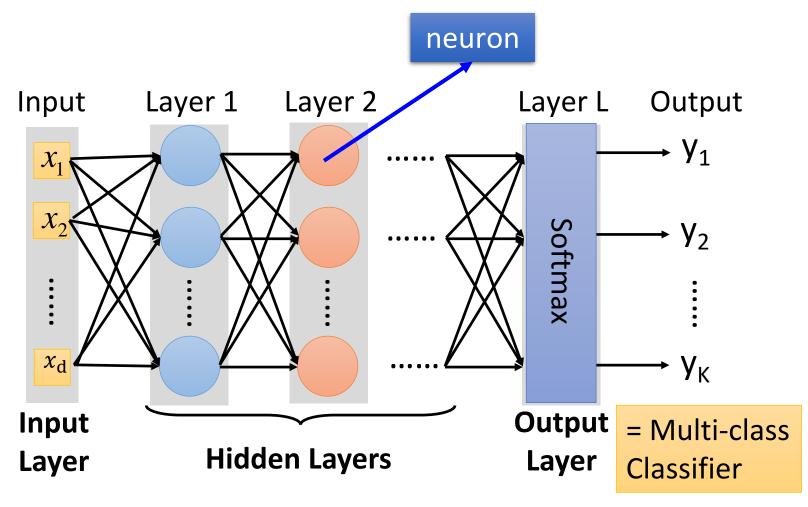
全链接前馈神经网络(Fully Connect Feedforward Network):



Source: NTU-LEE



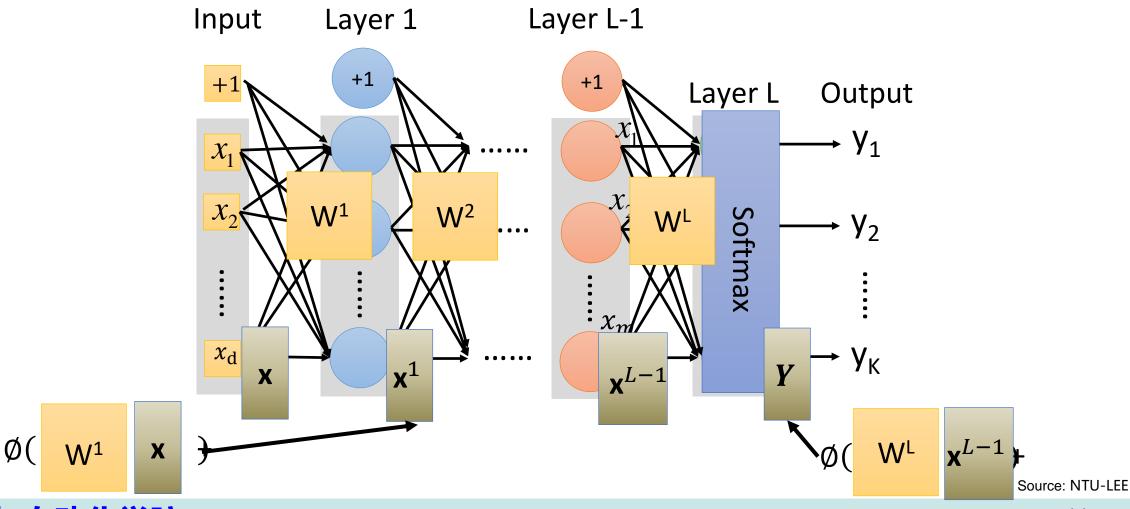
全链接前馈神经网络(Fully Connect Feedforward Network):

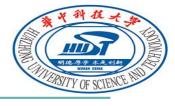


Source: NTU-LEE

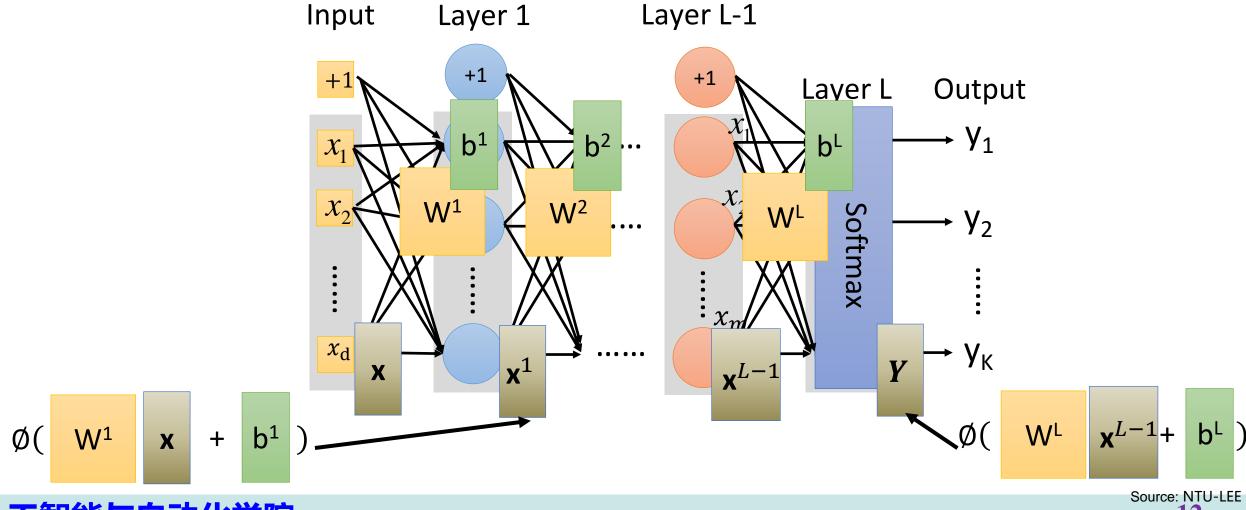


全链接前馈神经网络(Fully Connect Feedforward Network):





全链接前馈神经网络(Fully Connect Feedforward Network):

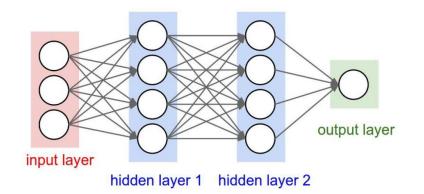


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Source: NTU-LEE 12



神经网络模型:

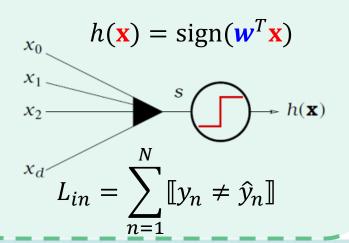


为简单起见, 输出端考虑二分类问题 为简单起见, 输出层选择线性回归模型

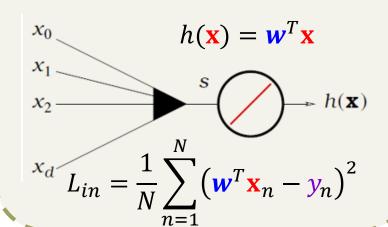
Output:
$$S = \mathbf{w}^T \phi^{(2)}(\phi^{(1)}(\mathbf{x}))$$

$$L_{in} = \frac{1}{N} \sum_{i=1}^{N} (S - y_n)^2$$

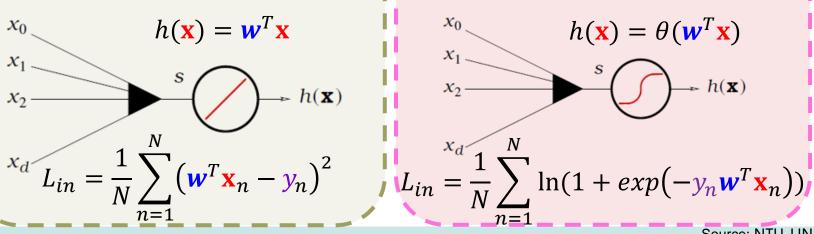
线性分类(感知器):



线性回归:



逻辑斯蒂回归:



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激活(activation)/变换(transformation)函数:

激活函数可以是任意形式吗?



