# 第3章 电路分析方程

# 3.1 结点分析法 Nodal Analysis

- 1.结点分析方程 Nodal Equations
- 2.观察法列写结点分析方程 Nodal Equations by Inspection
- 3.含电源支路的结点分析方程 Nodal Equations with source branch

# 3.2 网乳分析法 Mesh Analysis

- 1. 网乳分析方程 Mesh Equations
- 2.观察法列写网孔分析方程 Mesh Equations by Inspection
- 3.含电源支路的网孔分析方程 Mesh Equations with source branch

# 第3章 电路分析方程

目标: > 熟练应用结点分析法。

> 熟练应用网孔分析法。

根据电路特点选择最佳分析方法。

难点: > 含电压源支路电路的结点方程。

> 含电流源支路电路的网孔方程。

学时: 4

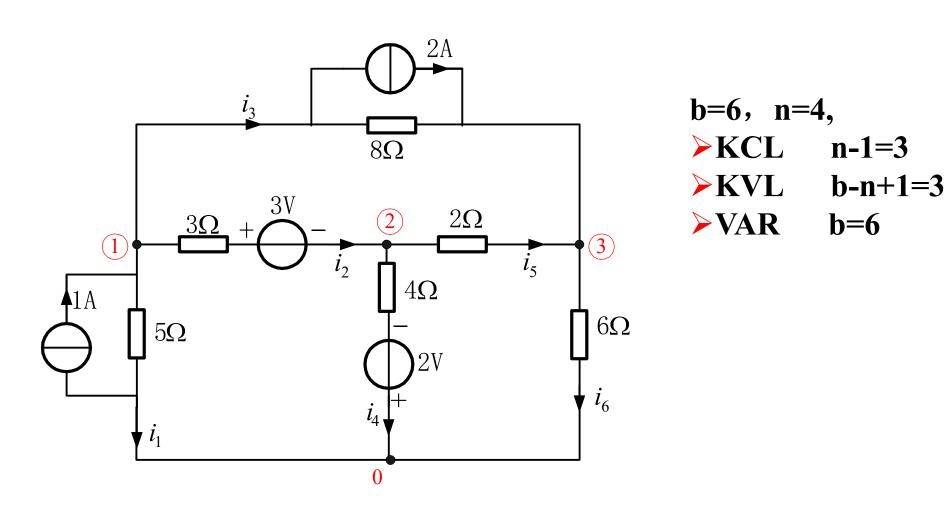
# 3.1 概述: 电路分析方法

直接方法 以支路电压、支路电流为变量列写方程。 分为支路电流法和支路电压法。

▶ 间接方法 求解一组独立变量方程来分析电路。 分为结点分析法、网孔(回路)分析法。

# 3.1 概述: 电路分析方法

#### 电路的基本方程

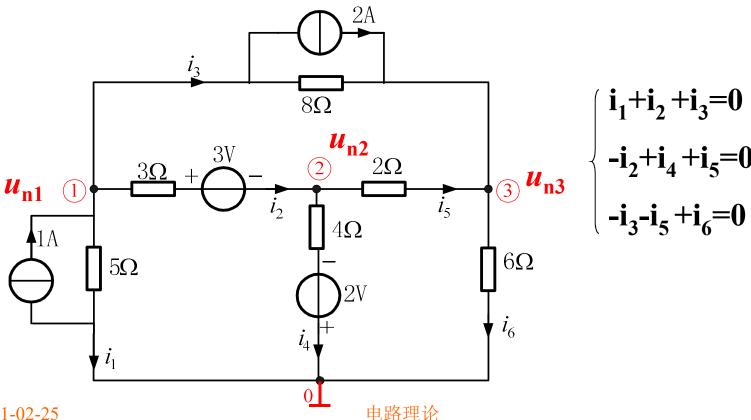


#### 3.3 结点分析法 (Nodal Analysis)

结点电压法的思想

以结点电压为变量,对各结点列写KCL方程并求解,称 为结点电压分析法,简称结点法。

对应于结点法列写的方程称为结点电压方程。



2021-02-25

5

3.1 结点分析法 Nodal analysis

1.结点方程 Nodal equations

$$i_1 = \frac{u_{n1}}{5} - 1$$

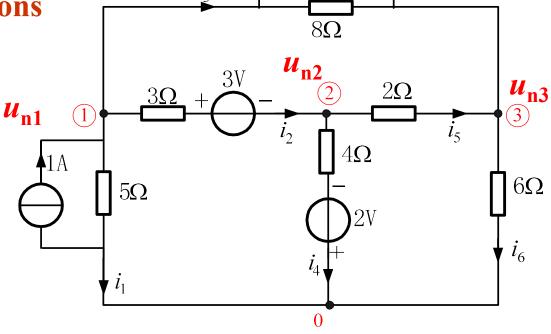
$$i_3 = \frac{u_{n1} - u_{n3}}{8} + 2$$

$$i_2 = \frac{u_{n1} - u_{n2} - 3}{3}$$

$$i_4 = \frac{u_{n2} + 2}{4}$$

$$i_5 = \frac{u_{n2} - u_{n3}}{2}$$

$$i_6 = \frac{u_{n3}}{6}$$



$$\left(\frac{u_{n1}}{5} - 1\right) + \left(\frac{u_{n1} - u_{n2} - 3}{3}\right) + \left(\frac{u_{n1} - u_{n3}}{8} + 2\right) = 0$$

$$-\left(\frac{u_{n1} - u_{n2} - 3}{3}\right) + \left(\frac{u_{n2} + 2}{4}\right) + \left(\frac{u_{n2} - u_{n3}}{2}\right) = 0$$

$$-\left(\frac{u_{n1}-u_{n3}}{8}+2\right)-\left(\frac{u_{n2}-u_{n3}}{2}\right)+\left(\frac{u_{n3}}{6}\right)=0$$

3.1 结点分析法 Nodal analysis

1.结点方程 Nodal equations

$$\left(\frac{1}{5} + \frac{1}{3} + \frac{1}{8}\right)u_{n1} - \frac{1}{3}u_{n2} - \frac{1}{8}u_{n3}$$

