Final Project Due: 05 July 2022

Notice:

- DO NOT copy any codes or reports from your classmates! Students are expected to do this project on their own. Students are expected to submit their work (reports + codes in a zip file) on time.
- Any confirmed plagiarism/cheating results in 59/100 score for the corresponding part.
- For every day that your assignment is late, your score is multiplied by 0.8. Submissions that are more than 3 days late will NOT be accepted.
- And, all standard HUST rules/policies on final examination apply.

1 Background description

1.1 Suspension system

The suspension system (see its broad classification in Fig. 1) connects the wheels of the vehicle to the chassis. Its main functions, which have to be fulfilled within the available working space, are:

- to support the vehicle chassis,
- to keep the rider's comfort within allowable limits,
- to control chassis and wheel attitudes,
- to maintain yaw and roll stability, and
- to ensure good road holding by minimizing the vertical force variation in the tyre-road interaction.

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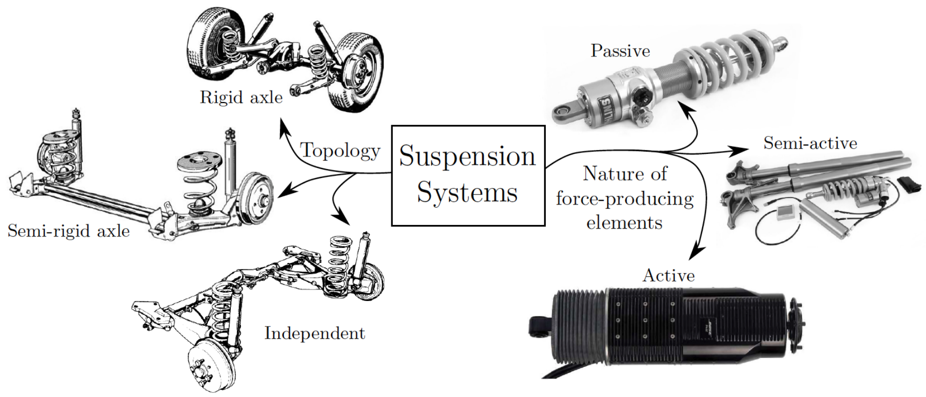

Figure 1: Broad classification of suspension systems according to mechanical layout and nature of the force-producing elements.

1.2 Passive suspension

To fulfil these functions, a large number of topologies and geometries have been developed. According to the nature of the force-producing elements, there are three main suspension types: passive, active and semi-active suspensions. In this project, we consider the most basic and widely used suspension type: passive suspension. A schematic of the passive suspension is shown in Fig. 2. The main passive suspension characteristics and modelling assumptions considered are:

- The passive suspension system comprises two rigid wishbones, a rigid wheel spindle, a vertically compliant tyre, a suspension spring and a suspension damper.
- The wheel motion is planar and remains constrained to the local y-z plane.
- All the unsprung mass is lumped at the wheel centre, H.
- All joints are ideal revolutes (i.e., rubber bushing are not included). The suspension spring and damper act in parallel between points E and F, and are assembled together to form a single unit, referred to as spring-damper (SD), that can only produce forces along its axis.
- The suspension spring is linear, and the suspension damper is fully nonlinear.
- The tyre is represented by a linear vertical spring and a linear vertical damper that act in parallel between the road and point I.

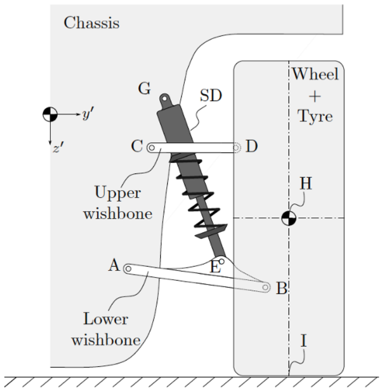

Figure 2: The chematic of a passive double-wishbone suspension.