线性网络,作用不大

$$s = \mathbf{w}^{(3)} (\mathbf{w}^{(2)} (\mathbf{w}^{(1)} \mathbf{x} + \mathbf{b}^{(1)}) + \mathbf{b}^{(2)}) + \mathbf{b}^{(3)}]$$

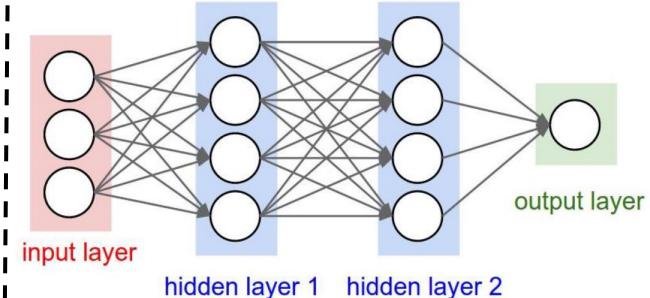
= $\mathbf{W} \mathbf{x} + \mathbf{b}$



NP难问题,难以获得最佳的w



非线性变换,增强网络的性能



filluderi layer i filluderi layer 2

$$s = \mathbf{w}^{(3)} \phi(\mathbf{w}^{(2)} \phi(\mathbf{w}^{(1)} \mathbf{x} + \mathbf{b}^{(1)}) + \mathbf{b}^{(2)}) + \mathbf{b}^{(3)}$$

Source: NTU-LIN



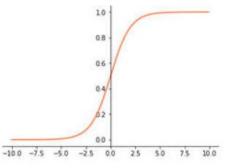
激活(activation)/变换(transformation)函数:

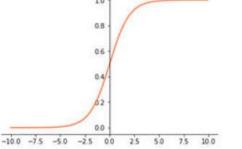
Sigmoid

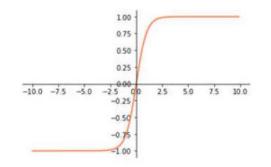
$$1/(1+e^{-x})$$

tanh

$$\frac{e^x - e^{-x}}{e^x + e^{-x}}$$

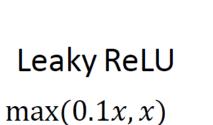


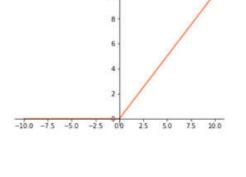


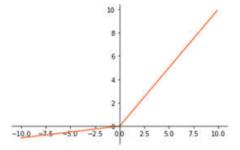


ReLU

 $\max(0,x)$

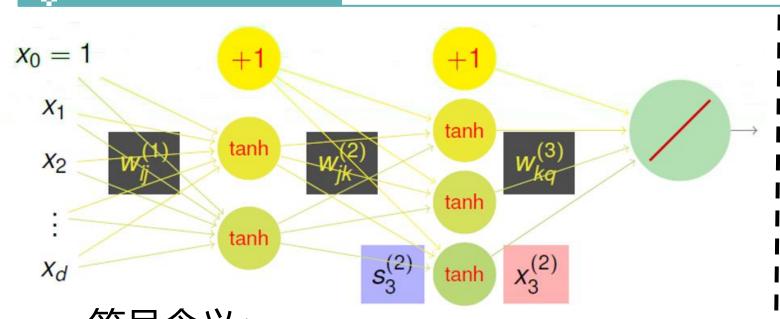






- 一个感知器单元都应具有非线性特性
- 非线性函数应该具有可导性质便于训练





符号含义:

 $d^{(l)}$:网络第l层的神经元个数

 $w_{ij}^{(l)}$: 网络的第l-1层到第l层的权系数矩阵元素

 $S_i^{(l)}$: 网络第l层第j个神经元的输入

 $x_i^{(l)}$: 网络第l层第j个神经元的输出

$$\begin{array}{c} \boldsymbol{w_{ij}^{(l)}} : \left\{ \begin{array}{l} 1 \leq l \leq L & \text{layers} \\ 0 \leq i \leq d^{(l-1)} & \text{inputs} \\ 1 \leq j \leq d^{(l)} & \text{outputs} \end{array} \right.$$

$$S_j^{(l)} = \sum_{i=0}^{d^{(l-1)}} w_{ij}^{(l)} x_i^{(l-1)}$$

$$x_j^{(l)} = \begin{cases} \tanh(S_j^{(l)}) & \text{if } l < l \\ S_j^{(l)} & \text{if } l = L \end{cases}$$

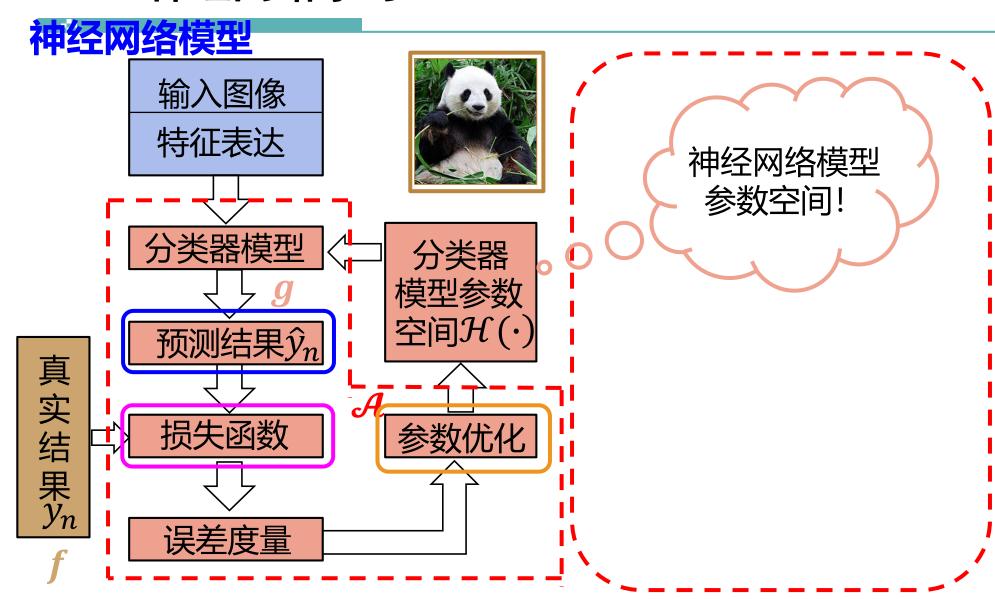
将输入样本x作为input layer, 经过hidden layers得到 $\mathbf{x}^{(l)}$, 在output layer去预测 $x_1^{(L)}$

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第十讲 神经网络到深度学习(From Neural Networks to Deep Learning)

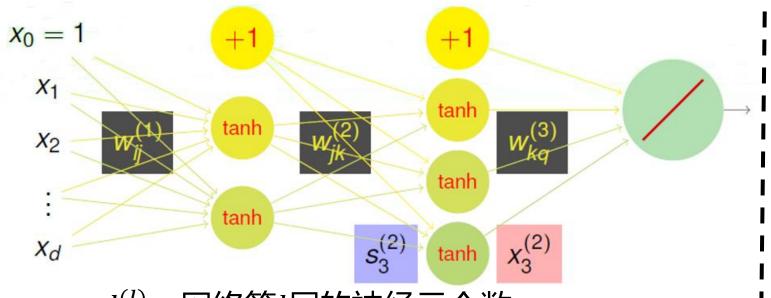
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如何学习网络权重 $w_{ij}^{(l)}$?