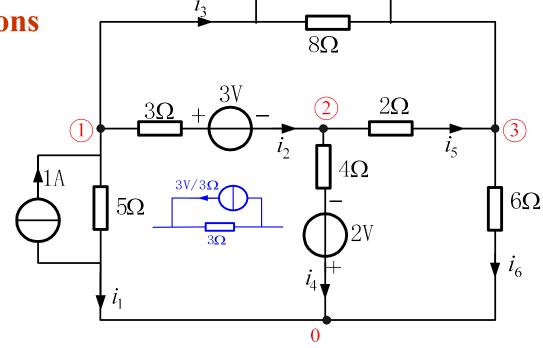
$$= 1 + \frac{3}{3} - 2$$

$$-\frac{1}{3}u_{n1} + (\frac{1}{3} + \frac{1}{4} + \frac{1}{2})u_{n2} - \frac{1}{2}u_{n3}$$

$$= -\frac{3}{3} - \frac{2}{4}$$

$$-\frac{1}{8}u_{n1} - \frac{1}{2}u_{n2} + (\frac{1}{8} + \frac{1}{2} + \frac{1}{6})u_{n3}$$

$$= 2$$



$$\left(\frac{u_{n1}}{5} - 1\right) + \left(\frac{u_{n1} - u_{n2} - 3}{3}\right) + \left(\frac{u_{n1} - u_{n3}}{8} + 2\right) = 0$$

$$-\left(\frac{u_{n1}-u_{n2}-3}{3}\right)+\left(\frac{u_{n2}+2}{4}\right)+\left(\frac{u_{n2}-u_{n3}}{2}\right)=0$$

$$-\left(\frac{u_{n1}-u_{n3}}{8}+2\right)-\left(\frac{u_{n2}-u_{n3}}{2}\right)+\left(\frac{u_{n3}}{6}\right)=0$$

3.1 结点分析法 Nodal analysis

1.结点方程 Nodal equations

$$(\frac{1}{5} + \frac{1}{3} + \frac{1}{8})u_{n1} - \frac{1}{3}u_{n2} - \frac{1}{8}u_{n3}$$

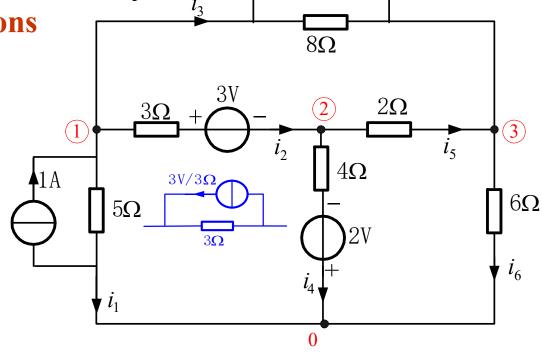
$$=1+\frac{3}{3}-2$$

$$-\frac{1}{3}u_{n1} + (\frac{1}{3} + \frac{1}{4} + \frac{1}{2})u_{n2} - \frac{1}{2}u_{n3}$$

$$=-\frac{3}{3}-\frac{2}{4}$$

$$-\frac{1}{8}u_{n1} - \frac{1}{2}u_{n2} + (\frac{1}{8} + \frac{1}{2} + \frac{1}{6})u_{n3}$$

$$= 2$$



$$\frac{1}{8}u_{n1} - \frac{1}{2}u_{n2} + (\frac{1}{8} + \frac{1}{2} + \frac{1}{6})u_{n3} = 2$$

$$\begin{bmatrix}
(\frac{1}{5} + \frac{1}{3} + \frac{1}{8}) & -\frac{1}{3} & -\frac{1}{8} \\
-\frac{1}{3} & (\frac{1}{3} + \frac{1}{4} + \frac{1}{2}) & -\frac{1}{2} \\
-\frac{1}{8} & -\frac{1}{2} & (\frac{1}{8} + \frac{1}{2} + \frac{1}{6})
\end{bmatrix}
\begin{bmatrix}
u_{n1} \\
u_{n1} \\
u_{n1}
\end{bmatrix} = \begin{bmatrix}
1 + \frac{3}{3} - 2 \\
-\frac{3}{3} - \frac{2}{4} \\
2
\end{bmatrix}$$

## 2.快速列写法

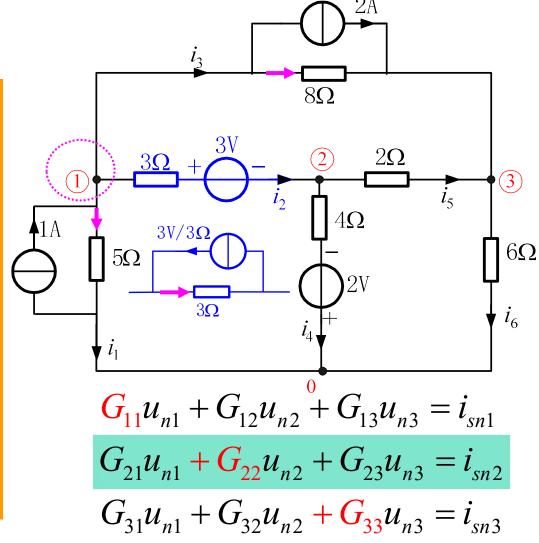
$$\left[ (\frac{1}{5} + \frac{1}{3} + \frac{1}{8})u_{n1} - \frac{1}{3}u_{n2} - \frac{1}{8}u_{n3} \right]$$

$$=1+\frac{3}{3}-2$$

$$-\frac{1}{3}u_{n1} + (\frac{1}{3} + \frac{1}{4} + \frac{1}{2})u_{n2} - \frac{1}{2}u_{n3}$$

$$=-\frac{3}{3}-\frac{2}{4}$$

$$-\frac{1}{8}u_{n1} - \frac{1}{2}u_{n2} + (\frac{1}{8} + \frac{1}{2} + \frac{1}{6})u_{n3}$$



 $G_{kk}$ : Self-conductance ——k结点上各支路电导之和

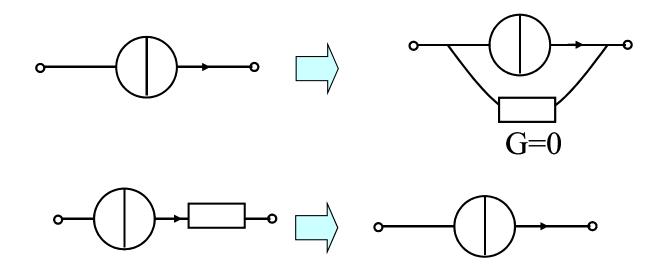
 $G_{ki}$ : Mutual-conductance ——k、 j结点间支路电导的负值