1.3 Simplified quarter-car linear model

Simplified linear models capturing the vertical and lateral dynamics of the vehicle (equipped with passive suspension only) have been used to guide the development and qualitatively validate more complex models. Fig. 3 shows a 2 DOF linear quarter-car model for the study of vertical dynamics. Even quarter-car model does not contain the whole-car geometric effects such as wheelbase filtering and lateral equivalent, and does not offer the longitudinal interconnection properties, it still preserves the most basic features of wheel mass and body mass that are required for meaningful suspension analysis.

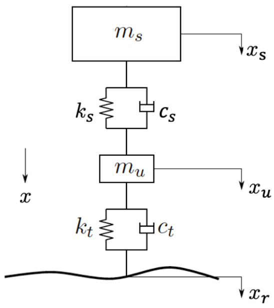

Figure 3: Linear quarter-car model for the study of vertical dynamics.

The equations of motion for the quarter car model can be found by applying Newton's second law to the sprung and unsprung masses (m_s and m_u):

$$\begin{aligned} m_s\ddot{x}_s &= -k_s\big(x_s - x_u - (h_{s0} - h_{u0})\big) - c_s(\dot{x}_s - \dot{x}_u) \\ m_u\ddot{x}_u &= k_s\big(x_s - x_u - (h_{s0} - h_{u0})\big) + c_s(\dot{x}_s - \dot{x}_u) - k_t(x_u - x_r - h_{u0}) - c_t(\dot{x}_u - \dot{x}_r) \end{aligned} \tag{1}$$

where subscripts s and u refer to the sprung and unsprung masses, h_{s0} and h_{u0} are the default height of the sprung and unsprung mass at their static states, k_s and c_s

are the equivalent suspension stiffness and damping coefficients, subscripts t and r refer to the tyre and road respectively.

Let the state variables be $x_1 = x_s$, $x_2 = \dot{x}_s$, $x_3 = x_u$, and $x_4 = \dot{x}_u$, and the road displacement and velocity be external disturbances $u_1 = x_r$ and $u_2 = \dot{x}_r$. The model for the quarter car can be expressed in differential equation form as indicated by

$$\dot{X} = AX + BU + C \tag{2}$$

where state vector and external disturbance vector are

$$X = \begin{bmatrix} x_s \\ \dot{x}_s \\ x_u \\ \dot{x}_u \end{bmatrix}, \quad U = \begin{bmatrix} x_r \\ \dot{x}_r \end{bmatrix} \tag{3}$$

with system matrices

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_s}{m_s} & -\frac{c_s}{m_s} & \frac{k_s}{m_s} & \frac{c_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_u} & \frac{c_s}{m_u} & -\frac{k_s+k_t}{m_u} & -\frac{c_s+c_t}{m_u} \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{k_t}{m_u} & \frac{c_t}{m_u} \end{bmatrix}, C = \begin{bmatrix} 0 \\ \frac{k_s(h_{s0}-h_{u0})}{m_s} \\ 0 \\ \frac{-k_s(h_{s0}-h_{u0})+k_t h_{u0}}{m_u} \end{bmatrix}.$$

1.4 Model parameters and initial conditions:

Model parameters:

- Sprung mass: m_s = 313.00 kg;
- Unsprung Mass: $m_u = m_{tyre} + m_{hub} + m_{sd}$; m_{tyre} = 39.00 kg, m_{hub} = 7.70 kg, m_{sd} = 1.20 kg;
- Spring Stiffness: k_s = 157614 N/m;
- Damper Coefficient: c_s = 5792 N/(m/s);
- Tyre's radial stiffness: k_t = 2.75×10^5 N/m;
- Typre's Damping Coefficient: c_t = 300 N/(m/s);
- Default height of sprung mass: h_{s0} = 0.7 m;
- Default height of unsprung mass: h_{u0} = 0.2 m;

Initial conditions:

$$x_0 = \begin{bmatrix} 0.66 & 0 & 0.42 & 0 \end{bmatrix}^T. \tag{4}$$

2 Problem 1: system simulation

Consider the dynamical system (1) or its state-space model (2), with three types of ground conditions:

- $x_r(t) = 0$, a perfectly flat road;
- $x_r(t) = 0.2\sin(0.5t)$, that is a road with sinusoidal shaped bumps;
- $x_r(t) = 0.01\sin(25t) + 0.02\sin(50t)$ to mimic a coarse road that presents numerous little bumps.