 $d^{(l)}$: 网络第l层的神经元个数

 $w_{ij}^{(l)}$: 网络的第l-1层到第l层的权系数矩阵元素

 $S_i^{(l)}$: 网络第l层第j个神经元的输入

 $x_i^{(l)}$: 网络第l层第j个神经元的输出

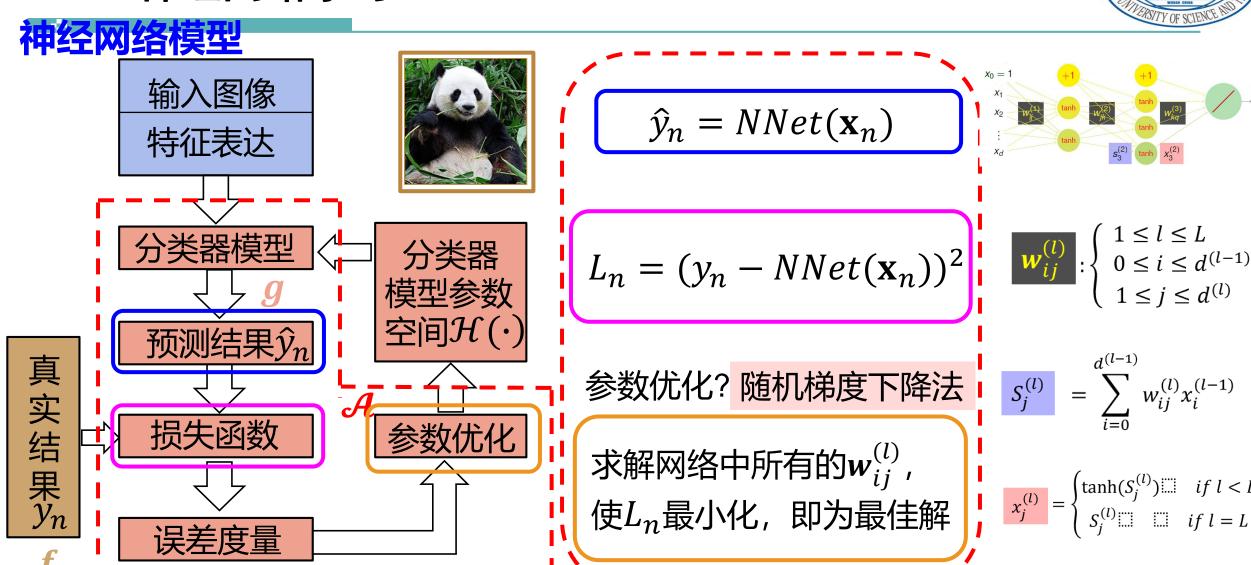
$$S_j^{(l)} = \sum_{i=0}^{d^{(l-1)}} w_{ij}^{(l)} x_i^{(l-1)}$$

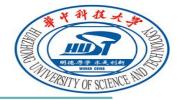
$$x_j^{(l)} = \begin{cases} \tanh(S_j^{(l)}) & \text{if } l < L \\ S_j^{(l)} & \text{if } l = L \end{cases}$$

将输入样本x作为input layer, 经过hidden layers得到 $\mathbf{x}^{(l)}$, 在output layer去预测 $x_1^{(L)}$

Source: NTU-LIN







神经网络中如何求解梯度:

$$L_n = (y_n - NNet(\mathbf{x}_n))^2 = (y_n - S_1^{(L)})^2 = (y_n - \sum_{i=0}^{n} \mathbf{w}_{i1}^{(L)} \mathbf{x}_i^{(L-1)})^2$$

「网络输出层:第
$$L$$
层)」 网络其他层:第 l 层

$$(0 \le i \le d^{(L-1)}, j = 1, l = L) \qquad (0 \le i \le d^{(l-1)}, 1 \le j \le d^{(l)})$$

$$\frac{\partial L_n}{\partial w_{i1}^{(L)}} = \frac{\partial L_n}{\partial S_1^{(L)}} \cdot \frac{\partial S_1^{(L)}}{\partial w_{i1}^{(L)}}$$

$$= -2(y_n - S_1^{(L)}) \cdot (x_i^{(L-1)}) = \delta_j^{(l)} \cdot (x_i^{(l-1)})$$
$$\delta_i^{(L)} = -2(y_n - S_1^{(L)}) \qquad \delta_i^{(l)} = ?$$

$$(0 \le i \le d^{(l-1)}, 1 \le j \le d^{(l)})$$

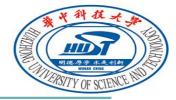
$$\frac{\partial L_n}{\partial w_{ij}^{(l)}} = \frac{\partial L_n}{\partial S_j^{(l)}} \cdot \frac{\partial S_j^{(l)}}{\partial w_{ij}^{(l)}}$$

$$= \delta_j^{(l)} \cdot (x_i^{(l-1)})$$
$$\delta_i^{(l)} = ?$$

$$\mathbf{w}_{ij}^{(l)} : \begin{cases} 1 \le l \le L \\ 0 \le i \le d^{(l-1)} \end{cases}$$

$$S_j^{(l)} = \sum_{i=0}^{a^{(l)}} w_{ij}^{(l)} x_i^{(l-1)}$$

$$x_j^{(l)} = \begin{cases} \tanh(S_j^{(l)})if \ l < L \\ S_j^{(l)} \quad if \ l = L \end{cases}$$



神经网络中如何求解梯度: $\delta_j^{(l)} = \frac{\partial L_n}{\partial S_i^{(l)}}$

$$S_{j}^{(l)} \xrightarrow{\tanh} x_{j}^{(l)} \xrightarrow{\mathbf{w}_{jk}^{(l+1)}} \begin{pmatrix} S_{1}^{(l+1)} \\ \vdots \\ S_{k}^{(l+1)} \end{pmatrix} \xrightarrow{\vdots} \cdots \longrightarrow L_{n}$$

$$\delta_{j}^{(l)} = \frac{\partial L_{n}}{\partial S_{j}^{(l)}} = \sum_{k=1}^{d^{(l+1)}} \frac{\partial L_{n}}{\partial S_{k}^{(l+1)}} \cdot \frac{\partial S_{k}^{(l+1)}}{\partial x_{j}^{(l)}} \cdot \frac{\partial x_{j}^{(l)}}{\partial S_{j}^{(l)}}$$

$$= \sum_{k=1}^{d^{(l+1)}} (\delta_{k}^{(l+1)}) \cdot (\mathbf{w}_{jk}^{(l+1)}) \cdot (\tanh'(S_{j}^{(l)}))$$

$$\delta_{j}^{(l)}$$
能够通过反传 $\delta_{k}^{(l+1)}$ 求得

 $\begin{vmatrix} \mathbf{v}_{ij}^{(l)} \\ \mathbf{v}_{ij}^{(l)} \\ 1 \le i \le L \\ 0 \le i \le d^{(l-1)} \\ 1 \le j \le d^{(l)}$ $S_j^{(l)} = \sum_{ij} w_{ij}^{(l)} x_i^{(l-1)}$ $x_j^{(l)} = \begin{cases} \tanh(S_j^{(l)})if \ l < L \\ S_i^{(l)} \quad if \ l = L \end{cases}$