 $i_{\mathit{snk}}$ : Equivalent nodal current source—流入k结点所有电流源代数和

#### 3、特殊支路的处理

a. 电流源支路 (With current source branch )

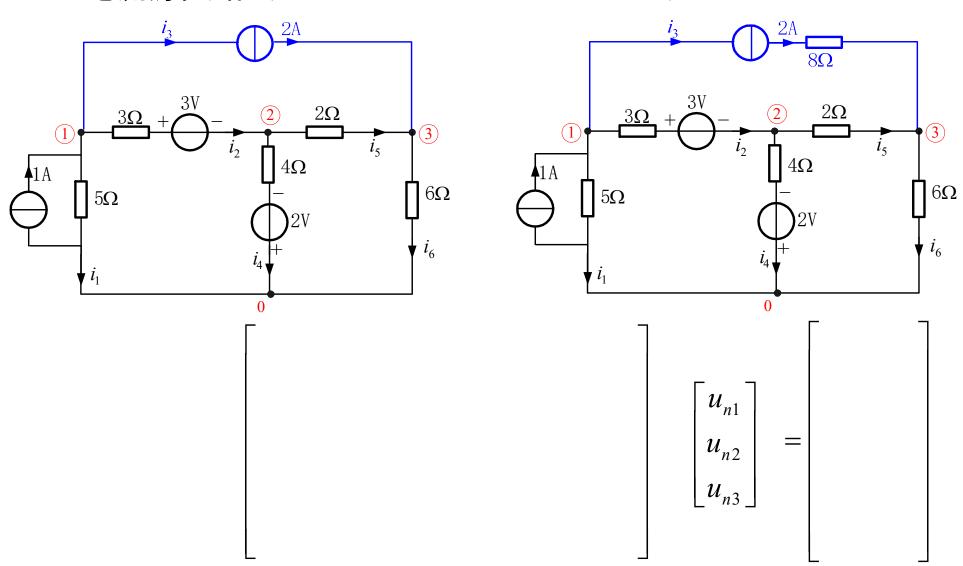
电流源支路视为电导为零的诺顿支路



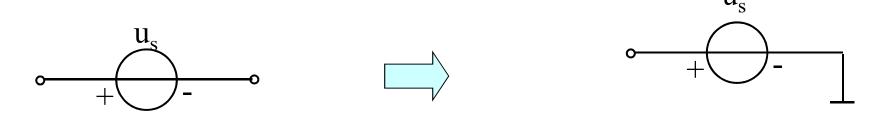
与无伴电流源串联的电阻不出现在结点方程中。

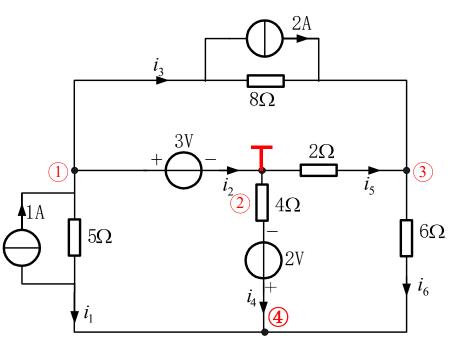
# 3、特殊支路的处理

# a. 电流源支路 (With current source branch )



#### b无伴电压源处理方法1: 无伴电压源的一端设为参考结点

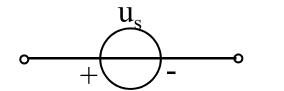




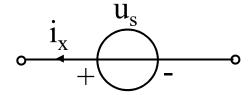
设结点2为参考结点,由结点法得;

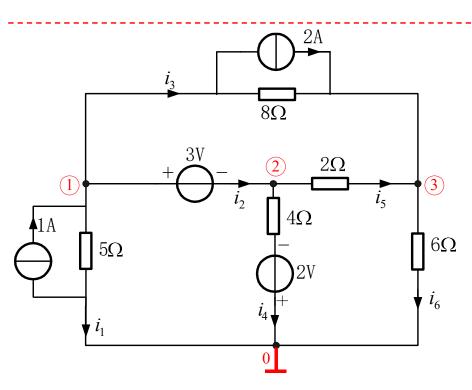
$$\begin{cases}
 u_{n1} = 3 \\
 -\frac{1}{8}u_{n1} - \frac{1}{6}u_{n4} + (\frac{1}{2} + \frac{1}{6} + \frac{1}{8})u_{n3} = 2 \\
 -\frac{1}{5}u_{n1} - \frac{1}{6}u_{n3} + (\frac{1}{5} + \frac{1}{4} + \frac{1}{6})u_{n4} \\
 = -1 + \frac{2}{4}
\end{cases}$$

