Homework 4 Due: 12:00 10 June 2022

Problem 1

Consider the optimization problem

$$\min_{x \in \mathbf{R}^2} f(x) = \frac{1}{4} x^T Q x + b^T x$$

where

$$Q = \begin{bmatrix} 3.65 & 0.37 \\ 0.37 & 5.84 \end{bmatrix}, \quad b = \begin{bmatrix} 1.2 \\ 4 \end{bmatrix}.$$

- Visualize the objective function $f(x)$ in MATLAB using x_1 `= -3:.1:3` and x_2 `= -3:.1:3`. (Hint: use built-in functions `meshgrid` and `surf`.)
- Write **your own gradient descent method** to solve this optimization problem, using a) exact line search and b) backtracking line search, with initial point $x^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. And verify your results by calling the built-in function `fminunc`.

  (Hint 1: You may use the built-in function `fminbnd` for exact line search.)

  (Hint 2: The gradient of $f(x)$ is

  $$\nabla f(x) = \frac{1}{2} Q^T x + b.$$

  If you are interested in its computation, see *The Matrix Cookbook*[*].)

[*] https://www.math.uwaterloo.ca/~hwolkowi/matrixcookbook.pdf