利用反向传播算法实现神经网络学习(Back propagation (Backprop) Algorithm)

初始化所有的权系数 $\mathbf{w}_{ij}^{(l)}$

$$for t = 0,1,...T$$

- ① 随机性(stochastic): 从训练样本集中任选一个 \mathbf{x}_n , $n \in \{1,2,...N\}$
- ② 前向传播(forward): 利用 $\mathbf{x}^{(0)} = \mathbf{x}_n$ 计算所有的 $\mathbf{x}_i^{(l)}$
- ③ 后向传播(backward): $\mathbf{c}\mathbf{x}^{(0)} = \mathbf{x}_n$ 基础上,计算所有的 $\delta_i^{(l)}$
- ④ 梯度下降(gradient descent): $\mathbf{w}_{ij}^{(l)} \leftarrow \mathbf{w}_{ij}^{(l)} \eta x_i^{(l-1)} \delta_j^{(l)}$

返回结果:
$$g_{NNET}(\mathbf{x}) = \left(\cdots \tanh \left(\sum_{j} \mathbf{w}_{jk}^{(2)} \cdot \tanh \left(\sum_{i} \mathbf{w}_{ij}^{(1)} x_{i} \right) \right) \right)$$

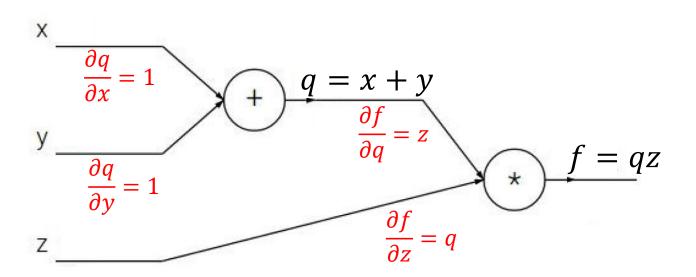
- 所有梯度下降法的技巧都可使用
- 步骤①②③可并行处理,提高效率

反向传播算法是当前神 经网络学习的有效技术



用计算图求解梯度反传(Computational Graph for Gradient Backpropagation):

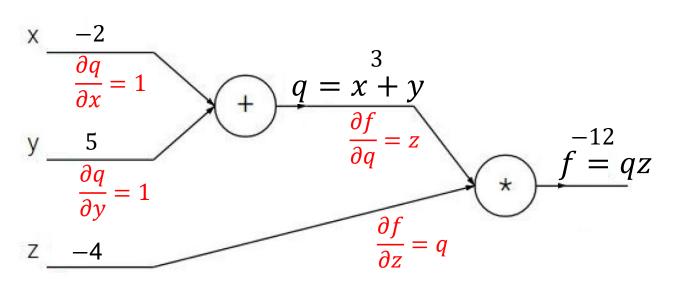
$$f = (x + y)z$$





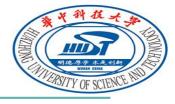
用计算图求解梯度反传(Computational Graph for Gradient Backpropagation):

$$f = (x + y)z$$

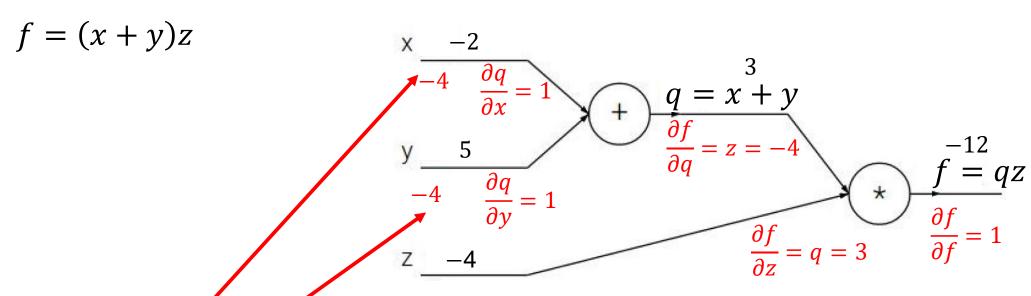


if:
$$x = -2, y = 5, z = -4$$

want:
$$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$$



用计算图求解梯度反传(Computational Graph for Gradient Backpropagation):



if:
$$x = -2, y = 5, z = -4$$

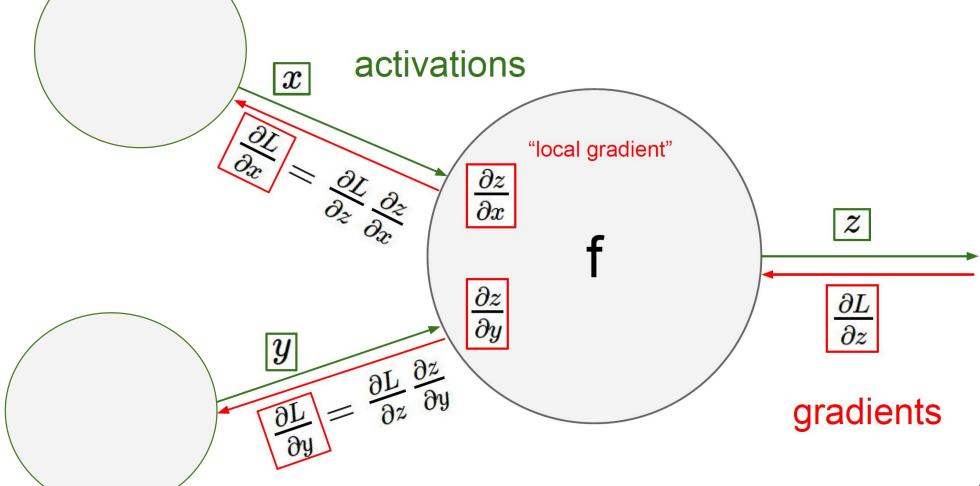
want:
$$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$$

Chain Rule:
$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial x}$$

 $\frac{\partial f}{\partial y} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial y}$

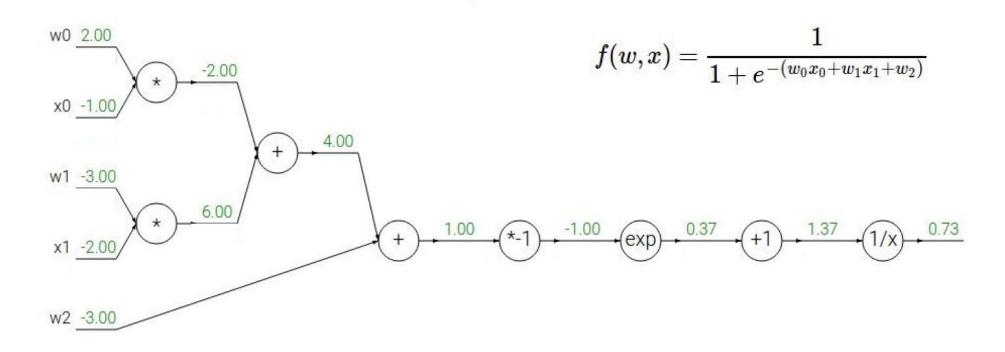


用计算图求解梯度反传(Computational Graph for Gradient Backpropagation):