#### b无伴电压源处理方法2: 增设无伴电压源电流变量





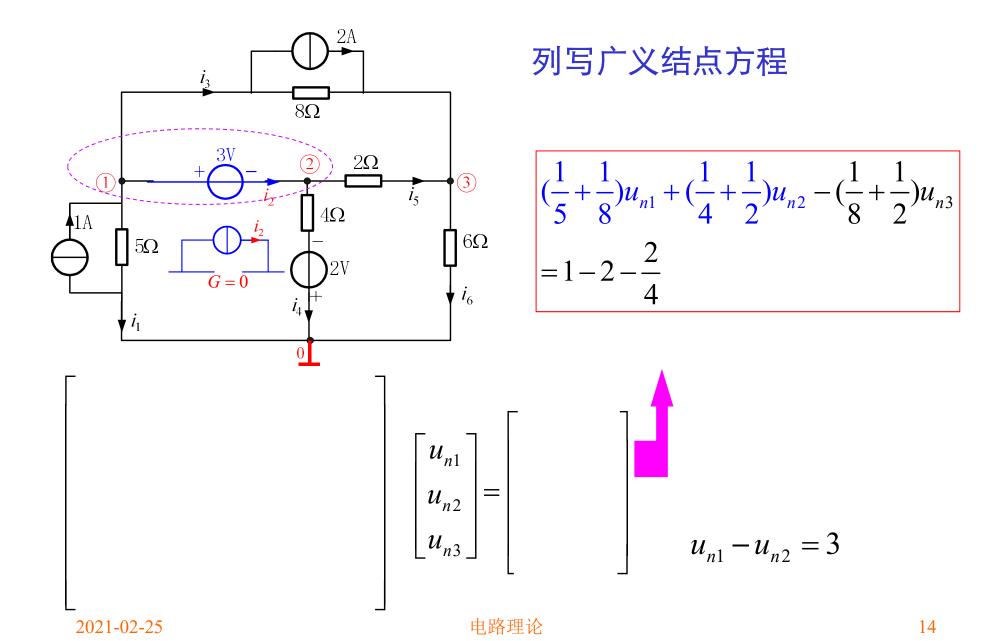




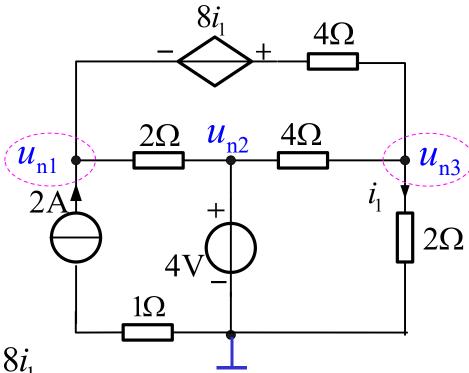
#### 由结点法得:

$$\begin{cases} (\frac{1}{5} + \frac{1}{8})u_{n1} - \frac{1}{8}u_{n3} = -2 + 1 - i_{2} \\ (\frac{1}{4} + \frac{1}{2})u_{n2} - \frac{1}{2}u_{n3} = +i_{2} - \frac{1}{2} \\ -\frac{1}{8}u_{n1} - \frac{1}{2}u_{n2} + (\frac{1}{2} + \frac{1}{6} + \frac{1}{8})u_{n3} = 2 \\ u_{n1} - u_{n2} = 3 \end{cases}$$

#### b无伴电压源处理方法3: 广义结点法



## 例2: 列写结点方程



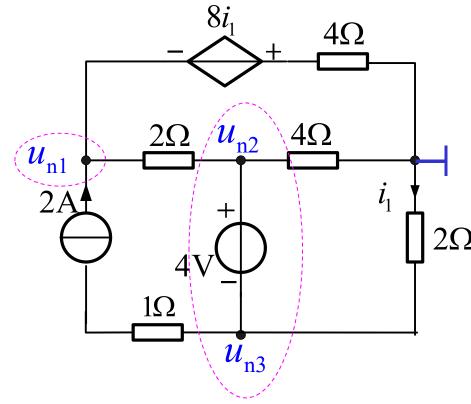
$$\left(\frac{1}{2} + \frac{1}{4}\right)u_{n1} - \frac{1}{2}u_{n2} - \frac{1}{4}u_{n3} = 2 - \frac{8i_1}{4}$$

$$-\frac{1}{4}u_{n1} - \frac{1}{4}u_{n2} + (\frac{1}{4} + \frac{1}{4} + \frac{1}{2})u_{n3} = \frac{8i_1}{4}$$

$$u_{\rm n2} = 4$$
  $i_1 = \frac{1}{2}u_{\rm n3}$ 

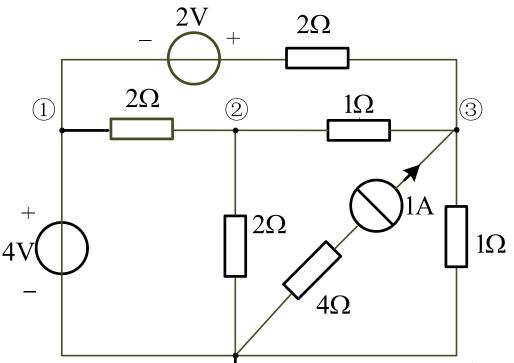
2021-02-25

## 例2: 列写结点方程



$$\begin{cases} (\frac{1}{2} + \frac{1}{4})u_{n1} - \frac{1}{2}u_{n2} = 2 - \frac{8i_1}{4} \\ -\frac{1}{2}u_{n1} + (\frac{1}{4} + \frac{1}{2})u_{n2} + \frac{1}{2}u_{n3} = -2 \\ u_{n2} - u_{n3} = 4 \qquad i_1 = -\frac{1}{2}u_{n3} \end{cases}$$

## 练习: 求独立电源提供的功率。



$$U_{n1} = 4V$$

$$-\frac{1}{2}U_{n1} + (\frac{1}{2} + \frac{1}{2} + 1)U_{n2} - U_{n3} = 0$$

$$-\frac{1}{2}U_{n1} - U_{n2} + (\frac{1}{2} + 1 + 1)U_{n3} = 1 + \frac{2}{2}$$

# 解方程得出:

$$U_{n1} = 4V, U_{n2} = 2.25V, U_{n3} = 2.5V$$

$$P_{1A} = (U_{n3} + 4) \times 1 = 6.5W;$$

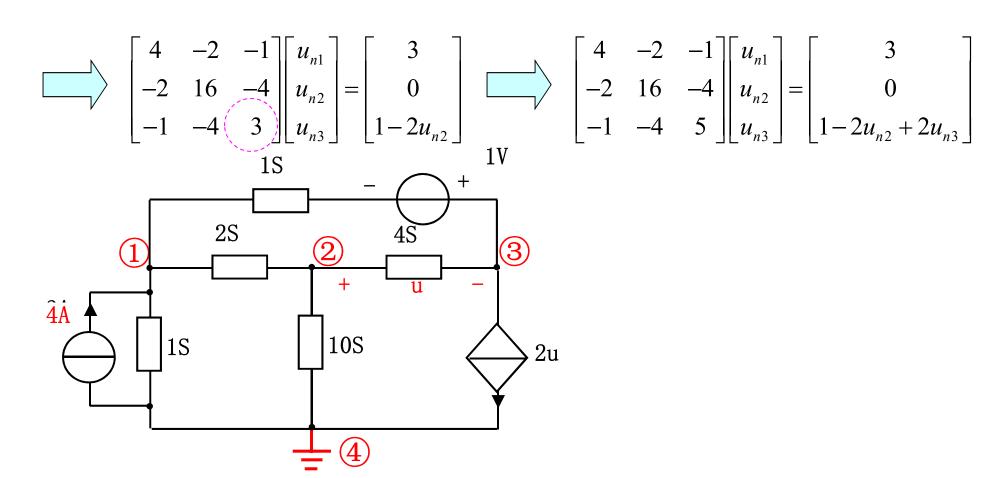
$$P_{2V} = \frac{U_{n1} - U_{n3} + 2}{2} \times 2 = 3.5W;$$