Solve the following problems:

1. Use the Runge-Kutta method (`ode45`) to simulate this mechanical system for the time interval `t = 0:0.01:2` given different road conditions (i.e. $x_r(t)$). Describe how your simulation is performed in your report and provide the figures of signals $x_s(t), x_u(t)$ (Note: in all figures in your report, x/y-labels must be added; if a figure has multiple curves, the legend should be added).

2. For $x_r(t) = 0$, try using the Forward-Euler and Backward-Euler approaches to perform simulation. Tell whether they have performed simulation properly; and, if not, explain the possible reasons for their failures.

3. When $x_r(t) = 0$ (ideally flat), compute the steady state of x_s, x_u by using an iterative method to solve systems of linear equations obtained by setting $\dot{X} = 0$ in (2). Note that you need to write your own Jacobi or Gauss-Seidel method to solve it.
   And redo the simulation for longer period, `t = 0:0.1:10` (only for this sub-problem). Does the simulation figure match the steady state that you computed? Try explaining the physical meaning that you read from the solution and the figure.

3 Problem 2: curve fitting

1. Consider the data of signal $x_s(t)$ given $x_r(t) = 0$ from Problem 1. Use the mean-square error (MSE) criterion to try fitting the following function

$$f_1(t) = c_1 \sin(c_2 t) + c_3$$

   If an optimization procedure is needed, write your own gradient descent method.
   Also try fitting the $x_s(t)$ with 1st and 2nd order polynomials.

Compare these fitted plots with the figure of $x_s(t)$, and explain why $f_1(t)$ and line/parabola are not good models for this curve fitting.

2. Due to the drawbacks of the above models in fitting $x_s(t)$, now we explore more patterns that show in the plot of $x_s(t)$. Notice that the magnitude of $x_s(t)$ keeps decaying and it shows a periodic oscillation over time. We propose the following function to fit $x_s(t)$:

$$f_2(t) = \exp(c_1 t)\big[c_2 \sin(c_3 t) + c_4 \cos(c_5 t)\big] + c_6.$$

Solve this curve fitting problem by an optimization procedure (you may use MATLAB built-in function `fminunc` here), and provide the result in your report.
Note that, due to the issue of local optimality, you may not easily get a good fit. Hence you need to try a few initial points to find a good one.

Hints (initialization):

- To reduce number of trials, you may set $c_2 = c_4, c_3 = c_5$, which describe the period of $x_s(t)$. So you may guess one from the plot of $x_s(t)$, or the information in fitting $f_1(t)$ may help you.
- c_6 is the displacement of whole curve, so you may guess a good from the figure of signal $x_s(t)$.
- The magnitude of $x_s(t)$ is decreasing over time, thus c_1 is negative.

4 Problem 3: model reduction

Consider the state signals $X(t)$ with $x_r = 0$.

- Perform PCA to obtain the signals of the 1st and 2nd principal components, denoted by $z_1(t), z_2(t)$.
- Remove the first 4 elements in the time vector, $z_1(t)$ and $z_2(t)$ (due to bad numerical difference computation). Then compute the derivative of z_1, z_2 numerically, denoted by $\dot{z}_1(t), \dot{z}_2(t)$. We now have data $\{z_1(t), z_2(t), \dot{z}_1(t), \dot{z}_2(t)\}$ $(t = 1, \ldots, T)$. We like to learn a reduced model by fitting the following model via minimizing its mean-square error:

$$\begin{bmatrix} \dot{z}_1(t) \\ \dot{z}_2(t) \end{bmatrix} = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix}.$$

In your report, describe how you formulate into an optimization problem and the way to solve it.