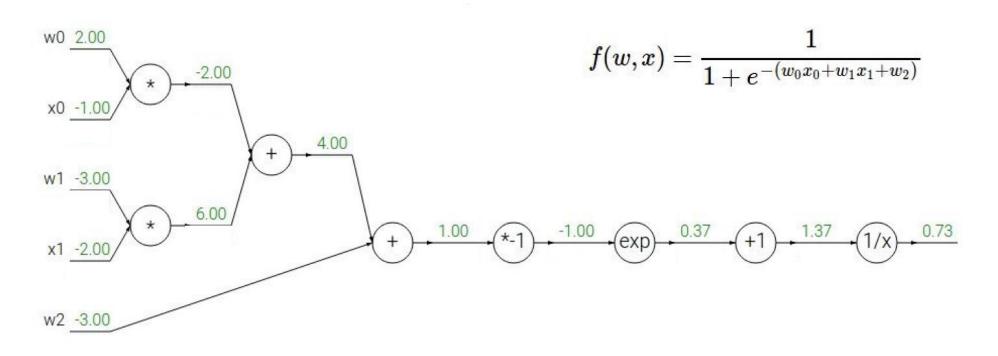


用计算图求解梯度反传(Computational Graph for Gradient Backpropagation):





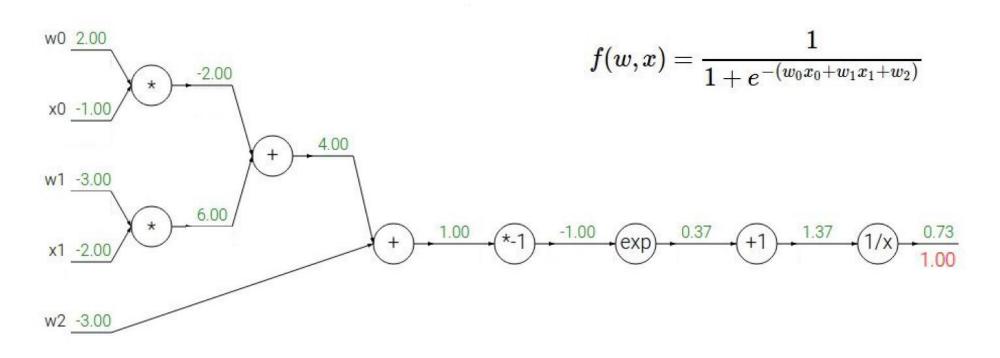
用计算图求解梯度反传(Computational Graph for Gradient Backpropagation):



$$egin{aligned} f(x) = e^x &
ightarrow & rac{df}{dx} = e^x & f(x) = rac{1}{x} &
ightarrow & rac{df}{dx} = -1/x^2 \ f_a(x) = ax &
ightarrow & rac{df}{dx} = a & f_c(x) = c + x &
ightarrow & rac{df}{dx} = 1 \end{aligned}$$



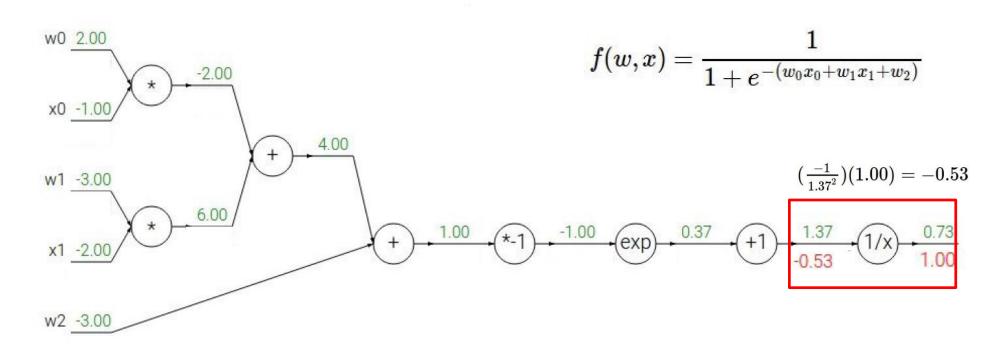
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ightarrow & rac{df}{dx} = -1/x^2 \ f_a(x) = ax &
ightarrow & rac{df}{dx} = a & f_c(x) = c + x &
ightarrow & rac{df}{dx} = 1 \end{aligned}$$



用计算图求解梯度反传(Computational Graph for Gradient Backpropagation):

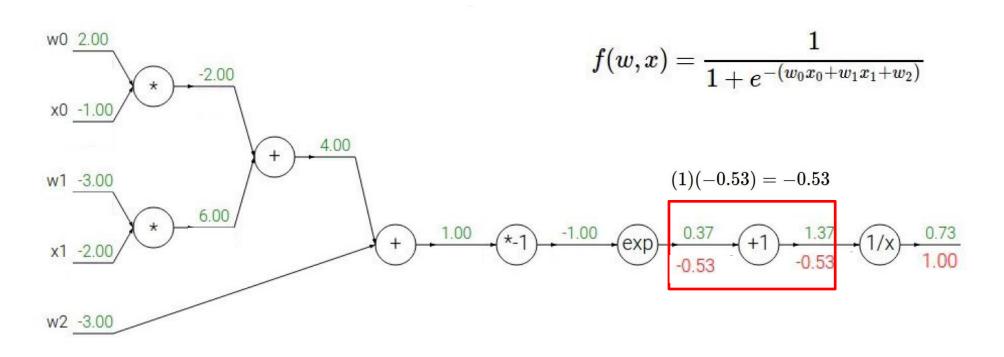


$$egin{aligned} f(x) = e^x &
ightarrow & rac{df}{dx} = e^x \ & & \ f_a(x) = ax &
ightarrow & rac{df}{dx} = a \end{aligned}$$

$$egin{aligned} f(x) = rac{1}{x} &
ightarrow & rac{df}{dx} = -1/x^2 \ & f_c(x) = c + x &
ightarrow & rac{df}{dx} = 1 \end{aligned}$$