# 例3:已知某电路的结点方程,画出电路图。

$$\begin{cases}
4u_{n1} - 2u_{n2} - u_{n3} = 3 \\
-2u_{n1} + 16u_{n2} - 4u_{n3} = 0 \\
-u_{n1} - 2u_{n2} + 3u_{n3} = 1
\end{cases}$$

$$\begin{cases}
4u_{n1} - 2u_{n2} - u_{n3} = 3 \\
-2u_{n1} + 16u_{n2} - 4u_{n3} = 0 \\
-u_{n1} - 2u_{n2} + 3u_{n3} = 1
\end{cases}$$

$$\begin{bmatrix}
4 & -2 & -1 \\
-2 & 16 & -4 \\
-1 & (-2) & 3
\end{bmatrix}
\begin{bmatrix}
u_{n1} \\
u_{n2} \\
u_{n3}
\end{bmatrix} = \begin{bmatrix}
3 \\
0 \\
1
\end{bmatrix}$$



练习:已知某电路的结点方程,根据要求分别修改结点方程: (1)在结点2、3之间并联0.5欧姆的电阻; (2)在结点2、3之间并联电压源为2V、电阻为0.5欧姆的戴维南支路,电压源正极接到结点2。(3)在结点2、3之间并联2V的电压源,电压源正极接到结点2; (4)将2、3结点短接。

解(1)在结点2、3之间并联0.5欧姆的电阻;

$$\begin{cases} 4u_{n1} - 2u_{n2} - u_{n3} = 3 \\ -2u_{n1} + 18u_{n2} - 6u_{n3} = 0 \\ -u_{n1} - 4u_{n2} + 5u_{n3} = 1 \end{cases}$$

(2) 在结点2、3之间并联2V、0.5欧姆的戴维南支路, 电压源正极接到结点2;

$$\begin{cases}
4u_{n1} - 2u_{n2} - u_{n3} = 3 \\
-2u_{n1} + 18u_{n2} - 6u_{n3} = 4 \\
-u_{n1} - 4u_{n2} + 5u_{n3} = -3
\end{cases}$$

(3) 在结点2、3之间并联2V的电压源;

$$\begin{cases}
4u_{n1} - 2u_{n2} - u_{n3} = 3 \\
-3u_{n1} + 14u_{n2} - 1u_{n3} = 1 \\
u_{n2} - u_{n3} = 2
\end{cases}$$

$$\begin{cases} 4u_{n1} - 2u_{n2} - u_{n3} = 3\\ -2u_{n1} + 16u_{n2} - 4u_{n3} = 0\\ -u_{n1} - 2u_{n2} + 3u_{n3} = 1 \end{cases}$$

(4) 结点2、3短路;

$$\begin{cases} 4u_{n1} - 2u_{n2} - u_{n3} = 3 \\ -3u_{n1} + 14u_{n2} - 1u_{n3} = 1 \\ u_{n2} = u_{n3} \end{cases}$$

思考题1:在结点2与参考结点之间接2V、0.5 欧姆的戴维南支路,电压源正极接到结点2?

思考题2: 结点2接参考结点?

$$\begin{cases} 4u_{n1} - 2u_{n2} - u_{n3} = 3 \\ -2u_{n1} + 18u_{n2} - 4u_{n3} = 4 \\ -u_{n1} - 2u_{n2} + 3u_{n3} = 1 \end{cases} \begin{cases} 4u_{n1} - (0) - u_{n3} = 3 \\ u_{n2} = 0 \\ -u_{n1} - (0) + 3u_{n3} = 1 \end{cases}$$