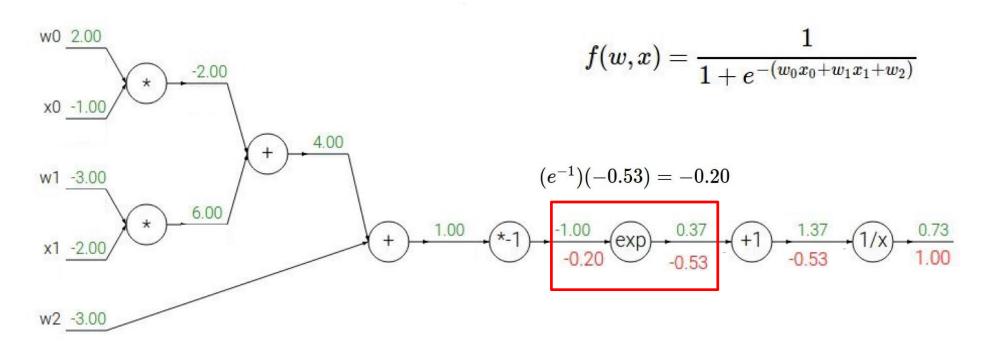
用计算图求解梯度反传(Computational Graph for Gradient Backpropagation):



$$f(x) = e^x \hspace{1cm} o \hspace{1cm} rac{df}{dx} = e^x \hspace{1cm} f(x) = rac{1}{x} \hspace{1cm} o \hspace{1cm} rac{df}{dx} = -1/x^2 \ f_a(x) = ax \hspace{1cm} o \hspace{1cm} rac{df}{dx} = a \hspace{1cm} f_c(x) = c + x \hspace{1cm} o \hspace{1cm} rac{df}{dx} = 1$$



用计算图求解梯度反传(Computational Graph for Gradient Backpropagation):

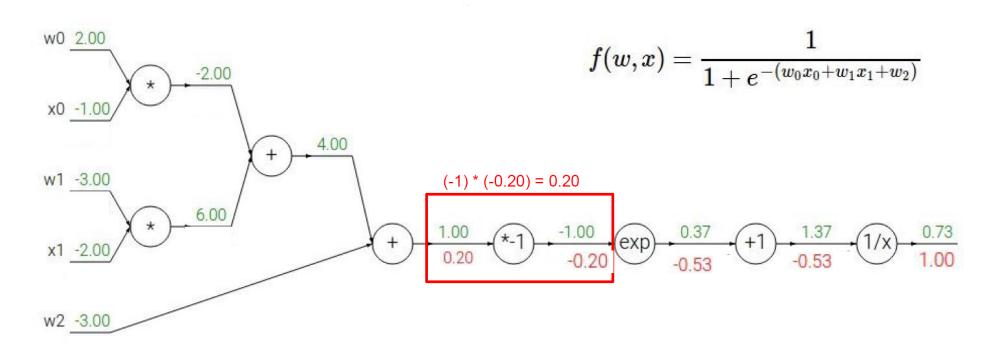


$$f(x)=e^x \qquad \qquad o \qquad \qquad rac{df}{dx}=e^x \ f_a(x)=ax \qquad \qquad o \qquad \qquad rac{df}{dx}=a$$

$$f(x)=rac{1}{x} \qquad \qquad
ightarrow \qquad rac{df}{dx}=-1/x^2 \ f_c(x)=c+x \qquad \qquad
ightarrow \qquad rac{df}{dx}=1$$



用计算图求解梯度反传(Computational Graph for Gradient Backpropagation):

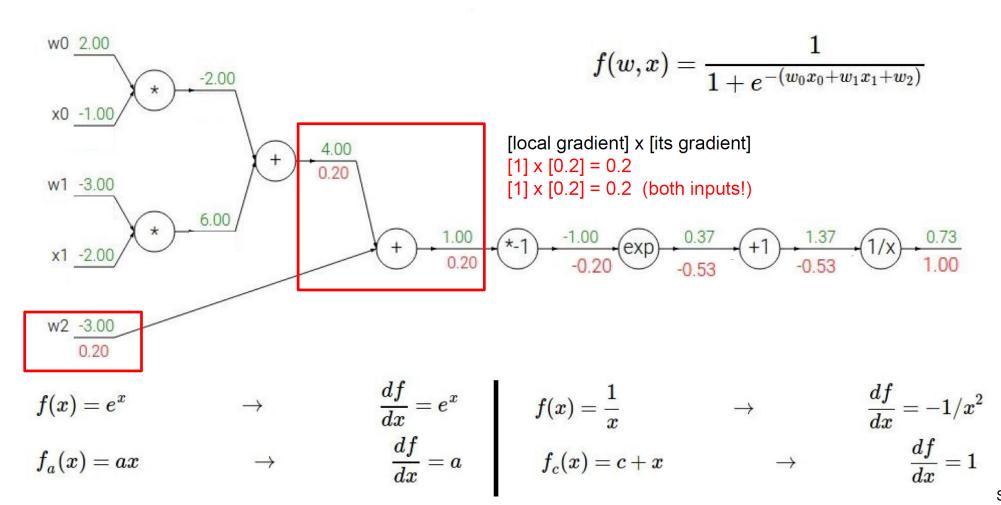


$$egin{aligned} f(x) = e^x &
ightarrow & rac{df}{dx} = e^x \ & & \ f_a(x) = ax &
ightarrow & rac{df}{dx} = a \end{aligned}$$

$$egin{aligned} rac{df}{dx} = e^x & f(x) = rac{1}{x} &
ightarrow & rac{df}{dx} = -1/x^2 \ rac{df}{dx} = a & f_c(x) = c + x &
ightarrow & rac{df}{dx} = 1 \end{aligned}$$

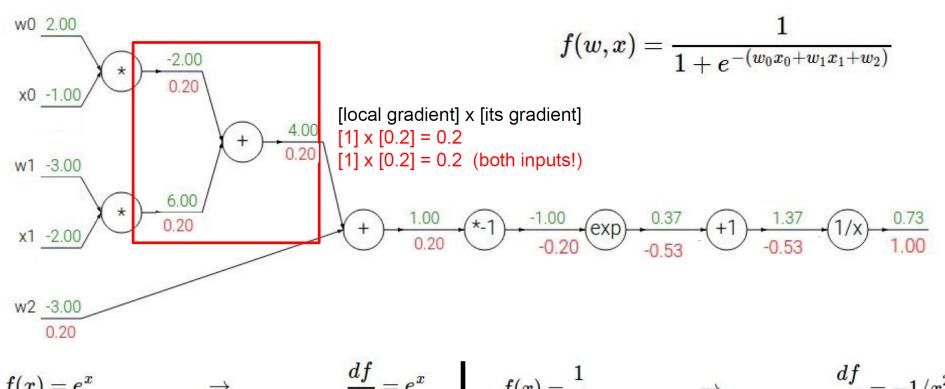


用计算图求解梯度反传(Computational Graph for Gradient Backpropagation):





用计算图求解梯度反传(Computational Graph for Gradient Backpropagation):

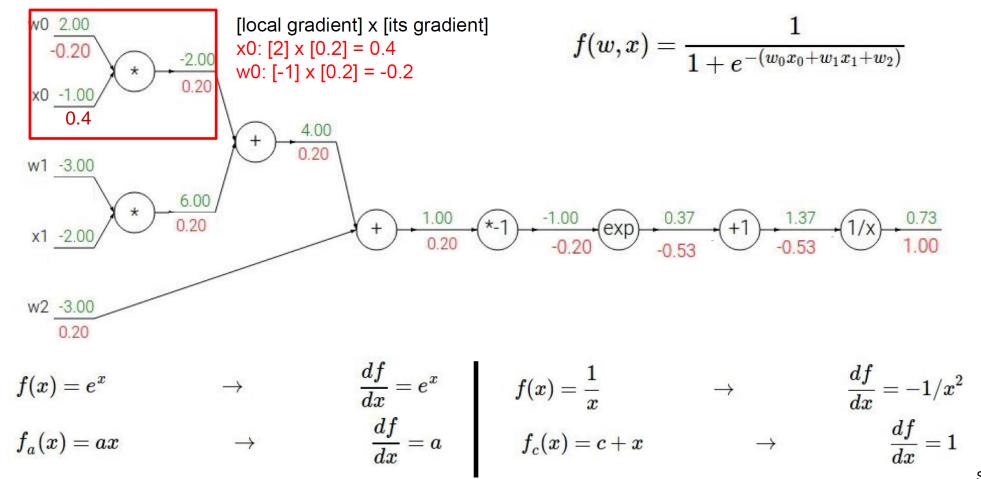


$$egin{aligned} f(x) = e^x &
ightarrow & rac{df}{dx} = e^x & f(x) = rac{1}{x} &
ightarrow & rac{df}{dx} = -1/x^2 \ f_a(x) = ax &
ightarrow & rac{df}{dx} = a & f_c(x) = c + x &
ightarrow & rac{df}{dx} = 1 \end{aligned}$$

10.3 神经网络学习



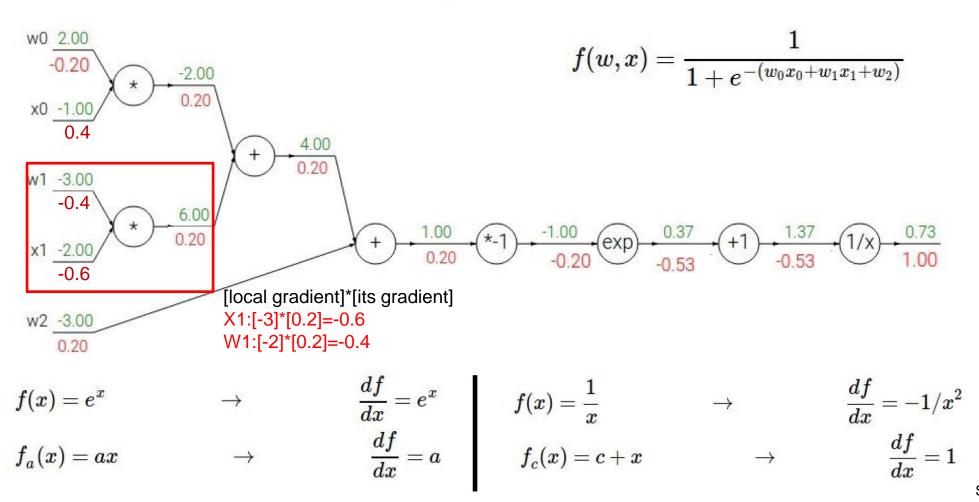
用计算图求解梯度反传(Computational Graph for Gradient Backpropagation):



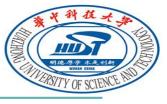
10.3 神经网络学习



用计算图求解梯度反传(Computational Graph for Gradient Backpropagation):



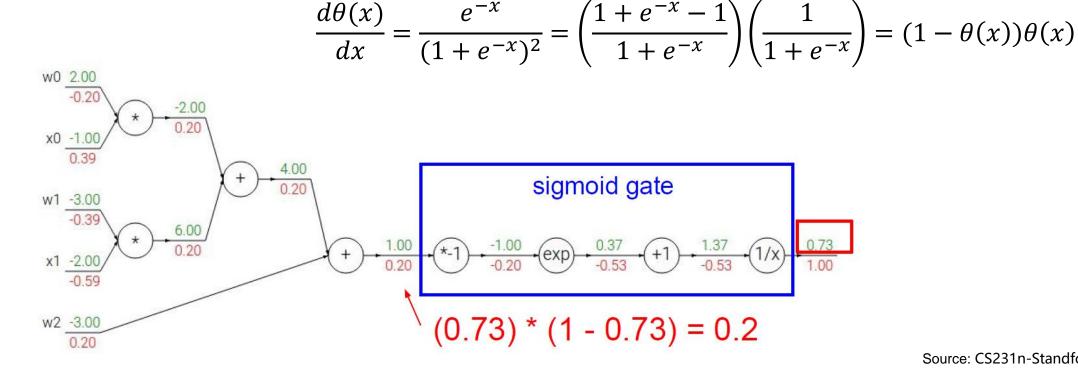
10.3 神经网络学习



用计算图求解梯度反传(Computational Graph for Gradient Backpropagation):

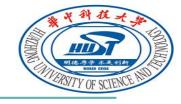
$$f(w,x)=rac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

$$f(w,x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$$
 $\theta(x) = \frac{1}{1 + e^{-x}}$ sigmoid function

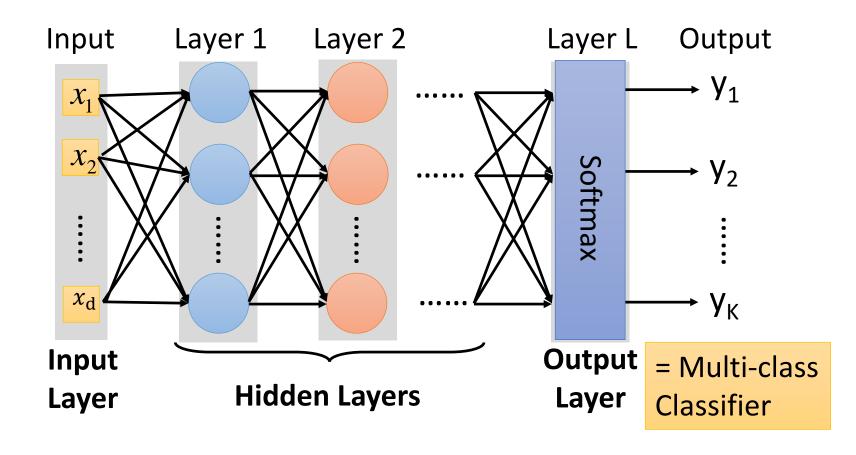


第十讲 神经网络到深度学习(From Neural Networks to Deep Learning)

- 10.1 神经网络动机 (Motivation of Neural Networks)
- 10.2 神经网络模型(Neural Network Hypothesis)
- 10.3 神经网络学习 (Neural Network Learning)
- 10.4 深度神经网络 (Deep Neural Networks)

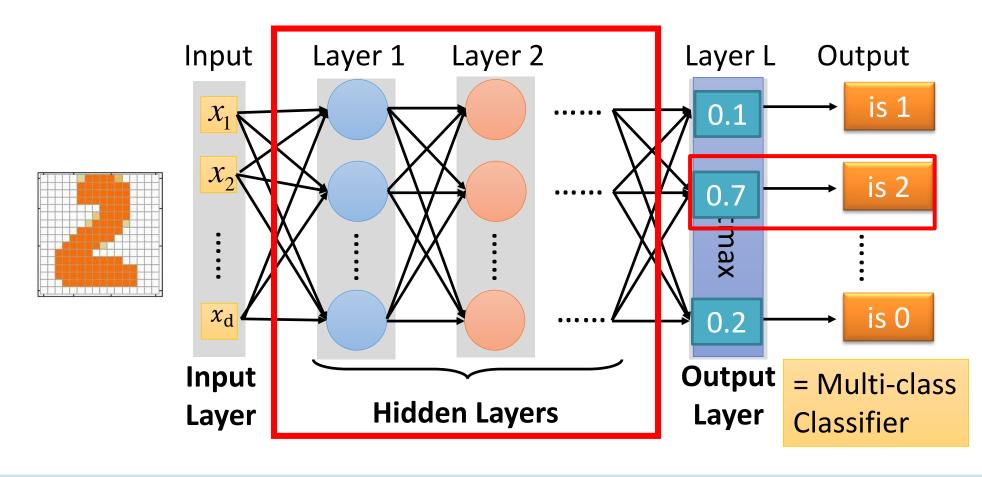


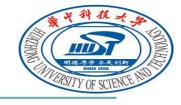
神经网络(Neural Networks):