# 3.4 网孔分析法 (Mesh analysis)

基本思想: 为减少未知量(方程)的个数,可以假想每个网孔中有一个网孔电流。则各支路电流可用网孔电流线性组合表示。

以<mark>网孔电流</mark>为变量,对各网孔列写KVL方程并求解,称为网孔分析法。

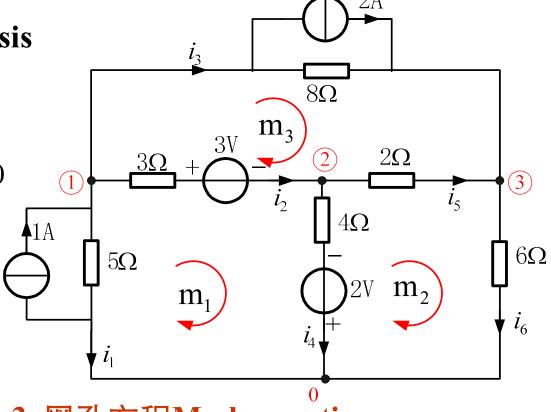
# 3.4 网孔分析法Mesh analysis

#### 1. 网孔KVL

$$5(-i_1 - 1) + [3i_2 + 3] + [4i_4 - 2] = 0$$

$$[-4i_4 + 2] + 2i_5 + 6i_6 = 0$$

$$8(i_3 - 2) - 2i_5 + [-3i_2 - 3] = 0$$



#### 2.网孔电流Mesh currents

$$i_1 = -i_{m1}$$
  $i_2 = i_{m1} - i_{m3}$ 

$$i_3 = i_{m3}$$
  $i_4 = i_{m1} - i_{m2}$ 

$$i_5 = i_{m2} - i_{m3}$$
  $i_6 = i_{m2}$ 

# 3. 网孔方程Mesh equations

$$5(i_{m1}-1) + [3(i_{m1}-i_{m3})+3] + [4(i_{m1}-i_{m2})-2] = 0$$

$$[4(i_{m2} - i_{m1}) + 2] + 2(i_{m2} - i_{m3}) + 6i_{m2} = 0$$

$$8(i_{m3}-2)+2(i_{m3}-i_{m2})+[3(i_{m3}-i_{m1})-3]=0$$

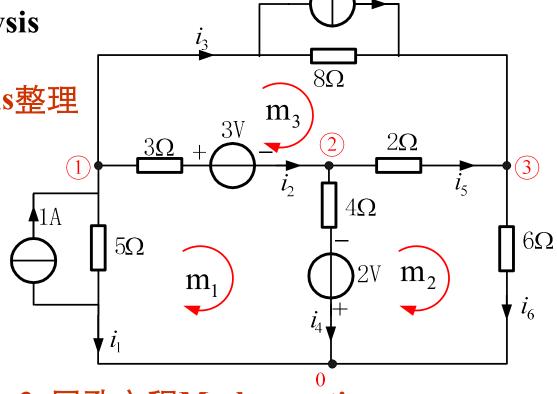
# 3.4 网孔分析法Mesh analysis

# 4. 网孔方程Mesh equations整理

$$(5+3+4)i_{m1} - 4i_{m2} - 3i_{m3}$$
$$= 5 \times 1 - 3 + 2$$

$$-4i_{m1} + (4+2+6)i_{m2} - 2i_{m3}$$
  
= -2

$$-3i_{m1} - 2i_{m2} + (8+2+3)i_{m3}$$
$$= 2 \times 8 + 3$$



# 3. 网孔方程Mesh equations

$$5(i_{m1}-1) + [3(i_{m1}-i_{m3})+3] + [4(i_{m1}-i_{m2})-2] = 0$$

$$[4(i_{m2} - i_{m1}) + 2] + 2(i_{m2} - i_{m3}) + 6i_{m2} = 0$$

$$8(i_{m3}-2)+2(i_{m3}-i_{m2})+[3(i_{m3}-i_{m1})-3]=0$$

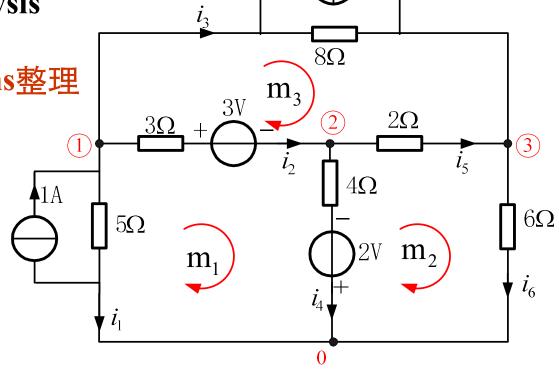
# 3.4 网孔分析法Mesh analysis

# 4. 网孔方程Mesh equations整理

$$(5+3+4)i_{m1} - 4i_{m2} - 3i_{m3}$$
  
= 5×1-3+2

$$-4i_{m1} + (4+2+6)i_{m2} - 2i_{m3}$$
  
= -2

$$-3i_{m1} - 2i_{m2} + (8+2+3)i_{m3}$$
$$= 2 \times 8 + 3$$



$$\begin{bmatrix} 5+4+3 & -4 & -3 \\ -4 & 4+2+6 & -2 \\ -3 & -2 & 8+2+3 \end{bmatrix} \begin{bmatrix} i_{m1} \\ i_{m2} \\ i_{m3} \end{bmatrix} = \begin{bmatrix} 5 \times 1 - 3 + 2 \\ -2 \\ 2 \times 8 + 3 \end{bmatrix}$$

# 5. 快速列写法

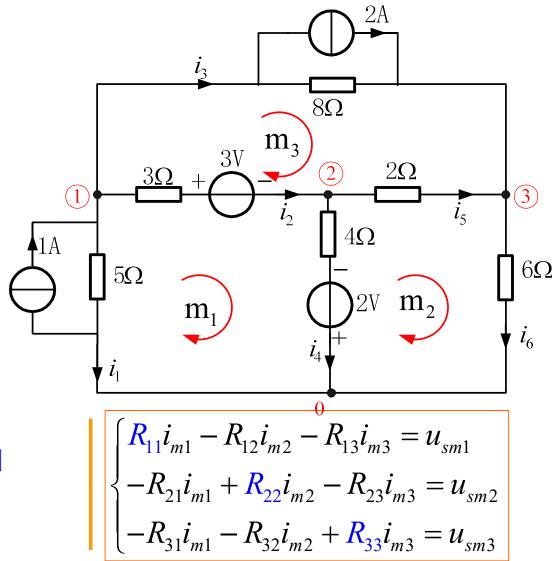
$$(5+3+4)i_{m1} - 4i_{m2} - 3i_{m3}$$
  
= 5×1-3+2

$$-4i_{m1} + (4+2+6)i_{m2} - 2i_{m3}$$
  
= -2

$$-3i_{m1} - 2i_{m2} + (8+2+3)i_{m3}$$
$$= 2 \times 8 + 3$$

R<sub>kk</sub>: k网孔内各支路电阻之和

 $R_{kk}$ : Self - resistance



 $R_{kj}$ : k、j网孔间各支路电阻之和的负值  $R_{kj}$ : Mutual-resistance

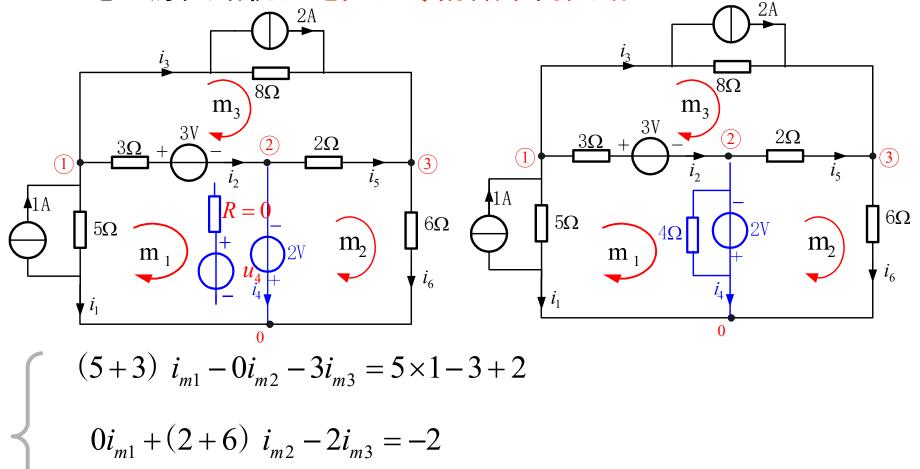
 $u_{smk}$ : k网孔内各电压源代数和,与网孔绕向反为正

2021-02mls: Mesh voltage source 出路理论

#### 4. 对电源支路的处理

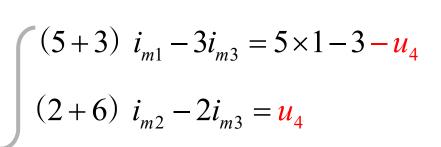
a. 电压源支路视为电阻为零的戴维南支路

 $-3i_{m1}-2i_{m2}+(8+2+3)i_{m3}=2\times8+3$ 



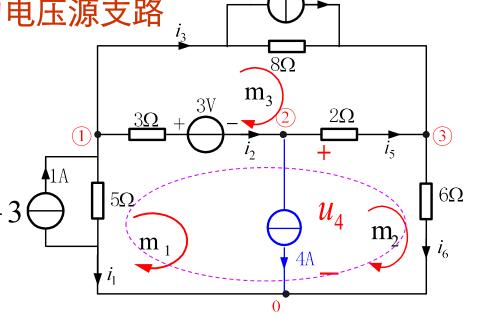
#### 4. 对电源支路的处理

b. 电流源支路视为电压为u4的电压源支路



$$-3i_{m1} - 2i_{m2} + (8+2+3)i_{m3} = 2 \times 8 + 3 \bigoplus^{\uparrow 1A}$$

$$i_{m1} - i_{m2} = 4$$
 (4个方程)

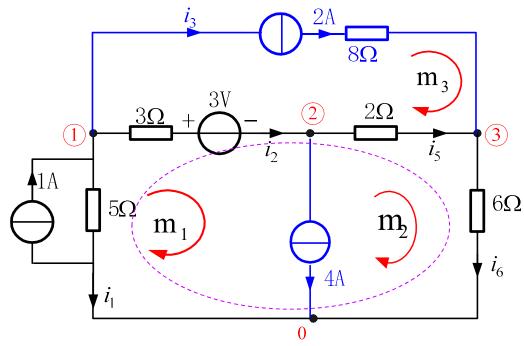


# •应用回路的KVL——广义网孔 (3个方程)

$$\begin{cases} (5+3) i_{m1} + (2+6)i_{m2} - (3+2)i_{m3} = 5 \times 1 - 3 \\ -3i_{m1} - 2i_{m2} + (8+2+3)i_{m3} = 2 \times 8 + 3 \end{cases}$$
$$i_{m1} - i_{m2} = 4$$

#### 讨论 ——目标2: 网孔分析法应用

例4: 列写网孔方程.

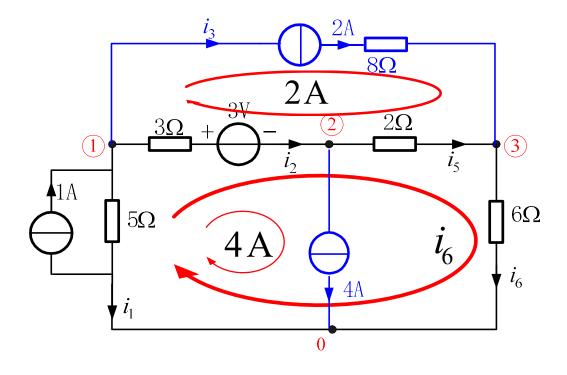


#### 网孔分析法:

$$\begin{cases} (5+3) i_{m1} + (2+6)i_{m2} - (3+2)i_{m3} = 5 \times 1 - 3 \\ i_{m1} - i_{m2} = 4 \\ i_{m1} = 2 \end{cases}$$

#### 讨论 ——目标2: 网孔分析法应用

例5: 列写回孔方程.



#### 回路分析法:

$$(5+3+2+6)$$
  $i_6 + (3+5) \times 4 - (3+2) \times 2 = 5 \times 1 - 3$ 

#### 讨论 ——目标2: 网孔分析法应用

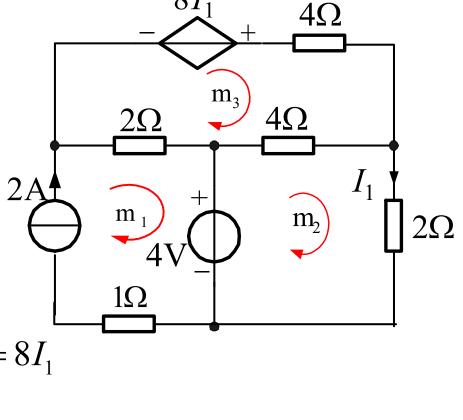
练习: 列写网孔方程.

$$I_{m1} = 2$$

$$(4+2)I_{m2} - 4I_{m3} = 4$$

$$-2I_{m1} - 4I_{m2} + (4+4+2)I_{m3} = 8I_{1}$$

$$I_{1} = I_{m2}$$



#### 例6: 计算电源功率

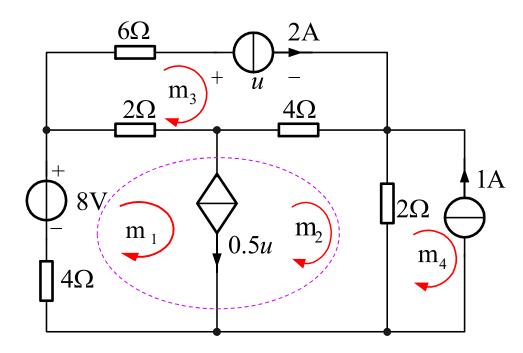
#### 结点法? 网孔分析法?