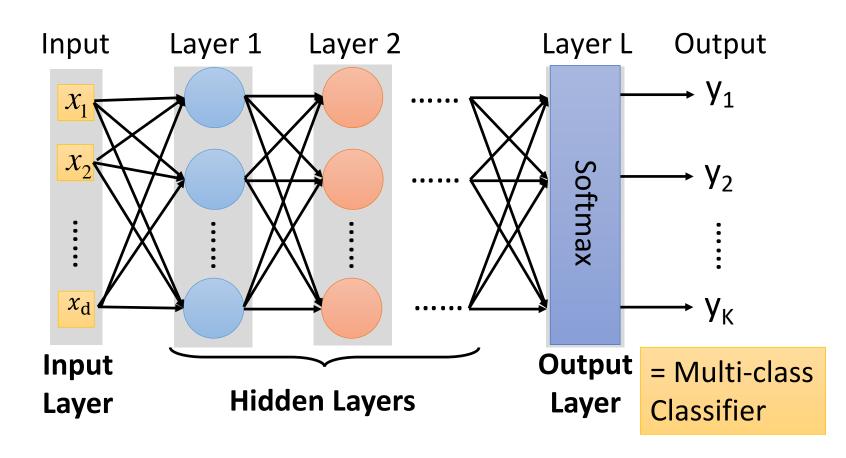
神经网络(Neural Networks):





神经网络(Neural Networks):

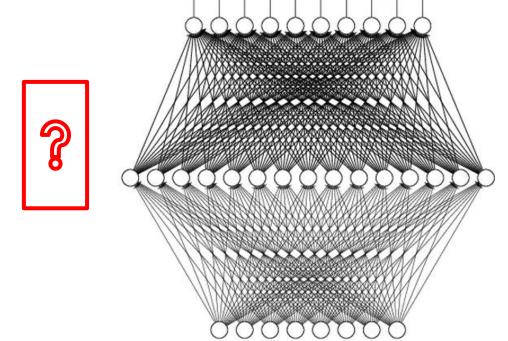
Deep = Many hidden layers





为什么要用深度网络?

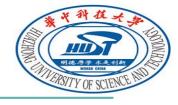
Layer X Size	Word Error Rate (%)	
1 X 2k	24.2	
2 X 2k	20.4	
3 X 2k	18.4	
4 X 2k	17.8	
5 X 2k	17.2	
7 X 2k	17.1	



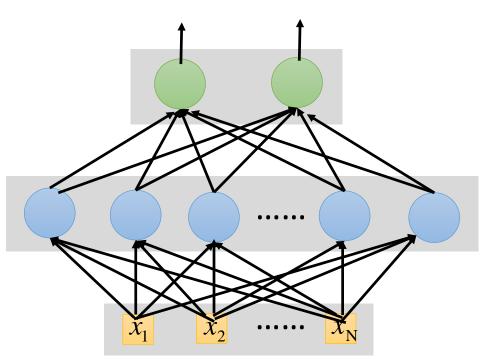
 $f: \mathbb{R}^N \to \mathbb{R}^M$

只要有足够多神 经元,一个隐含 层的网络结构也 能拟合出任意复 杂的连续曲面

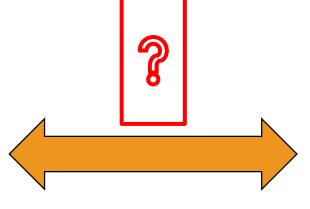
按照学习理论,参数多、性能好



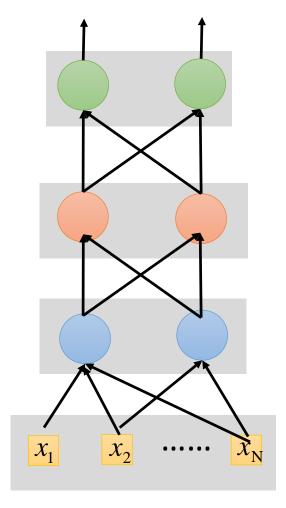
浅层网络好还是深层网络好?



Shallow



同等参数量条 件下比较性能



Deep



为什么要用深度网络?

Layer X Size	Word Error Rate (%)	Layer X Size	Word Error Rate (%)
1 X 2k	24.2		
2 X 2k	20.4		
3 X 2k	18.4		
4 X 2k	17.8		
5 X 2k	17.2	1 X 3772	22.5
7 X 2k	17.1	1 X 4634	22.6
		1 X 16k	22.1



深度学习存在的挑战和关键技术

- > 如何设计或确定网络结构
 - "领域知识"有助于选择网络结构,如计算机视觉中常常用卷积神经网络

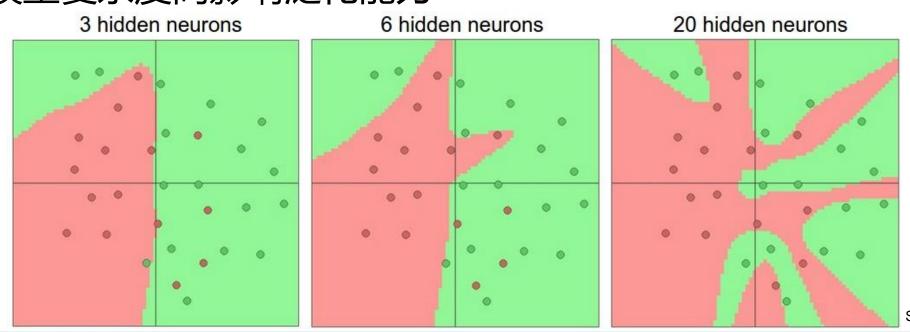


Source: NTU-LIN



深度学习存在的挑战和关键技术

- > 如何设计或确定网络结构
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- > 模型复杂度高影响泛化能力





深度学习存在的挑战和关键技术

- > 如何设计或确定网络结构
 - "领域知识"有助于选择网络结构,如计算机视觉中常常用卷积神经网络
- > 模型复杂度高导致过拟合
 - "大数据"有助于增加训练集,提升泛化能力
 - "正则化"—如Dropout等手段,增强对噪声的容忍度
- > 模型复杂难以找到最优解
 - 仔细选择初始值,避免落入局部极值,包括预训练等
 - 防止梯度消失,采用合适的激活函数、设计更好的网络结构
- > 计算复杂度高
 - 平行化、批量化、GPU等

Source: NTU-LIN

第十讲 神经网络到深度学习(From Neural Networks to Deep Learning)

- 10.1 神经网络动机(Motivation of Neural Networks) *多层网络结构具有更强的分类能力*
- 10.2 神经网络模型参数空间(Neural Network Hypothesis) 基于线性模型构建多层结构
- 10.3 神经网络学习 (Neural Network Learning)

通过反向传播法有效实现梯度计算

10.4 深度神经网络 (Deep Neural Networks)

介绍了其优越性和面临的挑战