#### Mesh analysis:

$$i_{\rm m1} - i_{\rm m2} = 0.5 u$$

$$i_{\rm m3} = 2$$

$$i_{m4} = -1$$



$$(4+2) i_{m1} + (4+2)i_{m2} - (2+4)i_{m3} - 2i_{m4} = 8$$

#### 解得

$$i_{m1} = -1A$$
,  $i_{m2} = 4A$ ,  $u = -10V$ 

## 计算独立源的功率

$$p_{8V} = 8i_{m1} = 8 \times (-1) = -8W$$
 吸收功率

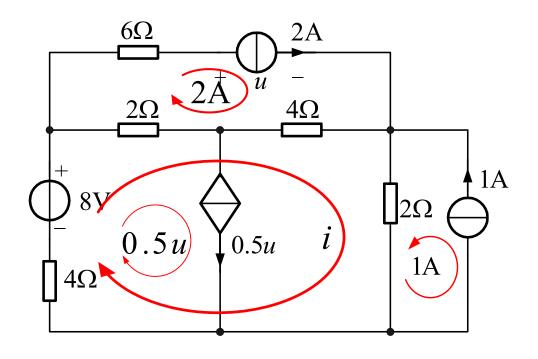
$$p_{2A} = \mathbf{u} \times i_{m3} = -10 \times 2 = -20 \text{W}$$
 发出功率 
$$p_{1A} = 1 \times 2 (i_{m2} - i_{m4}) = 10 \text{W}$$
 发出功率

2021-02-25 电路理论

例6: 计算电源功率

结点法? 网孔分析法?

回路分析法:



$$(4+2+2+4)$$
  $i+(4+2)\times 0.5u-(2+4)\times 2+2\times 1=8$ 

例7: 计算独立电源功率

结点法? 网孔分析法?

 $U_{n1} = 6$ 

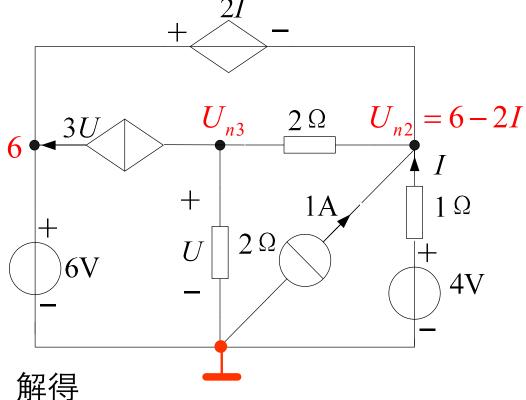
#### Nodal analysis:

$$(\frac{1}{2} + \frac{1}{2})u_{n3} - \frac{1}{2}u_{n2} = -3u$$

$$u = u_{n3}$$

$$I = -\frac{u_{n2} - 4}{1} = -\frac{6 - 2I - 4}{1}$$

$$u_{n1} = 6$$

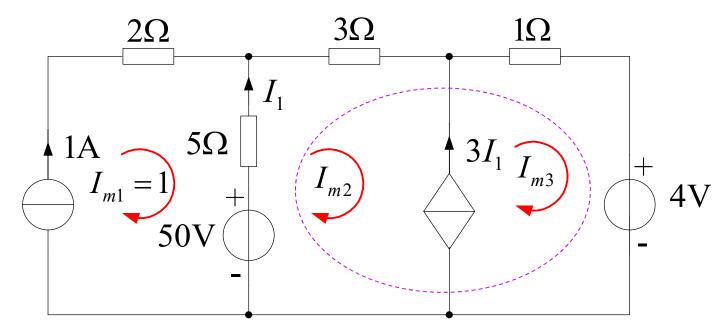


$$u_{n2} = 2V$$
,  $u_{n3} = 0.25V$ ,  $I = 2A$ 

$$P_{4V} = 4 \times I = 8$$
W 发出功率

$$P_{1A} = 1 \times u_{n2} = 2$$
W 发出功率

例8: 计算 $I_1$ 及各电源功率.



Mesh analysis:

$$\begin{cases} (3+5)I_{m2} + 1 \times I_{m3} - 5I_{m1} = 50 - 4 & I_{1} = 3.5 \\ I_{m3} - I_{m2} = 3I_{1} & I_{m2} = 4.5 \\ I_{m2} - I_{m1} = I_{1} & I_{m3} = 15 \end{cases}$$

$$P_{1A} = 1 \times (2 \times 1 - 5I_1 + 50)$$
  $P_{50V} = 50I_1$   $P_{4V} = -4I_{m3}$ 

## 课下练习: 求1

#### 回路方程为

$$I_{\rm m1}=1$$

$$I_{\rm m2} = 1.5 U_1$$

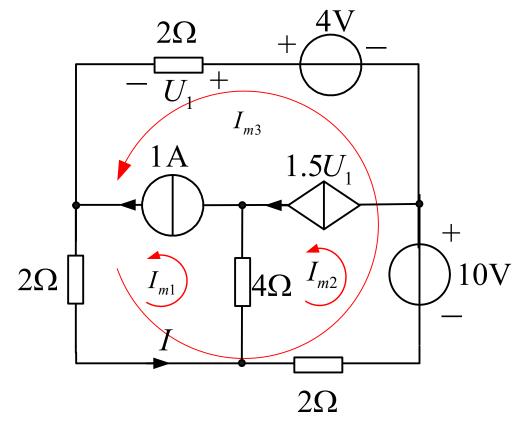
$$2I_{m1}+2I_{m2}+(2+2+2)I_{m3}=4+10$$

约束方程为

$$U_1 = 2 I_{m3}$$

#### 解方程得出:

$$I_{m3} = 1A$$
  $I = I_{m1} + I_{m3} = 2A$ 



#### 3.5 网孔法和结点法的比较:

(1) 方程数量的比较

	KCL方程	KVL方程	方程总数
支路法	<i>n</i> -1	<i>b</i> -( <i>n</i> -1)	b
网孔法	0	<i>b</i> -( <i>n</i> -1)	b-(n-1)
结点法	<i>n</i> -1	0	<i>n</i> -1

- (2) 对于非平面电路,选独立回路不容易,因此不用网孔法,而独立结点较容易。
- (3) 目前用计算机分析网络(电网,集成电路设计等)采用 结点法较多。

# 计划学时: 4学时; 课后学习12学时

# 作业:

- 3-7、3-11常规网络结点方程
- 3-14 含电源支路电路的结点方程
- 3-28 常规网络网孔方程
- 3-30 含电源支路电路的网孔方程
- 3-38、3-40 方法选择